

Realization of interacting quantum field theories in driven tight-binding lattices

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Abstract. It is theoretically shown that the coherent hopping dynamics of a non-relativistic particle in engineered tight-binding lattices subjected to combined ac and dc driving forces can mimic in Fock space relativistic quantum field theories (QFTs) of strongly interacting fields, enabling access to extreme dynamical regimes beyond weak coupling and perturbative predictions. In particular, the simulation of a QFT model describing a Dirac field strongly coupled to a scalar bosonic field via a Yukawa coupling is proposed, suggesting the possibility of implementing this model using engineered lattices of evanescently coupled optical waveguides with a bent optical axis.

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1. Introduction

The possibility of reproducing complex quantum phenomena using simple and fully controllable engineered quantum systems has received continuous and increasing interest in recent years [1, 2]. Quantum simulations consist in the intentional and artificial reproduction of a quantum dynamics, difficult to access experimentally and/or to study theoretically, onto an unnatural quantum system that is more controllable theoretically and experimentally. The objectives of quantum simulations are diverse, ranging from purely aesthetic, to communicating independent fields, to the possibility of calculating and predicting physics that cannot be done with existing computational power. Important models and phenomena of interacting non-relativistic particles of condensed-matter physics, such as the superfluid–Mott insulator quantum phase transition or the Lieb–Liniger and Dicke models, have been successfully reproduced using cold atoms or ions loaded in optical lattices, cavity photons strongly coupled to atomic media or nuclear magnetic resonance processors (see [2–9] and reference therein). Recent works have suggested the exciting possibility of simulating relativistic quantum field theories (QFTs) using cold atoms, trapped ions or solid-state systems [10–14]. One of the main motivations for quantum simulations of QFTs is the possibility of investigating the physics of strongly interacting fields beyond standard QFT perturbative methods, which is generally a challenging problem on a classical computer. Such a possibility would offer a probing tool to open issues of non-perturbative QFTs [13, 15], which is complementary to more traditional approaches such as QFTs on a lattice [16]. The interest in developing quantum analogue simulators is also to provide experimentally accessible test beds to explore extreme dynamical regimes in the matter or ideal models or exotic phenomena which are not currently accessible, regardless of the numerical complexity of the model [1, 2]. Along this line, simple textbook relativistic models and phenomena, such as the Zitterbewegung and Klein tunneling of relativistic particles, have been mimicked in the laboratory [17–21]. Quantum simulation can also provide a platform where we may not only reproduce existing QFT models that are of difficult access, but also create QFT models that might not exist in nature, or QFT models that show unpleasant features, such as the appearance of ghosts, i.e. the negative norm states and non-unitarity of the S -matrix for strong coupling [22]. Such QFT models have recently received renewed interest in the framework of \mathcal{PT} -symmetric extensions of quantum mechanics and QFTs [23]; however, their physical relevance remains controversial.

In this work, we propose a tight-binding lattice realization of a QFT model describing the interaction of scalar bosons with Dirac fermions/antifermions via a Yukawa coupling. The simulator is based on the coherent motion of a single non-relativistic particle hopping on a dc–ac-driven tight-binding lattice, in which the evolution of the interacting fields in Fock space is mapped into the hopping dynamics of the particle on the lattice. A classical implementation of the driven lattice model, based on the transport of classical light in curved optical waveguide arrays, is also proposed.

2. The quantum field theoretical model

The QFT model that we aim to simulate is a simplified version of the Yukawa QFT model describing the interaction of a Dirac field (fermions) and a scalar Klein–Gordon field (mesons), originally introduced by Yukawa to describe the strong nuclear force [24, 25]. Yukawa coupling of fermions arises in field-theoretic descriptions of few-nucleon systems (see, e.g., [26]) as well

as in the Yukawa sector of the standard model (see, e.g., [27]). In this model, the fermions are nucleons which interact with a scalar or pseudoscalar meson field (pions). The Yukawa interaction of nuclei via the exchange of massive particles is the analogue of the quantum electrodynamics (QED) interaction, except that the particles mediating the nuclear force have to be massive in order to have a finite range interaction. In the 1 + 1 dimension, a Dirac (fermionic) field $\hat{\Psi}(x)$ coupled to a scalar bosonic (Klein–Gordon) field $\hat{\phi}(x)$ via a Yukawa coupling is described by the Hamiltonian $\hat{H} = \int dx \hat{\mathcal{H}}(x)$ with density [25, 28]

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_D + \hat{\mathcal{H}}_{KG} + \hat{\mathcal{H}}_Y, \quad (1)$$

where

$$\hat{\mathcal{H}}_D = \hat{\Psi}^\dagger (-i\sigma_x \partial_x + \sigma_z m) \hat{\Psi}, \quad \hat{\mathcal{H}}_{KG} = \frac{1}{2} \hat{\pi}^2 + \frac{1}{2} \left(\frac{d\hat{\phi}}{dx} \right)^2 + \mu^2 \hat{\phi}^2 \quad (2)$$

are the Hamiltonian densities of the free Dirac and Klein–Gordon fields, respectively, and

$$\hat{H}_Y = g \hat{\Psi} \hat{\Psi} \phi \quad (3)$$

is the Yukawa coupling. In the previous equations, m and μ are the masses of the Dirac and Klein–Gordon particles, respectively, σ_x and σ_z are the usual Pauli matrices, $\hat{\bar{\Psi}} = \hat{\Psi}^\dagger \gamma^0$ is the adjoint spinor of $\hat{\Psi}$, g is the coupling constant, and we assumed $\hbar = c = 1$. The field operators satisfy the bosonic (fermionic) commutation (anti-commutation) relations $[\hat{\phi}(x), \hat{\pi}(x')] = i\delta(x - x')$, $\{\hat{\Psi}(x), \hat{\Psi}^\dagger(x')\} = \delta(x - x')$, $[\hat{\phi}(x), \hat{\phi}(x')] = [\hat{\pi}(x), \hat{\pi}(x')] = 0$ and $\{\hat{\Psi}(x), \hat{\Psi}(x')\} = \{\hat{\Psi}^\dagger(x), \hat{\Psi}^\dagger(x')\} = 0$. The quantum fields can be expanded on the plane-wave basis of a one-dimensional (1D) box of size L .

The simplest case that we consider here is that of one fermion of momentum p and one antifermion of opposite momentum $-p$, which are coupled to bosons at rest, i.e. with momentum $q = 0$. Such an assumption greatly restricts the Hilbert space of the QFT model. However, as in [14], our proposed quantum simulation of finite-number interacting quantized modes includes *all terms* in a finite-mode Dyson expansion, instead of the standard approach that takes into account all modes *but* with a reduced number of perturbative Feynman diagrams [14]. After setting $\hat{\Psi}(x) = \sqrt{m/(L\omega_p)} [u(p)\hat{b} + v(-p)\hat{d}^\dagger] \exp(-ipx)$ and $\hat{\phi}(x) = \sqrt{1/(2\Omega_p L)} (\hat{a} + \hat{a}^\dagger)$, the Hamiltonian that we wish to simulate then reads

$$\hat{H} = \omega_p (\hat{b}^\dagger \hat{b} + \hat{d}^\dagger \hat{d}) + \Omega_p \hat{a}^\dagger \hat{a} + \left[g_1 (\hat{b}^\dagger \hat{b} - \hat{d} \hat{d}^\dagger) + g_2 (\hat{b}^\dagger \hat{d}^\dagger + \hat{b} \hat{d}) \right] (\hat{a} + \hat{a}^\dagger), \quad (4)$$

where we have set $g_1 = g(m/\omega_p) \sqrt{1/(2\Omega_p L)}$, $g_2 = -(p/m)g_1$ and neglected the vacuum energy contributions. In the previous relations, \hat{b}^\dagger , \hat{d}^\dagger and \hat{a}^\dagger are the creation operators of the fermionic, antifermionic and bosonic particles, respectively, $\omega_p = (m^2 + p^2)^{1/2}$ and $\Omega_p = (\mu^2 + q^2)^{1/2} = \mu$ are the energies of the Dirac and Klein–Gordon particles, respectively, and $u(p)$, $v(-p)$ are the spinor components of the free fermionic and antifermionic particles with momentum p and $-p$, respectively [28]. Note that the Yukawa coupling of the bosonic field with a fermion–antifermion pair at rest ($p = 0$), previously considered in [29], is obtained from equation (4) by assuming $g_2 = 0$. This limiting case is exactly integrable and was introduced as a simple solvable model in non-perturbative QFTs [29]; it also describes other physical systems, such as the atomic limit of the Hubbard–Holstein model of solid-state physics (which is exactly integrable via a Lang–Firsov transformation [30]) or the deep strong coupling regime of circuit or cavity quantum electrodynamics with degenerate qubit energy levels [31]. System

realizations of the Hamiltonian (4) in the $g_2 = 0$ limiting case were proposed previously in [32, 33], and a recent experimental implementation based on engineered optical waveguide lattices has been reported in [34]. However, the more general case $p \neq 0$, which corresponds to $g_2 \neq 0$, was not considered in such previous works. A reduced Hamiltonian similar to (4) obtained from a QFT model analogous to that discussed in the present work, in which the Yukawa coupling is replaced by a QED interaction, has recently been derived in [14]; however, in that model the coupling g is time-varying, and the proposed physical implementation of the model was based on two trapped ions. Conversely, here we assume a constant coupling g and suggest a completely different physical implementation of the Hamiltonian (4), which is based on the hopping dynamics of a *single* non-relativistic particle moving on a 1D chain driven by combined ac and dc external fields.

The Hilbert space of the Hamiltonian \hat{H} is spanned by the Fock states $|m_1, m_2, n\rangle = (1/\sqrt{n!})b^{\dagger m_1}d^{\dagger m_2}a^{\dagger n}|0\rangle$, with $m_1, m_2 = 0, 1$ and $n = 0, 1, 2, 3, \dots$. Note that $|m_1, m_2, n\rangle$ is the state in which there are m_1 fermions, m_2 antifermions and n bosons and that m_1 and m_2 can assume solely the two values 0 or 1 according to the Pauli exclusion principle. If the state vector of the system $|\psi(t)\rangle$ is expanded on the basis of Fock states according to

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} a_n(t)|0, 0, n\rangle + b_n(t)|1, 1, n\rangle + c_n(t)|1, 0, n\rangle + d_n(t)|0, 1, n\rangle, \quad (5)$$

the temporal evolution of the probability amplitudes a_n , b_n , c_n and d_n , as obtained from equations (4) and (5), is governed by the following equations:

$$i\frac{da_n}{dt} = n\Omega_p a_n - g_1 \left(\sqrt{n}a_{n-1} + \sqrt{n+1}a_{n+1} \right) + g_2 \left(\sqrt{n}b_{n-1} + \sqrt{n+1}b_{n+1} \right), \quad (6)$$

$$i\frac{db_n}{dt} = (n\Omega_p + 2\omega_p) b_n + g_1 \left(\sqrt{n}b_{n-1} + \sqrt{n+1}b_{n+1} \right) + g_2 \left(\sqrt{n}a_{n-1} + \sqrt{n+1}a_{n+1} \right), \quad (7)$$

$$i\frac{dc_n}{dt} = (n\Omega_p + \omega_p) c_n, \quad i\frac{dd_n}{dt} = (n\Omega_p + \omega_p) d_n. \quad (8)$$

Note that the Yukawa interaction introduces an effective coupling of states $|0, 0, n\rangle$ and $|1, 1, n\rangle$ solely, whereas the states with either one fermion or one antifermion are not affected by the interaction. Hence we will focus our attention on the dynamical coupling among the states $|0, 0, n\rangle$ and $|1, 1, n\rangle$.

The interaction dynamics in the weak coupling limit $g \rightarrow 0$ is rather simple. In this case, the Yukawa interaction can induce transitions solely between the states $|0, 0, n\rangle$ and $|1, 1, n-1\rangle$ for some non-vanishing integer n , provided that $g_2 \neq 0$ and the energy conservation condition $\Omega_p = 2\omega_p$ is satisfied. Such transitions correspond to the annihilation (creation) of one fermion–antifermion pair, with simultaneous creation (annihilation) of one boson. For $g \rightarrow 0$ but in the non-perturbative regime, the occupation amplitudes $a_n(t)$ and $b_{n-1}(t)$ of states $|0, 0, n\rangle$ and $|1, 1, n-1\rangle$ periodically oscillate in time at the Rabi frequency $g_2\sqrt{n}$. In the deep strong coupling regime, i.e. when g_1, g_2 are of the same order as the frequencies ω_p and Ω_p , the dynamics becomes less intuitive. The case $g_2 = 0$, corresponding to the interaction of the bosonic field with a fermion–antifermion pair at rest (i.e. $p = 0$), decouples the dynamics of equations (6) and (7), and describes the process of creation and annihilation of bosons via the self-interaction terms $\hat{b}^\dagger\hat{b}\hat{a}$, $\hat{b}^\dagger\hat{b}\hat{a}^\dagger$, $\hat{d}\hat{d}^\dagger\hat{a}$ and $\hat{d}\hat{d}^\dagger\hat{a}^\dagger$. The $g_2 = 0$ limit is an exactly solvable model of non-perturbative QFT [29]: in this case the spectrum of \hat{H} is composed by a ladder

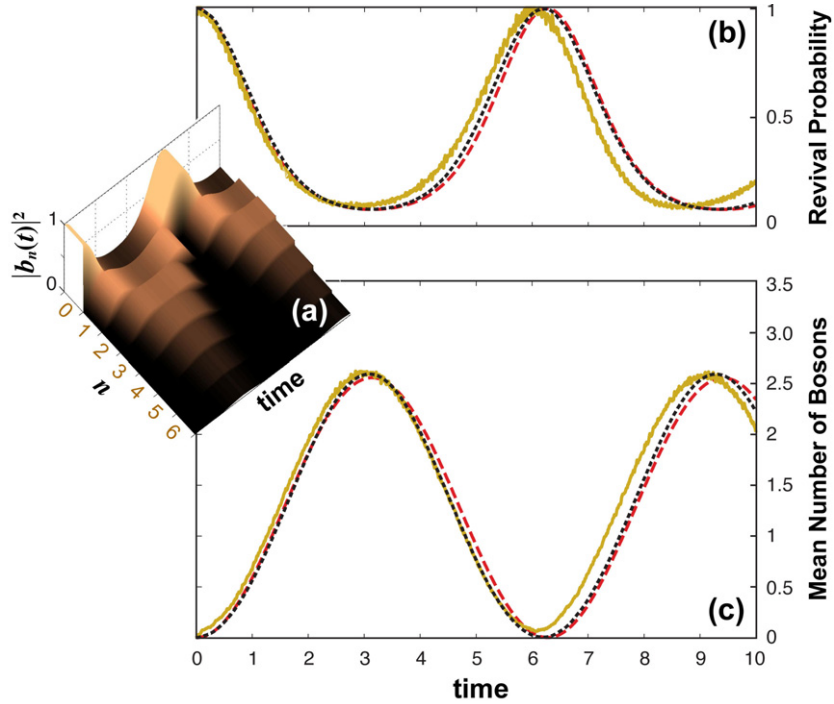


Figure 1. Evolution of (a) Fock state occupation probabilities $|b_n(t)|^2$, (b) revival probability and (c) the mean number of bosons (dashed curves) for the QFT model (4) for parameter values $g_1 = 0.8$, $g_2 = 0$, $\omega_p = 0.3$ and $\Omega_p = 1$. The system is initially in the state $|1, 1, 0\rangle$. The dotted curves in (b) and (c) refer to the results obtained by the ac–dc-driven tight-binding lattice model (equations (9) and (10)) for parameter values $\rho = 10.3$, $\kappa = 1.6$, $aF_1 = 1$, $aF_2 = 0$ and $\omega = 20$. The solid curves in (b) and (c) refer to the results obtained for the optical waveguide simulator (equation (12)).

of equally spaced energy levels, with energy spacing Ω_p , and the dynamics of occupation amplitudes a_n (or b_n) in Fock space is thus exactly periodic at frequency Ω_p . As an example, figure 1 shows the evolution of the initial state $|\psi(t=0)\rangle = |1, 1, 0\rangle$ for $g_1 = 0.8$, $g_2 = 0$, $\omega_p = 0.3$ and $\Omega_p = 1$. In the figure, the behavior of the revival probability $P_r(t) = |\langle\psi(0)|\psi(t)\rangle|^2$ and the mean number of bosons $\langle n(t)\rangle = \sum_n n|b_n(t)|^2$ are depicted, together with a snapshot of the evolution of occupation probabilities $|b_n(t)|^2$. Note the characteristic periodic bouncing dynamics of bosons in Fock space with period $2\pi/\Omega_p$. For a non-vanishing coupling g_2 , i.e. for bosons interacting with a moving fermion–antifermion pair, annihilation or creation of fermions/antifermions is allowed, and the periodic dynamics in Fock space breaks down. However, for values of g_2 comparable to or smaller than g_1 , the bouncing dynamics can be still observed. This is shown, as an example, in figure 2, in which the signature of the non-vanishing g_2 interaction term is clearly visible (compare figures 1 and 2). Note that in this case the mean number of bosons is given by $\langle n(t)\rangle = \sum_n n[|b_n(t)|^2 + |a_n(t)|^2]$.

3. Non-relativistic tight-binding lattice realization

The main result of this work is that the QFT Hamiltonian (4) can be simulated *in Fock space* by considering the coherent hopping motion of a non-relativistic quantum particle in a

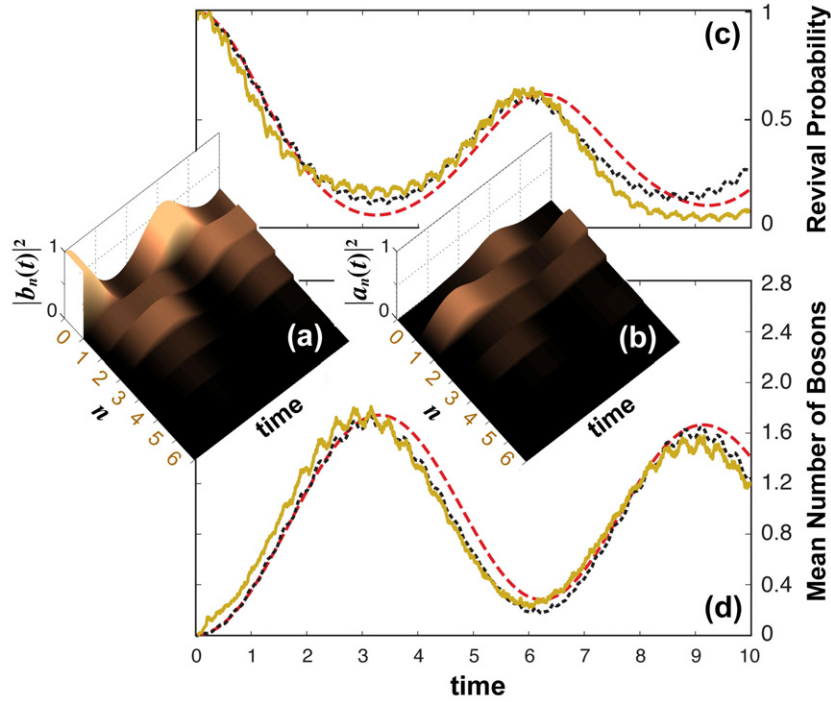


Figure 2. Evolution of Fock state occupation probabilities (a) $|b_n(t)|^2$, (b) $|a_n(t)|^2$ and (c) revival probability and (d) the mean number of bosons (dashed curves) for the QFT model (4) for parameter values $g_1 = 0.4095$, $g_2 = 0.4463$, $\omega_p = 0.3$ and $\Omega_p = 1$. The system is initially in the state $|1, 1, 0\rangle$. The dotted curves in (c) and (d) refer to the results obtained by the tight-binding lattice model (equations (9) and (10)) for parameter values $\rho = 10.3$, $\kappa = 1.6$, $aF_1 = 1$, $aF_2 = 30$ and $\omega = 20$. The solid curves in (c) and (d) refer to the results obtained for the optical waveguide simulator (equation (12)). In this case the physical units of couplings and frequencies are cm^{-1} , whereas the time variable on the horizontal axis is replaced by the spatial propagation length z (in units of cm).

tight-binding chain of ‘molecules’, driven by combined ac and dc forces. In our quantum simulator, the evolution of the amplitudes a_n and b_n of the Fock states $|1, 1, n\rangle$ and $|0, 0, n\rangle$ for the quantum field (the former corresponding to the state with one fermion of momentum p , one antifermion of momentum $-p$ and n bosons at rest and the latter corresponding to the state with n bosons at rest solely) are mapped onto the occupation amplitudes of the non-relativistic particle at the molecular sites of the lattice. The hopping rates are engineered to precisely mimic *all* kinds of interactions of the Yukawa coupling, i.e. boson creation and annihilation induced by self-interaction terms (g_1 terms in equation (4)) and fermion–antifermion pair creation or annihilation with simultaneous creation or annihilation of one boson (g_2 terms in equation (4)). Hopping engineering is attained by the introduction of a suitable combination of ac and dc forces. Driven tight-binding lattice models have been investigated for a long time, mainly in connection with phenomena such as Bloch oscillations, dynamic localization and quasi-energy band engineering (see, e.g., [35, 36]). Here we show that driven lattice models can mimic QFTs in Fock space of strongly interacting fields, a possibility that was not explored in previous works.

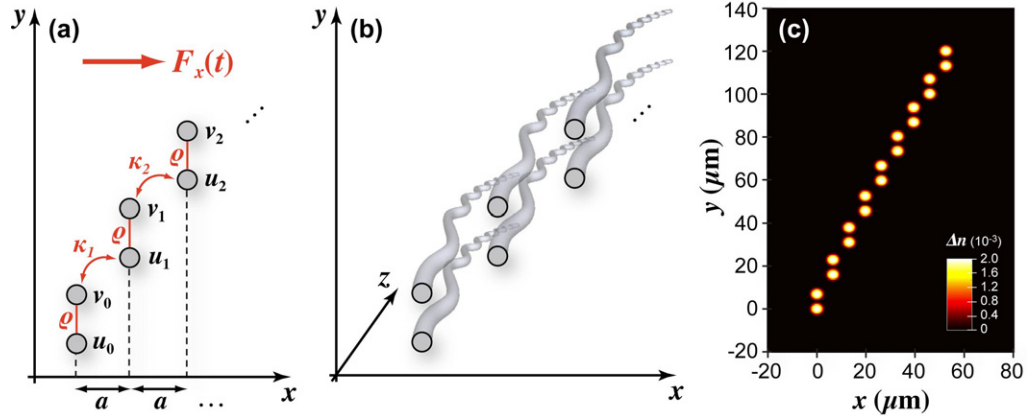


Figure 3. (a) Schematic diagram of a driven tight-binding lattice composed of a chain of coupled molecules and (b) the corresponding physical implementation based on the transport of classical light in an array of optical waveguides with a curved z -axis. The refractive index map (snapshot of $n(x, y) - n_s$) of the waveguide array used in the numerical simulations is shown in (c).

We note that as our quantum simulator mimics the QFT model in *Fock space* (not in real space) and deals with a *single* particle hopping on a lattice, it can be realized with any kind of particle, i.e. particle statistics is not important. As discussed in the next section, the single-particle-driven tight-binding lattice model can be even implemented using a *classical* (rather than a quantum) simulator based on the transport of *classical light waves* in engineered waveguide lattices.

To mimic the QFT model (4) in Fock space, we consider a tight-binding lattice composed of a semi-infinite chain of weakly coupled ‘molecules’, with non-homogeneous coupling rates $\kappa_n = \kappa\sqrt{n}$ ($n = 0, 1, 2, \dots$), in the geometrical setting shown in figure 3(a). Each molecule is realized by the strong coupling of two Wannier states (‘atoms’), with a hopping rate ρ , and thus sustains two bound states (the symmetric and antisymmetric molecular states) with energy separation $\simeq 2\rho$. The combined external driving forces are used to suitably control the hopping dynamics along the chain [37], realizing an effective lattice model that mimics our QFT model in Fock space. The tight-binding chain can be implemented using different physical (either quantum or classical) systems (see, e.g., [37]); in the next section we will discuss in detail a possible physical realization of the lattice model based on the transport of classical light in an array of optical waveguides. The non-homogeneous couplings κ_n are realized by suitable distance control of the molecules, keeping the horizontal spacing uniform and equal to a . Denoting by u_n and v_n the occupation amplitudes of the two ‘atoms’ of the n th molecule in the chain, in the nearest-neighbor tight-binding approximation their temporal evolution is governed by the coupled equations

$$i\frac{du_n}{dt} = -\rho v_n - \kappa_n v_{n-1} + naF_x(t)u_n, \quad (9)$$

$$i\frac{dv_n}{dt} = -\rho u_n - \kappa_{n+1}u_{n+1} + naF_x(t)v_n, \quad (10)$$

where $F_x(t) = F_1 + F_2 \cos(\omega t)$ is the external dc-ac force acting along the x -direction, as shown in figure 3(a). The idea is to transform the time-periodic system of equations (9) and (10) into an effective time-independent lattice system [37, 38], which reproduces our QFT model. With this aim, we consider the case of a strong and high-frequency ac force and perform a multiple scale asymptotic analysis of equations (9) and (10) [37, 38], assuming the scaling $\kappa/\rho \sim \epsilon$, $aF_1/\rho \sim \epsilon$, $\omega/\rho \sim 1$ and $aF_2/\omega \sim 1$, where ϵ is a smallness parameter. After the introduction of the occupation amplitudes of symmetric and antisymmetric modes of each molecule, $a_n = [(u_n + v_n)/\sqrt{2}] \exp[i\theta_1(t)]$, $b_n = [(u_n - v_n)/\sqrt{2}] \exp[i\theta_2(t)]$, where $\theta_1(t) = -\rho t + n(aF_2/\omega)\sin(\omega t)$, $\theta_2(t) = (\rho - 2\omega_p)t + n(aF_2/\omega)\sin(\omega t)$ and $\omega_p/\rho \sim \epsilon$, assuming the resonance condition $\omega = 2(\rho - \omega_p) \simeq 2\rho$, at leading order in the perturbative analysis one obtains that the slowly varying amplitudes a_n and b_n evolve according to equations (6) and (7), with couplings g_1 , g_2 and boson frequency Ω_p given by

$$g_1 = \frac{\kappa}{2} J_0 \left(\frac{aF_2}{\omega} \right), \quad g_2 = \frac{\kappa}{2} J_1 \left(\frac{aF_2}{\omega} \right), \quad \Omega_p = aF_1. \quad (11)$$

Hence, by tuning the ratio $\Gamma = (aF_2/\omega)$ of ac driving, the detuning $\omega_p = \rho - \omega/2$ and the amplitude aF_1 of the dc field, one can simulate the various coupling regimes of the QFT model (4). In particular, the dynamics of the QFT model for $g_2 = 0$ (figure 1) can be mimicked by considering a dc force solely. For example, the dotted curves in figure 1 show the behavior of revival probability and the mean number of bosons as obtained by numerical simulations of equations (9) and (10) for a relatively large detuning ($\rho/\kappa \simeq 6.44$). Note that the behavior of both revival probability and the mean number of bosons as obtained by the driven lattice model (equations (9) and (10)), dotted curves in figures 1(b) and (c)) reproduces well those calculated by the averaged lattice model (equations (6) and (7), dashed curves in figures 1(b) and (c)). To simulate the QFT model with non-vanishing momentum p , i.e. for $g_2 \neq 0$, the ac force is switched on. The dotted curves in figure 2 show the behavior of revival probability and the mean number of bosons as obtained by numerical simulations of equations (9) and (10) for the non-vanishing g_2 case. In this case the modulation frequency ω of the ac force has been chosen to satisfy the resonance condition $\omega = 2(\rho - \omega_p) \simeq 2\rho$, whereas its amplitude F_2 to give the target values of g_1 and g_2 according to equation (11).

4. Photonic simulator

The driven tight-binding model described by equations (9) and (10) can be implemented in principle using different physical (either quantum or classical) systems (see, e.g., [37]). In this section, we discuss in some detail an experimentally accessible realization of the driven lattice model, which is based on the propagation of classical light waves in an array of evanescently coupled optical waveguides with a bent axis. The recently developed direct waveguide laser writing technology, in which a femtosecond (fs) laser beam is used to create a guiding core inside a transparent glass (such as fused silica), has attained nowadays an excellent degree of maturity, enabling us to manufacture with great accuracy and reproducibility 3D low-loss waveguide circuits in rather arbitrary and complex geometrical settings (see, e.g., [39–41] and references therein). Indeed, several recent works have experimentally demonstrated that simulation of the coherent motion of a single non-relativistic quantum particle on a lattice driven by external fields is feasible using fs-laser-written waveguide arrays, in which the driving forces are effectively mimicked by a suitable geometric bending of the waveguide

axis [41–44]. Direct fs laser writing enables us to engineer coupling rates with great accuracy. In particular, optical waveguide arrays with non-uniform coupling rates following the law $\kappa_n = \kappa\sqrt{n}$ have already been realized in [45] to demonstrate classical analogues of the displaced Glauber–Fock states. Similarly, curved waveguide arrays with such non-uniform couplings have been manufactured very recently in [34]. The latter experiment provides the first photonic simulation of the ultrastrong deep coupling regime of the quantum Rabi model, which realizes our QFT Hamiltonian (4) as well in the special case $g_2 = 0$. Such recent experimental results indicate that fs-laser-written waveguide arrays should provide a feasible experimental platform to simulate the QFT model (4) in the more general case $g_2 \neq 0$. For non-vanishing values of g_1 and g_2 , an optical realization of the lattice model of figure 3(a) is provided by light transport in a chain of equal and weakly coupled optical waveguide couplers (the photonic analogue of the ‘molecules’ of figure 3(a)) with a bent optical axis and engineered spacing, as schematically shown in figure 3(b). The refractive index profile of the optical structure can be written as $n(x, y) = n_s + \sum_n \Delta n h(x - x_n, y - y_n)$, where n_s is the refractive index profile of the substrate dielectric medium, Δn and $h(x, y)$ are the index change and normalized profile of the guiding core, respectively, and (x_n, y_n) denote the positions of the various guides in the (x, y) -plane. In the waveguide reference frame, the propagation of a light beam at wavelength λ is described by the paraxial optical wave equation for the scalar field envelope $\psi(x, y, z)$ [43]

$$i \frac{\partial \psi}{\partial z} = -\frac{\lambda}{4\pi n_s^2} \nabla_{x,y}^2 \psi + \frac{2\pi}{\lambda} [n_s - n(x, y) + n_s \ddot{x}_0(z)x] \psi, \quad (12)$$

where z is the paraxial propagation distance, $\nabla_{x,y}^2$ is the transverse Laplacian and $x_0(z)$ is the bending profile of the waveguide axis. The coupled-mode equations (9) and (10), describing the propagation of the modal amplitudes u_n and v_n of light trapped in the various waveguides of the chain shown in figure 3(b), can be obtained from the paraxial wave equation (12) by standard variational techniques (see, e.g., [46]). Note that in our photonic simulator the time variable t of the original QFT model is mapped into the spatial propagation distance z along the axis of the waveguides, as shown in figure 3(b). The non-homogeneous coupling rates κ_n among nearest waveguides can be realized by appropriate control of waveguide distances [34, 45, 47, 48].

We have simulated light propagation in the optical waveguide array by assuming circular guiding cores of super-Gaussian shape as in [47], with a uniform horizontal spacing $a = 6.5 \mu\text{m}$; an index change $\Delta n = 0.002$, a wavelength $\lambda = 633 \text{ nm}$ and a substrate index $n_s = 1.45$ have been assumed, which typically apply to fs-laser-written waveguides in fused silica probed in the red [41, 45, 48]. The refractive index profile of the optical structure that reproduces the parameter values of simulations shown figures 1 and 2 is depicted in figure 3(c). The waveguide axis is bent in the (x, z) plane with a bending profile $x_0(z) = (1/2R)z^2 - A\cos(\omega z)$, as schematically shown in figure 3(b). The driven parameters aF_1 and aF_2 of the lattice model (equations (9) and (10)) are related to the waveguide bending parameters by the simple relations [43]

$$aF_1 = \frac{2\pi n_s a}{R\lambda}, \quad aF_2 = \frac{2\pi \omega^2 n_s A a}{\lambda}. \quad (13)$$

Excitation of the boundary waveguide coupler (at the bottom left of figure 3(b)) in its antisymmetric (odd) supermode at the input ($z = 0$) plane is accomplished to simulate the initial state $|1, 1, 0\rangle$ of the quantum system. In an experiment, such a kind of multi-waveguide excitation can be achieved using, for example, a phase grating, as demonstrated in the experiment of [49].

The numerical results obtained by numerical simulations of the paraxial optical wave equation (12), shown by the solid curves in figures 1 and 2, turn out to be in good agreement with the predictions of the driven tight-binding lattice model. The detailed evolution of light intensity distribution along the waveguide structure, from the input ($z = 0$) to the output ($z = 8$ cm) planes at steps of $\Delta z = 0.1$ cm, can be viewed in the supplementary multimedia, available from stacks.iop.org/NJP/14/053026/mmedia². In an optical experiment, the evolution of the mean number of bosons $\langle n(t) \rangle$, shown in figures 1(c) and 2(d), can be retrieved after mapping the light intensity distribution along the waveguide array. In fact, one has $\langle n(t) \rangle = \sum_n n[|a_n(t)|^2 + |b_n(t)|^2] = \sum_n n[|u_n(t)|^2 + |v_n(t)|^2]$, and $|u_n(t)|^2 + |v_n(t)|^2$ is the fractional light power trapped in the n th coupler of the array. Similarly, the evolution of the revival probability can be simply retrieved by monitoring the fractional light power that remains trapped in the initially excited coupler at the left boundary of figure 3(b). Light intensity distribution in the various waveguides along the propagation distance z can be obtained by a fluorescence imaging technique, as discussed, e.g., in [34, 41, 45, 48, 49]. In such experiments, light propagation along the array is directly visible due to the fluorescence of color centers created in OH-rich fused silica during the fs laser writing process, and can be accurately mapped on a CCD camera by means of a collecting microscope objective.

5. Conclusions

In this work, we have shown that the coherent hopping dynamics of a non-relativistic particle in ac–dc-driven tight-binding lattices can simulate in Fock space relativistic QFTs of strongly interacting fields, making it possible to mimic in a tabletop experiment extreme dynamical regimes of QFTs. In particular, we have suggested the possibility of simulating a simple QFT model describing the Yukawa coupling of a Dirac field with a scalar bosonic field using engineered lattices of evanescently coupled optical waveguides with a bent optical axis. As the external driving of tight-binding lattices has generally been investigated in connection with coherent control of tunneling, Bloch oscillations, dynamic localization, Anderson localization and related phenomena [35–37], here we have shown that driven tight-binding lattices can provide a test bed to simulate simple QFT models as well. As compared with the quantum simulator based on trapped ions proposed recently in [14], our driven tight-binding lattice simulator is not scalable; however, it may offer a simpler physical implementation (e.g. using a photonic lattice), as recently demonstrated in [34]. In this work, we have focused our attention on a simple version of the Yukawa QFT model; however, our findings could be extended to other QFT models, including QFTs showing ghosts which might not exist in nature and whose physical relevance is still debated (see, e.g., [22, 23, 50]). In this regard, photonic simulations of QFTs in Fock space might provide a rather unique test bed to realize in the laboratory \mathcal{PT} -symmetric QFTs, such as the QFT Lee model in the ghost regime [51].

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² See the supplementary material for movies visualizing light intensity evolution for the simulations of figures 1 and 2.

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