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Construction of $6D$ supersymmetric field models in $\mathcal{N} = (1,0)$ harmonic superspace

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Abstract

We consider the six-dimensional hypermultiplet, vector and tensor multiplet models in $(1,0)$ harmonic superspace and discuss the corresponding superfield actions. The actions for free $(2,0)$ tensor multiplet and for interacting vector/tensor multiplet system are constructed. Using the superfield formulation of the hypermultiplet coupled to the vector/tensor system we develop an approach to calculation of the one-loop superfield effective action and find its divergent structure.

1. Introduction

The construction of the non-Abelian $(1,0)$ and $(2,0)$ superconformal theories in $6D$ has attracted much attention for a long time (see e.g. [1,2]). Such models are considered as the candidates for dual gauge theories of the interacting multiple M5-branes [3] and can be related to near-horizon AdS_7 geometries. A crucial ingredient of this construction is the non-Abelian tensor multiplet gauge fields [4].[1] A solution to this problem has been found [6] in the framework of

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[1] Recently, a number of papers devoted to constructing the class of non-Abelian superconformal $(1,0)$ and $(2,0)$ theories in six dimensions have appeared (see [5] and references therein). These works were inspired by papers [2],

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a tensor hierarchy [7] which, besides the Yang–Mills gauge field and the two-form gauge potentials of the tensor multiplet, contains the non-propagating three- and four-forms gauge potentials. Construction of the $(2, 0)$ models can be realized on the base of coupling the $(1, 0)$ non-Abelian tensor/vector models to the superconformal hypermultiplets [8]. We also mention the work [9] where the Killing spinor equations of 6-dimensional $(1, 0)$ superconformal theories have been solved and the solutions for the configuration of the background fields preserving 1,2, 4 and 8 supersymmetries have been found.

Superfield formulation of the tensor hierarchy has been studied in the paper [10] where a set of constraints on the super-$(p+1)$-form field strengths of non-Abelian super-p-form potentials in $(1, 0)$ $D6$ superspace has been proposed. These constraints restrict the field content of the super-p-forms to the fields of the non-Abelian tensor hierarchy. The superfield formulation of the tensor hierarchy sheds light on a supersymmetric structure of the theory and can serve as a base for the various generalizations. They can be useful for searching the superfield action [11] and for studying the $(2, 0)$ superconformal theory by superspace methods. However, the superfield Lagrangian formulation of the theory under consideration has not been constructed so far.

In this paper we are going to develop the superfield methods for studying the open problems related to superfield formulation of the vector/tensor system and calculating the quantum effective action. Our consideration is based on the harmonic superspace technique formulated for four dimensions in [12,13] and extended to six dimensions in [14,15]. The superfield realization of the unitary representations of $(n, 0)$ superconformal algebras $OSp(8^\star/2n)$ in six dimensions [16] has been found in [17] and it was shown that the $D6$, $(1, 0)$ and $(2, 0)$ tensor multiplets are described by the analytic superfields in appropriately defined harmonic superspaces [18]. In this paper we are going to demonstrate that a harmonic superspace formalism can be efficiently implemented for the superfield Lagrangian construction of the tensor hierarchy models.

The paper is organized as follows. Section 2 is devoted to the basic notations of the $6D$ harmonic superspace. In Section 3 we are going to review the superfield formulations of the $6D$ $(1, 0)$ hypermultiplet [15], the vector multiplet [14] and the tensor multiplet [19], in harmonic superspace. We also discuss the structure of the $(2, 0)$ tensor multiplet. The material of Section 3 is used in the other sections to formulate the new superfield models. Section 4 is devoted to the superfield Lagrangian construction of the $(2, 0)$ tensor multiplet in terms of the $(1, 0)$ hypermultiplet and the $(1, 0)$ tensor multiplet. In Section 5 we are going to study the superfield Lagrangian formulation for the non-Abelian vector/tensor system. We begin with a harmonic superspace reformulation of the results of the paper [10], then we propose the superfield action for the superconformal models of tensor hierarchy and, using the results of Section 5, we derive the component structure of the superfield action and show that it coincides with the component Lagrangian which was constructed in [6]. Section 6 will demonstrate a power of superfield methods. This section is devoted to a study of the quantum effective action in the $(1, 0)$ hypermultiplet theory coupled to the Abelian vector/tensor system. We are going to develop the superfield proper time technique, which will allow us to calculate the effective action in manifestly supersymmetric and gauge invariant form, and calculate the divergent part of the effective action. We will prove that this divergent part contains a term, providing the charge renormalization in the vector/tensor action from Section 5, and a higher derivative action, found in [20]. In conclusion we will summarize the results obtained. In Appendix A we will describe the basic notations and conventions

which explored the 3-algebra gauge structure and used a non-propagating vector field of negative scaling dimension which transforms nontrivially under the non-Abelian gauge symmetry.

of $6D$ supersymmetry. Appendix B contains some details of deriving the component action from superfield action of vector/tensor system.

2. $6D, \mathcal{N}=(1,0)$ harmonic superspace

Harmonic superspace is a powerful formalism for the off-shell construction of extended supersymmetric field theories in four and six dimensions [12–15]. In this section we will briefly describe the basic notations and conventions which are used in this paper (see the details of $D=6$ superspace e.g. in [21]).[2]

It is well known that in six dimensions there are two independent supersymmetry generators. Therefore, the representations of the $6D$ superalgebra are defined by two integers (p,q) [21]. The corresponding supersymmetries are generally denoted as $\mathcal{N}=(p,q)$ or simply (p,q). In this paper we will construct the harmonic superfield models corresponding to $\mathcal{N}=(1,0)$ and $\mathcal{N}=(2,0)$ supersymmetries.

The six-dimensional superspace is parameterized by the coordinates $z=\{x^{\alpha\beta}\equiv x^m\gamma_m^{\alpha\beta}, \theta_I^\alpha\}$. Here the odd coordinates θ_I^α $(\alpha=1,\dots,4)$ are the right-handed chiral spinors of the group $SU^*(4)\sim SO(1,5)$ (left-handed spinors are denoted as ψ_α). The index I is the spinor one of the group $USp(2n)$ (below we use $n=1,2$) and n corresponds to the $\mathcal{N}=(n,0)$ supersymmetry. The properties of the matrices $\gamma_m^{\alpha\beta}$ are given in Appendix A. The index I is raised and lowered with the help of the $USp(2n)$ matrix Ω^{IJ} (see the properties of this matrix e.g. in [25]), $\psi_I=\Omega_{IJ}\psi^J$, $\psi^I=\Omega^{IJ}\psi_J$, $\Omega^{IJ}\Omega_{JK}=\delta^I_K$. The Grassmann coordinates obey the reality condition $\overline{\theta_I^\alpha}=\theta^{\alpha I}=\Omega^{IJ}\theta_J^\alpha$. The basic spinor derivatives of the $6D, \mathcal{N}=(n,0)$ superspace are

$$D^I_\alpha=\frac{\partial}{\partial\theta^\alpha_I}-i\theta^{I\beta}\partial_{\alpha\beta},\qquad \left\{D^I_\alpha, D^J_\beta\right\}=-2i\Omega^{IJ}\gamma^m_{\alpha\beta}\partial_m. \tag{2.1}$$

The symmetry group of the superspace involves $USp(2n)$ transformations of the R-symmetry.

The harmonic $D6$, $\mathcal{N}=(1,0)$ superspace was introduced in [14,15,20] and it is parameterized by the coordinates $(x^m,\theta^{\alpha i},u^{\pm i})$, where harmonics $u^{\pm i}$ $(\widetilde{u_i^\pm}=u^{\pm i}, u^{+i}u_i^-=1\ (i=1,2))$ live on the coset R-symmetry of the group $SU(2)/U(1)$. Besides the standard (or central) basis $(x^m,\theta_i^\alpha,u_i^\pm)$ one can introduce the analytical basis $(\zeta_A^M=\{x_A^m,\theta^{+\alpha}\},u_i^\pm,\theta^{-\alpha})$:

$$x_A^a=x^a+i\theta^-\gamma^a\theta^+,\qquad \theta^{\pm\alpha}=u_i^\pm\theta^{\alpha i}. \tag{2.2}$$

The important property of the coordinates $\zeta_A^M, u_i^\pm$ is that they form a subspace closed under $\mathcal{N}=(1,0)$ supersymmetry transformations. The covariant harmonic derivatives which form the Lie algebra of $SU(2)$ group ($[D^{++},D^{--}]=D^0$) in the analytic basis have the form

$$D^{++}=u^{+i}\frac{\partial}{\partial u^{-i}}+i\theta^+\not\partial\theta^++\theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}},\qquad D^{--}=u^{-i}\frac{\partial}{\partial u^{+i}}+i\theta^-\not\partial\theta^-+\theta^{-\alpha}\frac{\partial}{\partial\theta^{+\alpha}},$$
$$D^0=u^{+i}\frac{\partial}{\partial u^{+i}}-u^{-i}\frac{\partial}{\partial u^{-i}}+\theta^{+\alpha}\frac{\partial}{\partial\theta^{+\alpha}}-\theta^{-\alpha}\frac{\partial}{\partial\theta^{-\alpha}}.$$

By using the analytic subspace, one can define the analytical superfields, which do not depend on $\theta^{-\alpha}$, i.e. they satisfy the condition of the Grassmann analyticity $D^+_\alpha\phi=0$, where the spinor derivatives in the analytic basis have the form

[2] Harmonic superspace is closely related to projective superspace which is also successfully applied to off-shell formulations of extended supersymmetric theories (see e.g. the recent papers [22–24]).

$$D^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}}, \qquad D^-_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}} - 2i\partial_{\alpha\beta}\theta^{-\beta}, \qquad \{D^+_\alpha, D^-_\beta\} = 2i\partial_{\alpha\beta}. \tag{2.3}$$

The possibility to formulate the theory in terms of G-analytic superfields is a crucial advantage of the harmonic superspace formalism.[3]

3. Harmonic superfields and their interactions

It is known that the massless conformal $(1,0)$ and $(2,0)$ superfields in six dimensions are divided into two classes: (i) the superfields whose first component carries any spin but it is an $USp(2n)$ singlet; (ii) the 'ultrashort' analytic superfields in harmonic superspace, their first component is a Lorentz scalar but it carries $USp(2n)$ indices [17]. All these superfields satisfy some superspace constraints. In this paper we will consider the simplest superfields from both of the above classes, which correspond to the following three types of $(1,0)$ $6D$ multiplets: the hypermultiplet, the vector multiplet and the tensor multiplet.

3.1. Hypermultiplet

The $(1,0)$ and $(2,0)$ hypermultiplets are described by the superfields $q^I(x,\theta)$ and their conjugate $\bar{q}_I(x,\theta)$, $\bar{q}_I = (q^I)^\dagger$, both in the fundamental representation of $USp(2n)$ group. Here $i = 1,2$ for the $(1,0)$ case and $I = 1,\dots,4$ for the $(2,0)$ case. The corresponding constraint is

$$D^{(I}_\alpha q^{J)}(x,\theta) = 0. \tag{3.1}$$

In the case of $\mathcal{N} = (1,0)$ supersymmetry, the superfield $q^i(x,\theta)$ has a short expansion $q^i(z) = f^i(x) + \theta^{\alpha i}\psi_\alpha(x) + \dots$. The doublet of scalars f^i and the spinor ψ_α satisfy the equations $\Box f^i = 0$, $\partial^{\alpha\beta}\psi_\beta = 0$. As a result, the $\mathcal{N} = (1,0)$ hypermultiplet in six dimensions has 4 bosonic + 4 fermuinic real degrees of freedom.

The off-shell Lagrangian formulation of the hypermultiplet is based on the use of the analytic superfields in harmonic superspace. In this formulation the hypermultiplet is described by an unconstrained analytic superfield $q^+_A(\zeta,u)$ satisfying the reality condition $\widetilde{(q^{+A})} \equiv q^+_A = \varepsilon_{AB}q^{+B}$

$$D^+_\alpha q^+_A(\zeta,u) = 0. \tag{3.2}$$

Here $A = 1,2$ is a Pauli–Gürsey index lowered and raised by ε_{AB}, ε^{AB}. After the expansion of q^+ in θ^+ and u we obtain an infinite set of auxiliary fields which vanish on-shell due to the equations of motion

$$D^{++}q^+(\zeta,u) = 0. \tag{3.3}$$

The equations of motion follows from the action

$$S_q = -\frac{1}{2}\int d\zeta^{(-4)}du\, q^{+A}D^{++}q^+_A. \tag{3.4}$$

[3] In some cases it may be helpful to use an anti-analytic basis in which

$$x^a_{\bar{A}} = x^a - i\theta^+\gamma^a\theta^-, \qquad D^-_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}}, \qquad D^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}} - 2i\partial_{\alpha\beta}\theta^{+\beta},$$

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + \theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}} - i\theta^+\partial\theta^+, \qquad D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} + \theta^{-\alpha}\frac{\partial}{\partial\theta^{+\alpha}} - i\theta^-\partial\theta^-.$$

Here $d\zeta^{(-4)} = d^6x d^4\theta^+$. This formulation allows us to write down the most general self-couplings in the form of the arbitrary potential $\mathcal{L}^{(+4)}(q^+, \tilde{q}^+)$ [13]. The corresponding sigma models have the complex hyper-Kähler manifolds as their target manifolds [26].

3.2. Vector multiplet

The off-shell $(1,0)$ non-Abelian vector supermultiplet is realized in $6D$ conventional superspace as follows.[4] As usual one introduces the gauge-covariant derivatives

$$\mathcal{D}_M = D_M + \mathcal{A}_M, \qquad [\mathcal{D}_M, \mathcal{D}_N] = T_{MN}{}^L \mathcal{D}_L + F_{MN},$$

with $D_M = \{D_m, D^i_\alpha\}$ being the flat covariant derivatives obeying the anticommutation relations (2.1), and $\mathcal{A}_M$ being the gauge connection taking the values in the Lie algebra of the gauge group. The gauge-covariant derivatives under consideration obey the constraints $F^{ij}_{\alpha\beta} = 0$ and

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = -2i\varepsilon^{ij}\mathcal{D}_{\alpha\beta}, \qquad [\mathcal{D}^i_\gamma, \mathcal{D}_{\alpha\beta}] = -2i\varepsilon_{\alpha\beta\gamma\delta}W^{i\delta}. \tag{3.5}$$

Here $W^{i\alpha}$ is the superfield strength of the anti-Hermitian superfield gauge potential obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace. In this case, the integrability condition $\{\mathcal{D}^+_\alpha, \mathcal{D}^+_\beta\} = 0$ yields $\mathcal{D}^+_\alpha = e^{-ib} D^+_\alpha e^{ib}$ with some Lie-algebra valued harmonic superfield $b(z,u)$ of zero harmonic $U(1)$ charge. In the λ-frame, the spinor covariant derivatives $\mathcal{D}^+_\alpha$ coincide with the flat ones, $\mathcal{D}^+_\alpha = D^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}}$, while the harmonic covariant derivatives acquire the connection V^{++},

$$\mathcal{D}^{++} = D^{++} + V^{++}. \tag{3.6}$$

The connection is an unconstrained analytic potential of the theory. In the Wess–Zumino gauge, the component expansion of $V^{++}(\zeta, u)$

$$V^{++}_{WZ} = \theta^{+\alpha}\theta^{+\beta}A_{\alpha\beta}(x_A) + (\theta^+)^3_\alpha \lambda^{-\alpha}(x_A) + 3(\theta^+)^4 Y^{--}(x_A), \tag{3.7}$$

involves the physical gauge fields and the auxiliary fields.

The other non-analytic harmonic connection $V^{--}(z,u)$ is uniquely determined in terms of V^{++} as a solution of the zero-curvature condition [27,13]

$$D^{++}V^{--} - D^{--}V^{++} + [V^{++}, V^{--}] = 0. \tag{3.8}$$

The connection V^{--} transforms as $\delta V^{--} = -\mathcal{D}^{--}\Lambda$ under gauge transformations. Here the gauge parameter Λ is an analytic anti-Hermitian superfield.

Using the connection V^{--} one can build the spinor and the vector superfield connections as follows

$$\mathcal{A}^-_\alpha = -D^+_\alpha V^{--}, \qquad \mathcal{A}_{\alpha\beta} = \frac{i}{2} D^+_\alpha D^+_\beta V^{--}.$$

This yields

$$W^{+\alpha}_\lambda = -\frac{1}{4}(D^+)^{3\alpha} V^{--}, \tag{3.9}$$

[4] An incomplete list of the references includes [14,15,20,21,27,28,31].

where $W^{\alpha i}_{\lambda}$ is the field strength in the λ-frame. The Bianchi identities lead to relations

$$D^+_\alpha W^{-\alpha} = D^-_\alpha W^{+\alpha}, \qquad D^\pm_\alpha F_{ab} = i D_{[a}(\gamma_{b]})_{\alpha\beta} W^{\pm\beta}. \tag{3.10}$$

The vector superfield strength is defined as follows: $F^\beta_\alpha = (D^-_\alpha W^{+\beta} - D^+_\alpha W^{-\beta})$. The other useful consequences of the Bianchi identities are

$$\begin{aligned}
&D^+_\beta W^{+\alpha} = \frac{1}{4}\delta^\alpha_\beta Y^{++}, \qquad Y^{++} = -\left(D^+\right)^4 V^{--}, \qquad \mathcal{D}^{++} Y^{++} = 0,\\
&W^-_\alpha = \mathcal{D}^{--} W^+_\alpha, \qquad \frac{1}{2}\mathcal{D}^{--} Y^{++} = D^-_\alpha W^{+\alpha},\\
&D^{++} W^{+\alpha} = 0.
\end{aligned} \tag{3.11}$$

These relations define the superfield Y^{++} which will be used further.

The superfield action of $6D$ SYM theory is written in the form

$$S_{SYM} = \frac{1}{f^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\operatorname{tr}\int d^{14}z du_1 \ldots du_n \frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u^+_1 u^+_2)\ldots(u^+_n u^+_1)}. \tag{3.12}$$

Here f is the dimensional coupling constant ($[f] = -1$). The corresponding equations of motion have the form $Y^{++} = (D^+)^4 V^{--} = 0$. The component fields of $W^{+\alpha}$ and V^{++} are related to each other with help of the zero-curvature condition (3.8) and due to the definition (3.9).

It is known that the superfield action with dimensionless coupling constant [20] has the form

$$S = \frac{1}{2g^2}\operatorname{tr}\int d\zeta^{(-4)} du \left(Y^{++}\right)^2. \tag{3.13}$$

It possesses the superconformal symmetry and contains the higher derivatives.

To conclude this section, we give the decomposition of the superfield V^{--} in terms of the component fields

$$\begin{aligned}
&V^{--}\left(x_A,\theta^-,\theta^+,u\right) = \theta^{-\alpha}\theta^{-\beta} v_{\alpha\beta}\left(x_A,\theta^+\right) + \left(\theta^-\right)^3_\alpha v^{+\alpha}\left(x_A,\theta^+\right) + \left(\theta^-\right)^4 v^{++}\left(x_A,\theta^+\right),\\
&v_{\alpha\beta} = A_{\alpha\beta} + \frac{1}{4}\varepsilon_{\alpha\beta\gamma\delta}\theta^{+\gamma}\lambda^{-\delta} - \frac{1}{4}\varepsilon_{\alpha\beta\gamma\delta}\theta^{+\gamma}\theta^{+\delta} Y^{--},\\
&v^{+\alpha} = -\frac{1}{2}\lambda^{+\alpha} + \theta^{+\alpha} Y^{+-} + \theta^{+\beta} i f^\alpha_\beta + \theta^{+\gamma}\theta^{+\delta}\delta^\alpha_{[\gamma}\omega^-_{\delta]} + \left(\theta^+\right)^3_\beta \kappa^{(-2)\beta\alpha} + \left(\theta^+\right)^4 \sigma^{(-3)\alpha},\\
&\frac{1}{2} f^\alpha_\beta = (\gamma_{mn})^\alpha_\beta F_{mn}, \qquad F_{mn} = \partial_m A_n - \partial_n A_m - i[A_m, A_n], \qquad \mathcal{D}_{\alpha\beta} = \partial_{\alpha\beta} - i[A_{\alpha\beta},\cdot],\\
&3\omega^-_\alpha = i\mathcal{D}_{\alpha\beta}\lambda^{-\beta}, \qquad \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\kappa^{(-2)\gamma\delta} = 2i\mathcal{D}_{\alpha\beta} Y^{--} + \frac{1}{4}\varepsilon_{\alpha\beta\gamma\delta}\left\{\lambda^{-\gamma},\lambda^{-\delta}\right\},\\
&v^{++} = Y^{++} + \theta^{+\alpha}\chi^+_\alpha + \theta^{+\alpha}\theta^{+\beta}\Omega_{\alpha\beta} + \left(\theta^+\right)^3_\alpha \rho^{-\alpha} + \left(\theta^+\right)^4 \pi^{(-2)},\\
&\chi^+_\alpha = i\mathcal{D}_{\alpha\beta}\lambda^{+\beta}, \qquad \varepsilon^{\alpha\beta\gamma\delta}\Omega_{\gamma\delta} = -2i\mathcal{D}^{\alpha\beta} Y^{+-} - \mathcal{D}^{[\alpha\gamma} f^{\beta]}_\gamma - \frac{1}{4}\left\{\lambda^{+[\alpha},\lambda^{-\beta]}\right\},\\
&\mathcal{D}^{(\alpha\delta} f^{\beta)}_\delta = 0,\\
&\rho^{-\alpha} = 2i\mathcal{D}^{\alpha\beta}\chi^-_\beta + \frac{1}{2}\left[\lambda^{+\alpha}, Y^{--}\right] - \left[\lambda^{-\alpha}, Y^{+-}\right] + i\left[\lambda^{-\beta}, f^\alpha_\beta\right],\\
&\pi^{(-2)} = \mathcal{D}^{\alpha\beta}\mathcal{D}_{\alpha\beta} Y^{--} - \frac{1}{2}\left\{\lambda^{-\alpha},\mathcal{D}_{\alpha\beta}\lambda^{-\beta}\right\} + 3\left[Y^{+-}, Y^{--}\right].
\end{aligned} \tag{3.14}$$

The relation (3.14) defines the complete component structure of the superfield V^{--} in terms of the components of the superfield potential V^{++}. The component at $(\theta^-)^4(\theta^+)^4$ has already been calculated in [20] (at the different conventions). Further we will need all the components of $V^{--}(x_A, \theta^+, \theta^-, u)$.

3.3. Linear multiplet in harmonic superspace

In this subsection we will briefly review a self-dual tensor multiple and its description in harmonic superspace [19].

As it is well known, the so-called self-dual tensor multiplet contains a scalar ϕ, a spinor $\psi_{i\alpha}$ and an antisymmetric tensor B_{ab} subject to the self-dual constraint

$$\partial_{[a} B_{bc]} - \frac{1}{6}\varepsilon_{abcdef}\partial^d B^{ef} = 0. \tag{3.15}$$

There are two ways to describe the self-dual tensor multiplet in harmonic superspace.

Firstly, one introduces the superfield $\Phi(x, \theta)$ subject to the constraint

$$D^{(i}_\alpha D^{j)}_\beta \Phi = 0. \tag{3.16}$$

Such a superfield is also called a linear. This superfield has no external indices and obey the reality condition $\overline{\Phi} = \Phi$. In the case of $\mathcal{N} = (1, 0)$ supersymmetry, the component expansion of the superfield $\Phi(x, \theta)$ has the form

$$\Phi = \phi + \theta^\alpha_i \psi^i_\alpha + \theta^{\alpha i}\theta^\beta_i G_{(\alpha\beta)} + \dots, \tag{3.17}$$

where the component fields satisfy the massless equations of motion. Note that the field $G_{(\alpha\beta)}$ is related to 3-form field G_{abc}, $G_{(\alpha\beta)}(x) = (\gamma^{abc})_{\alpha\beta} G_{abc}(x)$ and is self-dual by definition. We see that on shell, the linear superfield contains all components of self-dual tensor multiplet.

The above $(1, 0)$ self-dual tensor multiplet (with the constraint (3.16)) is formulated in harmonic superspace [19] with the help of real superfield, which satisfy the constraints

$$D^+_\alpha D^+_\beta \Phi = 0, \qquad D^{++}\Phi = 0. \tag{3.18}$$

The first of them means that $\Phi(x, \theta^+, \theta^-, u)$ is linear in θ^-:

$$\Phi = l\big(x_A, \theta^+, u\big) + \theta^{-\alpha} f^+_\alpha\big(x_A, \theta^+, u\big), \tag{3.19}$$

where the coefficient functions are the analytic superfields

$$l = \phi + \theta^+\psi^-, \qquad f^+_\alpha = -\psi^+_\alpha - \theta^{+\beta} i\partial_{\alpha\beta}\phi + \theta^{+\beta}G_{(\alpha\beta)} - i\partial_{\alpha\beta}\theta^{+\beta}\theta^{+\gamma}\psi^-_\gamma. \tag{3.20}$$

The dynamical equations follow from the second constraint (3.18) which reduces the harmonic dependence of l and f^+ to a polynomial and thus produces a finite supermultiplet. It leads to the following harmonic constraints

$$\hat{D}^{++} l + \theta^{+\alpha} f^+_\alpha = 0, \qquad \hat{D}^{++} f^+_\alpha = 0, \tag{3.21}$$

from which we obtain the equations of motion for the components of the self-dual tensor multiplet

$$\partial^{\alpha\beta}\psi^+_\beta = 0, \qquad \Box\phi = 0, \qquad \partial^{\alpha\gamma} G_{\gamma\beta} = 0.$$

Note that all these components are the field strengths. Besides, the kinematical constraints (3.18) is solved [19] by introducing the prepotential

$$\Phi = \left(D^+\right)^{3\alpha}\Phi^{(-3)}_\alpha. \tag{3.22}$$

Another way to formulate the tensor multiplet in superfield form is based on the superfield $\mathcal{V}^{\alpha i}$ subject to the kinematical constraints [19]

$$D^{(i}_\beta \mathcal{V}^{j)\alpha} - \frac{1}{4}\delta^\alpha_\beta D^{(i}_\gamma \mathcal{V}^{j)\gamma} = 0. \tag{3.23}$$

In the harmonic superspace one gets the superfield $\mathcal{V}^{+\alpha}(x,\theta^+,\theta^-,u)$ under the following constraints

$$D^+_\alpha \mathcal{V}^{+\beta} - \frac{1}{4}\delta^\beta_\alpha D^+_\gamma \mathcal{V}^{+\gamma} = 0, \qquad D^{++}\mathcal{V}^{+\alpha} = 0. \tag{3.24}$$

In the analytic basis we have

$$\mathcal{V}^{+\alpha}\left(x_A,\theta^-,\theta^+,u\right) = v^{+\alpha}\left(\theta^+\right) + \theta^{-\alpha}v^{(+2)}\left(\theta^+\right), \tag{3.25}$$

here $v^{+\alpha}$ and $v^{(+2)}$ are the arbitrary analytic superfields. The second dynamical constraint $D^{++}\mathcal{V}^{+\alpha} = 0$ then becomes

$$\hat{D}^{++}v^{(+2)} = 0, \qquad \hat{D}^{++}v^{+\alpha} + \theta^{+\alpha}v^{(+2)} = 0. \tag{3.26}$$

The component expansions of these superfields are obtained from the above relations in the form

$$v^{(+2)} = f^{(+2)} + \theta^{+\alpha}\kappa^+_\alpha + \left(\theta^+\right)^{2\alpha\beta}a_{\alpha\beta} + \left(\theta^+\right)^3_\alpha\tau^{-\alpha} + \left(\theta^+\right)^4 C^{(-2)},$$
$$v^{+\alpha} = \rho^{+\alpha} + \theta^{+\beta}\left(B^\alpha_\beta + \delta^\alpha_\beta\Sigma\right) + \left(\theta^+\right)^{2\beta\gamma}\omega^{-\alpha}_{\beta\gamma} + \left(\theta^+\right)^3_\beta E^{(-2)\beta\alpha} + \left(\theta^+\right)^4\varphi^{(-3)\alpha}.$$

The kinematical constraints (3.24) are solved by introducing the prepotential [19]:

$$\mathcal{V}^{+\alpha} = \left(D^+\right)^{3\alpha}\mathcal{V}^{(-2)}, \tag{3.27}$$

where

$$\mathcal{V}^{(-2)} = \left(\theta^-\right)^3_\alpha v^{+\alpha} + \left(\theta^-\right)^4 v^{(+2)}. \tag{3.28}$$

This prepotential is defined up to the Abelian gauge transformations

$$\delta_\Lambda \mathcal{V}^{(-2)} = -D^{--}\Lambda. \tag{3.29}$$

If

$$\Lambda \sim \cdots + \frac{1}{2}\theta^{-\alpha}\theta^{+\beta}i\Lambda_{\alpha\beta} + \varepsilon_{\alpha\beta\gamma\delta}\theta^{-\alpha}\theta^{-\beta}\theta^{+\gamma}\rho^{+\delta} + \left(\theta^-\right)^3_\alpha\theta^{+\alpha}f^{++} + \ldots,$$

then

$$\delta\mathcal{V}^{+\alpha} \sim \theta^{+\beta}\partial^{\alpha\gamma}\Lambda_{\gamma\beta} - \rho^{+\alpha} - \theta^{-\alpha}f^{+2}, \qquad \delta B^\alpha_\beta = \partial^{\alpha\gamma}\Lambda_{\gamma\beta} - \frac{1}{4}\delta^\alpha_\beta\partial^{\gamma\delta}\Lambda_{\delta\gamma}.$$

The fields $f^{ij}(x)$, ρ^α_i form a multiplet of gauge degrees of freedom, they can be excluded by an appropriate gauge choice, i.e. one can use the Wess–Zumino gauge.

By substituting the quantities $v^{+\alpha}$, $v^{(+2)}$ in (3.26) we find the general solution

$$\tau^{i\alpha} = 2i\partial^{\alpha\beta}\kappa^i_\beta, \qquad \omega^{-\alpha}_{\beta\gamma} = -\frac{1}{2}\delta^\alpha_{[\beta}\kappa^-_{\gamma]}, \qquad a^{\alpha\beta} = -\frac{i}{2}\partial^{[\alpha\gamma}B^{\beta]}_\gamma - i\partial^{\alpha\beta}\Sigma,$$

as well as the on-shell conditions

$$\Box\Sigma = 0, \qquad \partial^{(\alpha\gamma}B_\gamma^{\beta)} = 0, \qquad \partial^{\alpha\beta}\kappa_\beta^i = 0. \tag{3.30}$$

At the same time the fields $C^{(-2)}$, $\varphi^{(-3)\alpha}$, $E^{(-2)\alpha\beta}$ are eliminated by the choice of gauge $f^{ij} = 0$, $\rho_\alpha^i = 0$.

The free action for the dynamical equations (3.26), (3.21) has been proposed in [19]

$$S_{TM} = \int d^6x\, d^8\theta\, du \Phi_\alpha^{(-3)} D^{++}\mathcal{V}^{+\alpha} = \int d^6x\, d^8\theta\, du \Phi D^{++}\mathcal{V}^{(-2)}. \tag{3.31}$$

This action is invariant under the above gauge transformations of the $\mathcal{V}^{(-2)}$ together with the gauge invariant condition for Φ ($\delta_\Lambda\Phi = 0$),

$$\delta S_{TM} = \int d^8\theta\, du \Phi D^{++}D^{--}\Lambda = \int d^8\theta\, du D^{++}\Phi D^{--}\Lambda = 0,$$

where the on-shell equation $D^{++}\Phi = 0$ has been used. Note also that all the constraints (3.23), (3.16) for the superfields $\mathcal{V}^{-\alpha} = D^{--}\mathcal{V}^{+\alpha}$ and Φ can be solved in the anti-analytic basis of the harmonic superspace, where $D_\alpha^- = -\frac{\partial}{\partial\theta^{+\alpha}}$:

$$\mathcal{V}^{-\alpha}\big(x_{\bar{A}}, \theta^-, \theta^+, u\big) = v^{-\alpha}\big(\theta^-\big) + \theta^{+\alpha}v^{(-2)}\big(\theta^-\big) = \big(D^-\big)^{3\alpha}\mathcal{V}^{+2}, \tag{3.32}$$

and

$$\Phi\big(x_{\bar{A}}, \theta^-, \theta^+, u\big) = l\big(\theta^-\big) + \theta^{+\alpha}f_\alpha^-\big(\theta^+\big) = \big(D^-\big)^{3\alpha}\Phi_\alpha^{(+3)}. \tag{3.33}$$

This BF-type action (3.31) describes two tensor multiplets one of which acts as a Lagrange multiplier for the equations of motion of the other multiplet:

$$S = \int d^6x \big(G_{\alpha\beta}^+\partial^{(\alpha\gamma}B_\gamma^{\beta)} + i\psi_\alpha^+\partial^{\alpha\beta}k_\beta^- + \phi\Box\Sigma\big). \tag{3.34}$$

We see that the superfield $\Phi_\alpha^{(-3)}$ describes those degrees of freedom, which are killed in 3-form G_{abc} by the self-duality condition. According to the work [19] the self-dual fields do not exist off-shell on their own.[5]

In the analytic subspace of the harmonic superspace the analytic superfields

$$\mathcal{G}^{++} = D_\alpha^+\Phi\mathcal{V}^{+\alpha} + \frac{1}{4}\Phi D_\alpha^+\mathcal{V}^{+\alpha}, \qquad D_\alpha^+\mathcal{G}^{++} = 0, \tag{3.35}$$

allow us to rewrite the action (3.31) in the form

$$S = \int d\zeta^{(-4)}du\bigg\{D_\alpha^+\Phi D^{++}\mathcal{V}^{+\alpha} + \frac{1}{4}\Phi D^{++}D_\alpha^+\mathcal{V}^{+\alpha}\bigg\}. \tag{3.36}$$

This expression completely corresponds to the standard recipe for constructing the superfield action in harmonic/projective superspace (see e.g. [13,23,24]) and will be used below for constructing the interacting superfield action of the vector/tensor system.

[5] It would be interesting to quantize such a theory and study the effective action analogously to the self-dual YM theory [32]. One can expect that these fields can be propagating due to quantum corrections.

3.4. (2, 0) *Tensor multiplet*

The field content of the six-dimensional (2, 0) tensor multiplet consists of a self-dual 3-form curvature $G_{(\alpha\beta)}(x) = (\gamma^{abc})_{\alpha\beta} G_{abc}$ with three on-shell degrees of freedom, four left-handed spinors $\psi^I_\alpha(x)$ and five scalars $\phi^{IJ}(x) = -\phi^{JI}(x)$ which satisfy the condition $\Omega_{IJ}\phi^{IJ} = 0$ [17]. All these component fields can be encoded into the Ω-traceless scalar superfield $L^{[IJ]}(x, \theta^I)$ ($I, J = 1, 2, 3, 4$; **5** of $USp(4)$), subject to the differential constraints

$$D^K_\alpha L^{IJ} - \frac{2}{5} D_{\alpha L}\left(\Omega^{KI} L^{LJ} - \Omega^{KJ} L^{LI} + \frac{1}{2}\Omega^{IJ} L^{LK}\right) = 0. \tag{3.37}$$

One can also impose the reality condition $\overline{L_{IJ}} = \Omega_{IK}\Omega_{JL} L^{KL}$. The constraints on the trace-free part of $D^K_\alpha L^{IJ}$ arise as a consistency condition on the embedding an M5-brane in an eleven-dimensional superspace [29]. Using the spinor derivative algebra (2.1) it is not difficult to show that this superfield has the following θ expansion

$$\begin{aligned} L^{IJ} &= \phi^{IJ} + \left(\theta^{\alpha[I}\psi^{J]}_\alpha + \frac{1}{2}\Omega^{IJ}\theta^{\alpha K}\psi_{\alpha K}\right) \\ &\quad + \left(\theta^{\alpha[I}\theta^{\beta J]} + \frac{1}{2}\Omega^{IJ}\theta^{\alpha K}\theta^\beta_K\right)\frac{1}{2} G_{(\alpha\beta)} + \ldots. \end{aligned} \tag{3.38}$$

The corresponding component fields satisfy the massless equations of motion

$$\Box\phi^{IJ} = 0, \qquad \partial^{\alpha\beta}\psi^I_\beta = 0, \qquad \partial^{\alpha\gamma} G_{\gamma\beta} = 0.$$

The latter equation implies that the 3-form G_{abc} is the curl of a 2-form $G_{abc} = \partial_{[a} B_{bc]}$, or $G_{\alpha\beta} = \partial_{(\alpha\gamma} B^\gamma_{\beta)}$. The gauge transformations now take the form $B^\alpha_\beta \to \partial^{\alpha\gamma}\Lambda_{\beta\gamma} - \frac{1}{4}\delta^\alpha_\beta\partial^{\gamma\delta}\Lambda_{\gamma\delta}$.

There are various complications in formulation of (2, 0) interacting theories with non-Abelian gauge group [2,3,5]. It is still unclear whether a superfield action for this multiplet actually exists.

4. (2, 0) tensor multiplet in $D6$, (1, 0) harmonic superspace

In this section we are going to show that the (2, 0) tensor multiplet can be formulated in (1, 0) harmonic superspace in terms of the (1, 0) tensor multiplet and hypermultiplet.

It is easy to see that the total on-shell field contents of the (1, 0) hypermultiplet and the (1, 0) tensor multiplet exactly coincides with one of the (2, 0) tensor multiplet. Therefore it seems natural that the dynamical theory of the (2, 0) tensor multiplet can be constructed in the (1, 0) harmonic superspace in terms of the (1, 0) hypermultiplet and the (1, 0) tensor multiplet. Consider a sum of actions for hypermultiplet (3.4) and (1, 0) tensor multiplet (3.26). We show that this total action possesses by extra hidden (1, 0) supersymmetry. Taking into account the manifest (1, 0) supersymmetry of the actions (3.4) and (3.26), one gets a (2, 0) supersymmetry of the total action.

Let us write the above total action in the form

$$S^{(2,0)} = S_q + S_T = \int d^6x d^8\theta du \Phi D^{++}\mathcal{V}^{(-2)} + \frac{1}{2}\int d\zeta^{(-4)} du q^+_A D^{++} q^{+A}. \tag{4.1}$$

Here $A = 1, 2$ is the index of the Pauli–Gürsey rigid $SU(2)$ symmetry. As mentioned in the previous section, there are two ways to interpret the action S_T. Therefore we can define two types of hidden supersymmetry transformations.

First, we treat the superfield Φ in action S_T as Lagrangian multiplier and $\mathcal{V}^{(-2)}$ as the basic superfield. We define the hidden supersymmetry transformations in the form

$$\delta q_A^+ = \left(D^+\right)^4 \epsilon_A^\alpha \Phi_\alpha^{(-3)}, \qquad \delta\mathcal{V}^{(-2)} = -\epsilon_A^\alpha \left(\theta^-\right)_\alpha^3 q^{+A}, \qquad \delta\Phi = 0, \tag{4.2}$$

where ϵ_A^α is the transformation parameter. Then, the variation of the hypermultiplet action is

$$\delta S_q = \int d^6x d^8\theta du \epsilon_A^\alpha \Phi_\alpha^{(-3)} D^{++} q^{+A}. \tag{4.3}$$

The variation of the tensor multiplet action looks like

$$\begin{aligned}\delta S_T &= \int d^6x d^8\theta du \Phi D^{++} \delta\mathcal{V}^{(-2)} = \int d^6x d^8\theta du \Phi_\alpha^{-3} D^{++} \left(D^+\right)^{3\alpha} \delta\mathcal{V}^{(-2)} \\ &= -\int d^6x d^8\theta du \epsilon_A^\alpha \Phi_\alpha^{(-3)} D^{++} q^{+A}.\end{aligned} \tag{4.4}$$

We see that $\delta S_q + \delta S_T = 0$.

Second, we treat the superfield $\mathcal{V}^{(-2)}$ in action S_T as the Lagrangian multiplier and the superfield Φ as the basic superfield. In this case we define the hidden supersymmetry transformations in the form

$$\delta q_A^+ = \left(D^+\right)^4 \epsilon_A^\alpha D_\alpha^- \mathcal{V}^{(-2)}, \qquad \delta\mathcal{V}^{(-2)} = 0, \qquad \delta\Phi = -\epsilon^{\alpha A} D_\alpha^- q_A^+. \tag{4.5}$$

Then

$$\delta S_q = \int d^6x d^8\theta du \epsilon_A^\alpha D_\alpha^- \mathcal{V}^{(-2)} D^{++} q^{+A}, \tag{4.6}$$

and

$$\delta S_T = -\int d^6x d^8\theta du \epsilon^{\alpha A} D^{++} q_A^+ D_\alpha^- \mathcal{V}^{(-2)} = \int d^6x d^8\theta du \epsilon_A^\alpha q^{+A} D^{++} D_\alpha^- \mathcal{V}^{(-2)}. \tag{4.7}$$

We see again that $\delta S_q + \delta S_T = 0$.

As a result we have constructed the free action for the $(2,0)$ tensor multiplet in the $(1,0)$ harmonic superspace in terms of the $(1,0)$ hypermultiplet and the $(1,0)$ tensor multiplet. This action is invariant under the manifest $(1,0)$ supersymmetry transformations and under the hidden $(1,0)$ supersymmetry transformations.

It is interesting to study whether the supersymmetry algebra is closed. Let us begin with formulation on the base of superfields q^+ and $\mathcal{V}^{(-2)}$. The transformation laws for these superfields are given by (4.2). Then it is not difficult to obtain that

$$[\delta_2, \delta_1]\Phi = 2i\epsilon_1^{\beta A} \epsilon_{2A}^\alpha \partial_{\alpha\beta} \Phi. \tag{4.8}$$

Here we have used the identity $(D_\alpha^+ D_\beta^- + D_\alpha^- D_\beta^+)\Phi = 0$, which follows from the constraints (3.16). For the hypermultiplet we have

$$[\delta_2, \delta_1] q_A^+ = 2i\epsilon_1^{\alpha B} \epsilon_{2B}^\beta \partial_{\alpha\beta} q_A^+. \tag{4.9}$$

We see that the algebra of the hidden supersymmetry transformations is closed. An analogous consideration can be carried out for the formulation with basic superfields q^+ and Φ. The corresponding algebra is also closed.

5. The interacting $D6$ $(1,0)$ vector and tensor multiplets in harmonic superspace

5.1. Non-Abelian vector/tensor system

In this subsection we will briefly mention the general non-Abelian couplings of vectors and antisymmetric p-form fields in six dimensions following [6]. The $(1,0)$ superconformal $6D$ field theory of [6] (vector/tensor system) describes a hierarchy of non-Abelian scalar, vector and tensor fields $\{\phi^I, A^r_a, Y^{ij\,r}, B^I_{ab}, C_{abc\,r}, C_{abcd\,A}\}$ and their supersymmetric partners that label by the indices $r = 1,\dots,n_V$ and $I = 1,\dots,n_T$. To label the $C_{abc\,r}$ field a dual index r is used since the vector fields are dynamically dual to the antisymmetric three-form tensors. The full non-Abelian field strengths of vector and two-form gauge potentials are given as

$$
\begin{aligned}
&\mathcal{F}^r_{ab} = \partial_{[a}A^r_{b]} - f_{st}{}^r A^s_a A^t_b + h^r_I B^I_{ab},\\
&\mathcal{H}^I_{abc} = \frac{1}{2}\mathcal{D}_{[a}B_{bc]} + d^I_{rs}A^r_{[a}\partial_b A^s_{c]} - \frac{1}{3}f_{pq}{}^s d^I_{rs}A^r_{[a}A^p_b A^q_{c]} + g^{Ir}C_{abc\,r}.
\end{aligned}
\tag{5.1}
$$

Here $f_{[st]}{}^r$ are the structure constants, $d^I_{(rs)}$ are the d-symbols, defining the Chern–Simons couplings, and h^r_I, g^{Ir} are the covariantly constant tensors, defining the general Stückelberg-type couplings among forms of different degrees. The existence of the non-degenerate Lorentz-type metric η_{IJ}, so that $h^r_I = \eta_{IJ}g^{Jr} \equiv g^r_I$, $b_{Irs} = 2\eta_{IJ}d^J_{rs} \equiv d_{Irs}$ is also assumed. The covariant derivatives are defined as $\mathcal{D}_m = \partial_m - A^r_m X_r$ with the gauge generators X_r acting on the different fields as follows: $X_r \cdot \Lambda^s \equiv -(X_r)^s_t\Lambda^t$, $X_r \cdot \Lambda^I \equiv -(X_r)^I_J\Lambda^J$. The covariance of the field strengths (5.1) requires that the gauge group generators in the various representations should have the form

$$
(X_r)^t_s = -f_{rs}{}^t + g^t_I d^I_{rs}, \qquad (X_r)^J_I = 2d^J_{rs}g^s_I - g^{Js}d_{Isr},
$$

in terms of the invariant tensors parameterizing the system (see the details in [6]). The field strengths (5.1) are defined such that they transform covariantly under the set of non-Abelian gauge transformations

$$
\begin{aligned}
&\delta A_m = \mathcal{D}_m\Lambda^r - h^r_I\lambda^I_m,\\
&\delta B^I_{mn} = \mathcal{D}_{[m}\Lambda^I_{n]} - 2d^I_{rs}\left(\Lambda^r\mathcal{F}^s_{mn} - \frac{1}{2}A^r_{[m}\delta A^s_{n]}\right) - g^{Ir}\Lambda_{mn\,r}.
\end{aligned}
\tag{5.2}
$$

The superspace realization of the tensor hierarchy has been developed in the paper [10] in conventional $6D$, $(1,0)$ superspace. In the next subsection we are going to consider the generalized Bianchi identities from [10] for the superfield vector/tensor system, reformulate them in the harmonic superspace and study the consistency conditions for the generalized Bianchi identities. For further use, it is convenient to introduce the generalized superfield strength

$$
\mathcal{W}^{i\alpha\,r} = W^{i\alpha\,r} + g^r_I\mathcal{V}^{i\alpha\,I},
\tag{5.3}
$$

where the $W^{i\alpha\,r}$ is the superfield strength of the super Yang–Mills theory (defined in Subsection 3.2) and $\mathcal{V}^{i\alpha I}$ is the superfield of the tensor multiplet (defined in Subsection 3.3), and write the generalized Bianchi identities in its terms. Then one can see that the conventional strength F_{mn} of the vector multiplet and the conventional strength B_{mn} of the tensor multiplet enter into $\mathcal{W}^{i\alpha\,r}$ in the form $F_{mn} + gB_{mn}$.

Using the generalized Bianchi identities and their consistency conditions we will formulate the superfield action for vector/tensor system and find its component form.

5.2. A harmonic superspace description of non-Abelian vector/tensor system

In this subsection we are going to formulate the superfield version of non-Abelian vector/tensor system using the harmonic superspace technique. A complete set of the constraints on superfield strengths of the p-form potentials has been proposed in [10] in conventional $6D$, $(1,0)$ superspace. Our aim is to reformulate these constraints in harmonic superspace and study their consistency conditions.

First of all, the SYM constraint $\mathcal{F}^{ij\,r}_{\alpha\beta} = 0$ is not deformed, therefore we can use a harmonic superfield technique. Then we consider the dimension 2 component of the generalized Bianchi identities

$$\left(\gamma^a\right)_{(\alpha\delta}\mathcal{D}^{(j}_{\beta)}\mathcal{W}^{i)\delta\,r} - 2\varepsilon^{ij}\left(\gamma^b\right)_{\alpha\beta}\mathcal{F}^r_{ab} = \frac{1}{2}\varepsilon^{ij}\gamma^a_{\alpha\beta}\Phi^I g^r_I. \tag{5.4}$$

This relation leads to the covariant derivatives of generalized superfield strength (5.3) in the form

$$\mathcal{D}^i_\alpha\mathcal{W}^{j\beta\,r} = \delta^\beta_\alpha\left(\mathcal{Y}^{ij\,r} + \frac{1}{2}\varepsilon^{ij}\Phi^I g^r_I\right) + \frac{1}{2}\varepsilon^{ij}(\gamma_{ab})^\beta_\alpha\mathcal{F}^r_{ab}. \tag{5.5}$$

This equation is equivalent to the following set of relations

$$\begin{aligned}
&\mathcal{D}^{(i}_\alpha\mathcal{W}^{j)\beta} = \frac{1}{4}\delta^\beta_\alpha\mathcal{D}^{(i}_\gamma\mathcal{W}^{j)\gamma}, \qquad \mathcal{Y}^{ij} = \frac{1}{8}\mathcal{D}^{(i}_\alpha\mathcal{W}^{j)\alpha},\\
&\Phi^I g^r_I = \frac{1}{4}\mathcal{D}_{i\alpha}\mathcal{W}^{i\alpha\,r}, \qquad \mathcal{F}_{ab} = -\frac{1}{8}(\gamma_{ab})^\beta_\alpha\mathcal{D}_{i\beta}\mathcal{W}^{i\alpha}.
\end{aligned} \tag{5.6}$$

We turn now to harmonic superspace formulation. In terms of the harmonic superfields, the relations (5.6) take the form

$$\begin{aligned}
&\mathcal{D}^+_\alpha\mathcal{W}^{+\beta\,r} = \delta^\beta_\alpha\mathcal{Y}^{++\,r}, \qquad \mathcal{D}^-_\alpha\mathcal{W}^{+\beta\,r} = \delta^\beta_\alpha\left(\mathcal{Y}^{+-\,r} + \frac{1}{2}\Phi^I g^r_I\right) + \frac{1}{2}\mathcal{F}^{\beta\,r}_\alpha,\\
&\mathcal{D}^-_\alpha\mathcal{W}^{+\beta\,r} - \mathcal{D}^+_\alpha\mathcal{W}^{-\beta\,r} = \delta^\beta_\alpha\Phi^I g^r_I + \mathcal{F}^{\beta\,r}_\alpha,\\
&\frac{1}{4}\left(\mathcal{D}^-_\alpha\mathcal{W}^{+\alpha\,r} - \mathcal{D}^+_\alpha\mathcal{W}^{-\alpha\,r}\right) = \Phi^I g^r_I.
\end{aligned} \tag{5.7}$$

Consider the dimension $5/2$ component of the generalized Bianchi identities

$$\mathcal{D}^i_\alpha\mathcal{F}^r_{ab} + i(\gamma_{[a})_{\alpha\delta}\mathcal{D}_{b]}\mathcal{W}^{i\delta\,r} = i(\gamma_{ab})^\beta_\alpha\Psi^{iI}_\beta g^r_I. \tag{5.8}$$

It yields $3\Psi^{iI}_\alpha g^r_I = \frac{i}{10}\mathcal{D}^i_\beta\mathcal{F}_\alpha{}^{\beta\,r} + \mathcal{D}_{\alpha\beta}\mathcal{W}^{i\beta\,r}$. In addition, the above deformed identity determines the transformation law for the potential of 2-forms

$$\delta B^I_{ab} = i\epsilon_i\gamma_{ab}\Psi^{iI}.$$

The self-consistency conditions $\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\}\mathcal{W}^{k\delta} = -2i\varepsilon^{ij}\mathcal{D}_{\alpha\beta}\mathcal{W}^{k\delta}$ leads to

$$\mathcal{D}^i_\alpha\Phi^I = 2i\Psi^{iI}_\alpha, \qquad i\mathcal{D}_{\alpha\beta}\mathcal{W}^{\beta i\,r} = -\frac{1}{3}\mathcal{D}_{\alpha l}\mathcal{Y}^{li\,r} + \mathcal{D}^i_\alpha\Phi^I g^r_I, \tag{5.9}$$

$$\mathcal{D}^k_\alpha\mathcal{Y}^{ij\,r} = -i\varepsilon^{k(i}\left(\mathcal{D}_{\alpha\beta}\mathcal{W}^{\beta j)\,r} - 2\Psi^{j)I}_\alpha g^r_I\right). \tag{5.10}$$

By rewriting the above relations in terms of harmonic projection, one gets $\mathcal{D}^+_\alpha\mathcal{Y}^{++\,r} = 0$,

$$\begin{aligned}
&\mathcal{D}^-_\alpha\mathcal{Y}^{++\,r} = -2i\left(\mathcal{D}_{\alpha\beta}\mathcal{W}^{+\beta\,r} - 2\Psi^{+I}_\alpha g^r_I\right),\\
&\mathcal{D}^\pm_\alpha\mathcal{Y}^{+-\,r} = \pm i\left(\mathcal{D}_{\alpha\beta}\mathcal{W}^{\pm\beta\,r} - 2\Psi^{\pm I}_\alpha g^r_I\right).
\end{aligned} \tag{5.11}$$

Acting on the relation $\mathcal{D}^i_\alpha \Phi^I = 2i\Psi^{iI}_\alpha$ by the spinor derivative, one obtains

$$\mathcal{D}_a\Phi^I = \frac{1}{4}\mathcal{D}_{\alpha i}\gamma_a^{\alpha\beta}\Psi^{iI}_\beta. \tag{5.12}$$

The dimension 3 component of the generalized Bianchi identities is

$$\mathcal{D}_{[a}\mathcal{F}_{bc]} = \mathcal{H}^I_{abc}g^r_I, \qquad \mathcal{H}_{abc} = \big(\mathcal{H}^{(+)} + \mathcal{H}^{(-)}\big)_{abc}. \tag{5.13}$$

In the spinor notations it has the form

$$\frac{1}{2}\mathcal{D}_{(\alpha\delta}\tilde{\mathcal{F}}^\delta{}_{\beta)}{}^r = \frac{1}{3}\mathcal{H}^{(-)I}_{\alpha\beta}g^r_I, \qquad \frac{1}{2}\mathcal{D}^{(\alpha\delta}\mathcal{F}^{\beta)\ r}_\delta = \frac{1}{3}\mathcal{H}^{(+)\alpha\beta I}g^r_I. \tag{5.14}$$

The symmetric in (i, j) parts of Eqs. (5.8) have the form

$$\mathcal{D}^{(i}_{(\alpha}(\gamma_{ab})^\delta_{\beta)}\Psi^{j)I}_\delta = 2id^I_{rs}\mathcal{W}^{\rho i\ r}\gamma^{[a}_{\rho(\alpha}\gamma^{b]}_{\beta)\delta}\mathcal{W}^{\delta j\ s}, \qquad \frac{1}{2}\mathcal{D}^{(i}_\alpha\gamma_a^{\alpha\beta}\Psi^{j)I}_\beta = 2i\mathcal{W}^{i\ r}\gamma_a\mathcal{W}^{j\ s}d^I_{rs}.$$

The antisymmetric in (i, j) parts of the same equations have the form

$$\mathcal{H}^{(+)I}_{abc} = \frac{i}{4}d^I_{rs}\mathcal{W}^{\alpha\ r}_i\gamma^{abc}_{\alpha\beta}\mathcal{W}^{i\beta\ s}, \qquad \mathcal{H}^{(-)I}_{abc} = \frac{1}{8}\mathcal{D}_{i\alpha}\gamma^{\alpha\beta}_{abc}\Psi^{iI}_\beta.\text{[6]} \tag{5.15}$$

The above equations for symmetric and antisymmetric parts together imply

$$\mathcal{D}^i_\alpha\Psi^{jI}_\beta = -\frac{1}{2}\varepsilon^{ij}\mathcal{D}_{\alpha\beta}\Phi^I - \frac{1}{12}\varepsilon^{ij}\gamma^{abc}_{\alpha\beta}\mathcal{H}^{(-)I}_{abc} + i\varepsilon_{\alpha\beta\gamma\delta}\mathcal{W}^{i\gamma\ r}\mathcal{W}^{j\delta\ s}d^I_{rs}. \tag{5.16}$$

In terms of the harmonic superfields these relations take the form

$$\begin{aligned}
&\mathcal{D}^\mp_\alpha\Psi^{\pm I}_\beta = \mp\frac{1}{2}\mathcal{D}_{\alpha\beta}\Phi^I \mp \frac{1}{12}\gamma^{abc}_{\alpha\beta}\mathcal{H}^{(-)I}_{abc} + i\varepsilon_{\alpha\beta\gamma\delta}\mathcal{W}^{-\gamma r}\mathcal{W}^{+\delta\ s}d^I_{rs},\\
&\mathcal{D}^\pm_\alpha\mathcal{D}^\pm_\beta\Phi^I = -2\varepsilon_{\alpha\beta\gamma\delta}\mathcal{W}^{\pm\gamma\ s}\mathcal{W}^{\pm\delta\ r}d^I_{sr},\\
&\mathcal{D}^-_\alpha\mathcal{D}^-_\beta\mathcal{Y}^{++\ r} = -i\left(\mathcal{D}_{\alpha\beta}\Phi^I g^r_I + \frac{1}{3}\gamma^{(3)}_{\alpha\beta}\mathcal{H}^{(-)\ I}_{(3)}g^r_I - \mathcal{D}_{\beta\gamma}\mathcal{F}^\gamma{}_{\alpha\ r} - 2\mathcal{D}_{\alpha\beta}\mathcal{Y}^{+-\ r}\right).
\end{aligned} \tag{5.17}$$

The spinor derivative of the 3-rank tensor superfield $\mathcal{H}^I_{abc}$ is

$$\mathcal{D}^i_\alpha\mathcal{H}^I_{abc} = i\gamma^{[a}_{\alpha\beta}\mathcal{W}^{i\beta\ r}\mathcal{F}^{bc]\ s}d^I_{sr} + \frac{i}{2}\mathcal{D}^{[a}\big(\gamma^{bc]}\big)^\beta_\alpha\Psi^{Ii}_\beta - i\gamma^{abc}_{\alpha\beta}\mathcal{W}^{i\beta\ s}\Phi^J d_{Jsr}g^{Ir}. \tag{5.18}$$

This relation also determines the transformation law of the 3-form potential

$$\delta C_{abcr} = -i\epsilon_i\gamma_{abc}\mathcal{W}^{i\ s}\Phi^J d_{Jsr}.$$

The corresponding degrees of freedom are not dynamic since the generalized 4-form field strength satisfies the duality conditions

$$-\frac{1}{4!}\varepsilon^{abcdef}\mathcal{H}_{abcd\ r} = \big(\mathcal{F}^{ef\ s}\Phi^I + i\mathcal{W}^{i\ s}\gamma^{ef}\Psi^I_i\big)d_{I\ rs}. \tag{5.19}$$

As shown in the paper [10], all other relations among the main superfield strengths and the equations of motion can be derived from the following relations

[6] An important feature of these equations is that the anti-self-dual part of the field strength $\mathcal{H}$ is fixed in terms of the dynamical vector multiplet.

$$
\begin{aligned}
&\left(\mathcal{Y}^{ij\ s}\Phi^I - 2i\mathcal{W}^{(i\ s}\Psi^{j)I}\right)d_{I\ rs} = 0,\\
&d_{Irs}\left\{\Phi^I\mathcal{D}_{\alpha\beta}\mathcal{W}_i^{\beta\ s} + \frac{1}{12}\mathcal{H}^{(-)I}_{\alpha\beta}\mathcal{W}_i^{\beta\ s} + \frac{1}{2}\mathcal{D}_{\alpha\beta}\Phi^I\mathcal{W}_i^{\beta\ s} + \frac{1}{2}\mathcal{F}_\alpha{}^{\beta\ s}\Psi^I_{\beta i} + \mathcal{Y}^s_{ij}\Psi^{I\ j}_\alpha\right\}\\
&\quad = +\frac{1}{2}\Psi^I_{\alpha i}\Phi^J\left(4g^s_I d_{Jrs} - g^s_J d_{Irs}\right) + \frac{2i}{3}\varepsilon_{\alpha\beta\gamma\delta}\mathcal{W}^{\beta js}\mathcal{W}_j^{\gamma u}\mathcal{W}_i^{\delta v}d^I_{rs}d_{Iuv},\\
&\mathcal{D}_e\left(\Phi^I\mathcal{F}^{ea\ s} + i\mathcal{W}^{i\ s}\gamma^{ea}\Psi^I_i\right)d_{Irs} + \frac{1}{6}\varepsilon^{abcdef}\mathcal{F}^s_{bc}\mathcal{H}^I_{def}d_{Irs}\\
&\quad + \left(-\frac{1}{4}\Phi^{[I}\mathcal{D}^a\Phi^{J]} + \frac{i}{2}\Psi^{iI}\tilde{\gamma}^a\Psi^J_i\right)X_{rIJ} - \frac{i}{2}\Phi^I\mathcal{W}^{is}\gamma^a\mathcal{W}^t_i(X_r)^u{}_{[s}d_{I\ t]u} = 0.
\end{aligned}
\tag{5.20}
$$

They lead to the Dirac equation for the fermions of the tensor multiplet

$$
\mathcal{D}^{\alpha\beta}\Psi^{iI}_\beta = -\mathcal{Y}^{ij\ r}\mathcal{W}^{\alpha\ s}_j d^I_{rs} + \frac{1}{2}\mathcal{W}^{i\alpha\ r}\Phi^J\left(4d_{Jrs}g^{Is} - g^s_J d^I_{sr}\right) - \frac{1}{2}\mathcal{W}^{i\beta\ r}\mathcal{F}_\beta{}^{\alpha\ s}d^I_{rs}, \tag{5.21}
$$

to scalar superfield equation of motion

$$
\begin{aligned}
\Box\Phi^I = {}& d^I_{rs}\left(\mathcal{F}^r_{ab}\mathcal{F}^{ab\ s} - \mathcal{Y}^r_{ij}\mathcal{Y}^{ij\ s} - i\mathcal{W}^{\alpha\ r}_i\mathcal{D}_{\alpha\beta}\mathcal{W}^{i\beta\ s}\right) + \frac{3}{2}\Phi^J g^r_J\Phi^K g^s_K d^I_{rs}\\
& - i\mathcal{W}^{i\alpha\ r}\Psi_{i\alpha}{}^J\left(4g^s_J d^I_{rs} - g^{Is}d_{J\ rs}\right),
\end{aligned}
\tag{5.22}
$$

and to the second order equations for the 3-form field strength $\mathcal{H}^I_{abc}$

$$
\begin{aligned}
\mathcal{D}^c\mathcal{H}^I_{abc} = {}& -\frac{1}{4}\varepsilon_{abcdef}\mathcal{F}^{cd\ r}\mathcal{F}^{ef\ s}d^I_{rs} + \mathcal{F}^r_{ab}\Phi^J d_{Jrs}g^{rI}\\
& + i\mathcal{W}^r_i\gamma_{abc}\mathcal{D}^c\mathcal{W}^{i\ s}d^I_{rs} - i\mathcal{W}^{i\ r}\gamma_{ab}\Psi^J_i g^s_J d^I_{sr}.
\end{aligned}
$$

These equations of motion allow us to construct a component action of the theory in the form

$$
\begin{aligned}
S = \int d^6x\Big\{ & \frac{1}{2}\mathcal{D}^a\Phi_I\mathcal{D}_a\Phi^I + \Phi_I d^I_{rs}\Big(-Y^{ij\ r}Y_{ij\ s} + \mathcal{F}^{ab\ r}\mathcal{F}^s_{ab} - iW^{\alpha\ r}_i\mathcal{D}_{\alpha\beta}W^{i\beta\ s}\\
& + \frac{1}{2}\Phi^J\Phi^L g^r_J g^s_L\Big) + \Phi_I iW^{\alpha\ r}_i\Psi^{i\ J}_\alpha\left(4g^s_J d^I_{rs} - g^{Is}d_{J\ rs}\right)\\
& + i\Psi^i_{\alpha I}\mathcal{D}^{\alpha\beta}\Psi^I_{i\beta} - 2i\Psi_{i\alpha\ I}W^{\alpha\ r}_j Y^{ij\ s}d^I_{rs} + i\Psi^i_{\alpha\ I}W^{\beta\ r}_i\mathcal{F}_\beta{}^{\alpha\ s}d^I_{rs} + \frac{1}{6}\mathcal{H}^{abc}_I\mathcal{H}^I_{abc}\\
& + \frac{i}{12}W^{i\alpha\ r}\gamma^{abc}_{\alpha\beta}W^{\beta\ s}_i\mathcal{H}^I_{abc}d_{Irs} - \frac{1}{3}\varepsilon_{\alpha\beta\gamma\delta}W^{\alpha\ r}_j W^{i\beta\ s}W^{\gamma\ u}_i W^{j\delta\ v}d^I_{rs}d_{Iuv}\Big\}.
\end{aligned}
\tag{5.23}
$$

Here we assume the existence of the non-degenerate symmetric metric $\eta_{IJ} = g_{Ir}g^r_J$. Also the first components of the superfields are denoted the same way as the corresponding superfields, e.g. $\mathcal{W}^{\alpha i}|_{\theta=0} = W^{\alpha i}$, $\mathcal{Y}^{ij}|_{\theta=0} = Y^{ij}, \ldots$. The action (5.27) coincides (up to field redefinition) with the component action of the superconformal vector/tensor system constructed in the papers [6,30].

The other set of equations given in [10] includes supersymmetrizations of the Hodge-duality relations between the 3-form potential and the non-Abelian vectors and scalar-4-forms relations. All the relations of this subsection are used in the next subsection for finding the component form of superfield action of the vector/tensor system.

5.3. Superfield Lagrangian formulation of the vector/tensor system

In this subsection we are going to propose the superfield action for the non-Abelian vector/tensor system in harmonic superspace and find its component form.

Let us introduce the superfield

$$\Upsilon^I = \Phi^I + \frac{1}{2} d^I_{rs}\big(D^+_\alpha V^{--r}\mathcal{W}^{+\alpha s} + 2V^{--r}\mathcal{Y}^{++s}\big), \tag{5.24}$$

where

$$\mathcal{Y}^{++s} = Y^{++s} + \frac{1}{4} D^+_\alpha \mathcal{V}^{+\alpha s}. \tag{5.25}$$

One should remember that the Y^{++} is defined in Subsection 3.2 and the $\mathcal{V}^{+\alpha s}$ is defined in Subsection 3.3. The expression Υ^I (5.24) is the only extension of Φ preserving linearity, $D^+_\alpha D^+_\beta \Upsilon = 0$, i.e. the Υ^I is a linear superfield. By using the superfield (5.24), one can define the superfield action in harmonic superspace as follows

$$S = \int d\zeta^{(-4)} du g_{Ir}\big\{\Upsilon^I \mathcal{D}^{++}\mathcal{Y}^{++\,r} + D^+_\alpha \Upsilon^I \mathcal{D}^{++}\mathcal{W}^{+\alpha r}\big\}. \tag{5.26}$$

The invariant tensor g_{Ir} has already been defined in [6]. The integrand of the expression (5.26) is the only (up to common coefficient) analytic superfield constructed from Υ^I, $\mathcal{Y}^{++}$, $\mathcal{W}^{+\alpha}$ and contains no higher derivatives of $\mathcal{D}^{++}$, D^+_α. The action (5.26) depends both on superfields of the vector multiplet and superfield Φ responsible for the tensor multiplet. If Φ is a constant $1/f^2$, this action takes the form (3.12) of SYM action $S \sim \frac{1}{f^2}\int d^6x d^8\theta du V^{++}V^{--}$. Besides, the proposed action possesses supersymmetry and gauge symmetries of vector/tensor system.

The action (5.26) is the natural generalization of the free action (3.36). Indeed if we put in (5.24) $\Upsilon = \Phi$ and use the relations (5.25), (5.3) and the identities $D^{++}Y^{++} = 0$, $D^{++}W^{+\alpha} = 0$, one gets the action (3.36). Thus, the action (5.26) is the only possible superfield action for the non-Abelian vector/tensor system which has the free action (3.36) in the Abelian limit.

Now we will derive the component form of the action (5.26). For simplicity we are only going to consider the Abelian case. By integrating over the anticommuting coordinates, one gets

$$S = \frac{1}{8}\int d^6x_A du \big(\mathcal{D}^-\big)^4 g_{I\,r}\big\{\Upsilon^I \mathcal{D}^{++}\mathcal{Y}^{++\,r} + D^+_\alpha \Upsilon^I \mathcal{D}^{++}\mathcal{W}^{+\alpha\,r}\big\}\big|_{\theta=0}. \tag{5.27}$$

Further we act by the derivatives D^-_α and put all the theta's equal to zero. Then act by the harmonic derivative ∂^{++}. After the cumbersome enough calculations,[7] we obtain all the functional structures which are present in the component action of the vector/tensor system (5.23).[8]

6. One-loop effective action in the hypermultiplet theory

In this section we will consider a calculation of the superfield quantum effective action in the hypermultiplet theory coupled to the external field of vector/tensor system. We will show that the $(1,0)$ super Yang–Mills action (3.12), the vector/tensor multiplet action (3.36) or (5.27) and higher derivative vector multiplet action [20] are generated as the divergent parts of the effective action. For simplicity we will assume that the background is Abelian.

[7] The intermediate calculations are given in Appendix B with use of the relations from the Subsection 5.2.

[8] Derivation of a component action in a non-Abelian case requires an additional study.

The classical conformal invariant action for a massless hypermultiplet of canonical dimension 2 coupled to a background $6D$ $\mathcal{N}=(1,0)$ vector/tensor system is written as

$$S=-\frac{1}{2}\int du d\zeta^{(-4)} q^{+A}\mathcal{D}^{++}q^{+}_{A}=-\int du d\zeta^{(-4)}\tilde{q}^{+}\mathcal{D}^{++}q^{+}, \tag{6.1}$$

with $\mathcal{D}^{++}=D^{++}+gV^{++}$ the analyticity-preserving covariant derivative and V^{++} the analytic potential. We want to emphasize that the superfield V^{++} here is not one for pure vector multiplet, the superfield strengths, involving the superfield V^{++}, obey the Bianchi identities which contain the superfield Φ related to tensor multiplet (see Subsection 5.2). As a result the action (6.1) describes interaction of hypermultiplet with vector/tensor system. The dynamical variable q^+ is a covariantly analytic superfield and $\tilde{q}^+$ is the conjugate of q^+ with respect to the analyticity preserving conjugation [13] $q^{A+}=\epsilon^{AB}q^{+}_{B}=\widetilde{(q^{+}_{A})}$, $q^{+}_{A}=(q^{+},-\tilde{q}^{+})$.

The hypermultiplet effective action Γ is defined by

$$e^{i\Gamma[V^{++}]}=\int\mathcal{D}q^{+}\mathcal{D}\tilde{q}^{+}\exp\left(-i\int d\zeta^{(-4)}\tilde{q}^{+}\mathcal{D}^{++}q^{+}\right). \tag{6.2}$$

The expression (6.2) yields

$$\Gamma\left[V^{++}\right]=i\,\mathrm{Tr}\ln\mathcal{D}^{++}=-i\,\mathrm{Tr}\ln G^{(1,1)}. \tag{6.3}$$

Here $G^{(1,1)}(\zeta_1,u_1|\zeta_2,u_2)=\langle\tilde{q}^{+}(\zeta_1,u_1)q^{+}(\zeta_2,u_2)\rangle$ is the superfield Green function in the τ-frame. This Green function is analytic with respect to both arguments and satisfies the equation

$$\mathcal{D}^{++}_{1}G^{(1,1)}_{\tau}(1|2)=\delta^{(3,1)}_{A}(1|2). \tag{6.4}$$

Here $\delta^{(3,1)}_{A}(1|2)$ is the appropriate covariantly analytic delta-function

$$\begin{aligned}\delta^{(q,4-q)}_{A}&=\left(D^{+}_{2}\right)^{4}\delta^{14}(z_1-z_2)\delta^{(q,-q)}(u_1,u_2)\\&=\left(D^{+}_{1}\right)^{4}\delta^{14}(z_1-z_2)\delta^{(q-4,4-q)}(u_1,u_2).\end{aligned} \tag{6.5}$$

The formal solution to this equation can be found analogously to four-dimensional case [33–35] and looks[9] like

$$G^{(1,1)}_{\tau}(1|2)=-\frac{1}{4\,\widehat{\Box}}\left(\mathcal{D}^{+}_{1}\right)^{4}\left(\mathcal{D}^{+}_{2}\right)^{4}\delta^{14}(z_1-z_2)\frac{1}{(u^{+}_{1}u^{+}_{2})^{3}}, \tag{6.6}$$

where $1/(u^{+}_{1}u^{+}_{2})^{3}$ is a special harmonic distribution. In Eq. (6.6) the $\widehat{\Box}$ is the covariantly analytic d'Alembertian ($[D^{+}_{\alpha},\widehat{\Box}]=0$) which arises when $(\mathcal{D}^{+})^{4}(\mathcal{D}^{--})^{2}$ acts on the analytical superfield

[9] As well as in the work [34] we will act by the operator $(\mathcal{D}^{--}_{1})^{2}$ on both sides of (6.4)

$$\mathcal{D}^{++}_{1}(\mathcal{D}^{--}_{1})^{2}G^{(1,1)}(1|2)=(\mathcal{D}^{--}_{1})^{2}\delta^{(3,1)}_{A}(1|2)=\mathcal{D}^{++}_{1}2(\mathcal{D}^{+}_{2})^{4}\frac{\delta^{14}(z_1-z_2)}{(u^{+}_{1}u^{+}_{2})^{3}}.$$

Now, since the equation $D^{++}f^{-|q|}=0$ has only the trivial solution $f^{-|q|}=0$, after the action of the operator $(\mathcal{D}^{+}_{1})^{4}$ we obtain:

$$(\mathcal{D}^{+}_{1})^{4}(\mathcal{D}^{--}_{1})^{2}G^{(1,1)}(1|2)=-8\,\widehat{\Box}\,G^{(1,1)}(1|2)=2(\mathcal{D}^{+}_{1})^{4}(\mathcal{D}^{+}_{2})^{4}\frac{\delta^{14}(z_1-z_2)}{(u^{+}_{1}u^{+}_{2})^{3}}.$$

and has the form

$$\widehat{\Box} = -\frac{1}{8}(\mathcal{D}^+)^4(\mathcal{D}^{--})^2\big| = \mathcal{D}_a\mathcal{D}^a + \mathcal{W}^{+\alpha}\mathcal{D}^-_\alpha + Y^{++}\mathcal{D}^{--} - \frac{1}{4}(\mathcal{D}^-_\alpha\mathcal{W}^{+\alpha}) - \frac{1}{2}\Phi. \tag{6.7}$$

Pay attention to the fact that the term Φ in the above relation is responsible for the tensor multiplet contribution. Like in four- and five-dimensional cases [35] one can obtain the useful identity

$$(\mathcal{D}^+_1)^4(\mathcal{D}^+_2)^4\frac{1}{(u^+_1u^+_2)^3} = (\mathcal{D}^+_1)^4\left\{(u^+_1u^+_2)(\mathcal{D}^-_1)^4 - (u^-_1u^+_2)\Delta^{--} - 4\widehat{\Box}\frac{(u^-_1u^+_2)^2}{(u^+_1u^+_2)}\right\}. \tag{6.8}$$

Here

$$\Delta^{--} = i\mathcal{D}^{\alpha\beta}\mathcal{D}^-_\alpha\mathcal{D}^-_\beta + 4\mathcal{W}^{-\alpha}\mathcal{D}^-_\alpha - (D^-_\alpha\mathcal{W}^{-\alpha}).$$

This identity is used later for computing the effective action.

The definition (6.3) of the one-loop effective action is purely formal. The actual evaluation of the effective action can be done in various ways (see e.g. [34,35]). Further we will follow [35] and use the relation

$$\Gamma(V) = \Gamma_{y=0} + \int_0^1 dy\,\partial_y\Gamma(yV) = -i\,\mathrm{Tr}\int_0^1 dy\big(V^{++}G^{(1,1)}(y)\big), \tag{6.9}$$

where

$$\mathrm{Tr}\big(V^{++}G^{(1,1)}\big) = \int du_1 d\zeta_1^{(-4)}V^{++}(1)G^{(1,1)}(1|2)\Big|_{1=2}. \tag{6.10}$$

Here $G^{(1,1)}(yV)$ means the Green function depending on the superfield yV^{++}.

The effective action in local approximation is represented as a series in powers of the background fields and their derivatives. Further we will consider the calculation of the effective action on the base of the superfield proper-time technique.

It is obvious that the leading non-vanishing contribution on the diagonal $(z_1, u_1) = (z_2, u_2)$ of the two-point function

$$-\frac{1}{4}\int du_1 d\zeta^{(-4)}V_1^{++}\frac{1}{\widehat{\Box}_1}(\mathcal{D}^+_1)^4(\mathcal{D}^+_2)^4\delta^{14}(z_1-z_2)\frac{1}{(u^+_1u^+_2)^3}\Big|_{1=2}, \tag{6.11}$$

arises when $\mathcal{D}^{--}_1$ from $\widehat{\Box}_1$ hits on $(u^+_1u^+_2)|_{u_1=u_2}$ and in addition at least eight spinor derivatives acting on the Grassmann delta-function are required to produce a non-vanishing result, $(\mathcal{D}^-)^4(\mathcal{D}^+)^4\delta^8(\theta_1-\theta_2)|_{\theta_1=\theta_2} = 1$. On the right hand side of (6.8) the third term contains a harmonic distribution which is singular at coincident points. However, this singular terms does not contribute to $\Gamma^{(1)}$ in the leading approximation since there is no necessary degree of D^-_α.

In the framework of the proper-time technique, the inverse operator $\frac{1}{\widehat{\Box}}$ is defined as follows

$$-\frac{1}{\widehat{\Box}} = \int_0^\infty d(is)e^{is\widehat{\Box}}. \tag{6.12}$$

To avoid the divergences on the intermediate steps it is necessary to introduce a regularization. We will use a variant of dimensional regularization (so-called ω -regularization) accommodative for regularization of the proper-time integral (see e.g. [36]). The ω-regularized version of the relation (6.12) is

$$-\left(\frac{1}{\widehat{\Box}}\right)_{reg} = \int_0^\infty d(is)\big(is\mu^2\big)^\omega e^{is\widehat{\Box}}, \tag{6.13}$$

where ω tends to zero after renormalization and μ is an arbitrary parameter of mass dimension. Taking into account the relation (6.13) and the relations (6.6), (6.9) one gets for effective action

$$\frac{1}{4}\int du_1 d\zeta_1^{(-4)} V^{++}(1)\int_0^\infty d(is)\big(is\mu^2\big)^\omega e^{is\widehat{\Box}_1}\big(\mathcal{D}_1^+\big)^4\big(\mathcal{D}_2^+\big)^4\frac{1}{(u_1^+u_2^+)^3}\delta^{14}(z_1-z_2)\bigg|_{1=2}. \tag{6.14}$$

Here $\delta^{14}(z_1-z_2)=\delta^6(x_1-x_2)\delta^4(\theta_1^+-\theta_2^+)\delta^4(\theta_1^--\theta_2^-)$. We use now the representation of the delta function

$$\delta^{14}(z_1-z_2)=\int\frac{d^6p}{(2\pi)^6}e^{ip_a\rho^a}\delta^8\big(\rho_i^\alpha\big),$$

where

$$\rho^a=(x_1-x_2)^a-2i\big(\theta_1^+-\theta_2^+\big)\gamma^a\theta^-, \qquad \rho^{\alpha i}=(\theta_1-\theta_2)^{\alpha i},$$

and $i=+,-$. In the expression (6.14) we commute the exponent $\exp ip_a\rho^a$ through all the operator factors to the left and then use the coincidence limit. This yields to $e^{is\widehat{\Box}_1(X)}\cdot\delta^8(\theta_1-\theta_2)$ where $X_a=\mathcal{D}_a+ip_a$, $X_\alpha^-=\mathcal{D}_\alpha^-+2p_{\alpha\beta}\rho^{-\beta}$. In order to get the expansion of effective action in background fields and their derivatives we should expand $e^{is\widehat{\Box}_1(X)}$ and calculate the momentum integrals. All these integrals have the standard Gauss form.

We will now concentrate on calculating the divergent part of the effective action. In the regularization scheme under consideration, the divergences mean the pole terms of the form $\frac{1}{\omega}$. By expanding the $e^{is\widehat{\Box}_1(X)}$ in the (6.14) and leaving only the terms relating to divergences one gets

$$\begin{aligned}&e^{is\widehat{\Box}_1}\big(u_1^+u_2^+\big)\big(\mathcal{D}_1^+\big)^4\big(\mathcal{D}_1^-\big)^4\delta^8(\theta_1-\theta_2)|_{1=2}\\&\quad=-\int_0^\infty\frac{d(is)}{(is)^3}\big(is\mu^2\big)^\omega e^{-ism^2}\bigg\{isY^{++}+\frac{(is)^2}{2}\big[\widehat{\Box},Y^{++}\big]\bigg\}.\end{aligned} \tag{6.15}$$

Here $m^2=\Phi$. By calculating the proper-time integral and extracting the pole terms one gets for the right hand side of the above expression

$$\frac{1}{\omega}m^2Y^{++}-\frac{1}{2\omega}\Box Y^{++}-\frac{1}{2\omega}\mathcal{W}^{+\alpha}D_\alpha^-Y^{++}. \tag{6.16}$$

By using the conditions (5.11), we obtain finally the divergent part of the effective action in the form

$$\frac{1}{4(4\pi)^3\omega}\int dud\zeta^{(-4)}V^{++}\big(\mathcal{D}_\alpha^+\Phi\mathcal{W}^{+\alpha}+\Phi Y^{++}\big)-\frac{1}{2(8\pi)^3\omega}S_{ISZ}. \tag{6.17}$$

In principle there is also the term $\frac{1}{\omega}\int du d\zeta^{(-4)} V^{++}\mathcal{W}^{+\alpha} i\mathcal{D}_{\alpha\beta}\mathcal{W}^{+\beta}$. However it is cancelled out with the corresponding term $-i\mathcal{D}^{\alpha\beta}\mathcal{D}^-_\alpha\mathcal{D}^-_\beta\frac{(is)^2}{2}\mathcal{W}^{+\gamma}\mathcal{D}^-_\gamma\mathcal{W}^{+\delta}\mathcal{D}^-_\delta$ from the second order expansion $\Delta^{--}e^{is\widehat{\Box}}$.

The divergent part of the effective action contains two contributions. The first of them is the part of the Abelian action (5.26) of the vector/tensor system proposed in Section 5. If we consider a sum of the action (5.26) and first term in (6.17), we will see that this term from (6.17) determines a renormalization of the coupling constant in the (5.26)[10] The second contribution is the Ivanov–Smilga–Zupnik Abelian higher derivative action of the vector multiplet [20][11]

$$S_{ISZ} = \frac{1}{2}\int du d\zeta^{(-4)} Y^{++}Y^{++}. \tag{6.18}$$

One should emphasize once more that we have considered only the divergent parts of the effective action. Of course, the effective action contains the finite part, the calculation of which is extremely interesting, but is a more difficult and delicate problem.

The divergent part of the effective action has been calculated within the ω-regularization. If we use the other regularization schemes, we can expect some extra terms in the divergent part of the effective action. For example, the application of the cut-off regularization results in the same two terms as in (6.17) only with a replacement of the term $\frac{1}{\omega}$ with $\sim \log L^2$ where L^2 is the cut-off on the lower limit of the proper-time integral (see the details e.g. in [36]). However, within this regularization we will get the extra contribution to divergent part of the effective action in the form

$$S_{L^2} \sim L^2\frac{1}{4(4\pi)^3}\int d\zeta^{-4} du V^{++}Y^{++}. \tag{6.19}$$

This term is generated from (6.15) when we take only the Y^{++} in the integrand, put $m^2 = 0$, $\omega = 0$ and cut the integral on the lower limit by L^2. It is easy to see that this result is (up to a coefficient) the Abelian $(1, 0)$ vector multiplet action (3.12).

As a result we see that the classical actions (3.12) and (5.26) of the Abelian theory are generated in quantum theory as the one-loop counterterms. The Abelian higher derivative action introduced in [20] is also generated as the one-loop counterterm. We emphasize once more that a coupling to tensor multiplet is stipulated by the superfield Φ, at $\Phi = 0$ such a coupling vanishes and (6.17) gives us the divergent part of the effective action for the hypermultiplet in a pure vector multiplet background. It is worth pointing out that the superfield calculation of the divergent part of the effective action is simple enough in comparison with the component calculation and demonstrates the power of superfield methods.

[10] Coupling constant g is defined through the covariant derivative $\mathcal{D}^{++} = D^{++} + gV^{++}$.

[11] The action (6.18) can be written in the different superfield forms

$$\begin{aligned} S_{ISZ} &= \int du d\zeta^{(-4)} V^{++}\Box Y^{++} = -\frac{1}{8}\int d^6x d^8\theta V^{++}(D^{--})^2 Y^{++} \\ &= -\frac{1}{8}\int d^6x d^8\theta V^{--}[D^{++}, D^{--}]Y^{++}. \end{aligned}$$

7. Conclusion

We have considered the superfield formulations of a class of six-dimensional supersymmetric models related to the low-energy dynamics of $M5$ branes. These models possess the $\mathcal{N} = (1, 0)$ supersymmetry and describe the hierarchy of interacting scalar, vector and tensor fields and their superpartners [6]. Our main aim was to construct the superfield actions for the above models. We have shown that this aim has been achieved in the framework of a six-dimensional harmonic superspace. As a demonstration of the power of the harmonic superspace approach we have considered the problem of effective action in the hypermultiplet model coupled to the background field of the vector/tensor system.

We have constructed the harmonic superfield Lagrangian formulation of the free $\mathcal{N} = (2, 0)$ tensor multiplet in $\mathcal{N} = (1, 0)$ superspace. The system of the $(1, 0)$ hypermultiplet and $(1, 0)$ tensor multiplet was considered. The corresponding action is a sum of actions for the corresponding $(1, 0)$ harmonic superfields. We have found the hidden $(1, 0)$ supersymmetry transformation which mixes the hypermultiplet and the tensor multiplet and have shown that this transformation leaves the action invariant.

We have proposed the superfield Lagrangian formulation of the non-Abelian tensor hierarchy in $(1, 0)$ harmonic superspace. The superfield action has been formulated in terms of harmonic superfields of vector and tensor multiplets. We reformulated the constraints on the superstrengths [10] in terms of harmonic superspace and by using these constraints we computed the component action corresponding to the proposed superfield action. It was shown that in an Abelian case this component action is analogous to the action of the tensor hierarchy [6].

To demonstrate a power of superfield methods we have considered a problem of quantum effective action in Abelian hypermultiplet theory coupled to background fields of vector/tensor system. Such an effective action is generated by hypermultiplet loop and depends on vector and tensor multiplet superfields. We have constructed the second order differential operator with coefficients, depending on background superfields, which acts on harmonic superfields and defines a form of effective action. The superfield proper-time technique for evaluating the effective action is developed. Such a technique allows to compute the effective action in manifestly supersymmetric and gauge invariant manner. We calculated the divergent part of the effective action and showed that it has a structure analogous to one of vector/tensor multiplet superfield action and defines a renormalization of coupling constant. Also it was shown that the actions of vector and tensor multiplet are generated as the parts of divergences of the effective action.

There are various ways to generalize and apply the obtained results. We will point out two of them. Firstly, in Section 4 we constructed the action of free $(2, 0)$ tensor multiplet in terms of $(1, 0)$ hypermultiplet and tensor multiplets. The main element of this construction was the existence of hidden $(1, 0)$ supersymmetry transformations. We hope that such transformations can also be found in the non-Abelian case which allows us to construct the non-Abelian superfield action for the $(2, 0)$ tensor multiplet. Secondly, in Section 6 we began to study the effective action of the $(1, 0)$ hypermultiplet coupled to the Abelian background field of the vector/tensor system. We developed the superfield proper-time technique for evaluating the effective action and calculated the divergent part of the effective action. It would be extremely interesting to find the finite part of this effective action since it be could a new $6D$ superconformal functional written in terms of harmonic superspace. Also, it would be interesting to study the effective action for the hypermultiplet in non-Abelian vector/tensor background. We hope to consider the above problems in the forthcoming papers.

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Appendix A. Notations and conventions

In six dimensions the $(1,0)$ and $(0,1)$ Weyl spinors belong to the fundamental representation of $SU^*(4) \sim SO(1,5)$ group and to the transpose representation, respectively. The 8×8 Dirac matrices Γ^a (where $a = 0, 1, \ldots, 5$) satisfy the Clifford algebra

$$\Gamma^a\Gamma^b + \Gamma^b\Gamma^a = 2\eta^{ab}. \tag{A.1}$$

The Dirac matrices for even dimensions can be chosen in the form

$$\Gamma^a = \begin{pmatrix} 0 & (\gamma^a)_{\alpha\beta} \\ (\tilde\gamma^a)^{\beta\alpha} & 0 \end{pmatrix},$$

with $\alpha = 1, \ldots, 4$.

Our notation and conventions follow to [10]. We use the metric $\eta^{ab} = \mathrm{diag}(+,-,-,-,-,-)$ as well as $\varepsilon_{abcdef}\varepsilon^{a_1a_2a_3def} = -6\delta^{a_1}_{[a}\delta^{a_2}_{b}\delta^{a_3}_{c]}$. Everywhere the antisymmetrization with the weight 1 is used. We chose the antisymmetric representation of the $6D$ Weyl 4×4 γ-matrices $\gamma^a_{\alpha\beta} = -\gamma^a_{\beta\alpha}$ and

$$\tilde\gamma_a^{\alpha\beta} = -\tilde\gamma_a^{\beta\alpha} = \frac{1}{2}\varepsilon^{\alpha\beta\sigma\delta}(\gamma_a)_{\sigma\delta}, \qquad \gamma^a_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\sigma\delta}(\tilde\gamma_a)^{\sigma\delta}, \tag{A.2}$$

where the $SU^*(4)$ invariant $\varepsilon^{\alpha\beta\sigma\delta}$ is the totally antisymmetric symbol ($\varepsilon_{1234} = \varepsilon^{1234} = 1$). The matrices $\gamma^a_{\alpha\beta}$ obey the relation

$$\left(\gamma^a\tilde\gamma^b + \gamma^b\tilde\gamma^a\right)_\alpha{}^\beta = 2\eta^{ab}\delta_\alpha{}^\beta, \qquad [\gamma_{ab}, \gamma_c] = 2\eta_{[bc}\gamma_{a]}. \tag{A.3}$$

The six-dimensional Pauli-type matrices $\{\gamma^a\}$ and $\{\tilde\gamma^a\}$ are two separate bases of 4×4 antisymmetric matrices so that

$$\gamma^a_{\alpha\beta}\tilde\gamma_a^{\sigma\delta} = -2\delta^\sigma_{[\alpha}\delta^\delta_{\beta]}, \qquad \gamma^a_{\alpha\beta}\gamma^a_{\sigma\delta} = -2\varepsilon_{\alpha\beta\sigma\delta}, \qquad \tilde\gamma_a^{\alpha\beta}\tilde\gamma_a^{\sigma\delta} = -2\varepsilon^{\alpha\beta\sigma\delta}. \tag{A.4}$$

The normalized antisymmetrized product of Pauli-type matrices

$$\gamma^{ab} = \frac{1}{2}\left(\gamma^a\tilde\gamma^b - \gamma^b\tilde\gamma^a\right), \quad (\tilde\gamma_{ab})^\alpha{}_\beta = -(\gamma_{ab})_\beta{}^\alpha,$$

$$\gamma_{abc} = \frac{1}{3!}\gamma_{[a}\gamma_b\gamma_{c]} = \gamma_a\gamma_b\gamma_c - \gamma_a\eta_{bc} + \eta_{ac}\gamma_b - \eta_{ab}\gamma_c$$
$$= \gamma^a\tilde\gamma_{bc} - \eta^{a[b}\gamma^{c]} = \gamma_{ab}\gamma_c + \eta_{c[a}\gamma_{b]},$$

$$\left(\gamma^{abc}\right)_{\alpha\beta} = \left(\gamma^{abc}\right)_{\beta\alpha} = \frac{1}{3!}\varepsilon^{abcdef}(\gamma_{def})_{\alpha\beta},$$

$$(\tilde\gamma_{abc})^{\alpha\beta} = (\tilde\gamma_{abc})^{\beta\alpha} = -\frac{1}{3!}\varepsilon_{abcdef}(\tilde\gamma_{def})^{\alpha\beta} = \tilde\gamma_a\gamma_{bc} - \eta_{a[b}\tilde\gamma_{c]},$$

$$\left(\gamma^{abcdef}\right)_\alpha{}^\beta = -\varepsilon^{abcdef}\delta_\alpha{}^b,$$

$$\left(\gamma^{abcde}\right)_{\alpha\beta} = -\varepsilon^{abcdef}(\gamma_f)_{\alpha\beta}, \qquad \left(\tilde\gamma^{abcde}\right)^{\alpha\beta} = \varepsilon^{abcdef}(\tilde\gamma_f)^{\alpha\beta},$$

$$\left(\gamma^{abcd}\right)_{\alpha\beta} = \frac{1}{2}\varepsilon^{abcdef}(\gamma_{ef})_{\alpha\beta}, \qquad \left(\tilde{\gamma}^{abcd}\right)^{\alpha\beta} = -\frac{1}{2}\varepsilon^{abcdef}(\tilde{\gamma}_{ef})^{\alpha\beta}, \tag{A.5}$$

form the basis of general 4×4 matrices with the completeness relation

$$\begin{aligned}
&\left(\gamma^{ab}\right)_\alpha{}^\beta(\gamma_{ab})_\sigma{}^\delta = 2\delta_\alpha^\beta\delta_\sigma^\delta - 8\delta_\alpha^\delta\delta_\sigma^\beta, \qquad \gamma^{abc}_{\alpha\beta}\tilde{\gamma}^{\sigma\delta}_{abc} = -24\delta_\alpha^{(\sigma}\delta_\beta^{\delta)}, \qquad \gamma^{abc}_{\alpha\beta}\gamma^{abc}_{\sigma\delta} = 0,\\
&\gamma^a_{\alpha\beta}(\gamma_{ab})^\rho_\delta = 2\delta^\rho_\alpha\gamma^b_{\beta\delta} + 2\delta^\rho_\beta\gamma^b_{\delta\alpha} + \delta^\rho_\delta\gamma^b_{\alpha\beta}, \qquad \tilde{\gamma}^{\alpha\beta}_a(\gamma_{ab})^\rho_\delta = 2\delta^\beta_\delta\tilde{\gamma}^{\alpha\rho}_b + 2\delta^\alpha_\delta\tilde{\gamma}^{\rho\beta}_b - \delta^\rho_\delta\tilde{\gamma}^{\alpha\beta}_b,\\
&\gamma^a_{\alpha\beta}(\gamma_{abc})_{\gamma\delta} = 2\varepsilon_{\alpha\beta\gamma\sigma}(\gamma_{bc})_\delta{}^\sigma - \gamma^{[b}_{\alpha\beta}\gamma^{c]}_{\gamma\delta}, \qquad \gamma^a_{\alpha\beta}(\tilde{\gamma}_{abc})^{\gamma\delta} = -2\delta^\gamma_{[\alpha}(\gamma_{bc})^\delta_{\beta]} - \gamma_{[b\alpha\beta}\tilde{\gamma}^{\gamma\delta}_{c]},\\
&(\tilde{\gamma}_{abc})^{\alpha\beta}(\gamma_{ab})^\rho_\beta = 20\tilde{\gamma}^{\rho\alpha}_c, \qquad \gamma^a_{\alpha\delta}(\tilde{\gamma}_{abc})^{\delta\beta} = 4(\gamma_{bc})^\beta_\alpha,\\
&(\gamma_{abc})_{\alpha\beta}\left(\gamma^{ab}\right)^\delta_\gamma = 4\delta^\delta_\alpha\gamma^c_{\beta\gamma} + 4\delta^\delta_\beta\gamma^c_{\alpha\gamma}.
\end{aligned} \tag{A.6}$$

The trace relations are

$$\begin{aligned}
&\mathrm{tr}\left(\gamma^a\tilde{\gamma}^b\right) = 4\eta^{ab}, \qquad \mathrm{tr}\,\gamma_{ab}\gamma^{cd} = -4\delta^c_{[a}\delta^d_{b]},\\
&\mathrm{tr}\left(\gamma_{abc}\tilde{\gamma}^{def}\right) = -4\varepsilon^{abc}{}_{def} - 4\delta^a_{[d}\delta^b_e\delta^c_{f]},\\
&\mathrm{tr}\left(\gamma^a\tilde{\gamma}^b\gamma^c\tilde{\gamma}^d\right) = 4\left(\eta^{ab}\eta^{cd} - \eta^{ac}\eta^{bd} + \eta^{ad}\eta^{bc}\right), \qquad \mathrm{tr}(\gamma_e\tilde{\gamma}_{abc}) = 0,\\
&\tilde{\gamma}_a\gamma^b\tilde{\gamma}_a = -4\tilde{\gamma}^b, \qquad \gamma_e\gamma_{ab}\gamma_e = 2\gamma_{ab},\\
&\gamma_e\gamma_{abc}\gamma_e = 0, \qquad \gamma_e\gamma^{def}\gamma_{ab}\gamma_e = -2\gamma_{[a}\gamma^{def}\gamma_{b]}.
\end{aligned} \tag{A.7}$$

A Minkowski six-vector can be written as the bi-spinor:

$$\begin{aligned}
&x_{\alpha\beta} = \gamma^a_{\alpha\beta}x_a, \qquad x^{\alpha\beta} = \tilde{\gamma}^{\alpha\beta}_a x^a, \qquad x^2 = \frac{1}{4}x^{\alpha\beta}x_{\beta\alpha}, \qquad x_a = \frac{1}{4}\mathrm{tr}(\tilde{\gamma}_a x) \equiv \frac{1}{4}\tilde{\gamma}^{\alpha\beta}_a x_{\beta\alpha},\\
&\partial_{\alpha\beta} = \gamma^a_{\alpha\beta}\partial_a, \qquad \partial_{\alpha\beta}x^{\gamma\delta} = -2\delta^\gamma_{[\alpha}\delta^\delta_{\beta]}, \qquad \partial_{\alpha\beta}x_{\gamma\delta} = -2\varepsilon_{\alpha\beta\gamma\delta}.
\end{aligned} \tag{A.8}$$

The supersymmetric covariant derivatives in the central basis of the $(1,0)$ $D6$ harmonic superspace have the form

$$D^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}} - i\partial_{\alpha\beta}\theta^{+\beta}, \qquad D^-_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}} - i\partial_{\alpha\beta}\theta^{-\beta}, \qquad \left\{D^+_\alpha, D^-_\beta\right\} = 2i\partial_{\alpha\beta}. \tag{A.9}$$

The definition of the vector superfield strength looks like

$$\begin{aligned}
&[\mathcal{D}_a, \mathcal{D}_b] = F_{ab}, \qquad \left\{\mathcal{D}^{(i}_\alpha, \mathcal{D}^{j)}_\beta\right\} = 0,\\
&F_{ab} = -\frac{1}{8}(\gamma_{ab})_\alpha{}^\beta F_\beta{}^\alpha, \qquad F_\alpha{}^\beta = (\gamma_{ab})_\alpha{}^\beta F^{ab}.
\end{aligned} \tag{A.10}$$

The field strength of the 2-form potential can be decomposed as follows

$$H^\pm_{abc} = \frac{1}{2}(H_{abc} \pm *H_{abc}), \qquad *H_{abc} = \frac{1}{6}\varepsilon_{abcdef}H^{def}, \tag{A.11}$$

where in the spinor representation the (anti-)self-dual parts of a 3-form H satisfy the relations

$$H^{(-)}_{\alpha\beta} = H_{abc}\left(\gamma^{abc}\right)_{\alpha\beta}, \qquad H^{(+)\alpha\beta} = H^{abc}(\gamma_{abc})^{\alpha\beta}.$$

We also used the following notations and conventions

$$
\begin{aligned}
&(D^{\pm})^4 = -\frac{1}{4!}\varepsilon^{\alpha\beta\rho\gamma} D^{\pm}_\alpha D^{\pm}_\beta D^{\pm}_\rho D^{\pm}_\gamma, \qquad (D^{+})^{3\alpha} = -\frac{1}{6}\varepsilon^{\alpha\beta\gamma\delta} D^{+}_\beta D^{+}_\gamma D^{+}_\delta,\\
&D^{\pm}_\alpha D^{\pm}_\beta D^{\pm}_\rho = \varepsilon_{\alpha\beta\rho\gamma}(D^{\pm})^{3\gamma}, \qquad D^{\pm}_\alpha D^{\pm}_\beta D^{\pm}_\gamma D^{\pm}_\delta = -\varepsilon_{\alpha\beta\gamma\delta}(D^{\pm})^4,\\
&D^{\pm}_\alpha (D^{\pm})^{3\beta} = \delta^\beta_\alpha (D^{\pm})^4,
\end{aligned}
\tag{A.12}
$$

and also

$$
\begin{aligned}
&\theta^{\pm\alpha}\theta^{\pm\beta}\theta^{\pm\gamma} = -\varepsilon^{\alpha\beta\gamma\delta}(\theta^{\pm})^3_\delta, \qquad (\theta^{\pm})^3_\alpha = \frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta}\theta^{\pm\beta}\theta^{\pm\gamma}\theta^{\pm\delta},\\
&\theta^{\pm\alpha}\theta^{\pm\beta}\theta^{\pm\gamma}\theta^{\pm\delta} = -\varepsilon^{\alpha\beta\gamma\delta}(\theta^{\pm})^4, \qquad (\theta^{\pm})^4 = -\frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\theta^{\pm\alpha}\theta^{\pm\beta}\theta^{\pm\gamma}\theta^{\pm\delta},\\
&\theta^{\pm\alpha}(\theta^{\pm})^3_\beta = -\delta^\alpha_\beta(\theta^{\pm})^4,\\
&(D^{+})^{3\alpha}(\theta^{-})^3_\beta = \delta^\alpha_\beta, \qquad (D^{+})^4(\theta^{-})^4 = 1,\\
&(D^{+})^{3\alpha}(\theta^{-})^4 = \theta^{-\alpha}.
\end{aligned}
\tag{A.13}
$$

Appendix B. Component expansion of the action (5.26)

We will now consider the main steps to derive the component decomposition of the action $S \sim \int d\zeta^{(-4)} du \mathcal{L}^{(+4)}$ in the Abelian case. Here $\mathcal{L}^{(+4)}$ is given by (5.26). To do that, we should integrate over harmonics and over all anticommuting coordinates.

Let us begin with the integration rule over anticommuting coordinates

$$
\int d\zeta^{(-4)} du = \int d^6x du \left(-\frac{1}{4!}\right)\varepsilon^{\alpha\beta\gamma\delta}\mathcal{D}^-_\alpha\mathcal{D}^-_\beta\mathcal{D}^-_\gamma\mathcal{D}^-_\delta . \tag{B.1}
$$

First, we act by spinor derivatives and kill all the theta's and then integrate over harmonics $u^\pm_i$. It is obvious that $\mathcal{D}^-_\alpha$ does not act on $\theta^{-\alpha}$, therefore the dependence on them in (5.26) can be omitted from the very beginning. Using the rules $[\mathcal{D}^{++}, \mathcal{D}^-_\alpha] = \mathcal{D}^+_\alpha$ in the expression $\mathcal{D}^-_\alpha\mathcal{D}^-_\beta\mathcal{D}^-_\gamma\mathcal{D}^-_\delta\mathcal{L}^{(+4)}$ we get a number of terms which are conveniently grouped into

$$
-\Phi\left(D^+_\alpha D^-_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^+_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^-_\beta D^+_\gamma D^-_\delta Y^{++}\right) \tag{B.2}
$$

$$
\begin{aligned}
&+4D^-_\alpha \Phi D^{++} D^-_\beta D^-_\gamma D^-_\delta Y^{++} - 4D^-_\alpha\Phi\left(D^+_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\beta D^+_\gamma D^-_\delta Y^{++}\right)\\
&\quad + D^+_\alpha\Phi D^-_\beta D^-_\gamma D^-_\delta Y^{++} - D^+_\rho\Phi\left(D^+_\alpha D^-_\beta D^-_\gamma D^-_\delta + D^-_\alpha D^+_\beta D^-_\gamma D^-_\delta\right.\\
&\quad \left. + D^-_\alpha D^-_\beta D^+_\gamma D^-_\delta\right) W^{+\rho}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
&\left\{+6D^-_\alpha D^-_\beta\Phi D^{++} D^-_\gamma D^-_\delta Y^{++} - 6D^-_\alpha D^-_\beta \Phi D^+_\gamma D^-_\delta Y^{++}\right\} + 4D^-_\alpha D^+_\beta\Phi D^-_\gamma D^-_\delta Y^{++}\\
&\quad + 4D^-_\alpha D^+_\rho\Phi\left(D^+_\beta D^-_\gamma D^-_\delta + D^-_\beta D^+_\gamma D^-_\delta\right)W^{+\rho} - 4D^-_\alpha D^+_\rho\Phi D^{++} D^-_\beta D^-_\gamma D^-_\delta W^{+\rho}
\end{aligned}
\tag{B.4}
$$

$$
\begin{aligned}
&\left\{+6D^-_\alpha D^-_\beta D^+_\gamma \Phi D^-_\delta Y^{++} + 6D^-_\alpha D^-_\beta D^+_\rho \Phi D^{++} D^-_\gamma D^-_\delta W^{+\rho}\right.\\
&\quad \left. - 6D^-_\alpha D^-_\beta D^+_\rho\Phi D^+_\gamma D^-_\delta W^{+\rho}\right\} - 4D^-_\alpha D^-_\beta D^-_\gamma D^+_\rho\Phi D^{++} D^-_\delta W^{+\rho}\\
&\quad + 4D^-_\alpha D^-_\beta D^-_\gamma D^+_\delta\Phi Y^{++} + \Phi D^{++} D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta\Phi D^{++} Y^{++}\\
&\quad + D^+_\rho\Phi D^{++} D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta W^{+\rho} + D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta D^+_\rho\Phi D^{++} W^{+\rho}\\
&\quad + 3D^-_\alpha D^-_\beta D^-_\gamma\Phi D^{++} D^-_\delta Y^{++}.
\end{aligned}
\tag{B.5}
$$

Further we will investigate each of these expressions separately.

First, note that the set of terms (B.5) obviously vanishes if we recall the properties of harmonic superfields. In order to transform the expression (B.2) to the component form we should commute the spinor derivative D^+_α to the right, use the fact that $D^+_\alpha Y^{++} = 0$ and take into account that $[\mathcal{D}^-_\gamma, \mathcal{D}_{\alpha\beta}] = -2i\varepsilon_{\alpha\beta\gamma\delta}W^{-\delta}$. As a result one gets

$$\frac{1}{4!}\varepsilon^{\alpha\beta\gamma\delta}\Phi\left(12iD_{\alpha\beta}D^-_\gamma D^-_\delta + 32\varepsilon_{\alpha\beta\gamma\rho}W^{-\rho}D^-_\delta + 12\varepsilon_{\beta\gamma\delta\rho}\left(D^-_\alpha W^{-\rho}\right)\right)Y^{++}.$$

Eqs. (5.11), (5.17) of Subsection 5.2 allow us to rewrite this equation as follows:

$$\Phi\Bigg(D^{\alpha\beta}D_{\alpha\beta}\Phi - 16iW^{-\alpha}D_{\alpha\beta}W^{+\beta} - 12Y^{--}Y^{++} + 32iW^{-\alpha}\Psi^+_\alpha - \frac{1}{2}\mathcal{F}_\alpha{}^\beta\mathcal{F}_\beta{}^\alpha - 2D^{\alpha\beta}D_{\alpha\beta}Y^{+-}\Bigg).$$

The last term of the above relation vanishes after integration over the harmonic variables $\varepsilon_{ij}Y^{ij} \equiv 0$. The obvious transformations of the other terms under the integral $\int du u^+_i u^-_j = \frac{1}{2}\varepsilon_{ij}$, $\int du u^+_i u^+_j u^-_k u^-_l = \frac{1}{6}\varepsilon_{(ik}\varepsilon_{j)l}$ give

$$\int d^6x\left\{4D^a\Phi D_a\Phi + \Phi\left(4\mathcal{F}^{ab}\mathcal{F}_{ab} - 4Y^{ij}Y_{ij} - 8iW^\alpha_i D_{\alpha\beta}W^{i\beta} + 16iW^\alpha_i\Psi^i_\alpha\right)\right\}. \tag{B.6}$$

The other terms in the expression (B.3) are considered analogously. As a result, one gets

$$-24i\Psi^+_\alpha W^{+\alpha}Y^{--} + 16i\Psi^+_\alpha W^{-\alpha}\left(Y^{+-} + \frac{1}{2}\Phi\right) + 16i\Psi^+_\alpha D^{\alpha\beta}\Psi^-_\beta - 16i\Psi^-_\alpha D^{\alpha\beta}\Psi^+_\beta - 8i\Psi^+_\alpha D^{\alpha\beta}D_{\beta\gamma}W^{-\gamma} - 4i\Psi^+_\rho D^{\alpha\beta}D_{\alpha\beta}W^{-\rho} + 8i\Psi^-_\alpha D^{\alpha\beta}D_{\beta\gamma}W^{+\gamma}.$$

Integrating over harmonics leads to

$$+4i\Psi^i_\alpha W^\alpha_i\Phi + 16i\Psi^i_\alpha D^{\alpha\beta}\Psi_{i\beta} + 4i\Psi^i_\alpha W^\beta_i\mathcal{F}_\beta{}^\alpha - \frac{32}{3}i\Psi^i_\alpha W^{j\alpha}Y_{ij}. \tag{B.7}$$

The expression (B.4) has a complicated structure. However, bear in mind that we have the properties $W^{\alpha i}W^\beta_i = W^{\beta i}W^\alpha_i$ in the Abelian case. This allows us to make cancellations of the potentially admissible terms $\varepsilon_{\alpha\beta\gamma\delta}W^{-\alpha}W^{+\beta}W^{-\gamma}W^{+\delta}$, $D_{\alpha\beta}\Phi W^{-\alpha}W^{+\beta}$. As a result, we have

$$-\frac{1}{12}\mathcal{H}^{(-)}{}_{\alpha\beta}D^{(\alpha\delta}\mathcal{F}_\delta{}^{\beta)} - \frac{8i}{3}\mathcal{H}^{(-)}_{\alpha\beta}W^{-\alpha}W^{+\beta} + D^{\alpha\beta}\Phi D_{\beta\rho}\mathcal{F}_\alpha{}^\rho - D_{\alpha\beta}\Phi D^{\alpha\delta}\mathcal{F}_\delta{}^\beta,$$

where the last two terms disappear. Finally, after using the identity (5.14) this expression takes the form

$$\int d^6x\left\{-\frac{1}{18}\mathcal{H}^{(-)}{}_{\alpha\beta}\mathcal{H}^{(+)\alpha\beta} - \frac{4i}{3}\mathcal{H}^{(-)}_{\alpha\beta}W^\alpha_i W^{j\beta}\right\}. \tag{B.8}$$

Thus we see that all the functional structures of the component action (5.23) are obtained from the superfield action (5.26).

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