Institute of Physics Publishing Journal of Physics: Conference Series **50** (2006) 95–102
doi:10.1088/1742-6596/50/1/011 5th International Conference on Physics and Astrophysics of Quark Gluon Plasma

# Fluctuations and Correlations in Heavy Ion Collisions

V. Koch

*Lawrence Berkeley National Laboratory*
*Berkeley, CA 94720, U.S.A.*

**Abstract.** An overview of the physics of event by event fluctuations in heavy ion collisions is provided. We will discuss what the measurement of fluctuations and correlation can tell us about the system created in these collisions.

## 1. Introduction

The study and analysis of fluctuations and correlations are an essential method to characterize a physical system. In general, one can distinguish between several classes of fluctuations. On the most fundamental level there are quantum fluctuations, which arise if the specific observable does not commute with the Hamiltonian of the system under consideration. These fluctuations probably play less a role for the physics of heavy ion collisions. Second, there are "dynamical" fluctuations reflecting the dynamics and responses of the system. They help to characterize the properties of the bulk (semi-classical) description of the system. Examples are density fluctuations, which are controlled by the compressibility of the system. Finally, there are "trivial" fluctuations induced by the measurement process itself, such as finite number statistics etc. These need to be understood, controlled and subtracted in order to access the dynamical fluctuations which tell as about the properties of the system.

While fluctuations are related to the variance of a given observable[1], correlations are accessible via the co-variances. The measurement of these co-variances will provide information about the independence of the degrees of freedom considered.

Fluctuations are also closely related to phase transitions. The well known phenomenon of critical opalescence is a result of fluctuations at all length scales due to a second order phase transition. First order transitions, on the other hand, give rise to bubble formation, i.e. density fluctuations at the extreme.

The most efficient way to address fluctuations of a system created in a heavy ion collision is the study of event-by-event (E-by-E) fluctuations, where a given observable is measured on an event-by-event basis and the fluctuations are studied over the ensemble of the events. In most cases (namely when the fluctuations are Gaussian) this analysis is equivalent to the measurement of two particle correlations over the same region of acceptance [1]. Consequently, fluctuations tell us about the 2-point functions of the system, which in turn determine the response of the system to external perturbations.

In the framework of statistical physics, which appears to describe the bulk properties of heavy ion collisions up to RHIC energies, fluctuations measure the susceptibilities of the system. These susceptibilities also determine the response of the system to external forces. For example, by measuring fluctuations of the net electric charge in a given rapidity interval, one obtains information on how this (sub)system would respond to applying an external (static) electric field. In other words, by measuring fluctuations one gains access to the same fundamental properties of the system as "table top" experiments dealing with macroscopic probes. In the later case, of course, fluctuation measurements would be impossible.

## 2. Correlations and Fluctuations in a thermal system

Given the free energy $F = -T \log Z$ of a system, its fluctuations are controlled by the susceptibilities, which are the second derivatives of the free energy with respect to the appropriate conjugate variables,

$$\chi_{i,j} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_i \mu_j} F. \tag{1}$$

As an example, the fluctuations of the net charge are given by

$$\left\langle \delta Q^2 \right\rangle = -T \frac{\partial^2}{\partial \mu_Q^2} = TV \chi_{Q,Q}. \tag{2}$$

Covariances are related to the off diagonal susceptibilities, $i \neq j$, for example

$$\langle (\delta B)(\delta S) \rangle = TV \chi_{B,S} = -T \frac{\partial^2 F}{\partial \mu_B \mu_S}. \tag{3}$$

Note that for vanishing baryon number chemical potential, these susceptibilities can be calculated in Lattice QCD. In this case it is more convenient to study quark number susceptibilities $\chi_{f,f'}$ where $f$ and $f'$ represent the different quark flavors u,d and s. These have been calculated in lattice QCD [2, 3] and provide interesting insight into the structure of the matter above the phase transition.

Recently the correlation between baryon number and strangeness has been proposed as a useful diagnostic for the structure of high temperature matter [4]. The authors observed that in case of uncorrelated quarks strangeness is carried exclusively by quarks, and thus is tightly correlated to the baryon number. In a hadron gas or in a bound-state picture of the QGP [5] strangeness can be carried by mesons or $s\bar{q}$ states, which do not carry baryon number. This will reduce the correlations. In ref. [4] the following ratio is considered

$$C_{BS} = \equiv -3 \frac{\sigma_{BS}}{\sigma_S^2} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle}, \tag{4}$$

which gives $C_{BS} = 1$ for uncorrelated quarks, and $C_{BS} \simeq 0.66$ and $C_{BS} \simeq 0.62$ for a hadron gas and a bound-state QGP, respectively. In terms of the flavor susceptibilities

$$C_{BS} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_{ss}}. \tag{5}$$

Using the results obtain in quenched Lattice QCD [2] one obtains $C_{BS} = 1$, consistent with uncorrelated quarks. At the same time the equation of state (EOS) differs from that of free quarks an gluons. However, the EOS can be reproduced in a quasi-particle picture [6, 7] suggesting an effective mean field description for the matter at high temperature. Incidentally the quasiparticle picture and the EOS indicate a predominantly repulsive interaction, which might partially account for the observed flow at RHIC.

Let us point out that $C_{BS}$ is accessible in experiment via event-by-event fluctuations [4], although a precise determination would require neutron detection capabilities.

## 3. Transverse momentum and charge fluctuations

The field of event-by-event fluctuations is relatively new to heavy ion physics and ideas and approaches are just being developed. So far, most of the analysis has concentrated on transverse momentum and the net charge fluctuations.

The pioneering event-by-event studies have been carried out by the NA49 collaboration. They have analyzed the fluctuations of the mean transverse momentum [8] and the kaon to pion ratio [9]. Both measurements have been carried out at at the CERN SPS at slightly forward rapidities. In both cases mixed events can essentially account for the observed signal, leaving little room for genuine dynamical fluctuations.

Transverse momentum fluctuations should be sensitive to temperature/energy fluctuations [10, 11]. These in turn provide a measure of the heat capacity of the system [12] since

$$\left\langle \delta E^2 \right\rangle = \frac{\partial^2}{\partial \beta^2} \log Z = -T^3 \frac{\partial^2}{\partial T^2} F = T^2 C_V . \tag{6}$$

As the QCD phase transition is associated with a maximum of the specific heat, the temperature fluctuations should exhibit a minimum in the excitation function. It has also been argued [13, 14] that these fluctuations may provide a signal for the long range fluctuations associated with the tri-critical point of the QCD phase diagram. In the vicinity of the critical point the transverse momentum fluctuations should increase, leading to a maximum of the fluctuations in the excitation function.

Transverse momentum fluctuations have been analyzed by several experiments at different bombarding energies. At SPS energies the NA49 collaboration measured transverse momentum fluctuations in the forward rapidity region and found no significant deviation from pure statistics [8]. Similarly, at RHIC energies, the PHENIX collaboration also reports no significant non-statistical transverse momentum fluctuations [15]. In contrast to that the CERES collaboration finds fluctuations larger than those from mixed events [16] at SPS energies and at RHIC the STAR collaboration reports significant deviations from mixed events [17]. To which extent this can be attributed to the different acceptance regions covered by these experiments remains to be investigated.

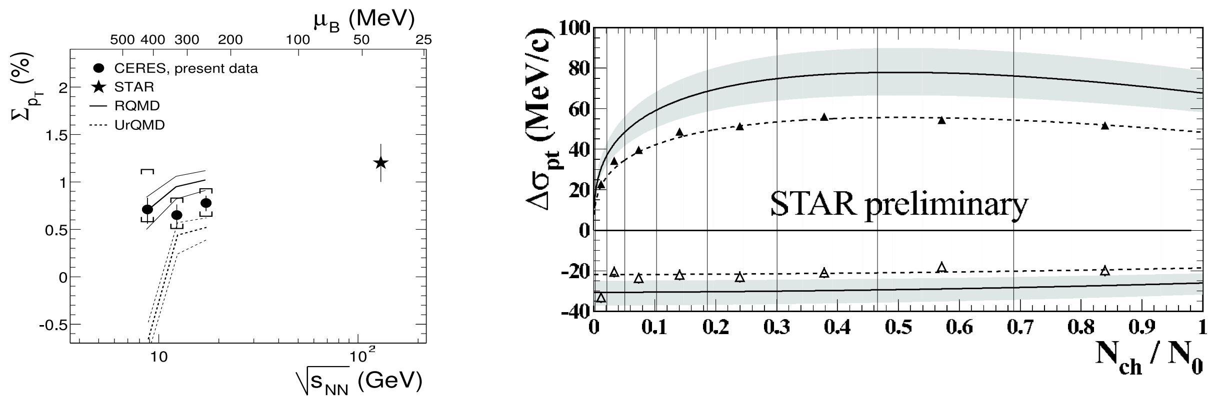


**Figure 1.** Left:Excitation function for transverse momentum fluctuations from [16]. Right: Centrality dependence of $p_t$ fluctuations (from [17]).

The obvious question is why are the $p_t$ fluctuations small and similar for all bombarding energies. This could have a simple explanation. Note, that experiments always subtract mixed events. If the system is similar to an ideal gas, i.e. does not carry significant correlations, the fluctuations are just those of the mixed events [24] and thus not "net" $p_t$ fluctuations remain. Now a hadron gas as well as a uncorrelated QGP are rather similar to an ideal gas. Only if the system is prepared close to the phase transition, where the heat capacity $C_V$ has a maximum, we expect the $p_t$ fluctuations to deviate significantly from that of an ideal gas. So it could very well be the case that the phase transition is either between SPS and RHIC energies, or, as suggested by the rather interesting preliminary data on $K/\pi$ fluctuations [18], it is located at the lower SPS energies (20-30 AGeV beam energy).

Another observable of interest are so-called charge fluctuations, since they provide a signature for the existence of a de-confined Quark Gluon Plasma phase [19, 20]. The essential idea is that in a QGP the charge carriers are the quarks, which posses fractional charge. Since charge fluctuations are proportional to the square of the charge

$$\left\langle \delta Q^2 \right\rangle \quad = \quad q^2 \left\langle (\delta N)^2 \right\rangle , \tag{7}$$

the ratio of charge fluctuation over entropy

$$\frac{\langle \delta Q^2 \rangle}{S} \sim \frac{\langle \delta Q^2 \rangle}{\langle N_{charge} \rangle} \tag{8}$$

is sensitive to the fractional charges in a QGP. In ref. [20] the observable

$$D \equiv 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{charge} \rangle} \tag{9}$$

has been proposed and is has been shown that $D = 4$ for an uncorrelated pion gas, $D \simeq 3$ for a resonance gas [21] and $D \simeq 1 - 1.5$ for a Quark Gluon Plasma, respectively. Since the electric charge is conserved globally, and thus does not fluctuate, experimental measurements need to be corrected for charge conservation effects. These become significant once a sizeable fraction of the final state particles are taken into account. Several prescriptions for these corrections have been proposed [22, 23] which all agree in the limit of small acceptance. A detailed discussion can be found in [24].

In the mean time charge fluctuation have been analyzed by several experiments. PHENIX [25] at RHIC which measures with a small rapidity acceptance, finds charge fluctuations consistent with a resonance gas, if extrapolated to larger acceptance. STAR, which has a large acceptance also finds charge fluctuations consistent with a resonance gas [26]. CERES [27] and NA49 [28], which both measure at SPS energies, report preliminary results on charge fluctuations, which are consistent with a pure pion gas. However, at the SPS the overall rapidity distribution is rather narrow, so that the correlation effect of the resonances gets lost when correcting for charge conservation [29]. But certainly, none of the measurements is even close to the prediction for the QGP.

These findings have prompted ideas, that possibly a constituent quark plasma, without gluons, has been produced [30]. However, the measurement of additional observables would be needed in order to distinguish this from a hadronic gas.

But maybe the present range of $\Delta y$ is so small, that the charge fluctuations have time to assume the value of the resonance gas. As shown in [20] and [31], the larger the rapidity interval considered, the longer the relaxation time for the charge fluctuations. Thus, maybe even larger acceptance is needed to recover the QGP value. This is also suggested by a model calculation using several event generators. As shown in Fig. 2, the results for the parton cascade arrive at the predicted value for the QGP only for $\Delta y \geq 3$. None of the present experiments has such a coverage yet and thus a detailed analysis of $D$ as a function of $\Delta y$ is needed, before any firm conclusions can be drawn.

We note that a recombination picture [33] for the hadronization should not affect the charge fluctuations, as recombination is local in momentum space and thus introduces negligible rapidity shifts of the charges.

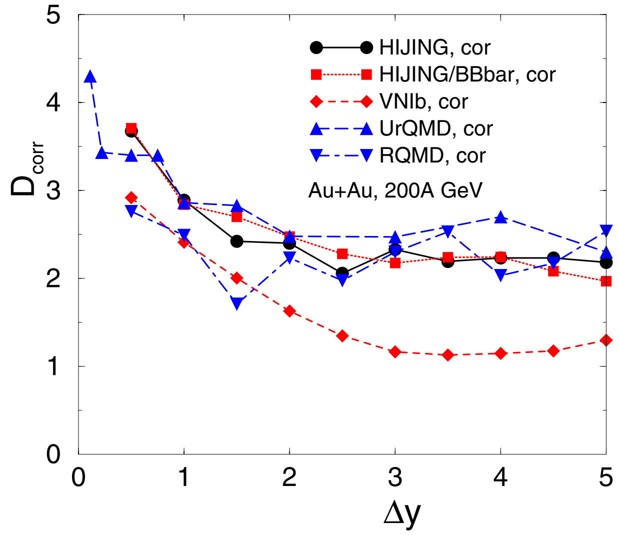


**Figure 2.** Charge fluctuation from several event generators [32]

Another way to access the non-trivial correlations of the system is the so called balance function [34, 35]. The balance function for charged particle for instance is defined as

$$B(\eta|\Delta\eta) = \frac{1}{2}\left[\frac{\langle N_{+-}(\eta|\Delta\eta)\rangle}{\langle N_{-}(\Delta\eta)\rangle} + \frac{\langle N_{-+}(\eta|\Delta\eta)\rangle}{\langle N_{+}(\Delta\eta)\rangle} - \frac{\langle N_{++}(\eta|\Delta\eta)\rangle}{\langle N_{+}(\Delta\eta)\rangle} - \frac{\langle N_{--}(\eta|\Delta\eta)\rangle}{\langle N_{-}(\Delta\eta)\rangle}\right], \tag{10}$$

where $N_{+-}(\eta|\Delta\eta)$ is the number of unlike-sign pairs which are $\eta$ apart from each other within the rapidity window $\Delta\eta$. It essentially measures the average distance in rapidity over which a given charge is neutralized (balanced). It is related to the above charge fluctuations in that the latter can be expressed as an integral over the charge balance function [35]. The balance function measurement at $\sqrt{s} = 130$ GeV has been reported by the STAR collaboration[36]. Going from peripheral to central collisions, the width of balance function steadily decreases. The trend is what one would expect if more of the system is filled with a QGP as the collision becomes more central. However, since the reduction is only about 20 % going from most peripheral to most central, it is not yet clear whether this signals the presence of a QGP, constituent quark clusters [37] or more mundane effect such as the strong flow. For instance in [38] the measured balance functions, along with particle ratios and spectra, could be explained in an expanding hadron gas model.

As detailed in [24] all these event-by-event fluctuation observables can be derived from underlying basic correlator

$$\Delta_{\alpha,\beta}(p,q) = \langle n_\alpha(p) n_\beta(q)\rangle\,, \tag{11}$$

which gives the correlation between particles with quantum numbers $\alpha$ and $\beta$ and momenta $p$ and $q$ respectively. The difference between momentum fluctuation charge fluctuations etc. is then simply the choice of $\alpha$ and $\beta$ and the weighting functions this correlator is folded with. Also, in order to remove effects from finite number statistics, so called dynamical fluctuations are extracted by either subtracting [39] or dividing [14] the result obtained with and uncorrelated basic "correlator",

$$\Delta^0_{\alpha,\beta}(p,q) = \delta_{\alpha,\beta}\delta_{p,q} n_\alpha(p). \tag{12}$$

Recently is has been pointed out [40, 41] that this can also be achieved by generalized factorial moments.

**4. Kaon to pion ratio and its fluctuation**

Recently NA49 has reported interesting results on the excitation function of the inclusive kaon to pion ratio as well as its fluctuations [18, 42]. Around 30 AGeV beam energy the inclusive $K/\pi$ ratio shows a maximum and at the same energy the preliminary data on the fluctuations of the $K/\pi$-ratio increase to about 8%, which is a factor of 4 larger then at 160 AGeV [18]. While the fluctuations at 160 AGeV can be accounted for by resonance decays [21] the value of 8% are impossible to reproduce in a hadron gas. Actually, as at lower bombarding energies global strangeness conservation plays a significant role, one would expect some small negative value for the dynamical fluctuations. Instead the fluctuations are large and positive. One possibility to account for these large fluctuation would be domain formation due to spinodal instabilities [43] with subsequent locking of strangeness [44]. In this scenario the large fluctuations of strangeness in the QGP are frozen in during the sudden breakup of the system due to spinodal instabilities. This scenario also provides for a small increase of the inclusive $K/\pi$-ratio.

**5. Equilibrium**

Another important question, which might be addressed by the study of fluctuation is equilibration. While measured particle abundances are well described by a hadron gas in chemical equilibrium [45] this is also the case for collision of elementary particle such as proton-proton or $e^+ - e^-$. Thus simple phase-space dominance a la Fermi [46] needs to be ruled out [47]. In other words, how can we distinguish between a superposition of essentially independent nucleon-nucleon collisions as depicted in Fig. 3(a) and a system which equilibrated over the entire volume (Fig. 3(b))? In the absence of any correlations, the partition functions factorizes and thus the two system are indistinguishable.

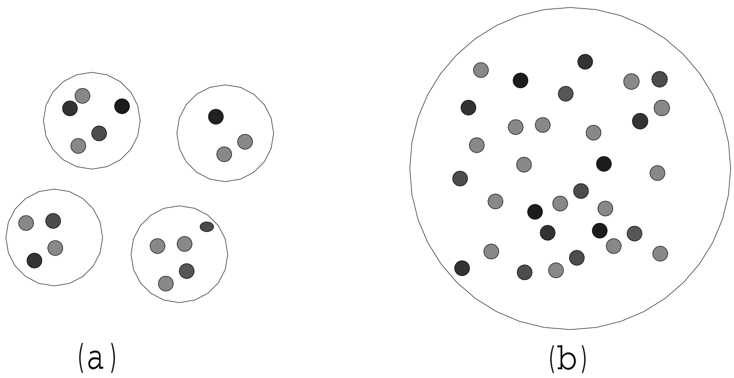


**Figure 3.** Individual nucleon-nucleon collisions (a) and "true" matter generated in a nucleus-nucleus collision (b).

At low energies ($\sim 1$ AGeV) strangeness conservation introduces such correlations and leads to unique predictions for the second factorial moment of the kaon abundance [48]. At higher energies, however, explicit strangeness conservation becomes less relevant and has only a small effect on the single particle yield. Only if, for some reason, the domain over which strangeness (or any other conserved quantum number) is conserved is so small, that it contains of the order of one conserved quantum, conservation laws still affect particle abundances. In case of strangeness this has been demonstrated in [49]. However, once the strangeness correlation volume, i.e the volume over which strangeness is conserved is larger than twenty times that of a nucleon, the particle abundances are simply a superposition of the sub-domains and sensitivity to the size of the correlation volume is lost. In order to establish strangeness correlation volumina comparable with the system size, many-particle correlations need to be measured. As demonstrated in [50] a definitive measurement of equilibration at RHIC energies would require the measurement of five Omega-baryon coincidences. To which extend this is feasible remains to be seen. Two-particle correlations, which are often discussed as a possible means to establish the degree of equilibrium, are dominated by Poisson statistics and thus are misleading.

## 6. Conclusions

In this contribution we have discussed the physics of fluctuations in the context of heavy ion collisions. As this is a developing field, this should be considered as a snapshot of our present understanding. We have argued that fluctuations are indeed a new tool to investigate the properties of the strongly interacting matter at high temperature. As an example we have shown that baryon-number strangeness correlations can be utilized to address the correlations among quark flavors, and that charge fluctuations are sensitive to the fractional charges of the quarks, and thus server as a signature for the QGP. The measurement of momentum fluctuations, on the other hand should give us an idea about the heat capacity of the system. Furthermore, if the system is created close to a second order phase transition point, the associated long range fluctuations should be observable in event-by-event observables. Spinodal instabilities associated with a first order phase transition are also detectable in fluctuation observables. Also the question of equilibration can be addressed via fluctuations. At RHIC energies, however, this requires rather difficult measurements of many ($> 5$) particle correlations.

## Acknowledgments

The author thanks A. Majumder and J. Randrup for a fruitful collaboration on some of the topics discussed here. This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, and by the Office of Basic Energy Sciences, Division of Nuclear Sciences, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

## References

[1] A. Bialas and V. Koch, Phys. Lett. **B456**, 1 (1999), [arXiv:nucl-th/9902063].
[2] R.V. Gavai and S. Gupta, Phys. Rev. D **67**, 034501 (2003) [arXiv:hep-lat/0211015].
[3] C.R. Allton *et al.*, Phys. Rev. D **71**, 054508 (2005) [arXiv:hep-lat/0501030].
[4] V. Koch, A. Majumder, and J. Randrup [arXiv:nucl-th/0505052].
[5] E.V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004) [arXiv:hep-ph/0403127].
[6] J.P. Blaizot, E. Iancu and A. Rebhan, Quark Gluon Plasma 3, 60-122, Hwa, R.C. (ed.), World Scientfic, Singapore 2004. [arXiv:hep-ph/0303185].
[7] A. Peshier, B. Kämpfer and G. Soff, Phys. Rev. D **66**, 094003 (2002) [arXiv:hep-ph/0206229].
[8] NA49, H. Appelshauser *et al.*, Phys. Lett. **B459**, 679 (1999), [arXiv:hep-ex/9904014].
[9] NA49, S. V. Afanasev *et al.*, Phys. Rev. Lett. **86**, 1965 (2001), [arXiv:hep-ex/0009053].
[10] L. Stodolsky, Phys. Rev. Lett. **75**, 1044 (1995).
[11] E. V. Shuryak, Phys. Lett. **B423**, 9 (1998), [arXiv:hep-ph/9704456].
[12] L. Landau and L. Lifshitz, *Statistical Physics* (Pergamon Press, New York, 1980).
[13] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998), [arXiv:hep-ph/9806219].
[14] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. **D60**, 114028 (1999), [arXiv:hep-ph/9903292].
[15] PHENIX, K. Adcox *et al.*, Phys. Rev. **C66**, 024901 (2002), [arXiv:nucl-ex/0203015].
[16] CERES, D. Adamova *et al.*, Nucl. Phys. **A727**, 97 (2003), [arXiv:nucl-ex/0305002].
[17] STAR, J. Adams *et al.*, Adams *et al.* [STAR Collaboration], arXiv:nucl-ex/0308033.
[18] C. Roland *et al.* [NA49 Collaboration], J. Phys. G **30**, S1381 (2004) [arXiv:nucl-ex/0403035].

[19] M. Asakawa, U. W. Heinz, and B. Muller, Phys. Rev. Lett. **85**, 2072 (2000), [arXiv:hep-ph/0003169].
[20] S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000), [arXiv:hep-ph/0003168].
[21] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999), [arXiv:nucl-th/9906074].
[22] M. Bleicher, S. Jeon, and V. Koch, Phys. Rev. **C62**, 061902 (2000), [arXiv:hep-ph/0006201].
[23] C. Pruneau, S. Gavin, and S. Voloshin, (2002), [arXiv:nucl-ex/0204011].
[24] S. Jeon and V. Koch, Quark-Gluon Plasma 3, 430-490, Hwa, R.C. (ed.), World Scientfic, Singapore 2004 [arXiv:hep-ph/0304012].
[25] PHENIX, K. Adcox *et al.*, Phys. Rev. Lett. **89**, 082301 (2002), [arXiv:nucl-ex/0203014].
[26] J. Adams *et al.*, Phys. Rev. **C68**, 044905 (2003).
[27] H. Appelshauser *et al.* [CERES Collaboration], Nucl. Phys. A **752**, 394 (2005) [arXiv:nucl-ex/0409022].
[28] C. Blume *et al.*, (2002), [arXiv:nucl-ex/0208020].
[29] J. Zaranek, Phys. Rev. **C66**, 024905 (2002), [arXiv:hep-ph/0111228].
[30] A. Bialas, Phys. Lett. **B532**, 249 (2002), [arXiv:hep-ph/0203047].
[31] E. V. Shuryak and M. A. Stephanov, Phys. Rev. **C63**, 064903 (2001), [arXiv:hep-ph/0010100].
[32] Q. H. Zhang, V. Topor Pop, S. Jeon, and C. Gale, Phys. Rev. **C66**, 014909 (2002), [arXiv:hep-ph/0202057].
[33] R.J. Fries, B. Müller, C. Nonaka, and S.A. Bass Phys. Rev. Lett. **90**, 202303 (2003).
[34] S. A. Bass, P. Danielewicz, and S. Pratt, Phys. Rev. Lett. **85**, 2689 (2000), [arXiv:nucl-th/0005044].
[35] S. Jeon and S. Pratt, Phys. Rev. **C65**, 044902 (2002), [arXiv:hep-ph/0110043].
[36] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **90**, 172301 (2003) [arXiv:nucl-ex/0301014].
[37] A. Bialas, (2003), [arXiv:hep-ph/0308245].
[38] P. Bozek, W. Broniowski, and W. Florkowski, (2003),[arXiv:nucl-th/0310062].
[39] M. Gazdzicki and S. Mrowczynski, Z. Phys. **C54**, 127 (1992).
[40] F. Jinghua and L. Lianshou, (2003), [arXiv:hep-ph/031030].
[41] A. Bialas, (2003), [arXiv:hep-ph/0310348].
[42] M. Gazdzicki *et al.* [NA49 Collaboration], J. Phys. G **30**, S701 (2004) [arXiv:nucl-ex/0403023].
[43] J. Randrup, Phys. Rev. Lett. **92** 122301 (2004).
[44] V. Koch, A. Majumder, and J. Randrup in preparation for Phys. Rev. C
[45] P. Braun-Munzinger, K. Redlich and J. Stachel, Quark Gluon Plasma 3, 491-599, Hwa, R.C. (ed.), World Scientfic, Singapore 2004. [arXiv:nucl-th/0304013].
[46] E. Fermi, Progr. Theoret. Phys. **5**, 570 (1950).
[47] V. Koch, Nucl. Phys. **A715**, 108 (2003),[arXiv:nucl-th/0210070].
[48] S. Jeon, V. Koch, K. Redlich, and X. N. Wang, Nucl. Phys. **A697**, 546 (2002), [arXiv:nucl-th/0105035].
[49] S. Hamieh, K. Redlich, and A. Tounsi, Phys. Lett. **B486**, 61 (2000), [arXiv:hep-ph/0006024].
[50] A. Majumder and V. Koch, Phys. Rev. C **68**, 044903 (2003) [arXiv:nucl-th/0305047].