Neutrino Mass Constraints from WMAP and SDSS

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Abstract

We discuss the constraints on the neutrino mass from recent cosmological data. Assuming the flat LCDM model with power-law adiabatic perturbations, we find $m_\nu < 0.7$ eV (95% C.L.) from the WMAP data alone, without the aid of any other cosmological data. We also investigate how much the SDSS LRG power spectrum can improve this WMAP limit.

1 Introduction

The neutrino mass is directly searched by tritium β-decay experiments and the current upper bound for the electron neutrino mass is 2 eV (95% C.L.) [1]. At present, it is difficult to push down the limit in this way, but, as is well known, cosmological considerations give more stringent constraints.

For example, Ref. [2] obtained $m_\nu < 0.21$ eV and Ref. [3] obtained $m_\nu < 0.58$ eV, using the WMAP 1st year data combined with the galaxy power spectrum (the former used the data from the 2dFGRS and the latter from the SDSS main sample. The difference can be ascribed to the use of the bias information by Ref. [2]). By contrast, we found $m_\nu < 0.66$ eV from the WMAP 1st year data alone as reported in Ref. [4]. We note that this is the first to point out CMB data (the WMAP 1st year data) alone can give a sub-eV upper bound on the neutrino mass, which is comparable to the limits obtained from the CMB and galaxy clustering data combined.

Before Ref. [4] (and some time after that too), there seems to be a lack of consensus about whether the CMB experiment with the WMAP-level precision can derive a sub-eV neutrino mass limit and, in fact, the WMAP alone limit reported in Ref. [3], $m_\nu < 3.8$ eV, which allows 100% HDM was somewhat accepted (incidentally, the WMAP group did not report the WMAP 1st year data alone limit on the neutrino mass). We, on the contrary, have derived the upper limit 0.66 eV as quoted above from the same data by the χ^2 minimization method which is independent from the MCMC method adopted by Ref. [3]. Our conclusion is later confirmed by Refs. [5–7] (Ref. [5] does not report the WMAP alone limit in a number but judging from their likelihood figure, it looks less than 1 eV. Refs. [6] and [7] gives $m_\nu < 0.70$ eV and $m_\nu < 0.63$ eV respectively).

Below, we first discuss the WMAP alone limit comparing results from 1st year data and 3rd year data. Then, we will see how the limit is tightened by the SDSS Luminous Red Galaxy (LRG) power spectrum [8] which is obtained from a galaxy sample with the largest effective volume to date (about 6 times larger than that of the main sample).

We here summarize our notations. We derive neutrino mass constraint in the flat ΛCDM model with the power-law adiabatic perturbations. Namely, cosmological parameters we consider are: baryon density ω_b, matter density ω_m, hubble parameter h, reionization optical depth τ, spectral index of primordial spectrum n_s, its amplitude A and massive neutrino density ω_ν. Here, $\omega \equiv \Omega h^2$ where Ω is the energy density normalized to the critical density and $\omega_m \equiv \omega_b + \omega_c$ where ω_c is cold dark matter density [2]. ω_ν is related to neutrino masses by $\omega_\nu = \sum m_\nu/(94.1 \text{ eV})$ and we assume three generations of massive neutrinos with degenerate masses so that $\omega_\nu = m_\nu/(31.4 \text{ eV})$. The flatness condition is expressed as $\omega_m + \omega_\nu + \omega_\Lambda = h^2$ where ω_Λ is the cosmological constant energy density.

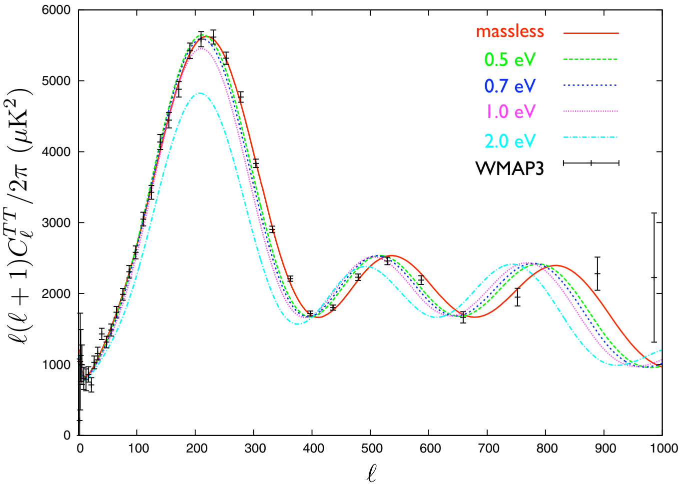

Figure 1: Effects of massive neutrinos on the CMB TT power spectrum. The labels denote the neutrino mass for single species m_ν and we assume three degenerate species. When we vary the neutrino mass, we fix ω_b, ω_c, h and $\Omega_{\rm tot} = 1$ so that Ω_Λ varies. We also fix the amplitude (normalization) for the primordial power spectrum A. Errorbars are from the WMAP 3-year observation.

2 WMAP alone limit

We begin with showing how CMB power spectrum is modified by increasing neutrino mass in Fig. 1. The other cosmological parameters are fixed here. Also, we assume equal mass m_ν for three neutrino species, which is justified because, from the oscillation experiments, the mass differences are known to be much smaller than one eV. In this figure, we see horizontal shift, and suppression around the first peak. We will briefly discuss whether these variations are degenerate with other cosmological parameters.

As for the horizontal shift, this comes from the fact that the larger m_ν implies smaller amount of cosmological constant, since we assume that the universe is flat. Then the distance to the last scattering surface is shorter and the peaks move to smaller ℓ (to larger angular scales). However, this shift is easily cancelled by the shift in the hubble parameter. Therefore this does not produce a neutrino mass signal.

Next, the suppression of the 1st peak takes place only when m_ν is larger than about 0.6 eV. This corresponds to 0.3 eV in terms of photon temperature T. Meanwhile, the recombination takes place at $z \approx 1088$ or $T \approx 0.3$ eV. In other words, massive neutrinos become non-relativistic before the epoch of recombination if they are heavier than 0.6 eV. Therefore, only in this case, the neutrino mass can imprint a characteristic signal in acoustic peaks (to be specific, the matter-radiation equality occurs earlier due to less relativistic degrees of freedom and the enhancement of the 1st peak by the early-integrated Sachs-Wolfe effect is smaller).

In passing, notice that such separation of the neutrino mass effect into the horizontal shift and the suppression of the 1st peak for $m_\nu > 0.6$ eV can be accomplished by our parametrization that $\omega_m \equiv \omega_b + \omega_c$ is fixed (Ω_Λ is varied) when m_ν is varied.

This signal, however, could be accidentally mimicked by some combination of other cosmological parameters. So we searched a large cosmological parameter space in order to find the degree of degeneracy between m_ν and the other cosmological parameters. The results are shown in Fig. 2. We calculated WMAP χ^2 (log likelihood) as functions of m_ν. For each value of m_ν, we varied 6 other ΛCDM cosmological parameters to find minimum χ^2. In Fig. 2, the different lines represent different data sets. The blue dotted line is the WMAP 1st year result which I mentioned in the introduction. The others use WMAP 3-year data. The red solid line uses full data sets including temperature and polarization and the green dashed line uses only temperature power spectrum. We find that three lines are quite similar.

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[2] Caution that many literatures define $\omega_m \equiv \omega_b + \omega_c + \omega_\nu$ to include the massive neutrino in the matter density.

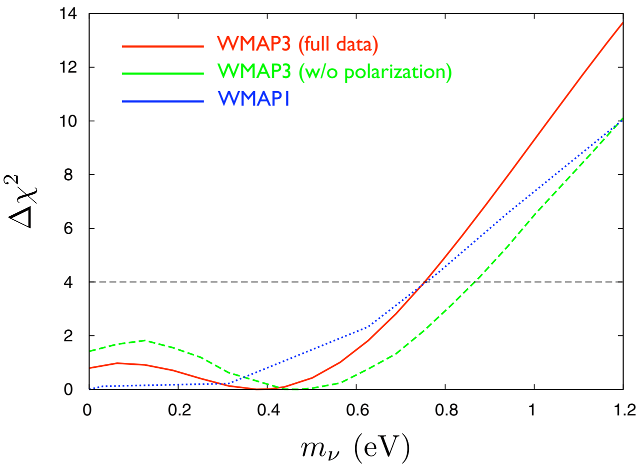

Figure 2: $\Delta\chi^2$ of WMAP data as functions of the neutrino mass m_ν [4,9]. $\Delta\chi^2 = 4$ roughly corresponds to 95% C.L. limit. Each curve represents different data sets. The blue dotted line use the WMAP 1st year data (TT+TE), the red solid line uses the full WMAP 3-year data including temperature and polarization and the green dashed line uses only temperature power spectrum of the WMAP 3-year data.

In summary, we obtained the upper bound of 0.7 eV from WMAP alone and two points are worth mentioning. First is that the WMAP 3-year constraint is not improved from the 1st year limit. Next is that the polarization data does not contribute to the limit much. These results are reasonable because the neutrino mass (larger than 0.6 eV) characteristically modifies the acoustic peaks around 1st and 2nd peaks in the temperature power spectrum and these regions are already well measured by the WMAP 1st year.

We stress that this bound is robust in a sense that it is obtained from CMB data of the WMAP which is considered to be the cleanest cosmological data and that it is obtained from a single experiment. Also, CMB can be dealt with the linear perturbation theory so it does not suffer from non-linearity or biasing which appear in galaxy clustering data.

However, we have to combine other data sets to improve this limit. This is because, as we mentioned earlier, CMB is insensitive to the neutrino mass lighter than 0.6 eV [3] , whose effects being absorbed in the shift of the hubble parameter.

3 WMAP + SDSS LRG limit

Now, we try to improve the WMAP alone limit by combining with the newest galaxy power spectrum data of SDSS based on the luminous red galaxy samples. We are mostly interested in how combined limit is affected by systematic effects, especially by uncertainties in non-linear modeling. We show the result of the χ^2 analysis first in Fig. 3. We find SDSS data improves the limit down to 0.2 eV shown by the light-blue dot-dashed line from 0.7 eV of the WMAP alone limit shown by the red solid line. Here, when we marginalize over the other cosmological parameters, we marginalized also over two parameters for non-llinear modeling following the SDSS group's analysis [8]. Next I will explain non-linear modeling we adopted.

Our non-linear modeling follows a simple model of Ref. [10]: $P_{\rm gal}(k) = (1 + \mathcal{Q}k^2)/(1 + Ak)P_{\rm lin}(k)$, where k is the wavenumber. $P_{\rm lin}$ is the linear power spectrum computed by *e.g.* CMBFAST and $P_{\rm gal}$ is the galaxy power spectrum to be compared with the observation. A is fixed to 1.4 and we marginalize over $\mathcal{Q}$. We also marginalize over scale-independent "bias factor" b in addition to the above correction factor for Fig. 3.

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[3] The lensed CMB is known to be sensitive to smaller neutrino masses if we obtained polarization data of next (or next-to-next) generation experiments, but we do not discuss this possibility here.

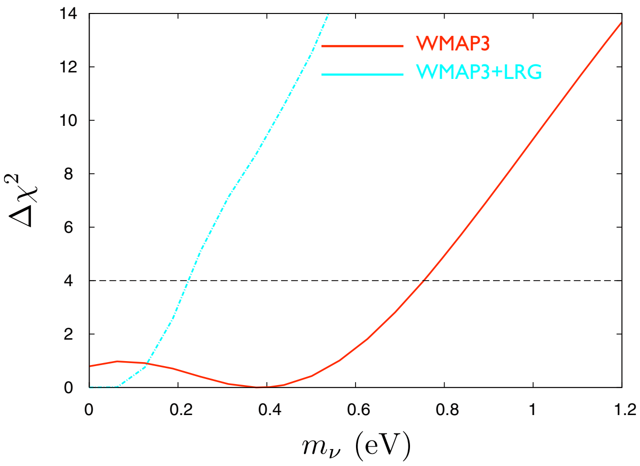

Figure 3: $\Delta\chi^2$ of WMAP 3-year and SDSS LRG power spectrum data as functions of the neutrino mass m_ν. The red solid line uses the full WMAP 3-year data and the light-blue dot-dashed line uses the WMAP 3-year + LRG.

To investigate how the neutrino mass and the non-linear correction are correlated, as a very simple test, we begin by fixing $\mathcal{Q}$. This is because the value such as $\mathcal{Q}$ is in principle determined by theoretical calculation if we could calculate how galaxies form in the dark matter halo. We performed χ^2 analysis with several fixed values of $\mathcal{Q}$ and we found that the upper bound does not move very much from 0.2 eV (0.2-0.25 eV for $\mathcal{Q}$ =25-35). However, note that, in such a simple way, systematic effects of non-linear corrections can not be fully investigated, of course.

In conclusion, the WMAP 3-year alone limit is 0.7 eV and the SDSS new galaxy data of LRG improves this limit down to around 0.2 eV. This combined limit does not seem to change much by simple systematic effects of non-linear correction like the one explained above. However, we need to investigate more the effects of the non-linear modeling on the neutrino mass constraints to increase the robustness of limits from the galaxy clustering data.

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