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Quark Mixing: The CKM Picture*

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ABSTRACT

We review the current status of the determination of the CKM matrix elements. We explain the concept of quark mixing and describe various parametrizations of the mixing matrix. We give the ranges for the parameters as determined by direct measurements (tree-level processes), unitarity and indirect measurements (one-loop processes). We explain how schemes for mass matrices may give relations among the CKM parameters. Finally, we overview further progress in the determination of the matrix elements which can be expected in the near future.

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1. INTRODUCTION

The minimal Standard Model with three generations of fermions and a single Higgs doublet is the simplest theoretical framework which is consistent with all experimental measurements of electroweak phenomena. The predictions of the minimal Standard Model can all be given in terms of eighteen independent parameters. Of these, ten are related to the quark sector of the theory: six quark masses and four mixing parameters. The latter ones parametrize the Cabibbo - Kobayashi - Maskawa (CKM) mixing matrix (1, 2). In this work, we describe our present knowledge of the CKM matrix.

The concept of quark mixing is a result of quark mass eigenstates being different from quark interaction eigenstates. We explain this in the second section. The CKM matrix, which connects them, is unitary and consequently its nine complex elements are given by only four independent parameters. We explain this counting of parameters and how it generalizes to any number of generations.

In the third section we present the specific parametrization with which we work, and mention other possible parametrizations. The physics related to the mixing matrix is, of course, independent of any specific parametrization. We explain what are the parametrization independent quantities, with special emphasis on CP violating observables.

The main source of information on the CKM elements are *direct* measurements, namely measurements of processes which occur at the tree level in the Standard Model. In the fourth section we survey the determination of the six matrix elements which have been directly measured.

Based on knowledge of some of the CKM elements from direct measurements, one can obtain further information on additional matrix elements from unitarity. In the fifth section we explain the assumptions that are made when using unitarity; we calculate the unitarity constraints; and we present the unitarity triangle, which is a convenient geometrical presentation of these constraints.

Additional information on the CKM elements is achieved from *indirect* measurements, namely measurements of processes which occur at one loop level in the Standard Model. The allowed ranges for the CKM elements depend on the yet unknown mass of the top quark. In the sixth section we discuss measurements of mixing in the K^0 and B^0 systems and of the CP violating parameter ϵ.

The number of parameters in the quark sector is reduced in a class of models that go beyond the Standard Model. In particular, various schemes for quark mass matrices suggest relations among quark masses and mixing angles. In the seventh section we describe various approaches to this problem. We discuss in some detail the Fritzsch scheme.

Finally, we give an overview on experimental and theoretical progress which could be expected in the near future.

2. WEAK AND MASS EIGENSTATES

The gauge group of electroweak interactions is $SU(2)_L \times U(1)_Y$. Left-handed quarks are in doublets of $SU(2)_L$, while right-handed quarks are singlets of $SU(2)_L$. Consequently, only left-handed quarks take part in charged weak interactions:

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L^I} \gamma^\mu \mathbf{1} D_L^I W_\mu^+ + \text{h.c.} \tag{2.1}$$

where $U_L^I(D_L^I)$ is a vector in generation-space of the left-handed up- (down-) quark interaction-eigenstates, and 1 is a unit-matrix in the generation-space. This unit-matrix is written explicitly to emphasize that the W-interactions are, by definition, diagonal in the interactioneigenbasis.

If $SU(2)_L$ x $U(1)_Y$ were an exact symmetry, all quarks would be massless, and there would be no physical distinction between the interaction eigenbasis and the mass eigenbasis. However, in the physical world, this symmetry is broken. In the Standard Model, the breaking is spontaneous due to a vacuum expectation value (VEV) of a Higgs doublet. The Yukawa interactions of H^0, the neutral member of the Higgs doublet, are given by:

$$-\mathcal{L}_Y = \overline{U_L^I} F U_R^I H^{0*} + D_L^I G D_R^I H^0 + \text{h.c.} \tag{2.2}$$

where F and G are the Yukawa matrices. The most important feature of these matrices for our discussion is that they cannot be simultaneously diagonalized. When the neutral member of the Higgs doublet assumes a VEV, $\langle H^0 \rangle = v/\sqrt{2}$, the quarks acquire masses:

$$M_U = Fv/\sqrt{2}; \ M_D = Gv/\sqrt{2}. \tag{2.3}$$

Without loss of generality, we can choose to identify the up quark interaction eigenstates with the mass eigenstates (quark mass eigenstates will carry no superscript), $U_{L,R} = U^I_{L,R}$. This means that we work in a basis where F is diagonal, and consequently so is M_U. However, in this basis G is non-diagonal, and consequently so is M_D. We can always use a bi-unitary transformation to diagonalize M_D:

$$M_D^{diag} = V_L M_D V_R^\dagger. \tag{2.4}$$

As-the mass eigenstates $D_{L,R}$ fulfill $\overline{D_L} M_D^{diag} D_R = \overline{D^I_L} M_D D^I_R$, we identify

$$D_L = V_L D^I_L; \ D_R = V_R D^I_R. \tag{2.5}$$

The gauge interaction term becomes

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} \gamma^\mu V_L^\dagger D_L W_\mu^+ + \text{h.c.} \tag{2.6}$$

The matrix $V_L^\dagger$ is the mixing matrix for quarks. For n generations it is an n x n unitary matrix. As such, it generally contains n^2 parameters, of which $n(n-1)/2$ can be chosen as real angles, and $n(n+1)/2$ are phases. By the transformation

$$V = P_U V_L^\dagger P_D^*, \tag{2.7}$$

with P_U and P_D unitary diagonal matrices, we may eliminate (2n – 1) phases. The number of physically meaningful phases in V is, therefore, $(n-1)(n-2)/2$.

With two generations V is the Cabibbo matrix (2) of one real angle. With three generations,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.8}$$

is the KM matrix (1) of three real angles and one complex phase.

3. PARAMETRIZATIONS

3.1. THE "STANDARD" PARAMETRIZATION

The specific choice of parameters for V is quite arbitrary. Many different choices have been proposed for the three generation case as well as for the $n > 3$ case. A "good" choice depends on the intended use of that parametrization: For different purposes, different parametrizations may prove to be convenient. We use a parametrization first introduced by Chau & Keung (3) on the basis of suggestions by Maiani (4) and Wolfenstein (5), generalized to four generations by Botella & Chau (6) and to any number of generations by Harari & Leurer (7) and Fritzsch & Plankl (8) and chosen by the Particle Data Group (9) as standard. The advantages of this parametrization are explained in detail in ref. (7), and it is particularly suitable for our purposes.

For three generations the matrix is [in the notation of ref. (7)]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{3.1}$$

where $c_{ij} \equiv \cos\,\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. In the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle. The real angles $\theta_{12}, \theta_{23}, \theta_{13}$ can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases. Then all s_{ij} and c_{ij} are positive, $|V_{us}| = s_{12}c_{13}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}c_{13}$. As c_{13} is known to deviate from unity only in the fifth decimal place, $|V_{us}| = s_{12}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}$ to an excellent approximation. We often use the ratio

$$q \equiv |V_{ub}/V_{cb}| = s_{13}/s_{23}. \qquad (3.2)$$

The phase δ lies in the range $0 \leq \delta \leq 2\pi$, with non-zero values generally breaking CP invariance for the weak interactions.

3.2. THE KM PARAMETRIZATION

Kobayashi & Maskawa (1) originally chose-a parametrization involving the four angles, $\theta_1, \theta_2, \theta_3, \delta$:

$$\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \qquad (3.3)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for i = 1, 2, 3. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle (2). Slightly different forms of the Kobayashi-Maskawa parametrization are found in the literature. The CKM matrix used in the 1982 Review of Particle Properties is obtained by letting $s_1 \to -s_1$ and $\delta \to \delta + \pi$ in the matrix given above. An alternative is to change Eq. (3.3) by $s_1 \to -s_1$ but leave δ unchanged. With this change in s_1, the angle θ_1 becomes the usual Cabibbo angle, with the "correct" sign (i.e. $d^I = d\cos\theta_1 + s\sin\theta_1$) in the limit $\theta_2 = \theta_3 = 0$. The angles $\theta_1, \theta_2, \theta_3$ can, as before, all be taken to lie in the first quadrant by adjusting quark field phases. Since all these parametrizations are referred to as "the" Kobayashi-Maskawa form, some care about which one is being used is needed when the quadrant in which δ lies is under discussion.

Other parametrizations, mentioned above, are due to Maiani (4) and to Wolfenstein (5). The latter emphasizes the relative sizes of the matrix elements by expressing them in powers of the Cabibbo angle. Still other parametrizations have come into the literature in connection with attempts to define "maximal CP violation" [see e.g ref. (10)]. No physics can depend on which of the above parametrizations (or any other) is used as long as a single one is used consistently and care is taken to be sure that no other choice of phases is in conflict.

3.3. CP violation

Physical observables are independent of the parametrization of the mixing matrix. Though it is convenient to use specific parametrizations in actual calculations, it is helpful to demonstrate that one could use parametrization-invariants in the various analyses.

The absolute values of the various matrix elements are all parametrization-invariant. This is obvious, since these are physical observables, determined for example from hadron semileptonic decay rates. It is somewhat more complicated to understand how various CP violating observables can be expressed in a parametrization-independent manner.

The CKM matrix for three quark generations and its generalization to n $\geq$ 3 generations suggest that CP-invariance may be broken by complex phases in the matrix.

Within the three generation model, there is only *one* physical phase, δ of the matrix (3.1). There are several conditions that have to be fulfilled in order that CP is violated with three generations:

1. sin6 $\neq$ 0.

2. There should be no mass degeneracy among the three up-sector quarks or among the three down-sector quarks. We explain this by an example: suppose that the u and the c quarks are degenerate in mass. Then any linear combination of the mass eigenstates u and c is still a mass eigenstate. Consequently, instead of a *diagonal* unitary-matrix P_U [Eq. (2.7)], we may use a more general form:

$$P_U = \begin{pmatrix} U & \\ & e^{i\phi} \end{pmatrix} \tag{3.4}$$

   where U is a general 2 x 2 unitary matrix. This allows us to remove one more phase (and an angle) from V, thus making V real.

3. All mixing angles, s_{12}, s_{23} and s_{13}, and all the cosines, c_{12}, c_{23} and c_{13}, should be different from zero.

The above conditions can be summarized in one equation, which must be fulfilled if CP is violated:

$$T(M_u^2) \cdot B(M_d^2) \cdot J \neq 0, \tag{3.5}$$

where

$$T(M_u^2) = (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)$$

$$B(M_d^2) = (m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \tag{3.6}$$

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13} \sin\delta = c_1 c_2 c_3 s_1^2 s_2 s_3 \sin\delta_{KM}$$

The above condition can be restated in another way (11), which is explicitly independent of the parametrization of V. Choose any basis for the quarks, and assume that the mass matrices in this basis are M_D and M_U. Then the l.h.s of Eq. (3.5) equals (up to a possible sign) $\mathrm{Im}\{\det[M_D M_D^\dagger, M_U M_U^\dagger]\}/2$. Thus, for an arbitrary basis, CP is violated in the three generation Standard Model if and only if (11):

$$\mathrm{Im}\{\det[M_D M_D^\dagger, M_U M_U^\dagger]\} \neq 0. \tag{3.7}$$

J, the function of the mixing angles and the phase, can also be written in a form which is explicitly parametrization-independent:

$$|J| = |\mathrm{Im}(V_{ij} V_{lk} V_{ik}^* V_{lj}^*)| \tag{3.8}$$

for any choice of i, j, k, l.

The numerical values that we later find for the different mixing angles imply that $|J| \leq 10^{-4}$, even for $\sin\delta = 1$. This value is to be compared with the maximum possible value of $1/(6\sqrt{3})$. The Standard Model predicts a small intrinsic value for this measure of CP-violation.

In experiments, one really measures the ratio between the CP-violating part of a process to its CP-conserving part. This ratio is the CP-asymmetry. The asymmetry is given by the l.h.s of Eq. (3.5), divided by the total rate of the process. As the l.h.s of Eq. (3.5) does not depend on the specific process, the product of the CP-asymmetry and the branching ratio is roughly of the same order of magnitude in all measurements of CP-violation. The result of that is that processes with large asymmetries tend to have small branching ratios, while those with large branching ratios usually have small asymmetries.

4. DIRECT MEASUREMENTS (12)

In direct measurements we measure processes which occur at the tree level within the Standard Model. The assumption made here is that there are no processes from new physics which compete with Standard Model tree-level processes. This assumption holds in most models which go beyond the Standard Model. Thus, we expect the values of CKM matrix elementswhich are extracted from direct measurements to hold even if the Standard Model is only a low-energy effective theory.

As the top-quark has not been experimentally observed, its mixings, V_{ti}, cannot be directly measured. At present we have direct information on the six elements of the first two rows in the CKM matrix, and in this section we describe this information.

The simplest example of a model in which direct measurements would lead to wrong values for the CKM elements is a two Higgs doublet model, with a light charged Higgs. The two body decay into a charged Higgs and a quark could dominate over W-mediated decays. However, the present limits on the mass of a charged Higgs allow this decay mode for only the top-quark, whose mixings have not been directly measured anyway.

Our present knowledge of the matrix elements comes from the following sources:

1. Nuclear beta decay, when compared to muon decay, gives (13 – 16)

$$|V_{ud}| = 0.9744 \pm 0.0010 \ . \tag{4.1}$$

This includes refinements in the analysis of the radiative corrections, especially the order $Z\alpha^2$ effects, which have brought the ft-values from low and high Z Fermi transitions into good agreement.

2. Analysis of K_{e3} decays yields (17)

$$|V_{us}| = 0.2196 \pm 0.0023 \ . \tag{4.2}$$

The isospin violation between K^+_{e3} and- K^0_{e3} decays has been taken into account, bringing the values of $|V_{us}|$ extracted from these two decays into agreement at the 1% level of accuracy. The analysis of hyperon decay data has larger theoretical uncertainties because of first order $SU(3)$ symmetry breaking effects in the axial-vector couplings, but due account of symmetry breaking (18) applied to the WA2 data (19) gives a corrected value (J. M. Gaillard and G. Sauvage, private communication) of 0.222 ± 0.003 . We average these two results to obtain:

$$|V_{us}| = 0.2205 \pm 0.0018 \ . \tag{4.3}$$

3. The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections

of the CDHS group (20) yield $\overline{B}_c\,|V_{cd}|^2 = (0.41 \pm 0.07) \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced, The corresponding preliminary value from a recent Tevatron experiment (21) is $\overline{B}_c\,|V_{cd}|^2 = (0.534^{+0.052}_{-0.078}) \times 10^{-2}$. Averaging these two results gives $\overline{B}_c\,|V_{cd}|^2 = (0.47 \pm 0.05) \times 10^{-2}$. Supplementing this with measurements of the semileptonic branching fractions of charmed mesons (22), weighted by a production ratio of $D^0/D^+ = (60\pm10)/(40\mp10)$, to give $\overline{B}_c$ = 0.113f0.015, yields

$$|V_{cd}| = 0.204 \pm 0.017. \tag{4.4}$$

4. Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, leads to a lower bound (20), $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression [The result for M = 2.2 GeV is found in ref. (23)] that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \bar{K}e^+\nu_e) = |f_+^D(0)|^2|V_{cs}|^2(1.54 \times 10^{11}\mathrm{sec}^{-1}). \tag{4.5}$$

Here $f_+^D(q^2)$, with $q = p_D - p_K$, is the form factor relevant to D_{e3} decay; its variation has been taken into account with the parametrization $f_+^D(t)/f_+^D(0) = M^2/(M^2 - t)$ and $M = 2.1\ GeV/c^2$, a form and mass consistent with Mark III and E691 measurements (24, 25). Combining data on branching ratios for $D_{\ell 3}$ decays (24, 25) with accurate values (26) for τ_{D^+} and τ_{D^0}, gives the value (0.78f0.11) x 10^{11} sec^{-1} for $\Gamma(D \to \bar{K} e^+ \nu_e)$. Therefore

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.51 \pm 0.07. \tag{4.6}$$

A very conservative assumption is that $|f_+^D(0)| < 1$, from which it follows that $|V_{cs}| > 0.66$. Calculations of the form factor either performed (27, 28) directly at $q^2 = 0$, or done (29) at the maximum value of $q^2 = (m_D - m_K)^2$ and interpreted at $q^2 = 0$ using the measured q^2 dependence, yield $f_+^D(0) = 0.7 \pm 0.1$. It follows that

$$|V_{cs}| = 1.02 \pm 0.18 . \tag{4.7}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

5. The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to$ u and $b \to$ c. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor depends on the quark masses, but is roughly one-half (in suppressing $b \to$ c compared to $b \to u$). Both the CLEO (30) and ARGUS (31) collaborations have reported evidence for $b \to u$ transitions in semileptonic B decays. The interpretation of the result in terms of $|V_{ub}/V_{cb}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially for $b \to$ u transitions (28, 29, 32). Combining the experimental and

theoretical uncertainties, we quote

$$q \equiv |V_{ub}/V_{cb}| = 0.11 \pm 0.05. \tag{4.8}$$

6. The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction:

$$\Gamma(b \to c\ell\bar{\nu}_\ell) = \frac{BR(b \to c\ell\bar{\nu}_\ell)}{\tau_b} = \frac{G_F^2 m_b^5}{192\pi^3}\, \eta F(m_c/m_b)|V_{cb}|^2 \;, \tag{4.9}$$

where τ_b is the b lifetime, η is a QCD correction factor and $F(m_c/m_b)$ is the phase space factor noted above as approximately one-half. Most of the error on $|V_{cb}|$ derived from Eq. (4.9) is not from the experimental uncertainties, but in the theoretical uncertainties in choosing a value of m_b and in the use of the quark model to represent inclusively semileptonic decays which, at least for the B meson, are dominated by a few exclusive channels. Instead we quote the value derived from $B_{\ell 3}$ decay, $\bar{B} \to D\ell\bar{\nu}_\ell$, by comparing the observed rate with the theoretical expression that involves a form factor, $f_+^B(q^2)$. This is analogous to what gives the most accurate values for $|V_{us}|$ (from K_{e3} decay) and $|V_{cs}|$ (from $D_{\ell 3}$ decay). It avoids all questions of what masses to use, and the heavy quarks in both the initial and final states give more confidence in the accuracy of the theoretical calculations of the form factor. With account of a number of models of the form factor,. the data (33) yield

$$|V_{cb}| = 0.044 \pm 0.009 \,. \tag{4.10}$$

The central value and the error are now comparable to what is obtained from the inclusive semileptonic decays, but ultimately, with more data and more confidence in the calculation of the form factor, exclusive semileptonic decays should provide the most accurate value of $|V_{cb}|$.

5. UNITARITY

5.1. INTRODUCTION

The requirement of unitarity can be simply stated as $V^\dagger V = \mathbf{1}$. This imposes the following conditions on the matrix elements:

$$\sum_{j=1}^{n} |V_{ij}|^2 = 1; \quad \sum_{i=1}^{n} |V_{ij}|^2 = 1; \quad \sum_{k=1}^{n} V_{ik}^* V_{kj} = 0. \tag{5.1}$$

Unitarity may be used in several ways:

1. If we directly measure enough of the matrix elements, we may check whether their values are consistent with the unitarity constraint. We illustrate this in a two generation model, and explain the present situation in the three generation case.

2. Within the minimal Standard Model, where neutrinos are all massless, the number of generations is known to be three. We may then find values (or allowed ranges) for the matrix elements which have not been directly measured.

3. In extensions of the Standard Model, where neutrinos of higher generations may be very massive, the number of generations is only known to be a.? *least* three. We may still give upper bounds on the unmeasured matrix elements.

5.2. TWO GENERATIONS

To demonstrate how unitarity gives a consistency check on our measurements, we now assume that we know of two generations only. The mixing matrix is the 2 x 2 Cabibbo matrix. Direct measurements give the following range for the absolute values of its elements:

$$V_C = \begin{pmatrix} 0.9744 \pm 0.0010 & 0.2205 \pm 0.0018 \\ 0.204 \pm 0.017 & 1.02 \pm 0.18 \end{pmatrix}. \tag{5.2}$$

Unitarity implies that the above matrix depends on one parameter only:

$$V_C = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}. \tag{5.3}$$

With the above measurements we have certainly overdetermined the Cabibbo angle. The test of the two generation Standard Model is the following: Can we find a range for the Cabibbo angle which is consistent with all measurements? The answer is in the affirmative if

$$0.220 \leq s_{12} \leq 0.221. \tag{5.4}$$

Thus, a two generation picture is consistent within the experimental errors on the matrix elements; we could not tell that there is a third generation if it were not for its direct observation (or from CP violation). From our knowledge about $|V_{cb}|$ and $|V_{ub}|$ we know that the third generation mixings would be probed only if we reached an accuracy level of 10^{-4} in the determination of $|V_{ui}|$ or 10^{-3} in the determination of $|V_{ci}|$ $(i = d, s)$; this is well beyond the present level of accuracy. At present, the values in Eq. (5.2) imply only the following mild bounds on the possible mixings of a third generation:

$$V = \begin{pmatrix} & & 0-0.07 \\ & & 0 \quad 0.51 \\ 0-0.13 & 0-0.50 & 0-1 \end{pmatrix} \tag{5.5}$$

The-results derived here have some bearing on the three generation analysis. We have directly measured six elements, which should give a consistency check on the four parameters of the CKM matrix. However, due to the small values of $|V_{ub}|$ and $|V_{cb}|$ together with the present level of accuracy, four of the elements overdetermine s_{12}, just as described above for two generations, and there is no overdetermination yet of the other parameters.

5.3. THREE GENERATIONS

The recent measurements of the number of light neutrinos (34 – 38) imply that, within the minimal Standard Model and some of its extensions, the number of generations is three. This makes it very likely that the 3 x 3 CKM matrix exactly fulfills the unitarity constraints. Consequently, we may deduce the allowed ranges for the V_{ti} elements, and also further restrict the allowed ranges for elements which were directly measured.

1. The value of $|V_{tb}|$ is derived from

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1. \tag{5.6}$$

As both $|V_{ub}|$ and $|V_{cb}|$ are measured to be much smaller than 1, the $|V_{tb}|$

value is very close to 1:

$$0.9986 \leq |V_{tb}| \leq 0.9994. \tag{5.7}$$

2. The value for $|V_{ts}|$ is derived from

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0. \tag{5.8}$$

As the first term on the left hand side is much smaller than the other two, and as both $|V_{cs}|$ and $|V_{tb}|$ are very close to 1, the $|V_{ts}|$ value is very close to $|V_{cb}|$:

$$0.034 \leq |V_{ts}| \leq 0.055. \tag{5.9}$$

3. The allowed range for $|V_{td}|$ is derived from

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \tag{5.10}$$

As both V_{ud} and V_{tb} are very close to 1, and as $V_{cd} \approx -s_{12}$, we may approximate Eq. (5.10) by

$$V_{ub}^* + V_{td} = s_{12} V_{cb}. \tag{5.11}$$

This gives:

$$0.002 \leq |V_{td}| \leq 0.020. \tag{5.12}$$

Full information on the ranges for the absolute values of the CKM elements (at one sigma) from both direct measurements and unitarity is summarized by:

$$V = \begin{pmatrix} 0.9749-0.9754 & 0.2187-0.2223 & 0.002-0.009 \\ 0.187-0.221 & 0.974-0.982 & 0.035-0.053 \\ 0.002-0.020 & 0.034-0.055 & 0.9986-0.9994 \end{pmatrix} \quad (5.13)$$

In terms of the parameters of Eq. (3.1) we get:

$$s_{12} = 0.2205 \pm 0.0018; \quad s_{23} = 0.044 \pm 0.009; \; q \equiv s_{13}/s_{23} = 0.11 \pm 0.05. \quad (5.14)$$

There are no direct constraints on the phase δ. Among the three real angles, there are large uncertainties in s_{13} only. Therefore, it is useful to present the information coming from indirect measurements as constraints in the $q-\delta$ plane.

5.4. THE UNITARITY TRIANGLE

As is apparent from Eq. (5.13), the only poorly determined matrix elements are V_{td} and V_{ub}. They are related to each other by the unitarity constraint (5.10) or, to a very good approximation, (5.11). The information that we may get from indirect measurements will have to comply with this constraint. It is very convenient to present such information and to discuss further predictions by using the *unitarity* triangle, which is just a geometrical representation of the relation (5.11) in the complex plane: The three complex quantities, V_{ub}^*, V_{td} and $s_{12}V_{cb}$ should form a triangle, as shown in Figure 1. Rescaling the triangle by $[1/(s_{12}|V_{cb}|)]$, the coordinates of the three vertices A, B and C become

$$A\left[\frac{\text{Re } V_{ub}}{s_{12}|V_{cb}|}, -\frac{\text{Im } V_{ub}}{s_{12}|V_{cb}|}\right], \; B(1,0), C(0,0). \quad (5.15)$$

In the Wolfenstein parametrization (5), which is just the small mixing-angle approximation of the parametrization (3.1), the coordinates of the vertex A are (ρ, η).

5.5. MORE THAN THREE GENERATIONS

The existence of more than three generations is possible only if neutrinos of higher generations are massive, and their masses are $\gtrsim M_Z/2$. If, indeed, there are more than three generations, we could still derive useful limits on the elements of the 3 x 3 sub-matrix:

$$\sum_{j=1}^{3} |V_{ij}|^2 \leq 1; \sum_{i=1}^{3} |V_{ij}|^2 \leq 1. \tag{5.16}$$

This would give the following ranges for the elements of the sub-matrix:

$$V = \begin{pmatrix} 0.9734-0.9754 & 0.2187-0.2223 & 0.002-0.009 \\ 0.187-0.221 & 0.84-0.98 & 0.035-0.053 \\ 0-0.13 & 0-0.50 & 0-0.9994 \end{pmatrix} \tag{5.17}$$

6. INDIRECT MEASUREMENTS

6.1. INTRODUCTION

With indirect measurements we are able to probe physics at a high energy scale due to loop effects. Most important, loop contributions from an intermediate top quark allow us to measure its mass and mixings. The assumption made is that there are no important contributions from new physics beyond the Standard Model. Note, however, that this does not hold in many extensions of the Standard Model: loop diagrams within the Standard Model are suppressed by being of higher order in the weak coupling and by the GIM mechanism, while tree level processes from beyond the Standard Model are suppressed by the high energy scale, but are lower order in the couplings and may be free of GIM suppression. Thus, they may compete with or even dominate over the Standard Model contributions. This makes indirect measurements an important test of the Standard Model, and at the same time a sensitive probe of physics beyond it.

A useful demonstration of the use of indirect measurements is the analysis of $K-\bar{K}$ mixing. The existence of a third generation is not necessary, and we show how the two generation Standard Model is tested.

All other indirect measurements that we study have significant contributions from the top quark. Within the Standard Model, the strongest lower bound on the top mass comes from the CDF collaboration (39):

$$m_t \geq 77 \ GeV. \tag{6.1}$$

This bound may be evaded in specific extensions of the Standard Model. For example, within a two Higgs doublet model, the present limits from hadron colliders may not hold, and the strongest limit comes from measurements by OPAL (40), $m_t \geq 43.5 \ GeV$. Another measurement by CDF which determines the W-width (41), gives a bound that is independent of the decay modes of the top quark, $m_t \geq 41 \ GeV$. An upper limit on m_t is derived from the virtual effects of the top quark on the gauge sector parameters (42 – 44). The main ingredients in this analysis are the consistency through one-loop electroweak corrections of the precisely known values of α, G_F and M_Z with the measurements of M_W/M_Z and deep inelastic neutrino scattering on nucleons. Though this is an indirect determination of m_t, the only assumption made on the CKM elements is a very mild one, $|V_{tb}| \sim 1$. It gives:

$$m_t \lesssim 200 \ GeV. \tag{6.2}$$

As long as the top quark is not experimentally observed, it is impossible in principle to directly measure the values of the three elements $|V_{ti}|$. (It is important to emphasize that even when the top quark is finally observed, it may be very difficult, if not impossible, to measure some of these elements.) Consequently, the only way to improve our knowledge of these parameters beyond the unitarity constraints, is by using indirect measurements. In all these measurements, the results depend on both the top-quark mixings and its mass. We study mixing in the $B-\bar{B}$ system and CP violation in the neutral K system to further constrain the V_{ti} elements.

6.2. $K-\bar{K}$ MIXING

The mass difference ΔM_K between the two neutral K-mesons is a result of" mixing, $M_{12}(K^0)$, between the two isospin eigenstates. With two generations, the short distance contribution to the mass difference is given by (45):

$$\Delta M_K \cdot (1-D)=\frac{G_F^2}{6\pi^2}\eta_1 M_K(B_K f_K^2)M_W^2 y_c \mathrm{Re}[(V_{cd}^* V_{cs})^2] \qquad (6.3)$$

where $y_i = m_i^2/M_W^2$. We divide the parameters in this equation (other than m_c, V_{cd} and V_{cs}) into two categories:

1. Parameters which are known to a high level of accuracy. We collect many of them into

$$C_K \equiv \frac{6\pi^2 \Delta M_K}{G_F^2 f_K^2 M_K M_W^2} = 1.8 \times 10^{-5} \qquad (6.4)$$

where we use:

$$G_F = 1.166 \times 10^{-5} GeV^{-2},\ M_W = 80\ GeV$$

$$f_K^2 = (0.165\ GeV)^2,\quad M_K = 0.498\ GeV,\ \Delta M_K = 3.52 \times 10^{-15} GeV. \tag{6.5}$$

The parameter η_1 is a QCD correction (46): $\eta_1 = 0.7$.

2. Parameters with large theoretical uncertainties. The D parameter gives the relative part of long distance contributions to ΔM_K. We use

$$0 \le D \le 0.5. \tag{6.6}$$

The B_K parameter gives the ratio between the short distance contribution to ΔM_K and its value in the vacuum insertion approximation. We use

$$\frac{1}{3} \le B_K \le 1 \tag{6.7}$$

Eq. (6.3) can then be rewritten as

$$C_K \frac{(1-D)}{B_K} = \eta_1 y_c (V_{cd} V_{cs})^2. \tag{6.8}$$

(In the two generation case all matrix elements are real.) When the original study of $K - \bar{K}$ mixing (45) was performed, the c-quark was not yet experimentally discovered. Thus, one could use Eq. (6.8) to predict the mass of the c-quark. In the original calculation, the vacuum saturation approximation was used ($B_K = 1$), and neither long-distance contributions nor QCD corrections were taken into account

$(D = 0, \eta_1 = 1)$. This led, somewhat coincidentally, to the correct prediction (45): $m_c = 1.5\ GeV$. With the full range of B_K and D,

$$0.9 \times 10^{-5} \leq C_K \frac{(1-D)}{B_K} \leq 5.3 \times 10^{-5}, \tag{6.9}$$

and with the correct value for η_1, one would have predicted $1.3\ GeV \leq m_c \leq 3.2\ GeV$. If, on the other hand, we use $m_c = 1.4\ GeV$, we get $0.20 \leq V_{cd}V_{cs} \leq 0.50$. This is to be compared with the constraints from direct measurements and unitarity as follow from Eq. (5.4).

With three generations one has to take into account contributions from intermediate t-quarks. The r.h.s of Eq. (6.8) becomes more complicated (47) and depends also on m_t, V_{ts} and V_{td}. However, the contribution of diagrams involving the t-quark is suppressed by more than an order of magnitude compared to the c-quark contribution. With the large theoretical uncertainties, it is impossible to derive any useful information on m_t and V_{td}.

6.3. $B - \bar{B}$ MIXING: x_d

Mixing in the $B - \bar{B}$ system is characterized by the parameter $x_d \equiv$ AM/I'. Within the Standard Model it is given to a very good approximation by

$$x_d = \tau_b \frac{G_F^2}{6\pi^2} \eta M_B (B_B f_B^2) M_W^2 y_t f_2(y_t) |V_{td}^* V_{tb}|^2 \tag{6.10}$$

where

$$f_2(y_t) = 1 - \frac{3}{4} \frac{y_t(1+y_t)}{(1-y_t)^2} \left[1 + \frac{2y_t}{1-y_t^2} \ln(y_t) \right]. \tag{6.11}$$

Corrections of order m_b^2/m_t^2 due to external momenta are neglected. This affects the result by 1% or less. We note two important differences from the $K - \bar{K}$ case:

1. For neutral mesons AM $\propto$ Re $\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2}$. In the K^0 system, both M_{12} and Γ_{12} are almost real, and the mixing depends on $\mathrm{Re}[M_{12}(K^0)]$. In the B^0 system we have $|\Gamma_{12}| \ll |M_{12}|$, and the mixing depends on $|M_{12}(B^0)|$.

2. In the K system, the contribution from intermediate t quarks is suppressed -because the external d and s quarks are of the first and second generations, with small mixing to the third generation quark. In the B system, the external d and b quarks are of first and third generations. Thus the t quark couplings are not suppressed, and with its large mass the diagram with two intermediate t quarks is dominant. In fact, the contribution of u and c intermediate quarks can be safely neglected.

The parameters in Eq. (6.10) (other than m_t and V_{td}) can be divided into:

1. Parameters which are known to a high level of accuracy. We collect them into

$$C_B \equiv \frac{6\pi^2}{G_F^2 M_B M_W^2} = 1.3 \times 10^7\ GeV \tag{6.12}$$

where, in addition to the previously given parameters, G_F and M_W, we use

$$M_B = 5.28\ GeV. \tag{6.13}$$

The parameter η is a QCD correction (48): η = 0.85. To a very good approximation $V_{tb} = 1$.

2. Parameters with relatively large theoretical ambiguities ($B_B f_B^2$), experimental errors (x_d), or both ($\tau_b|V_{cb}|^2$).

One should note that $|V_{cb}|$ and τ_b appear Only in the combination $\tau_b|V_{cb}|^2$, which does not depend on τ_b, as can be readily seen from Eq. (4.9). Therefore, the error on this combination is somewhat smaller than on $|V_{cb}|^2$ alone:

$$\tau_b|V_{cb}|^2 = (3.5 \pm 0.6) \times 10^9\, GeV^{-1}. \tag{6.14}$$

The hadronic parameter B_B (analogous to B_K of the Kaon system) is believed to be close to 1. However, there is much uncertainty involved in the calculation of the B decay constant f_B. (Note that for the K, the decay constant f_K is experimentally determined.) A range of values for f_B has been derived from QCD sum rules and lattice calculations:

$$\sqrt{B_B} f_B = 0.15 \pm 0.05\, GeV. \tag{6.15}$$

The ARGUS (49) and CLEO (50) collaborations observe $B_d - \bar{B}_d$ mixing with

$$r_d = \begin{cases} 0.21 \pm 0.06 & \text{ARGUS} \\ 0.14 \pm 0.05 & \text{CLEO} \end{cases} \tag{6.16}$$

The x_d parameter is related to r_d by $r_d = x_d^2/(2 + x_d^2)$. We take the combined result of the two experiments, $r_d = 0.18 \pm 0.05$, and get

$$x_d = 0.66 \pm 0.11. \tag{6.17}$$

Eq. (6.10) then can be rewritten as

$$C_B \frac{x_d}{(\tau_b|V_{cb}|^2)(B_B f_B^2)} = \eta y_t f_2(y_t)|V_{td}/V_{cb}|^2. \tag{6.18}$$

With the parametrization (3.1) we get

$$C_B \frac{x_d}{(\tau_b|V_{cb}|^2)(B_B f_B^2)} = \eta y_t f_2(y_t)(s_{12}^2 + q^2 - 2s_{12} q \cos\delta). \tag{6.19}$$

From the ranges given above:

$$0.044 \leq C_B \frac{x_d}{(\tau_b|V_{cb}|^2)(B_B f_B^2)} \leq 0.35. \tag{6.20}$$

Eq. (6.18) together with the unitarity constraints on $|V_{td}|$ gives $m_t \gtrsim 50\ GeV$, which is below the bound from direct searches, but may be useful in extensions of the Standard Model (51). Eq. (6.18) together with the upper bound on m_t gives

$$|V_{td}| \geq 0.004. \tag{6.21}$$

For $m_t \gtrsim 185\ GeV$ the upper limit on $|V_{td}|$ that follows from Eq. (6.18) is stronger than the unitarity bound.

6.4. THE ϵ PARAMETER

The expression for ϵ is (47)

$$\begin{aligned}|\epsilon| =& \frac{G_F^2}{12\pi^2} \frac{M_K}{\sqrt{2}\Delta M_K}(B_K f_K^2) M_W^2 \\ &\times \left\{ \eta_1 y_c \mathrm{Im}\left[(V_{cd}^* V_{cs})^2\right] + \eta_2 y_t f_2(y_t) \mathrm{Im}\left[(V_{td}^* V_{ts})^2\right] + 2\eta_3 f_3(y_t) \mathrm{Im}\left[V_{cd}^* V_{cs} V_{td}^* V_{ts}\right]\right\}\end{aligned} \tag{6.22}$$

where

$$f_3(y_t) = \ln\left(\frac{y_t}{y_c}\right) - \frac{3}{4}\frac{y_t}{1-y_t}\left[1 + \frac{y_t}{1-y_t}\ln(y_t)\right]. \tag{6.23}$$

The parameters (other than those in the curly brackets) are divided into:

1. Well-known parameters, which we collect into

$$C_\epsilon \equiv \sqrt{2}|\epsilon|C_K = 5.6 \times 10^{-8} \tag{6.24}$$

where, in addition to the parameters given previously, we have used

$$|\epsilon| = 2.27 \times 10^{-3} \tag{6.25}$$

2. Parameters with large uncertainties. The long distance contribution to ϵ is small and introduces.an uncertainty smaller than 5%. Thus, the large uncertainty is in B_K, which we have already encountered in Eq. (6.7).

Eq. (6.22) can be rewritten as

$$\frac{C_\epsilon}{B_K} = -|V_{cb}|\mathrm{Im}(V_{td}) \ \{[\eta_3 f_3(y_t) - \eta_1]y_c|V_{cd}| + \eta_2 y_t f_2(y_t)|V_{cb}|\mathrm{Re}(V_{td})\} \, . \tag{6.26}$$

or, using the parametrization (3.1),

$$\frac{C_\epsilon}{B_K} = (s_{23})^2 q \sin\delta \left\{ [\eta_3 f_3(y_t) - \eta_1] y_c s_{12} + \eta_2 y_t f_2(y_t)(s_{23})^2 (s_{12} - q\cos\delta) \right\} . \tag{6.27}$$

For the terms in the curly brackets we use $m_c = 1.4\ GeV$ and (46) $\eta_1 = 0.7$; $\eta_2 = 0.6$; $\eta_3 = 0.4$ (the η_i are QCD corrections).

6.5. RESULTS

We present our results for the CKM parameters in two equivalent ways:

1. Allowed regions in the $q-\delta$ plane (52, 53). We use Eq. (6.19) involving x_d and Eq. (6.27) involving ϵ. For each relation, we use the full range of parameters as given in Eqs. (6.20) and [(6.7),(4.10)], respectively. For a fixed top quark mass, we get allowed bands in the $q-\delta$ plane. The final allowed region is within the two bands and within the direct limits on q [eq. (4.8)]. We show the constraints for m_t = 80, 120, 160 and 200 GeV in Figure 2.

2. Allowed region for the vertex A of the unitarity triangle (54). The analysis is done using the x_d relation as given in Eq. (6.18) and the ϵ relation as given in Eq. (6.26). The constraints are shown in Figure 3.

7. RELATIONS AMONG QUARK MASSES, MIXING ANGLES AND PHASES

Within the Standard Model, the quark sector is described by ten free parameters. In the physical (mass) basis, these are the six quark masses, three mixing angles and one phase. These parameters can all be experimentally determined. Whatever their experimental values are, the Standard Model remains self-consistent.

In the interaction basis, our parameters are entries of the yet undiagonalized mass matrices. If we had some theoretical principle from which we could determine the mass matrices, we would predict the values of the physical parameters. In several schemes of mass matrices, the number of independent entries in the mass matrices is less than ten; some entries vanish or there are relations among the non-vanishing entries. Such schemes consequently provide us with relations among quark masses, angles and phases.

One needs to check whether the relations are consistent with experimental data. If the relations suggested by a certain scheme were not compatible with the experimental constraints, then either the scheme is incorrect, or new physics must intervene to alter the parameters derived from indirect measurements. We note that the existence of new physics beyond the Standard Model is already inherent in the suggestion of schemes for quark mass matrices. The validity of the discussion on the experimental consistency of various schemes given below lies in the assumption that this new physics itself does not significantly contribute to CP violation or to $B - \bar{B}$ mixing. This is the case, for example, if the new physics takes place at a high enough energy scale.

Various approaches to the problem have been tried. Some simply give a qualitative explanation for the smallness of the angles, e.g. the work by Froggatt and Nielsen (55). Others try to derive the form of the mass matrices from a fundamental theoretical framework such as string theory [see e.g. ref. (56) and references therein]. Still others postulate a certain form for the mass matrices (or equivalently a discrete symmetry) and study the resulting predictions. Many of these schemes are inconsistent with the $B - \bar{B}$ mixing measurement (52) or with the recent lower bounds on the top mass [Eq. (6.1)], e.g. the Stech scheme (57). We demonstrate the general approach using a scheme due to Fritzsch (58); which is consistent with present data.

Fritzsch suggested the following form for the mass matrices (58):

$$M^u = \begin{pmatrix} 0 & a^u & 0 \\ a^u & 0 & b^u \\ 0 & b^u & c^u \end{pmatrix} \quad M^d = \begin{pmatrix} 0 & a^d e^{i\phi_1} & 0 \\ a^d e^{-i\phi_1} & 0 & b^d e^{i\phi_2} \\ 0 & b^d e^{-i\phi_2} & c^d \end{pmatrix} \tag{7.1}$$

The six real parameters $(a^u, b^u, c^u, a^d, b^d, c^d)$ can be expressed in terms of the six quark masses:

$$\begin{aligned} a^u &\approx \sqrt{m_u m_c}\ ;\ b^u \approx \sqrt{m_c m_t}\ ;\ c^u \approx m_t \\ a^d &\approx \sqrt{m_d m_s}\ ;\ b^d \approx \sqrt{m_s m_b}\ ;\ c^d \approx m_b \end{aligned} \tag{7.2}$$

The approximation is good to $O(m_d/m_s) \sim (1/20)$. The two phases (ϕ_1, ϕ_2) can be expressed in terms of the quarks masses and the two *known* mixing angles, s_{12} and s_{23}

$$s_{12} \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \right| \tag{7.3}$$

$$s_{23} \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right| \tag{7.4}$$

Thus, the eight parameters of the Fritzsch scheme are expressible in terms of eight other parameters: seven known (five quark masses and two mixing angles) and one unknown (the mass of the top quark). Consequently, if we select a value of m_t, we get predictions for $q \equiv s_{13}/s_{23}$ and for the phase 6:

$$q \equiv \frac{s_{13}}{s_{23}} = \left| \frac{1}{s_{23}} \left(\frac{m_s}{m_b}\right)^{3/2} \sqrt{\frac{m_d}{m_s}} + e^{-i(\phi_1+\phi_3)} \sqrt{\frac{m_u}{m_c}} \right| \tag{7.5}$$

where the phase ϕ_3 is defined through $s_{23} e^{-i\phi_3} = \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2}\sqrt{\frac{m_c}{m_t}}$;

$$\frac{\sin\delta}{(s_{12}/q) - \cos\delta} \approx \frac{\sin\phi_1}{\cos\phi_1 - \sqrt{\frac{m_d m_c}{m_s m_u}}} \tag{7.6}$$

A detailed comparison between the predictions of the Fritzsch scheme and the allowed ranges for the CKM parameters given in previous sections, shows that in order to get a consistent solution (a) the mass ratio m_s/m_b should be close to its lower limit, $m_s/m_b \sim 0.021$, (b) the mixing angle s_{23} should be close to its upper limit, $s_{23} \sim 0.053$, (c) the $B-\bar{B}$ mixing parameter x_d should be close to its lower limit, $x_d \sim 0.55$, (d) the B decay constant f_B should be close to the upper limit of its theoretical range, $B_B\, f_B^2 \sim (0.20\ GeV)^2$, and (e) the B_K constant should be close to the upper limit of its theoretical range, $B_K \sim 1$. Only if all of these conditions are simultaneously fulfilled will there be a narrow region of (m_t, q, δ) space which is consistent with both the Fritzsch relations and the experimental. data. This region is within the following bounds:

$$
\begin{aligned}
77\ GeV \leq & m_t \leq 96\ GeV \\
0.06 \leq & q \leq 0.08 \\
90^o \leq & \delta \leq 110^o
\end{aligned}
\tag{7.7}
$$

These constraints give many specific predictions that can be tested in the near future (53).

8. FURTHER PROGRESS

What further progress can be expected in the determination of the CKM matrix elements? In what follows we give a brief overview of the on-going and near-future studies, both experimental and theoretical, which may improve our understanding of quark mixing.

1. The determination of $|V_{cd}|$ and $|V_{cs}|$ from semileptonic D decays, and the determination of $|V_{ub}|$ and $|V_{cb}|$ from semileptonic B decays.

With higher statistics, the experimental errors on the relevant branching ratios will become smaller. Moreover, we will have detailed information on exclusive semileptonic decays: branching ratios, polarization, angular distribution, etc. This will help to test the various phenomenological models and to tune their parameters. Consequently, we will have a more reliable theoretical interpretation of the experimental results.

2. The determination of $|V_{ts}|$ and $|V_{td}|$ from $B-\bar{B}$ mixing.

A B factory or a Z factory may give a first measurement of $B_s-\bar{B}_s$ mixing. In addition, the mixing in $B_d-\bar{B}_d$ will be measured more accurately. The accuracy in the determination of V_{ts} and V_{td} will be mainly limited by the theoretical uncertainties in the calculation of f_B. However, the ratio x_d/x_s is well-approximated by

$$\frac{x_d}{x_s}=\frac{|V_{td}|^2}{|V_{ts}|^2}\frac{f_{B_d}^2}{f_{B_s}^2}. \tag{8.1}$$

The ratio $f_{B_d}/f_{B_s}=1$ in the $SU(3)$ symmetry limit. One expects the deviation from the symmetry limit to make the ratio smaller than 1 by $O(0.1)$. Lattice calculations (extrapolating from lower energy scales) may give a better understanding of this deviation.

Within the Standard Model, x_s is predicted to be large. Consequently, its measurement can be done only in a time-dependent way. Studies of future experimental facilities show (59, 60) that the measurement is feasible only if $x_s \lesssim 15$. However, if such a measurement is made, and if the theoretical understanding of the ratio f_{B_d}/f_{B_s} improves to the 10% level, it will provide us with very strong constraints on the CKM parameters: within the Standard Model, where $|V_{ts}|=|V_{cb}|$, we will have the $|V_{td}/(V_{cb}V_{cd})|$ side of the triangle determined with 10% accuracy. This measurement, though indirect, will be independent of m_t and V_{cb}.

3. CP violation in B^0 decays.

The main goal of future B factories is to measure CP asymmetries in B^0 decays. This tests the CKM description of quark mixing, and in particular the mechanism of CP violation. The decay rate of a timeevolved, initially pure B^0 ($\bar{B}^0$) into a CP-eigenstate, j, is [see ref. (61) and references therein]:

$$\begin{aligned} \Gamma(B^0_{\text{phys}}(t) \to \text{j}) &\propto e^{-\Gamma t}\, [1 - \text{Im}\,\lambda\, \sin(\text{Am t})] \\ \Gamma(\bar{B}^0_{\text{phys}}(t) \to \text{j}) &\propto e^{-\Gamma t}\, [1 + \text{Im}\,\lambda\, \sin(\text{ Am } t)]\,. \end{aligned} \tag{8.2}$$

CP-violating effects are manifest through the presence of the interference term Im λ. For the processes under consideration here, the CP violation arises from the quantum mechanical interference of amplitudes corresponding to two paths to the same final state, one of which involves $B^0 - \bar{B}^0$ mixing. The CP asymmetry, Im λ, depends only on the CKM matrix elements. More specifically, Im λ = $\sin(2\phi)$ where ϕ stands for one of the angles of the unitarity triangle, α, β or γ (see Figure 1). The best known examples are the measurement of $\sin(2\beta)$ in $B_d \to \psi K_S$; the measurement of $\sin(2\alpha)$ in $B_d \to \pi^+\pi^-$; and the measurement of $\sin(2\gamma)$ in $B_s \to \rho K_S$. [For a recent study of the Standard Model predictions for these processes, see ref. (54).]

There are many other measurements that may provide us with additional information on the CKM elements. For some, we need to improve our theoretical understanding, (e.g. ϵ'/ϵ). For others, we need higher energy (e.g. finding the top quark and measuring m_t) or higher statistics [e.g. BR($K \to \pi\nu\nu$)] experiments.

The Standard Model describes quark mixing and CP violation through the CKM matrix. When we measure the values of the CKM elements and CP asymmetries in many different ways, we test this description. Even if the Standard Model remains consistent with all measurements, the values of the various parameters may give us a window into a more fundamental framework, one which is able to-predict them from first principles.

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REFERENCES

1. Kobayashi, M., Maskawa, T., *Prog. Theo. Phys.* 49:652 (1979)
2. Cabibbo, N., *Phys. Rev. Lett.* 10:531 (1963)
3. Chau, L.-L., Keung, W.-Y., *Phys. Rev. Lett.* 53:1802 (1984)
4. Maiani, L., *Phys. Lett.* 62B:183 (1976)
5. Wolfenstein, L., *Phys. Rev. Lett.* 51:1945 (1983)
6. Botella, F. J., Chau, L.-L., *Phys. Lett.* 168B:97 (1986)
7. Harari, H., Leurer, M., *Phys. Lett.* 181B:123 (1986)
8. Fritzsch, H., Plankl, J., *Phys. Rev.* D35:1732 (1987)
9. Particle Data Group, *Phys. Lett.* 204B:1 (1988)
10. Gronau, M., Schechter, J., *Phys. Rev. Lett.* 54:385 (1985)
11. Jarlskog, C., *Phys. Rev. Lett.* 55:1039 (1985); *Z. Phys.* C29:491 (1985)
12. Gilman, F. J., Kleinknecht, K., Renk, B., *Review of Particle Properties.* In press
13. Marciano, W. J., Sirlin, A., *Phys. Rev. Lett.* 56:22 (1986)
14. Sirlin, A., Zucchini, R., *Phys. Rev. Lett.* 57:1994 (1986)
15. Jaus, W., Rasche, G., *Phys. Rev.* D35:3420 (1987)
16. Sirlin, A., *Phys. Rev.* D35:3423 (1987)
17. Leutwyler, H., Roos, M., Z. *Phys.* C25:91 (1984)
18. Donoghue, J. F., Holstein, B. R., Klimt, S. W., *Phys. Rev.* D35:934 (1987)

19. Bourquin, M., et al, Z. *Phys.* C21:27 (1983)

20. Abramowicz, H., et al, Z. *Phys.* C15:19 (1982)

21. Shaevitz, M., 1989. Presented at Int. Workshop on Weak Interactions and Neutrinos, 12th, Ginosar

22. Hitlin, D., *Nucl. Phys. (Proc. Suppl.)* B3:179 (1988)

23. Bletzacker, F., Nieh, H. T., Soni, A., *Phys. Rev.* D16:732 (1977)

24. Coffman, D. M., California Institute of Technology Ph.D. thesis, 1986

25. Anjos, J. C., et al, *Phys. Rev. Lett.* 62:1587 (1989)

26. Raab, J. R., et al, *Phys. Rev.* D37:2391 (1988)

27. Aliev, T. M., et al, Yad. *Fiz.* 40:823 (1984) [*Sov. J. Nucl. Phys.* 40:527 (1984)]

28. Bauer, M., Stech, B., Wirbel, M., Z. *Phys.* C29:637 (1985)

29. Grinstein, B., Isgur, N., Wise, M. B., *Phys.* Rev. *Lett.* 56:298 (1986); Grinstein, B., Isgur, N., Scora, D., Wise, M. B., *Phys. Rev.* D39:799 (1989)

30. Fulton, R., et al, *Phys. Rev. Lett.* 64:16 (1989)

31. Albrecht, H., et al, *Phys. Lett.* 234B:409 (1990)

32. Altarelli, G., et al, *Nucl. Phys.* B208:365 (1982)

33. Albrecht, H., et al, *Phys. Lett.* 229B:175 (1989)

34. Abrams, G. S., et al, *Phys. Rev. Lett.* 63:2173 (1989)

35. Adeva, B., et al, *Phys. Lett.* 231B:509 (1989)

36. Decamp, D., et al, *Phys. Lett.* 231B:519 (1989)

37. Akrawy, M. Z., et al, *Phys. Lett.* 231B:531 (1989)

38. Aaranio, P., et al, *Phys. Lett.* 231B:539 (1989)

39. Abe, F., et al, *Phys. Rev. Lett.* 64:142 (1990)

40. Akrawy, M. Z., et al, CERN-EP/89-154 (1989)

41. Abe, F., et al, *Phys. Rev. Lett.* 64:152 (1990)

42. Amaldi, U., et al, *Phys. Rev.* D36:1385 (1987)

43. Costa, G., et al, *Nucl. Phys.* B297:244 (1988)

44. Langacker, P., *Phys. Rev. Lett.* 63:1920 (1989)

45. Gaillard, M. K., Lee, B. W., *Phys. Rev.* D10:897 (1974)

46. Gilman, F. J., Wise, M. B., *Phys. Lett.* 93B:129 (1980); *Phys. Rev.* D27:1128 (1983)

47. Inami, T., Lim, C. S., *Prog. Theo. Phys.* 65:297 (1981); (E) 65:1772 (1982)

48. Hagelin, J. S., *Nucl. Phys.* B193:123 (1981)

49. Danilov, M., 1989. Presented at Int. Symposium on Lepton-Photon Interactions at High Energies, Stanford

50. Kreinick, D., 1989. Presented at Int. Symposium on Lepton-Photon Interactions at High Energies, Stanford

51. Nir, Y., 1990, *Phys. Lett.*. In press

52. Harari, H., Nir, Y., *Phys. Lett.* 195B:586 (1987)

53. Nir, Y., *Nucl. Phys.* B306:14 (1988)

54. Dib, C. O., Dunietz, I., Gilman, F. J., Nir, Y., 1990, *Phys. Rev.* D. In press

55. Froggatt, C. D., Nielsen, H. B., *Nucl. Phys.* B147:277 (1979)

56. Ross, G. G., *Nucl. Phys. (Proc. Suppl.)* B5:199 (1988)

57. Stech, B., *Phys. Lett.* 130B:189 (1983)

58. Fritzsch, H., *Phys. Lett.* 70B:436 (1977); 73B:317 (1978)

59. Hitlin, D., (editor), SLAC report SLAC-353 (1989)

60. Browder, T., et al, SLAC report SLAC-354 (1989)

61. Dunietz, I., Rosner, J. L., *Phys. Rev.* D34:1404 (1986)

FIGURE CAPTIONS

Figure 1. Representation in the complex plane of (a) the triangle formed by the CKM matrix elements V_{ub}^*, $V_{cd} \cdot V_{cb}^*$ and V_{td}, and (b) the rescaled triangle with vertices at $A(\rho,\eta)$, B(l, 0) and C(O,O). A relevant B^0 decay mode is indicated for the angle involved in the corresponding CP-violating asymmetry, as explained in section 8.

Figure 2. Constraints from $|V_{ub}/V_{cb}|$ (dotted lines), x_d (dashed curves) and ϵ (solid curves) on the parameters $q = s_{13}/s_{23}$ and δ for $m_t = 80, 120, 160$ and 200 GeV. The shaded region is the finally allowed range.

Figure 3. Constraints from $|V_{ub}/V_{cb}|$ (dotted circles), x_d (dashed circles) and ϵ (solid hyperbolas) on the rescaled unitarity triangle for $m_t = 80, 120, 160$ and 200 GeV. The shaded region is that allowed-for the vertex $A(\rho,\eta)$.

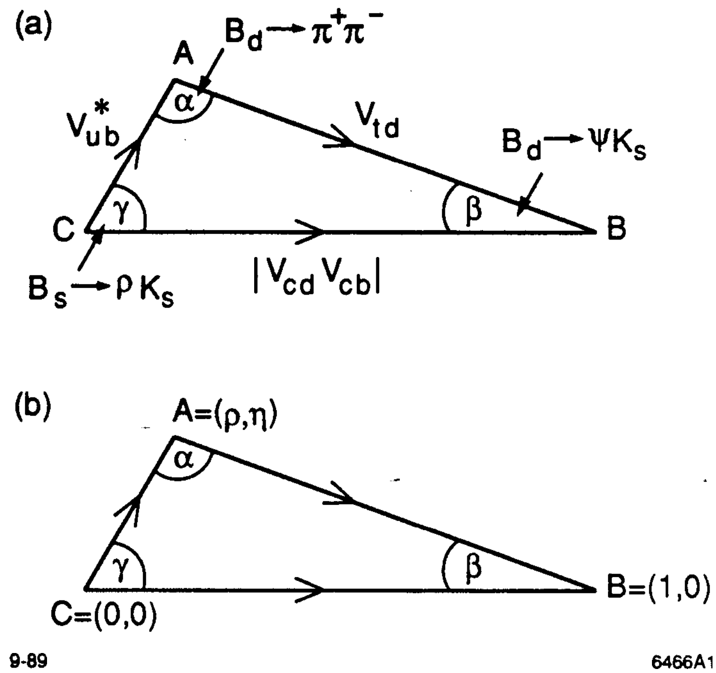
(a)
A
$B_d \rightarrow \pi^+\pi^-$
α
V_{ub}^*
V_{td}
$B_d \rightarrow \Psi K_s$
γ
β
C
B
$B_s \rightarrow \rho K_s$
$|V_{cd} V_{cb}|$
(b)
A=(ρ,η)
α
γ
β
B=(1,0)
C=(0,0)
9-89
6466A1

Fig. 1

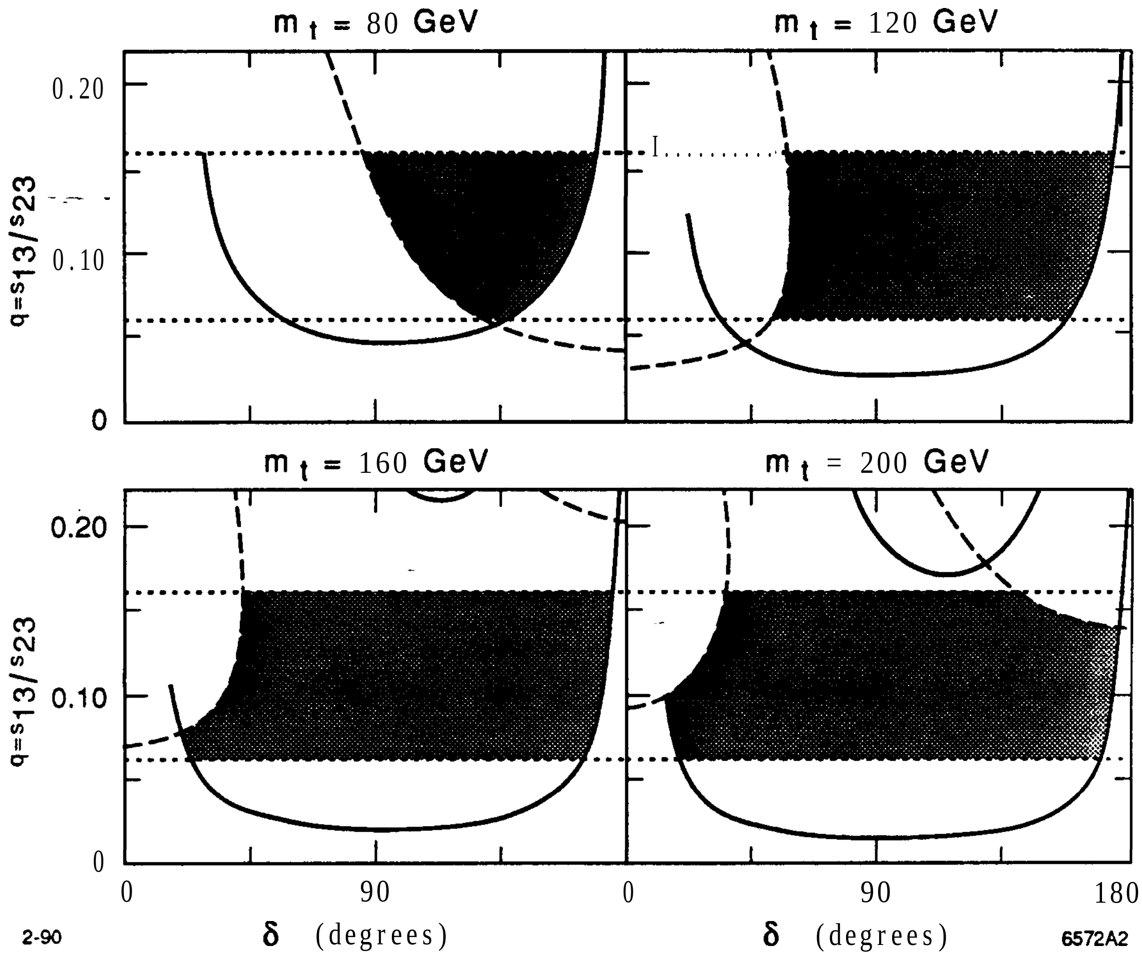
m t = 80 GeV
m t = 120 GeV
m t = 160 GeV
m t = 200 GeV
0.20
0.10
0
q=s13/s23
90
180
δ (degrees)
2-90
6572A2

Fig. 2

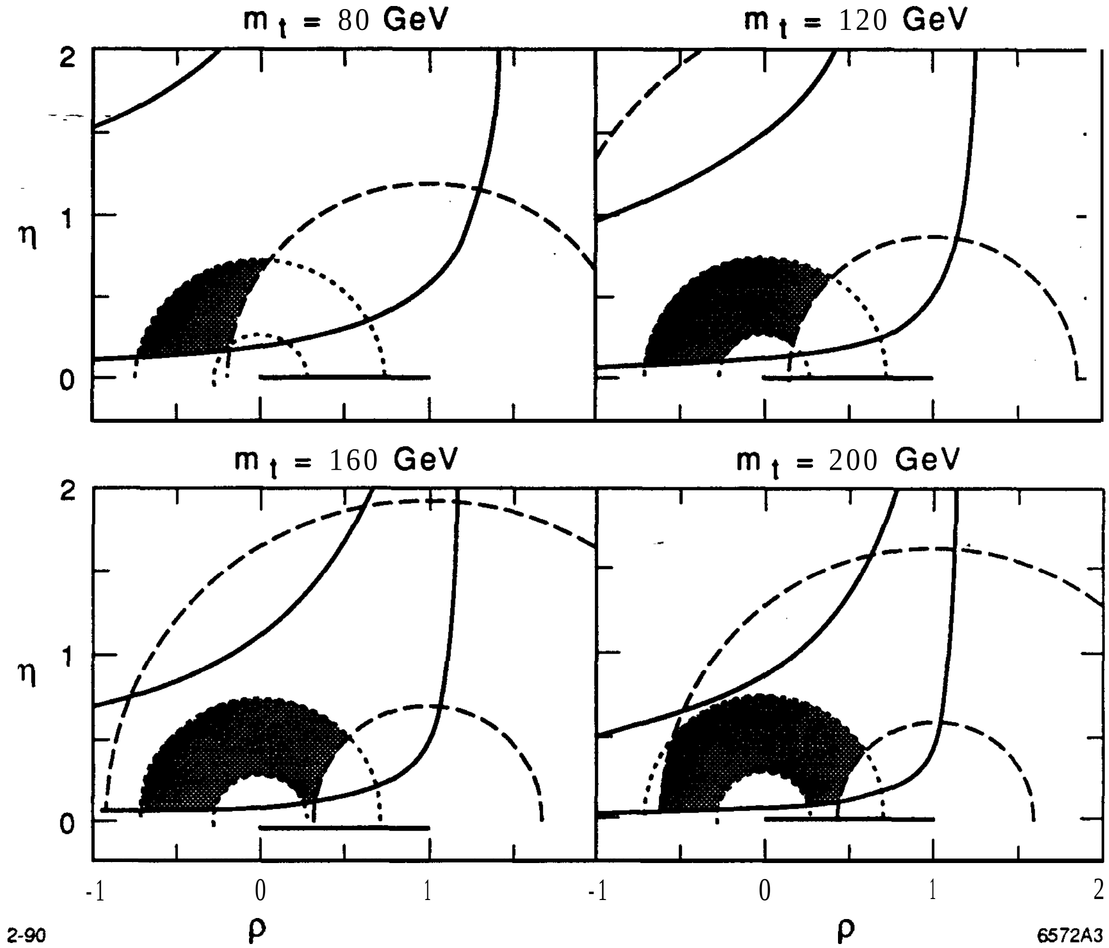

Fig. 3