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METHOD FOR CALCULATION OF LONGITUDINAL AND TRANSVERSE COUPLING IMPEDANCES OF PERIODIC STRUCTURES IN ACCELERATORS

S. S. KURENNOY and S. V. PURTOV

USSR State Committee for Utilization of Atomic Energy, Institute for High Energy Physics, USSR, 142284, Serpukhov, Moscow region

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A method for calculating the coupling impedance of periodic structures with quite general boundaries has been developed. This technique is applicable at any frequency and for arbitrary beam energy. It is especially convenient for short-period structures. To illustrate the method's possibilities, we calculate longitudinal and transverse impedances of a corrugated vacuum chamber for a cyclic accelerator, both in the resonance region and at low frequencies, for various beam energies. Comparison with results obtained by other methods, in the regions of their applicability, shows good agreement.

1. INTRODUCTION

The appearance of various instabilities in modern accelerators and storage rings is the most serious constraint on their beam intensities. Stability conditions are usually formulated as limitations on the allowed value of beam-chamber coupling impedances.[1–3] Calculation of the impedance for a given vacuum-chamber geometry remains an important problem despite many methods offered for its solution. Unfortunately, we are not familiar with reviews treating the calculations of coupling impedance; the papers mentioned below do not at all constitute an exhaustive list of what has been written on this subject.

The majority of the authors investigate the simplest chamber models: a periodic sequence of identically shaped cylinders connected by beam tubes;[4,5] one cylinder on an infinite pipe,[6,7] etc. Others consider only very high frequencies (diffraction models) or low frequencies,[8] or proceed from additional assumptions about the impedance behavior, the broadband resonator model.[9] There are some methods applying "equivalent" boundary conditions: non-smooth chamber walls are replaced with smooth ones but having a dielectric coating whose permittivity is coordinate-dependent; see Ref. 10. The approaches of some papers[11–14] using perturbation theory are applicable only for small variations of the chamber's transverse cross section. Numerical methods for the calculation of fields and impedances (the best known program packages are SUPERFISH[15] and URMEL[16]) are used to study trapped modes in cavity-like chamber elements. Sometimes, one manages to apply them to find the impedance of periodic structures near resonances, e.g., MULTIMODE[17] in Ref. 14.

The aforementioned papers use the frequency representation of fields. There is a completely different approach—the numerical calculation of a wake potential in the time domain with the help of the TBCI package.[18] The wake potential is related to the impedance through a Fourier transform.[2] This method is sufficiently universal to calculate the impedance of a single chamber element,[19] but from our viewpoint it is far from perfect. It does not provide good accuracy, especially for smooth boundaries and/or small boundary variations; it is not quite suitable at low frequencies; and, more important, it requires a lot of computing capacity. The last point is a specific feature of all purely numerical methods. In this connection, semianalytic[4–9] and analytic[10–14] calculational methods still remain important.

This paper develops the matrix method of calculating the impedance of periodic structures that have sufficiently smooth boundaries of arbitrary shape. The approach has been offered[8] to calculate the impedance in a low-frequency range. The authors of that paper investigated the limiting case of extremely high energies for perfectly conducting chamber walls, which allows one to find the imaginary part of the impedance at low frequencies but provides no possibility of investigating resonances. The method, as we interpret it, considers the wall material to have finite conductivity and the beam to have an arbitrary energy. This makes it possible to calculate the impedance for any frequency including the resonance range and also to study its dependence on the beam energy. In addition, boundary conditions are modified at low frequencies, which allows the method to take into account the finite thickness of the vacuum chamber walls.

The paper is arranged as follows. Section 2 explains our method of impedance calculation. Section 3 studies in detail the example of a periodic corrugated structure simulating the chamber of the Serpukhov U-70 accelerator. The results are compared with those obtained by other methods. In conclusion we discuss briefly the possibilities for further development of the approach.

2. THE MATRIX METHOD OF CALCULATING THE IMPEDANCE

2.1 Calculation of Longitudinal Impedance

As an example, we will calculate the longitudinal impedance for a flat vacuum chamber of the accelerator with a period $2\pi R$. The chamber is symmetric with respect to the plane $x=0$. There is no y-dependence, and the walls have the shape described by $x=b(z)$. A transversely uniform flat beam having vertical size, $2a$, travels in the chamber center along the z-direction at a velocity βc.

Consider the nth mode of the longitudinal perturbation of the beam current density

$$j_z = \rho_n \beta c \exp(ikz - i\omega t), \tag{1}$$

where $k=n/R$ and $\omega=\beta ck$. With symmetry taken into account, the fields produced by this current have the form

$$E_z = i \sum_{m=-\infty}^{\infty} e^{ik_m z}\left[A_m ch\chi_m x - \delta_{mn}\begin{pmatrix} 1 \\ ch\chi(x-a)\end{pmatrix}\right],$$
$$E_x = \sum_{m=-\infty}^{\infty} e^{ik_m z}\frac{k_m}{\chi_m}\left[A_m sh\chi_m x - \delta_{mn}\begin{pmatrix} 0 \\ sh\chi(x-a)\end{pmatrix}\right], \tag{2}$$

where E_y, H_x, $H_z = 0$; the factor $\rho_n/(\varepsilon_0 k)\exp(-i\omega t)$ is omitted. The expression $Z_0 H_y$ differs from E_x in replacing k_m/χ_m by $\omega/(\chi_m c)$. Here $Z_0 = 120\pi\ \Omega$; $k_m = m/R$; $\chi_m^2 = k_m^2 - (\omega/c)^2$; $\chi \equiv \chi_n = k/\gamma$; and $\gamma = 1/\sqrt{1-\beta^2}$. The upper line in (. . .) corresponds to $0 \le x \le a$ and the lower one to $x \ge a$. A_m represents unknown coefficients.

In order to find A_m, let us impose on fields (2) the Leontovich boundary condition for $x = b(z) > a$:

$$\left[E_z + b'(z)E_x + (1-i)\frac{\delta\omega}{2c}\sqrt{1+(b'(z))^2}\, Z_0 H_y\right]_{x=b(z)} = 0, \tag{3}$$

where $\delta = \sqrt{2/(\mu_0\sigma\omega)}$ is the skin depth at frequency ω, and σ is the conductivity of the wall material. The applicability domain of boundary condition (3) is confined by the condition $\delta\omega/2c \ll 1$ at high frequencies and by $\delta \le \Delta$ at low ones, where Δ is the chamber wall thickness. One may extend this range into the region of lower frequencies by replacing δ with $\delta cth[(1-i)\Delta/\delta]$, which is Balbekov's result.[20] This modification works well until the inequality $\delta^2 \ll b\Delta$ takes effect.

After substituting Eq. (2) into the boundary condition of Eq. (3), we decompose the result over the system of functions $e^{ik_m z}$ which is complete in the segment $\{z : 0 < z < 2\pi R\}$. As a result, we obtain an infinite system of linear equations for the coefficients A_m.

Consider now the case when N periods of the vacuum chamber fit into an accelerator ring of length $2\pi R$. We will use the following notation:

$$D = 2\pi R/N \text{ is the chamber period}$$
$$\vartheta = 2\pi z/D$$
$$w(\vartheta) = b(\vartheta)/b$$

where $b = \langle b(\vartheta)\rangle \equiv (2\pi)^{-1}\int_0^{2\pi} d\vartheta b(\vartheta)$ is the mean half-height of the chamber,

$$G = 2\pi b/D$$
$$\xi = \omega b/\beta c$$
$$\eta = \delta/2b(\omega b/c)^2$$
$$x_0 = \chi b = \xi/\gamma$$

and

$$x_p^2 \equiv (\chi_{n+pN}b)^2 = x_0^2 + 2pG\xi + (pG)^2.$$

Then the system of equations for the field coefficients $F_p \equiv A_{n+pN}$ ($A_m = 0$, if $m \neq n + pN$; $p = 0, \pm 1, \ldots$) takes the form

$$\sum_{p=-\infty}^{\infty} M_{sp} F_p = R_s, \qquad s = 0, \pm 1, \ldots, \tag{4}$$

with

$$\begin{aligned} M_{sp} &= \left\langle e^{i(p-s)\vartheta}\left(f(x_p w) - \frac{f'(x_p w)}{x_p}\left[iw'G(\xi + pG) + (1+i)\eta\sqrt{1 + G^2 w'^2}\right]\right)\right\rangle, \\ R_s &= \left\langle e^{-is\vartheta}\left(g(x_0 w) - \frac{g'(x_0 w)}{x_0}\left(iw'G\xi + (1+i)\eta\sqrt{1+G^2 w'^2}\right]\right)\right\rangle. \end{aligned} \tag{5}$$

The functions f and g enter Eq. (5). For the flat chamber considered above,

$$\begin{aligned} f(u) &= \begin{cases} \cosh u, & u^2 \geq 0 \\ \cos |u|, & u^2 < 0 \end{cases} \\ g(u) &= \cosh\left(u - \frac{a}{b} x_0\right). \end{aligned} \tag{5a}$$

For an axisymmetric case (b is the mean radius of the chamber, a is the beam radius) we have the same system (4), but in Eqs. (5), we have

$$\begin{aligned} f(u) &= \begin{cases} I_0(u), & u^2 \geq 0 \\ J_0(|u|), & u^2 < 0 \end{cases} \\ g(u) &= \frac{a}{b} x_0 \left[K_1\left(\frac{a}{b} x_0\right) I_0(u) + I_1\left(\frac{a}{b} x_0\right) K_0(u)\right], \end{aligned} \tag{5b}$$

where I_i and K_i are modified Bessel functions and J_i is an ordinary one.

The longitudinal impedance (this notion seems to have been introduced for the first time by Lebedev and Zhilkov[21]) is determined according to[4] as follows:

$$Z_n = -\frac{1}{\rho_n \beta c S_\perp} \int_0^{2\pi R} dz e^{-ikz} \bar{E}_z, \tag{6}$$

where $\bar{E}_z$ is the amplitude of the field z component averaged over the beam transverse cross section and $S_\perp$ is the area of beam cross section.

For a flat chamber, Eqs. (2) and (6) suggest that

$$\frac{Z_n}{n} = -\frac{iZ_0\pi}{\beta kakL}\left[\frac{sh\chi a}{\chi a} F_0 - 1\right], \tag{6a}$$

where L is taken to mean the beam width in the y direction.

For the case of an axisymmetric chamber,

$$\frac{Z_n}{n} = -\frac{2iZ_0}{\beta(ka)^2}\left[\frac{2I_1(\chi a)}{\chi a} F_0 - 1\right]. \tag{6b}$$

Note that only the coefficient of the wave synchronous with the beam, i.e., $F_0 = A_n$, enters Eqs. (6a) and (6b).

2.2 Calculation of Transverse Impedance (Dipole Mode)

The transverse impedance (dipole mode) is calculated in a similar way. Consider, for an axisymmetric chamber, the following perturbation of the beam charge density

$$\rho = \rho_n d \sin\varphi \delta(a-r) \exp(ikz - i\omega t).$$

Here (r, φ, z) are the cylindrical coordinates, d is the maximum transverse displacement, and the remaining notations are as introduced above.

The relevant current density is $\vec{j} = (j \sin\varphi, j\cos\varphi, \rho\beta_p c)$, where

$$j = i\rho_n dkc(\beta_p - \beta)\vartheta(a-r)\exp(ikz - i\omega t);$$

$\vartheta(u)$ is the Heaviside function and $\beta_p c$ is the longitudinal beam velocity, which differs, generally speaking, from the phase velocity βc of the perturbation wave.

The transverse impedance is defined as

$$Z_t = -\frac{i}{\beta_p d\rho_n \beta_p c\pi a^2}\int_0^{2\pi R} dz\overline{\{\vec{E} + [\vec{\beta}_p * \vec{H}]\}_\perp}\,|_{\omega t = kz}. \tag{7}$$

In the case under consideration, all the components of E and H fields are nonzero. To find the two sequences of unknown coefficients B_m and C_m in the field expansion, another boundary condition (in addition to Eq. (3), in which x is replaced by r and y by φ) is imposed:

$$\left[E_\varphi\sqrt{1+(b'(z))^2} - (1-i)\frac{\delta\omega}{2c}Z_0(H_z + b'(z)H_r)\right]_{r=b(z)} = 0. \tag{3'}$$

It is not difficult to get the system of equations for B_m and C_m. For a chamber period of $D = 2\pi R/N$, it is

$$\begin{cases} \sum\limits_{p=-\infty}^{\infty}(D_{sp}E_p + S_{sp}H_p) = R_s, \\ \sum\limits_{p=-\infty}^{\infty}(\tilde{S}_{sp}E_p + \tilde{D}_{sp}H_p) = \tilde{R}_s, \end{cases} \quad s = 0, \pm 1, \ldots, \tag{8}$$

where

$$\begin{aligned}
&E_p \equiv B_{n+pN} \\
&H_p \equiv C_{n+pN}, \\
&D_{sp} = \langle e^{i(p-s)\vartheta}\{x_p^2 wt(x_p w) - t_1(x_p w)[iw'G(\xi + pG) + (1+i)\eta\sqrt{1+G^2w'^2}]\}\rangle, \\
&S_{sp} = \beta\xi\left\langle e^{i(p-s)\vartheta}t(x_p w)\left[iw'G + (1+i)\frac{\delta}{2b}(\xi + pG)\sqrt{1+G^2w'^2}\right]\right\rangle, \\
&\tilde{S}_{sp} = \langle e^{i(p-s)\vartheta}t(x_p w)[(\xi + pG)\sqrt{1+G^2w'^2} - (1+i)\eta iw'G]\rangle, \\
&\tilde{D}_{sp} = \beta\xi\left\langle e^{i(p-s)\vartheta}\left[-t_1(x_p w)\sqrt{1+G^2w'^2} + \right.\right. \\
&\qquad \left.\left. + (1+i)\frac{\delta}{2b}(iw'G(\xi + pG)t_1(x_p w) - x_p^2 wt(x_p w))\right]\right\rangle.
\end{aligned} \tag{9}$$

$$R_s = \left\langle e^{-is\vartheta}\left\{ -\beta_2 \xi w g_1(x_0 w) + iw' G[-g_1(x_0 w) + \beta_2 \gamma^2 g(x_0 w)] + \right.\right.$$
$$\left.\left. + (1+i)\frac{\delta\omega}{2c}\sqrt{1 + G^2 w'^2}[-\beta_p g_1(x_0 w) + \beta_2 \beta \gamma^2 g(x_0 w)] \right\}\right\rangle,$$

and

$$\tilde{R}_s = \left\langle e^{-is\vartheta}\left\{ \sqrt{1 + G^2 w'^2}[-g_1(x_0 w) + \beta_1 \beta \gamma^2 g(x_0 w)] + \right.\right.$$
$$\left.\left. + (1+i)\frac{\delta\omega}{2c}[\beta_1 \xi w g_1(x_0 w) + iGw'(\beta_p g_1(x_0 w) - \beta_1 \gamma^2 g(x_0 w))] \right\}\right\rangle.$$

The following notation is used in Eqs. (9):

$$t(u) = \begin{cases} I_1(u)/u, & u^2 > 0 \\ 1/2, & u^2 = 0 \\ J_1(|u|)/|u|, & u^2 < 0 \end{cases}$$
$$t_1(u) = f(u) - t(u)$$
$$g_1(u) = g'(u)/u$$

$g(u)$ and $f(u)$ are as defined in (5b)

$$\beta_1 = \beta_p - \beta$$
$$\beta_2 = 1 - \beta_p \beta.$$

The value of β is obtained from the relationship $\beta = (1 - Q/n)\beta_p$, where Q is a betatron frequency, using the specified value of β_p.

It follows from Eq. (7) that

$$Z_t = -i\frac{Z_0 R}{\beta_p^2 a^2}\left[\frac{2I_1(\chi a)}{\chi a}\xi(\beta_2 E_0 + \beta_1 H_0) + 2\beta_1^2 \gamma^2\right], \tag{10}$$

so in order to calculate Z_t it is necessary to find E_0 and H_0 from Eq. (8).

3. AN EXAMPLE

In practical calculations, the infinite systems of linear equations in Eqs. (4) and (8) are replaced with finite ones. For example, instead of Eq. (4) we have

$$\sum_{p=-P}^{P} M_{sp} F_p = R_s, \qquad s = 0, \pm 1, \ldots, \pm P. \tag{11}$$

The value of P is chosen from the following considerations. In the expansion in Eq. (2) the term numbered $m_p = n - pN$ yields the pth longitudinal resonance. In order to obtain all resonances in the frequency range $(0, f)$ we have to take into account all terms numbered $0 \le p < 2Df/c$. In reality, the terms having larger numbers affect the values of these resonances, therefore P should be chosen larger than $2Df/c$. Numerical experiments have shown that the impedance value is established quickly with an increase of P. The estimate of P shows that the method is convenient for short-period structures. For long-period structures, large Ps are required and therefore calculations become more time-consuming.

To solve the system of Eq. (11), the code NML has been developed, in which the quantities M_{sp} and R_s are computed from Eq. (5) by numerical integration. For solving the system of complex linear equations, the procedure CMLIN is used.

A similar program, NMT, computing the transverse impedance, solves Eq. (8) in which the matrices D, S, $\tilde{D}$, and $\tilde{S}$ are truncated in the same way as M in Eq. (11).

The impedance of the corrugated chamber of the U-70 accelerator has been calculated in order to illustrate and test the capabilities of this method. A rigorously periodic structure is assumed (irregular insertions like smooth sections of the chamber, etc., are neglected). It has a cosine-shaped corrugation with a period D of 1.1 cm and a depth h of 0.55 cm. In addition, the elliptical chamber is replaced with an axisymmetric one whose radius, b, is 7 cm[10].

Figure 1 presents some results from calculations of the longitudinal impedance in the frequency range up to 16 GHz. In these calculations, a 7×7 matrix was used, and it took 2 seconds of ICL-1906A CPU time to compute one frequency point. Figure 2 shows the values of impedance and resonance frequency versus the matrix truncation size with a lower-order resonance used as an example. These dependences reach the asymptotes rapidly. The values obtained for $\gamma = 70$ ($Z/n = 3620\,\Omega$; $f_0 = 6.26$ GHz; $2\Delta f = 14$ MHz) are in good agreement with results obtained in earlier analytical work[10] with the help of an approximate conformal mapping (3400 Ω, 6.1 GHz, 15 MHz). The energy dependence of the resonance parameters also coincides with that found earlier in Ref. 10. As shown in that work, this resonance is caused by a slow wave, i.e., the corrugations serve as a

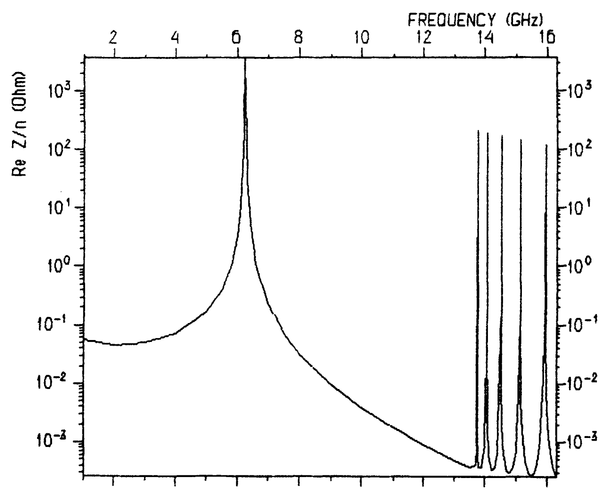

FIGURE 1 The longitudinal impedance of the corrugated chamber.

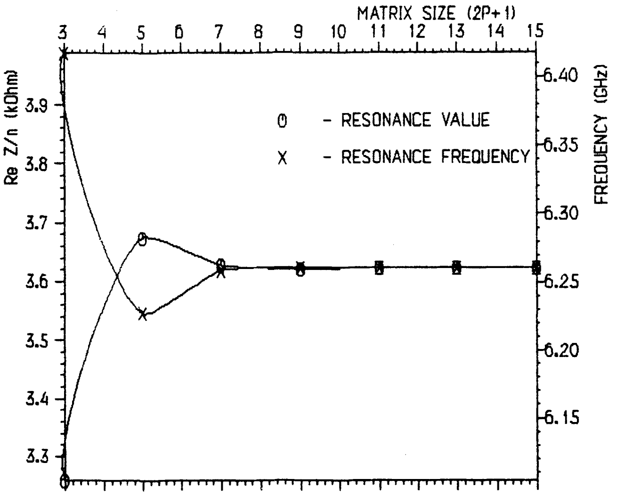

FIGURE 2 The dependence of the results on the matrix truncation size.

decelerating structure. It is worth noting that a decrease of the corrugation depth increases the resonance frequency f_0 noticeably, simultaneously reducing the value Z/n.

The resonances close to and above 14 GHz (note that at these frequencies the method presented in Ref. 10 is inapplicable), as shown in Fig. 1, have a different nature. These are various radial modes ($r = 1, 2, 3 \ldots$) of a lower-order longitudinal resonance whose wavelength satisfies the condition $p\lambda_{p,r}/2 \simeq D$ for $p = 1$. This can be established directly by constructing the field pattern with the help of the coefficients F_p found from Eq. (11) or by comparison with the results of the analytical method, meaning the expansion in terms of a small parameter in the boundary conditions [11, 13]. The expansion parameter is $\varepsilon \equiv h/2b = 0.039 \ll 1$. The resonance frequencies $f_{p,r}$ (determined by simple formulas of the ε-expansion method [11, 13]) agree with those obtained with the help of the matrix method. The widths of these resonances range from 0.05 MHz ($r = 1$) to 0.8 MHz ($r = 5$), whereas the range predicted by the ε-expansion method is 0.3–0.4 MHz.

It is worthwhile noting that the ε-expansion formulae are easily obtained from Eqs. (4) and (5). For this to be accomplished, it is sufficient to expand the matrix M_{sp} and R_s into a series over powers of ε and search for the solution to F_p in the form $F_p^{(0)} + \varepsilon F_p^{(1)} + \varepsilon^2 F_p^{(2)} + \cdots$ Solving the systems of equations for ε^0, ε^1, ε^2, etc. recurrently, we obtain $F_p^{(i)}$ in an analytical form. As a result, the general expression for the impedance of small chamber perturbations obtained earlier by one of us [13] is reproduced easily. However, the necessary additional conditions for this perturbation theory to be applicable are the inequalities

$$\varepsilon \ll 1, \tag{12a}$$

$$\varepsilon |x_p| \ll 1. \tag{12b}$$

See the discussions in Refs. 8 and 13. When they are fulfilled, M_{sp} and R_s can be expanded into a series in ε. Strictly speaking, the ε-expansion method is inapplicable for the structure under consideration: the inequality Eq. (12b) is violated already for $|p| = 1$ because $\varepsilon G = \pi h/D = \pi/2 > 1$. Still, this method yields the correct values of resonance frequencies $f_{1,r}$.

In terms of the matrix approach, the resonances obtained by the ε-expansion method are determined by the condition $Re\ M_{-p,-p} = 0$ for the pth resonance. The roots of this equation, $f_{p,r}$ with $r = 1, 2, 3 \ldots$, correspond to various radial modes. As seen from numerical calculations, the value of $\mathrm{Re}\, M_{-1,-1}$ is really small near resonances in the 14-GHz band. The situation for the resonance at a slow wave, i.e., when no element of the matrix M is small, is completely different. Note that frequencies $f_{1,r}$ actually do not change with the variation of the corrugation depth, providing ε remains small as compared with unity.

To test the method additionally, we have calculated the resonances of the longitudinal impedance of the structure under consideration using a finite-element method. For this purpose we applied the MULTIMODE package;[17] we describe the technique briefly elsewhere.[14] The comparison showed good agreement between the matrix method and the numerical one for the frequencies and values of all resonances presented in Fig. 1. For instance, for a lower-order resonance, MULTIMODE yields 6.33 GHz, 4027 Ω, and for the greatest high-frequency one it yields 13.75 GHz, 236 Ω (the matrix method yields 13.75 GHz, 208 Ω).

Figure 3 shows the energy dependence of the imaginary part of the longitudinal impedance of the U-70 corrugated chamber at low frequencies (more precisely, in the nonresonance range, below the cutoff frequency). It is interesting to note that $\mathrm{Im}\, Z/n$ changes sign in the vicinity of the transition energy (for U-70 $\gamma_{cr} = 9.46$). The part of the impedance that is connected with losses in the walls is subtracted from the data presented in Fig. 3. This addition is weakly dependent on energy, varies with frequency as $1/\sqrt{f}$, and is equal to $(1 - i)0.53\ \Omega$ for 10 MHz. The effect of the finite wall thickness ($\Delta = 0.4$ mm in U-70) becomes noticeable for

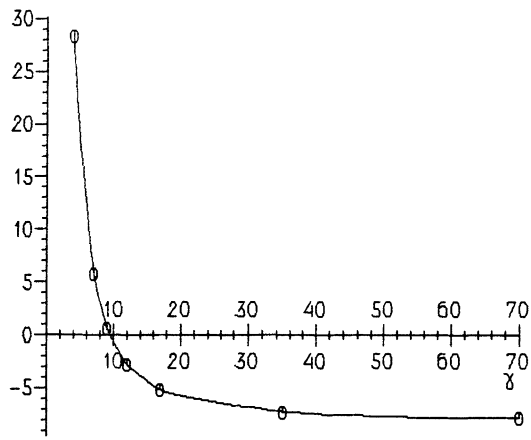

FIGURE 3 The longitudinal impedance versus energy at low frequencies.

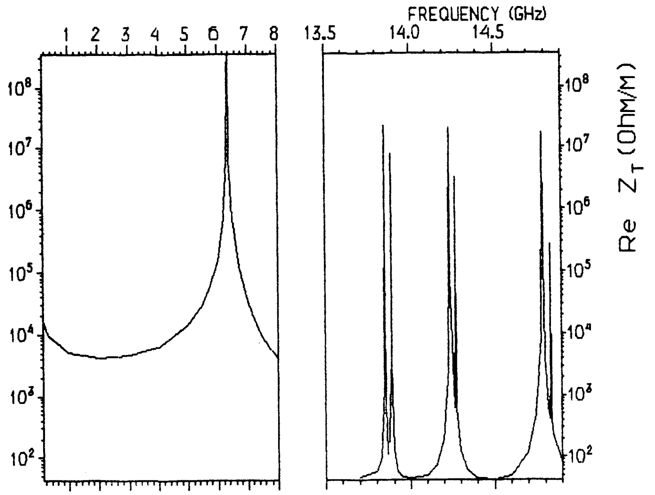

FIGURE 4 The transverse impedance of the corrugated chamber.

$f \leq 1$ MHz. In particular, the frequency dependence of $\mathrm{Re}\, Z/n$ is replaced by $1/f$, with $\mathrm{Im}\, Z/n$ being almost independent of f. For $\gamma = 70$ ($a = 0.5$ cm) and $f = 100$ kHz $Z/n = (17.5 - 18.8)\ \Omega$, whereas for a smooth chamber and under the same conditions $Z^{(0)}/n = (11.9 - 10.5)\ \Omega$.

The longitudinal impedance of a similar corrugated structure with a corrugation depth $h = 1$ cm was calculated at low frequencies by the matching technique.[9] The corrugation shape was approximated by a set of step-like transitions in this approach, and the chamber walls were considered perfectly conducting. The result of the numerical calculations,[9] $\mathrm{Im}\, Z/n \simeq -15.5\ \Omega$ for $\gamma \gg 1$, is close to the one obtained for the same case with the matrix method, $\mathrm{Im}\, Z/n \simeq -19\ \Omega$.

Figure 4 presents the results from calculations of transverse impedance for our model of the U-70 corrugated chamber. The computation time was only 1.5 times larger than for the computation of the longitudinal impedance with the same size of matrix cutoff. For $\gamma = 70$ the parameters of a lower-frequency resonance caused by a slow wave are: $\mathrm{Re}\, Z_t = 348\ \mathrm{M}\Omega/\mathrm{m}$, $f = 6.35$ GHz, $2\Delta f = 13$ MHz.

This figure also shows two sets of narrow-band resonances (their width varies from 0.05 to 0.4 MHz) whose frequencies almost coincide (the shift is 30–40 MHz). These resonances are various radial modes ($r = 1, 2, 3$) of the transverse resonances with $p = 1$ of E- and H-types. Comparison of Figs. 4 and 1 shows that the frequencies of the transverse resonances increase, as compared with those of the relevant longitudinal ones, by only 1–2%. It is interesting to note that for all of them, with the exception of the H-type resonances, the commonly used relationship $\mathrm{Re}\, Z_t = (2R/b^2)\, \mathrm{Re}\, Z/n$ (in our model $2R/b^2 = 9.63 \cdot 10^4 \mathrm{m}^{-1}$) holds true to a good level of accuracy. Note also that the resonance values are weakly dependent on the betatron frequency Q ($Q = 9.7$ in U-70).

The behavior of the transverse impedance of the U-70 corrugated chamber is shown in Figs. 5 and 6. Figure 5 shows how the dependence of $\mathrm{Re}\, Z_t$ on the frequency $1/\sqrt{f}$ is replaced with $1/f$ for $f < 1$ MHz. Here $\mathrm{Re}\, Z_t$ is independent of

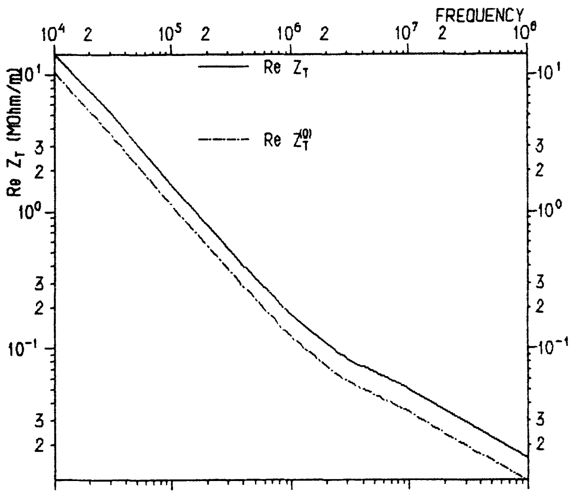

FIGURE 5 The transverse impedance versus frequency in the nonresonance range.

γ. By contrast, Im Z_t is dependent weakly on frequency and strongly on energy (see Fig. 6). It is easy to calculate that the correction introduced by corrugation into Im Z_t causes a tune shift of $\delta Q \simeq -3.10^{-3}/\gamma$ for a beam intensity of 5×10^{13} particles per cycle. Note that everywhere in the nonresonance region the ratio Re Z_t/Re $Z_t^{(0)}$, where $Z_t^{(0)}$ is the impedance of a smooth chamber of the same radius ($b = 7$ cm), is equal to the ratio of the areas of the corrugated and smooth chamber surface: $\sqrt{4+\pi^2}E(\pi/\sqrt{4+\pi^2}) \simeq 1.46$. The same is also true for the longitudinal impedance at low frequencies (see above). This result is quite clear from fairly simple considerations related to screening of the field by the chamber walls.

The relationship between the transverse and longitudinal impedances, Re $Z_t =$ $(2R/b^2)$ Re Z/n, works well at low frequencies, too.

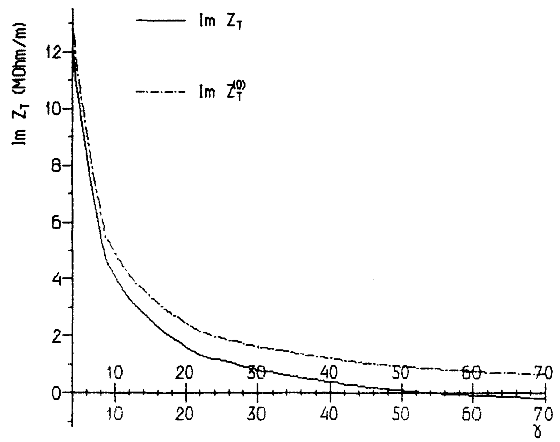

FIGURE 6 The transverse impedance versus energy at low frequencies.

4. CONCLUSION

The matrix method (as we refer to it) for calculating the impedance of periodic structures in accelerators is applicable for a rather general case. For example, a small value of the boundary variation is not required, in contrast to some methods discussed previously in the literature.[11,13] However, certain limitations on the shape of the boundary $b(z)$ remain: the function $r = b(z)$ should be single-valued, i.e., only one point should correspond to each $z \in [0, D]$. Generalization for the case when the structure includes steplike transitions (where δ functions enter $b'(z)$) would require, during calculation of the matrix elements of Eqs. (5) and (9), additional definition of the distribution product at singular points, taking into account the boundary conditions at the end walls.

The matrix method can be used to calculate the impedance for any frequency. The example presented in Section 3 demonstrates the convenience of the approach for the case of short-period structures. Comparison of the results obtained using the matrix method with those obtained by other methods in their applicability domain (see Section 3) shows a good agreement.

It is expedient to generalize the method for the case of violated periodicity (Refs. 10, 13, 14). This would make it possible to obtain more rigorous estimates of the beam-chamber coupling impedance of real accelerators.

Note that, when applying a similar approach to nonperiodic structures, integral equations must be solved rather than the matrix ones, Eqs. (4) or (8). We hope to consider this case in our further investigations.

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Note added in proof. An integral-equation method for calculating the coupling impedance of nonperiodic structures has already been developed. See more details in Ref. 22.

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