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ABSTRACT

Recent progress in Hamiltonian lattice gauge theory is reviewed. Methods considered include variational, weak and strong coupling expansions and finite scaling techniques.

1. HISTORY

Kogut and Susskind [1] proposed the discrete space/continuous time Hamiltonian lattice approach to the numerical solution of quantum field theories. The theory is described by a quantum Hamiltonian $\hat{H}$ corresponding to a collection of coupled Schroedinger equations. The connection with the Euclidean lattice partition function Z is via the transfer matrix

$$Z \sim \lim_{a_0 \to 0} \mathrm{Tr}\left[e^{-a_0 \hat{H}} \right]^{T/a_0}$$

(a_0 is the temporal lattice spacing and T its extent). For a typical gauge theory (e.g. QED), $\hat{H}$ takes the schematic form (in some gauge):

$$\hat{H} = \hat{H}^{e\ell} + \hat{H}^{mag.} = \sum_{\ell} \hat{E}_{\ell}^{2} - \frac{1}{g^4} \sum_{\square} (\hat{\square} + \hat{\square}^{+})$$

where ℓ denotes links and $\square$ plaquettes. Standard NRQM methods apply and so, for example, series expansions in $x = 1/g^4$ can be obtained. Weak coupling estimates ($x \to \infty$) can be extracted from these using Padé approximant or other extrapolation techniques (modulo intervening singularities/critical points).

The Hamiltonian (in particular strong coupling series) approach appeared very promising up to about the time of the Lisbon Conference (1981). For QCD series expansions in D=3+1 dimensions, the state of the art was

glueball masses	$O(g^{-16})$	[2]
baryon masses	$O(g^{-16})$	[3]
axial string tension	$O(g^{-24})$	[4]

In 1981, worries about (a) the roughening transition, (b) the presence of nearby singularities in the cross-over region and (c) the amount of work needed to do the calculations severely curtailed activity in this field. Although roughening was soon realised to be a curiosity of the axial string sector and (b,c) could possibly be overcome by technical developments, the Hamiltonian approach was halted in its tracks.

What has happened since?

2. VARIATIONAL METHODS

One can make a variational anzatz for the vacuum or other eigenstate of the Hamiltonian. Following Drell et al. [5] many authors have written product (or Jastrow) -type solutions of the Schroedinger equations, e.g.

$$\Psi_0^{trial} = \exp\left\{\alpha \sum_{\square} (\hat{\square} + \hat{\square}^{+})\right\} |0\rangle \qquad (1)$$

where α is a variational parameter minimised to improve the bound

$$E_0^{trial} \geq E_0, \quad \hat{H}\Psi_0^{trial} = E_0^{trial}\Psi_0^{trial}$$

Matrix elements with respect to Ψ_0^{trial} for a D=d+1 theory can be evaluated as an equivalent d dimensional Euclidean problem. For d=2, in particular, this means exact solutions are available for the variational problem.

A recent example of U(1) and SU(2) in D=2+1 was provided by Hofsäss and Horseley [6] using a trial wave functional of the form (1). For these theories the weak coupling behaviour can be anticipated by analytic approximations and, indeed, the variational anzatz gives a satisfactory description of the vacuum (energy density E_0/N and specific heat C_V) from strong (s.c.) to weak coupling (w.c.). However, the string tension σ in both theories behaves as $\sigma a^2 \sim g^4$ at w.c. rather than $\sim g^4$ for SU(2) (which is super-renormalisable and gives trivial dimensional scaling for its observables [7]) and $\sim e^{-c/g^2}$ for U(1) (which has an analogous confining mechanism to QCD in D=3+1).

Recently several attempts to go beyond the basic "uncorrelated product" (1) have been made. Langguth [8] studied U(1) in D=2+1 both with (1) and with a modified form

$$\Psi_0^{trial} = \prod_{\square} \sum_{k=-\infty}^{\infty} e^{-k^2/2\alpha} (\hat{\square})^k \quad |0\rangle \qquad (2)$$

allowing higher plaquette excitations which he concluded were essential to obtain correct w.c. behaviour for the mass-gap (Ma) and string tension. Suranyi [9] has produced similar results for U(1) using a non-product form. Although these various calculations give qualitatively correct w.c. behaviour for Ma and σa^2 ($\sim e^{-c/g^2}$) they disagree quantitatively with each other and with the series and scaling methods discussed below (§4,5).

For SU(2) in D=2+1, Arisue et al.[10] tried to improve the w.c. behaviour of M and σ by perturbatively introducing □ and ◫ correlations. Some success was achieved while, as expected, the vacuum observables were little changed. However, the ◫ terms necessary were large and the prospects for a systematic improvement along these lines looks bleak.

Work in D=3+1 is complicated by the need for considerable numerical effort and the more subtle constraints of gauge invariance. Horn and Karliner [11] using product type wave functionals (1) in □ space and in link space (with suitable gauge invariant projections) confirmed the pattern already found in 2+1, i.e. satisfactory vacuum quantities but incorrect string tension at w.c. A further undesirable feature of the link-product anzatz is the production of a critical point at finite g^2 (s.c. to w.c. cross-over region). Hellmund [12] studying SU(3) found a similar result and attempted to "smear" away this critical point by a partial introduction of link correlations.

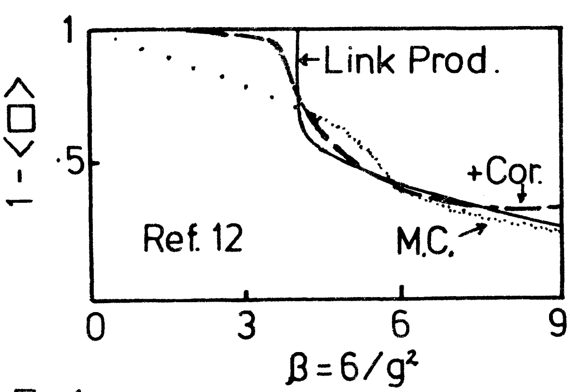


Fig. 1

Fig. 1 [12] shows the effect of this smearing for the mean plaquette energy 1 - □ in comparison with Monte "data". The agreement is still rather poor.

Theoretical objections to the Jastrow-type wave functional (1) have been raised by Anishetty and Bovier [13] who claim that such a form cannot regain Lorentz invariance in the continuum limit. They give an example from D=1+1 to demonstrate this.

What appears to be needed is a systematic scheme for the improvement of Ψ_o^{trial}. Heys and Stump [14] have used Ψ_o^{trial} as input to an M.C. importance sampling procedure. As yet the initial variational wave functions have proved inadequate at weak coupling. Recently, Horn and Weinstein [15] noted that as the parameter $\tau\to\infty$ the new trial wave function

$$\Psi_\tau = \exp\left\{-\frac{\tau}{2}H\right\}\Psi_o^{trial} \qquad (3)$$

contains an ever-increasing projection onto the true vacuum. By extrapolating series in τ they proposed to obtain a systematic improvement on the initial trial function. Tests on simple models are encouraging although care in choosing Ψ_o^{trial} is obviously needed. Technically, it appears at least as demanding as S.C. series (§4). An attractive feature of the method is that it provides a link between variational and s.c. formalisms.

3. WEAK COUPLING CALCULATIONS

Direct w.c. calculations in Hamiltonian formalism are mainly restricted to D=2+1. Müller and Rühl [16] used a flux-loop variable formalism [7,17] to calculate the mass gap in second order w.c. perturbation theory. Their result Ma ≃ .2637 g^2 is almost an order of magnitude smaller [18] than S.C. extrapolations (§4) and M.C. estimates. The origin of this disagreement has not yet been established.

Suranyi [19] has used W.K.B. methods to study D=2+1 U(1) and finds qualitatively correct W.C. behaviour for the mass-gap.

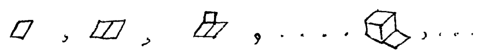
Luscher and Münster [20] have presented a W.C. expansion of SU(2) eigenvalues. This is not a lattice but a spatially cut-off field theory approach. Convergence is not wholly satisfactory and a detailed comparison with M.C. data has yet to be made.

4. STRONG COUPLING

Little progress on QCD in D=3+1 has been made beyond the 1981 limits (§1). Kimura [21] has made $O(g^{-8})$ estimates of excited glueball masses to be compared with M.C. calculations. The idea of recasting existing S.C. series in terms of physical quantities to improve extrapolation has been implemented by Münster [22] (e.g. using the correlation length (1/Ma) rather than x).

The major barrier to progress with S.C. is in reaching high enough order. With this in mind, the linked-cluster expansion method proposed by Nickel [23] has been extended by Hamer and Irving [24] to obtain a higher order series for lattice gauge theories in 2+1 and 3+1 dimensions.

Calculations are performed on a sequence of linked clusters of plaquettes

 (4)

For example, in D=2+1 the string tension and mass-gap are now known to $O(g^{-48})$ sufficient to observe clear evidence of weak coupling behaviour. As seen in fig. 2a the results [25] are considerably more accurate than variational estimates (§3)

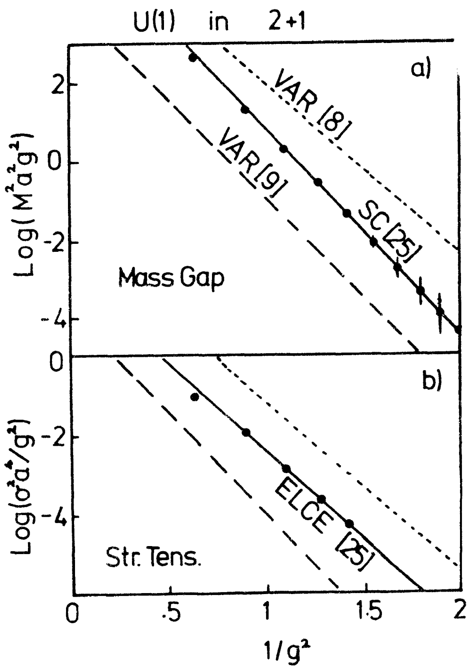


Fig 2

For SU(2) in D=2+1 [18], $O(g^{-40})$ series for Ma and $O(g^{-56})$ for C_V again show the expected weak coupling behaviour. The Ma results agree with M.C. estimates but disagree with the W.C. results of ref. [16] (§3). Related string tension results are discussed in §5.

In D=3+1, further S.C. series for U(1) and Z_N abelian theories have been obtained by such linked-cluster methods [24,26] and used to determine critical parameters. The non-abelian theories SU(2) and SU(3) are now being subjected to these techniques [28]. Before turning to scaling methods which are particularly relevant to the treatment of roughening, we note another proposed technique to deal with artifacts induced by the loss of rotational symmetry. Schlereth [27] has proposed a Hamiltonian version of the Gaussian smeared action whose rotational properties are expected to be improved at finite g^2. S.C. expansions are proposed but not yet implemented.

5. SCALING METHODS

The finite lattice technique of Hamer and Barber [29] which proved so successful in D=1+1 was pushed to the limit with the study of U(1) in D=2+1 [30]. Its advantage over S.C. and similar methods is that it has guaranteed convergence to the correct bulk limit at all g^2 (phase transition or not). Thus it can be used to estimate the axial string tension even in the presence of roughening e.g. [30].

In D=3+1 this approach is impracticable (too many states) but a similar procedure involving the exact computation of cluster (rather than full lattice) eigenvalues has been developed [24,25,18,26]. As its name suggests (ELCE: exact linked cluster expansion) the alogrithm is analogous to the series linked cluster expansion of §4 (it uses the same sequence of clusters (4)) and gives reliable estimates of the ground state energy and axial string tension in D=2+1 [24,18] and 3+1 [25,26]. The stability of these results for σa^2 (e.g. fig. 2b) is in constrast to the uncertainties found in the various M.C. methods of extracting this quantity [31]. A similar algorithm for masses has been developed but is less useful.

6. CONCLUSIONS AND OUTLOOK

As well as being a complementary approach to Monte Carlo simulation, the Hamiltonian method has two valuable features. First, in its variational form it allows physical insight into the structure of the vacuum via the explicit construction of the latter's wave function. Second, it is capable of extremely high accuracy in numerical work with S.C. expansions and scaling methods.

In the variational approach, we now recognise the increasing complexity and correlated structure of the vacuum at weak coupling. Systematic methods for improving trial wave functions are now badly needed. The series approach when pushed to higher orders using new "automated" methods and supplemented by new scaling algorithms now allow us access to the weak coupling regime and gives reliable estimates of non-trivial physical quantities in the continuum limit.

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POTENTIALS FROM LATTICE GAUGE THEORY

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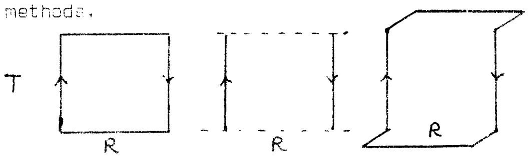
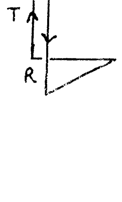
The energy increase from static sources of colour in the fundamental representation at separation R is just the quark-antiquark potential V(R) for heavy quarks which is relevant to the spectroscopy of $c\bar{c}$, $b\bar{b}$ mesons, etc., if light quark virtual states are relatively unimportant. This static interquark potential can be studied by lattice Monte Carlo methods and provides a calibration of the lattice method. Potentials corresponding to excited gluonic fields, to adjoint representation sources, and to spin-spin interactions are then discussed.

1. STATIC QUARK POTENTIAL. This potential can be extracted by studying rectangular Wilson loops, correlations of Wilson lines, or by variational methods.

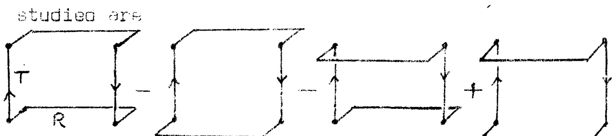


Results from the first two methods were in apparent disagreement but a reanalysis[1] of the raw results has resolved this discrepancy. The ground state potential extracted[1] has been confirmed by two recent evaluations using rectangular Wilson loops and large lattices[2,3]. Thus at $\beta=6$ for SU(3) the string tension K is found to be $\sqrt{K}/\Lambda_L = 105\pm10$[1], 105 ± 1[2], 106 ± 3[3]. A wide range of β values is studied so one can check whether the lattice extraction of aV for different values of R/a is consistent with a continuum limit V(R) as lattice spacing a decreases with increasing β. It is found that results are consistent and so $a(\beta)$ can be extracted. For $\beta>6$ up to $\beta \approx 6.5$ where the finite size of the lattice will cause modifications, the result for $a(\beta)$ is in agreement with the perturbative expectation - this is asymptotic scaling. Below this, one finds consistency with scaling but with $a(\beta)/a(\beta)_{PERT} = 1.38$ at $\beta=5.6$[2]; 1.28 at $\beta=5.7$[4] and 1.08 at $\beta=5.8$[2]. This conclusion that the lattice may be reliable down to $\beta=5.6$ if $a(\beta)$ is interpreted suitably is supported by a study[5] of the restoration of rotational invariance by looking



at potentials for $R = \sqrt{2}a$, $\sqrt{3}a$, etc., from Wilson loops like that shown. For $\beta=5.7$ but not $\beta=5.4$ these potentials interpolate smoothly.

2. EXCITED GLUONIC FIELD/HYBRID MESONS. In a non-abelian theory such as QCD it is possible that in the presence of a static quark and antiquark at separation R, the gluonic field can exist in several different configurations with different energies. Some of these configurations will have a component of angular momentum about the $q\bar{q}$ axis and so can bind mesons with exotic J^{PC} values (such as 0^{+-}, 1^{-+}; 2^{+-} etc.). Such mesons are called hybrid since the gluonic field is contributing in an essential way to them. Lattice gauge theory allows a search for such excited gluonic states with discrete energies. Typical Wilson loop combinations studied are



A lattice variational technique is used[6,7] and the states are classified under the discrete group D4h. This classification can be related to the continuum group D∞h and orbital angular momentum can be added to give the resulting mesonic J^{PC}. The lowest energy state is of course the most symmetric (A1g with longitudinal colour electric field) which has been studied in the preceding section. The first excited states are in Eu (transverse colour magnetic field) and in A1u (longitudinal colour magnetic field) representations. The Eu potential is the best determined (the Wilson loop combination illustrated above is in this representation) and results[7] for this potential in SU(3) are given in fig. 1. Here the points are upper limits since a variational method has been used.

Solving Schroedinger's equation in the A1g and Eu potential then allows the excited state spectrum of mesons to be determined. The results[7] are shown in fig. 2 where one sees that the usual radial and orbital $q\bar{q}$ excitations are quite well described by the A1g potential and that the lowest Eu excited states lie about 1 to 1.5 GeV above the ground state. Such hybrid mesons are characterized by the presence

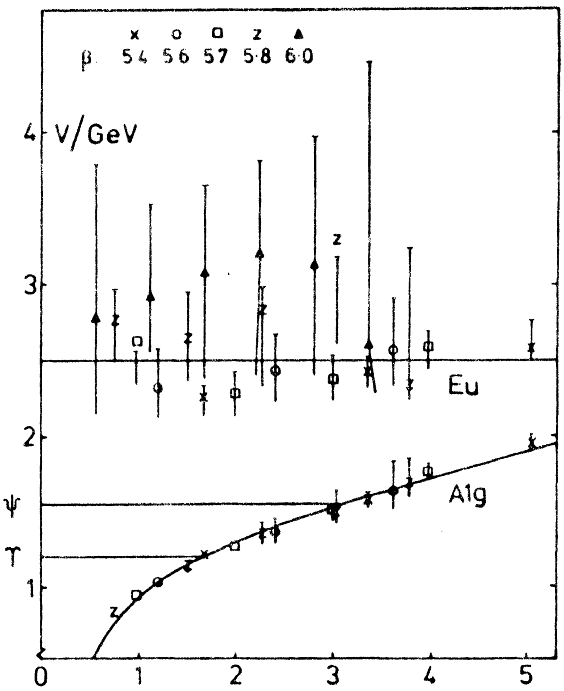


FIGURE 1

of exotic J^{PC} assignments. Even so they will be difficult to detect experimentally since they lie above the open $C\bar{D}$ or $B\bar{B}$ channels so are likely to be broad resonances. They have a small overlap with e^+e^- since they have a large radius in such a flat potential and, moreover, the spectrum will be dense since radial and orbital excitations of the lowest such Eu-state will lie close in energy.

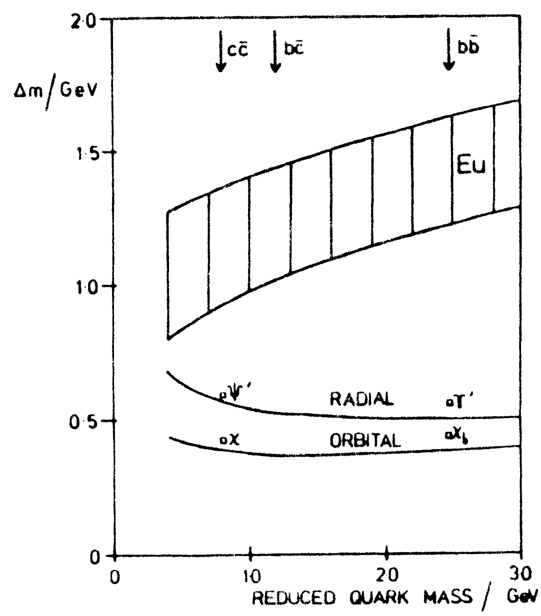
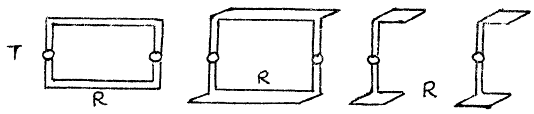


FIGURE 2

3. ADJOINT STATIC POTENTIAL. The potential between adjoint colour sources should not exhibit confinement since it can be screened by glueballs. It is of interest to see if a region of linearly rising potential does, nevertheless, exist between the coulombic region at small R and the saturation at large R. In such a case a gluonic string tension K_{ADJ} could be defined and it is of relevance to gluon jet evolution. Another application of the adjoint potential is to gluino bound states if a massive gluino exists.

The adjoint potential can be studied by evaluating Wilson loops such as



where the circle refers to an adjoint projection operator. The last case clearly yields an R-independent potential at large separation and this is the energy of the deconfinement. Using a variational method, preliminary results from the Liverpool group in SU(2) on a 16^4 lattice are shown in fig. 3.

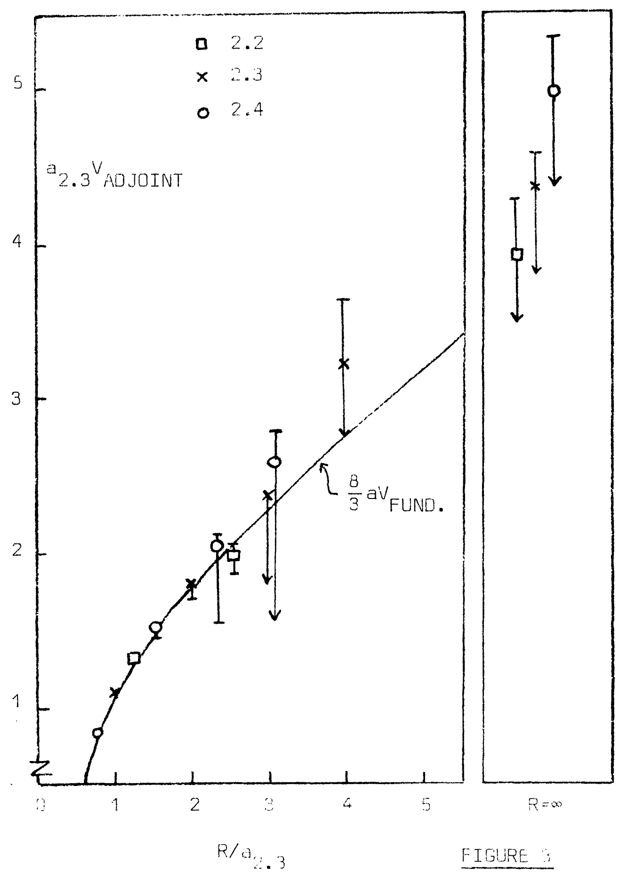
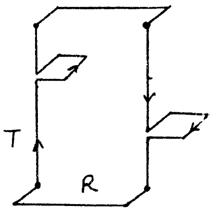


FIGURE 3

These results as well as those obtained using rectangular loops[8] find that $V_{ADJ} \approx \frac{8}{3}V_{FUND}$. This is the perturbative expectation in the Coulombic region but contains new information beyond, namely that a linear rise is present and that $K_{ADJ} \simeq \frac{8}{3}K$. Our evaluation of the R=∞ potential allows us to estimate that for R≥7 GeV^{-1} the linear rise must cease (in SU(2) using $\sqrt{K}$ = 0.44 GeV to set the scale in GeV). These results illustrate that confinement and a linearly rising potential are separate features.

4. SPIN-SPIN POTENITAL. Peskin[9] has reviewed the connection between Wilson loops and spin-spin and spin-orbit potentials for non-relativistically moving quarks. Essentially the interaction between colour magnetic moments at separation R is mediated by fluctuations in the gluon colour magnetic field and this is determined by Wilson loops such as that shown.



A preliminary study by the Liverpool group in SU(2) using a 16^4 lattice and a variational method to project out the ground state potential at each extreme T-value gives the results shown in fig. 4 for the scalar and tensor spin-spin potentials.

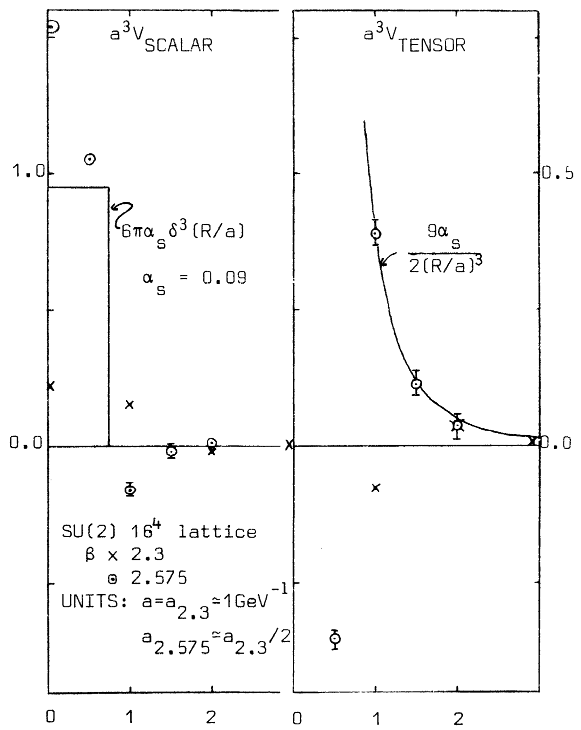


FIGURE 4 $R/a_{2.3}$

In these results each component of the colour magnetic field is measured by taking the average of four 1×1 loops which have a corner at the static source site. Other B measuring combinations are found to give different results for R≤2a but similar results for R>2a. This is in agreement with model calculations. For R>2a the results are in qualitative agreement with scaling (also taking account of the appropriate anamalous dimension) and with perturbation theory (shown by the curve falling as R^{-3} and the rectangle which has the area of the δ-function expected). In conclusion, our preliminary look at spin-spin forces in SU(2) suggests that perturbation theory is a good approximation for R>1 GeV^{-1}.

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Recent progress in the Monte Carlo calculation of the hadron spectrum of QCD has involved the use of larger and larger lattices to explore the approach to the continuum limit for the quenched theory (in which internal quark loops are ignored), and the first simulation of the fully interacting theory on a modest-sized lattice ($8^3 \times 16$)[(1)]. Here, I will concentrate on the former development; the results for dynamical quarks are described in the review by Kripfganz.

Within the quenched approximation, the different lattice regularisations of the fermionic action, due to Wilson[(2)] and Susskind[(3)], may be compared using the same set of gauge field configurations. If both are in the scaling regime they should lead to the same values for hadron masses in lattice units (apart from a possible finite renormalization of the quark mass). At $\beta = 5.7$ ($= \frac{6}{g^2}$) this comparison has been made by the Edinburgh group on an $8^3 \times 16$ lattice[(4)]. Our results may be compared with those of other groups on the same size[(1)] or bigger lattices[(5,6)] to expose any finite-size effects. These appear to be small at this β-value for these lattice sizes. This is further supported by the observation that different lattice definitions of hadron operators lead to predictions for the corresponding masses which agree within errors. Our results[(4)], together with higher statistics measurements using the hopping parameter expansion[(1)] are given in table 1.

Wilson $\beta = 5.7$	ref(1) $8^3 \times 16$	ref(4) $8^3 \times 16$
K_c	$0.1696 \pm .0016$	$0.1695 \pm .0007$
$m_\rho a$	$0.57 \pm .01$	$0.53 \pm .03$
$(m_\pi a)^2/ma$	$2.5 \pm .3$	$2.85 \pm .15$
$m_p a$	$0.97 \pm .14$	$1.11 \pm .10$
$(m_\Delta - m_p) a$	$0.25 \pm .08$	$0.02 \pm .01$

Table 1

The latter agree with a similar calculation on a 16^4 lattice[(5)]. Note that the proton-to-rho mass ratio is much bigger than the experimental value. (The disagreement on the delta-proton splitting is not understood.)

The calculations using Susskind fermions have been done at much smaller quark masses, so any systematic errors coming from the extrapolation to zero quark mass are small. In fact, both groups[(4,6)] presenting results at $\beta = 5.7$ obtain clear evidence of the Goldstone nature of the pion. However, they disagree on the rho mass in lattice units (and hence on the lattice spacing in physical units) as shown in table 2.

Susskind $\beta = 5.7$	ref(7) $8^3 \times 16$	ref(6) $10^3 \times 16$
$m_\rho a$	$0.80 \pm .04$	0.98
$(m_\pi a)^2/ma$	7.0	7.6

Table 2

The explanation for this discrepancy comes not from the slight difference in spatial lattice size, but from a disagreement on the amount of contamination by radial excitations at the lighter quark masses ($0.01 \lesssim ma \lesssim 0.05$). Because these cannot be easily included in the fit to the rho propagator (from which the rho mass is deduced), timeslices nearest to the origin must be successively removed from the fit until the mass estimate stabilizes. The results obtained by us[(7)] at a typical quark mass value are shown in fig. 1. In contrast to the pion mass, which is extracted from a fit which includes a radial excitation, the estimated rho mass falls as timeslices close to the origin are removed, indicating the diminishing influence of the radial excitation. Gilchrist et al.[(6)] drop only one timeslice (G in fig. 1), whereas our[(7)] study suggests it is necessary to drop at least two. Our results for the rho meson and pion masses at different quark masses are shown in fig. 2. Note that our results agree completely with Gilchrist et al. (G in fig. 2) if analysed in the same way they do ($m_\rho(13)$ in fig. 2). However, there is a second source of systematic error: an artificially small rho mass estimate may result if the fit includes mostly timeslices far from the origin where the small signal-to-noise ratio tends to flatten the propagator. In the results presented here we[(7)] have tried to improve the signal by computing the sum

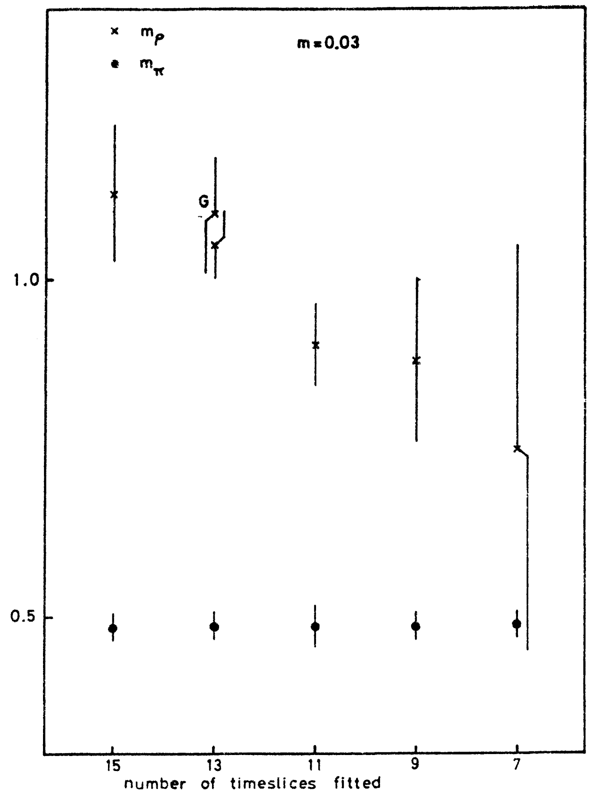

Figure 1

of the quark propagators from 64 origins on one timeslice. This requires no extra computation. However, since the quark propagator enters quadratically in the meson propagator, this introduces an unwanted gauge-dependent contribution which should average to zero. We have checked that this cancellation occurs to within the error in the propagator in the average over 16 gauge configurations. As a result, the rho propagator is exposed above the noise one or two more timeslices further from the source compared to the results from using one origin. On account of these two systematic effects, we can only conclude that

$$0.80 \lesssim m_\rho a \lesssim 0.98.$$

This precludes any agreement between Wilson and Susskind formulations at $\beta = 5.7$ e.g. the lattice spacings we[4,7] obtain from the rho mass are

a(Wilson) = 0.136 ±.008 fm, a(Susskind) = 0.205 ± .010 fm.

Our results for the nucleon mass are in rough agreement with Gilchrist et al. (marked G) as shown in fig 3, although the statistical errors are much too large to give anything more than a hint that the Susskind formulation is in better agreement with experiment.

There is an unphysical critical point in the upper half of the fundamental-adjoint gauge coupling plane, which lies close to the points on the fundamental axis where most hadron mass measurements have been made. We[4] explored lines of constant string tension in the lower half plane, and observed

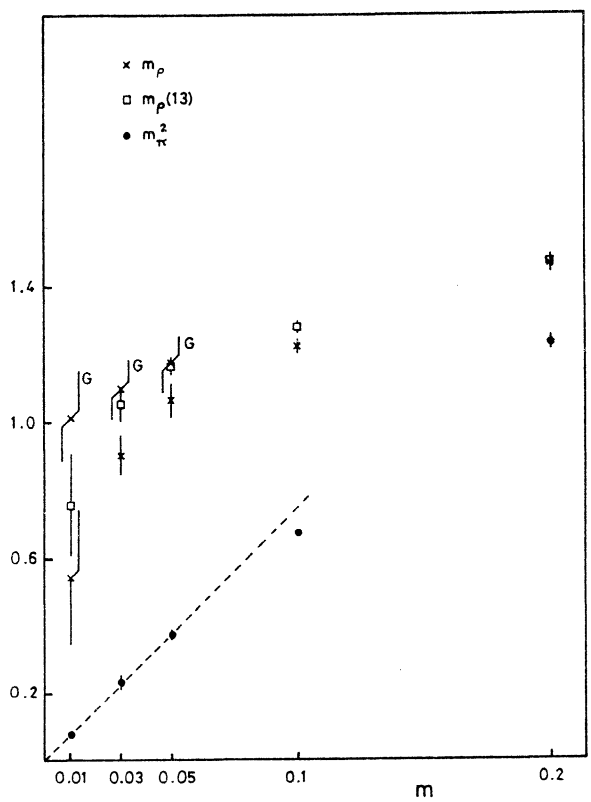

Figure 2

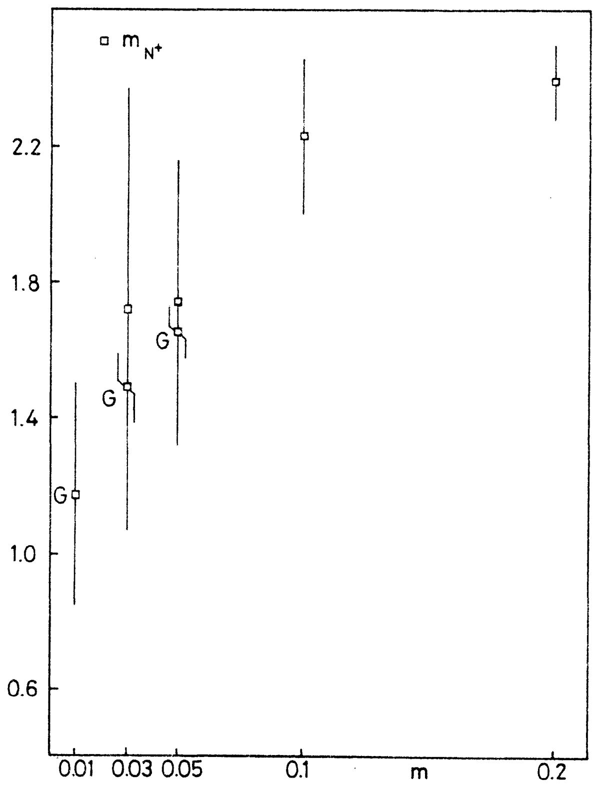

Figure 3

that the specific heat peak disappears as we move away from the critical point. However, hadron physics on an $8^3 \times 16$ lattice is approximately constant, apart from a slight improvement for Wilson baryons: there is an order-of-magnitude increase in the delta-proton mass splitting, although the proton-to-rho mass ratio remains too big. Thus, the unphysical critical point is not responsible for the discrepancy with experiment and the fault appears to lie with the Wilson fermion prescription.

At $\beta = 6.0$ there are large finite-size effects on an $8^3 \times 16$ lattice[4]. This is signalled by inconsistencies between the results from different lattice operators for the same hadron and by an approximate pion-rho degeneracy. On a $10^3 \times 20$ lattice the situation is better[8,9], and a comparison between the Wilson and Susskind formulations has been carried out[9]. This time the quark mass values used in the Susskind calculation are much larger than in the Wilson calculation, so it is the results of the former that are more sensitive to the choice of extrapolation to zero quark mass. Approximate agreement is claimed for the meson spectra obtained in the two schemes, with a lattice spacing

$$a \simeq 0.09 \text{ fm}.$$

However, the quark mass estimates differ by about a factor of 2 due to the difference in slope of $(m_\pi a)^2$ versus ma (c.f. tables 1 and 2). For the baryons there are large statistical and systematic uncertainties, particularly in the Susskind formulation. For Wilson fermions again the proton-to-rho mass ratio is too big and the delta-proton splitting is too small, results which have been confirmed by a recent calculation on a $16^3 \times 28$ lattice[10]. This last calculation uses a block diagonalization technique to reduce the lattice on which the quark propagator is calculated to $4^3 \times 7$, while preserving the long-distance properties of the original lattice.

Little attempt has been made to test scaling in hadron mass calculations because of the problems already mentioned. The Susskind formulation offers the best agreement with experiment and Gilchrist et al.[6] claim that the hadron masses computed at $\beta = 5.7$ and 5.9 on a $10^3 \times 16$ lattice, when related by the 2-loop β-function, all lie approximately on universal curves (as functions of quark mass), as do the experimental values.

Susskind fermions, being the spin diagonalisation of the naive lattice action, introduce corrections of order a^2 and so, in some sense, already represent a tree improvement over Wilson fermions which have corrections of order a coming from the r-term. This may account for their better behaviour. However, it should be noted that recent results[1] in which dynamical quarks are incorporated in a hopping parameter expansion calculation on an $8^3 \times 16$ lattice, correct the quenched Wilson results in the right direction, producing agreement with experiment within large error bars. Before a clear picture can emerge, though, the baryon sector must be cleaned up and results must be obtained on larger lattices (at correspondingly larger β-values).

The calculation of quark propagators on large lattices is very demanding of computer time and memory. Block diagonalization[10], mentioned previously, and partitioning[11] have been proposed in order to drastically reduce the size of the matrix which must be inverted. The latter effectively eliminates the time direction and, in the simplest case, computes the propagator to a single spatial hyperplane. It is based on the observation that, for an action involving only nearest-neighbour fermionic couplings, the propagator on all timeslices is completely determined by the propagator on any two neighbouring timeslices. With Dirichlet boundary conditions in time, one of these can be taken to be the fixed boundary, leaving its neighbour to be determined. This requires a column of the inverse of a matrix, which is defined through a second order recursion relation involving only products of the inverse propagators on each of the 3-dimensional timeslices, which can be done using, for example, the conjugate gradient algorithm. The matrix which must be inverted becomes increasingly ill-conditioned as the number of timeslices grows, so it is important to scale its diagonal elements to order unity and to use an adequate word length (e.g. 64-bit words for a $16^3 \times 28$ lattice). This both speeds up convergence and avoids rounding errors. With the added refinement of preconditioning by means of the free fermion propagator, this method is currently being used at Edinburgh to compute hadron propagators in both the Wilson and Susskind formulations on a $16^3 \times 28$ lattice.

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CHIRAL SYMMETRY BREAKING IN LATTICE QCD

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It is essential that we have a clear understanding of the chiral condensate as calculated in lattice QCD and hence the Goldstone mechanism and the associated spontaneous chiral symmetry breaking so necessary if QCD is the theory of the strong interactions. Here I shall present methods for the calculation of an results on $\langle\bar{\psi}\psi\rangle$ at light and zero quark masses below and at the chiral phase transition.

Our action consists of the standard EuclideanWilson[1] gauge field action together with the Kogut-Susskind[2] formulation for the fermionic contribution which has the form

$$S_F = -\bar{\psi}_i(M_{ij}(U)+2ma)\psi_j$$

where $M_{ij}(U)$ is essentially the link connecting nearest neighbour sites i and j, and a is the lattice spacing. On the lattice this action has an explicit $U_A(1)$ chiral symmetry (m=0) for which $\langle\bar{\psi}\psi(0)\rangle$ is an order parameter.[3]

In the quenched approximation[4]

$$\langle\bar{\psi}\psi(m)\rangle = \frac{3}{N}\sum_{i=1}^{N}\frac{2ma}{\lambda_i^2+(2ma)^2}$$

with $N = 3N_S^3N_t$, λ_i the eigenvalues of the matrix M(U) and the condensate is averaged over an ensemble of gauge field configurations in thermal equilibrium.

The eigenvalues of the fermion matrix are imaginary and appear (because of the γ_5 symmetry) in conjugate pairs. They were calculated[5] using the Lanczos algorithm[6] on 4^38, 6^38 and 8^4 lattices. In Fig. 1 we show the calculated $\langle\bar{\psi}\psi(m)\rangle$ at $\beta = 6/g^2 = 5.7$.

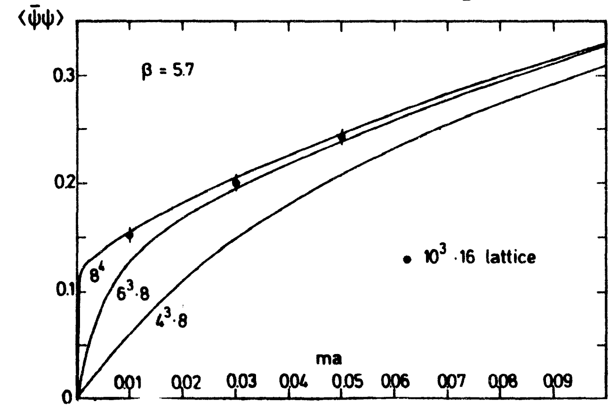


Fig.1: $\langle\bar{\psi}\psi\rangle$ as a function of ma as the volume increases

As the lattice volume increases there is a clear signal that an envelope is developing with non-zero intercept at zero quark mass and hence the chiral symmetry is spontaneously broken.

We have repeated this calculation for values of β between 0.1 and 5.9. Fig. 2 shows excellent agreement with the strong coupling expansion at low β and with asymptotic scaling for $\beta > 5.3$. This early onset of good continuum behaviour for the chiral condensate is unexpected when compared with the Monte Carlo renormalization group studies[7] of the pure gauge theory.

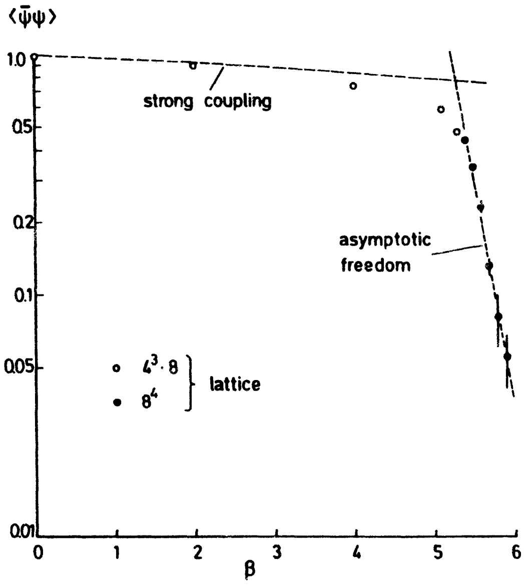


Fig. 2: The intercept $\langle\bar{\psi}\psi(0)\rangle$ for various values of β together with the O(1/d) strong coupling and scaling (asymptotic freedom) curves.

The renormalization group invariant $\langle\bar{\psi}\psi\rangle_{inv}$ is related to the measured condensate by

$$\langle\bar{\psi}\psi(0)\rangle = 2\alpha_{Mom}^{-4/11}\, a^3\langle\bar{\psi}\psi\rangle_{inv}$$

with $\alpha_{Mom} \simeq \frac{3}{2\pi(\beta-2.75)}$ for $\Lambda_{Mom} = 200$ MeV

and $a(\beta) = \frac{83.5}{\Lambda_{Mom}} e^{-\frac{4\pi^2}{33}\beta}\left(\frac{8\pi^2}{33}\beta\right)^{51/121}$

The experimental estimate of $\langle\bar{\psi}\psi\rangle^{1/3}$ is in good agreement with our measured $\langle\bar{\psi}\psi\rangle^{1/3}_{inv}$ if $\Lambda_{Mom} \simeq 200$ MeV.

(Fig. 3)

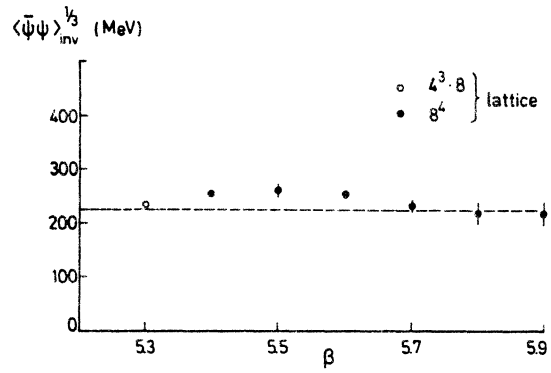


Fig. 3: The renormalization group invariant $\langle\bar\psi\psi\rangle^{1/3}_{inv}$ for $5.3 \leq \beta \leq 5.9$. The dashed line represents the experimental value[8] $\langle\bar u u\rangle = (225(\pm 25))^3$ MeV; x is $\langle\bar\psi\psi\rangle$ on $12^3 4$.

In the infinite volume limit the chiral condensate $\langle\bar\psi\psi(0)\rangle = 3\pi\rho(0)$ where $\rho(\lambda)$ is the normalized spectral density for the fermion matrix M(U). On a finite lattice the spectral density $\rho(\lambda)$ is given by

$$\rho(\lambda) = \frac{1}{N}\,\frac{\Delta n}{\Delta\lambda} \qquad \text{where} \qquad \Delta n$$

is the number of eigenvalues between λ and $\lambda + \Delta\lambda$

Hence

$$\langle\bar\psi\psi(0)\rangle = \frac{3\pi}{N}\,\frac{\Delta n}{\Delta\lambda}\Big/_{\lambda = 0}$$

Fig. 4 shows that the small eigenvalues on an 8^4 lattice at $\beta = 5.7$ give a linear plot and the chiral condensate is well determined on a finite lattice at $m = 0$ from the predominantly non-perturbative contribution on the lattice. The results of Fig. 2 are reproduced by this method.

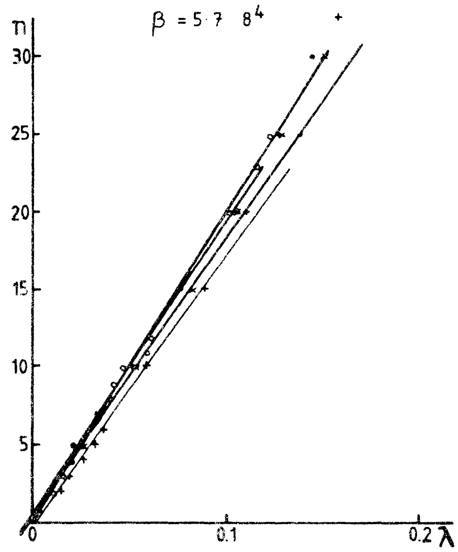
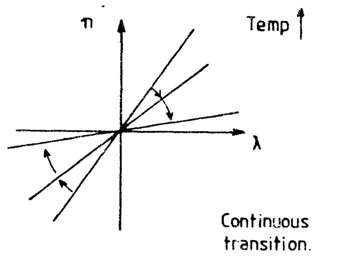
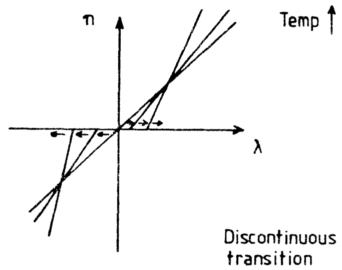


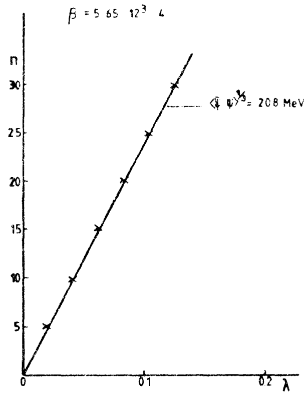
Fig. 4: Some of the lowest eigenvalues for the 4 configuration shown in Fig. 1. All low eigenvalues lead to a clear linear interpolation.

At temperatures close to the phase transition we have used this method to determine the order of the transition and T_c. If the transition is continuous then one would expect the slope $\frac{\Delta n}{\Delta\lambda}\Big/_{\lambda = 0}$ to change smoothly to zero as one raises the temperature through T_c. A discontinuous transition would require a discontinuous change in the slope at $\lambda = 0$, as illustrated below.





We have studied the zero mass behaviour of $\langle\bar\psi\psi\rangle$ on a $12^3 4$ lattice for β between 5.65 and 5.73. As can be seen from Figs. 5 the symmetry is clearly broken at $\beta = 5.65$ with $\langle\bar\psi\psi\rangle^{1/3}_{inv} \simeq 210$ MeV in remarkable agreement with the zero temperature results described above.



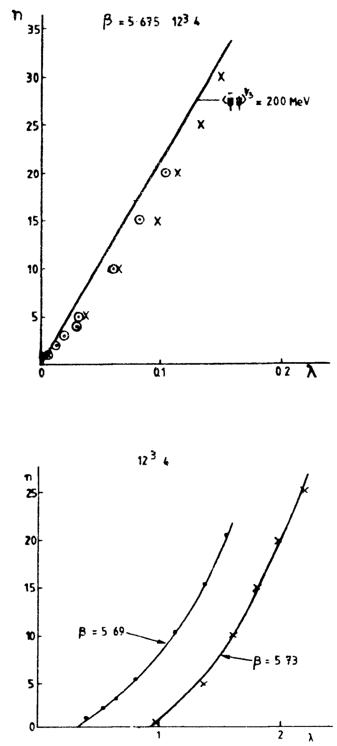


Fig. 5: the lowest eigenvalues on a $12^3 4$ lattice as the temperature rises from the broken phase to and above the transition temperature T_c. Only every fifth eigenvalue is plotted.

Again there is strong evidence for the surprising scaling behaviour of the condensate. At higher temperatures the results are consistent with a first order phase transition but, not unexpectedly, finite volume effects are large at the transition. We therefore find the phase transition at $\beta \simeq 5.675$ with

$$T_c = \frac{1}{4a} \simeq 74\ \Lambda_L = 177\ \text{MeV}$$

if $\Lambda_{MOM} = 200$ MeV, in agreement with other estimates.[9]

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DYNAMICAL FERMIONS AT NON-ZERO CHEMICAL POTENTIAL AND TEMPERATURE: MEAN FIELD APPROACH

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The deconfining phase transition of lattice gauge theory without dynamical fermions is numerically known for a couple of years /1/, and now there is also rigorous proof of it /2/. The string tension σ and the Wilson line W are acceptable order parameters in a world of static, infinitely heavy quarks only. In the presence of light quarks, the force experienced by static quarks will be screened, and the global Z_N symmetry spontaneously broken $(W \neq 0)$ at $T > T_d^*$ is explicitely broken by the effective fermionic action. During the last year the fate of the deconfining phase transition in real QCD has been a top issue /3/. The non-quenched Monte-Carlo investigation of the phases of QCD has become feasible /4/. The purpose of this talk is, however, to recall analytical attempts and to present paper # 139 (submitted by J. Kripfganz and myself) concerning the extension to $\mu \neq 0$.

The deconfining transition is well described in terms of an effective spin model derivable at strong coupling /5/. For an anisotropic lattice $(a_t \neq a_s,\ r = a_t/a_s)$ it is defined by

$$Z_{\mathrm{Spin}} = \int \prod_{\vec{x}} d\Omega_{\vec{x}}\, e^{\frac{\beta'}{2}\sum_{\langle xy\rangle}(tr\Omega_{\vec{x}}\, tr\Omega_y^+ + c.c.)} \tag{1}$$

$$\beta' = 2e^{-\frac{\sigma a_s}{T}} = 2\left(\frac{1}{g_t^2 N}\right)^{N_t}, \quad N_t a_t = 1/T$$

The spin model is known to reproduce correctly the order of the transition (e.g. 1^{st} order for SU(3) at $\beta' = 0.27$) /6/. Rephrased in terms of this model the role of dynamical fermions was argued about as follows: lowering the mass of (Wilson) fermions from infinity, the fermion determinant

$$\det(1-KM)^{n_f} \longrightarrow \prod_{\vec{x}} \det(1+\gamma^2+\gamma(\Omega_{\vec{x}}+\Omega_{\vec{x}}^+))^{2n_f} \tag{2}$$
$$\sim \exp\sum_{\vec{x}} H(tr\Omega + tr\Omega^+)$$

is switched on, and the "magnetic field" $H = (2K)^{N_t}\ 2N_f/(1+(2K)^{N_t})$ turns out to be strong enough to destroy the 1^{st} order transition experienced by W at a mass of a few times T_d^*. For really heavy quarks the influence is negligible, and their thermal density

$$n_Q/T^3 = 2N_t^3\,(2K)^{N_t}\ W \tag{3}$$

reflects merely the behaviour of the Wilson line. The extrapolation of the finding above to light quarks is obscured by the phenomenon of chiral symmetry breaking and restoration.

For massless quarks there is rigorous proof /7/ that at sufficiently high temperature chiral symmetry is restored. Hamiltonian lattice approaches /8/ establish the analogy to an antiferromagnetic Ising model. The chiral transition is second order at $m = 0$ and disappears for $m \neq 0$ (staggered magnetic field). Nonvanishing μ (uniform magnetic field) can destroy the chiral condensate, probably in a first order transition. The phase diagram in the m-T plane will be available soon from MC simulation. Nevertheless, qualitative understanding provided by effective field models is welcome. The case of $\mu \neq 0$, while unproblematical in those models, seems to be not manageable in non-quenched MC (except for N=2).

Starting from the Kogut-Susskind Lagrangian on the anisotropic lattice and integrating out the spatial links, one obtains besides of the spin-spin interaction (1) an antiferromagnetic next neighbour interaction between the color singlet bilinear $\bar{\chi}^a(x)\cdot\chi^a(x)$ within each time slice. Linearizing this by means of an auxiliary random field λ one gets again the fermion determinant factorized in 3-space (the label $\vec{x}$ is suppressed henceforth, $\eta_4 = (-1)^{x_1+x_2+x_3}$ from the KS Lagrangian):

$$\prod_{\vec{x}} \det \begin{bmatrix} m+\eta_4\lambda(1) & \frac{\eta_4}{2}U_4(1)e^{\mu a_t} & & & \frac{\eta_4}{2}U_4^+(N_t)e^{-\mu a_t} \\ -\frac{\eta_4}{2}U_4^+(1)e^{-\mu a_t} & m+\eta_4\lambda(2) & \frac{\eta_4}{2}U_4(2)e^{\mu a_t} & & 0 \\ & -\frac{\eta_4}{2}U_4^+(2)e^{-\mu a_t} & \ddots & \ddots & \\ & 0 & \ddots & \ddots & \ddots \\ -\frac{\eta_4}{2}U_4(N_t)e^{\mu a_t} & & & & m+\eta_4\lambda(N_t) \end{bmatrix}_{NN_t\times NN_t} \qquad (4)$$

The chemical potential enters like an imaginary Abelian gauge potential. It pays to use the gauge $U_4(\vec{x},\tau)$ independent of τ. We restrict ourselves to the massless case. Looking for the (homogeneous) saddle point of the integral over λ while treating the spin-spin interaction in a mean-field fashion amounts to minimize the free energy per site $(\sin h(s) = \lambda r)$:

$$\begin{aligned} F = {} & (d-1)\beta' W^2 + NN_t\,\lambda^2/(d-1) \\ & - \log \int d\Omega\, e^{2(d-1)\beta' W\,\mathrm{tr}\,\Omega} \det\left(2\cos h(N_t s) + e^{\frac{\mu}{T}}\Omega + e^{-\frac{\mu}{T}}\Omega^+\right)_{(N\times N)} \end{aligned} \qquad (5)$$

The integral over Haar's measure is taken numerically. We choose $N_t = 4$ and four light flavours. The input parameters are $T_d^* = 0.2$ GeV (deconfinement of pure gauge theory) and $\sigma = (0.4\ \mathrm{GeV})^2$. This, together with $\beta'^*_d = 0.27$, gives $a_s = 2.5\ \mathrm{GeV}^{-1}$ (kept fixed). As a function of β' and μ, we search for the values of W and λ making (5) minimal. λ acts as a dynamically generated mass and is related to the quark condensate $\langle \eta_4 \bar{\chi}\chi \rangle = 2N\lambda/(d-1)$, i.e. $\langle \bar{\psi}\psi \rangle = -0.13\,\lambda\ \mathrm{GeV}^{-3}$. Fig. 1 shows the temperature dependence of the chiral condensate λ, the Wilson line W and the thermal light quark density n_q at $\mu = 0$. Clearly discernible is the second order chiral transition at $T_c = 0.215$ GeV. n_q turns to zero at $T_d = 0.164$ GeV in a smooth way (confinement). The Wilson line rises gradually over the whole temperature range. Fig. 2 depicts the μ dependence of λ and W together with the baryon density at T = 0.174 GeV. The first order nature of the transition is evident, corresponding to $\Delta n_B \lesssim 0.02\ \mathrm{GeV}^3$.

Effective field approaches (at $\mu = 0$) have been advocated also in Refs. /9,10/. The interrelation of the (may be) two transitions was not considered at all in /9/, and denied in /10/ with the result that the different temperatures differed so much. We find that, confronted with MC results /4/, the effective field approach provides even a semiquantitatively correct picture of the phases of QCD.

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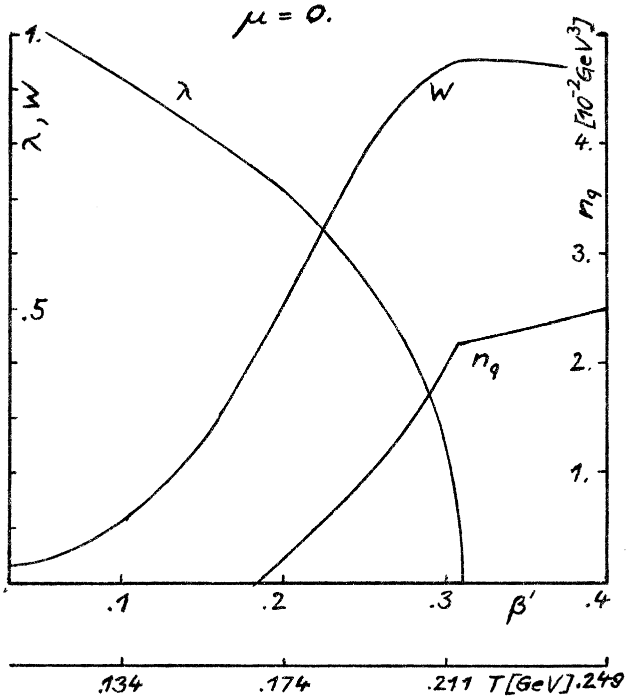

Figure 1

Chiral condensate λ , Wilson line W, and light quark density n_q vs. temperature at zero chemical potential

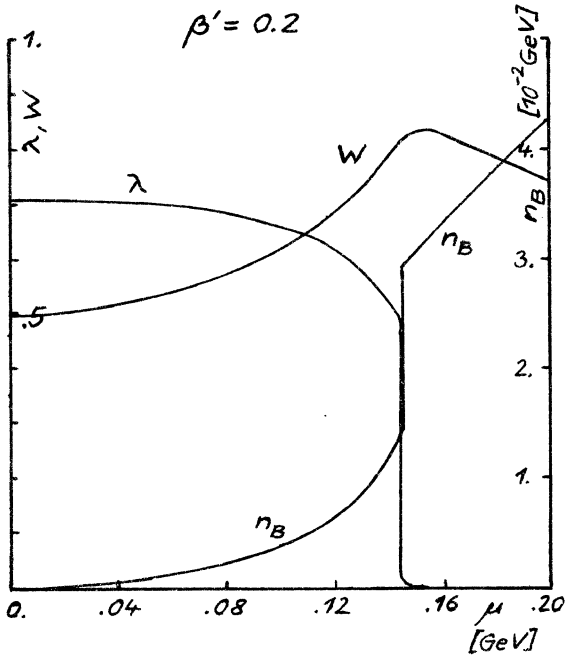

Figure 2

Chiral condensate λ , Wilson line W,and baryon density n_B vs. chemical potential (at T = .174 GeV)

UNIVERSALITY IN LATTICE CALCULATIONS: TWO YEARS AFTER PARIS CONFERENCE

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Values of dimensionful quantities calculated in QCD by the Monte Carlo (MC) method should not depend on the choice of the lattice action (universality). A simplest alternative to the standard Wilson action (WA) is the mixed action (MA) that combines characters in the fundamental and adjoint representations of SU(N) (with couplings β and β_A, respectively).

At Paris Conference it was widely discussed work by Bhanot and Dashen whose calculations with SU(2) MA at some choice of β and β_A led, at first glance, to 400 p.c. violation of universality. During two years after the Conference, many authors related this discrepancy to the fact that weak coupling formulae are not applicable at these values of β and β_A. Most promising is non-perturbative approach based on reduction of MA to WA which is motivated by large-N limit [1-5].

Large-N reduction. At large N, MA is reduced to WA with the effective coupling $\bar{\beta}$ given by [1]

$$\bar{\beta} = \beta + 2\beta_A \omega(\bar{\beta}) \quad , \qquad (1)$$

where $\omega(\bar{\beta})$ stands for mean plaquette for WA. This implies that all quantities coincide for MA and WA providing couplings are related by eq. (1). In other words, those remain constant in β,β_A plane along the lines of constant $\bar{\beta}$.

This fact quarantees universality in the continuum. However, large-N reduction is more stronger property since it holds, say, in the strong coupling region as well.

Extension to finite N. At finite N, exact reduction of MA to WA is no longer possible in the whole β,β_A plane because different quantities would be constant along, generally speaking, different lines. However, in the region where the continuum limit sets in, those should coincide for different dimensionful quantities in order that their ratios do not depend on β or β_A. Explicit calculation of finite-N correction to eq. (1) describing these lines of "constant physics" has been performed in two-loop order of the weak coupling expansion [4].

Before this result available, an approximate approach motivated by large N had been suggested [3]. It uses lines of constant value of mean plaquette as lines of "constant physics" in β,β_A plane. For small β_A, the exact formula reads

$$\bar{\beta} = \beta + \beta_A \left(2\omega(\bar{\beta}) + \rho'(\bar{\beta})/\omega'(\bar{\beta}) \right) \quad , \qquad (2)$$

where $\rho'(\bar{\beta})$ is the derivative of irreducible correlator of two same plaquettes w.r.t. $\bar{\beta}$.

Application to MC data. a) SU(2) MA. Eq. (2) describes MC data for lines of "constant physics" in rather wide range of β_A. More subtle point is the ratio of scale parameters that depend on coupling exponentially. Using reduction to WA, we get

$$\Lambda_L(\beta,\beta_A) = \left(\frac{6\pi^2}{11}\bar{\beta}\right)^{51/121} exp\left(-\frac{3\pi^2}{11}\bar{\beta}\right) \quad (3)$$

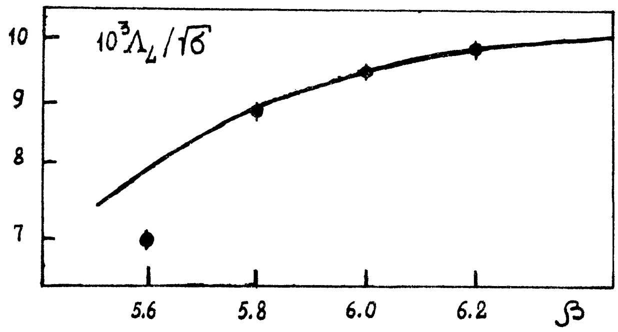
for the scale parameter for MA. At $\beta_A = 1.21$, near the phase transition, bending of the lines of "constant physics" becomes essential so that $\bar{\beta}$ versus β was determined in refs. 3,5 by mapping mean plaquette along the line $\beta_A = 1.21$ onto $\omega(\bar{\beta})$. The obtained dependence of Λ_L on β is much steeper than asymptotic scaling (AS). Such a behavior agrees with MC data for string tension, deconfinement temperature and mass gap.

The major source for deviations from AS is dynamics of Z_2- degrees of freedom (fluxons) that leads to the phase transition. Contribution of these fluctuations to β-function can be calculated directly considering the nontrivial local extrema of the lattice action. In the given approach, it enters via ω which is taken from Monte Carlo that simulates these fluctuations as well.

b) SU(3) WA. Recent MC studies of SU(3) WA have shown deviations from AS in analogy with SU(2) MA at $\beta_A = 1.21$. Many authors tried to relate these deviations to the nearby phase transition in β,β_A plane. The nontrivial local extrema contribute again to β -function but now not so strongly as for SU(2) MA at $\beta_A = 1.21$ because distance from the phase transition is larger.

An attempt of quantitative analysis is given in ref. 6 where the contribution of Z_3-degrees of freedom is assumed to be the smaller the larger is the distance from the phase transition so that AS is assumed to be exact at some $\beta_A < 0$ (this is to be confirmed a posteriori by comparison with MC data). Then eq. (2) leads to an improved scale parameter for SU(3) WA.

This scale parameter [6], represented by the bold line, is compared with MC data [7] for string tension, σ , in the figure.



The predicted deviation from AS (AS would be a horizontal line) agrees fairly with the data at $\beta \geqslant 5.8$. While $10^3\Lambda_L/\sqrt{\sigma} = 9.84$ at $\beta = 6.2$, the bold line tends asymptotically, as $\beta \to \infty$, to bigger value 10.3 that yields $\Lambda_{MOM} = 340 \div 350$ MeV in the continuum. It is important that the improved scale parameter allows to extract this number near $\beta \approx 6$.

There is similar agreement with MC data [8] for deconfinement temperature, T_c,(yielding $T_c \approx 60\Lambda_L = 240 \div 250$ MeV) but those presented at this Conference by S.Meyer contradict this picture. Therefore, definite conclusions can not be drawn. I checked, however, qualitative agreement with MC data for 2^{++} glueball mass as well as for quark condensate and rho mass in the quenched approximation.

Discussion and conclusions. The above agreement seems to indicate that approach to AS is related to the nearby phase transition. The discrepancy at $\beta = 5.6$ shows that either (i) eq. (2) is not applicable or (ii) AS does not hold at $\beta_A < 0$. Being approximate, eq. (2) agrees, nevertheless, numerically well with the exact formula [4] which, in turn, describes the MC data for lines of "constant physics". On the contrary, assumption (ii) is probably not valid. The MC data of ref. 9 for MA at $\beta_A = -\beta/6$ confirm this conclusion. Those possess indeed smaller deviation from AS than for WA but it persists. The agreement at $\beta \geqslant 5.8$ has been reached because AS sets in, however, a little bit earlier for $\beta_A < 0$.

The smoother approach to AS is an advantage of MA at $\beta_A < 0$ over WA which might be tried to be utilized. An interesting question is whether the deviation from AS can be decreased further by moving toward larger negative β_A . While it is to be answered by comparing MC data at various β_A , some qualitative insight can be reached by the following thought experiment. Let us look at the lines of "constant physics" which correspond to same lattice spacings in physical units. Because these lines become more rare for larger negative β_A , the deviation from AS is expected to decrease. I do not mean, however, that it can be reduced completely by this procedure, rather the contamination of β-function which is due to the nearby phase transition can be reduced.

Since dimensionful quantities become smoother function of $\beta+2\beta_A$ for $\beta_A < 0$, it is easier to determine their values in units of Λ_L on small lattices. Such a programme may be tried as an alternative to nonperturbative study of β -function for WA by MC renormalization group method reported at this Conference by D.Wallace. Whether it would be useful practically depends on the fact that whether the deviation from AS could be made small enough for some not too big negative β_A . The point is that one-loop relation between Λ_L and $\Lambda_{continuum}$ ceases to be applicable near the line $\beta + 2\beta_A = 0$, where one needs higher orders in $1/(\beta+2\beta_A)$ to determine $\Lambda_{continuum}$. Anyhow the problem for such values of β_A is to manage perturbative corrections only. Presumably, the approach described above might turn out to be useful for this purpose.

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