The spin-current tensor contribution in collision dynamics

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Abstract. Spin polarization, which arises mostly from the spin-orbit force, spontaneously takes place in the early stage of heavy-ion reactions, and affects the equilibration process to a large extent. On the other hand, the tensor force, whose origin can be found in the one pion exchange potential, has been shown to play a crucial role in nuclear structure. In this paper, based on time-dependent density functional calculations, the difference between spin-orbit and spin-current tensor contributions is shown in the dynamics of low-energy heavy-ion collisions.

1. Introduction

Spontaneous spin polarization, whose amplitude is suggested to depend on the N/Z ratio of colliding nuclei [1], has been shown to appear due to the time-odd part of the spin-orbit force [2]. On the other hand, the tensor force attracts special attention recently, because it has turned out to play an essential role in the existence limit of exotic nuclei (for example, see [3, 4, 5, 6, 7, 8, 9, 10]). These two forces are quite different in their origin, while resulting in the same dynamical effect, namely, spin polarization. In this paper, the role of the spin-current tensor contribution in collision situations is investigated based on time-dependent density functional calculations.

2. Framework for measuring the spin-current tensor contribution

2.1. Bilinear spin-current tensor contribution

Let ρ and **J** represent the number density and spin-orbit density, respectively. The contribution of the spin-orbit type force has the form

$$\mathbf{W}_q(\mathbf{r}) \cdot (-i)(\nabla \times \boldsymbol{\sigma}),\tag{1}$$

where q = n, p (*n* and *p* stand for neutron and proton, respectively), σ denotes the spin, and $\mathbf{W}_q(\mathbf{r})$ corresponds to the form factor of the spin mean-field [11]. The contribution from the spin-orbit force is represented by

$$\mathbf{W}_q(\mathbf{r}) = \frac{1}{2} W_0(\nabla \rho(\mathbf{r}) + \nabla \rho_q(\mathbf{r})).$$

On the other hand, according to Stancu-Brink-Flocard [12], the spin-current tensor contribution is represented by

$$\Delta \mathbf{W}_q(\mathbf{r}) = \alpha \mathbf{J}_q(\mathbf{r}) + \beta \mathbf{J}_{q'}(\mathbf{r})$$

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Figure 1. (color online) Snapshots of a composite nucleus for ${}^{40}\text{Ca} + {}^{40}\text{Ca}$ at the bombarding energy 130 MeV (c.m.) are shown on the reaction plane (time = 6.0×10^{-22} s; the time corresponds to briefly after the contact), where the density and spin distributions (the spin direction is projected on the *y*-axis) are shown in the left and the right panels, respectively (for the definition of spin polarization used here, see [1]).

where q' = n, p satisfying $q \neq q'$, and $\rho = \rho_q + \rho_{q'}$. Although the full introduction of the tensor force includes more terms compared to $\Delta \mathbf{W}_q(\mathbf{r})$ [13], it is sufficient to investigate $\Delta \mathbf{W}_q(\mathbf{r})$ with respect to study the spin-current tensor contribution to the spin polarization. Indeed, only the spin-current tensor terms contribute directly to the spin-polarization, where it is readily seen that the spin-current tensor contribution corresponds to a quantitative modification of the effect due to the spin-orbit force.

2.2. Spin-current tensor contribution in collision situations

The spin-orbit coupling is defined by the scalar triple product

$$\mathbf{L} \cdot \mathbf{S} = -i\hbar \, \left(\mathbf{r} \times \mathbf{p} \right) \cdot \left(\boldsymbol{\sigma} + \boldsymbol{\sigma'} \right) = -i\hbar \, \mathbf{r} \cdot \left(\mathbf{p} \times \left(\boldsymbol{\sigma} + \boldsymbol{\sigma'} \right) \right), \tag{2}$$

where σ and σ' denote the spins of the two nucleons. In collision situations $\mathbf{r} \times \mathbf{p}$ is related to the impact parameter. Comparing Eqs. (1) and (2), $\mathbf{W}_q(\mathbf{r})$ in Eq. (1) plays the role of the vector \mathbf{r} in Eq. (2), where the momentum \mathbf{p} is replaced/approximated using ∇ in the Skyrme energy density functional.

In order to evaluate the spin-current tensor contribution to spontaneous spin polarization, we introduce a proper theoretical setting of heavy-ion collisions. Let the reaction plane be (x, z) with the initial collision direction z, and the direction perpendicular to the reaction plane be y. For simplicity, the spin direction of the initial state is assumed to be parallel to the y-axis. In this setting, because only the z-component of \mathbf{p} and the y-component of $\boldsymbol{\sigma}$ are non-zero, we have

$$\mathbf{L} \cdot \mathbf{S} = -i\hbar \ x \left(p_y \left(\sigma + \sigma' \right)_z - p_z \left(\sigma + \sigma' \right)_y \right) = i\hbar \ x p_z \left(\sigma + \sigma' \right)_y.$$
(3)

We see that only the x-component of the vector \mathbf{r} , and thus the x-component of $\mathbf{W}_q(\mathbf{r})$ play a role. In this setting, the spin-current tensor contribution to the spin polarization can be evaluated by the corresponding x-component of $W_q(\mathbf{r})$.



Figure 2. (color online) Snapshots of the x-component of $\mathbf{W}_p(\mathbf{r})$ projected on the reaction plane for the case shown in Fig. 1. The left and right panels show the components for the spin-current tensor and spin-orbit components, respectively.

3. Spin-current tensor contribution

3.1. Spatial average of the tensor operator

The tensor force is known to be necessary to explain the properties of the deuteron. Here we recall a general feature of the tensor operator (for example, see [14]). It is represented by

$$S_{12} = \left(v_0(r) + v_1(r)\tau \cdot \tau'\right) \left[\frac{(\mathbf{r} \cdot \boldsymbol{\sigma}) (\mathbf{r} \cdot \boldsymbol{\sigma'})}{r^2} - \frac{1}{3}\boldsymbol{\sigma} \cdot \boldsymbol{\sigma'}\right],$$

where τ and τ' denote the isospins of the two nucleons. Integrating up $(\mathbf{r} \cdot \boldsymbol{\sigma})$ $(\mathbf{r} \cdot \boldsymbol{\sigma'})$ over solid angle, we see that

$$\frac{1}{4\pi} \int d\Omega \, \left(\mathbf{r} \cdot \boldsymbol{\sigma} \right) \, \left(\mathbf{r} \cdot \boldsymbol{\sigma'} \right) = \frac{r^2}{3} \boldsymbol{\sigma} \cdot \boldsymbol{\sigma'}.$$

The spatial average of the tensor operator is equal to zero. Therefore the spin-current tensor contribution bring about localized attraction and repulsion.

3.2. Spontaneous spin polarization

A systematic three-dimensional time-dependent density functional calculation is carried out in a spatial box $48 \times 48 \times 48$ fm³ with a unit spatial spacing 0.8 fm, in which a Skyrme-force parameter SV-tls [15] is adopted for the spin-current tensor part, and SLy4d [16, 17] for the remainder including the spin-orbit force; W_0 , $\alpha = C_1 - C'_1$ and $\beta = C_1 + C'_1$ correspond to 128, 71.102 and 35.142 MeV fm⁵, respectively. The relative velocity of the collisions is set to 10 percent of the speed of light, and the initial distance of the colliding nuclei to 20.0 fm. In order to pay special attention to the mass-dependent general feature, we consider central collisions between identical N=Z nuclei: ¹⁶O + ¹⁶O, ⁴⁰Ca + ⁴⁰Ca and ⁵⁶Ni + ⁵⁶Ni.

The appearance of spontaneous spin polarization, which has already been shown even in low-energy central collisions [1, 2], and the corresponding density distribution are shown in Fig. 1. The appearance of spin polarization ensures the validity of the present theoretical framework for examining the spin-current tensor contribution in spin polarization. Because the right panel of Fig. 1 corresponds to $(\sigma + \sigma')_y$ in Eq. (3), the localized pattern of spin distribution leads to the complicated localization of attraction and repulsion. Note that the spatial average of spin polarization for the spin-saturated system is equal to zero.



Figure 3. (color online) The ratio between spin-current tensor and spin-orbit contributions for protons (left panel) and neutrons (right panel). The maximum values of \mathbf{W}_q at time = 6.0 × 10^{-22} s are compared.

3.3. Comparison between spin-current tensor and spin-orbit contributions

Figure 2 compares the x-component of $\mathbf{W}_p(\mathbf{r})$ for the spin-current tensor and spin-orbit components. First, it is clearly seen from Fig. 2 that the spin-current tensor contribution is opposite to the spin-orbit contribution. This is also confirmed to be valid in all the reactions shown above, while not in the y and z-components. Second, the spin-current tensor contribution is small, less than 10 percent of those from spin-orbit force. It follows that the total contribution both from spin-current tensor and spin-orbit terms is not so different from the contribution of the spin-orbit terms alone. The same conclusion is also true for $\mathbf{W}_n(\mathbf{r})$.

The smallness of the spin-current tensor contribution compared to the spin-orbit contribution can be quantitatively generalized to the other cases. It is seen from the left panel of Fig. 3 that the contribution from the spin-current tensor component is generally small, at most 15 percent of that from the spin-orbit component. Furthermore, the spin-current tensor contribution becomes larger for reactions involving a heavier nucleus.

Turning now to the properties of the composite nucleus formed in one of these collisions. For the heaviest case: ${}^{56}\text{Ni} + {}^{56}\text{Ni}$, the corresponding mass number of the composite nucleus is 112. In view of the remarkable spin-orbit splitting even in the ground states of heavy nuclei [18], 15 percent difference due to the tensor force is not negligible. Therefore the spin-current tensor contribution can be much more important to heavy composite nuclei.

4. Conclusion

The major role of the spin-current tensor contribution has been studied in the context of collision dynamics. Its contribution is small and precisely opposite to that of the spin-orbit force. This implies that the spin-current tensor contribution can be obtained by adjusting the strength of the spin-orbit force. The contribution from the spin-orbit force, and thus the spin-orbit splitting, tends to decrease sharply for heavier nuclei, and the spin-current tensor contribution becomes prominent instead.

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