Dynamics of Unruh-DeWitt detectors in a relativistic quantum field

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Abstract. Based on our results for the detector-field system in Gaussian states, I am addressing the following issues about the quantum dynamics of atoms or detectors moving in a relativistic quantum field: First, the dynamics of the detectors as an open quantum quantum system, hence the entanglement creation and sudden death processes between the detectors, are non-Markovian in general. Second, the excitations of the accelerated detectors in the Unruh effect depend only on the kinematics of the detectors. The event horizon for the detector is not essential in this effect, and the particle notion of the quantum field is not needed here. Third, entanglement dynamics of the detectors are independent of time-slicing scheme. Local projective measurements on point-like detectors are also consistent in different time-slicing schemes even in the presence of relativistic quantum fields.

1. Introduction

Recent years have seen growing interest in relativistic quantum informaion (RQI) science, in which people are applying new knowledge from quantum information science to relativistic systems, while using relativity to examine the consistency of quantum information. The most intriguing notion of the RQI is the tension between relativistic locality and quantum nonlocality. Such a tension manifests when performing a quantum measurement on two spacelike seperated, spatially localized but quantum entangled objects. The sudden transition of quantum entangled state at a distance ("wave-function collapse") before and after measurement appears to violate the principles of relativity.

To study this notion, one of the simplest tools is the Unruh-DeWitt detector model [1, 2, 3]. In (3+1)D Minkowski space, it is described by the action

$$\begin{aligned} S &= -\frac{1}{2}\int d^4x\sqrt{-g}\partial_\alpha\Phi\partial^\alpha\Phi + \frac{1}{2}\sum_\Gamma\left\{\int d\tau_\Gamma\left[(\partial_\Gamma Q_\Gamma)^2 - \omega_\Gamma^2 Q_\Gamma^2\right] + \right. \\ &\quad \left. \lambda\int d\tau_\Gamma Q_\Gamma(\tau_\Gamma)\int d^4x\Phi(x)\delta^4(x^\alpha - z_\Gamma^\alpha(\tau_\Gamma))\right\}, \end{aligned} \tag{1}$$

which consists of harmonic oscillators (HO) Q_Γ as internal degrees of freedom of point-like objects in space, called the detectors, moving along prescribed trajectories $z_\Gamma^\alpha(\tau_\Gamma)$ in a massless scalar field Φ. Here $\alpha = 0, 1, 2, 3$, Γ is labelling the detectors: $\Gamma = A$ for one-detector case, $\Gamma = A, B$ for two-detector case, and $\partial_\Gamma \equiv d/d\tau_\Gamma$. This simple atom-field interacting system is explicitly covariant. It is a linear system, so the solutions once obtained are crystal clear. Indeed, in simple cases analytic results have been obtained in the whole parameter range without any approximation. Still it is complicated enough to give nontrivial results and insights. Note that, with the point-like detectors and the retarded field sourced from them present in spacetime, the field state does not respect the full Lorentz symmetry.

Below I am discussing three issues in the quantum dynamics of atoms or detectors moving in a relativistic quantum field: (i) Non-Markovian dynamics of the detector as an open quantum system, (ii) relations between the Unruh effect and the event horizon, and (iii) the scheme independence of time-slicing for the reduced dynamics of the detectors and the projective measurement on the detectors. Although most of our insights are coming from specific models and the calculations are restricted for Gaussian states, I expect the following discussions would be valid for more general cases.

2. Non-Markovian dynamics in open system

For simplicity, the initial state in our model is usually taken as a factorized state,

$$| \psi(0) \rangle = | q \rangle \otimes | 0_M \rangle , \tag{2}$$

which is a direct product of a Gaussian state of the detector $| q \rangle$ and the Minkowski vacuum of the field $| 0_M \rangle$, which is also a Gaussian state in Schrödinger picture and is the no-particle (field quanta) state for an inertial observer in Minkowski space. The quantum state (2) is not a stationary state of the detectors -field interacting system. Thus the solution of the Schrödinger equation with this initial state will be time-varying, and there are interesting dynamics after the combined system started to evolve.

One can express a Gaussian state of the combined system in the (K, Δ)-representation of the Wigner functional [11] as

$$\begin{aligned} \rho(K,\Delta) &= \int \mathcal{D}\Sigma e^{iK\cdot\Sigma/\hbar} \psi\left[\Sigma - (\Delta/2)\right] \psi^*\left[\Sigma + (\Delta/2)\right] \\ &= N \exp - \frac{1}{2\hbar} \left[K_\mu \mathcal{Q}^{\mu\nu} K_\nu - \Delta_\mu \mathcal{R}^{\mu\nu} K_\nu + \Delta_\mu \mathcal{P}^{\mu\nu} \Delta_\nu \right] \end{aligned} \tag{3}$$

where the indices μ, ν are running over $\{\Gamma\}$ and $\{\mathbf{x}\}$, N is the normalization factor, and the time-dependent factors $\mathcal{Q}^{\mu\nu}$, $\mathcal{P}^{\mu\nu}$, and $\mathcal{R}^{\mu\nu}$ are completely determined by the symmetric two-point correlators $\langle A, B \rangle \equiv \langle AB + BA \rangle /2$ as

$$\langle \Phi_\mu, \Phi_\nu \rangle = \left[\frac{\hbar\delta}{i\delta K_\mu} \frac{\hbar\delta}{i\delta K_\nu} \rho(K,\Delta) \right]_{\Delta=K=0} = \hbar \mathcal{Q}^{\mu\nu}, \tag{4}$$

$$\langle \Pi_\mu, \Pi_\nu \rangle = \left[\frac{i\hbar\delta}{\delta\Delta_\mu} \frac{i\hbar\delta}{\delta\Delta_\nu} \rho(K,\Delta) \right]_{\Delta=K=0} = \hbar \mathcal{P}^{\mu\nu}, \tag{5}$$

$$\langle \Pi_\mu, \Phi_\nu \rangle = \left[\frac{i\hbar\delta}{\delta\Delta_\mu} \frac{\hbar\delta}{i\delta K_\nu} \rho(K,\Delta) \right]_{\Delta=K=0} = \frac{\hbar}{2} \mathcal{R}^{\mu\nu}, \tag{6}$$

where Π_μ are conjugate momenta to Φ_μ, and we have written $\Phi_\Gamma \equiv Q_\Gamma$ and $\Pi_\Gamma \equiv P_\Gamma$ for convenience. Because of the linearity of the Hamiltonian derived from the action (1), a Gaussian state of the combined system will always be a Gaussian state during the evolution, which can always be fully determined by the two-point correlators. So it is sufficient to calculate the evolution of the the two-point correlators, either in Schödinger picture or in Heisenberg picture, to see how the quantum state evolves.

In Fig. 1 we illustrate the time-evolution of the self correlator $\langle Q^2 \rangle$ of a single uniformly accelerated detector with the initial state $| q \rangle$ being its ground state. One can see that the value

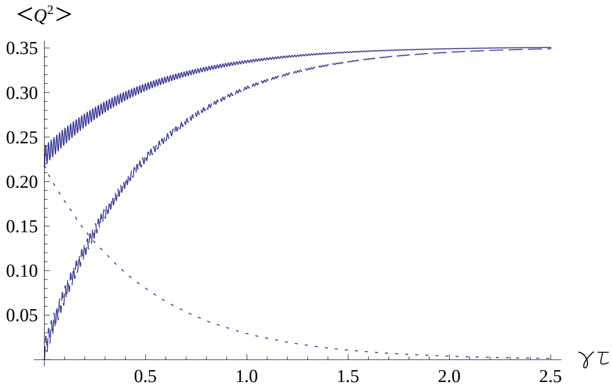

Figure 1. Evolution of the two-point correlator $\langle\, Q^2(\tau)\,\rangle$ of a single uniformly accelerated UD detector (solid curve). The initial state (2) at $\tau = 0$ is a direct product of the ground state of the detector and the Minkowski vacuum of the field. The dotted curve represents how the zero-point fluctuations of the detector dissipate to the field, the dashed curve represents the response of the detector driven by vacuum fluctuations of the field, and the solid curve is the sum of these two. The small oscillation on top of the growing solid curve is due to the sudden switching-on of the coupling. Here $\gamma = 0.01$, $\Omega = 2.3$, $\Lambda_0 = 20$, $a = 10$, and τ is the proper time of the detector.

of $\langle\, Q^2\,\rangle$ evolves from a value for the ground state of a HO, $\hbar/2\Omega m$ (≈ 0.2174 in Fig. 1) with the mass $m = 1$ and the Planck constant $\hbar = 1$, to a value for a HO in equilibrium with the heat bath at the Unruh temperature $T_U = a/2\pi$. In the weak-coupling limit $\gamma\Lambda_0 \ll a, \Omega$, where $\gamma \equiv \lambda^2/8\pi m$ is the strength of coupling and Λ_0 is a constant corresponding to the time-scale of switching-on the coupling [4], the latter value is approximately $\hbar \coth(\pi\Omega/a)/2\Omega m$ (≈ 0.3515 in Fig. 1). At early times ($\gamma(\tau - \tau_0) \ll 1$ but $\tau - \tau_0 \gg a^{-1}$ with τ_0 the initial moment, roughly in the interval $0.01 < \gamma\tau < 0.2$ in Fig. 1, where $\gamma = 0.01$ is not quite weak to see its effect in the plot, though), the value of $\langle\, Q^2\,\rangle$ grows almost linearly in time in the weak-coupling limit. This is the regime where the conventional result, the transition probability

$$P_{0\to 1} = \frac{\hbar}{4\pi m}\frac{\tau - \tau_0}{e^{2\pi\Omega_r/a} - 1}\ , \tag{7}$$

from the ground state of the detector to its first excited state by the Unruh effect, is valid [5]. Indeed, this transition probability is obtained by time-dependent perturbation theory (TDPT), which describes the transient behavior only at early times of the evolution and only in perturbative regime.

Dynamics of the correlators of two detectors, hence the dynamics of quantum entanglement between them, are non-Markovian in general. One origin of the non-Markovian property is the correlations in the field state at zero or low temperature. If the field is not initially at high temperature and the coupling of the field and the detectors are not weak enough, then such long-range correlations will manifest in the dynamics. The most obvious case is observed in the Rob-AntiRob problem [8], where the cross correlators started to evolve at initial moment $-t_0$ will have a sudden change of behavior at $+t_0$, just like they have the memory of the initial moment (see Fig. 2).

The other origin of the non-Markovian or memory effect is the mutual influences mediated by the retarded field sourced from one point-like detector then hitting the other detector [6, 7]. The influenced detectors then generate the secondary or higher-order back reactions to the field, which propagate back and forth between the detectors. Note that these retarded mutual influences are of quantum origin and explicitly causal.

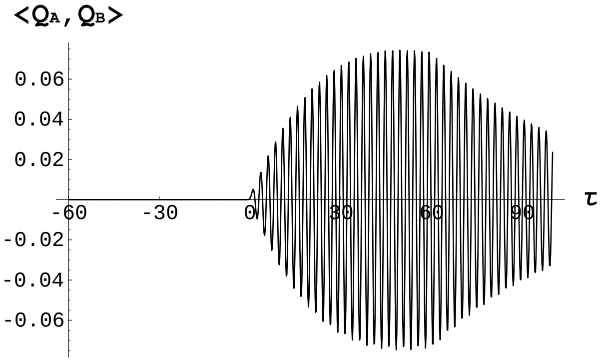

Figure 2. Evolution of the cross correlator $\langle Q_A, Q_B \rangle$ for a pair of back-to-back uniformly accelerated detectors A and B initially in their ground state (For more details, see Ref. [8].) Here $\gamma = 0.01$, $\Omega = 1.3$, $a = 2$, $\hbar = 1$, and the coupling was started at $\tau_0 = -60$. One can see that when $\tau \approx -\tau_0 = 60$, the bahavior of $\langle Q_A, Q_B \rangle$ shows a transition.

Non-Markovian property from both origins can decrease, increase, eliminate or create entanglement, which is measured by the logarithmic negativity E_N or the quantity Σ [7]. For example, quantum entanglement will have a sudden death very quickly after the coupling of the field and the detectors is turned on in strong coupling regime [6]. In other examples, entanglement between two spacelike separated detectors initially in a separable state can be created either by the field correlations they experienced in specific motion (the Rob-AntiRob problem in [8]), or by the mutual influences between two detectors at rest and sitting very close to each other [7].

Of course, such entanglement generation between two detectors may not improve the fidelity of quantum teleportation using these detectors. Suppose Alice and Bob share an entangled pair of detectors, and Alice want to teleport the quantum state of Charlie's detector to Bob. Recall that to perform a successful quantum teleportation, Bob needs the knowledge of the initial state to do the correct operation (post-selection) on his detector (or qubit) according to the outcome of the Gaussian (or Bell) measurement on Alice and Charlie's detectors. Once the quantum state of the Alice-Bob pair deflects from the initial state, Bob's ability of reproducing Charlie's quantum state (i.e. the fidelity of quantum teleportation) will decrease, no matter quantum entanglement between Alice and Bob's detectors (or qubits) right before Alice's measurement was increased or decreased compared with the initial state.

3. Event Horizon and Unruh Effect

There are mainly two approaches to the Unruh effect in literature [1, 2, 3]. In the first approach the Minkowski vacuum of a quantum field is re-expressed as a two-mode squeezed state in Rindler space. When the degrees of freedom behind the event horizon for an observer living in one of the left and right Rindler wedges are traced out, the field state in the observer's wedge will appear to be a thermal state of Rindler particle at certain Unruh temperature. In the other approach one applies the TDPT to see a uniformly accelerated Unruh-DeWitt detector initially in the ground state can be excited by the Minkowski vacuum.

Event horizon is essential in the former approach, but not in the latter. Indeed, in [9], we calculated the deviation from the self correlator $\langle Q^2(\tau) \rangle_{\rm v} |_{T_U=0}$ for inertial UD detectors in the Minkowski vacuum,

$$\delta \langle Q^2(\tau) \rangle \equiv \lim_{\tau' \to \tau} \left[\langle Q(\tau)Q(\tau') \rangle - \langle Q(\tau)Q(\tau') \rangle_{T_U=0} \right] \quad (8)$$

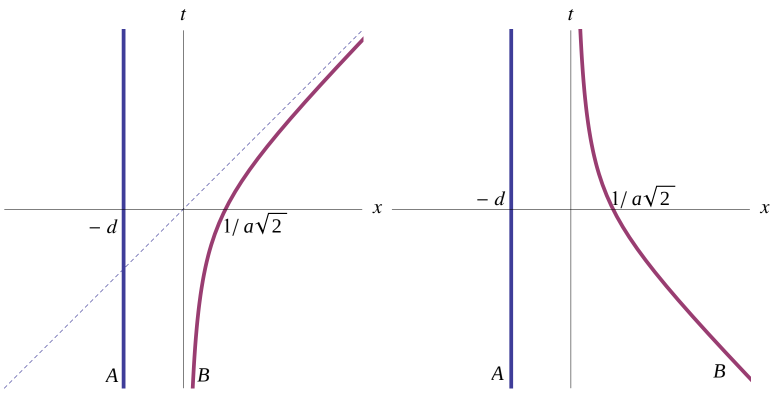

Figure 3. Two setups of the trajectories of the detectors A and B in non-uniform acceleration in [9]. The detector A is at rest in both setups. In the left plot, the detector B is almost at rest when $t < 0$, and almost uniformly accelerated when $t > 0$. The dashed line is the event horizon for the detector B. In the right plot, the detector B is linearly accelerated when $t < 0$ and becomes inertial when $t > 0$, so there is no event horizon for the detector B.

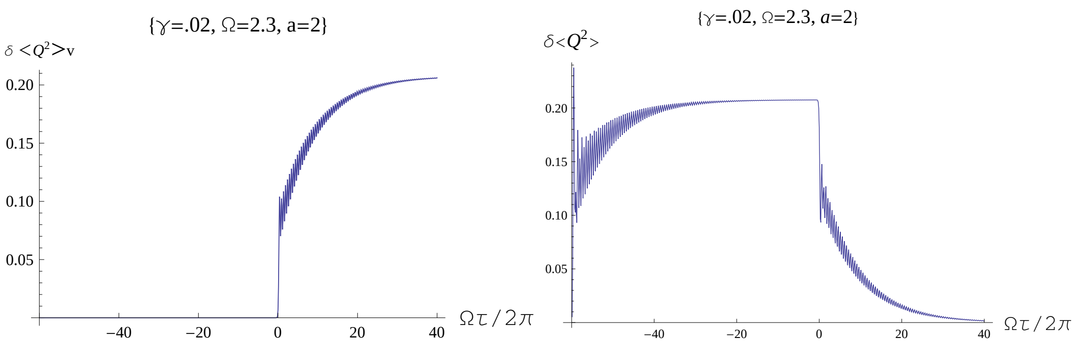

Figure 4. Evolutions of $\delta \langle\, Q^2(\tau)\, \rangle$ defined in (8) for the detector B in the left and the right plots in Fig. 3, respectively.

by numerical method. We found that during each session of almost-uniform acceleration with some constant instantaneous proper acceleration, the self correlators of the accelerated detector behave the same as those uniformly accelerated in the same proper acceleration, as illustrated in Fig. 4. As we showed in [5], the correlators of a UD detector can be related to the transition probability of excitation in TDPT. So our results imply that the excitations in the detector moving in the Minkowski vacuum at some proper acceleration will look the same as those due to the Unruh effect.

If the acceleration of the detector eventually vanishes, there will be no event horizon for the detector, but those excitations in the detector driven by vacuum fluctuations of the field will still occur at early times when the detector is almost-uniformly accelerated. Therefore it is the kinematics of the detector rather than the global structure of spacetime relevant to the excitations in the detector.

One may wonder, without the event horizon, how can one incorporate the notion of particles such as the Rindler particles of quantum fields in a non-inertial frame? The answer is simple. Recall that the concept of particle of a quantum field is not fundamental and sometimes not well-defined in curved space [3], and even in inertial frame, the particle notion of the field is not clear in the non-perturbative regime. In both cases one may consider the transitions between the neighboring energy levels of an atom or a detector as an operational definition of the particle (field quanta), but it works only in the TDPT regime. Off the TDPT regime and the asymptotic region, the particle notion of the field will not be well defined and one should look at the overall response of the detector rather than the transitions between the neighboring energy levels.

4. Time-slicing scheme independence

Since the UD detectors are point-like objects in space, quantum entanglement of the detectors at each single moment is independent of the time-slicing scheme we are using in calculation to specify that moment. Those quantum fields $\Phi_{\mathbf{x}}$ living on different choices of time slices $(t, \mathbf{x})$ but passing through the same two spacelike separated events of the detectors will be traced out to give the same reduced density matrix (RDM) of the detectors at the time associated with that time slice, namely,

$$\rho_R(Q, Q'; t) = \text{Tr}_\Phi \rho(Q, \Phi_{\mathbf{x}}, Q', \Phi'_{\mathbf{x}}; t) \equiv \int \mathcal{D}\Phi_{\mathbf{x}} \rho(Q, \Phi_{\mathbf{x}}, Q', \Phi_{\mathbf{x}}; t) \ , \tag{9}$$

where ρ is the density matrix of the combined system. Here the RDM ρ_R of the detector can be completely determined by the two-point correlators of the detectors by virtue of the Gaussianity. Now the RDM is not explicitly dependent on the data over the time slice $(t, \mathbf{x})$ except those right at the position of the detectors. From these RDM one obtains the same correlators of the detectors at the moment t from different time-slicing schemes, which give the same measure of quantum entanglment at that moment.

Note that, as we have observed in [6], the evolution of the two-point correlators of the detectors in one coordinate (e.g. $\langle\, Q_A(\tau_A(t)) Q_B(\tau_B(t)) \,\rangle$ in t) could be quite different from the ones in a different coordinate (e.g. $\langle\, Q_A(\tau_A(t')) Q_B(\tau_B(t')) \,\rangle$ in t'), since τ_A and τ_B evolve in different ways in different coordinates. So entanglement dynamics of the detectors in a time interval do depend on the coordinate of the observer.

A more interesting question is, whether the projective measurement (or wave function collapse) is consistent in different time-slicing scheme. For example, if we choose an alternative coordinate in Minkowski space,

$$\eta = t - A \sin t \cos x \ , \tag{10}$$

$$\xi = x - A \sin x \cos t \ , \tag{11}$$

with a constant $A < 1$. Then

$$ds^2 = -dt^2 + dx^2 = \gamma(\eta, \xi) \left(-d\eta^2 + d\xi^2 \right) \ , \tag{12}$$

where

$$\gamma(\eta, \xi) = \left[1 - 2A \cos t \cos x + A^2 \cos(t + x) \cos(t - x) \right]^{-1} \tag{13}$$

with $t = t(\eta, \xi)$ and $x = x(\eta, \xi)$ according to (10) and (11). η-slices and t-slices will overlap whenever $t = n\pi$ with integer n. Off these time-slices, η-slices are the wavy ones in Fig. 5, where t-slices would be the horizontal straightlines. Now, the wave functional of a quantum field in Minkowski space is defined on the whole time-slice at each moment. Suppose the projective measurement works in both frames, and a local measurement is done on a point-like detector coupled with the field at some moment $t_1 \neq n\pi$, then which time-slice, t_1-slice or η_1-slice, would the wave functional of the combined system collapse onto? If both happen in their coordinates, will they produce different post-measurement states?

An explicit calculation in a Raine-Sciama-Grove model [12], which is an Unruh-DeWitt like detector theory in (1+1) dimensional Minkowski space, has been done in [10] where the case with a detector A at rest at $x = \xi = 0$ was considered. Suppose the combined system is initially

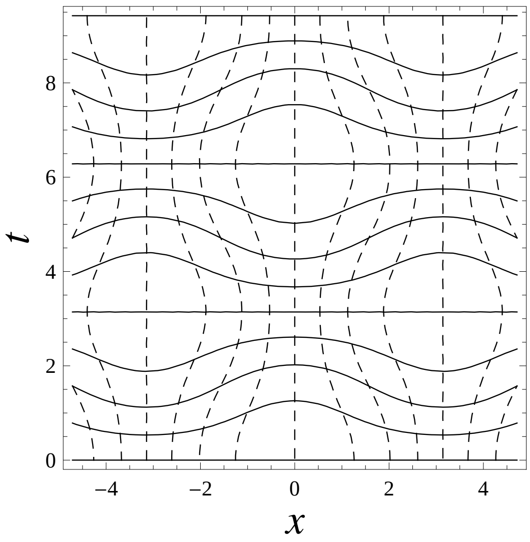

Figure 5. An alternative coordinate (η, ξ) of Minkowski space given by (10) and (11) with $A = 1/2$. The solid curves are constant η slices while the dashed curves are constant ξ hypersurfaces.

in a Gaussian state at $t = \eta = 0$, and at some moment $t = t_1$, $\eta_1 \equiv \eta(t_1) \in (0, \pi)$, a projective Gaussian measurement collapses the wave functional of the combined system to a new Gaussian state,

$$\tilde{\rho} = \rho_A \otimes \rho_\Phi, \tag{14}$$

instantaneously on the whole t_1-slice or η_1-slice, depending on which coordinate the observer is using in Minkowski space. Here the detector state ρ_A after measurement is supposed to be a squeezed Gaussian state and the field is also in a Gaussian state ρ_Φ which is derived according to ρ_A. After t_1 or η_1, the post-measurement state (14) keeps evolving to, say, $t = t_2 = \pi$ and $\eta = \eta_2 = \pi$, when t and η-slices overlap so we can make a direct comparison between these two quantum states of the combined system collapsed over two different time-slices by the same local Gaussian measurement on the detector.

In the conventional coordinate (t, x) of Minkowski space, the two-point correlators at $t = t_2$ determining the wave functional (3) can be expressed as combinations of the mode-functions evolving from t_1 to t_2, together with the initial data on the t_1-slice in the form of the correlators of the field at space points on the slice. For instance,

$$\begin{aligned} \langle\, \hat{\Phi}_x, \hat{\Phi}_y \,\rangle_\pi &= \mathrm{Tr}\hat{\Phi}_x(t_2 - t_1)\hat{\Phi}_y(t_2 - t_1)\tilde{\rho}(\Phi, \Pi; t_1) \\ &\sim \int dx' dy' \phi_x^{x'}(t_2 - t_1)\phi_y^{y'}(t_2 - t_1) \,\langle\, \hat{\Phi}_{x'}, \hat{\Phi}_{y'} \,\rangle_{t_1} + \cdots \end{aligned} \tag{15}$$

where x' and y' are points on the t_1-slice, and $\phi_x^{x'}(t_2 - t_1)$ is a real mode-function in the expansion

$$\hat{\Phi}_x(t) = \sum_{\Gamma'} \left[\phi_x^{\Gamma'}(t)\hat{Q}_{\Gamma'} + \Phi_x^{\Gamma'}(t)\hat{P}_{\Gamma'} \right] + \int dx' \left[\phi_x^{x'}(t)\hat{\Phi}_{x'} + \Phi_x^{x'}(t)\hat{\Pi}_{x'} \right], \tag{16}$$

$$\hat{Q}_\Gamma(\tau_\Gamma) = \sum_{\Gamma'} \left[\chi_\Gamma^{\Gamma'}(\tau_\Gamma)\hat{Q}_{\Gamma'} + X_\Gamma^{\Gamma'}(\tau_\Gamma)\hat{P}_\Gamma \right] + \int dx' \left[\chi_\Gamma^{x'}(\tau_\Gamma)\hat{\Phi}_{x'} + X_\Gamma^{x'}(\tau_\Gamma)\hat{\Pi}_{x'} \right]. \tag{17}$$

According to the equations of motion for $\phi_x^{x'}(t_2 - t_1)$, it can be interpreted as a retarded field $\phi(t_2, x)$ sourced from the point-like detector driven by the vacuum fluctuations at (t_1, x'), which is a point on the t_1-slice. In the alternative coordinate (η, ξ), the form of the correlator is similar, except that there η_1-slice is different from t_1-slice. Now the two-point correlators, or equivalently, the wave functional at $t_2 = \eta_2$, appear to depend on the time-slicing scheme.

Nevertheless, the dependence of the data on the t_1-slice or η_1-slice can be removed. If the measurement at t_1 or η_1 was absent, one would have

$$\langle\, \hat{\Phi}_x, \hat{\Phi}_y \,\rangle_{t_2} = \langle\, \hat{\Phi}_x(t_{20}), \hat{\Phi}_y(t_{20}) \,\rangle_{t_0} = \phi^A_x(t_{20})\phi^A_y(t_{20}) \langle\, \hat{Q}^2_A \,\rangle_{t_0} + \cdots, \tag{18}$$

with $t_{mn} \equiv t_m - t_n$, while

$$\begin{aligned} \langle\, \hat{\Phi}_x, \hat{\Phi}_y \,\rangle_{t_2} &= \langle\, \hat{\Phi}_x(t_{21}), \hat{\Phi}_y(t_{21}) \,\rangle_{t_1} = \phi^A_x(t_{21})\phi^A_y(t_{21}) \langle\, \hat{Q}^2_A \,\rangle_{t_1} + \cdots \\ &= \phi^A_x(t_{21})\phi^A_y(t_{21}) \langle\, \hat{Q}^2_A(t_{10}) \,\rangle_{t_0} + \cdots \\ &= \phi^A_x(t_{21})\phi^A_y(t_{21})\chi^A_A(t_{10})\chi^A_A(t_{10}) \langle\, \hat{Q}^2_A \,\rangle_{t_0} + \cdots \end{aligned} \tag{19}$$

is also true. A comparison between the above two expressions implies a set of mathematical identities such as

$$\begin{aligned} \phi^\mu_x(t_{20}) &= \sum_{\Gamma'} \left[\phi^{\Gamma'}_x(t_{21})\chi^\mu_{\Gamma'}(t_{10}) + \Phi^{\Gamma'}_x(t_{21})p^\mu_{\Gamma'}(t_{10}) \right] + \\ &\quad \int dx' \left[\phi^{x'}_x(t_{21})\phi^\mu_{x'}(t_{10}) + \Phi^{x'}_x(t_{21})\pi^\mu_{x'}(t_{10}) \right], \end{aligned} \tag{20}$$

in this initial value problem, where $p^\mu_\Gamma(\tau_\Gamma(t))$, $P^\mu_\Gamma(\tau_\Gamma(t))$, $\pi^\mu_x(t)$, and $\Pi^\mu_x(t)$ are conjugate variables to the mode-functions χ^μ_Γ, X^μ_Γ, ϕ^μ_x, and Φ^μ_x, respectively. These identities can be interpreted as the Huygens principle of the mode functions, and can be verified explicitly by inserting the solutions for those mode-functions into the identities.

It turns out that all the dependence in the wave functional at $t = \eta = \pi$ on the data living on the t_1-slice or η_1-slice, namely, those $\int dx'$ terms, can be replaced using these identities, e.g., one has $\int dx'[\cdots] = \phi^\mu_x(t_{20}) - \sum_{\Gamma'}[\cdots]$ in (20). Then the wave functional at $t = \eta = \pi$ can be expressed as a functional of the functions representing the point-like detector as well as the retarded field sourced from the detector. All these objects are explicitly covariant, so that the two wave functionals in different coordinates and collapsed on different time-slices can be identified with each other by a coordinate transformation at $t_2 = \eta_2 = \pi$. This means that these two quantum states after measurement are actually the same, and the projective measurement is consistent in the detector theory even in the presence of relativistic quantum fields.

This consistency still holds if the initial Gaussian state is a mixed state, and if one has two or more successive Gaussian measurements on the detector before comparison. It will be interesting to see if this consistency still hold for non-Gaussian states, non-Gaussian measurements, or in nonlinear systems.

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