THE AUTORESONANT ACCELERATOR CONCEPT

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I. Introduction

The autoresonant accelerator principle[1] offers a conceptually simple and compact method for the generation of energetic pulsed ion beams in the multi-ampere current range. This accelerator scheme utilizes the lower branch of the Doppler shifted cyclotron mode of a relativistic electron beam propagating along a guide magnetic field inside a cylindrically symmetric conducting guide to serve as a traveling wave for the acceleration of ions loaded into the potential troughs of such a wave. This accelerator scheme thus combines the basic concepts of traveling wave and collective acceleration,[2,3] in that while a traveling wave is used for the acceleration process, this wave is a collective eigenmode of the electron beam-magnetic guide field-cylindrical guide system rather than a vacuum wave guide mode as in conventional traveling wave accelerators.

Due to the collective nature of the medium of propagation much higher effective accelerating fields can be sustained than in a conventional accelerator, allowing a substantial economy of machine size. Two further characteristics of the lower branch of the Doppler-shifted cyclotron mode make it well suited for use as the traveling wave for the acceleration process:

1) Of the 8 eigenmodes of the electron beam system, this mode is the only one with a phase velocity variable from zero to (asymptotically) the velocity of the electron beam. Hence, proton energies upwards of 10 GeV (and larger energies for heavier ions) would be achievable using present day electron beam devices which typically operate in the 5-10 MeV energy range.[4]

Furthermore, the phase velocity can be varied simply by spatially varying the magnetic field along the length of the accelerator, the phase velocity of the wave varying inversely with the strength of the magnetic guide field. Thus control of the phase velocity is relatively easy to achieve.

2) Conventional traveling wave accelerators employing vacuum wave guide modes must supply large amounts of RF power to the wave to compensate for losses since such waves are positive energy waves and hence their electric field energy is degraded by dissipative processes, such as the acceleration process or cavity losses. However, the cyclotron wave used in the Autoresonant Accelerator is a negative energy wave. Hence in the acceleration process where energy is delivered to the ions, rather than being degraded, the electric field energy of the wave actually grows. The reason this can occur is that the wave is not propagating in a passive medium, but rather an active medium with a large free energy source; namely the relativistic electron beam. The process of acceleration automatically extracts energy from the electron beam to both accelerate the ions and to increase the electric field energy of the wave. Hence the need for large RF sources to maintain the wave is removed. Furthermore, present day pulsed relativistic electron beam devices output pulsed power typically in the range of 10^{12} watts over a pulse time of typically 10^{-7} s, with the larger machines achieving power levels well over 10^{13} watts.[5] Thus, if the autoresonant accelerator achieves only a few percent efficiency of conversion of electron beam energy to ion energy, one could anticipate pulsed ion currents in the tens of ampere range or larger.

In the following sections, a technical discussion of the accelerator proper and its operating parameter constraints, methods of generation of the desired eigenmode, and problem areas remaining to be investigated are presented. In Section II, the equilibrium and stability requirements for the electron beam configuration are presented. Section III contains a discussion of the eigenmodes, with the basic operating principles of the accelerator discussed in Section IV. In Section V a method for generation of the wave is presented. Section VI contains an example of the performance one might optimistically expect from such an accelerator. Problem areas one might anticipate in the efficient operation of such an accelerator are also briefly discussed.

II. Electron Beam Propagation Requirements

We consider a cold relativistic electron beam of current I_e and relativistic factor γ propagating interior to an azimuthally symmetric conducting vacuum wave guide along a guide magnetic field. $\vec{B} = B\hat{z}$, where $\hat{r}$, $\hat{\theta}$, $\hat{z}$ comprise the usual cylindrical coordinate unit vector triad. Let the electron beam radius be given by a and the radius of the conducting guide given by $b \geq a$, where we shall consider $b \approx a$. We further specialize to the case $\gamma \gg 1$.

It is well known, both from theoretical investigation[6,7] and experimental verification,[8] that such an electron beam may be propagated provided that the equilibrium and stability conditions of such a beam are satisfied.

In equilibrium, the beam propagates along the z axis with a velocity v_e while undergoing an azimuthal rotation with velocity v_θ. For such an equilibrium to exist, it is necessary that the $\vec{v} \times \vec{B}$ restoring force, including the self magnetic field, be larger than either the destabilizing forces due to the self radial electric field or the centrifugal force associated with the azimuthal rotation. The following conditions can be shown to be sufficient for such an equilibrium to exist:[9]

$$\omega_p^2 < \gamma\Omega c/a \quad \text{or} \quad I_e < 5Ba\gamma \qquad (1a)$$

$$2\omega_p^2 < \gamma^2\Omega^2 \qquad I_e < .6B^2a^2\gamma \qquad (1b)$$

Here $\omega_p^2 \equiv 4\pi n_o e^2/(\gamma m)$ is the usual relativistic plasma frequency, where n_o is the beam density as measured in the laboratory frame; m, the electron mass, and e, the magnitude of the electron charge. The quantity $\Omega \equiv eB/(\gamma mc)$ is the usual relativistic electron gyrofrequency.

The equilibrium equations have also been stated in terms of the total electron current I_e, beam radius a, and guide magnetic field strength B, where I_e is measured in kiloamperes; a, in centimeters; and B, in kilogauss.

Furthermore, if we consider the anode of the electron beam diode as electrically convected to the walls of the conducting guide so that it resides at the same potential as the conducting guide, then the electron beam must have sufficient energy to overcome its own self potential as it propagates away from the anode plane. This leads to the usual Lawson criterion

$$\omega_p^2a^2 < 4c^2 \quad \text{or} \quad I_e < 17\gamma \qquad (1c)$$

Such an electron beam equilibrium has furthermore been investigated for stability both for the special case of electrostatic perturbations[10] and the more general case of electromagnetic perturbations.[11] A sufficient condition for stability is given by:

$$\omega_p^2 < \gamma\Omega c/b \quad \text{or} \quad I_e < 5Ba^2\gamma/b \qquad (2)$$

which reduces effectively to Eq. (1a) for b≈a.

III. Electron Beam System Eigenmodes

The linearized fluid equations governing the electron beam coupled with Maxwell's equations and boundary conditions at r=a appropriate to a conducting wall boundary ($E_z(a) = E_\theta(a) = B_r(a) = 0$) allow a determination of the linear eigenmodes of the system. Expanding the z component of the electric field in an appropriate Fourier-Bessel series:

$$E_z = \sum_n E_n J_o(k_\perp r)\exp(-i\omega t + ik_z z)$$

and expressing the $\hat{r}$ and $\hat{\theta}$ components in a similar suitable fashion, one obtains an eigenenvector equation of the usual form:

$$\overleftrightarrow{T}\bullet\vec{E} = 0$$

with the dispersion relation given by:

$$\varepsilon(\omega,k) \equiv \|\overleftrightarrow{T}\| = \frac{\omega^2}{\Delta\omega^2(\Omega^2-\Delta\omega^2)}\left\{\Omega^2(\omega^2-k^2c^2)\left[\Delta\omega^2(\omega^2-k^2c^2) - \omega_p^2/\gamma^2(\omega^2-k_z^2c^2)\right] - \Delta\omega^2(\Delta\omega^2-\omega_p^2/\gamma^2)(\omega^2-k^2c^2-\omega_p^2)^2\right\} \qquad (3)$$

Here the quantity $\Delta\omega \equiv \omega - k_z v_e$ and $k^2 \equiv k_z^2 + k_\perp^2$.

The boundary conditions require that

$$J_o(k_\perp a) = 0 \qquad (4)$$

We now wish to determine the zeroes of $\varepsilon(\omega,k)$ and the appropriate sign of the wave energy of various modes.

A. Definition of Wave Energy

The energy density and energy density flux of a vacuum electromagnetic wave is given by the usual formulas

$$(E^2+B^2)/8\pi \qquad (5)$$

and

$$\frac{c}{4\pi}(\vec{E}\times\vec{B}) \qquad (6)$$

respectively. However, for waves propagating in an active medium, such as in the electron beam background here considered, energy resides not only in the bare electric and magnetic fields, but also in the "sloshing" motion of the particles comprising the background medium, i.e., the relativistic electrons, as they move under the influence of the electric and magnetic field.

The wave energy of such a system is defined as the change in total energy of the electric and magnetic fields plus particles from the state where the wave is absent to the state where the wave is present. If this change is positive, then the wave is defined as a positive energy wave; if negative, then a negative energy wave. In coordinate systems in which the active medium is at rest, the introduction of a wave always causes an increase in the sloshing energy of the particles and hence is a positive energy wave in that reference frame. However, in reference frames where the medium is not at rest, the introduction of a wave, in addition to increasing the sloshing energy of the particles, may also tend to slow the velocity of the beam. If this second effect is strong enough, then the introduction (or growth) of such a wave actually results in less energy in the system than before. Such waves are called negative energy waves. The introduction of any dissipative effect into such a system will then result in the growth of this wave, rather than its damping as in the case of a positive energy wave.

In terms of the Fourier components of the electric field, the wave energy and wave energy flux can be found in the following simple manner:[12,13,14]

$$U = \text{wave energy} = \frac{\partial G}{\partial\omega}$$

and

$$\vec{S} = \text{wave energy flux} = -\vec{\nabla}_k G$$

$$\text{where } G \equiv \frac{E_k^2}{8\pi}\ \frac{\varepsilon(\omega,k)}{\omega \sum_i [\text{cofactors of } T_{ii}]}$$

The quantities U and $\vec{S}$ obey the usual energy conservation law

$$\partial_t U + \vec{\nabla}\bullet\vec{S} = R \qquad (7)$$

where R is the energy density loss rate due to dissipative effects.

The above definitions reduce to the usual case Eqs. (5) and (6) for vacuum electromagnetic waves.

B. Character of the Eigenmodes

The 8 eigenmodes of Eq. (3) may be identified as follows:

1) Four electromagnetic modes of the general form

$$\omega^2 = k^2c^2 + \text{[correction terms due to the presence of the electron beam]} \tag{8}$$

where the effect of the electron beam is to remove the degeneracy in the forward traveling and backward traveling electromagnetic modes. The phase velocity of these waves is always greater than c for this wave guide configuration, and hence they cannot serve as traveling waves for acceleration for this configuration. These are furthermore all positive energy modes, which might be expected since they are nothing more than slightly modified vacuum electromagnetic modes.

2) Two Doppler-shifted plasma modes, with a dispersion relation appropriate for a strong magnetic field limit. For such a case the magnetic field effectively prevents transverse oscillation (except near the cyclotron frequency) and hence the mode involves longitudinal electron mass effects:

$$\omega = k_z v_e \pm (\omega_p/\gamma_e)(k_z/k) \tag{9}$$

The phase velocity of the upper branch (positive sign) is always greater than v_e, the electron flow velocity. Furthermore, it is a positive energy mode. The lower branch (negative sign) which is a negative energy mode, unfortunately has a phase velocity range bounded by $c(1-\frac{1}{\gamma}) < v_\phi \le v_e \approx c$, so that it is not useful as a traveling wave.

3) The final two modes are the Doppler shifted cyclotron modes. The dispersion relation for this mode is to good approximation given by

$$\omega \cong k_z v_e \pm \Omega \frac{k^2c^2}{k^2c^2 + \omega_p^2} \tag{10}$$

The upper branch suffers from the same deficiencies as the upper branch of the plasma mode. However, the lower branch (negative sign) has the proper features of being both a negative energy wave and having a phase velocity variable from $0 \le v_\phi < v_e$. Of the 8 eigenmodes, this wave alone is suitable for use as the wave of a traveling wave accelerator for the electron beam configuration considered.

IV. Principle of Acceleration

We now consider the acceleration process proper. Let the electron beam be traveling from left to right, with the low phase velocity cyclotron wave with the ions trapped in the troughs of the wave entering the system at the left also. Furthermore, let the magnetic field be slowly decreased spatially from its initial value B_0 at the left hand boundary to a final value B_1. The electron beam, because it tends to be tied to the magnetic field lines, will expand in a flux preserving manner, i.e., "a" will scale as $B^{-\frac{1}{2}}$. The walls of the conducting guide are thus also considered to be expanded spatially in a similar manner, $b \approx a \propto B^{-\frac{1}{2}}$.

Because only a spatial change is being made in the magnetic field, the frequency of the mode stays fixed at its initial frequency ω_0, while the wave vector changes in order to continue to satisfy the dispersion relation:

$$\omega_0 \cong k_z v_e - \Omega \tag{11}$$

and the boundary condition $k_\perp a = 2.4$, the first zero of J_0. Thus the phase velocity is given by:

$$v_\phi = \frac{\omega_0}{k_z} = v_e \frac{\omega_0}{\omega_0 + \Omega} \hat{z} \tag{12}$$

and, therefore, the phase velocity of the wave increases down the accelerator. By tailoring the magnetic field appropriately so that there is no sudden acceleration which would cause the ions to be spilled from the potential well in which they are trapped, the ions may be brought up to a velocity comparable to v_e.

Equations (11) and (12) govern the adiabatic change in wave number and phase velocity of the wave. In order to calculate the change in the electric field amplitude, Eq. (7) must be used. For the temporally independent system considered here we obtain

$$\vec{\nabla}\bullet\left[\frac{E_k^2}{8\pi} \frac{\vec{v}_g \frac{\partial\varepsilon}{\partial\omega}}{\omega\Sigma \text{ cofactors of } T_{ii}}\right] = R = -\vec{\nabla}\bullet \left[\hat{z}\,\frac{1}{2} M_i n_i v_\phi^3\right] \tag{13}$$

where the acceleration of the ions provide the dissipation. Here M_i, n_i are the mass and density of the accelerated ions, respectively. Integrating Eq. (14) over the volume of the accelerator, we obtain the following equation governing the change of the electric field strength down the accelerator:

$$\pi a^2\left[\frac{E_k^2}{4\pi} c \frac{\omega_0(\omega_0 - k_z v_e)\, k^2}{\omega_p^2 k_\perp^2} + \frac{1}{2} M n_i v_\phi^3\right] = \text{constant} \tag{14}$$

In the foregoing analysis the response of the electrons was considered within the linear approximation. We therefore require that the perturbed velocity and position of the electrons remain small compared to the equilibrium values. Such consideration gives rise to the following constraints:

$$e\phi \ll \gamma m_c^2 \tag{15}$$

and

$$I_e > 1.68\gamma \left[e\phi/(\gamma mc^2)\right] \tag{16}$$

where ϕ is the electric potential of the wave. Furthermore, total energy flux must of course be conserved; which places a kinematical upper bound on the ion energy:

$$I_e \gamma m \gg I_i M_i (\gamma_i - 1) \tag{17}$$

where γ_i is the final ion relativistic gamma factor.

Equations (11), (14)-(17), along with the equilibrium and stability constraints, Eqs. (1) and (2), delineate the parameter range of operation of the accelerator.

V. Generation of the Cyclotron Eigenmode

It is evident that rather large amounts of power must be invested in the traveling wave to effect the acceleration of ions in the relatively short distances the extremely high electric fields would indicate possible. The pulsed electron beam source provides a convenient source of energy for growing the desired eigenmode at the desired amplitude and phase velocity through use of the negative energy character of the wave, without having to rely on large amounts of external RF power and attendant coupling problems.

The introduction of a resistive liner within a section of the guide introduces a dissipation within the system and hence renders all negative energy modes unstable. Analysis indicates that such a scheme can indeed be employed to grow the desired eigenmode.

In particular, it was found that a resistive liner must be constructed to be highly conducting in the $\hat{\theta}$ direction, but with a predetermined conductivity σ in the $\hat{r}$, $\hat{z}$ directions. The high conductivity in the $\hat{\theta}$ direction is required to stabilize the non-axisymmetric m=1 kink instability which, for a scalar conductivity, has a higher growth rate than the purely axisymmetric mode. Such a liner could be constructed, for example, by using metallic rings of high conductivity alternated with rings of material with the appropriate resistivity. Another possibility for partially quenching the growth of the m=1 mode has been offered by M. Rosenbluth, who suggests that the resistive liner be surrounded by a highly conducting material, with the liner chosen thinner than the skin depth of the cyclotron wave. Such a configuration would partially short out the $\hat{\theta}$ electric field component associated with the m=1 mode, thereby lowering its growth rate below that for the axisymmetric mode.

Here we only consider the growth of the axisymmetric mode. For a low phase velocity, the electric field is given roughly as the gradient of a potential

$$E = -\nabla\psi, \quad \psi = \Phi(r)\exp(ik_z z - i\omega t) \tag{18}$$

Interior to the beam (r<a) we have $\Phi(r) \propto J_o(k_\perp r)$, while in the interior of the resistive liner (r>a) $K_o(k_z r)$ is the appropriate form for $\Phi(r)$.

Using the appropriate matching conditions across the beam-liner interface, one arrives at the following dispersion relation for $k_\perp$:

$$\frac{k_z^2}{k_\perp^2}\,\frac{\partial_a J_o(k_\perp a)}{J_o(k_\perp a)} = -\left(1 + i\,\frac{4\pi\sigma}{\omega}\right)\frac{\partial_a K_o(k_z a)}{K_o(k_z a)} \tag{19}$$

where the frequency ω is governed by the cyclotron dispersion relation

$$\omega \cong k_z v_e - \Omega\frac{k^2c^2}{k^2c^2 + \omega_p^2} \tag{20}$$

Equations (19) and (20) can be solved approximately in the limit that $4\pi\,\sigma/\Omega << 1$ by expanding $J_o(k_\perp a)$ about its first zero to obtain the following estimate for the growth rate Γ of the wave:

$$\Gamma \approx 2\Omega\,\frac{I_e}{4\nu}\,\frac{k_z a}{(6+k_z^2a^2)^2}\,\frac{4\pi\sigma(k_z v_e - \Omega)}{16\pi^2\sigma^2 + (k_z v_e - \Omega)^2} \tag{21}$$

which is strongly peaked about $k_z v_e \approx \Omega$. By choosing $\frac{\Omega a}{v_e} = 1.4$ this growth rate can be maximized to yield:

$$\Gamma\ \max \approx \frac{I_e}{1288}\,\frac{c}{a} \quad \text{at } Ba \cong \sqrt{6}\,\gamma \tag{22}$$

The phase velocity of the most unstable wave is given by:

$$v_\phi \cong c\,\frac{4\pi\sigma}{\Omega} \tag{23}$$

Thus by an appropriate choice of $4\pi\sigma/\Omega$, the phase velocity of the wave may be chosen as required. Furthermore, the amplitude of the wave may also be selected by an appropriate choice of the length of the liner. Finally, we note that such a resistive liner also renders the negative energy plasma wave unstable. However its growth rate can be shown to be substantially smaller and hence the inclusion of an appropriately tailored resistive liner interior to the conducting guide allows a cyclotron mode with the proper phase velocity and amplitude to be generated.

VI. Operating Parameters and Problem Areas

As an example to illustrate the operating parameters for the autoresonant accelerator, we consider a 12 MV, 100 kamp electron beam with an initial beam radius of 1 centimeter. Then with a magnetic field decreasing from 200 kG to 2.5 kG, acceleration of upwards of 500 amperes of ions to the 1 GeV energy level can be achieved in an accelerator length of less than 5 meters. Of course, the electron beam and magnetic field requirements are fairly severe for this case; however, these numbers are meant only to be exemplary of the performance one might expect from an autoresonant accelerator.

There are of course numerous problem areas that remain to be completely investigated. Of particular concern are 1) nonlinear instabilities associated with the electron response to the cyclotron eigenmode for large values of electric field strength, and 2) trapped particle instabilities between the electrons and the ions trapped in the potential troughs of the accelerating electric field. The so-called decay instability has been briefly examined for the cyclotron mode and appears not to be a case of concern. Furthermore, because the electrons are used in a one pass fashion, one might anticipate that most instabilities would be convective in nature and possibly convect out of the system before growing to an amplitude large enough to be disruptive. However, the complete answer to such questions of stability and performance will have to await further analysis, computer simulation, and experimental determination.

This work was supported by ARALCO, Austin, Texas.

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