SLAC-PUB-1426
(T)
May 1974

FINITE WIDTH EFFECTS IN RESONANCE DECAY NEAR THRESHOLD*

Eugene Golowich†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The decay of a resonance into a final state containing two particles, the sum of whose mean masses exceeds that of the parent particle, is investigated. Alternative methods for calculating the transition rate are compared. Two specific decays, $Y_0(1518) \rightarrow Y_1(1385)\pi$ and $A_2(1310) \rightarrow B(1237)\pi$, are studied numerically.

(Submitted to Phys. Rev.)

*Work supported by the U.S. Atomic Energy Commission.
† On sabbatical leave from the University of Massachusetts.

I. INTRODUCTION

Recently, separate measurements of the transition rate for $Y_0(1518) \to Y_1(1385)\pi$ were performed by groups from Berkeley[1] and the University of Massachusetts.[2] A noteworthy feature of this decay process is that the sum of the pion mass and the mean mass of $Y_1(1385)$ exceeds the mean mass of $Y_0(1518)$. Thus, the physical transition takes place only because of the finite resonance widths.

The purpose of this communication is to comment upon certain questions[3] which arose in the course of the analysis of this system due to its somewhat delicate kinematics. Let us phrase the situation as follows. Suppose we are given the probability amplitude for $Y_0(1518) \to Y_1(1385)\pi$ and wish to calculate the transition rate. Clearly, in the course of integrating over phase space, some averaging over the baryon mass is called for. However, there is more than one way to proceed. One may either fix the initial baryon mass at its central value and average over the mass of the final baryon or alternatively, average over the masses of both initial and final baryons. What is the relation between the rates calculated these two ways? Can the difference ever be significant? Another type of question which can occur concerns the nature of the averaging variable. Is it more "natural" to use mass or squared mass, in the event that the amplitude depends only upon the latter?

For any individual situation, one can, of course, use a computer to answer all the above questions numerically. However, an analytic treatment of the problem is more instructive in revealing the basic parameters occurring in the analysis, and in determining the way in which they interrelate to give the final result.

In the following, we shall define and then analyze a model appropriate for dealing with these questions. Two specific resonance decays, $Y_0(1518) \rightarrow Y_1(1385)\pi$ and $A_2(1310) \rightarrow B(1237)\pi$, will be studied numerically. Finally, we shall comment on theoretical aspects of these transitions.

II. THE MODEL

The physical situation under consideration here is that of an unstable particle of central mass $\overline{M}_R$, width Γ_R, decaying into a zero-width meson of mass μ and a second unstable particle of mean mass $\overline{M}$, width Γ. The mass of each unstable particle is described in terms of some distribution function ρ, which for definiteness, we shall take in the numerical part of our analysis as Lorentzian. For simplicity, we shall assume both unstable particles to have the same mass distribution function.[4] Thus we describe the mass spectrum of the parent and daughter resonances in terms of $\rho(M_R)$ and $\rho(M)$ respectively. The effect of this assumption on our numerical work is expected to be slight. Let $f(M_R, M, \mu)$ represent the transition rate for the decay; the parent and daughter masses are M_R and M, μ respectively, and we work in the rest frame of the parent particle. If the resonances were narrow and if $M_R >> M + \mu$, then the function $f(\overline{M}_R, \overline{M}, \mu)$ would accurately describe the transition rate. However, for the situation under investigation, we must instead consider a quantity like

$$\langle f(M_R) \rangle = \frac{\int_{\overline{M}-\Gamma}^{M_R-\mu} dM\, f(M_R, M, \mu)\ \rho(M)}{\int_{\overline{M}-\Gamma}^{M_R-\mu} dM\, \rho(M)} . \tag{1}$$

We have arbitrarily decided[4] to average all masses in this analysis over the range $\overline{M} - \Gamma \leq M \leq \overline{M} + \Gamma$. This explains the lower limit in the integrals of Eq. (1).

The upper limit is a consequence of the bound $M \leq M_R - \mu$ arising from powers of the decay momentum of final state particles which invariably appear in transition rates. At this point, we are ready to define two alternative ways of calculating the transition rate. Either we may simply fix the mass M_R at its mean value $\overline{M}_R$,

$$<f> \; = \; <f(\overline{M}_R)> \tag{2}$$

or we may average over the variable M_R,

$$<<f>> \; = \frac{\displaystyle\int_{\overline{M}_R-\Gamma_R}^{\overline{M}_R+\Gamma_R} dM_R \; \rho(M_R) \; <f(M_R)>}{\displaystyle\int_{\overline{M}_R-\Gamma_R}^{\overline{M}_R+\Gamma_R} dM_R \; \rho(M_R)} \; . \tag{3}$$

Our task is to relate the two definitions of transition rate, $<<f>>$ and $<f>$.

Let us begin by expanding $<f(M_R)>$ in powers of $M_R - \overline{M}_R$,

$$<f(M_R)> \; = \; <f> + <f>^{(1)}(M_R - \overline{M}_R) + \frac{1}{2} <f>^{(2)} (M_R - \overline{M}_R)^2 + \cdots \tag{4}$$

where

$$<f>^{(n)} \; = \; \frac{d^n}{dM_R^n} <f(M_R)> \Bigg|_{M_R = \overline{M}_R} \; . \tag{5}$$

At this point, it would be helpful to have a definite form for the mass distribution function ρ. We have chosen to work with

$$\rho(M_R) \; = \; \frac{1}{(M_R - \overline{M}_R)^2 + \frac{\Gamma_R^2}{4}} \; . \tag{6}$$

Upon inserting Eq. (4) into Eq. (3) and using Eq. (6), we obtain[5]

$$\langle\langle f \rangle\rangle = \langle f \rangle + \frac{(2 - \tan^{-1} 2)}{8 \tan^{-1} 2} \Gamma_R^2 \langle f \rangle^{(2)} + \cdots \quad . \tag{7}$$

The numerical factor in the second term of Eq. (7) is rather small, equaling about 0.1. Thus, given our assumptions, the difference between $\langle\langle f \rangle\rangle$ and $\langle f \rangle$ depends upon the function $\langle f \rangle^{(2)}$. We can obtain an expression for $\langle f \rangle^{(2)}$ from Eqs. (1), (5), and (6),

$$\langle f \rangle^{(2)} = \left[\frac{I^{(2)}}{N} - \frac{2}{\frac{\Gamma^2}{4} + \Delta^2} \cdot \frac{I^{(1)}}{N^2} + \frac{2}{\left(\frac{\Gamma^2}{4} + \Delta^2\right)^2} \cdot \left(\frac{1}{N} - \Delta\right) \cdot \frac{I}{N^2} \right]_{M_R = \overline{M}_R} \tag{8}$$

where

$$\Delta = \overline{M} + \mu - \overline{M}_R \quad , \tag{9}$$

$$N = \frac{2}{\Gamma} \cdot \left(\tan^{-1} 2 - \tan^{-1} \frac{2\Delta}{\Gamma}\right) \quad , \tag{10}$$

and

$$I(M_R) = \int_{\overline{M} - \Gamma}^{M_R - \mu} dM \, f(M_R, M, \mu) \, \rho(M) \quad . \tag{11}$$

The first and second derivatives of I with respect to M_R are denoted by $I^{(1)}$ and $I^{(2)}$.

It should be noted that the quantities which appear in our analysis can be divided into two distinct numerical classes, large ($\overline{M}_R$ and $\overline{M}$) and small (μ, Γ, and Γ_R). For the calculation of $f^{(2)}$, it turns out that the most convenient parameterization is in terms of one large parameter, $\overline{M}_R$, and the three small ones μ, Γ and Δ. This is evidenced to some extent in Eqs. (8) and (10).

III. EXAMPLES

In order to proceed further, we must adopt a particular form for the transition rate $f(M_R, M, \mu)$. This step is not without ambiguity.[6] For example, consider the $Y_0(1518) \to Y_1(1385)\pi$ decay. In principle this reaction can proceed via S-wave and D-wave amplitudes. Given the kinematical situation, we may neglect the D-wave contribution. One way to calculate the transition rate is to start with a local field theory interaction,

$$\mathscr{L}(x) = \sqrt{4\pi}\, g\, \overline{Y}_{1\mu}(x)\, Y_0^{\mu}(x)\, \pi(x) \quad . \tag{12}$$

For the moment, we shall ignore internal symmetry considerations and assume that the parent mass exceeds the sum of the decay particle masses. We then find for the parent decay width

$$\Gamma_R = g^2 \quad \frac{(E + M)\, q}{M_R} \tag{13}$$

where E, q, M are the energy, momentum, and mass of the decay baryon. Contrast Eq. (13) with the result obtained from the "barrier penetration factor" approach based on potential theory,

$$\Gamma_R = \frac{g^2\, M_0 q}{M_R} \tag{14}$$

where M_0 is a scale mass which allows g^2 to be dimensionless. The difference between Eqs. (13), (14) can be considerable when used to test the predictions of a symmetry scheme with reactions of widely varying kinematics.[6] Unfortunately, aside from pointing out the existence of this ambiguity in the choice of $f(M_R, M, \mu)$, we have no suggestion for resolving it. For definiteness, we shall consider the function

$$f(M_R, M, \mu) = \frac{q}{M_R} \; , \tag{15}$$

essentially the barrier penetration formula. This approach appears to generally be employed in phenomenological analyses.

It is important to explicitly display the mass dependence contained in Eq. (15),

$$f(M_R, M, \mu) = \left[(M_R-M-\mu)(M_R+M+\mu)(M_R-M+\mu)(M_R+M-\mu)\right]^{1/2}/2M_R^2 \tag{16}$$

This must now be inserted into Eq. (11) and the differentiations indicated in Eq. (8) performed. An approximation which makes use of the inequality $\overline{M}_R >> \mu, \Gamma, \Delta$ simplifies things considerably. First we make a change of variable in (11),

$$M \rightarrow z = -M - \mu + M_R \tag{17}$$

which facilitates differentiation of the integrals in Eq. (8), as well as setting up the desired approximation. Equation (11) becomes

$$I(M_R) = \frac{1}{2M_R^2} \int_0^{M_R-\overline{M}-\mu+\Gamma} dz \; \rho(M_R-z-\mu)\,(z(2\mu+z)(2M_R-z)(2M_R-2\mu-z))^{1/2}$$

$$\cong \frac{1}{M_R} \int_0^{M_R-\overline{M}-\mu+\Gamma} dz \quad \rho(M_R-z-\mu)\;(z(2\mu+z))^{1/2} \; . \tag{18}$$

The function I and its first two derivatives are to be evaluated at $M_R = \overline{M}_R$. Upon scaling all quantities in terms of Γ, and hereafter denoting $I(\overline{M}_R)$ as I_1, we obtain

$$I_1 = \frac{1}{\overline{M}_R} \int_0^{1-\frac{\Delta}{\Gamma}} dz \quad \frac{(z(z+\frac{2\mu}{\Gamma}))^{1/2}}{(z+\frac{\Delta}{\Gamma})^2+\frac{1}{4}} \; . \tag{19}$$

In calculating the derivatives $I^{(1)}$ and $I^{(2)}$, it is consistent with our approximation to keep only those terms of leading order in inverse powers of $\overline{M}_R$. This gives

$$I^{(1)}(\overline{M}_R) = \frac{1}{\overline{M}_R \Gamma} \left[\frac{4}{5} \left((1 - \frac{\Delta}{\Gamma})(1 + \frac{2\mu - \Delta}{\Gamma}) \right)^{1/2} + I_2 \right] , \tag{20}$$

$$I_2 = 2 \int_0^{1 - \frac{\Delta}{\Gamma}} dz \, \frac{(z + \frac{\Delta}{\Gamma})(z(z + \frac{2\mu}{\Gamma}))^{1/2}}{((z + \frac{\Delta}{\Gamma})^2 + \frac{1}{4})^2} , \tag{20a}$$

and

$$I^{(2)}(\overline{M}_R) = \frac{1}{\overline{M}_R \Gamma^2} \left[\frac{32}{25} (1 + \frac{2\mu}{\Gamma})^{1/2} + \frac{2}{5} \frac{2 + \frac{2\mu}{\Gamma}}{(1 + \frac{2\mu}{\Gamma})^{1/2}} - 2 I_3 + 8 I_4 \right] , \tag{21}$$

where

$$I_3 = \int_0^{1 - \frac{\Delta}{\Gamma}} dz \, \frac{(z(z + \frac{2\mu}{\Gamma}))^{1/2}}{\left((z + \frac{\Delta}{\Gamma})^2 + \frac{1}{4}\right)^2} , \tag{21a}$$

$$I_4 = \int_0^{1 - \frac{\Delta}{\Gamma}} dz \, \frac{(z + \frac{\Delta}{\Gamma})^2 (z(z + \frac{2\mu}{\Gamma}))^{1/2}}{\left((z + \frac{\Delta}{\Gamma})^2 + \frac{1}{4}\right)^3} . \tag{21b}$$

Our main reason for exhibiting the somewhat lengthy expressions (19)-(21b) is that it is now clear how to characterize a given kinematical situation, even when higher powers of the momentum occur. The integrals $I_1, \cdots, I_4$ as well as the other factors in Eqs. (20), (21) are seen to be functions only of the parameters Δ/Γ and μ/Γ. Let us approximately simulate the $Y_0(1520) \to Y_1(1385)\pi$ conditions by taking $\Delta/\Gamma = 0$, $\mu/\Gamma = 4$. It is found that[7]

$$\langle f \rangle^{(2)} = \frac{1.39}{\overline{M}_R \Gamma} ,$$

so Eq. (7) becomes

$$\langle\langle f \rangle\rangle = \langle f \rangle + .14 \frac{\Gamma_R^2}{\overline{M}_R \Gamma} + \cdots \quad . \tag{22}$$

An estimate of $\langle f \rangle$ comes from Eqs. (2), (10), and (19),

$$\langle f \rangle = 1.62 \frac{\Gamma}{\overline{M}_R} \quad . \tag{23}$$

Our final result is then

$$\langle\langle f \rangle\rangle = 1.62 \frac{\Gamma}{\overline{M}_R} \left[1 + .087 \left(\frac{\Gamma_R}{\Gamma} \right)^2 + \cdots \right] \quad . \tag{24}$$

Since Γ_R is less than half as large as Γ for the $Y_0(1518) \to Y_1(1385)\pi$ decay, we conclude that the difference between $\langle\langle f \rangle\rangle$ and $\langle f \rangle$ is insignificant for this example in the context of our approximations.

Another reason for presenting Eqs. (19)-(21b) is that with the entire calculation displayed, we can better comment upon the use of squared mass instead of linear mass as an averaging variable. The motivation for using M^2 stems from the fact that the transition rate (16) may be written as a function of M_R^2 and M^2. Thus, one could conceive of using a distribution function in the variable M^2, such as

$$\rho(M^2) = \left[(M^2 - \overline{M}^2)^2 + \overline{M}^2 \Gamma^2 \right]^{-1} \quad . \tag{25}$$

It is difficult to see how this could change our results qualitatively, although small quantitative effects are to be expected. In response to the suggestion that squared mass is a more natural averaging variable, we can point out that the upper limit, $M_R - \mu$, of Eq. (1) (and of most of the subsequent integrals) exhibits a linear dependence upon M_R. This can be readily obtained from Eq. (16), where for fixed M_R and μ, the lowest positive zero of the decay momentum as a function of M occurs for $M = M_R - \mu$. In this sense, the apparent dependence of $f(M_R, M, \mu)$ upon M_R^2 and M^2 is somewhat misleading, and it is our feeling that the linear mass is an entirely appropriate variable.

We shall conclude this Section with discussion of a second numerical example, the mesonic decay $A_2(1310) \to B(1237)\pi$. Experimental evidence for this transition is not on as firm a footing as the $Y_0(1518) \to Y_1(1385)\pi$ decay. Several groups have observed a significant $\omega\pi\pi$ decay mode of the A_2.[8] To the extent that a three body final state is expressible as a two body state in which one of the particles is itself a resonance, it is reasonable to expect at least part of the $\omega\pi\pi$ mode to arise from a $B(1237)\pi$ composite. Recall that the B meson decays almost entirely into $\omega\pi$. Moreover, although both $Y_0(1518) \to Y_1(1385)\pi$ and $A_2(1310) \to B(1237)\pi$ share the property that finite width effects make both decays possible, these two reactions are rather different in their kinematics, thus providing an instructive contrast.

Analogous to $Y_0(1518) \to Y_1(1385)\pi$, the $A_2 \to B\pi$ transition can proceed through two partial waves, in this case P-wave and F-wave. Again, it is safe to ignore the effect of the higher partial wave. The notation is carried through to this case as expected, M_R, Γ_R representing the $A_2(1310)$ meson and M, Γ the B(1237) meson. Although our choice of transition rate $f(M_R, M)$ is afflicted with the same kind of ambiguity as mentioned earlier, we shall work with

$$f(M_R, M, \mu) = \frac{q^3}{M_R^2} . \tag{26}$$

As before, we use Eq. (7) to relate $\langle\langle f \rangle\rangle$ to $\langle f \rangle$, so most of the work lies in computing $\langle f \rangle^{(2)}$. Equations (8)-(11) are still operative but there are modifications in Eqs. (18)-(21b). However, the appearance of Δ/Γ and μ/Γ as parameters does not change. The $A_2 \rightarrow B\pi$ kinematics corresponds approximately to $\Delta/\Gamma = \frac{1}{2}$ and $\mu/\Gamma = 1.0$. This leads to the numerical result

$$< f >^{(2)} = 0.84 \frac{\Gamma}{\overline{M}_R^2}$$

which together with

$$< f > = 0.41 \frac{\Gamma^3}{\overline{M}_R^2}$$

gives finally

$$<< f >> = 0.41 \frac{\Gamma^3}{\overline{M}_R^2} \left(1 + 0.21 \left(\frac{\Gamma}{\Gamma_R}\right)^2 + \cdots\right) . \qquad (27)$$

Since Γ and Γ_R are comparable in this case, the difference between $<< f >>$ and $< f >$ can be as large as 20%.

IV. CONCLUSION

In an attempt to comment upon some questions of procedure which arose in a recent analysis of the $Y_0(1518) \rightarrow Y_1(1385)\pi$ transition,[3] we have examined a simple model which we feel contains the essential ingredients of the process. Simplifying assumptions, such as regarding resonance shape, were made to keep the mathematical complexity to a minimum. Our main purpose was to establish a relation between the transition rates $<< f >>$ and $< f >$, the former corresponding to integration over both initial and final state baryon masses, the latter to integration over just the final state baryon mass, with the initial baryon mass being fixed at its central value. We found that $< f >$ is the leading term in an expansion[9] of $<< f >>$ in powers of Γ_R/Γ. Given our assumptions, the first correction to $<f>$ appeared to 2nd order. The parameters which turned out to be most appropriate to our calculation were $\overline{M}_R$, Γ, and the ratios Γ_R/Γ, μ/Γ, and Δ/Γ, where $\Delta = \overline{M} + \mu - \overline{M}_R$. A numerical study of the $Y_0(1518) \to Y_1(1385)\pi$ system showed that the difference in approach studied here between the analyses of Ref. 1 and Ref. 2 cannot account for the (roughly) factor of two difference found for the branching ratio. However, for the $A_2(1310) \to B(1237)\pi$ decay, the difference between $<<f>>$ and $<f>$ is significant, amounting to about 20% in our model. In our opinion, should the difference between $<<f>>$ and $<f>$ be appreciable for a given transition, it is the former, $<<f>>$, which is the more appropriate to employ.

The status of our theoretical understanding of the $J^P = \frac{3}{2}^-$ baryons is not entirely clear. Conventionally, SU(3) symmetry has been used to characterize the lowest lying $\frac{3}{2}^-$ baryons in terms of an octet and a singlet with mixing between $Y_0(1518)$ and $Y_0(1690)$, expressed in terms of an angle θ. Uncertainty in the $\frac{3}{2}^-$ Ξ^* baryon mass hinders an accurate determination of θ in terms of a mass-matrix analysis. A conservative estimate is that $|\theta| \lesssim 30^{\circ}$. Incidentally, the criterion $\cos 2\theta \leq 1$ can be used to obtain an inequality for the Ξ^* mass,

$$M(\Xi^*) \leq \frac{3}{2} M(Y_0(1690)) + \frac{1}{2} M(Y_1(1670)) - M(N^*(1520))$$

or

$$M(\Xi^*) \leq 3370 - M(N^*). \tag{28}$$

The current limits on the N^* mass, $1510 \leq M(N^*) \leq 1540$, place upper bounds on the Ξ^* mass of 1860 and 1830 MeV, respectively. D-wave decays of the $\frac{3}{2}^-$ baryons into 0^- $\frac{1}{2}^+$ meson-baryon final states have also been used to obtain an estimate of θ, yielding $\theta \cong 25^{\circ}$,[10] a value not inconsistent with the mass-matrix analysis. However, decays of the type $\frac{3}{2}^- \to \frac{3}{2}^+ 0^-$ have caused some consternation among theorists because a much larger mixing angle is suggested. Perhaps the clearest example of this is seen in the decay of $Y_0(1690)$. Roughly speaking, a small mixing angle implies that $\Gamma[Y_0(1690) \rightarrow Y_1(1385)\pi]$ should be comparable to $\Gamma[N^*(1520) \rightarrow \Delta(1236)\pi]$. Instead, the former appears to be subtantially less than the latter. There is, however, considerable room for improvement in the quality of the data.[11]

Regarding theoretical efforts to explain this situation, the effort of Faiman and Plane[12] appears worthy of mention. Noting that the $SU(6)_W$ classification of $Y_0(1518)$ and $Y_0(1690)$ includes a third $I = Y = 0$ $\frac{3}{2}^-$ baryon, these authors expand the space in which mixing occurs from two to three dimensions. They find that the SU(3) wave function of $Y_0(1518)$ is practically undisturbed whereas that of $Y_0(1690)$ is modified in such a way as to suppress the $Y_1(1385)\pi$ decay mode. This seemingly successful resolution of the problem should be viewed with some caution, however. Part of the input to their mixing matrix involves a decay mode of $Y_0(1690)$, whose properties still appear in a state of flux.[10] Moreover, the only SU(3) breaking allowed in Ref. 11 is the effect of mixing. In view of the SU(3) breaking observed both in particle masses and decays of unmixed hadronic states,[13] this approach seems highly optimistic. Finally, the third $\frac{3}{2}^-$ Y_0 baryon has yet to be observed experimentally. It would be worthwhile to search for the huge decay width predicted in Ref. 11 for the $Y_1(1385)\pi$ mode of this as yet unobserved baryon.

Our final comments relate to the conjecture made in the previous Section that at least part of the $A_2 \rightarrow \omega\pi\pi$ mode can be attributed to the decay chain $A_2 \rightarrow B\pi \rightarrow \omega\pi\pi$. We can use existing information on branching ratios to place an upper bound on the coupling strength associated with the $A_2 \rightarrow B\pi$ system.

We define a dimensionless coupling parameter G,

$$\mathscr{L}(x) = G\,\epsilon_{abc}\,A^{\mu\nu}_{2a}(x)\,B_{\mu b}(x)\,\partial_\nu\,\pi_c(x) \tag{29}$$

whose relation to the decay width is

$$\Gamma(A_2 \to B\pi) = \frac{G^2}{12\pi}\,\frac{q^3}{M_A^2}\ . \tag{30}$$

From Ref. 8 and 10, we have $\Gamma(A_2 \to \omega\pi\pi)/\Gamma(A_2 \to \text{all}) \cong 0.1$, with $\Gamma(A_2 \to \text{all}) \cong 100.$ MeV. This implies the upper bound $\Gamma(A_2 \to B\pi) \lesssim 10$ MeV. Inserting the phase space estimate Eq. (27) into Eq. (30), we obtain $G^2/4\pi \lesssim 70$. We do not suggest that the actual $A_2 \to B\pi$ coupling is this large, but rather conclude that even a small part of the measured $A_2 \to \omega\pi\pi$ rate can imply an appreciable $A_2 \to B\pi$ coupling in view of the limited phase space.

ACKNOWLEDGMENTS

The author expresses his appreciation to Professor Janice B. Shafer for informative discussions, and to the Cornell University Physics Department for their hospitality during the initial stages of this work.

REFERENCES

1. T. S. Mast et al., Phys. Rev. D7, 5 (1973).
2. S. Chan et al., Phys. Rev. Letters 28, 256 (1972).
3. Professor Janice B. Shafer, private communication.
4. Extension to a more general situation is straightforward, although our results are not expected to change qualitatively.
5. The nonappearance of a first order term in this expansion is solely attributable to the symmetric form of ρ exhibited in Eq. (6) for the parent resonance.
6. This problem is discussed by E. Golowich, Phys. Rev. 177, 2295 (1969).
7. The author thanks Dr. M.-S. Chen for his assistance in the numerical evaluation of certain integrals.
8. For example, U. Karshon et al., Phys. Rev. Letters 32, 852 (1974), J. Diaz et al., Phys. Rev. Letters 32, 260 (1974), and references cited therein.
9. One might be concerned about the rate of convergence of our expansion for the case $\Gamma_R > \Gamma$. This does not seem to be a problem, not only because of the fact that $\Gamma_R \leq \Gamma$ in the applications which we considered, but also because the coefficients in the expansion decrease quite rapidly in higher orders, e.g. in 4th order, we have $0.004\ \Gamma_R^4 f^{(4)}$ contributing to $\langle\langle f \rangle\rangle$.
10. Our phase is opposite to that given in Ref. 1.
11. Particle Data Group, Rev. Mod. Phys. Suppl. 45, S1 (1973).
12. D. Faiman and D. Plane, Phys. Letters 39B, 358 (1972).
13. E. Golowich and V. Kapila, Phys. Rev. D8, 2180 (1973).