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COMPOSITE MODELS OF HADRONS AND LARGE TRANSVERSE PROCESSES*

MODELES COMPOSES DES HADRONS ET PROCESSUS A UNE GRANDE IMPULSION TRANSVERSALE

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I. INTRODUCTION

Throughout the history of physics, our knowledge of the underlying structure of matter at smaller and smaller distances has progressed by the scattering of particles to ever larger momentum transfer. Now we are at the stage where the internal structure of the hadrons is being resolved through measurements of deep inelastic scattering of electrons, muons, and neutrinos, and, most recently, through large transverse momentum processes involving the collisions of hadrons. The most recent measurements, at the CERN-ISR p-p storage rings[1] and the proton accelerator at FNAL[2] have involved particle production at transverse momentums $p_\perp$ up to 9 GeV, corresponding to a resolving length $\ell \sim p_\perp^{-1}$ of order 10^{-15} cm. These measurements have shown that the cross section for hadron production at large transverse momentum is much larger than what would be extrapolated from the fast exponential fall-off of low momentum transfer reactions. Moreover, a striking pattern of scaling laws observed in both exclusive and inclusive processes at large $p_\perp$ has given support to the quark-parton field-theoretic models of composite hadrons, and some basic properties of the underlying dynamics of the constituents are now beginning to emerge.

II. THE QUARK-PARTON MODEL

The evidence for a composite model of the hadrons based on SU(3)-quark degrees of freedom has been steadily accumulating. The quarks of Gell-Mann and Zweig seem to be the underlying common denominator of a whole range of hadronic phenomena:

(1) The SU(6) spectroscopy - which associates baryon states with bound states of three spin 1/2 quarks and mesonic states with $q\bar{q}$ bound states - is extraordinarily successful - although not quite on par with the Mendeléeff table of the elements. There do remain classes of missing baryonic resonance states, and some disarray among the mesons.

(2) The quantitative features and symmetry properties of the electromagnetic and weak interactions of the hadrons, their decays and production amplitudes, are beautifully summarized by Gell-Mann's "current algebra"; the weak and electromagnetic currents are constructed exactly as if the fundamental carriers of the charges within the hadrons are elementary Dirac-spin 1/2 quarks. The vector nature of the electromagnetic current and vector-axial nature of the weak interactions leads to a closed SU(6) × SU(6) symmetry algebra among the currents. Recently,

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extremely elegant field theories (developed by Weinberg and Salam) which can incorporate the quark current algebra, have been shown to lead to a renormalizable unified theory of the weak and electromagnetic interactions of the leptons and hadrons. Thus, for the first time we can contemplate real theories of the weak interactions which are a natural extension of quantum electrodynamics.

(3) The most dramatic evidence for an elementary-constituent structure of the hadron comes from the scale-independent behavior of deep inelastic electron (and muon) nucleon scattering[3] measured at SLAC and NAL. The scaling of the inclusive cross section is of the form ($s >> M^2$, $t/s, \mathcal{M}^2/s$ fixed)

$$\frac{d\sigma}{d^3p/E}\,(ep \to eX) = \frac{1}{s^N}\, f(t/s, \mathcal{M}^2/s) \tag{1}$$

where f is a function of ratios of invariants [$t = q^2 = (p_e - p_e')^2$ is the invariant momentum transfer squared, $s = (p_e + p)^2 = E_{cm}^2$ and $\mathcal{M}^2 = (p+q)^2$ is the invariant missing mass-squared]. The measured value, $N \cong 2$, which was predicted from current algebra by Bjorken,[4] indicates there is no internal length scale [larger than the resolving distance of $\sim 10^{-15}$ cm]. To see what this dramatic absence of a length scale means in more detail, we can use the parton model of Feynman[5] and Bjorken and Paschos,[6] and use a Fock-space representation of the proton wave function (which is an eigenstate of the total Hamiltonian) in terms of quark states (which are eigenstates of the free Hamiltonian). Deep inelastic scattering then resembles electron disintegration in nuclear physics (see Figure 1).

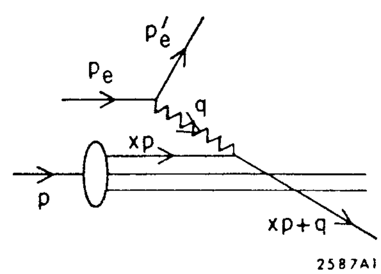

FIGURE 1

It is convenient to choose the Lorentz frame so that $|\vec{p}| = p_z$ is very large. The constituents of the n-particle state have momenta $\vec{p}_i = x_i\vec{p} + \vec{k}_{i\perp}$, $\sum_{i=1}^{n} x_i = 1$ $\sum_{i=1}^{n} \vec{k}_{i\perp} = 0$. If a quark can recoil from the electron and stay close to the mass shell, then it must have $x = -q^2/2p\cdot q$ [ignoring corrections of order $((m^2 + k_\perp^2)/q^2)$]. We thus have, in agreement with Eq. (1),

$$\frac{d\sigma}{dtdx} \sim \frac{4\pi\alpha^2}{t^2} \sum_a \lambda_a^2 f_a(x)\Big|_{x = -\frac{q^2}{2p\cdot q}} \tag{2}$$

where $4\pi\alpha^2/t^2$ represents elementary electron-quark scattering, and $f_a(x)$ is the probability that the quark of type a (and charge λ_a) has longitudinal momentum fraction x (we ignore inessential spin complications here). Thus, in the parton picture, the absence of a length scale corresponds directly to point-like electron-constituent scattering. In the analogous case of $\nu p \to \mu X$, the predictions of the simplest quark model using an elementary V-A Fermi interaction at the quark level have been recently confirmed by NAL and CERN experiments. In particular, the angular dependence, predicted from the Dirac-like structure of the quark, has been found to be, within rough errors, consistent with experiment.[7]

The quantity $F_2(x) = \sum_a \lambda_a \, x \, f_a(x)$, $0 < x < 1$ is referred to as the structure function of the proton. Note that if the hadronic binding were turned off we would have $f_a(x) \to \delta(x_a - \frac{m_a}{M_{tot}})$ since $p_a \to (m_a/M_{tot})p$ in the rest frame. Thus $f_a(x)$ includes the effects of Fermi motion of the quarks in the proton, but within a relativistic model. I will discuss further applications of this approach below.

Of course, from the above description there is no apparent reason why quarks could not be easily knocked out of the nucleon. A number of attempts are now under way to construct consistent quark-containment models in which the large-distance binding effects are separable from the short-distance features which yield scale independence. In the approach discussed by Weinberg, Casher, Kogut and Susskind, Bjorken, and others, a gauge theory for quark interactions of the type used to unify weak and electromagnetic interactions is constructed.[8] At short distances the quark interactions become weak ("asymptotic freedom") and Bjorken scaling (modulo logarithmic corrections) results.[9] However at large distances the strength of the quark-quark interaction apparently grows so fast that meson production is more favored than quark separation. In other approaches (the "bag" models) boundary conditions [10] or specific scalar field potentials[11] are introduced to achieve the same goals. The quark model is thus an extraordinary type of bound state theory! Hadron physics seems to be at the stage that nuclear physics was just before the shell model.

III. EXCLUSIVE PROCESSES AT LARGE TRANSVERSE MOMENTUM

Since point-like constituents carry a finite fraction of the hadronic momentum, one expects that hadrons should be able to scatter to large transverse momentum by the hard, large-angle scattering of their constituents. This leads in general to power-law behavior for cross sections at large transverse momentum. Recently, it has been found that the ansatz of scale-invariance at short distances leads to dimensional-counting scaling laws of the form[12-14]

$$\frac{d\sigma}{dt}(A + B \to C + D) \to \frac{1}{s^{n-2}} f(t/s) \qquad (3)$$

i.e.,

$$\frac{d\sigma}{d\Omega_{cm}}(A + B \to C + D) \to \frac{1}{s^{n-3}} f(\Theta_{cm})$$

for the asymptotic behavior of fixed center-of-mass angle scattering processes. The integer n is given by the minimum total number of elementary lepton, photon, or quark fields carrying a finite fraction of the momentum in the particles A, B, C, D; $n = n_A + n_B + n_C + n_D$. These scaling laws represent, in the simplest possible manner, a connection between the degree of complexity of a hadron and its dynamical behavior.

We can derive these counting rules in the following heuristic way:[12,13] Assuming limited hadronic binding, the computation of the hadronic amplitudes $M_{A+B\to C+D}$ is asymptotically identical to the computation of the n-particle scattering amplitude M_n obtained by replacing each hadron by a collection of quarks with the appropriate spin, each constituent carrying a finite fraction of the hadronic momentum. Note that M_n has dimension $[\text{Length}]^{n-4}$. If there is no indimensionless coupling constants) - then we expect $M_n \to 0\ [\sqrt{s}]^{4-n}$ since (barring infrared corrections) only $s^{-1/2}$ sets the length scale at fixed angle. Eq. (3) then follows from $d\sigma/dt \sim s^{-2}|M|^2$.

Applied to hadron scattering, the scaling laws (3) using quark counting seem to fare well, being consistent with a range of experiments for meson-baryon scattering, and more accurately verified for photoproduction $\gamma p \to \pi p$ and $pp \to pp$. We predict $d\sigma/dt \sim s^{-7}$ and s^{-10}, respectively, at fixed cm angle; experiment gives $s^{-7.3\pm0.4}$ and $s^{-9.7\pm0.5}$, respectively (see Figure 2). In general, using quark-counting, we have for any exclusive cross section integrated over a fixed cm region[12] (fixed invariant ratios):

$$\Delta\sigma \sim s^{-1-N_M-N_B} \tag{4}$$

where N_M (N_B) is the total number of mesons (baryons) in the initial and final states. Note that for processes involving only leptons and photons, $\Delta\sigma$ is scale invariant; each meson (baryon) introduces one (two) extra constrained fields which suppress the cross section.

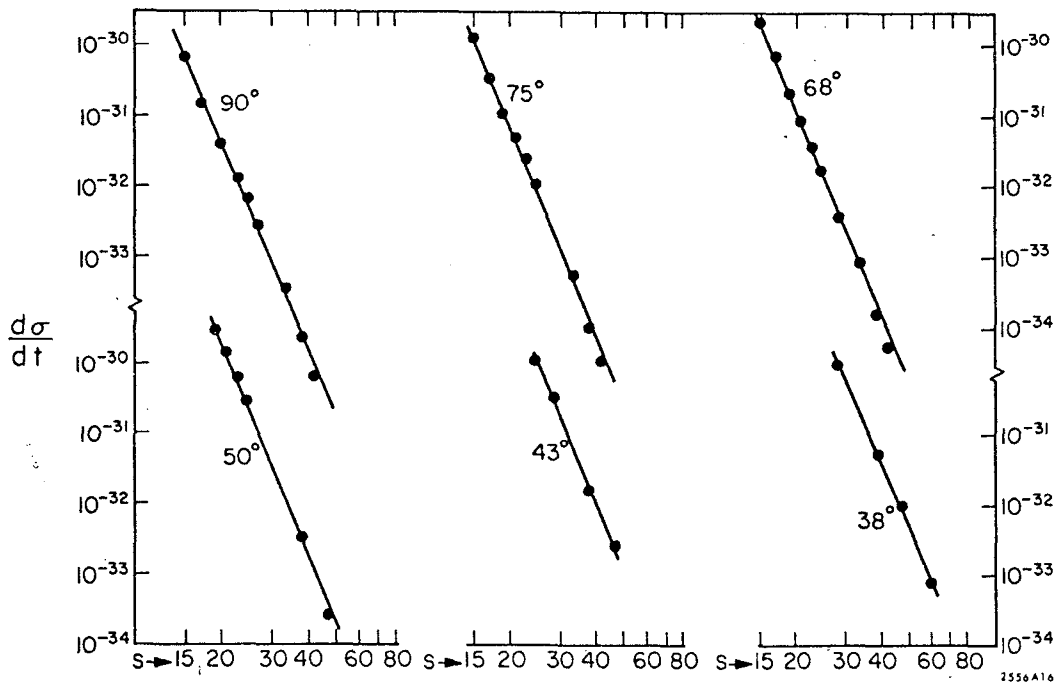

FIGURE 2

IV. FORM FACTORS AT LARGE t

One of the most important consequences of Eq. (3) is its application to elastic electron-hadron scattering; the rule immediately connects the asymptotic dependence of the spin-averaged form factor to the minimum number of fields in the hadron:[12,13,15]

$$F_M(t) \sim t^{1-n_M} \tag{5}$$

changed from p to p+q. Using the quark model, we predict $F(t) \sim t^{-1}$ for mesons and $F_1(t) \sim t^{-2}$ for baryons. We also find in field-theory models $F_2 \sim t^{-3}$ and thus $G_E \sim G_M \sim t^{-2}$ scaling.[12] These results are consistent with the dependence indicated by present experiments, although the data for the meson form factors (which come from timelike t in $e^+e^- \to \pi^+\pi^-$) are not at all conclusive. A plot of $t^2 G_M(t)$ is shown in Figure 3.

A simple illustration of how the dimensional counting rule arises in the Bethe-Salpeter computation of the meson form factor is illustrated in Figure 4.[12]

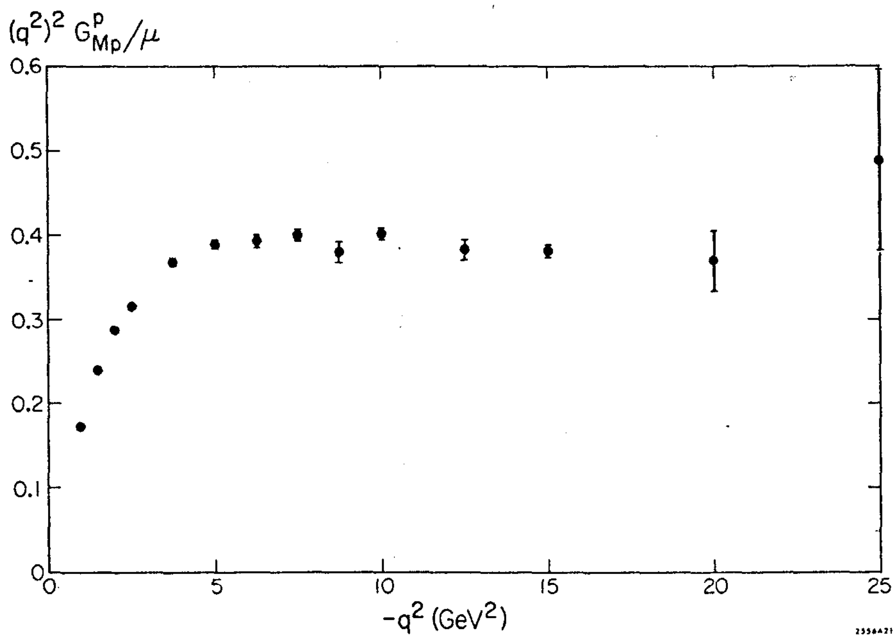

FIGURE 3

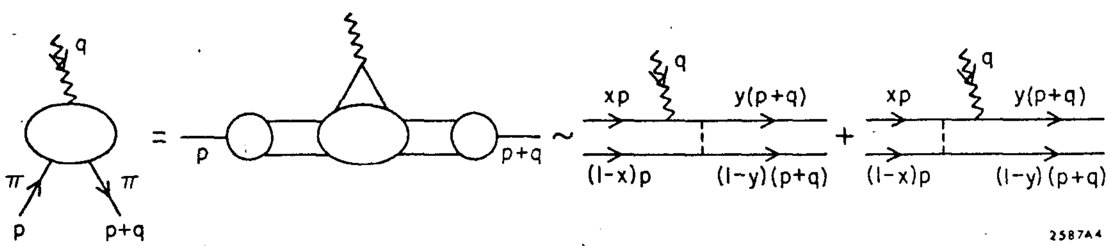

FIGURE 4

If we assume sufficient fall-off in the dependence of the Bethe-Salpeter wavefunctions at large relative momentum corresponding to a finite wavefunction in coordinate space, then the leading contribution to the asymptotic form factor comes from the iteration of the Bethe-Salpeter kernel wherever large relative momentum is required, as indicated in the figure. A simple computation then gives $F_\pi(t) \sim t^{-1}$ (modulo a logarithm) assuming a scale-invariant kernel. The inverse facgiving the result (5). Note that the minimum field contribution gives the leading asymptotic behavior.

If we are bold, we can apply the dimensional counting rule to the deuteron form factor (see Figure 5):

$$F_D(t) \sim t^{-5}$$

$$\frac{d\sigma}{dt}(eD \rightarrow eD) \sim \frac{4\pi\alpha^2}{t^2} F_D^{\,2}(t) \qquad (s >> t >> M^2) \qquad (6)$$

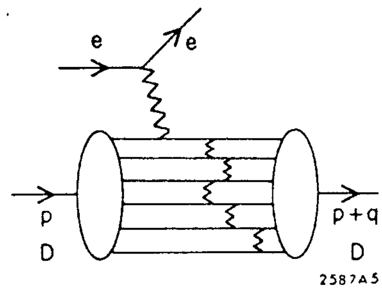

FIGURE 5

counting six quarks at short distance. A measurement of large momentum transfer elastic electron-deuteron scattering is now in progress at SLAC by a group headed by N. Chertok. The asymptotic prediction, however, is probably not useful until each of the off-shell quark lines is off-shell by $\gtrsim \frac{1}{2}$ GeV, i.e., for $|t| \gtrsim 10$ GeV^2. A convenient phenomenological approach which should be applicable at smaller $|t|$ is to plot $F_D(t)/F_{1p}^2(t/4)$ to see if a $(1-t/m^2)^{-1}$ behavior is observed.

The dimensional counting rule for form factors has recently been looked at more carefully within the context of specific renormalizable field theories.[12,16] In the case of asymptotic freedom theories, Appelquist and Poggio[17] have shown that the asymptotic behavior of the full Bethe-Salpeter kernel is effectively one logarithm more convergent at large momentum transfers than indicated by ladder approximation. They can then show that the quark-antiquark Bethe-Salpeter wavefunction falls with the required asymptotic dependence when both legs go off-shell at a constant rate; the coordinate space wavefunction is finite at the origin up to a calculable logarithm. Assuming that the wavefunction has no anomalous infrared behavior (which would be inconsistent with a bound system of finite size) when one leg goes on-shell, one predicts $F_\pi(t) \sim t^{-1}$ (modulo logarithms). A similar proof holds when a renormalizable theory has a small anomalous dimension. In the case of QED, the full Bethe-Salpeter kernel again clearly falls faster than indicated by simple ladder approximation, leading to a finite wavefunction at the origin. The true asymptotic dependence of the kernel to all orders in perturbation theory is not known, but to any finite order in perturbation theory the form factor of positronium obeys the dimensional counting rule - modulo powers of log (-t). [It should be noted that the singular behavior of the Bethe-Salpeter ladder approximation, where the wavefunction singularity at $x_\mu \rightarrow 0$ depends on the coupling constant (which in turn is restricted ad hoc by hermiticity), is misleading for the analysis of asymptotic behavior.] Schierholz and Alabiso[18] have shown explicitly how the dimensional counting analyses[19] go through in the case of three-body bound states.

V. THE CONSTITUENT INTERCHANGE MODEL

The angular dependence of the scaling cross section is of great fundamental interest itself. Note that for $s >> M^2$, the function becomes a "universal" function independent of s, no matter how large! The possibility of making asymptotic predictions such as this of course stems from the ultimate point-like structure of the constituents. It would not be surprising to see logarithmic modifications to the scaling laws (as expected in asymptotic freedom theories, for example).

Models that have been proposed for understanding the form of $f(\theta_{cm})$ are of two general categories: (1) gluon exchange and (2) quark interchange. In the first case, which has evolved from the early work of Wu and Yang,[20] hadron-hadron scattering derives from an elementary Yukawa interaction between quarks, and one predicts (for vector-gluon exchange)

$$\frac{d\sigma}{dt} \sim \left| F_{AC}(t) \frac{1}{t-\lambda^2} F_{BD}(t) + \text{u-channel terms} \right|^2 \qquad (8)$$

Although this can be an excellent description of (diffractive) low momentum transfer, high energy hadron scattering, there are several difficulties which become critical at large $p_\perp$: First, the effective trajectory $\alpha_{eff}(t)$, defined via the parametrization $\frac{d\sigma}{dt} = \beta^2(t)\, s^{2\alpha(t)-2}$ for $s >> t$, stays at $\alpha \sim 1$ (i.e., energy independence at fixed t) even at large t, whereas the data indicate α becomes negative at large t (i.e., strong energy dependence at fixed t). Second, scale-invariant quark-quark interactions lead to an essential conflict with the observed large $p_\perp$ hadron production data, as we discuss in the next section. Third, as shown by Landshoff,[21] multiple Glauber-like gluon exchange between on-shell quarks actually will violate the dimensional counting rules because of a linear infrared behavior in the quark mass. One finds

$$M_{had} \sim \left(\frac{i\sqrt{s}}{m}\right)^{L-1} \left(\frac{1}{\sqrt{s}}\right)^{n-4} \qquad (9)$$

where L is the number of on-shell quark pair interactions. This yields $d\sigma/dt \sim s^{-8}$ for p-p scattering (since L = 3), contrary to the data. Coleman[22] has shown that the only graphs with infrared behavior worse than logarithmic are these multiple, scale-invariant, on-shell quark scattering contributions. Thus, on empirical grounds, it appears that gluon exchange is not an important scattering mechanism at large transverse momentum.

Regardless of the importance of gluon exchange, composite states can always scatter by the interchange of their common constituents. In atomic physics, atoms can scatter via electron rearrangement, and in nuclear physics, deuteron-deuteron scattering can occur via nucleon rearrangement. In the case of p-p scattering, the nucleons can scatter via the interchange of a common p or n quark. The simplest example is $K^+p \to K^+p$ scattering which goes by p-quark interchange (see Figure 6).

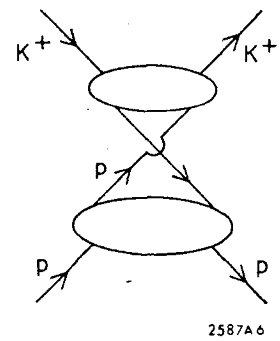

FIGURE 6

A simple calculation gives ($z = \cos\theta_{cm}$)[14,23]

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{u^2}\frac{1}{t^4} \quad \alpha \quad \frac{1}{s^8}\frac{1}{(1+z)^2}\frac{1}{(1-z)^4} \tag{10}$$

and $\alpha_{eff}(t \to -\infty) = -1$. A comparison with K^+p data shows excellent agreement. Other cases assume further model dependence, but the general features of the angular distributions of the constituent interchange model (CIM), including the prediction of negative values of $\alpha_{eff}(-\infty)$, seem to be confirmed. Further, the CIM diagrams are consistent with the dimensional counting rules, assuming the two-body Bethe-Salpeter kernel for the quark binding is scale-invariant. We will discuss applications to inclusive processes in the next section.

VI. THE IMPULSE APPROXIMATION IN HADRON PHYSICS[24]

One of the crucial differences between particle physics and nuclear physics is the presence of Regge behavior, and the resulting general inapplicability of the impulse approximation in high energy hadronic reactions. Because of the unlimited number of particles possible in a virtual intermediate state, the probability function f(x) diverges as $x \to 0$: $f(x) \sim x^{-\alpha}$ (with $\alpha \sim 1$); i.e., there is a very large probability for a particle to have a very small fraction of the target hadron momentum. Thus a typical forward hadronic interaction occurs on a constituent at an effective energy $s_{eff} = xs$. Integration over the spectrum gives

$$M \sim \int_0^\infty \frac{dx}{x} f(x)\, M_0(xs) \sim s^\alpha \tag{11}$$

and thus $\sigma_{TOT} \sim s^{-1}\,\mathrm{Im}\,M \sim \mathrm{const.}$ (for $\alpha \sim 1$) with most of the scattering occurring at the minimum possible s_{eff}. Thus high energy is not sufficient to study short distances in hadronic processes since the interactions can occur between particles with only a small fraction of the available energy. In contrast, when high transverse momentum particle production is required we must have $s_{eff} > 4p_\perp^2$, and a true high energy collision is involved. Further, in a large t exclusive process, coherence of the reaction allows only the fewest particle number states to contribute, α_{eff} is negative, and s_{eff} is of order s_{eff}.

It is interesting to note that at large momentum transfer, Compton scattering $\gamma p \to \gamma p$ and $\gamma p \to \gamma X$ is dominated by Thomson scattering off the point-like constituents of the hadron - exactly in analogy to high energy Compton scattering on atoms.[25] The calculations are extremely simple using the infinite momentum frame. For intermediate states with a finite number of particles, the high energy spin-averaged forward elastic Compton scattering amplitude is given by

$$M_{\gamma p \to \gamma p} \to -\sum_a \lambda_a^2 \frac{e^2}{m} \int_0^1 \frac{f_a(x)}{x}\, dx = -\sum_a \lambda_a^2 \frac{e^2}{m_{eff}} \tag{12}$$

where $m_{eff}^{-1} = m^{-1}\langle x^{-1}\rangle$ plays the role of an effective mass of the quark in the hadron. This can which account for hadronic Regge behavior entail a Regge subtraction consistent with analytic continuation in the trajectory α. The infinite-momentum frame analyses serve as a natural extension—of the Schroedinger many-body theory to the relativistic domain, and have many applications to nuclear physics problems. Some time-ordered perturbation calculations for quantum electrodynamics using the $P \to \infty$ method are discussed in Ref. 26.

VII. INCLUSIVE REACTIONS AT LARGE TRANSVERSE MOMENTUM

One of the most exciting areas of particle physics is the study of inclusive large transverse momentum reactions $A+B \to C+X$. For example, at the CERN intersecting storage rings, measurements of the reaction $pp \to \pi^0 X$ have been made at a center of mass energy of 52.7 GeV (equivalent to a lab energy of ~ 1300 GeV) where the π^0 is detected at $\theta_{cm} \sim 90^o$ with $p_\perp$ up to 9 GeV.[1] In terms of $t = (p - p_{\pi^0})^2$ this is $|t| \sim 475\ GeV^2 \sim 12000\ f^{-2}$. Over the range of the experiment, the results are consistent with a scaling law

$$\frac{d\sigma}{d^3p/E}(pp \to \pi^o X) \sim \frac{1}{(p_\perp^2)^N} f(\theta_{cm}, \frac{\mathcal{M}^2}{s}) \tag{13}$$

where $2N = 8.24 \pm 0.05$ ($\pm$ 0.70 including systematic errors). Here $\mathcal{M}^2/s$ measures the distance from the edge of phase space: $\mathcal{M}^2/s \cong 1 - p_{cm}/p_{max}$. Thus, as in the case of exclusive processes, the cross section has a power law scaling in $p_\perp^2$ (or s) at fixed ratios of invariants. For the ISR data $0.1 < p_{cm}/p_{max} < 0.4$.

In order to see what is involved physically let us imagine an experiment where two muonium atoms collide in the cm system and a high energy photon is detected at 90^o. The process is clearly due to the hard (Mott) scattering of the constituent leptons followed by the bremsstrahlung from a scattered lepton (see Figure 7). Since the electron only carries a fraction $x_e \sim m_e/(m_\mu + m_e)$ of the atomic momentum, it is evident that only muon scattering can produce a photon at an energy which is a sizable fraction of E_{cm}. Thus we expect ($x_\gamma = p_\gamma/\frac{1}{2}E_{cm} = 1 - \mathcal{M}^2/s$)

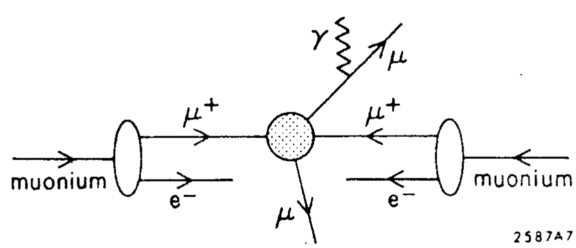

FIGURE 7

$$\frac{d\sigma}{d^3p/E} \sim \frac{\alpha^3}{p_\perp^4} \log \frac{p_\perp^2}{m_\mu^2} f(x_\gamma, \theta_{cm}) \tag{14}$$

where $f(x_\gamma)$ decreases with a bremsstrahlung spectrum from the maximum value $x_\mu = m_\mu/(m_e + m_\mu)$. More generally, we could include the Fermi motion due to atomic binding. Note that the scale-invariant $p_\perp^{-4}$ behavior is derived from the scale-invariant $\mu^+ - \mu^+$ scattering.

The fact that we do not see $p_\perp^{-4}$ behavior in $pp \to \pi^o X^-$ is thus strong evidence against scale-invariant quark-quark scattering[27] being responsible for this process. Presumably the scattering of particles with form factors[28] is involved. This suggests that perhaps quark-hadron scattering and even hadron-hadron scattering are the important underlying hard scattering processes. In order to allow for all of the possible parton model processes we can use a description based on Figure 8, where particle C is produced by the fragmentation of particle C after a hard collision of a+b → c+d: Using dimensional counting,[12] the cross section has the form of Eq. (13) with a power law $(p_\perp^2)^{2-n_{active}}$ where $n_{active} = n_a + n_b + n_c + n_d$ are the number of active fields involved in the large angle collision. In fact, we can go further: for $\mathcal{M}^2/s$ small, i.e., where $p_\perp^c$ is a large fraction of the available momentum, we have[29]

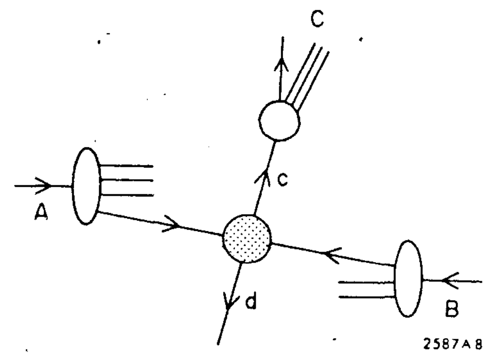

FIGURE 8

$$f(\theta_{cm}, \frac{m^2}{s}) \sim f(\theta_{cm})\left(\frac{m^2}{s}\right)^{2n_{passive}-1} \quad (15)$$

where $n_{passive}$ is the number of passive spectators of A, B, and C not involved in the production process - but which use up the available phase space. This rule accounts for the distribution of momentum of quarks in the hadrons, and can be easily derived from simple graphs in renormalizable perturbation theory. As a special case we can rederive the expression (2) for deep inelastic electron scattering and predict the threshold behavior

$$F_{2H}(x) \sim (1-x)^{2n_{spec}-1} \quad (16)$$

for the Bjorken structure functions; here $n_{spec} = n_M - 1$ is the number of non-interacting quarks in deep inelastic scattering. The reader can check that these results are consistent with the Drell-Yan-West[30] relation since $F_H(t) \sim t^{1-n_H}$. As a special case of Eqs. (14) and (15), we recover the dimensional counting rules for A+B → C+D using $n_{passive} = 0$.

The fact that 2N ~ 8 suggests that the dominant hard processes for the ISR measurements involve $n_{active} = 6$, perhaps processes such as q+M → q+M or q+q → p+$\bar{q}$. The dependence of the data on $\mathcal{M}^2/s$ given by Eq. (15) with $n_{spec} = 5$ is also consistent with this interpretation. Much more work relating quantum number characteristics, correlations, and multiparticle features of the cross section will be needed before we can become confident that this is indeed a correct explanation. Note that the dominance of quark-hadron scattering is the central hypothesis of the CIM model.[14]

The physics become much more complicated when the FNAL data for hadron production at large $p_\perp$ from proton-heavy target collisions are taken into account.[2] The unraveling of the nuclear physics effects is a fascinating problem here, especially in regard to the particle ratios. The π-production data, which cover $p_{cm}/p_{max} > 0.4$, are outside the ISR range, and yield 2N ~ 11 for $p_\perp$ > 4 GeV. Assuming that nuclear effects do not influence the value of N appreciably, we operating. In fact, as p_{cm}/p_{max} increases, we should expect that more active quarks are required in order that they have a sufficiently large fraction of the available energy. A $p_\perp^{-12}$ ($n_{active} = 8$) process like $q + (qq) \to M + B^*$ (with $n_{spec} = 3$) has just this feature; because we are close to the edge of phase space, its contribution is expected to be of comparable importance to the $p_\perp^{-8}$ terms.

Quite good fits[31] to the 90^o NAL, ISR, and low energy ($p_{lab} \sim 24$ GeV/c) data have been made using the predicted form[29]

$$\frac{d\sigma}{d^3p/E} = \frac{A}{(p_\perp^2 + m_8^2)^4}\left(1 - \frac{p}{p_{max}}\right)^9 + \frac{B}{(p_\perp^2 + m_{12}^2)^6}\left(1 - \frac{p}{p_{max}}\right)^5 \qquad (17)$$

with m_8^2, $m_{12}^2 \sim 1$ GeV^2. The effective power of $p_\perp$ in Eq. (13) thus changes from $p_\perp^{-8}$ to $p_\perp^{-12}$ over the range of the data.

VIII. CONCLUSIONS

Evidence from many different areas - particularly, deep inelastic scattering and large transverse momentum processes - supports the hypothesis that quarks determine the essential degrees of freedom of hadronic physics. The dimensional counting rules and the constituent interchange model seem to provide a convenient, simple, description of the scaling behavior of exclusive and inclusive processes at large transverse momentum. Despite the well-known present difficulties with the interpretation of the $e^+e^- \to$ hadron results of CEA and SLAC, it does seem that the quark model is the common denominator of many diverse hadronic phenomena.

As we have seen, there are some intriguing consequences for nuclear physics, e.g., the predicted t^{-5} asymptotic behavior of the spin-averaged deuteron form factor. A general consequence of the counting rules discussed above is the prediction[29] $G_{A/B}(x) \sim (1-x)^{2n-1}$ where $G_{A/B}$ is the probability of finding hadron A with a fraction x of the momentum of a fast-moving hadron B, and $n = n(\bar{A}B)$ is the number of quarks left behind. This gives a number of consequences for hadron physics - particularly in the triple Regge region of inclusive reactions. For deuterons we have $G_{p/d}(x) \sim (1-x)^5$ for the tail of the Fermi distribution of the nucleon momentum. This enters into the impulse calculations on a deuteron target, as well as being directly measurable in a diffractive breakup of a fast-moving deuteron. We also might remark here that since quarks carry the electromagnetic current, the Z-graph contribution to the meson exchange current amplitude in $e+p \to e'+p+$meson required in the nuclear form factors involves a quark pair rather than a $p\bar{p}$ pair.

It is now clearly important to establish theorems on which aspects of nuclear physics are indeed independent of nucleon substructure - e.g., low energy theorems, such as the Kroll-Ruderman theorem, and rigorous results valid for large nucleon mass which can establish the validity of the effective potential approach.[32] It seems clear that results requiring infinite sums over resonance states, subtraction constants, or questions of asymptotic behavior will require input from a theory of the underlying nucleon structure.

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