THE MUON ABSORPTION INTERACTION

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1. INTRODUCTION

The basic reaction responsible for muon capture by nuclei is believed to be

$$\mu^- + p \rightarrow n + \nu \qquad (A)$$

This is believed to be due to a coupling of the form used in β-decay theory.

$$\mathfrak{H}' = \sum_i C_i \bar{\psi}_n O_i \psi_p \bar{\psi}_\nu (1-\gamma_5) O_i \psi_\mu$$

More particularly the theory of the universal Fermi interaction (UFI) states that the coupling of the muon is identical to that of the electron; that is, we have only to replace ψ_e in the β-decay coupling by ψ_μ to get the mu-capture coupling.

The consequences of this assumption are not unambiguous because of uncertainties due to the strong interactions; in particular, virtual pions. If we write $\mathfrak{H}'$ in the form

$$\mathfrak{H}' = J^\lambda \bar{\psi}_\nu (1-\gamma_5) \gamma_\lambda \psi_\mu$$

where J^λ is the strangeness-conserving weak interaction current containing baryon field operators and perhaps pion field operators, observations on muon capture and β-decay depend upon

$$\langle n | J^\lambda | p \rangle$$

where $\langle n|$ and $|p\rangle$ represent physical nucleon states. Goldberger and Treiman [1] showed that the most general form of this matrix element correspond to four effective couplings between physical nucleons and the lepton fields.

These couplings are proportional to

g_A = renormalized axial vector coupling
g_V = renormalized vector coupling
g_P = effective pseudoscalar coupling
g_M = effective weak magnetism.

Each of these is a function of q^2, where q is the four-momentum transfer at the weak vertex; furthermore, g_P is proportional to the lepton mass and the weak magnetism coupling is linear in the momentum transfer.

The difficulty in directly comparing muon capture and β-decays has two origins :

(1) The momentum transfer is higher in muon capture. Therefore, the effective coupling constants may be different and the pseudoscalar and weak magnetism couplings are more important for muon capture. Furthermore, nuclear matrix elements are different than those in β-decay.

(2) The muon is at rest in the capture process so that there exists no direct analogues of the many important (v/c) effects of β-decay (such as Fierz term, electron-neutrino correlation, β-ray polarization, Wu experiment).

Looking more closely at the effects of virtual pions using dispersion relations or perturbation theory we note :

(1) Intermediate one-pion states contribute only to the g_P coupling and yield a calculable result

$$g_P^{(\mu)} = 8 g_A^{(\beta)} \qquad (1a)$$

where $g_A{}^{(\beta)}$ is the value of g_A measured in β-decay. The result depends on the magnitude of the pion-nucleon coupling constant and so has an uncertainty of at least 10%. The sign (but not the magnitude) depends on the assumption that pi-decay proceeds primarily via an intermediate nucleon-antinucleon pair, or on an assumption about the divergence of the axial-vector current as discussed by Gell-Mann.

(2) Intermediate two-pion states. These yield a momentum dependence of g_V and should give the main contribution to g_M. These should be considered as uncalculated, although a very rough calculation [1] indicated they were small in the normal *V-A* theory. However, in the Feynman-Gell-Mann conserved vector current theory, both the momentum dependence of g_V and the value of g_M may be related to electromagnetic properties of nucleons; the result is

$$g_V^{(\mu)} = 0.97 g_V^{(\beta)} \tag{1b}$$

$$g_M^{(\mu)} = (\mu_P - \mu_N) g_V^{(\mu)} = 3.7 g_V^{(\mu)} \tag{1c}$$

where μ_P, μ_N are the anomalous nucleon moments.

(3) Intermediate three-pion states. These are the first states that contribute to the momentum-dependence of g_A so that the momentum dependence should be at most of the order $\frac{q^2}{(3m_\pi)^2}$ which is about 0.05 for muon capture. Thus we may seem fairly safe in assuming

$$g_A^{(\mu)} = g_A^{(\beta)}. \tag{1d}$$

These states also modify the value of g_P.

In complex nuclei on which most observations are made there may be modifications in the effective couplings due to exchange pions. These might be particularly important for the one-pion term $g_P{}^{(\mu)}$.

We now wish to look at the evidence. By far the strongest evidence in favor of the identity of muon and electron couplings today is the evidence that was missing two years ago: namely, the ratio of the decay $\pi \to e + \nu$ to $\pi \to \mu + \nu$. This is strong evidence that the axial-vector couplings in $\mathfrak{H}'$ are identical for e and μ and this is the one piece of evidence that has no ambiguity arising from the effects of strong interactions.

Beyond this we must look at experiments on muon capture. An excellent review [2] of this subject by Primakoff, who originated much of this work, exists in the Review of Modern Physics. I shall try to bring this up to date, which should not be difficult in view of the slow progress in this field.

Our aims are

(1) To test the validity of UFI, keeping in mind the uncertainties of virtual pion effects.

(2) To detect, if possible, the most important predicted virtual pion effects; in particular, to verify the sign of $g_P{}^{(\mu)}$ and the magnitude of $g_M{}^{(\mu)}$.

2. EXPERIMENTS ON CAPTURE RATES

I. Partial Capture Rates

Measurements of total or partial capture rates depend almost entirely on the magnitudes and relative sign (or relative phase, if time-reversal invariance were not assumed) of an effective Fermi and an effective Gamow-Teller coupling constant:

$$G_F = g_V^{(\mu)} \tag{2a}$$

$$G_G = g_A^{(\mu)} - \frac{\nu}{6M}[g_P^{(\mu)} + 2(g_M^{(\mu)} + g_V^{(\mu)})] \tag{2b}$$

where ν is the neutrino momentum $[(\nu/(6M) - 0.017)]$. Effects of the order of 5% ($g_P{}^2$ terms and relativistic $\nu/(2M)$ terms) are neglected in the use of these effective coupling constants.

$$\mu^- + C^{12} \to B^{12} + \nu \tag{B}$$

The direct comparison of muon captive to β-decay has been accomplished most clearly in comparing the capture reaction

$$\mu^- + C^{12} \to B^{12} + \nu$$

to

$$B^{12} \to C^{12} + e^- + \bar{\nu}$$

These are allowed Gamow-Teller reactions and so primarily involve g_A; however, in the muon capture case, g_P and g_M make contributions of about 20% to the capture rate. The most precise experiment by the CERN group [3] gives a capture rate of $9.2 \pm 0.5 \times 10^3$ sec^{-1} in agreement with a Los Alamos measurement but significantly higher (by about 30%) than a bubble chamber measurement [4] and a new preliminary counter measurement from the Carnegie Tech. group. Using the CERN result to illustrate the obtainable information we find

$$G_G = (1.16 \pm 0.13) g_A^{(\beta)}) \tag{3a}$$

$$g_P^{(\mu)} + 2g_M{}^{(\mu)} = (-8 \pm 8) g_A^{(\beta)} \tag{3b}$$

where the last equality depends upon (1d) and to a minor extent (1b). If we assume on theoretical grounds $g_P^{(\mu)}+2g^{(\mu)} = (2\pm 10)g_A^{(\beta)}$, we deduce

$$g_A^{(\mu)} = (1.16\pm 0.22)g_A^{(\beta)} \tag{3c}$$

The errors are estimates [5] of the uncertainty of the theoretical calculation due to lack of knowledge of the nuclear wave functions and are not statistical in character. To these must be added a sizeable experimental uncertainty arising from (1) the discrepancy between the different experiments and (2) the uncertainty as to the fraction of the observed captures going directly to the ground state of B^{12} instead of an excited state; we have assumed on the basis of sparse data that 10% of the observations went to excited states, although this seems somewhat less than might be expected. Thus, we expect the true capture rate is lower than the value used so that we have an upper limit on $g_A^{(\mu)}$ or a lower limit on $g_P^{(\mu)}+2g_m^{(\mu)}$. The uncertainties render this measurement of marginal value in obtaining any information on $g_P^{(\mu)}$ and $g_M^{(\mu)}$. We can conclude, however, that the assumption $g_A^{(\mu)} = g_A^{(\beta)}$ is verified within about 30%.

II. Total Capture Rates

Detailed measurements on the capture rate of muons by complex nuclei [6] have been used in an attempt to deduce the magnitude of the basic coupling constants responsible for this process. The theory of Primakoff [2] is based on the closure approximation: each proton in the nucleus captures a muon yielding some final state; when all final states are summed over, the total capture rate is found to be proportional to the capture rate of an isolated (unpolarized) proton. The final Primakoff result for the capture rate for non-light nuclei may be written

$$\Lambda = (Z_{\text{eff}})^4\gamma\left(1-\delta\frac{A-Z}{2A}\right)\Lambda(1,1) \tag{4}$$

where

(1) $(Z_{\text{eff}})^4$ represents the integral over the nucleus of the product of proton density times muon density relative to that of a massive point nucleus with $Z = 1$.

(2) γ is the ratio of the neutrino phase space for the complex nucleus to that for capture in hydrogen.

(3) $\delta\dfrac{A-Z}{2A}$ represents a decrease in the capture rate due to the presence of the neutrons in the nucleus occupying final states into which the proton wishes to go.

(4) $\Lambda(1,1)$ is related to the capture rate in hydrogen (averaged over the hyperfine states statistically) by

$$\Lambda_p = \frac{\Lambda(1,1)}{(1+\mu/M)^4}$$

neglecting $(\mu/M)^z$ corrections. It is $\Lambda(1,1)$ that is directly proportional to $(G_F^2+3G_G^2)$.

From the days of the first experiments on muon capture it has been recognized that there is a rough equality of the muon capture coupling strength with the β-decay coupling strength. The experiments on the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$ have indicated a quantitative equality within limits that have been analyzed (about 30%). In spite of the attempts that have been made, we are extremely skeptical that such a quantitative analysis can be made of the total capture rate in the heavier nuclei.

The difficulty lies in the extreme sensitivity of the Primakoff formula to the value of δ because of the large exclusion principle effect. Thus a 10% error in δ causes a 30% error in Λ for the lighter nuclei and a 100% error in Λ for heavier nuclei. To determine δ, the Chicago group [6] fitted the experimental Λ for a large number of natural isotopes to Eq. (4). This fit requires that in going from Ca to U for which $\dfrac{A-Z}{2A}$ varies from 0.25 to 0.31, δ remains constant to a good degree of accuracy in spite of the great change in A and Z. This constancy of δ follows to a fair approximation from a homogeneous nuclear matter model assumed by Primakoff, but this model ignores physical effects such as the following: (1) Muons may be expected to be preferentially captured by protons spatially located near the surface of the nucleus because of the Pauli principle. As one goes from Ca to U the muon wave function becomes less concentrated at the surface of the nucleus relative to the center; this would thus tend to decrease the capture rate independent of the neutron excess. (2) The nuclear wave function is increasingly affected by the Coulomb potential as one goes from Ca to U so that the single-particle neutron and proton wave functions become increasingly different. The result of Sens (assuming $\gamma = 0.64$) is

$$\Lambda_p = 1.9 \times 10^2 \ \text{sec}^{-1}$$

in good agreement with the prediction of the universal interaction (using Eq. (1)) of $\Lambda_p = 1.7 \times 10^2 \ \text{sec}^{-1}$. This agreement, I believe, is fortuitous.

An alternative procedure for determining δ is to look at the variation in Λ in going from one isotope to another for a given element. There is then a significant change in $\dfrac{A-Z}{2A}$ with no change in Z and a minimum change in the nuclear wave function. A recent experiment at Carnegie Tech.[7] measuring the capture rates in separated isotopes gives a ratio of about 0.7 for capture in Cl^{37} to capture in Cl^{35}. This can be fitted with Eq. (4) with

$$\Lambda_p = 2.6 \times 10^2 \ \text{sec}^{-1}$$

The major drawback in this determination is its dependence on one nucleus for which there may be significant fluctuations from Primakoff's formula. In fact, in going from Cl^{35} to Cl^{37} we are closing a neutron shell, the very shell in which many of the capturing protons are originally; therefore, these added neutrons may have an " above-average " exclusion principle effect.

The analysis of the capture rates in a few light nuclei for which shell model wave functions are available would appear to be more fruitful. Such an analysis of the total capture rates[8] in N^{14}, O^{16}, and F^{19} does not give completely consistent results. It does, however, demonstrate clearly the equality $g_A^{(\mu)} = g_A^{(\beta)}$ within about 25%.

III. Hyperfine Structure Effect

Primakoff first pointed out that the muon capture rate could be very dependent on the relative orientation of muon and proton spin. Indeed for a simple V-A theory all capture occurs only when the spins are antiparallel, and this conclusion is not significantly modified by the corrections to the simple V-A theory that we have considered. In complex nuclei with spin $I \neq 0$, muons are expected to be captured from both hyperfine states of the K-shell: $F = I + \frac{1}{2}$ and $F = I - \frac{1}{2}$. The difference between these rates would be detected by the departure of the muon decay curve from a simple exponential or possibly by the dependence of the rate on the physical environments. In a simple shell model picture for odd Z nuclei one may immediately deduce the relative orientation of the muon spin and the odd proton spin for each value of F; the difference in capture rates may then be calculated provided one knows what fraction of the muon capture is due to this odd proton and what fraction is due to the rest of the protons. A recent calculation using the simple shell model and the closure approximation has been made by Überall[9] with the results that

$$\frac{\Delta\Lambda}{\Lambda_c} = \frac{\Lambda(F=3) - \Lambda(F=2)}{{}^7/_{12}\Lambda(F=3) + {}^5/_{12}\Lambda(F=2)} \simeq -0.50 \quad \text{for } Al^{27}$$

$$\frac{\Delta\Lambda}{\Lambda_c} = \frac{\Lambda(F=1) - \Lambda(F=0)}{{}^3/_4\Lambda(F=1) + {}^1/_4\Lambda(F=0)} \simeq -0.45 \quad \text{for } P^{31}$$

These effects are considerably larger than previous calculations because the greater probability of the outer proton to capture the muon is taken into account. Of course, configuration mixing could seriously change these predictions. Quite apart from any quantitative analysis, it may be noted that the observation of the hyperfine effect in an odd-Z nucleus rules out the possibility of a $V+A$ theory, thus definitely fixing the relative sign of G_F and G_G. This hyperfine effect has been observed by Telegdi, who will report later in this conference on his results.

IV. Muon Capture in Hydrogen and Deuterium

(1) Pure hydrogen. As is well known, exceedingly pure hydrogen with deuterium removed must be used to be sure that all captures really occur in hydrogen. It is generally believed that in pure hydrogen (at liquid hydrogen density) the muon is captured almost entirely from (1) the $F = 0$ hyperfine state of the ground state of the mesic atom or (2) the state of the $(p\mu p)$ molecule in which the proton spins are parallel and the " total spin " is $\frac{1}{2}$. The ratio of these two final states depends on the rate of the reaction

$$(\mu^- p) + (ep) \rightarrow (p\mu^- p) + e$$

Calculations by Cohen, Judd, and Riddell[10] indicate that most (93%) of the muons end up in the molecule, but such calculations may not be reliable. With the preferred coupling constants Weinberg[11] has pointed out that the capture rates from these two states are not very different (less than a factor of 2) so that an error of a factor of 2 in the rate of molecule formation would change the predicted capture rate in pure hydrogen by less than 7%. It is to be concluded that the uncertainties as to molecule formation and molecule wave functions may make a precise interpretation of such results impossible, but that the results would be very interesting. In particular we note that for any rate of molecule formation, with the preferred coupling constants the capture rate is greater than 300 $\sec^{-1}$, whereas for a $V+A$ theory the capture rate would be less than 200 $\sec^{-1}$. This difference is due to the fact that in both the atomic and molecular states considered, the muon spin is expected to be preferentially aligned oppositely to the proton spin. These conclusions do depend, however, on the reliability of estimates indicating negligible transition rates between various molecular states. Experimental tests of these various assumptions require experiments as a function of hydrogen density.

(2) Pure deuterium. The situation is similar to pure hydrogen with these differences: (1) the rate of $(d\mu d)$ molecules can be determined by observations on the products of the catalyzed d-d reactions. Preliminary observations[12] indicate that most muons never form molecules in liquid deuterium; (2) it is believed that the d-d reaction occurs so rapidly that there is essentially no capture from the molecular state, but that there is expected to be a significant amount of capture of muons bound in He^3, a product of the d-d reactions. The greatest uncertainty in calculating the total capture rate may be in the fractions of muons ending up on He^3; although this is estimated to be only 3 or 4%, the capture rate in He^3 is expected to be 6 times that in deuterium. The uncertainty in the calculation of the capture rate due to uncertainty in the high-momentum components and D-state amplitude of the deuteron wave-function is less than 10%.

(3) Hydrogen-deuterium mixtures. From the saturation of muon catalysis it is deduced that in liquid hydrogen containing more than about 2% deuterium all muons reach the molecular state $(p\mu d)$. In this case then in contrast to pure hydrogen or pure deuterium essentially all muons reach a unique chemical state before decay. An analysis of the capture in this case would be very interesting.

Two major problems face such an analysis: (1) It is necessary to know the distribution of the muons among the four hyperfine states of the $(p\mu d)$ molecule. This in turn depends upon whether the (μd) atom reaches the lower $(F=\frac{1}{2})$ hyperfine state before the molecule is formed. Since the transition to $F=\frac{1}{2}$ states is due to exchange collisions with deuterons, the probability of this transition depends very much on deuterium concentration. Since the rate of catalysis also depends on the distribution among the hyperfine states[10], a careful study of the number of muon regenerations as a function of deuterium concentration should yield light on this question; such an experimental study has been initiated by the Carnegie Tech. group[12]. If one studies solely the captures on a proton from $(p\mu d)$ molecules (by measuring the resultant neutron energy) this most likely would correspond to capture from a mixture of (μp) hyperfine states in contrast to the case in pure hydrogen. (2) The total capture rate in this case is greatly influenced by the fact that after the reaction $p+d\rightarrow He^3+\gamma$ is catalyzed the muon may be bound to He^3. Indeed it is expected that more than half of the captures observed would be on He^3. The amount of capture on He^3 depends on *(a)* the fraction of molecules that undergo catalysis which can be determined to a fair extent from a study of the emitted γ-rays, *(b)* the fraction of He^3 nuclei that hold on to the muon after catalysis, and *(c)* the hyperfine state of the final He^3-mesic atom. A study of the capture from $(p\mu d)$ molecules should make it possible to deduce from experiments done in normal liquid hydrogen the capture rate in pure hydrogen. The expected total capture rates for muons in liquid hydrogen are roughly: (1) Pure hydrogen: one in 800; (2) Pure deuterium: one in 800; (3) Hydrogen-deuterium mixtures: one in 500.

V. Muon Capture in He^3

The experiment on muon capture which can be calculated most accurately (for assumed coupling constants) now is the rate

$$\mu^-+He^3\rightarrow H^3+\nu \qquad (C)$$

For heavier nuclei we have seen there are uncertainties of the order of 20% from the nuclear wave functions, whereas for hydrogen (at least at the fixed density of liquid hydrogen) there are uncertainties as to the molecular state from which capture takes place. A calculation by Werntz [13)] indicates that the rate for capture in He^3 (given the *ft* value for H^3 β-decay) can be calculated with an accuracy of better than 5%. The main uncertainty could be reduced by a measurement of the charge distribution in He^3. The result of Werntz (with the coupling of Eq. (1)) corrected for relativistic terms omitted by him is 1.51×10^3 sec^{-1}, or about 1 in 300 μ stops. We emphasize that this experiment can be analyzed in terms of effective coupling constants almost as accurately as capture in hydrogen. It should be noted that it is assumed that the hyperfine states of the mesic atom of He^3 are populated statistically with no relaxation during the muon lifetime. The total capture rate in He^3 can be calculated quite accurately also.

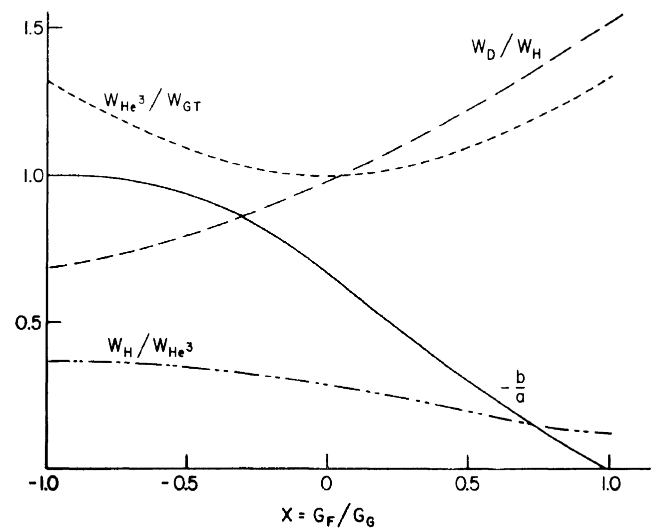

Fig. 1 Ratios of muon capture rates.

If this reaction *(C)* were used to determine the magnitude $(G_F{}^2+3G_G{}^2)$ other capture experiments would then serve to determine the ratio $X = G_F/G_G$. In Fig. 1 we show the ratios of various capture rates as a function of X. The hyperfine structure effect is directly proportional to $(-b/a)$. W_H is the capture rate in pure liquid hydrogen assuming all muons form molecules, but neglecting the decoupling of muon and proton spins due to their independent precession about the molecular rotational angular momentum. W_D is the capture rate in pure deuterium, neglecting capture from He^3. The dashed curve shows on an arbitrary scale the ratio of capture in He^3 (going to H^3) to a pure Gamow-Teller capture (as C^{12} going to B^{12} or Li^6 to He^6).

The following point is to be emphasized. All the quantities plotted in Fig. 1 except the last mentioned are rather dependent on the hyperfine structure effect, and show large differences between a $(V+A)$ theory, and a $(V-A)$ theory. In fact, Telegdi's observations of the hyperfine effect in odd-Z nuclei tells us that it certainly is not $(V+A)$ but something like a $(V-A)$ theory. But it is difficult to deduce anything about the magnitude of $G_F/G_G = X$ without very precise experiments, because of the flatness of the curves near $X = -1$.

Finally we note the special interest in looking for specific transitions satisfying special selection rules. Of particular interest would be (1) a $0 \rightarrow 0$ transition that would indicate directly the presence of Fermi-type coupling and (2) a $^1/_2 \rightarrow {}^3/_2$ " allowed Gamow-Teller " transition in which the two initial hyperfine states ($F = 0$ and $F = 1$) can be distinguished since the transition from the $F = 0$ state is " forbidden " if $G_P = G_M = 0$ and relativistic effects are ignored [14)].

3. PARITY-VIOLATION EXPERIMENTS

We hope to confirm by means of experiments involving parity violation that the basic process (*A*) does involve a two-component neutrino with the same helicity as that in positron decay. Furthermore, parity-violation experiments involve the induced couplings g_P and g_M in a manner distinct from Eq. (2*b*).

I. Neutron Asymmetry

If the muon retains some of its polarization before being captured then we may observe parity violation in the asymmetry of an outgoing neutron with respect to the muon spin direction. For a simple $V-A$ theory this effect vanishes due to cancellation of the A and V contributions, but with the effective couplings given by Eqs. (1) (particularly the effect of g_P) an asymmetry of about 40% is predicted for capture by an unpolarized proton. In complex nuclei the asymmetry is partially washed out by the initial proton motion and the final-state neutron interaction. Crude calculations indicate that the reduction to these effects is less than a factor of two, provided low energy evaporation neutrons are not observed [15,16]. An effect of the predicted order of magnitude has been reported by the Liverpool group, but Columbia has obtained a null result. Telegdi will report a measurement with good accuracy which indicates an asymmetry with the right size and sign. This result would be the strongest indication of the presence of the expected pseudoscalar interaction.

II. Asymmetry of Recoil Nuclei

In a specific transition as reaction (B) the recoiling nucleus may have an asymmetry with respect to the muon spin even in the simple $V-A$ theory. For example in a pure Gamow-Teller transition such as (B) the asymmetry is $\frac{1}{3}$. With the effective couplings (Eq. (1)) this is increased for reaction (B) because of the effective pseudoscalar interaction to about 0.8. Thus this observation would be of interest both as to parity violation and for observing the effect of g_P, although to some extent forbidden matrix elements may give an effect similar to g_P. These asymmetry results are different for nuclei with spin due to the hyperfine coupling effect.

III. Neutron Longitudinal Polarization

This observation has many advantages over the neutron asymmetry : (1) it is predicted to be 100% on the $V-A$ theory and this result is not seriously modified by the virtual pion effects, (2) it does not depend on retention of polarization by the muon, and (3) it directly measures the neutrino helicity independent of knowledge of the initial muon helicity. The experiment is difficult since it requires a rotation of the neutron spin and a subsequent rescattering, but nevertheless it is now seriously proposed to do this experiment. In complex nuclei the longitudinal polarization is washed out for the same reasons as the asymmetry and to the same extent if the spin dependence of the final state neutron interaction is ignored. A very rough estimate of the spin-orbit coupling in the final state indicates a sizeable longitudinal polarization would be retained [17].

4. RADIATIVE MUON CAPTURE

The internal bremsstrahlung accompanying muon capture corresponds to the basic process

$$\mu^- + p \rightarrow n + \nu + \gamma \qquad (D)$$

A major reason for the interest in this process lies in its intrinsic relativistic character. If we consider only the muon current it follows from a theorem of Cutkosky that such observables as the circular polarization, and angular correlation (with respect to the neutrino) of the photon are the same as in the non-radiative emission of a positive lepton in the extreme relativistic limit ($v \rightarrow c$). In particular, the ratio of radiative to non-radiative capture involves interference between A and T or S and V interactions (Fierz term); the photons are all right circularly-polarized for the standard V, A interaction, etc. Also considering diagram *(a)* of Fig. 2 we expect a 100% asymmetry of the photons with respect to the muon spin direction. A measurement of both the asymmetry and the circular polarization, in fact, would determine the muon spin direction just before capture.

This simple picture is modified even for the basic process (D) by the following considerations :

(1) In diagram *(a)* we must add the effective couplings g_P and g_M. In the case of g_M for which the coupling is linear in the momentum transfer a " catastrophic " type interaction (in which the photon appears at the four-fermion vertex) must be included to insure gauge-invariance. The effective g_P also arises from a momentum-dependent vertex, but it has been shown [18] that it is correctly treated as if it were truly a pseudoscalar coupling.

(2) Contributions of the nucleon anomalous magnetic-moment interactions (diagrams *(c)* and *(d)* Fig. 2) must be included.

(3) Contributions of the proton current (diagram *(b)* Fig. 2) should be included in a relativistic calculation. One of these is of special importance. It is the term in which the weak vertex is the g_P coupling, which is basically a relativistic interaction itself.

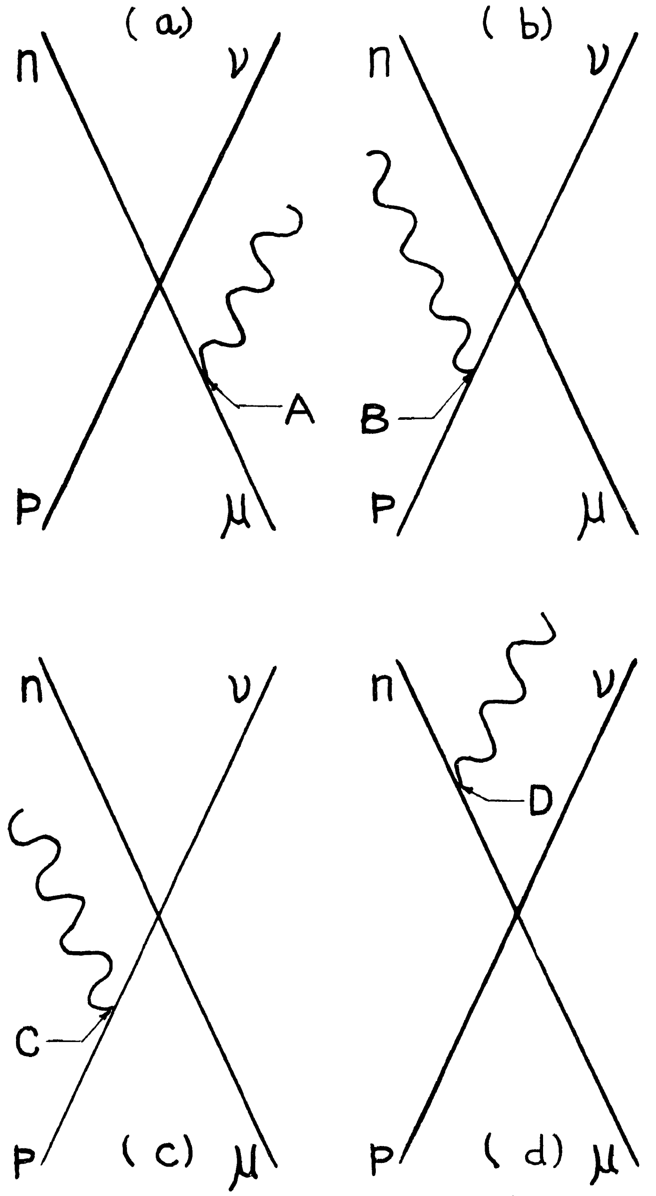

Fig. 2 Diagrams for radiative muon capture.

In this case, the proton current contribution is just as large as the muon current term rather than being reduced by the factor (m_μ/M). But this term is further greatly enhanced by the momentum-transfer dependence of the effective pseudoscalar coupling, which is proportional to the one-pion propagator $g_P \sim (m_\pi^2+q^2)^{-1}$. In non-radiative capture $q^2 \simeq m_\mu{}^2$, whereas for radiative capture at the top of the photon spectrum [for diagram (2*b*)] $q^2 \simeq m_\mu{}^2$. (Note that q is the four-momentum transfer at the nucleon vertex and therefore also at the lepton vertex.) Thus, this term is increased by a factor of about 10 at the top of the photon spectrum and has an anomalous spectral dependence. In non-radiative capture the g_P coupling may be thought of as the virtual emission of a pion and its subsequent " decay "; so in radiative capture this term may be considered as the virtual radiative emission of an S-wave pion (due to the gauge term in the static model) and its subsequent decay.

Fig. 3 shows the spectrum of photons from muon capture by an unpolarized proton relative to the total capture rate, that is, the integral gives the ratio of radiative to non-radiative capture. Curve A includes only diagram *(a)* with g_V and g_A couplings; curve B includes diagrams *(b)* and *(c)*, with g_V, g_A, g_P and g_M given by Eqs. (1*a*)-(1*d*); curve C is the same except that the sign of g_P is changed. Neutron recoil effects are not included. It is seen that the inclusion of the g_P term with the " proper " sign distorts the spectrum and increases the relative radiative capture rate over the top half of the spectrum by a factor of about two.

The circular polarization and asymmetry of the gamma-rays is expected to remain close to 100% as long as the g_P contribution is ignored. With this

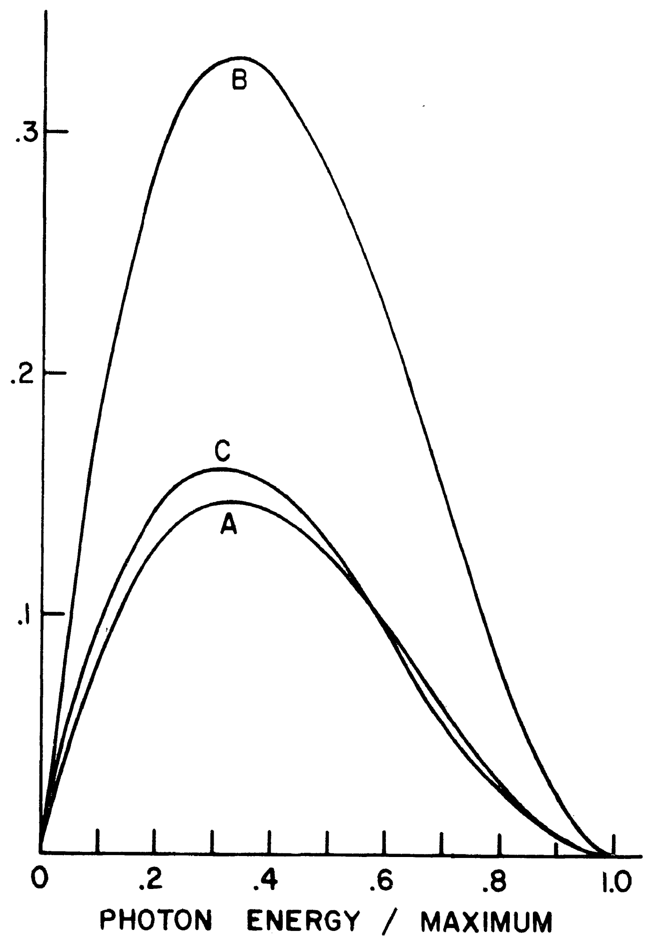

Fig. 3 γ-Spectrum for radiative capture.

contribution with the " proper " sign these parity-violating effects are 70% near the top of the spectrum and approach close to 100% as the photon energy decreases. With the opposite sign of g_P these parity-violating effects are further decreased.

For the observation of the process (D) in hydrogen these results are completely modified by the hyperfine structure. At the present time, however, our interest is in process (D) occurring inside complex nuclei. Cantwell has shown, considering only the muon current, that the relative rate of radiative capture in a complex spin-zero nucleus is similar to that for capture on an unpolarized proton except that the rate is reduced by an extra factor γ (see Eq. 4) due to the loss in phase space, and the spectrum is slightly flattened in the center due to a greater exclusion principle effect there. The effects of nucleon currents in the case of complex nuclei remain to be investigated. We expect, however, that the large contribution of the effective g_P should show up in the complex nucleus case also. This appears to be one of the most interesting ways of looking for the effective pseudoscalar interaction.

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DISCUSSION

TELEGDI : I would like to offer some comments to justify our use of Primakoff's formula. Our measurements extended not from calcium to uranium but from Be to U.

WOLFENSTEIN : I intentionally considered from calcium upwards because I have even greater doubts about the approximations for elements below calcium.

TELEGDI : The final results of our experiments is a table of capture rates. One can amuse himself and use the table to check the Primakoff formula. We divide the capture rates by Z^4_{eff}, plot them against the neutron excess, and observe that over the entire range the points are scattered about a straight line. It seems to me that this effect (that of the Pauli principle) which is a factor of 10 in the case of uranium and is at least a factor of four everywhere, would not *a priori* be expected to follow a straight line, unless the theory is a pretty good approximation to the truth. The approximation, as I understand it, is that δ is not a universal constant but a kind of fudge factor, and it is marvelous that in the mean it should hold so well. As the proof of the theory from the experimentalist's point of view is that these points lie along a straight line, we may next proceed to extract the constant factor in front of the Primakoff formula. The uncertainty in the determination of the interaction strength, as far as we can see it, is that we do not know the proportionality constant because we do not know the mean neutrino momentum. Any claims as to the validity of the determination of the sum of the squares of the coupling constants from this survey of complex nuclei was not from us. Our business is to supply correct capture rates.

I would like also to talk about the difference in the capture rates from two hyperfine states. There is an added difficulty there. The experiment depends upon two things, on muon physics and also on atomic physics, because it depends upon the internal conversion rate from one hyperfine state to the other. The observed experimental effect is a function of the ratio of the differences in the lifetimes to the internal conversion rate. So these experiments, like the hydrogen experiment, contain factors which transcend muon physics.

WOLFENSTEIN: Let me comment first on your second remark. Since there is a large uncertainty in the nuclear physics anyway, the major important result is a large effect, which you certainly find, as well as the sign of the effect.

I do not want to go into details concerning the analysis of the Primakoff theory. It certainly fits the experiment very well, but I think that one has to check its theoretical foundations and I think that the approximations involved are such that I do not feel that I can be confident of the interpretation of the constants that one deduces. We are trying to re-examine this derivation.

PRIMAKOFF: I would like to make one comment about an approximation that enters into this expression, and which I think dominates its character. This is essentially the approximation that the quantity δ is pretty much the same for all nuclei, i.e. that this δ is effectively independent of Z and A. Now physically, δ is a measure of the rate with which the probability of finding two parallel spin nucleons at a given distance goes to zero when they approach each other—the Pauli principle. It is a parameter which enters into the nucleon-nucleon correlation function. The fact that the experimental points fit the straight line whose slope is given by δ implies among other things that δ is indeed pretty much the same for all nuclei. In other words, as far as this formula is concerned, nuclei are very featureless. Now, Wolfenstein has suggested that the outer protons play a much greater role in the μ-meson capture than the core protons, because the resultant neutrons will be much less inhibited by the Pauli principle. If that is so, then the Z_{eff}^4 varies in a manner quite different from that of Wheeler, and the δ also varies, and presumably the two variations compensate. One rather indirect way of seeing which of these pictures is qualitatively correct in the case of muon capture involves the determination of the actual numerical value of the hyperfine structure effect. Now, Bernstein, Lee, Yang and myself gave a formula for the magnitude of this effect. The formula involves a parameter which gives the difference in the inhibition of the Pauli principle on a core proton and on an outer proton, and if one substitutes a value for the parameter corresponding to equal inhibitions, one obtains a magnitude for the hyperfine effect which is about $\frac{1}{3}$ (for Al) of the value that Überall obtains using shell model wave functions, which therefore effectively predict that the exclusion principle inhibits capture on outer protons much less than on core protons. Well, if one can deduce the actual numerical magnitude of the hyperfine effect from the observations, taking into account the internal conversion effects that Telegdi mentioned, one will have a better idea than one has now of which of these two general pictures—a featureless or a structured nucleus—is correct for a treatment of muon capture.

WOLFENSTEIN: Again to refer to what Telegdi said, this formula has other terms in it when calculated consistently without dropping terms of order 1/A, terms which could make the formula very different. I think the mathematical approximations made in deriving this formula are already quite significant, apart from the question of the constancy of δ.