Spectroscopy of the hidden-charm $[qc][\bar{q}\,\bar{c}]$ and $[sc][\bar{s}\,\bar{c}]$ tetraquarks in the relativized diquark model

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We calculate the spectrum of $q\bar{q}c\bar{c}$ and $s\bar{s}c\bar{c}$ tetraquarks, where q, s, and c stand for light (u, d), strange, and charm quarks, respectively, in a relativized diquark model, characterized by one-gluon-exchange plus confining potential. In the diquark model, a $q\bar{q}c\bar{c}$ ($s\bar{s}c\bar{c}$) tetraquark configuration is made up of a heavy-light diquark, qc (sc), and anti-diquark, $\bar{q}\,\bar{c}$ ($\bar{s}\,\bar{c}$). According to our results, 13 charmonium-like observed states can be accommodated in the tetraquark picture, both in the hidden-charm ($q\bar{q}c\bar{c}$) and hidden-charm hidden-strange ($s\bar{s}c\bar{c}$) sectors.

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I. INTRODUCTION

For a few decades after the formulation of the quark model, it was believed that baryons and mesons could be described as the bound states of three valence quarks and a constituent quark-antiquark pair, respectively. The classification of ground-state hadrons could be easily carried out in terms of group theoretical techniques and the quark model formalism, while resonances might be sorted by making use of effective potentials to describe the spatial excitations related to the interquark motion. For example, see Refs. [1–4].

However, more recent data from both e^+e^- and hadron colliders shed light on hadrons which do not fit well into this standard picture. They are the so-called *exotic hadrons*, namely, multiquark states (tetraquarks and pentaquarks) and particles including gluonic degrees of freedom (d.o.f.) (hybrids and glueballs). We are especially interested in tetraquarks, which are mesons containing two valence quarks and two antiquarks. Among tetraquark candidates, we can mention $Z_c(3900)$ [5,6], $Z_c(4020)$ [7,8], $Z_b(10610)$, $Z_b(10650)$ [9], and the well-known $X(3872)$ [10]. The tetraquark nature is still unclear and several different interpretations have been proposed. They include (i) Tightly bound objects, just as in the case of normal hadrons, but with more constituents [11–22], (ii) Hadro-quarkonia (hadro-charmonia) [23–29], (iii) Loosely bound meson-meson molecules similar to the deuteron [30–38], (iv) The result of kinematic or threshold effects caused by virtual particles [39–43], (v) The rescattering effects arising by anomalous triangular singularities [44–46]. More details on the previous interpretations can be found in Refs. [47–52]. Here, we focus on the first one.

Four quark states can, in principle, be bound by one-gluon-exchange (OGE) forces. However, their possible emergence and stability is controversial because of the lack of reliable and univocal experimental data. As a consequence, tetraquark model predictions are strongly model dependent and rely on the choice of a specific Hamiltonian, and also on the parameter fitting procedure. Despite this, the tetraquark hypothesis is worth investigating.

It is worth noting that in the tetraquark hypothesis one obtains a four-quark spectrum that is richer than those generated by molecular models or the inclusion of dynamical or threshold effects in the quark model formalism. In particular, in molecular models one only expects to get bound states in the proximity of meson-meson decay thresholds; radial excitations cannot take place because of the smallness of meson-meson binding energies. On the contrary, if one includes threshold effects in the quark model formalism, one gets radial excitations, but exotic charged states of the type $q\bar{q}Q\bar{Q}$, where Q is a heavy quark, are

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forbidden. A comparison between theoretical predictions for the spectrum and the main decay modes of four-quark states and the existing experimental data may allow us to distinguish between the previous hypotheses. The possible emergence of the fully heavy $QQ\bar{Q}\bar{Q}$ bound states may provide a strong indication in favor of the tetraquark one [22,53–57].

We also point out that the heavy-light tetraquarks in a $QQ\bar{q}\bar{q}$ configuration are also of considerable interest.[1] It would be very interesting to test the possibility of a $QQ\bar{q}\bar{q}$ tetraquark that remains stable against strong decays, but unfortunately there is no experimental evidence yet. Theoretically, $QQ\bar{q}\bar{q}$ was first shown to be stable against strong decays by Lipkin [58] and Ader *et al.* [59] long ago. Very recently, $bb\bar{q}\bar{q}$ was shown to be stable against strong decays but not its charm counterpart $cc\bar{q}\bar{q}$, nor the mixed (beauty + charm) $bc\bar{q}\bar{q}$ state [60,61]. For detailed discussions on the stability of different heavy-light tetraquarks, see Refs. [62,63].

In this paper, we compute the spectrum of $q\bar{q}c\bar{c}$ ($q = u, d$) and $s\bar{s}c\bar{c}$ tetraquarks. The calculations are performed within a relativized diquark-antidiquark model, characterized by one-gluon-exchange potential. The effective d.o.f. of diquark describes two strongly correlated quarks, with no internal spatial excitation. The tetraquark spectrum is obtained in a two-step process. First of all, the diquark masses are obtained by solving the Schrödinger equation with the relativized quark-quark potential [64]. In a second stage, the tetraquark spectrum is calculated by means of the relativized diquark-antidiquark potential [22]. Finally, by comparing our results to the data, we are able to provide some tentative assignments to XYZ-type states, including $X(3872)$, $Z_c(3900)$, $Z_c(4020)$, $Z_c(4240)$, $Z_c(4430)$, $Y(4008)$, $Y(4260)$, $Y(4360)$, $Y(4630)$, and $Y(4660)$ in the $q\bar{q}c\bar{c}$ sector, plus $X(4140)$, $X(4500)$, and $X(4700)$ in the $s\bar{s}c\bar{c}$ sector. The next step of our study of fully [22] and doubly heavy tetraquarks will be an analysis of the ground-state energies and dominant decay modes, including estimates of the total decay widths and production cross sections.

The paper is organized as follows. In Sec. II, we describe our relativized diquark model and the details of the calculation. Section III is devoted to a discussion of our results and a comparison with the experimental data and the previous theoretical studies. Here, we also provide some tentative assignments to the XYZ states and compare them with other theoretical interpretations (if available). Finally, we provide a short summary.

II. RELATIVIZED DIQUARK MODEL

In a diquark-antidiquark model, the effective d.o.f. of the diquark, describing two strongly correlated quarks with no internal spatial excitations, is introduced. Tetraquark mesons are then interpreted as the bound states of a diquark $\mathcal{D}$ and an antidiquark $\bar{\mathcal{D}}$.

The $\mathcal{D}-\bar{\mathcal{D}}$ relative motion is described in terms of a relative coordinate $\mathbf{r}_{\rm rel}$ (with conjugate momentum $\mathbf{q}_{\rm rel}$), thus neglecting the internal diquark (antidiquark) structure. As a result, one turns a four-body problem into a two-body and gets a spectrum that is less rich than that of a four-body system. Something similar also happens in the baryon sector, where the spectrum of a quark-diquark system is characterized by a smaller number of states than that of a three quark one. For example, see Refs. [65–70].

A. Diquark-antidiquark states

The diquark (antidiquark) can be found in two different $SU_c(3)$ color representations, $\bar{\mathbf{3}}_c$ ($\mathbf{3}_c$) and $\mathbf{6}_c$ ($\bar{\mathbf{6}}_c$). As the tetraquark must be a color singlet, there are two possible diquark-antidiquark combinations:

(1) diquark in $\bar{\mathbf{3}}_c$, antidiquark in $\mathbf{3}_c$

(2) diquark in $\mathbf{6}_c$, antidiquark in $\bar{\mathbf{6}}_c$.

Diquarks (antidiquarks) are made up of two fermions so they have to satisfy the Pauli principle, i.e., the diquark (antidiquark) total wave function,

$$\Psi_{\mathcal{D}} = \psi_c \otimes \psi_{\rm sf} \otimes \psi_{\rm sp}, \tag{1}$$

where ψ_c, $\psi_{\rm sf}$, and $\psi_{\rm sp}$ are the color, spin-flavor, and spatial wave functions, must be antisymmetric.

Moreover, if for simplicity we neglect the diquarks' internal spatial excitations, their color-spin-flavor wave functions must be antisymmetric. This limits the possible representations to being only [69,71]

$$\text{color in } \bar{\mathbf{3}}_c; \quad \text{symmetric } \psi_{\rm sf}, \text{ and}, \tag{2a}$$

$$\text{color in } \mathbf{6}_c; \quad \text{antisymmetric } \psi_{\rm sf}. \tag{2b}$$

In the study of $q\bar{q}c\bar{c}$ and $s\bar{s}c\bar{c}$ tetraquarks, we consider diquarks (antidiquarks) of the cq and cs type, where $q = u$, d, with isospin $I_{\mathcal{D}} = \frac{1}{2}$ or 0, respectively. Because of this, for $q\bar{q}c\bar{c}$ states both $I = 0$ and $I = 1$ tetraquark isospin combinations are possible (degeneracy in the isospin basis is the leading feature of diquark models [72]), while in the $s\bar{s}c\bar{c}$ case one necessarily has $I = 0$. We can determine the J^{PC} quantum numbers of the tetraquarks by applying the restrictions for the diquark-antidiquark limit, i.e., $L_{\mathcal{D}} = L_{\bar{\mathcal{D}}} = 0$ and color $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$. This is because we expect that color-sextet diquarks, Eq. (2b), will be higher in energy than color-triplet ones or even that they will not be bound at all [18,71,73]. Thus, we are left with the Eq. (2a) diquark representation which can be further decomposed in terms of the diquark spin and flavor content. As a result, we get a spin-0, flavor-antisymmetric representation, the scalar diquark, and a spin-1, flavor-symmetric representation, the

[1] All the other possible heavy-light tetraquarks can decay strongly by annihilating at least a quark-antiquark pair of the same flavor.

axial-vector diquark. The parity of a tetraquark meson having orbital angular momentum L can be defined as

$$P = (-1)^L. \tag{3}$$

The convenient basis for identifying the charge conjugation (C) quantum numbers are ones in which the quark spins are recoupled into spin of the charm-anticharm $s_{c\bar{c}}$ and spin of the light quark-antiquark $s_{q\bar{q}}$ [74]. The C parity (obviously only for its eigenstates) is then defined as

$$C = (-1)^{L+s_{c\bar{c}}+s_{q\bar{q}}}. \tag{4}$$

For different tetraquark configurations, $s_{c\bar{c}}$ and $s_{q\bar{q}}$ can be worked out [72,74]. For more details on the tetraquark basis, see the Appendix.

B. Relativized model Hamiltonian

We consider the following Hamiltonian:

$$\begin{aligned}\mathcal{H}^{\mathrm{REL}} &= T + V(r_{\mathrm{rel}}) \\ &= \sqrt{q_{\mathrm{rel}}^2 + m_{\mathcal{D}}^2} + \sqrt{q_{\mathrm{rel}}^2 + m_{\bar{\mathcal{D}}}^2} + V(r_{\mathrm{rel}}),\end{aligned} \tag{5}$$

where $\sqrt{q_{\mathrm{rel}}^2 + m_{\mathcal{D},\bar{\mathcal{D}}}^2}$ are the diquark (antidiquark) kinetic energies, with diquark (antidiquark) masses $m_{\mathcal{D}}$ ($m_{\bar{\mathcal{D}}}$), and $V(r_{\mathrm{rel}})$ the OGE plus confining potential. The usual form for $V(r_{\mathrm{rel}})$ is

$$\begin{aligned}V(r_{\mathrm{rel}}) = \Bigg[&\frac{\alpha_s}{r_{\mathrm{rel}}} - \frac{3\beta}{4} r_{\mathrm{rel}} - \frac{8\pi\alpha_s \delta(\mathbf{r}_{\mathrm{rel}})}{3 m_{\mathcal{D}} m_{\bar{\mathcal{D}}}} \mathbf{S}_{\mathcal{D}} \cdot \mathbf{S}_{\bar{\mathcal{D}}} \\ &- \frac{\alpha_s}{m_{\mathcal{D}} m_{\bar{\mathcal{D}}} r_{\mathrm{rel}}^3} \left(\frac{3 \mathbf{S}_{\mathcal{D}} \cdot \mathbf{r}_{\mathrm{rel}} \mathbf{S}_{\bar{\mathcal{D}}} \cdot \mathbf{r}_{\mathrm{rel}}}{r_{\mathrm{rel}}^2} - \mathbf{S}_{\mathcal{D}} \cdot \mathbf{S}_{\bar{\mathcal{D}}} \right) \\ &- \frac{3}{4}\Delta E \Bigg] \frac{\lambda_{\mathcal{D}}^a}{2} \frac{\lambda_{\bar{\mathcal{D}}}^a}{2},\end{aligned} \tag{6}$$

where $\lambda_{\mathcal{D},\bar{\mathcal{D}}}^a$ are Gell-Mann color matrices, ΔE a constant, α_s the strength of the color-Coulomb interaction, and β that of the linear confining potential.

The hyperfine interaction of Eq. (6) is an illegal operator in the Schrödinger equation; moreover, the Coulomb-like potential should be regularized in the origin [13]. To overcome these difficulties, we follow the prescriptions of Refs. [64,65,75] and rewrite Eq. (6) as

$$\begin{aligned}V(r_{\mathrm{rel}}) = &\beta r_{\mathrm{rel}} + G(r_{\mathrm{rel}}) + \frac{2 \mathbf{S}_{\mathcal{D}} \cdot \mathbf{S}_{\bar{\mathcal{D}}}}{3 m_{\mathcal{D}} m_{\bar{\mathcal{D}}}} \nabla^2 G(r_{\mathrm{rel}}) \\ &- \frac{1}{3 m_{\mathcal{D}} m_{\bar{\mathcal{D}}}} (3 \mathbf{S}_{\mathcal{D}} \cdot \hat{r}_{\mathrm{rel}} \mathbf{S}_{\bar{\mathcal{D}}} \cdot \hat{r}_{\mathrm{rel}} - \mathbf{S}_{\mathcal{D}} \cdot \mathbf{S}_{\bar{\mathcal{D}}}) \\ &\times \left(\frac{\partial^2}{\partial r_{\mathrm{rel}}^2} - \frac{1}{r_{\mathrm{rel}}} \frac{\partial}{\partial r_{\mathrm{rel}}} \right) G(r_{\mathrm{rel}}) + \Delta E,\end{aligned} \tag{7a}$$

where the Coulomb-like potential is given by [64,65]

$$G(r_{\mathrm{rel}}) = -\frac{4\alpha_s(r_{\mathrm{rel}})}{3 r_{\mathrm{rel}}} = -\sum_k \frac{4\alpha_k}{3 r_{\mathrm{rel}}} \mathrm{Erf}(\tau_{\mathcal{D}\bar{\mathcal{D}}k} r_{\mathrm{rel}}). \tag{7b}$$

Here, Erf is the error function [76] and $\tau_{\mathcal{D}\bar{\mathcal{D}}k}$ [64,65] is

$$\tau_{\mathcal{D}\bar{\mathcal{D}}k} = \frac{\gamma_k \sigma_{\mathcal{D}\bar{\mathcal{D}}}}{\sqrt{\sigma_{\mathcal{D}\bar{\mathcal{D}}}^2 + \gamma_k^2}}, \tag{7c}$$

with

$$\sigma_{\mathcal{D}\bar{\mathcal{D}}} = \sqrt{\frac{1}{2}\sigma_0^2 \left[1 + \left(\frac{4 m_{\mathcal{D}} m_{\bar{\mathcal{D}}}}{(m_{\mathcal{D}} + m_{\bar{\mathcal{D}}})^2}\right)^4\right] + s^2 \left(\frac{2 m_{\mathcal{D}} m_{\bar{\mathcal{D}}}}{m_{\mathcal{D}} + m_{\bar{\mathcal{D}}}}\right)^2}. \tag{7d}$$

The values of the parameters α_k and γ_k ($k = 1, 2, 3$), σ_0 and s, extracted from Refs. [64,65], are given in Table I. The value of the qc scalar diquark mass M_{qc}^s is extracted from Ref. [16]. The values of the qs scalar and axial-vector diquark masses, M_{sc}^s and M_{sc}^{av}, are estimated by binding a sc ($\bar{s}\,\bar{c}$) pair via the OGE plus confining potential [22,64]. The only free parameters of our calculation are thus the strength of the linear confining interaction β, the qc axial-vector diquark mass M_{qc}^{av}, and ΔE (see Table I); they are fitted to the reproduction of the experimental data [77], as discussed in Sec. III A.

Finally, it is interesting to compare our model Hamiltonian, Eqs. (5) and (7), to those used in other relativized diquark model calculations [19,78]. In particular, in Refs. [19] the authors made use of a relativized tetraquark model with non-point-like diquarks, where the internal diquark structure enters the calculation via a diquark form factor. To calculate the masses of heavy tetraquarks, they

TABLE I. Parameters of the model Hamiltonian of Eq. (5). The values denoted by the symbol † are extracted from previous studies. In the upper part of the Table, we give the values of the Coulomb-like potential parameters, α_1, α_2, α_3, γ_1, γ_2, γ_3, σ_0, and s, extracted from Refs. [64,65]. The value of M_{cq}^s ($q = u$, d) is extracted from Ref. [16]; those of β, M_{cq}^{av}, and ΔE are fitted to the reproduction of the experimental data [77]. The values of the qs scalar and axial-vector diquark masses, M_{sc}^s and M_{sc}^{av}, are estimated by binding a sc ($\bar{s}\,\bar{c}$) pair via a OGE plus confining potential [22,64].

Parameter	Value	Parameter	Value
α_1	0.25 †	γ_1	2.53 fm^{-1} †
α_2	0.15 †	γ_2	8.01 fm^{-1} †
α_3	0.20 †	γ_3	80.1 fm^{-1} †
σ_0	9.29 fm^{-1} †	s	1.55 †
β	3.90 fm^{-2}	ΔE	−370 MeV
M_{cq}^s	1933 MeV †	M_{cq}^{av}	2250 MeV
M_{cs}^s	2229 MeV	M_{cs}^{av}	2264 MeV

used the quasipotential approach in quantum field theory, where the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by a diquark wave function of the bound quark-quark state and the tetraquark wave function of the bound diquark-antidiquark state, respectively, satisfying a quasipotential equation of the Schrödinger type [79]. On the contrary, in Ref. [78] the calculations were performed in a relativized diquark-antidiquark model[2] based on the Godfrey-Isgur relativized QM Hamiltonian [64], which is characterized by confinement, contact, tensor, and spin-orbit contributions. In addition, to absorb the unquenched effects, the linear confining potential was replaced with a screened one, viz. $\beta r_{\rm rel} \to \beta(1 - e^{-\mu r_{\rm rel}})/\mu$, where β denotes the string tension and μ is the screening parameter [80].

III. RESULTS AND DISCUSSIONS

A. $q\bar{q}c\bar{c}$ tetraquark spectrum

In Table III and Figs. 1 and 2, our theoretical predictions for the masses of $q\bar{q}c\bar{c}$ ($q = u, d$) and $s\bar{s}c\bar{c}$ 0^{++}, 1^{++}, 1^{+-}, 1^{--}, 0^{--}, and 0^{-+} tetraquark states are compared to the existing experimental data [77]. Our results are obtained by solving the eigenvalue problem of the model Hamiltonian [Eq. (5)] via a numerical variational procedure with harmonic oscillator trial wave functions. The model parameters, reported in Table I, are partly extracted from those of previous studies and partly fitted to the reproduction of the spectrum of suspected charmonium-like exotic states [77]. Furthermore, we summarize the experimental measurements on the XYZ states considered in this study in Table II.

1. X_c and Z_c states

It is worth noting that we are able to make some clear assignments, as in the case of $X(3872)$, $Z_c(3900)$, $Z_c(4020)$, $Z_c(4240)$, and $Z_c(4430)$. This is because the mass difference between the predicted and experimental masses is within the typical error of a quark model calculation, of the order of 30–50 MeV.

The $X(3872)$, discovered by Belle in $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays [10], was the first example of a quarkonium-like candidate for a nonstandard or exotic meson. This is a well-established meson [77,81–83] with extremely peculiar features: its mass is 80–100 MeV below quark model predictions [64] and very close to the $D^0\bar{D}^{*0}$ threshold, it is quite narrow ($\Gamma < 1.2$ MeV) and exhibits strong isospin violation in its decays.

In the present study, the $X(3872)$ is interpreted as an S-wave[3] scalar diquark axial-vector antidiquark bound state

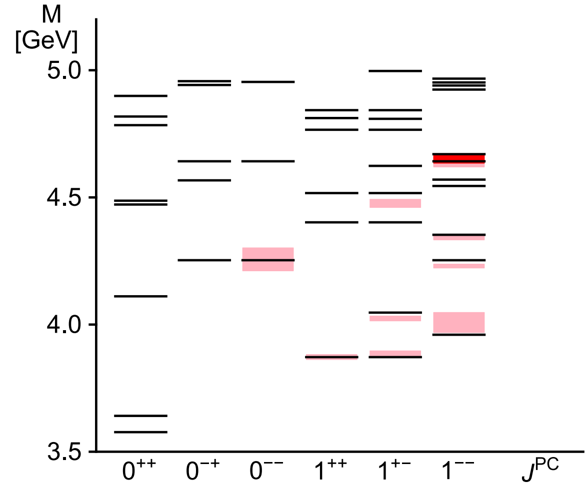

FIG. 1. The $qc\bar{q}\bar{c}$ tetraquark spectrum (lines), obtained by solving the eigenvalue problem of Eq. (5), is compared to the existing experimental data for XYZ exotics (boxes). For the numerical values, see Tables II and III.

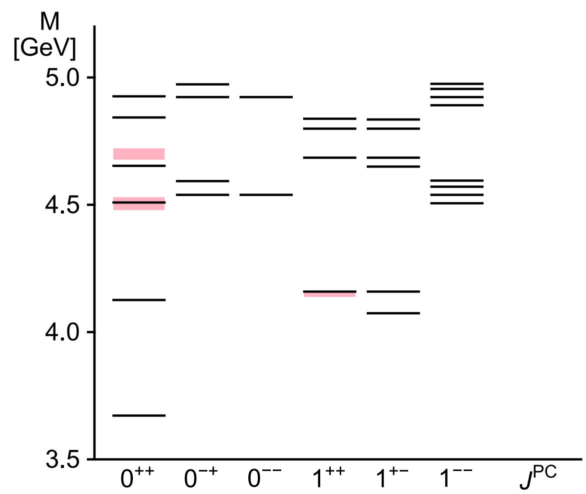

FIG. 2. As Fig. 1, but for $s\bar{s}c\bar{c}$ tetraquark states.

with $J^{PC} = 1^{++}$ quantum numbers. This is the same interpretation as Ref. [16], where the authors calculated the spectrum of $c\bar{q}c\bar{q}$ tetraquarks by means of an algebraic mass formula, giving the $X(3872)$ mass as input, and Ref. [19], where the authors calculated the tetraquark spectrum in a relativistic diquark-antidiquark model with one-gluon exchange and long-range vector and scalar linear confining potentials. In the molecular model, the $X(3872)$ is described as a $D^0\bar{D}^{*0}$ meson-meson bound state [32–38].

The $Z_c(3900)$ is a charged charmonium-like meson, with 1^{+-} quantum numbers, observed at about the same time by BESIII [5] and Belle [6]. Its exotic quantum numbers and the value of its mass, about 12 MeV above the D^0D^{*+} threshold, is incompatible with both the charmonium and molecular model interpretations. In Ref. [24], the $Z_c(3900)$ was interpreted as a hadro-charmonium state, namely, as a

[2]However, only the $s\bar{s}c\bar{c}$ tetraquark configuration was studied in Ref. [78].

[3]In the following, S, P, and D wave excitation refers to the orbital angular momentum of the tetraquark.

TABLE II. Experimental details on hidden-charm exotica which are discussed in this study. The last two columns show the first observation mode and the experiment where the discovery took place, respectively. The enlisted values are taken from the PDG [77].

State	J^{PC}	$M_{\rm exp}$ (MeV)	Γ (MeV)	Observing process	Experiment
$X(3872)$	1^{++}	3871.69 ± 0.17	< 1.7	$B^\pm \to K^\pm\pi^+\pi^- J/\psi$	Belle
$Z_c(3900)$	1^{+-}	3886.6 ± 2.4	28.1 ± 2.6	$e^+e^- \to \pi^+\pi^- J/\psi$	BESIII
$Y(4008)$	1^{--}	4008 ± 40	226 ± 44	$e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^- J/\psi$	Belle
$Z_c(4020)^\pm$	1^{+-}	4024.1 ± 1.9	13 ± 5	$e^+e^- \to \pi^+\pi^- h_c$	BESIII
$X(4140)$	1^{++}	4146.8 ± 2.5	19^{+8}_{-7}	$\gamma\gamma \to \phi J/\psi$	CDF
$Z_c(4240)^\pm$	0^-	$4239 \pm 18^{+45}_{-10}$	$220 \pm 47^{+108}_{-74}$	$B^0 \to K^+\pi^-\psi(2S)$	LHCb
$Y(4260)$	1^{--}	4230 ± 8	55 ± 19	$e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^- J/\psi$	*BABAR*
$X(4274)$	1^{++}	4273^{+19}_{-9}	56^{+14}_{-16}	$B^+ \to J/\psi\phi K^+$	CDF, LHCb
$Y(4360)$	1^{--}	4341 ± 8	102 ± 9	$e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^-\psi(2S)$	Belle
$Z_c(4430)^\pm$	1^+	4478^{+15}_{-18}	181 ± 31	$B \to K\pi^\pm\psi(2S)$	Belle
$X(4500)$	0^{++}	4506^{+16}_{-19}	92 ± 29	$B^+ \to J/\psi\phi K^+$	LHCb
$Y(4630)$	1^{--}	4634^{+8}_{-7}	92^{+40}_{-24}	$e^+e^- \to \Lambda_c^+\Lambda_c^-$	Belle
$Y(4660)$	1^{--}	4643 ± 9	72 ± 11	$e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^-\psi(2S)$	Belle
$X(4700)$	0^{++}	4704^{+17}_{-26}	120 ± 50	$B^+ \to J/\psi\phi K^+$	LHCb

J/ψ embedded in an S-wave spinless excitation of the light-quark matter with the quantum numbers of a pion, $J^P = 0^-$. Our interpretation is the same as Refs. [16,19], namely, as the C-odd partner of the $X(3872)$ [Eq. (22), [16]].

The $Z_c(4020)$ was seen by BESIII in a study of $h_c(1P)\pi^+\pi^-$ final states [7]; its quantum numbers are $J^{PC} = 1^{+-}$. We interpret the $q\bar{q}c\bar{c}$ state of Table III, with $1[(1,1)1,0]1$ and 1^{+-} quantum numbers,[4] as $Z_c(4020)$. Other interpretations include a $D^*\bar{D}^*$ molecular state with 1^{+-} quantum numbers [32,37], binded by one-pion-exchange and/or contact interactions, or a tightly bound tetraquark configuration [16].

In 2014, LHCb confirmed the existence of the $Z_c(4430)$ in $\pi^\pm\psi(2S)$ and, within the same data set, also observed a lighter and wider structure named the $Z_c(4240)$ [84,85]. Further experimental confirmation of the $Z_c(4240)$ would be helpful. In our study, we interpret the $Z_c(4240)$ as a P-wave scalar diquark, axial-vector antidiquark bound state with $J^{PC} = 0^{--}$.

Finally, the $Z_c(4430)$ was the first established candidate for a charged charmonium-like meson. It was observed by Belle as a peak in the invariant mass of the $\psi(2S)\pi^+$ system in $\bar{B} \to \psi(2S)\pi^+K$ [86]. In our study, we interpret the $Z_c(4430)$ as a D-wave scalar diquark, axial-vector antidiquark bound state with $J^{PC} = 1^{+-}$ quantum numbers. However, in this case the assignment is more dubious, because the experimental mass of the meson falls in the energy interval between the $2[(1,0)1,0]1$ and $1[(1,0)1,2]1$, $J^{PC} = 1^{+-}$ states of Table III. It is worth noting that the $Z_c(4430)$ was interpreted as a $2S$ scalar diquark, axial-vector antidiquark bound state in Ref. [19]. In Ref. [87], the $Z_c(3900)$ and $Z_c(4430)$ were assigned to a $1S$ state (with $J^P = 1^+$) and its radial excitation, respectively; several strong decay channels were also explored [87].

[4]The tetraquark quantum numbers notation is the same as Table III.

2. Y_c states

There is a rich spectrum of charmonium-like $J^{PC} = 1^{--}$ vector states, the so-called Y states. Below, we discuss our tetraquark model assignments.

Starting from $Y(4008)$, the presence of a broad structure, with mass $4008 \pm 40^{+114}_{-28}$ MeV and width $226 \pm 44 \pm 87$ MeV, was indicated by Belle in the measured $\pi^+\pi^- J/\psi$ mass spectrum [88]. However, *BABAR* did not find the $Y(4008)$ signal in the same $e^+e^- \to \pi^+\pi^- J/\psi$ process [89]. Future experiments will give a concluding answer about the $Y(4008)$ existence. In our tetraquark model calculation, the $Y(4008)$ is interpreted as a P-wave scalar diquark-antidiquark bound state.

$Y(4260)$ was discovered by *BABAR* in $e^+e^- \to Y \to \pi^+\pi^- J/\psi$ [86] and then confirmed by CLEO-c [90] and Belle [88]. We interpret it as a P-wave scalar diquark and axial-vector antidiquark bound state. In Ref. [19] it was described as a P-wave scalar diquark-antidiquark bound state, in Ref. [91] as the first orbital excitation of a diquark-antidiquark state $cs\bar{c}\bar{s}$, but it was also interpreted as a hybrid charmonium in Refs. [92–94]. The authors of Refs. [26,95] also interpreted $Y(4260)$ as $\bar{D}D_1(2420)$ molecule with a binding energy of 29 MeV. Very recently, a possible molecular scenario was discussed in a coupled-channel analysis [96].

BABAR found evidence of the $Y(4360)$ in $e^+e^- \to Y \to \pi^+\pi^-\psi(2S)$ [97]; later, the $Y(4360)$ was confirmed by Belle, which also found another peak, corresponding to $Y(4660)$ [98]. Analogously as in Ref. [72], we interpret $Y(4360)$ and $Y(4660)$ as the second and third radial

TABLE III. The $q\bar{q}c\bar{c}$ ($q = u$, d) and $s\bar{s}c\bar{c}$ tetraquark spectrum (up to 5 GeV), obtained by solving the eigenvalue problem of Eq. (5) with the model parameters of Table I, is compared to the existing experimental data [77]. In the third column, we give the quantum numbers of the predicted tetraquark states: N stands for the radial quantum number, $S_{\mathcal{D}}$ and $S_{\bar{\mathcal{D}}}$ are the spin of the diquark and antidiquark, respectively, coupled to the total spin of the meson S; the latter is coupled to the orbital angular momentum L to get the total angular momentum of the tetraquark J. For more details on the tetraquark basis, see the Appendix.

State ($q\bar{q}c\bar{c}$)	J^{PC}	$N[(S_{\mathcal{D}},S_{\bar{\mathcal{D}}})S,L]J$	$E^{\rm th}$ [MeV]	$E^{\rm exp}$ [MeV]	State ($s\bar{s}c\bar{c}$)	J^{PC}	$N[(S_{\mathcal{D}},S_{\bar{\mathcal{D}}})S,L]J$	$E^{\rm th}$ [MeV]	$E^{\rm exp}$ [MeV]
	0^{++}	1[(0,0)0,0]0	3577			0^{++}	1[(1,1)0,0]0	3672	
	0^{++}	1[(1,1)0,0]0	3641			0^{++}	1[(0,0)0,0]0	4126	
	0^{++}	2[(0,0)0,0]0	4111		$X(4500)$	0^{++}	2[(1,1)0,0]0	4509	$4506 \pm 11^{+12}_{-15}$
	0^{++}	3[(0,0)0,0]0	4480		$X(4700)$	0^{++}	2[(0,0)0,0]0	4653	4704^{+17}_{-26}
	0^{++}	2[(1,1)0,0]0	4482			0^{++}	3[(1,1)0,0]0	4926	
	0^{++}	4[(0,0)0,0]0	4784			0^{++}	1[(1,1)2,2]0	4843	
	0^{++}	1[(1,1)2,2]0	4818						
	0^{++}	3[(1,1)0,0]0	4899						
$X(3872)$	1^{++}	1[(1,0)1,0]1	3872	3871.69 ± 0.17	$X(4140)$	1^{++}	1[(1,0)1,0]1	4159	4146.8 ± 2.5
	1^{++}	2[(1,0)1,0]1	4402			1^{++}	2[(1,0)1,0]1	4685	
	1^{++}	1[(1,0)1,2]1	4517			1^{++}	1[(1,0)1,2]1	4799	
	1^{++}	3[(1,0)1,0]1	4766			1^{++}	1[(1,1)2,2]1	4838	
	1^{++}	1[(1,1)2,2]1	4812						
	1^{++}	2[(1,0)1,2]1	4843						
$Z_c(3900)$	1^{+-}	1[(1,0)1,0]1	3872	3886.6 ± 2.4		1^{+-}	1[(1,1)1,0]1	4074	
$Z_c(4020)$	1^{+-}	1[(1,1)1,0]1	4047	4024.1 ± 1.9		1^{+-}	1[(1,0)1,0]1	4159	
	1^{+-}	2[(1,0)1,0]1	4402			1^{+-}	2[(1,1)1,0]1	4650	
$Z_c(4430)$	1^{+-}	1[(1,0)1,2]1	4517	4478^{+15}_{-18}		1^{+-}	2[(1,0)1,0]1	4685	
	1^{+-}	2[(1,1)1,0]1	4624			1^{+-}	1[(1,0)1,2]1	4799	
	1^{+-}	3[(1,0)1,0]1	4766			1^{+-}	1[(1,1)1,2]1	4835	
	1^{+-}	1[(1,1)1,2]1	4809						
	1^{+-}	2[(1,0)1,2]1	4843						
	1^{+-}	3[(1,1)1,0]1	4997						
$Y(4008)$	1^{--}	1[(0,0)0,1]1	3960	4008 ± 40		1^{--}	1[(0,0)0,1]1	4506	
$Y(4260)$	1^{--}	1[(1,0)1,1]1	4253	4230 ± 8		1^{--}	1[(1,0)1,1]1	4539	
$Y(4360)$	1^{--}	2[(0,0)0,1]1	4353	4341 ± 8		1^{--}	1[(1,1)0,1]1	4571	
	1^{--}	1[(1,1)0,1]1	4545			1^{--}	2[(0,0)0,1]1	4891	
	1^{--}	1[(1,1)2,1]1	4570			1^{--}	1[(1,1)2,1]1	4595	
$Y(4630)$	1^{--}	2[(1,0)1,1]1	4642	4634^{+8}_{-7}		1^{--}	2[(1,0)1,1]1	4923	
$Y(4660)$	1^{--}	3[(0,0)0,1]1	4670	4643 ± 9		1^{--}	2[(1,1)0,1]1	4955	
	1^{--}	2[(1,1)0,1]1	4929			1^{--}	2[(1,1)2,1]1	4975	
	1^{--}	4[(0,0)0,1]1	4946						
	1^{--}	2[(1,1)2,1]1	4949						
	1^{--}	3[(1,0)1,1]1	4954						
$Z_c(4240)$	0^{--}	1[(1,0)1,1]0	4253	$4239 \pm 18^{+45}_{-10}$		0^{--}	1[(1,0)1,1]0	4539	
	0^{--}	2[(1,0)1,1]0	4642			0^{--}	2[(1,0)1,1]0	4923	
	0^{--}	3[(1,0)1,1]0	4954						
	0^{-+}	1[(1,0)1,1]0	4253			0^{-+}	1[(1,0)1,1]0	4539	
	0^{-+}	1[(1,1)1,1]0	4567			0^{-+}	1[(1,1)1,1]0	4593	
	0^{-+}	2[(1,0)1,1]0	4642			0^{-+}	2[(1,0)1,1]0	4923	
	0^{-+}	3[(1,0)1,1]0	4954						
	0^{-+}	2[(1,1)1,1]0	4947			0^{-+}	2[(1,1)1,1]0	4973	

excitations of $Y(4008)$, respectively. There are also other possible descriptions for these states, e.g., $Y(4260)$ and $Y(4360)$ were embedded into the hadro-charmonium picture [25], $Y(4260)$, $Y(4360)$, and $Y(4660)$ in a baryonium description [99], while in Ref. [100] $Y(4660)$ is assumed to be a $f_0(980)\psi(2S)$ bound state.

Finally, the $Y(4630)$ was seen in $e^+e^- \to Y \to \Lambda_c\bar{\Lambda}_c$ by Belle [101]. We interpret it as the $Y(4260)$ radial excitation. In Ref. [102], the authors discussed the $Y(4630) \to \Lambda_c \bar{\Lambda}_c$ decay mode in the $^3P_0$ model formalism, under the hypothesis that the $Y(4630)$ is a 1^{--} charmonium-like tetraquark. Because of its peculiar decay mode, $Y(4630)$ was also described as a baryonium state, namely, as a $\Lambda_c \bar{\Lambda}_c$ bound state [103].

B. $s\bar{s}c\bar{c}$ tetraquark spectrum

There are charmonium-like mesons whose decay modes and production mechanisms suggest the presence of $s\bar{s}$ d.o.f. in the tetraquark wave function. A typical example is the $X(4140)$, observed in $B \to KY(4140)$, with $Y(4140) \to \phi J/\psi$, by CDF [104]. In addition to the $X(4140)$, the CDF Collaboration found evidence of the $X(4274)$ with approximate significance of 3.1σ [105]. The related peaks of $J/\psi\phi$ mass structures around 4.3 GeV were also reported by LHCb, CMS, D0, and *BABAR* Collaborations [106–109], which may be the same state as the $X(4274)$. Very recently, the $X(4140)$ and $X(4274)$ were confirmed by LHCb, which also found evidence of two more structures, the $X(4500)$ and the $X(4700)$ [110].

We interpret the $X(4140)$ as the $s\bar{s}c\bar{c}$ counterpart of the $X(3872)$; $X(4500)$ and $X(4700)$ as 0^{++} radial excitations of S-wave axial-vector diquark-antidiquark and scalar diquark-antidiquark bound states, respectively. We cannot provide any assignment for the $X(4274)$.

An investigation similar to ours was conducted in Ref. [78]. There, the authors studied $s\bar{s}c\bar{c}$ tetraquarks within the relativized quark model [64] and discussed possible assignments for $X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$. In the $X(4140)$ case, their interpretation coincides with ours. They also obtained 0^{++} radial excitations of S-wave scalar diquark-antidiquark and axial-vector diquark-antidiquark bound states characterized by similar energies: one of them can be assigned to $X(4700)$. They could not accommodate the $X(4274)$. Stancu calculated the $s\bar{s}c\bar{c}$ tetraquark spectrum within a simple quark model with chromomagnetic interaction [111]. She interpreted the $X(4140)$ as the strange partner of the $X(3872)$, but she could not accommodate the other $s\bar{s}c\bar{c}$ states, $X(4274)$, $X(4500)$, and $X(4700)$.[5] In Refs. [112], a molecular model description for the $X(4140)$ as $D_s^{*+}D_s^{*-}$ was proposed.

By using QCD sum rules, the $X(4140)$ and $X(4274)$ were interpreted as S-wave $c\bar{c}s\bar{s}$ tetraquark states with opposite color structures [113], and, analogously, the $X(4500)$ and $X(4700)$ as the D-wave $c\bar{c}s\bar{s}$ tetraquark states with opposite color structures [114]. Maiani *et al.* suggested to accommodate $X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$ within two tetraquark multiplets. In particular, they suggested that the $X(4500)$ and $X(4700)$ are $2S$ $cs\bar{c}\bar{s}$ tetraquark states, the $X(4140)$ the 1^{++} ground state, and that the $X(4274)$ may have 0^{++} or 2^{++} quantum numbers [115].

IV. SUMMARY

We calculated the spectrum of $q\bar{q}c\bar{c}$ ($q = u, d$) and $s\bar{s}c\bar{c}$ tetraquarks in a relativized diquark model, characterized by one-gluon-exchange (OGE) plus confining potential [22]. According to our results, we were able to make some clear assignments, as in the case of $X(3872)$, $Z_c(3900)$, $Z_c(4020)$, $Y(4008)$, $Z_c(4240)$, $Y(4260)$, $Y(4360)$, $Y(4630)$, and $Y(4660)$ in the $q\bar{q}c\bar{c}$ sector. Our interpretation of the $Z_c(4430)$ is dubious, because the experimental mass of the meson falls in the middle of the energy interval between our $2[(1,0)1,0]1$ and $1[(1,0)1,2]1$ tetraquark model predictions of Table III, with $J^{PC} = 1^{+-}$. In the $s\bar{s}c\bar{c}$ sector, we could accommodate the $X(4140)$, $X(4500)$, and $X(4700)$. We could not provide any assignment for the $X(4274)$.

As a possible test of our model predictions, we suggest the experimentalists look for D-wave axial-vector diquark-antidiquark bound states with $J^{PC} = 1^{++}$ and 1^{+-} quantum numbers (namely, the $1[(1,1)2,2]1$ and $1[(1,1)1,2]1$ configurations) around the 4.8 GeV energy region. They could be interpreted as spin partners of the $X(3872)$ and $Z_c(4430)$; the C-even partner of the $Z_c(4430)$ is also worth searching for. Moreover, if the $X(4500)$ and $X(4700)$ are really the radial excitations of the S-wave axial-vector diquark-antidiquark and scalar diquark-antidiquark bound states, we expect their ground states to be found at energies of the order of 3.67 and 4.13 GeV, respectively; see Table III. This may represent a strong indication in favor of the diquark picture of tetraquarks. Finally, similarly to the $X(3872)$ and its odd partner case, there might be a C-odd partner of the $X(4140)$ which would have enough phase space to decay into the $J/\psi\eta^{(\prime)}$, $\eta_c\phi$, and $D_s\bar{D}_s^*$ final states. This is worthwhile to be searched for both on the experimental and theoretical sides.

Our relativized diquark-antidiquark model results are strongly model dependent. The possible sources of theoretical uncertainties lie in the choice of the effective Hamiltonian and model parameter fitting procedure, and also in the approximations introduced in the tetraquark wave function. The latter are strictly related to the possible ways of combining the quark color representations to obtain a color singlet wave function for the tetraquark. A study of the main decay modes of XYZ-type exotics in the diquark model will be important to provide a more precise identification of tetraquark candidates.

The next step of our study of fully and doubly heavy tetraquarks will be an analysis of the ground-state energies, dominant decay modes, and production mechanisms, including estimates of total decay widths and production cross sections. More precise experimental data for the exotic meson masses and properties and a detailed

[5] The $X(4500)$ and $X(4700)$ were observed at LHCb in 2016 [110], and the $X(4274)$ was first observed in 2011 by CDF with a small significance of 3.1σ [105], while Stancu's analysis dates back to 2010.

comparison between the calculated observables in the main interpretations (tetraquark, molecular model, hadro-quarkonium, and so on) may help to rule out one or more of these pictures.

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APPENDIX: CLASSIFICATION OF TETRAQUARK STATES

We report a classification of possible tetraquark states. In the following, we use the notation

$$|J^{PC}\rangle = |[(S_D, S_{\bar{D}})_S, L]_J\rangle, \tag{A1}$$

where the diquark S_D and antidiquark $S_{\bar{D}}$ spins are coupled to the total spin S, and the total spin and orbital angular momentum L are coupled to the total angular momentum J.

(i) $J^{PC} = 0^{++}$

$$|0^{++}\rangle = |[(0,0)_0, 0]_0\rangle(^1S_0), \tag{A2a}$$

$$|0^{++}\rangle = |[(1,1)_0, 0]_0\rangle(^1S_0), \tag{A2b}$$

$$|0^{++}\rangle = |[(1,1)_2, 2]_0\rangle(^5D_0). \tag{A2c}$$

(ii) $J^{PC} = 0^{-+}$

$$|0^{-+}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 1]_0\rangle + |[(1,0)_1, 1]_0\rangle](^3P_0), \tag{A3a}$$

$$|0^{-+}\rangle = |[(1,1)_1, 1]_0\rangle(^3P_0). \tag{A3b}$$

(iii) $J^{PC} = 0^{--}$

$$|0^{--}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 1]_0\rangle - |[(1,0)_1, 1]_0\rangle](^3P_0). \tag{A4}$$

(iv) $J^{PC} = 1^{++}$

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 0]_1\rangle + |[(1,0)_1, 0]_1\rangle](^3S_1), \tag{A5a}$$

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 2]_1\rangle + |[(1,0)_1, 2]_1\rangle](^3D_1), \tag{A5b}$$

$$|1^{++}\rangle = |[(1,1)_2, 2]_1\rangle(^5D_1). \tag{A5c}$$

(v) $J^{PC} = 1^{+-}$

$$|1^{+-}\rangle = |[(1,1)_1, 0]_1\rangle(^3S_1), \tag{A6a}$$

$$|1^{+-}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 0]_1\rangle - |[(1,0)_1, 0]_1\rangle](^3S_1), \tag{A6b}$$

$$|1^{+-}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 2]_1\rangle - |[(1,0)_1, 2]_1\rangle](^3D_1), \tag{A6c}$$

$$|1^{+-}\rangle = |[(1,1)_1, 2]_1\rangle(^3D_1). \tag{A6d}$$

(vi) $J^{PC} = 1^{--}$

$$|1^{--}\rangle = |[(0,0)_0, 1]_1\rangle(^1P_1), \tag{A7a}$$

$$|1^{--}\rangle = |[(1,1)_0, 1]_1\rangle(^1P_1), \tag{A7b}$$

$$|1^{--}\rangle = \frac{1}{\sqrt{2}}[|[(0,1)_1, 1]_1\rangle - |[(1,0)_1, 1]_1\rangle](^3P_1), \tag{A7c}$$

$$|1^{--}\rangle = |[(1,1)_2, 1]_1\rangle(^5P_1). \tag{A7d}$$

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