# POINT-LIKE STRUCTURE IN STRING THEORY

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## ABSTRACT

A possible framework for describing hadrons in terms of relativistic strings is reviewed. The modification of the conventional string perturbative vacuum by a gas of point-like closed strings induces dynamical point-like structure, leading to fixed-angle scattering cross-sections that are power-behaved functions of energy.

## 1. Introduction

The origins of relativistic string theory lie in the dual resonance model of the strong interactions of mesons. The spectrum of hadronic resonances discovered in the 1960's suggested that the mesons might be described as extended relativistic strings with a quark at one end and an antiquark at the other. The quantum states of a relativistic string display the characteristic dependence of spin ($J$) on mass ($M$) of straight-line Regge pole trajectories,

$$J = \alpha(M^2) - n, \tag{1}$$

where $n$ is a non-negative integer and $\alpha(s) = \alpha_0 + \alpha' s$. This is in excellent agreement with observation for most of the meson states on leading ($n = 0$) Regge trajectories (such as the degenerate $\rho$, $\omega$, $f$ and $a_2$ trajectories for which $\alpha_0 \sim 0.5$ and $\alpha' \sim 1\text{Gev}^{-2}$). Furthermore, high-energy scattering at fixed momentum transfer also fits the Regge pole parametrization, in which the two-particle $\rightarrow$ two-particle amplitude behaves as $(-t)^{\alpha(s)}$ as $t \rightarrow \infty$ with fixed $s < 0$ (where $t = -(k_1 + k_4)^2$ and $s = -(k_1 + k_2)^2$ are the Mandelstam invariants, $k_r^\mu$ are the $D$-dimensional particle momenta and $\alpha(s) = \alpha_0 + \alpha' s$). More recent data has reinforced this evidence for the string-like structure of the mesons. However, the original bosonic string theory suffers from several problems which make it unsatisfactory as a theory of hadrons:

1. The bosonic string perturbation theory rules do not describe a well-defined second-quantized theory (even ignoring questions relating to the divergences of loop diagrams) due to the presence of tachyons in the spectrum of both the open-string and the closed-string sectors. At best, this problem has yet to be resolved by discovering a stable ground state for the theory and at worst a stable ground state does not exist (as has often been argued over the years). Superstrings do not suffer from this problem.
2. The unrealistic dimensionality of space-time (namely, $D = 26$), is a problem for critical string theory although extra dimensions may be interpreted in terms of internal symmetries or by adding additional world-sheet fields (such as the dilation mode of the world-sheet metric).
3. Another characteristic of conventional formulations of all 'critical' string theories, whether or not they have tachyons, is the presence in the spectrum of a massless spin-2 particle. This may be of profound significance in formulating a unified theory of gravity together with the other forces but it is unfortunate in a theory that attempts to describe the strong interactions. It has been conjectured that 'sub-critical' theories may describe hadrons rather than gravity although current ideas suggest that these theories describe gravity in non-constant background space-times.
4. Any theory that separates the strong interaction from the other forces must possess well-defined gauge-invariant off-shell amplitudes describing the coupling of weak and electromagnetic currents (as is natural in quantum chromodynamics). In conventional string theories of gravity such amplitudes need not exist since these theories supposedly unify all the forces and there is no need to couple the strings to external sources. In contrast, a string theory of hadrons must have such interactions.

5. Conventional strings do not possess point-like substructure, which is in dramatic contradiction with hadronic experiments. Fixed-angle cross sections of string theory decrease exponentially with energy (as was shown at tree level in Veneziano's original paper [1] and, more recently, to all orders in loops [2]), whereas experimental fixed-angle hadronic cross-sections decrease like powers of the energy. This is the most persuasive evidence for the existence of quarks and gluons and their interactions as embodied in QCD. Consequently, QCD has been almost universally accepted as the theory of hadrons.

Despite the aesthetic appeal of QCD, the fact that quarks and gluons are confined makes large-scale properties, such as the spectrum of hadrons, notoriously difficult to calculate from the theory. Various procedures for approximating QCD (notably, the lattice approximation and the large $n$ limit of the continuum theory with a $U(n)$ gauge symmetry) suggest a string-like description of hadrons. It might be that such a description is an approximation that is valid only for long, massive strings. However, it would be appealing if there were some kind of quantum string theory that is precisely equivalent to QCD, even though the theories would look very different in their respective perturbative approximations. In such a case the colour-singlet strings could be thought of as fundamental entities and the confined point-like quarks and gluons would be manifestations of the string dynamics.

I will adopt the latter viewpoint in this talk which will consist of an overview of one approach to developing such a string theory.* This discussion will focus particularly on points 3-5 above (with a few comments on problems 1 and 2).

## 2. Dirichlet String Theory

The scattering amplitudes of string perturbation theory are defined by correlation functions of vertex operators on world-sheets of arbitrary topology. The world-sheets of theories with open strings have boundaries representing the trajectories of the string endpoints. Each boundary is associated with a factor of $g$ (the closed-string coupling constant) in the amplitude. The embedding coordinates, $X^\mu(\sigma,\tau)$, satisfy Neumann conditions $\partial_n X^\mu|_B = 0$ on any boundary labelled $B$ (where $\partial_n$ denotes the derivative normal to the boundary). The endpoints of a single open string may join to form a closed string so that the interacting theory necessarily includes closed strings as well as open ones. Also, the endpoints of open strings carry the quantum numbers of the defining (quark) representation of any classical Lie group, namely, $SO(n), U(n)$ or $Sp(2n)$. For theories with orientable world-sheets the only consistent choice of group is $U(n)$ (most of this paper will be concerned with this case) and open-string states are $n \times n$ representations of this group. In the original conception of string theory the open strings represented meson states and the endpoint quantum numbers were supposed to be the flavour charges carried by the quarks.

The search for well-defined off-shell amplitudes of conventional string theory motivated the study of open string theories with constant Dirichlet boundary conditions

* These ideas are based on suggestions in [3], which have been further developed in [4,5].

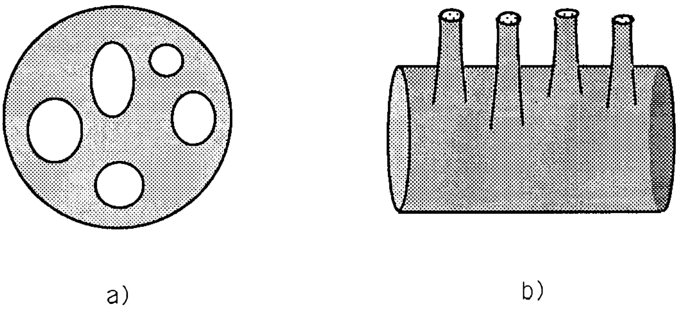


Figure 2.1. a) A world-sheet with an arbitrary number of boundaries. In conventional theories the embedding coordinates satisfy Neumann conditions at all boundaries. In the Dirichlet theory these coordinates are constant on all boundaries, so that each boundary labelled $B$ is mapped into a space-time point, $y_B$. These positions are integrated, so that no energy-momentum enters or leaves a boundary. External on-shell states may be included by attaching vertex operators to the world-sheet. b) In another parametrization the world-sheet is represented as a cylinder emitting closed strings into the vacuum via the Neumann or Dirichlet boundaries.

($\partial_t Y^\mu|_B = 0$, where $\partial_t$ denotes the tangential derivative to the boundary)† so that $Y^\mu|_B = y^\mu_B$, where $y_B$ is the constant position of the boundary $B$. Amplitudes are functions of these positions (and are invariant under a common shift of all the $y_B$'s). After Fourier transformation with respect to $y_B$ they are functions of the momenta flowing through the boundaries, describing off-shell scalar closed-string currents coupling to the boundaries [6]

This construction of amplitudes with external closed-string scalar sources suggests a method for modifying the string vacuum. The modified string theory is obtained (as in fig 2.1) by summing over orientable world-sheets with arbitrary numbers of boundaries at which the coordinates satisfy constant Dirichlet conditions (as well as summing over arbitrary numbers of handles). Gauge invariance dictates that the positions $y_B$ (zero modes of $Y^\mu$ at the boundaries) should be integrated so that no momentum flows through the boundaries, leading to Poincaré-invariant amplitudes. This is analogous to the emission of zero-momentum tadpoles into the vacuum in perturbative quantum field theory (fig 2.1b). More generally, there may be boundaries satisfying the usual Neumann conditions in addition to the Dirichlet boundaries. The presence of Dirichlet world-sheet boundaries is equivalent to a modification of the usual perturbative string vacuum that allows for a condensate of point-like closed strings. The modified theory exhibits point-like short-distance structure. Inserting only a single Dirichlet boundary into the usual world-sheets already dramatically alters the fixed-angle scattering amplitudes so that they decrease like powers of the energy.

† The space-time coordinates will be called $X$ in the theory with Neumann conditions and $Y$ in the theory with constant Dirichlet conditions.

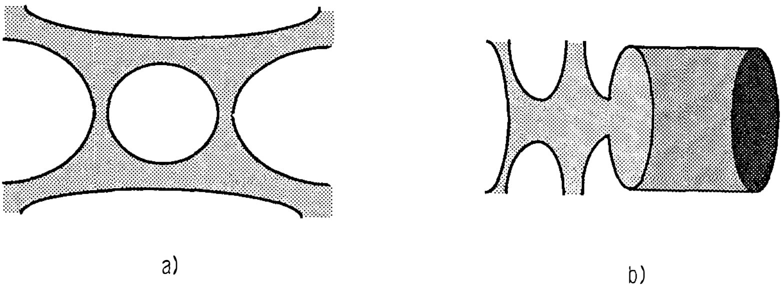


Figure 2.2. a) A one-boundary correction to the tree diagram contributing to the four open-string amplitude. In the usual Neumann theory the free boundary represents the trajectory of endpoints of intermediate open strings and the diagram describes a loop with circulating open strings. With a Dirichlet condition on the free boundary the intermediate strings each have one endpoint fixed at the same space-time point which is integrated over (so no momentum leaks out of the boundary). In this case there are no intermediate two-string states. b) By transforming to a different conformal frame the diagram is a cylinder terminating in the free Dirichlet boundary which represents a point-like closed-string state.

For example, fig 2.2 shows a diagram with one Neumann and one Dirichlet boundary that contributes to the amplitude with four external open strings. In [3] it was shown that this diagram behaves as $(-s)^{-5/2}(-t)^{-5/2}$ at fixed angle and high energy ($s \to \infty$ with $t/s = (\cos\theta - 1)/2$, where $\theta$ is the centre-of-mass scattering angle). This power behaviour arises from the non-locality of the mapping of the boundary into the target space-time. The region of moduli space in which the vertex operators approach the boundary is one in which these operators are close to each other in space-time even though they may be far apart on the world-sheet. A physical feeling for this process is obtained by a conformal transformation to a light-cone parametrization of this process, in which $Y^+ = x^+ + p^+\tau$ (where $v^\pm = v^0 \pm v^{D-1}$ for any $D$-component vector $v^\mu$).

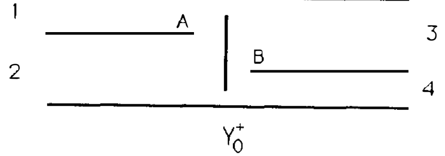


Figure 2.3. The representation of the process depicted in fig 2.2 in the light-cone parametrization in the case of Dirichlet conditions on the free boundary. The widths of the incoming and outgoing strings are $2\pi|p_r|^+$. The vertical slit at $\tau_0 \equiv Y_0^+$ represents the free boundary along which $\partial_\sigma Y^i = 0$ ($i = 1, \dots, D-2$).

In this parametrization the world-sheet is a strip of infinite length in $\tau$ and of width $2\pi p^+$ in $\sigma$, where $p = p_1 + p_2$ is the total momentum carried by the incident strings. Each incoming and outgoing string, carrying momentum $p_r$, is a narrower strip of width $2\pi p_r^+$. Interactions take place at the points (A and B in fig 2.3) at which these strings join and split apart. These interaction times are to be integrated in the expression for the scattering amplitude. A conventional open-string one-loop diagram (fig 2.2 with Neumann conditions on all boundaries) is obtained by inserting a horizontal slit in the world-sheet along which $\partial_\sigma X = 0$ and integrating over its length and its horizontal and vertical positions. The slit denotes the endpoints of intermediate open-string states. However, a Dirichlet condition on the free boundary in fig 2.2 maps it into a single space-time point. The boundary therefore becomes the vertical slit at fixed $\tau = \tau_0 \equiv Y_0^+$ in fig 2.3, along which the transverse embedding coordinates, $Y^i$, are required to be constant. The length of the slit and its horizontal and vertical positions are again to be integrated as is the constant space-time position of the boundary, $y_B^i$. As can be seen from fig 2.3 (the detailed analysis is given in [5]) the integration over the moduli for this surface (*i.e.*, the interaction points and the position and length of the slit) includes an endpoint at which the interaction points may be far apart on the world-sheet but are touching in space-time. This corresponds to histories in which a finite fraction of the total $p^+$ in each incoming string accumulates at one end point — these point-like energy densities then scatter from each other leading to power-behaved cross sections.

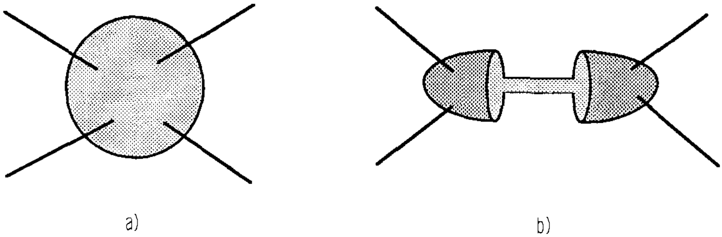

Figure 2.4. a) The one-boundary correction to the scattering of four closed strings is a disk diagram with the vertex operators attached to the interior. b) Another parametrization of the disk which illustrates the presence of an intermediate open string which is a group singlet. With a Dirichlet boundary condition the open string has both endpoints fixed at the same space-time point which is integrated (there are no propagating open-string states).

The simplest theory of this type is the one obtained by including only boundaries with Dirichlet conditions in the sum over orientable world-sheets (this will be referred to as the 'Dirichlet theory'). In this case open strings do not carry any centre-of-mass energy-momentum (since the endpoints of the open strings are fixed in space-time) and there are no open-string states of non-zero momentum in the spectrum. The theory only describes propagating closed bosonic strings with a spectrum that is radically modified by the presence of boundaries to all orders in perturbation theory. The amplitudes diverge at the boundary of moduli space illustrated by fig 2.4b), in the limit where the intermediate open string becomes infinitely long. This divergence is attributable to the presence of the isolated physical intermediate open-string group-singlet state of zero momentum. Regularizing the integral over the modulus suggests that the divergence generates a rather drastic mass shift but this is ill-understood as yet.

In order to avoid the complication of this divergence it is convenient to consider amplitudes with external massless tensor states. For example, denoting the polarization vector of the state of momentum $k_r^\mu$ by $\zeta_r^{\mu\nu}$, the four-particle scattering amplitude has a finite piece proportional to

$$\zeta_{1\mu}^{\nu}\zeta_{2\nu}^{\rho}\zeta_{3\rho}^{\omega}\zeta_{4\omega}^{\mu}\frac{1}{stu}, \tag{2}$$

at high energy and fixed angle. As we saw for the process depicted in fig 2.2, this power behaviour illustrates that the Dirichlet boundary condition generates point-like structure on the string. This is again particularly clear from the light-cone picture for this process (the analogue of fig 2.3).

It is important to verify that the Dirichlet theory amplitudes are gauge invariant. For example, an amplitude with external massless tensor states should be invariant under the linearized gauge transformations,

$$\delta\zeta^{\mu\nu} = k^\mu\zeta^\nu + k^\nu\tilde{\zeta}^\mu, \tag{3}$$

where $k^2 = 0$ and $\zeta$ and $\tilde{\zeta}$ are arbitrary transverse vector gauge parameters. For this to be true it is essential that the boundary positions be integrated (so that no momentum escapes through any boundary).

## 3. Duality of Neumann and Dirichlet theories

The above discussion illustrates that the Dirichlet theory has very different properties from the usual Neumann theory. However, the two theories are related by a space-time (or, target-space) duality transformation.

Target-space duality has been particularly well-studied in the context of compactifications of closed-string theory. Its simplest manifestation arises in considering closed-string theory in a flat space-time with one circular dimension (labelled $i$) of radius $R^i$, so that $Y^i \equiv Y^i + 2\pi R^i$. The string may wind around the compact dimension so the string states are characterized by an integer winding number $n^i$ in addition to the integer, $m^i$, that characterizes the discrete momentum $p^i = m^i/R^i$ of any quantum mechanical system in a periodic box. The masses of the closed-string states are given by

$$\alpha'(\text{Mass})^2 = \frac{\alpha' m^2}{R^2} + \frac{n^2R^2}{\alpha'} + 2N + 2\tilde{N} - 4, \tag{4}$$

where $N$ and $\tilde{N}$ are the integer-valued contributions to the mass from the right-polarized and left-polarized normal modes. This formula is symmetric under the interchanges

$$R^i \to \frac{\alpha'}{R^i}, \qquad m^i \leftrightarrow n^i, \tag{5}$$

which is the discrete $Z_2$ duality transformation. All scattering amplitudes are likewise invariant under these transformations (provided the coupling constant is scaled in an appropriate manner). Duality may also be expressed as the interchange of the space-time coordinates $Y^i(\sigma,\tau)$ with the 'dual space-time' coordinates, $X^i(\sigma,\tau)$, which are related by

$$\partial_\alpha X^i = \epsilon_{\alpha\beta}\partial^\beta Y^i, \tag{6}$$

where $\epsilon_{\alpha\beta}$ is the two-dimensional Levi–Cevita symbol (and $\alpha,\beta$ are two-dimensional world-sheet vector indices).

More generally, duality relates one string theory at radius $R^i$ to a different one at radius $\alpha'/R^i$. For example, the type 2b superstring (which is chiral) is related to the type 2a theory (which is not chiral). When $d$ flat dimensions are compactified the transformation is summarized by [7,8],

$$G_{(ij)} + B_{[ij]} \to (G_{(ij)} + B_{[ij]})^{-1}, \tag{7}$$

where $G$ and $B$ are the symmetric and antisymmetric components of the space-time metric, which are taken to be constant. In this case the discrete symmetry of the bosonic theory is enlarged to $O(d,d;Z)$. The duality property generalizes to constant curved space-time backgrounds, as is illustrated by the relation between 'mirror' Calabi–Yau spaces. More generally still, duality relates solutions of string theory in non-constant gravitational and dilaton backgrounds [9,10]. These ideas are currently the subject of intense study since they may provide a hint of the appropriate description of space-time in a more fundamental, non-perturbative, description of string theories of gravity, even though they are properties that are deduced from string perturbation theory.

The compactification of open-string theory raises new issues [11-17]. One obvious feature is that open strings with Neumann conditions do not possess winding numbers but their centre-of-mass momenta in compact dimensions are quantized as with closed strings. In addition, it is important to consider the non-trivial embeddings of the world-sheet boundaries in the target space [5]. In the simplest case, in which the target space is flat and a subset of the dimensions are compactified on a torus, there is a winding number, $n^i_B$, associated with each Neumann boundary. The variables conjugate to these integers, $y^i_B$, are angular variables and the amplitudes of the compactified theory have a phase factor $e^{\sum_B i y^i_B n^i_B R^i/\alpha'}$ $(0 \le y^i_B \le 2\pi\alpha'/R^i)$. This may be obtained by adding to the usual string action the topological term,

$$\frac{1}{2\pi\alpha'} \sum_B i y^i_B \oint_B dX^i. \tag{8}$$

This term does not affect the $X$ equation of motion but it does modify the momenta of open strings in the compact directions by terms that contribute only at the endpoints.

The overall momentum is quantized in units of $m^i/R^i$ and the resulting mode expansion for the compact coordinates of an open string joining a Neumann boundary with angular variables $y_1^i$ to one with $y_2^i$ is given by,

$$X^i(\sigma,\tau) = x^i + \left(\frac{2m^i\alpha'}{R^i} + \frac{y_2^i - y_1^i}{\pi}\right)\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}^{\infty}\frac{1}{n}\alpha_n^i \cos n\sigma e^{-in\tau} \tag{9}$$

(where $0 \leq \sigma \leq \pi$). The non-compact coordinates have the usual mode expansion, involving the continuous centre-of-mass momentum, $p^\mu$. Open string states are therefore labelled by the continuous $y_B^i$ variables in addition to $p^\mu$, the integers $m^i$ and the oscillator occupation numbers. For example, the open-string world-sheet hamiltonian, $L_0^{open}$, is given by

$$L_0^{open} = \alpha' p^2 + \frac{1}{4\alpha'}\left(\frac{2\alpha' m}{R} + \frac{y_2 - y_1}{\pi}\right)^2 + \sum_{n=1}^{\infty}\alpha_n^\dagger \cdot \alpha_n. \tag{10}$$

The propagator, $1/(L_0^{open} - 1)$ has singularities that depend on the value of $(y_2 - y_1)$ .

The loop amplitude for open strings in the compactified theory with the toplogy of fig 2.2 contains intermediate states of arbitrary $(y_2^i - y_1^i)$ (since the free boundary has an arbitrary value of $y^i$). External open-string states carry definite values of $y^i$ at their endpoints. Therefore, unitarity can only be satisfied in general if the values of $y_B^i$ on any free boundary are integrated over their full angular ranges. It is possible to restrict the $y_B^i$ to discrete subsets, such as the values $y_B^i = 2\pi\alpha' q^i/pR^i$, with integer $p^i$ and $q^i$ satisfying $0 \leq q^i \leq p$. These values of $y_B^i$ must then be summed in order for unitarity to be satisfied.

Target-space duality relates the space-time coordinates, $X^i$, to those of the dual space-time, $Y^i$, by (6). Since $X^i$ satisfies the Neumann conditions $\partial_n X^i = 0$ it follows that the dual coordinates satisfy constant Dirichlet conditions $\partial_t Y^i = 0$. The angular variables, $y_B^i$, conjugate to boundary winding numbers of the Neumann theory are now interpreted as the positions of the point-like boundaries of the Dirichlet theory. The mode expansion for $Y^i$ may be obtained from (9) using (6) and has the form [5],

$$Y^i(\sigma,\tau) = y_1^i + \left(\frac{2m^i\alpha'}{R^i} + \frac{y_2^i - y_1^i}{\pi}\right)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}^{\infty}\frac{1}{n}\alpha_n^i \sin n\sigma e^{-in\tau}. \tag{11}$$

This describes an open string with endpoints fixed at $y_1^i$ and $y_2^i$ winding $m^i$ times around the dual torus (of radius $\alpha'/R^i$).

We may now consider the situation in which *all* space-time dimensions of the Neumann theory are compactified on a torus of scale $R$ (so that $p^\mu = 0$). From target-space duality this is equivalent to a Dirichlet theory on the dual torus of scale $\alpha'/R$. In the limit in which $R \to 0$ the dual torus becomes infinitely big and states with non-zero $m^i$ decouple, giving the Minkowski-space Dirichlet theory considered earlier. This theory can therefore be thought of as the Neumann theory compactified on a 26-dimensional torus with all radii vanishingly small!

We might contemplate the possibility of an open-string theory (with compact radii $R^i$) which transforms into itself (at radii $\alpha'/R^i$) under a duality transformation of this type. Evidently such a theory would have both Neumann and Dirichlet boundaries that transform into each other and the symmetry groups associated with both types of boundary would be the same (*i.e.*, $U(n)$). However, the ghost coordinates (that encode geometrical features of the world-sheet) are inert under target-space duality transformations — a feature that ruins the duality symmetry. It may be that it can be restored by a more subtle transformation involving the ghosts as well as the target-space coordinates.

## 4. Analyticity of Position-Space Amplitudes

We now return to consider the Minkowski-space Dirichlet theory. The open-string propagator for this theory, $1/(L_0^{open} - 1)$, is defined by (10) with $R^\mu \to 0$, $p^\mu = 0$ and $m^\mu = 0$, so that

$$L_0^{open} = \frac{1}{\alpha'}\left(\frac{y_2 - y_1}{2\pi}\right)^2 + \sum_{n=1}^{\infty} \alpha_n^\dagger \cdot \alpha_n, \qquad (12)$$

where the $y_B$ now parametrize the dual Minkowski space. This propagator, together with suitable interaction vertices, can be used to build an arbitrary world-sheet (fig 2.1 with Dirichlet boundaries).

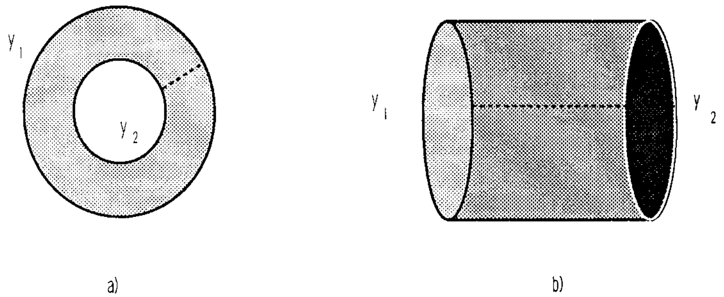


Figure 4.5. A diagram with two Dirichlet boundaries represented a) as a loop of circulating open string with boundary variables $y_1$ and $y_2$ (the dashed line indicates an open string joining the two boundaries) and b) as a cylinder which illustrates the transition between an initial closed string state at $y_1$ to a final state at $y_2$.

For example, the amplitude on a world-sheet with two boundaries, $G(y_2 - y_1)$, can be constructed as a loop of open string summed over all the circulating states (fig 4.5a)) or as a closed string evolving from a space-time point $y_1$ to a point $y_2$. This equivalence is expressed by the equality,

$$G(y_2 - y_1) = \frac{\pi}{(2\pi^2\alpha')^{13}} \int_0^\infty \frac{dl'}{l'} \mathrm{Tr}\left(e^{-(L_0^{open}-1)l'}\right) = \int_0^\infty dl \langle y_1 | e^{-(L_0+\tilde{L}_0-2)l} | y_2 \rangle, \qquad (13)$$

(where the end-states are defined by $Y(\sigma)|y\rangle = y|y\rangle$). These two expressions for $G(y_2 - y_1)$ are related by the transformation $l = 2\pi^2/l'$, where $l'$ is the evolution parameter around the loop of open string and $l$ is the length of the cylinderical world-sheet swept out in the evolution of the closed string (the closed-string hamiltonian being given by $L_0 + \tilde{L}_0$). From the expression (12) it is clear that this amplitude has a rich position-space singularity structure, with logarithmic branch points in $y \equiv y_1 - y_2$ at $y^2 = 4\pi\alpha'(1 - N)$ [18].

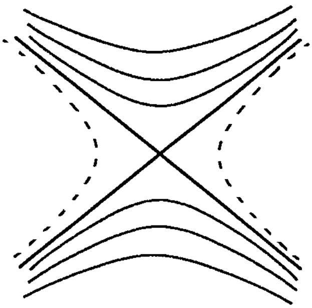

Figure 4.6. The singularities of the Green function, $G(y)$, lie on curves $y^2 = 4\pi^2\alpha'(1 - N)$ (integer $N \geq 0$). The singularity outside the light-cone (indicated by the dashed curve) is analogous to the tachyon singularity in momentum space (causing certain expressions to be ill-defined) and is absent in the corresponding supersymmetric construction.

The presence of singularities at non light-like separations is quite unfamiliar in conventional approximations to quantum field theory. However, it should be borne in mind that string perturbation theory contains an infinite number of stable excited states with degeneracy increasing exponentially with mass. The singularities inside (and outside) the light-cone in fig 4.6 arise from fourier transformation of this exponentially degenerate sequence of stable momentum-space poles. After the perturbation expansion is summed almost all momentum-space states develop decay widths and become unstable. Similarly, it might be expected that the non light-like position-space singularities will disappear from the full theory. In fact, fig 4.5 is modified by higher-order perturbative corrections due to the insertion of arbitrary numbers of zero-momentum Dirichlet boundaries. These lead to branch cuts in $y^2$ that should shield the singularities, just as usual loop diagrams in momentum space generate momentum-space branch cuts that shield the unstable particle poles. The singularity outside the light-cone remains problematic. In the analogous description of superstring theory this singularity is absent (much as the tachyon is absent in superstring theories). However, it is unclear whether a consistent supersymmetric Dirichlet theory can be formulated.

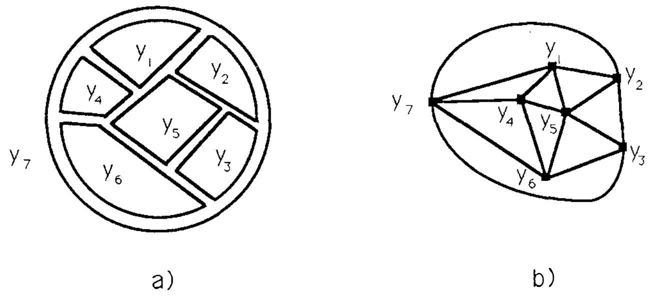


Figure 4.7. a) One of the contributions from the world-sheet depicted in fig 2.1 (in the case of Dirichlet boundaries) in the limit of zero string tension ($\alpha' \to \infty$). The double lines indicate strips of open string of zero width with boundaries fixed at $y_B$. b) The dual diagram, in which the lines indicate poles $1/(y_B - y_{B'})^2$ joining vertices at $y_B$ and $y_{B'}$, with arbitrary polynomial interactions.

The analysis of position-space singularities generalizes to abitrary world-sheets contributing to the Dirichlet theory perturbation series. A general world-sheet (fig 2.1) is composed of strips of open-string sheet with fixed boundaries that lead to position-space poles and branch cuts.

If Dirichlet string theory can be described in terms of a constituent field theory one would expect that in the limit $\alpha' \to \infty$ (high energy or vanishing string tension) the string would break up into its constituent fields. In this limit the position-space singularities away from the light-cone move to infinitely large $y^2$ and decouple, leaving a theory with conventional light-cone singularities (poles and branch points). The string diagram decomposes into a sum of fishnet diagrams such as the one illustrated in fig 4.7a (much as the usual string loop diagrams decompose into sums of Feynman diagrams in the limit $\alpha' \to 0$). The dual of this diagram (fig 4.7b) consists of links intersecting the strips in fig 4.7a, joining pairs of space-time points, $y_B$ and $y_{B'}$. Associated with each link is a pole $1/(y_B - y_{B'})^2$, so the diagram looks like a conventional Feynman diagram in position space. Since the theory is defined in a 26-dimensional space-time these poles correspond to fields with bizarre scaling dimensionality. However, one might envisage that some four-dimensional version of this kind of theory making contact with conventional quantum field theory in this manner.

## 5. A schematic string model of hadrons

The Dirichlet theory (constant coordinates on all boundaries) has qualitative properties reminiscent of the pure glue sector of QCD (pure Yang–Mills theory) with a $U(n)$ gauge group in the $n \to \infty$ limit. The physical states may be thought of as glueball states and the non-trivial condensate of point-like closed strings is somewhat akin to the gas of plaquette-sized flux loops in the strong coupling series approximation to lattice QCD. The spectrum of states of the exact theory is obscure due to the divergent behaviour induced by the mixing of closed strings with group-singlet open strings. It would be gratifying if the point-like behaviour of the Dirichlet theory was connected with the generation of a mass for the graviton. In [5] it was noted that the gauge symmetries of the Dirichlet theory have a structure that suggests a mixing of the graviton with an open-string vector and the scalar dilaton which may induce a graviton mass, but there is no direct evidence for this from low-order calculations. The suggestion that hard-parton behaviour might be correlated with the generation of a mass term for the graviton compliments the suggestion [19] that the absence of hard-parton behaviour results in a theory with a massless spin-2 particle.

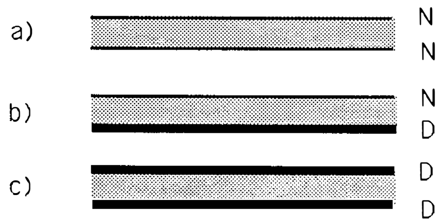


Figure 5.8. Strips of open string that arise in the world-sheet for a theory with both Dirichlet (D) and Neumann (N) boundaries. a) A 'meson string' generates a strip with two Neumann boundaries carrying 'flavour' indices. b) A 'gluon string' generates a strip with two fixed boundaries carrying 'colour' indices. c) A 'quark string' generates a strip with one Neumann boundary (carrying a flavour index) and one Dirichlet boundary (carrying a colour index).

In view of this analogy between Dirichlet theory and large $n$ Yang–Mills theory it is natural to designate the symmetry group associated with a Dirichlet boundary the 'colour' group in order to distinguish it from the 'flavour' $U(n')$ group associated with the Neumann boundaries. Open strings with fixed Dirichlet endpoints (which do not propagate) could then be denoted 'gluon strings'. More generally, in a theory with both Neumann and Dirichlet boundaries the world-sheet (fig 2.1) is composed of strips of open string world-sheet with various combinations of boundary conditions (depicted in fig 5.8). Strings with two Neumann endpoints propagate singlet and non-singlet flavour states and may therefore be referred to as 'meson strings'. Open strings with mixed boundaries carry one flavour index and one colour index. These half-integrally moded strings also do not propagate and may be referred to as 'quark strings'. Closed strings (or 'glueball strings') are propagating flavour and colour singlets. Interactions mix pairs of quark strings with both the meson and the gluon strings while the singlet gluons and mesons both mix with the glueballs. Baryons are soliton states of large-$n$ QCD [20] and may be described as the union of $n \to \infty$ string bits joined together at one end with Neumann conditions at the $n$ free endpoints.

The preceding nomenclature is very speculative and should be taken as an illustration of qualitative ideas. The bosonic theory should not be expected to account for details of real hadronic phenomena.

Apart form a sensible description of the spectrum and interaction of hadrons it is essential that a theory of the strong force allow for the coupling of hadrons to the electroweak force. Assuming the validity of perturbative electroweak theory, this means that the hadrons must couple to point-like flavoured electroweak spin-1 currents.

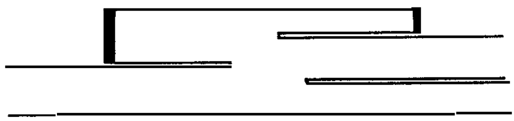

Figure 5.9. A possible scheme for coupling flavour currents to open meson (Neumann) strings in order to include the interaction of mesons with the electroweak force. The figure illustrates two off-shell strings coupling to three on-shell states. The thick lines denote segments of the boundary which satisfy Dirichlet boundary conditions and correspond to space-time points where momentum enters or leaves the external source.

One ansatz for such off-shell current amplitudes is illustrated in fig 5.9. In this the external source is taken to couple to a point-like open string which then evolves with Neumann boundary conditions in the usual manner. The resulting world-sheet is one with segments of boundary satisfying Dirichlet conditions (modified by Neumann and Dirichlet closed-boundary insertions, as well as handles, at higher order in perturbation theory), through which momentum can enter or leave the system. These point-like string configurations couple to external sources with non-trivial flavour quantum numbers. This construction (for scalar sources) was suggested in the 1970's [21-23] but has inconsistencies which have not yet been resolved. In the case of two-dimensional large-$n$ QCD an analogous picture of the off-shell amplitudes works in detail. For example, the large momentum behaviour of the two-current correlation function may be expressed either as a free quark loop or as a sum over all the poles in the 'meson' propagator that couple directly to the currents. This provides an illustration of how a quark constituent picture may be equivalent to a theory expressed entirely in terms of gauge-invariant bound states (although this model lacks many important features of four-dimensional physics).

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