Nuclear reactions at near-barrier energies with quantum diffusion approach

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Abstract. The role of neutron transfer in the fusion (capture) reactions is discussed.

1. Introduction

The nuclear deformation and neutron-transfer process have been identified as playing a major role in the magnitude of the sub-barrier fusion (capture) cross sections [1]. There are a several experimental evidences which confirm the importance of nuclear deformation on the fusion. The influence of nuclear deformation is straightforward. If the target nucleus is prolate in the ground state, the Coulomb field on its tips is lower than on its sides, that then increases the capture or fusion probability at energies below the barrier corresponding to the spherical nuclei. The role of neutron transfer reactions is less clear. The importance of neutron transfer with positive Q-values on nuclear fusion (capture) originates from the fact that neutrons are insensitive to the Coulomb barrier and therefore they can start being transferred at larger separations before the projectile is captured by target-nucleus. Therefore, it is generally thought that the sub-barrier fusion cross section will increase because of the neutron transfer.

2. Quantum diffusion approach for capture

In the quantum diffusion approach [2, 3, 4, 5, 6] the capture of the projectile by the targetnucleus is described with a single relevant collective variable: the relative distance R between the colliding nuclei. This approach takes into consideration the fluctuation and dissipation effects in collisions of heavy ions which model the coupling of the relative motion with various channels (for example, the non-collective single-particle excitations, low-lying collective dynamical modes of the target and projectile). The nuclear static deformation effects are taken into account through the dependence of the nucleus-nucleus potential on the deformations and mutual orientations of the colliding nuclei. We have to mention that many quantum-mechanical and non-Markovian effects accompanying the passage through the potential barrier are taken into consideration in our formalism [2, 3].

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Journal of Physics: Conference Series 515 (2014) 012001

The capture cross section is a sum of partial capture cross sections [2, 3]

$$\sigma_{cap}(E_{c.m.}) = \sum_{J} \sigma_{cap}(E_{c.m.}, J) = = \frac{\pi \hbar^2}{2\mu E_{c.m.}} \sum_{J} (2J+1) \int_0^{\pi/2} d\theta_1 \sin(\theta_1) \int_0^{\pi/2} d\theta_2 \sin(\theta_2) P_{cap}(E_{c.m.}, J, \theta_1, \theta_2), (1)$$

where $\mu = m_0 A_1 A_2 / (A_1 + A_2)$ is the reduced mass (m_0 is the nucleon mass), and the summation is over the possible values of angular momentum J at a given bombarding energy $E_{\text{c.m.}}$. Knowing the potential of the interacting nuclei for each orientation with the angles $\theta_i (i = 1, 2)$, one can obtain the partial capture probability P_{cap} which is defined by the passing probability of the potential barrier in the relative distance R coordinate at a given J. The value of P_{cap} is obtained by integrating the propagator G from the initial state (R_0, P_0) at time t = 0 to the final state (R, P) at time t:

$$P_{\rm cap} = \lim_{t \to \infty} \int_{-\infty}^{r_{\rm in}} dR \int_{-\infty}^{\infty} dP \ G(R, P, t | R_0, P_0, 0) = \lim_{t \to \infty} \frac{1}{2} \operatorname{erfc} \left[\frac{-r_{\rm in} + R(t)}{\sqrt{\Sigma_{RR}(t)}} \right].$$
(2)

The second line in (2) is obtained by using the propagator $G = \pi^{-1} |\det \Sigma^{-1}|^{1/2} \exp(-\mathbf{q}^T \Sigma^{-1} \mathbf{q})$ $(\mathbf{q}^T = [q_R, q_P], q_R(t) = R - \overline{R(t)}, q_P(t) = P - \overline{P(t)}, \overline{R(t=0)} = R_0, \overline{P(t=0)} = P_0,$ $\Sigma_{kk'}(t) = 2q_k(t)q_{k'}(t), \Sigma_{kk'}(t=0) = 0, \{k, k'\} = \{R, P\}, P$ is a momentum) calculated for an inverted oscillator which approximates the nucleus-nucleus potential V in the variable R. The frequency ω of this oscillator with an internal turning point $r_{\rm in}$ is defined from the condition of equality of the classical actions of approximated and realistic potential barriers of the same hight at given J. This approximation is well justified for the reactions and energy range, which are here considered.

We assume that the sub-barrier capture mainly depends on the optimal one-neutron $(Q_{1n} >$ Q_{2n}) or two-neutron $(Q_{2n} > Q_{1n})$ transfer with the positive Q-value. Our assumption is that, just before the projectile is captured by the target-nucleus (just before the crossing of the Coulomb barrier) which is a slow process, the transfer occurs and can lead to the population of the first excited collective state in the recipient nucleus [7] (the donor nucleus remains in the ground state). So, the motion to the N/Z equilibrium starts in the system before the capture because it is energetically favorable in the dinuclear system in the vicinity of the Coulomb barrier. For the reactions under consideration, the average change of mass asymmetry is connected to the one- or two-neutron transfer (1n- or 2n- transfer). Since after the transfer the mass numbers, the isotopic composition and the deformation parameters of the interacting nuclei, and, correspondingly, the height $V_b = V(R_b)$ and shape of the Coulomb barrier are changed, one can expect an enhancement or suppression of the capture. If after the neutron transfer the deformations of interacting nuclei increase (decrease), the capture probability increases (decreases). When the isotopic dependence of the nucleus-nucleus potential is weak and after the transfer the deformations of interacting nuclei do not change, there is no effect of the neutron transfer on the capture. In comparison with Ref. [8], we assume that the negative transfer Q-values do not play visible role in the capture process. Our scenario was verified in the description of many reactions [3, 4, 5, 6].

3. Results of calculations

Because the capture cross section is equal to the complete fusion cross section for the reactions treated, the quantum diffusion approach for the capture is applied to study the complete fusion. All calculated results are obtained with the same set of parameters as in Ref. [2]. Realistic friction coefficient in the relative distance coordinate $\hbar\lambda=2$ MeV is used. Its value is

close to that calculated within the mean-field approaches [9, 10]. For the nuclear part of the nucleus-nucleus potential, the double-folding formalism with the Skyrme-type density-dependent effective nucleon-nucleon interaction is used [2, 3]. The parameters of the nucleus-nucleus interaction potential V(R) are adjusted to describe the experimental data at energies above the Coulomb barrier corresponding to spherical nuclei. The absolute values of the experimental quadrupole deformation parameters β_2 of even-even deformed nuclei in the ground state and of the first excited collective states of nuclei are taken from Ref. [11]. For the nuclei deformed in the ground state, the β_2 in the first excited collective state is similar to the β_2 in the ground state. For the quadruple deformation parameter of an odd nucleus, we choose the maximal value from the deformation parameters of neighboring even-even nuclei (for example, $\beta_2(^{231}\text{Th}) = \beta_2(^{233}\text{Th}) = \beta_2(^{232}\text{Th}) = 0.261)$. For the double magic and neighboring nuclei, we take $\beta_2 = 0$ in the ground state. Since there are uncertainties in the definition of the values of β_2 in light-mass nuclei, one can extract the ground-state quadrupole deformation parameters of these nuclei from a comparison of the calculated capture cross sections with the existing experimental data. By describing the reactions ${}^{12}C+{}^{208}Pb$, ${}^{18}O+{}^{208}Pb$, ${}^{32}S+{}^{90}Zr$, ${}^{58}Ni$, ${}^{58}Ni$, and ${}^{64}Ni$ + ⁶⁴Ni, where there are no neutron transfer channels with positive Q-values, we extract the ground-state quadrupole deformation parameters β_2 =-0.3, 0.1, 0.312, 0.05, and 0.087, for the nuclei ¹²C, ¹⁸O, ³²S, ⁵⁸Ni, and ⁶⁴Ni, respectively, which are used in our calculations.

3.1. Role of neutron transfer in capture process at sub-barrier energies

After the neutron transfer in the reaction ${}^{40}\text{Ca}(\beta_2 = 0) + {}^{48}\text{Ca}(\beta_2 = 0) \rightarrow {}^{42}\text{Ca}(\beta_2 = 0.247) + {}^{46}\text{Ca}(\beta_2 = 0) [Q_{2n} = 2.6 \text{ MeV}] \text{ or } {}^{40}\text{Ca}(\beta_2 = 0) + {}^{116}\text{Sn}(\beta_2 = 0.112) \rightarrow {}^{42}\text{Ca}(\beta_2 = 0.247) + {}^{114}\text{Sn}(\beta_2 = 0.121) [Q_{2n} = 2.8 \text{ MeV}], \text{ or } {}^{40}\text{Ca}(\beta_2 = 0) + {}^{124}\text{Sn}(\beta_2 = 0.095) \rightarrow {}^{42}\text{Ca}(\beta_2 = 0.247) + {}^{122}\text{Sn}(\beta_2 = 0.121) [Q_{2n} = 5.4 \text{ MeV}], \text{ the deformation of the nuclei increases and the mass asymmetry of the system decreases, and, thus, the value of the Coulomb barrier decreases and the capture cross section becomes larger (Fig. 1). In Fig. 2, we observe the same behavior in the reactions <math>{}^{58}\text{Ni}(\beta_2 = 0.05) + {}^{64}\text{Ni}(\beta_2 = 0.087) \rightarrow {}^{60}\text{Ni}(\beta_2 = 0.207) + {}^{62}\text{Ni}(\beta_2 \approx 0.1) [Q_{2n} = 3.9 \text{ MeV}] \text{ and } {}^{64}\text{Ni}(\beta_2 = 0.087) + {}^{132}\text{Sn}(\beta_2 = 0.158) + {}^{130}\text{Sn}(\beta_2 = 0) [Q_{2n} = 2.5 \text{ MeV}]. \text{ One can see a good agreement between the calculated results and the experimental data [12, 13, 14, 15, 16]. So, the observed capture enhancement at sub-barrier energies in the reactions mentioned above is related to the two-neutron transfer channel. One can see that at energies above and near the Coulomb barrier the cross sections with and without two-neutron transfer are almost similar.$

One can find reactions with a positive Q-values of the two-neutron transfer where the transfer weakly influences or even suppresses the capture process. This happens if after the transfer the deformations of the nuclei do not change much or even decrease. For instance, in the reactions ${}^{60}\text{Ni}(\beta_2 \approx 0.1) + {}^{100}\text{Mo}(\beta_2 = 0.231) \rightarrow {}^{62}\text{Ni}(\beta_2 = 0.198) + {}^{98}\text{Mo}(\beta_2 = 0.168) [Q_{2n} = 4.2 \text{ MeV}], {}^{64}\text{Ni}(\beta_2 \approx 0.087) + {}^{100}\text{Mo}(\beta_2 = 0.231) \rightarrow {}^{66}\text{Ni}(\beta_2 = 0.158) + {}^{98}\text{Mo}(\beta_2 = 0.168) [Q_{2n} = 4.2 \text{ MeV}], {}^{64}\text{Ni}(\beta_2 \approx 0.087) + {}^{100}\text{Mo}(\beta_2 = 0.231) \rightarrow {}^{66}\text{Ni}(\beta_2 = 0.158) + {}^{98}\text{Mo}(\beta_2 = 0.168) [Q_{2n} = 0.94 \text{ MeV}], and {}^{60}\text{Ni}(\beta_2 \approx 0.1) + {}^{150}\text{Nd}(\beta_2 = 0.285) \rightarrow {}^{62}\text{Ni}(\beta_2 = 0.198) + {}^{148}\text{Nd}(\beta_2 = 0.204) [Q_{2n} = 6 \text{ MeV}]$ we expect a weak dependence of the capture cross section on the neutron transfer (Fig. 3). There is the experimental evidence [17] of such an effect for the {}^{60}\text{Ni} + {}^{100}\text{Mo}, {}^{150}\text{Nd} are close to each other in contrast to those in the reactions ${}^{64}\text{Ni} + {}^{58}\text{Ni}, {}^{132}\text{Sn}$. The ${}^{60}\text{Ni} + {}^{150}\text{Nd}$ reaction has even a small suppression due to the neutron transfer.

Figures 4 and 5 show the capture excitation function for the reactions ${}^{32}S+Pd$, Ru as a function of the bombarding energy. One can see a relatively good agreement between the calculated results and the experimental data [18]. The Q_{2n} -values for the 2*n*-transfer processes are positive for all reactions with ${}^{32}S$. After the 2*n*-transfer (before the capture) in the reactions ${}^{32}S(\beta_2 = 0.312) + {}^{106}Pd(\beta_2 = 0.229) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{104}Pd(\beta_2 = 0.209)$,



Figure 1. The calculated (lines) and experimental (symbols) [12, 13, 14] capture cross sections vs $E_{\text{c.m.}}$ for the reactions ${}^{40}\text{Ca}{+}^{48}\text{Ca}$ (a) and ${}^{40}\text{Ca}{+}^{116,124}\text{Sn}$ (b). The calculated capture cross sections without taking into account the neutron pair transfer are shown by dotted lines.

 ${}^{32}S(\beta_2 = 0.312) + {}^{104}Pd(\beta_2 = 0.209) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{102}Pd(\beta_2 = 0.196)$ or ${}^{32}S(\beta_2 = 0.312) + {}^{104}Ru(\beta_2 = 0.271) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{102}Ru(\beta_2 = 0.24)$, ${}^{32}S(\beta_2 = 0.312) + {}^{102}Ru(\beta_2 = 0.24) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{100}Ru(\beta_2 = 0.215)$, the deformations of the nuclei decrease and the values of the corresponding Coulomb barriers increase. As a result, the transfer suppresses the capture process in these reactions at the sub-barrier energies. The suppression becomes stronger with decreasing energy (Figs. 4 and 5).

Figures 6 and 7 show the excitation functions for the reactions ${}^{18}O + {}^{74}Ge, {}^{112,118,124}Sn$ and ${}^{32}S^{+112,116}Sn$. For the ${}^{32}S^{-112,116}Sn$ The agreement between the varying from 1.4 MeV for ${}^{18}O + {}^{124}Sn$ up to 5.5 MeV for ${}^{18}O + {}^{112}Sn$. The agreement between the calculated results and the experimental data [19, 21] is rather good. As seen in Fig. 7, the cross sections increase systematically with the target mass number and run nearly similarly down to the lowest energy treated. In the reactions ${}^{32}S(\beta_2 = 0.312) + {}^{112}Sn(\beta_2 = 0.123) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{110}Sn(\beta_2 = 0.122), {}^{32}S(\beta_2 = 0.312) + {}^{116}Sn(\beta_2 = 0.112) \rightarrow {}^{34}S(\beta_2 = 0.252) + {}^{114}Sn(\beta_2 = 0.121), {}^{18}O(\beta_2 = 0.12), {}^{32}S(\beta_2 = 0.283) \rightarrow {}^{16}O(\beta_2 = 0) + {}^{76}Ge(\beta_2 = 0.262), {}^{18}O(\beta_2 = 0.121), {}^{18}O(\beta_2 = 0.123) \rightarrow {}^{16}O(\beta_2 = 0.121), {}^{18}O(\beta_2 = 0.121), {}^{12}Sn(\beta_2 = 0.121), {}^{18}O(\beta_2 = 0.252), {}^{16}O(\beta_2 = 0.25$



Figure 2. The same as in Fig. 1, but for the reactions ${}^{58}\text{Ni}+{}^{64}\text{Ni}$ (a) and ${}^{64}\text{Ni}+{}^{132}\text{Sn}$ (b). The experimental data are from Refs. [15, 16].

was observed in the recent experiments ${}^{16,18}O + {}^{76,74}Ge$ [19].

3.2. Influence of neutron pair transfer on capture

The choice of the projectile-target combination is crucial in the understanding of pair transfer phenomenon in the capture process. In the capture reactions with $Q_{1n} < 0$ and $Q_{2n} > 0$, the two-step sequential transfer is almost closed before capture. So, choosing properly the reaction combination, one can reduce the successive transfer in the process [26].

In the reactions 40 Ca + 48 Ca, 116,124 Sn, 64 Ni + 58 Ni, 132 Sn, 32 S + 102,104 Ru, 104,106 Pd, and 18 O + 112,118,124 Sn, 1n-neutron transfer is closed ($Q_{1n} < 0$) and Q_{2n} -values for the 2n-transfer processes are positive (Figs. 1, 2, 4, 5, and 7). The enhancement or suppression arises not from the coherent successive transfer of two single neutrons, but from the direct transfer of one spatially correlated pair (the simultaneous transfer of two neutrons). Our results show that the capture (fusion) cross section of the reactions under consideration can be described by assuming the preformed dineutron-like clusters in the ground state of the nuclei 18 O, 48 Ca, 64 Ni, 116,124,132 Sn, 102,104 Ru, and 104,106 Pd. Since the dominance of the dineutron-like clusters is found in the surface of double magic, semimagic, and nonmagic nuclei, one can conclude that this effect is general for all stable and radioactive nuclei. Note that the strong spatial two-neutron correlation and the strong surface enhancement of the neutron pairing in the cases of a slab, a semi-infinite nuclear matter, and the finite superfluid nuclei are well known. Previously, the importance of the neutron pair transfer in the capture (fusion) process was stressed in Refs. [8, 18, 23, 24, 25].

One can make unambiguous statements regarding the neutron pair transfer process in the



Figure 3. The same as in Fig. 1, for the indicated reactions ${}^{60}\text{Ni} + {}^{100}\text{Mo}, {}^{150}\text{Nd}$ (solid lines), and ${}^{64}\text{Ni} + {}^{100}\text{Mo}, {}^{150}\text{Nd}$ (dashed lines). For the reactions ${}^{60}\text{Ni} + {}^{100}\text{Mo}$ and ${}^{60}\text{Ni} + {}^{150}\text{Nd}$, the calculated capture cross sections without the neutron transfer are shown by dotted lines. The experimental data for the reactions ${}^{60}\text{Ni} + {}^{100}\text{Mo}$ (closed squares) and ${}^{64}\text{Ni} + {}^{100}\text{Mo}$ (open squares) are from Ref. [17].

reactions ⁴⁰Ca + ⁶²Ni $[Q_{1n} = -2.23 \text{ MeV}, Q_{2n} = 1.43 \text{ MeV}]$, ⁴⁰Ca + ⁶⁴Ni $[Q_{1n} = -1.29 \text{ MeV}, Q_{2n} = 3.45 \text{ MeV}]$, ⁴⁰Ca + ¹¹⁴Sn $[Q_{1n} = -1.94 \text{ MeV}, Q_{2n} = 1.8 \text{ MeV}]$, ⁴⁰Ca + ¹¹⁸Sn $[Q_{1n} = -1.55 \text{ MeV}, Q_{2n} = 3.56 \text{ MeV}]$, ⁴⁰Ca + ¹²⁰Sn $[Q_{1n} = -0.75 \text{ MeV}, Q_{2n} = 4.25 \text{ MeV}]$, ⁴⁰Ca + ¹²²Sn $[Q_{1n} = -0.45 \text{ MeV}, Q_{2n} = 4.86 \text{ MeV}]$, ⁵⁸Ni + ⁶²Ni $[Q_{1n} = -1.6 \text{ MeV}, Q_{2n} = 1.94 \text{ MeV}]$, ⁶⁰Ni + ⁶⁴Ni $[Q_{1n} = -1.84 \text{ MeV}, Q_{2n} = 1.95 \text{ MeV}]$, ⁶⁴Ni + ¹²⁸Sn $[Q_{1n} = -1.8 \text{ MeV}, Q_{2n} = 1.6 \text{ MeV}]$, and ⁶⁴Ni + ¹³⁰Sn $[Q_{1n} = -1.52 \text{ MeV}, Q_{2n} = 2.1 \text{ MeV}]$. There is a considerable difference between the sub-barrier capture cross sections with and without taking into consideration the neutron pair transfer in these reactions [26]. After two-neutron transfer, the deformation of light nucleus strongly increases and the capture cross section enhances. The neutron pair transfer induces the effect of the quadrupole deformation in the light nucleus. The study of the capture reactions following the neutron transfer will provide a good test for the effects of the neutron pair transfer.

3.3. Neutron pair transfer phenomenon in heavy-ion sub-barrier reactions

The Time-Dependent Hartree-Fock (TDHF) plus BCS approach [27, 28] has been recently used [28, 26] to extract the one- and two-neutron transfer probabilities (P_{1n}, P_{2n}) in heavyion scattering reactions. It was shown that, when the energy is well below the Coulomb barrier, the one-nucleon channel largely dominates. This is further illustrated here for the reactions ⁴⁰Ca + ^{116,124,130}Sn that have been discussed above and where the tin isotopes are superfluid. In Fig. 8, the one- and two-neutron transfer probabilities are displayed as functions of $B_0 - E_{c.m.}$ for the



Figure 4. The same as in Fig. 1, but for the reactions ${}^{32}S+{}^{106}Pd$ (a) and ${}^{32}S+{}^{104}Pd$ (b). The experimental data are from Ref. [18].



Figure 5. The same as in Fig. 1, but for the reactions ${}^{32}S+{}^{104}Ru$ (a) and ${}^{32}S+{}^{102}Ru$ (b). The experimental data are from Ref. [18].

sub- and near-barrier binary collisions of 40 Ca and tin isotopes. The Coulomb barrier (capture threshold energy) B_0 is deduced from the mean-field transport theory. This barrier are equal to



Figure 6. (Color online) The calculated (solid line) capture cross sections vs $E_{c.m.}$ for the reactions ${}^{16}O+{}^{76}Ge$ and ${}^{18}O+{}^{74}Ge$ (the curves coincide). For the ${}^{18}O+{}^{74}Ge$ reaction, the calculated capture cross sections without neutron transfer are shown by dotted line. The experimental data for the reactions ${}^{16}O+{}^{76}Ge$ (open circles) and ${}^{18}O+{}^{74}Ge$ (open squares) are from Ref. [19]. The experimental data for the ${}^{16}O+{}^{76}Ge$ reaction (solid circles) are from Ref. [20].

116.41 ± 0.07 (¹¹⁶Sn), 114.69 ± 0.04 (¹²⁴Sn) and 113.92 ± 0.02 (¹³⁰Sn) MeV. It was found that the calculated B_0 are insensitive to the introduction of pairing and in a good agreement with the barriers extracted from the experimental data [28]. Note that the presented calculation are shown for the mixed pairing interaction only. The use of other interaction (surface or volume) leads to similar conclusions. Figure 8 gives an interesting insight in the one- and two-neutron transfers. As seen, a strong enhancement of P_{1n} and P_{2n} occurs with increasing bombarding energy. Since the enhancement of P_{2n} is stronger than that of P_{1n} , these probabilities become close to each other with decreasing $B_0 - E_{c.m.}$. This is indeed observed experimentally in Refs. [29, 30] where it was found that P_{2n} grows faster than P_{1n} with decreasing $B_0 - E_{c.m.}$ at energy relatively far below the Coulomb barrier. In all cases, as the energy approaches the capture barrier energy, there exist an energy range where $P_{2n} > P_{1n}$ dominates (shaded area). We also note that the energy windows where the two-nucleon channel becomes dominant increases as the neutron nucleus become more exotic.

This evidently supports our assumption about important role of the two-neutron transfer (compared to the one-neutron transfer) in the capture process, because in the TDHF calculation the scattering trajectory of two heavy ions at energy near the Coulomb barrier is close to the capture trajectory. Note that in the capture process the system trajectory crosses the barrier position $R = R_b$ at any energies. The results of our calculations predict that there is the crossing point of P_{2n} and P_{1n} at energy very close to the Coulomb barrier. Just before reaching R_b the neutron-pair transfer becomes the dominant channel. Thus, our assumption about two-neutron transfer before the capture is correct. The transfer more than two neutrons mainly occurs at $R < R_b$, i.e., just after the capture.

3.4. Neutron transfer in reactions with weakly bound nuclei

After the neutron transfer in the reactions ${}^{13}C+{}^{232}Th(\beta_2 = 0.261)\rightarrow{}^{14}C(\beta_2 = -0.36)+{}^{231}Th(\beta_2 = 0.261)$ $[Q_{1n} = 1.74 \text{ MeV}], {}^{15}C+{}^{232}Th(\beta_2 = 0.261)\rightarrow{}^{14}C(\beta_2 = -0.36)+{}^{233}Th(\beta_2 = 0.261)$ $[Q_{1n} = 3.57 \text{ MeV}]$ the deformations of the target or projectile nuclei in these reactions and in the ${}^{14}C+{}^{232}Th(\beta_2 = 0.261)$ $(Q_{1n,2n} < 0)$ reaction are the same. In Fig. 9 the calculated cross sections slightly increase with the mass number of C, and are nearly



Figure 7. The calculated capture cross sections vs $E_{c.m.}$ for the reactions ${}^{18}O+{}^{112,118,124}Sn$ (solid, dashed and dotted lines, respectively) (a) and ${}^{32}S+{}^{112,116,120}Sn$ (solid, dashed and dotted lines, respectively) (b). The experimental data (symbols) are from Ref. [21, 22].

parallel down to the lowest energy treated. There is a relatively good agreement between the calculated results [6] and the experimental data [31, 32] for the reactions $^{12,13,14}C+^{232}Th$, but the experimental enhancement of the cross section in the $^{15}C+^{232}Th$ reaction at sub-barrier energies cannot be explained with our and other [31] models. Because we take into account the neutron transfer ($^{15}C\rightarrow^{14}C$), one can suppose that this discrepancy is attributed to the influence of the breakup channel [1] which is not considered in our model. However, it is unclear why the breakup process influences only two experimental points at lowest energies. Different deviations of these points in energy from the calculated curve in Fig. 9 create doubt in an influence of the breakup on the kinetic energy. So, additional experimental and theoretical investigations are desirable.

The question is whether the fusion of nuclei involving weakly bound neutrons is enhanced or suppressed at low energies. This question can been addressed to the systems ${}^{12-15}C+{}^{208}Pb$ [33]. After the neutron transfer in the reactions ${}^{13}C+{}^{208}Pb(\beta_2 = 0) \rightarrow {}^{14}C(\beta_2 = -0.36)+{}^{207}Pb(\beta_2 = 0)$ $[Q_{1n} = 1.74 \text{ MeV}], {}^{15}C+{}^{208}Pb(\beta_2 = 0) \rightarrow {}^{14}C(\beta_2 = -0.36)+{}^{209}Pb(\beta_2 = 0.055) [Q_{1n} = 3.57 \text{ MeV}]$ the deformations of the light nuclei are the same as in the ${}^{14}C+{}^{208}Pb(\beta_2 = 0) [Q_{1n,2n} < 0]$ reaction. The heavy nuclei are almost spherical. This means that the slopes of the excitation functions are almost the same (Fig. 10). As in the case of the ${}^{15}C+{}^{232}Th$ reaction, we do not expect enhancement of the capture cross section in the ${}^{15}C+{}^{208}Pb$ reaction owing to the neutron transfer. The same effect was observed in Ref. [33]. The study of the reactions ${}^{15}C+{}^{208}Pb,{}^{232}Th$ at sub-barrier energies provides a good test for the verification of the effect of weakly bound



Figure 8. (Color online) The calculated one- (symbols connecting by solid lines) and twoneutron (symbols connecting by dotted lines) transfer probabilities vs $B_0 - E_{c.m.}$ for the reactions ${}^{40}\text{Ca} + {}^{116}\text{Sn}$ (circles), ${}^{40}\text{Ca} + {}^{124}\text{Sn}$ (triangles), and ${}^{40}\text{Ca} + {}^{130}\text{Sn}$ (squares).



Figure 9. (Color online) The calculated (lines) and experimental (symbols) capture cross sections vs $E_{\rm c.m.}$ for the reactions ${}^{12}\text{C}+{}^{232}\text{Th}$ (dash-dotted line, solid triangles), ${}^{13}\text{C}+{}^{232}\text{Th}$ (dotted line, open triangles), ${}^{14}\text{C}+{}^{232}\text{Th}$ (solid line, open squares), and ${}^{15}\text{C}+{}^{232}\text{Th}$ (dashed line, solid squares). The experimental data are from Refs. [31, 32].

nuclei on fusion and capture because it reveals the role of other effects besides neutron transfer.

After the 2*n*-transfer in the reactions ${}^{6}\text{He}+{}^{206}\text{Pb}\rightarrow{}^{4}\text{He}(\beta_{2}=0)+{}^{208}\text{Pb}(\beta_{2}=0.055)$ [$Q_{2n}=13.13 \text{ MeV}$], ${}^{9}\text{Li}+{}^{86}\text{Zn}\rightarrow{}^{7}\text{Li}(\beta_{2}\approx0.4)+{}^{70}\text{Zn}(\beta_{2}=0.248)$ [$Q_{2n}=9.60 \text{ MeV}$] they become equivalent to the reactions ${}^{4}\text{He}+{}^{208}\text{Pb}$ and ${}^{7}\text{Li}+{}^{70}\text{Zn}$. Therefore, the slopes of the excitation functions in the reactions with ${}^{6}\text{He}$ (${}^{9}\text{Li}$) and ${}^{4}\text{He}$ (${}^{7}\text{Li}$) should be similar. This conclusion supports the experimental data of Ref. [35], where the authors concluded that the fusion enhancement in the ${}^{6}\text{He}+{}^{206}\text{Pb}$ reaction (with respect to the ${}^{4}\text{He}+{}^{208}\text{Pb}$ reaction) is rather



Figure 10. The calculated (lines) and experimental (symbols) capture cross sections vs $E_{c.m.}$ for the reactions ${}^{12}C+{}^{208}Pb$ (dash-dotted line), ${}^{13}C+{}^{208}Pb$ (dotted line), ${}^{14}C+{}^{208}Pb$ (solid line), and ${}^{15}C+{}^{208}Pb$ (dashed line). The experimental data (solid squares) for the ${}^{12}C+{}^{208}Pb$ reaction are from Ref. [34].



Figure 11. The calculated (solid line) and experimental (symbols) capture cross sections vs $E_{\rm c.m.}$ for the reaction ${}^{9}{\rm Li}{+}^{70}{\rm Zn}$. The experimental data are from Ref. [36].

small or absent.

By assuming that the 2*n*-transfer process occurs, we calculated the capture cross sections for the ${}^{9}\text{Li}+{}^{70}\text{Zn}$ reaction (Fig. 11). At lowest energies, the calculated cross section is by factor of ~ 5 less than the experimental value. The experimental data are reproduced by the model [37] where two-neutron transfer from the ${}^{70}\text{Zn}$ leads to ${}^{11}\text{Li}$ halo structure and molecular bond between the nuclei in contact enhances the fusion cross section. However, the two-neutron transfer ${}^{9}\text{Li}+{}^{70}\text{Zn}\rightarrow{}^{7}\text{Li}+{}^{72}\text{Zn}$ with $Q_{2n} = 8.6$ MeV is much energetically favorable than the two-neutron transfer ${}^{9}\text{Li}+{}^{70}\text{Zn}\rightarrow{}^{11}\text{Li}+{}^{68}\text{Zn}$ with $Q_{2n} = -15.4$ MeV. These observations deserve further experimental and theoretical investigations including the breakup channel.

4. Summary

The quantum diffusion approach was applied to study the role of the neutron transfer with positive Q-value in the capture reactions at sub-, near- and above-barrier energies.

We demonstrated a good agreement of the theoretical calculations with the experimental data. We found, that the change of the magnitude of the capture cross section after the neutron transfer occurs due to the change of the deformations of nuclei. The effect of the neutron transfer is an indirect effect of the quadrupole deformation. When after the neutron transfer the deformations of nuclei do not change or slightly decrease, the neutron transfer weakly influences or suppresses the capture cross section. Good examples for this effect are the capture reactions $^{60}\text{Ni} + ^{100}\text{Mo}, ^{150}\text{Nd}, ^{18}\text{O} + ^{64}\text{Ni}, ^{112,114,116,118,120,122,124}\text{Sn}, ^{204,206}\text{Pb}, and <math display="inline">^{32}\text{S}+^{96}\text{Zr}, ^{94,96,98,100}\text{Mo}, ^{100,102,104}\text{Ru}, ^{104,106,108,110}\text{Pd}, ^{112,114,116,118,120,122,124}\text{Sn}$. at sub-barrier energies. Thus, the general point of view that the sub-barrier capture (fusion) cross section strongly increases because of the neutron transfer with a positive Q-values has to be revised. The neutron transfer effect can lead to a weak influence of halo-nuclei on the capture. We demonstrated the important role of two-neutron transfer channel in the heavy-ion scattering at sub-barrier energies close to the Coulomb barrier. One can suggest the experiments $^{40}\text{Ca} + ^{116,124}\text{Sn}, ^{48}\text{Ca}$ to check our predictions.

This work was supported by DFG, NSFC, RFBR, and JINR grants. The IN2P3(France)-JINR(Dubna) and Polish - JINR(Dubna) Cooperation Programmes are gratefully acknowledged.

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