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# DATA ANALYSIS FOR THE RESONANT GRAVITATIONAL WAVE DETECTOR AURIGA: OPTIMAL FILTERING, $\chi^2$ - TEST, EVENT TIMING AND RECONSTRUCTION

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We describe the possibilities in signal processing opened by the new fully numerical data analysis system developed for the ultracryogenic resonant gravitational wave detector AURIGA. First we review the main features of the AURIGA data acquisition system. These include 5 *kHz* data sampling, data synchronization with the Universal Time Coordinate (UTC) within 1  $\mu$ sec and full raw data storage on high density media for retrieval and off-line data reprocessing. Then we discuss some relevant points of the AURIGA data analysis system such as the data filtering and data compression algorithms, the computation of the  $\chi^2$  time series for the suppression of spurious events and a novel method to reconstruct the input signals based on the Karhunen-Loève expansion.

## 1 Introduction

AURIGA is a resonant gravitational wave (g.w.) detector located at the INFN-National Laboratories of Legnaro in Italy, designed to detect short bursts of gravitational radiation at characteristic frequencies of about 1 *KHz*.<sup>1</sup> Furthermore, AURIGA could be part of an array of g.w. detectors, interferometers and resonant cryogenic bars, which in perspective could operate as a g.w. observatory in such a way to obtain: i) almost isotropic full sky coverage, ii) reconstruction of direction of propagation and polarization, iii) measurement of the velocity of propagation, and vi) internal vetos against spuria. Notice that the object of gravitational wave experiments is to make astronomical observations and a networks of at least 3 or preferably more detectors guarantees the complete reconstruction of a g.w. event.<sup>2</sup> This task poses a first demanding

request to the data acquisition and analysis system of the AURIGA detector namely an accurate synchronization with the Universal Time Coordinate (UTC) at least within few  $\mu sec$ . Other requests include: the capability of handling of a large amount of data ( $\approx 3 Gbytes$  per day including the auxiliary data which ensure that the detector is working appropriately); continuous data collecting, 24 hours a day, for many years; real time data analysis searching for rare weak events of short duration ( $\sim 1 msec$ ); the data have to be analyzed searching for periodic signal from galactic pulsars; it must be possible to cross correlate the outputs of two different detectors searching for the stochastic g.w. background.

This paper is organized as follows. In Section 2, we present the hardware configuration of the data acquisition system (daq) we have set up for the AURIGA detector together with a brief description of the data synchronization apparatus with the UTC. In Section 3, we illustrate the features of the AURIGA on-line data analysis, including the Wiener filtering for  $\delta$ -like signals, the event searching algorithms, the  $\chi^2$  test, the Karunen Loéve expansion for signals of unknown waveform and the impulsive event search algorithms. Finally, our conclusions are given in Section 4.

## 2 Data acquisition system

The AURIGA daq system acquires and archives the signal of the antenna without the usual lock-in down conversion. In fig. 1 we report a schematic of this system. The antenna output is sampled at  $\sim 5 KHz$  with a 23 bit AD converter (HP E1430A) housed into a VXI crate (VXI is an industrial standard bus for electronic instrumentation). The data from the accessory instrumentation, such as accelerometers, seismometers, electromagnetic probes etc. are sampled at rates between 1 and 200 Hz with a 32 multiplexed channels, 16 bit, AD converter (HP E1413A) housed in the same VXI crate. The thermometers are acquired and controlled by a GPIB interface. The UTC is acquired by the synchronization apparatus. A dedicated UNIX workstation (SUN Spark 10) reads out the converted data from MXI, GPIB and RS232 interfaces and feeds them first to a 9 *Gbytes* hard disk as a safety buffer, then to a 35 *GBytes* cassette, to the on-line analysis workstation (DEC Alpha) and finally to a shared memory provided for the on-line monitoring of the raw data. To avoid dead times due to system failures or to system calibrations the acquisition chain has been completely duplicated.

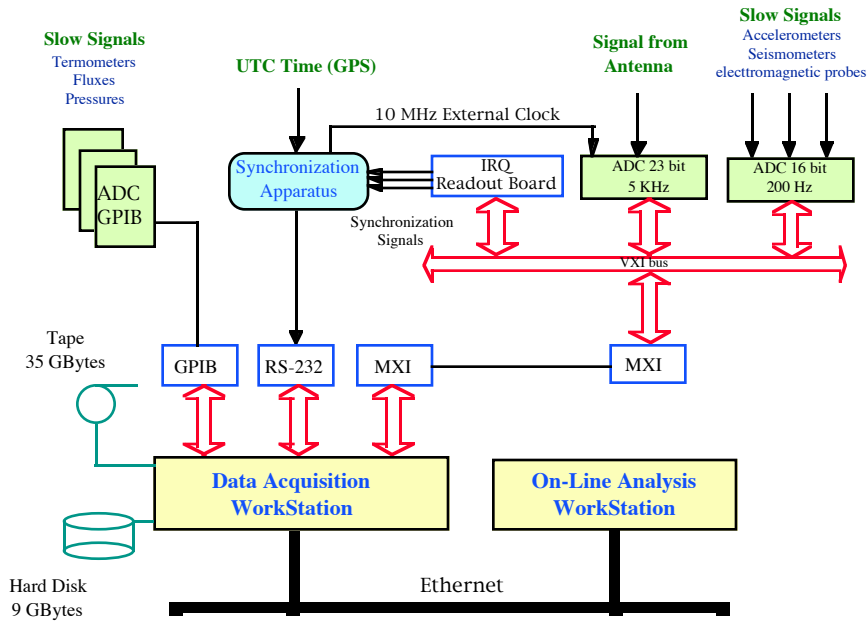


Figure 1: Schematic of the data acquisition and handling system for the AURIGA detector.

### 2.1 The AD Converter of the AURIGA output

The AD converter HP E1430A provides very high performances and flexibility for digitalizing band limited analog signals in a computer compatible format.<sup>3</sup> The normal ADC sample rate is 10 MHz and we make use of the external GPS referenced clock to keep a  $\mu\text{sec}$  synchronization with the UTC. To eliminate higher frequency components the analog signal is filtered by an analog low-pass filter which rejects signals above 5MHz by at least 110 dB of attenuation. Then the signal is digitalized by a 23-bit ADC, so that the effect of finite quantization levels can be completely ignored, leaving only two error sources: linearity error and electronic white noise. The measured total noise can be expressed as  $-137 \text{ dBfs/Hz}$ . This corresponds, for the 2.5 kHz bandwidth of the AURIGA signal to a signal to noise ratio of  $-103.1 \text{ dBfs}$ ; this dynamic range should be enough even when a quantum limited electronic chain of the AURIGA detector will be available. The samples of the signal are processed by a Digital Signal Processor (DSP) which provides a cascade chain of digital low-pass filters, each of which reduces the bandwidth of a factor 2. This

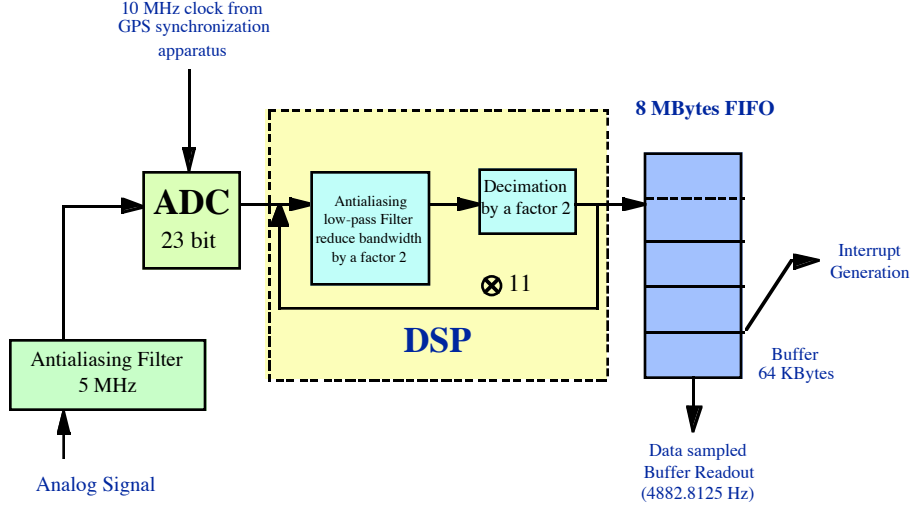


Figure 2: Schematic of the AD converter HP1413A.

is sufficient to avoid any aliasing due to the subsequent data decimation of a factor 2. For the AURIGA output we decided to use 11 of these filters reducing the sample rate to  $10/2^{11} \text{ MHz} = 4882.8125 \text{ Hz}$ . The HP E1430A stores the sampled data in the onboard 8 MBytes FIFO memory which has been divided into 64 kBytes blocks. When a data block is ready an interrupt signal (IRQ) is generated. The IRQ notifies to the read out process in the acquisition workstation that a data block is ready. Thus, the data from the FIFO memory are directed toward a VXI register and read through the MXI bus (see fig. 1). The same IRQ is sent to the GPS synchronization apparatus to date the block-ready event. The IRQ generation mechanism has an intrinsic jitter of  $\sim 0.1 \mu\text{sec}$  while the IRQ propagation lines introduce a fixed delay in the time associate to each data block of  $(1104.4 \pm 0.4) \mu\text{sec}$ .

To take into account of the effect of antialiasing filtering, we have measured the HP E1430A transfer function (amplitude and phase) in the range of the detector mode frequencies  $900 \div 950 \text{ Hz}$ . We found that the transfer function is dominated by the digital antialiasing filters and that it is flat in amplitude within 1/1000. The phase shift has a well understood frequency dependence that can be translated into a fixed delay of  $(875.6 \pm 0.1) \mu\text{sec}$  for the AURIGA signal which is a band limited signal with a carrier frequency of  $\sim 920 \text{ Hz}$ .

## 2.2 Synchronization apparatus

Nowadays the Universal Time can be conveniently obtained by the Global Positioning System (GPS). It consists of 24 satellites orbiting the Earth in 12 *hours* orbits and equipped with atomic clocks which are continuously synchronized with the UTC. This system provides the so-called GPS time that coincides with UTC with an accuracy of 0.1  $\mu\text{sec}$ .<sup>4</sup> The synchronization apparatus GPS100/S80 of the AURIGA antenna has been constructed by ESAT<sup>5</sup> on the specific requirements of our daq system. A schematic of this apparatus and its integration in the AURIGA data acquisition system is shown in fig. 3. The internal oscillator of the GPS100 clock is a 10 *MHz* VCO with a frequency stability of  $10^{-10} \text{ day}^{-1}$ . A dedicated CPU handles the GPS signals and continuously corrects the phase drifts of the internal oscillator so that the phase error of the local second is kept within 0.1  $\mu\text{sec}$  with respect to UTC. The calculus of the phase correction is based on statistical algorithms that limit the typical jitter of the GPS signal down to one hundred of *nsec*. The GPS100/S80 is equipped with three RS232 interfaces which are devised to monitor and setup the GPS receiver, to collect the statistical informations about the clock operation and to send to the acquisition workstation the synchronization strings. The S80 board allows to date up to 8 different events with a total rate limited to 50 events per second by the readout system of the RS232 interface. To synchronize the data flow with UTC we use the IRQs generated by the AD converters when a data block is ready for the data acquisition workstation. These signals, captured by the IRQ readout board, are sent to the S80 event board which returns to the workstation an ASCII string containing the time of the interrupt generation; then the data acquisition software associates this string to the corresponding data buffer.

## 2.3 Acquisition software

We have modelled the whole AURIGA daq system - readout and control - by means the Protob formalism (allowing the definition of object oriented models) based on high level Petri Nets.<sup>6</sup> The model has been then simulated to validate its logical consistency and then translated (with automatic code production) into our processing unit. The acquisition software is divided into 6 parallel processes: “*Read out antenna and auxiliary channels*”, “*Read out UTC*”, “*Read out temperatures*” which acquires the digitized signals from ADCs, GPS apparatus and GPIB instruments, “*Collect data*” which collects, formats and writes to disk the data coming from the readout processes, “*Write to tape*” which copies the data files from the disk to the cassette and “*Send data to analysis*” which sends the data buffer to the analysis workstation via TCP/IP.

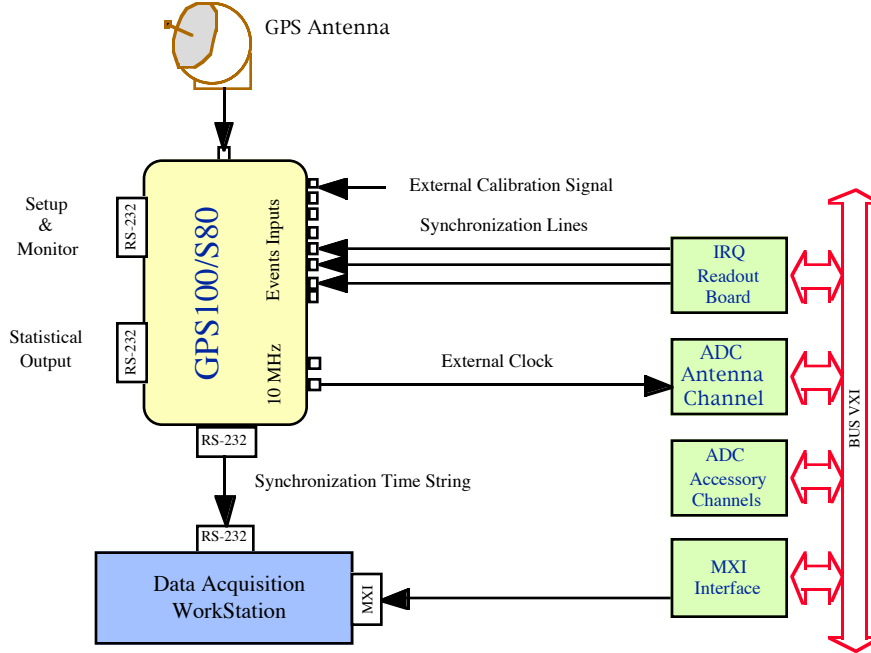


Figure 3: Schematic of the synchronization apparatus GPS100/S80 of the AURIGA data acquisition system.

We have developed software interfaces, based on Labview (National Instruments) and Paw (CERN), to setup and control the daq system and to monitor the raw data.

### 3 Data Analysis

The goal of the on-line data analysis is to implement linear filtering algorithms which make the signal to noise ratio (SNR) of a g.w. signal (defined as the amplitude of the signal divided by the root mean square fluctuation of the filter output) as large as possible. Up to now a well established and robust practice for the data analysis of resonant detectors consists in the lock-in demodulation of the data and their subsequent optimal filtering as required by the standard Wiener-Kolmogoroff theory.<sup>7</sup> This procedure can be successfully applied when the noise power spectrum of the detector and the signal transfer function can be separated into independent lorentzian curves which coincide with the normal modes of the mechanical and electrical oscillators. One can tune the

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lock-in amplifiers at each mode frequency extracting the two components of the output with respect an external sinusoidal reference. The filtering is then performed on the sampled “in phase” and “in quadrature” channels of the lock-ins by means of numerical algorithms including the Zero-th and First Order Prediction algorithms (ZOP and FOP) and the Wiener-Kolmogoroff optimum filter.<sup>7</sup> The data analysis procedure outlined above becomes somewhat inadequate when the normal modes (electrical or mechanical) are tightly tuned and the wideband noise level of the read-out electronics (a d.c. SQUID amplifier) is very low. We have found that this is indeed the case for the optimized configuration of the design parameters for the AURIGA detector that should be available in the near future.<sup>8</sup> Moreover, the predicted post-detection bandwidth for the AURIGA detector will be wide enough to allow for the recognition of some structures of the gravitational wave pulse, relaxing the assumption of a  $\delta$ -like signal. To this aim we have developed a more general approach to data filtering than the usual lock-in analysis.<sup>9</sup> However, digital lock-ins have been also implemented to be used as diagnosis tools, to determine the energy of each mode or to monitor a calibration sinusoidal signal.

### 3.1 Signal filtering

Let us remind some relevant properties of the standard Wiener filtering theory. The best linear estimate of the amplitude  $A$  of a given signal  $Af(t - t_a)$  (with  $\max\{f(t)\} = 1$  and maximum occurring at  $t_a$ ) buried into an additive, zero mean, stationary, gaussian noise  $\eta$  can be obtained by linearly correlating the detector output  $Y(\omega) = F(\omega)H(\omega) + \eta(\omega)$  with the matched Wiener filter

$$M(\omega) = \sigma_A^2 \frac{H^*(\omega)F^*(\omega)}{S(\omega)} e^{-i\omega t_a} , \quad (1)$$

where  $H(\omega)$  is the signal transfer function,  $S(\omega)\delta(\omega - \omega') \equiv \langle \eta(\omega)\eta(\omega') \rangle$  is the noise power spectrum and  $\sigma_A^{-2} \equiv \int_{-\infty}^{+\infty} d\omega |H(\omega)F(\omega)|^2 / S(\omega)$  is the variance of the noise after the filtering. It can be shown that the noise power spectrum of a minimum-phase system can be factorized as  $S(\omega) = L(\omega)L^*(\omega)$ , where  $L(\omega)$  and  $1/L(\omega)$  are called respectively “innovation” and “whitening” filters.<sup>10</sup> Within this assumption we can separated the filter in eq. (1) into a three steps procedure: i) a whitening filter  $M^C(\omega) \equiv 1/L(\omega)$ , ii) a pattern matching to a  $\delta$ -function filter  $M^A(\omega) \equiv H^*(\omega)/L^*(\omega)$  which correspond respectively to the *Casual* and *Anticasual* parts of the Wiener filter matched to a  $\delta$  function, and iii) a pattern matching to the waveform  $F^*(\omega)$  filter. Notice that the last step may be substituted by the Karhunen-Loève expansion (see §3.4) if the waveform  $F(\omega)$  is not known. If we restrict ourselves to a  $\approx 100$  Hz

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bandwidth centered around the mode frequencies, resonant detectors are well described by linear, constant coefficients, coupled differential equations<sup>8</sup> so that both  $H(\omega)$  and  $S(\omega)$  can reasonably well approximated by real coefficient polynomials

$$H(\omega) = \prod_{k=1}^{N_P} \frac{-i\omega}{(p_k - i\omega)(p_k^* - i\omega)}$$

$$S(\omega) = S_0 \prod_{k=1}^{N_P} \frac{(q_k + i\omega)(q_k - i\omega)(q_k^* + i\omega)(q_k^* - i\omega)}{(p_k + i\omega)(p_k - i\omega)(p_k^* + i\omega)(p_k^* - i\omega)}, \quad (2)$$

where  $S_0$  is a constant representing the wideband noise level,  $N_P$  is the number of normal modes of the system ( $N_P=2$  for the present AURIGA electromechanical transducing scheme),  $\text{Im}\{p_k\}$  and  $\text{Re}\{p_k\}/\pi$  are the resonance frequency  $\omega_k$  and bandwidth  $\Delta_k$  of each mode; moreover, as we can see below,  $\text{Im}\{q_k\}$  and  $\text{Re}\{q_k\}/\pi$  represent the resonance frequency  $w_k$  and the optimal bandwidth  $\Delta_k^{opt}$  of the Wiener filter matched to a  $\delta$  signal. Eqs. (1) and (2) show that the Wiener  $\delta$ -filter can be build by recursively applying to the data a cascade chain of  $N_P$  causal and  $N_P$  anticausal filters  $M_k^C(\omega) \equiv (p_k - i\omega)(p_k^* - i\omega)/(q_k - i\omega)(q_k^* - i\omega)$  and  $M_k^A(\omega) \equiv (i\omega)/(q_k + i\omega)(q_k^* + i\omega)$ , where  $k = 1 \dots N_P$ . Each filter can be easily translated into the discrete time domain by mapping its poles and zeroes into the complex  $z$ -plane, where  $z = e^{i\omega T}$  and  $T$  is the sampling time. The digital form of these filters has been then recast into autoregressive, moving average (ARMA) filters

$$y(n) = Q_1 y(n-1) + Q_2 y(n-2) + x(n) + P_1 x(n-1) + P_2 x(n-2) \quad (\text{Casual})$$

$$y(n) = Q_1 y(n+1) + Q_2 y(n+2) + x(n) - x(n+1) \quad (\text{Anticausal}), \quad (3)$$

where  $x(n)$  and  $y(n)$  are respectively the input and the output samples of the  $k$ -th filter,  $Q_1 \equiv -2e^{-T\Delta_k^{opt}} \cos(\omega_k T)$ ,  $Q_2 \equiv e^{-2T\Delta_k^{opt}}$ ,  $P_1 \equiv -2e^{-T\Delta_k} \cos(w_k T)$  and  $P_2 \equiv e^{-2T\Delta_k}$  are constants. The output of the last filter is then normalized by means of the constant  $\sigma_A^2 = 4S_0[\sum_{k=1}^{N_P} \text{Im}\{C_k\}]^{-1}$ , where  $C_k \equiv q_k^{2N_P-1} / [\text{Re}\{q_k\}\text{Im}\{q_k\} \prod_{l \neq k}^{N_P} (q_l + q_k)(q_l^* - q_k)(q_l^* + q_k)(q_l - q_k)]^{-1}$ . We remark that the two digital filters in eq. (3) require 7 additions and 9 multiplications per data sample so that the  $\delta$  filter of the AURIGA output requires 14 additions and  $18 + 1$  multiplications per each data sample; these figures have to be compared with  $\sim 80$  additions and  $\sim 60$  multiplications required by the FFT algorithm. It is worth to notice that the products  $C_k$  can be approximated to 1 when we have well separated post-detection bandwidths centered around the normal mode frequencies; within this approximation we recover, for each



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mode, the standard result of a single mode detector i.e.  $\sigma_{A_k}^2 = 4S_0\Delta_k^{opt}$ , where  $\sigma_{A_k}$  is the contribution to the noise of the  $k$ -th mode and  $\sigma_A^2 = \sum_{k=1}^{N_P} \sigma_{A_k}^2$ . We can also easily calculate the gain  $G_k$  of the Wiener  $\delta$ -filter in respect to the direct (lock-in) one; in fact, the variance of the lock-in filter is just the narrow band noise  $\sigma_{L_k}^2 = S_0(\Delta_k^{opt})^2/\Delta_k$  (see eq. (2)) and therefore  $G_k \equiv \sigma_{A_k}/\sigma_{L_k} = (\Delta_k^{opt}/2\Delta_k)^{1/2} = \Gamma_k^{-1/2}/2$ , where  $h_k = 2\text{Im}\{\}$   $\Gamma_k$ , defined as in ref. [7], is the ratio of the wideband noise  $S_0$  and the narrow band noise  $S(|p_k|) = S_0(\Delta_k^{opt}/\Delta_k)^2$ .

### 3.2 Adaptive procedure

Unfortunately, the parameters  $p_k = i\omega_k + \Delta_k$  and  $q_k = iw_k + \Delta_k^{opt}$  entering in eq. (2) are subject to slow time variations mainly caused by a drift of the mode frequency (variations of the bias electrical field in the transducer) or by a slight non stationarity of the detector noise. This requires some continuous, though slow, retuning of the filter; the typical time scale for the AURIGA detector is few hours. We studied, with the help of Monte Carlo and analytical calculations, how sensitive is the Wiener  $\delta$ -filter on the tuning of  $q_k$  and  $p_k$  in order to give its tolerance to the change of the AURIGA physical parameters. The real physical situation can be described by a given Wiener  $\delta$ -filter  $M(\omega; p_k, q_k)$  which is not perfectly matched to the signal transfer function  $H(\omega; p'_k)$  and detector noise  $S(\omega; p'_k, q'_k)$ , where  $p'_k$  and  $q'_k$  indicates the “true” detector parameters which differ slightly from the  $p_k$  and  $q_k$  used to build up the filter. In the range of parameter variations we are interested in, we can expand  $A$  and  $\sigma_A$  in power series of  $\delta p_k \equiv p'_k - p_k$  and  $\delta q_k \equiv q'_k - q_k$  up to the second order and the SNR variation can be expressed by

$$\frac{\delta(SNR^2)}{SNR^2} = - \sum_{k=1}^{N_P} \frac{1}{2} \frac{|\delta q_k|^2}{(\Delta_k^{opt})^2} + \Gamma_k^{1/2} \frac{|\delta p_k|^2}{(\Delta_k)^2}. \quad (4)$$

$\delta SNR$  depends weakly on the shifts of frequency and bandwidth of the normal modes as the  $\Gamma_k$  constants are very small while the variation of the post detection parameters  $\delta q_k$  sensibly affect the  $SNR$ . These results have been verified using the AURIGA data and we have seen that to achieve a 10 % of maximum variation of  $SNR$  it is required  $|\delta q_k| \leq 0.6\Delta_k^{opt}$ , a condition that has been fulfilled by our data analysis algorithms. It is worth pointing out that also the arrival time of an event is affected by the filter detuning but the corresponding error  $\delta t_a \simeq \sum_{k=1}^{N_P} \delta w_k / (\Delta_k^{opt} w_k)$  is well below 1  $\mu sec$  for normal operating conditions of the AURIGA detector.

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### 3.3 $\chi^2$ test, timing and event search algorithm

Let us assume that a g.w. signal is a broad band burst whose spectral density has no structure into the whole bandwidth of the detector. The only way to detect such a class of impulsive signals (SN explosions, to give an example) is to see if the output of the Wiener  $\delta$ -filter crosses a predetermined threshold (fixed by the detector noise statistics), if there is no local veto and if there is a time coincidence with other g.w. detectors. To this aim, we have implemented algorithms that i) search for threshold crossing which are statistically independent, ii) gives the arrival time of an event and iii) calculate a veto that discriminates a  $\delta$ -like excitation of the bar against other spurious signals (e.g. from the readout electronics). The theoretical framework of the arrival time estimate and  $\chi^2$  test has been reported elsewhere;<sup>2, 11, 12, 13</sup> here we report some results and discuss how the test has been implemented in the AURIGA online data analysis.

– Events are local maxima of the filtered data representing distinct  $\delta$ -like excitations. To distinguish these maxima among the noise peaks we use some properties of the output of the  $\delta$  Wiener filter  $o(t) = As(t - t_a) + \eta_w(t)$ ; here  $\eta_w$  is a stochastic process with zero mean and with correlation  $\langle \eta_w(t)\eta_w(t') \rangle \equiv \sigma_A^2 s(t - t')$  and

$$s(t) = \text{Re} \left\{ \sum_{k=1}^{N_P} C_k q_k e^{q_k |t|} \right\} / \text{Re} \left\{ \sum_{k=1}^{N_P} C_k q_k \right\}. \quad (5)$$

From the above equation we can deduce that  $s(t)$  is a superposition of  $N_P$  exponentially damped oscillating functions with close by frequencies. As a consequence  $s(t)$  is again an oscillating function with a carrier frequency  $w_0$ , close to the bar resonance frequency, with a amplitude modulation frequency and decaying time of the order of  $w_* = \max\{|w_k - w_l|\}$  and  $\tau_* = \max(1/\Delta_k^{opt})$ . These properties of  $o(t)$  has been used to build an algorithm which first looks for the maxima of the envelope within a decaying time of  $s(t)$  and then estimate, by interpolating the signal samples, the time to be associate with this maximum with a precision of 1  $\mu sec$ .

– The “time of arrival”  $t_a$  of a  $\delta$ -like signal is defined as the time of maximum output of the Wiener  $\delta$ -filter. The arrival time detection with resonant detectors can be split in two separate measurements. As the output of the  $\delta$ -filter  $|o(t)|$  is an oscillating function of time, first the phase of the oscillation can be detected by writing  $t_a = t_\phi + nT_0$  where  $T_0 = \pi/w_0$  is the oscillation period and  $n$  is an integer. The error associate with the phase part of timing is given by  $\sigma_\phi = 1/(w_0 SNR)$  and therefore the phase resolution capability of AURIGA is  $\simeq 173/SNR \mu sec$ . Absolute timing implies to identify the

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maximum absolute value of the  $\delta$ -filter output i.e. to determine also the integer  $n$ . We have shown<sup>13</sup> that the error in the  $n$  estimate can be written as  $\sigma_n = w_0/(\pi w_* SNR)$  so that the absolute timing ( $\sigma_n < 1$ ) for the present AURIGA configuration requires  $SNR > 18$ .

– We have shown<sup>11</sup> that in the discrete time domain the reduced  $\chi^2$  of an event can be written as

$$\chi_r^2 = \frac{1}{N-2} \left[ \sum_{i=1}^N \frac{w_i^2}{\sigma_w^2} - \frac{A^2}{\sigma_A^2} \right], \quad (6)$$

where  $w_i$  are the whitened data and  $\sigma_w^2$  the respective variance. Due to electromagnetic interferences or mechanical excesses noises, the whitening filter matches the noise power spectrum of eq. (2) only within a given band centered in the mode frequencies. However, the Wiener  $\delta$ -filter is a band-pass filter and we can achieve the frequency translation of this bandwidth by a simple decimation process. To be more specific, the decimation by a factor 70 of the AURIGA filtered data allows to translate the band  $906.8 \div 941.7 \text{ Hz}$  to the origin, so reducing the sampling rate to  $69.75 \text{ Hz}$ . The whitened data stream  $w_i$  is then calculated by applying the inverse of the filter  $M^C(\omega)$  to the decimate filtered data. It is worth to notice that these data contains all the physical information of any g.w. signal and that to store these data requires  $8.8 \text{ GBytes}$  of mass storage memory per year, greatly simplifying off-line analysis such as all sky pulsar search, stochastic background searches, refiltering of the data, etc.

### 3.4 The Karhunen Loéve expansion

We have developed<sup>14</sup> a method of data filtering which allows an optimal reconstruction of the g.w. signal impinging a resonant detector with the only assumption that the gravitational pulse has a finite duration  $T_s$ . The method consists of the estimate of the amplitude of the Karhunen-Loéve components of the signal at the antenna input. The incoming gravitational signal  $f(t)$  is expanded as

$$f(t) = \sum_{m=1}^{\infty} c_m \varphi_m(t), \quad (7)$$

where  $\varphi_m(t)$  are the eigenfunctions of the autocorrelation of the noise after the Wiener  $\delta$ -filter  $\sigma_A^2 \int_{-T_s/2}^{T_s/2} s(t-t') \varphi_m(t') dt' = \lambda_m \varphi_m(t)$ . The amplitudes  $c_m$  of this expansion are statistically independent and the error on their estimate turns out to be inversely proportional to the square root of the respective

eigenvalues, i.e. the eigenfunctions corresponding to the highest eigenvalues have the smallest errors in their amplitude estimate. In fact, the signal to noise ratio of each amplitude is given by  $SNR_m \equiv c_m/\sigma_m = \lambda_m^{1/2} c_m/\sigma_A$ .<sup>10</sup> A simplified analysis, where the modes are well separated lorentzian curves, shows that for short g.w. pulses most of the information is carried on by the first coefficients which are detected with larger signal to noise ratio. On the contrary the components corresponding to higher eigenfunctions are only excited by signals which exhibit some structures in post detection bandwidth. The Karhunen-Loève expansion gives also an efficient data compression as we can store on a database only the few relevant coefficients of the expansion for subsequent data retrieve and off-line data analysis.

#### 4 Conclusions

The fully numerical data analysis described in this paper, that substitute conventional semi-analogic methods, allows to make a fully optimal filtering of the data, to make an accurate timing of events, to perform a posteriori consistency test and to characterize events with a waveform pattern in the detector bandwidth. The implementation of a complete Data Base for the AURIGA experiment is currently under development with the aim of a real-time and automatic candidate event production. This effort will be of paramount importance for the upcoming global network of bars (AURIGA, NAUTILUS, EXPLORER, ALLEGRO, NIOBE) which will provide the first actual gravitational wave observatory.

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