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Dear Dr. Nauenberg :

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ON THE POSSIBLE EXISTENCE OF QUARK STARS

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An important feature of the colored gluon theory of quark interactions (quantum chromodynamics) is that for low momentum transfers, i.e., large distances, the interaction between quarks becomes large. It is speculated that this "infrared slavery" is responsible for the permanent confinement of quarks.[13] Conversely, at high momentum transfers, i.e. at short distances, the interaction between quarks becomes weak.[11,15]

One interesting consequence of this asymptotic freedom for quarks is that at high densities where the long-range forces between quarks are weakened, matter should behave like a gas of free quarks.[8] Of course, one knows experimentally that only baryons exist up to matter densities as high as those existing in atomic nuclei. Thus, the transition to the state where free quarks exist must take place at densities higher than those occurring in nuclei. The question of where the baryon-quark phase transition takes place is of considerable interest to astrophysics because of the possibility that this transition might take place inside neutron stars.[3,4] It might be the case, for example, that the central regions of the most massive neutron stars consist primarily of quarks. Another possibility is that free quarks do not occur in the interiors of neutron stars but are the principal constituents of a new regime of stable cold stars.[12] These questions have been previously investigated[1,6] using the MIT bag model theory of quark matter with the result that neither of the two possibilities just mentioned appeared likely to be realized in nature. In this letter we would like to report on the results of a study of these questions based on quantum chromodynamics (Q.C.D.).

In order to investigate the question of whether a star can exist with an interior consisting primarily of free quarks we need to have a theory for quark matter. If we neglect quark masses (which is a good approximation except when one is near the strangeness or charm thresholds) then the energy density ϵ of quark matter will be given by an expression of the form

$$\epsilon = An^{4/3} \tag{1}$$

where n is the conserved baryon number density and A depends only logarithmically on density. To second order in the quark-gluon coupling constant g one has

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$$A = \frac{9}{4}\left(\frac{3\pi^2}{K}\right)^{1/2}\left(1 + \frac{8\alpha_c}{3\pi}\right) \tag{2}$$

where K is the number of quark flavors contributing and $\alpha_c = g^2/16\pi$. As long as α_c is small, Equation 2 should be an adequate approximation for A. The value of α_c will depend on the average momentum transfer in quark-quark collisions.

Renormalization group arguments suggest that for small α_c one can write:[10]

$$\alpha_c = \frac{\pi}{\left(22 - \frac{4}{3}K\right)} \frac{1}{\ln \mu_F/\lambda_F} \tag{3}$$

where μ_F is the Gibb's energy per quark and λ_F is a constant. For calculational purposes μ_F can be replaced by the quark Fermi momentum k_F since the two quantities are nearly the same when α_c is small. The value of the scale parameter λ_F can in principle be determined from high-energy lepton-nucleon scattering and e^+e^- annihilation experiments, although in practice determination of an exact value for λ_F has not yet been possible. Analysis of hadron production between 2 and 8 GeV in e^+e^- annihilation[9] suggests that λ_F should lie in the range 0.2 to 1.0 Gev. Recent analysis of scale invariance in high-energy electroproduction[16] suggests that λ_F is in the range 0.3 to 0.5 GeV.

From Equations 1–3 one finds that the pressure P and Gibb's energy per unit baryon number μ in a cold dense three-flavor ($K = 3$) quark gas are

$$P = \frac{3\lambda_F^4}{4\pi^2}\left[1 + \frac{4}{27 \ln x}\left(1 - \frac{1}{\ln x}\right)\right]x^4 \tag{4a}$$

$$\mu = 3\lambda_F\left[1 + \frac{4}{27 \ln x}\left(1 - \frac{1}{4 \ln x}\right)\right]x \tag{4b}$$

where $x = k_F/\lambda_F$. We note that these equations imply that as a function of ϵ/λ_F^4, both P/λ_F^4 and μ/λ_F are independent of λ_F. Equation 4a differs from the usual scale-invariant equation of state for massless particles $P = \frac{1}{3}\epsilon$ because renormalization effects introduced a scale λ_F into the theory, even for zero quark masses.

Given an equation of state $P(n)$ for quark matter, one can determine the quark densities and pressures where quark stars are stable against gravitational collapse from the condition $\gamma > \gamma_c$, where $\gamma = d \log P/d \log n$ is the adiabatic index for quark matter and γ_c is the critical adiabatic index for a cold star in general relativity.[5] From Equation 4a one finds that

$$\gamma = \frac{4}{3} + \frac{(2 - \ln x)}{3 \ln x\left[\frac{27}{4}\ln^2 x + \ln x + 1\right]} \tag{5}$$

Using the fact that γ_c is a universal function of P/ϵ where ϵ is the mean energy density in a quark star,[14] one finds from Equation 5 and the condition $\gamma > \gamma_c$ that quark stars become gravitationally unstable when P/ϵ reaches the critical value:

$$(P/\epsilon)_c = .174 \tag{6}$$

corresponding to $\gamma_c = 2.27$. The values obtained from the MIT bag model are $(P/\epsilon)_c = .173$ and $\gamma_c = 2.26$ (Chapline and Nauenberg[6]), the close agreement with Q.C.D. values being due to the fact that the quark matter equation of state (4a) is very similar in shape for $P > 0$ to the bag model equation of state.[7] The critical values of ϵ, P, and μ are:

$$\epsilon_c \equiv 2.09\,\lambda_F^4 \tag{7a}$$

$$P_c = 0.363\,\lambda_F^4 \tag{7b}$$

$$\mu_c = 5.61\,\lambda_F^4 \tag{7c}$$

From Equations 6 and 7a one finds[14] that the upper limit for the mass of a quark star is

$$M_{max} \leq \frac{0.10}{\lambda_F^2}\,M_\odot \tag{8}$$

and the radius of a maximum mass quark star is:

$$R \simeq \frac{0.47}{\lambda_F^2}\ \text{km}. \tag{9}$$

when λ_F is in units of GeV.

In order for quark stars to exist, it is clear that the maximum pressure P_c for quark stars must exceed the transition pressure P_T for the baryon-quark phase transition. The transition pressure P_T will depend on both the scale parameter λ_F and the equation of state for baryonic matter. Since there is at the present time considerable uncertainty concerning the equation of state of baryonic matter at supernuclear densities, one must consider a range of possible baryon matter equations of state. Bethe and Johnson[2] have obtained equations of state for dense baryonic matter where the matter energy density has the form

$$\epsilon = mn + an^\nu \tag{10}$$

where a and ν are constants and m is the nucleon mass. The pressure derived from Equation 8 is

$$P = (\nu - 1)an^\nu \tag{11}$$

so that the adiabatic index $\gamma = \nu$ is a constant for these equations of state. The Gibb's energy per baryon will be given by

$$\mu = m + \nu a^{1/\nu}\left(\frac{P}{\nu - 1}\right)^{(\nu-1)/\nu} \tag{12}$$

The baryon-quark phase transition will occur when the value of μ for a quark gas (Equation 4b) falls below the baryonic matter μ as calculated, for example, from Equation 12. This is illustrated in FIGURE 1, where we show the crossing of the baryonic matter μ vs P curves for $\nu = 2.33$ and several values of a with the quark matter μ vs P curve for $\lambda_F = 446$ MeV. As one compresses matter one proceeds along a baryonic matter curve until one reaches the quark matter curve and thence to the right along the quark matter curve as shown. Whether a stable cold star can exist after one reaches the quark matter curve depends on whether the value of P at the crossing point is greater than or less than the critical value of P_c for a quark star, Equation 7b. For $\lambda_F = 446$ MeV one has $P_c = 3 \times 10^{36}$ dynes cm^{-2}. Thus, for $a = 3.03$ in units where the energy ϵ, Equation 10, is in 10^{35} erg/cm^3 and n is in fm^{-3} (model VH of Bethe-Johnson), the value of P at the crossing of the baryonic and quark matter curves is higher than P_c and thus no stable quark star can be formed. For $a = 7$, on the other hand, the value of the transition pressure is smaller than P_c and thus we see that a range of stable stars can be formed after one reaches the quark matter curve. For a certain value of the parameter a (namely, $a = 4$ for the case illustrated) the crossing point of the baryonic and quark matter μ vs P curves will occur exactly at the critical point (μ_c, P_c).

In general, for any given λ_F and ν there is a value for the parameter a, $a(\lambda_F, \nu)$, such that the transition pressure $P_T = P_c$. The function $a(\lambda_F, \nu)$ can be easily computed setting $\mu = \mu_c$ and $P = P_c$ in Equation 12 and using equations 7b and 7c. The resultant value for $\nu = 2.33$ are shown in FIGURE 2. As was illustrated in FIGURE 1, for any given value of λ_F stable quark stars can exist for $a > a(\lambda_F, \nu)$ and conversely will not exist for $a < a(\lambda_F, \nu)$. Thus, the $a(\lambda_F, \nu)$ curve shown in FIGURE 2 divides the a, λ_F space into two regions: one where stable quark stars can exist and one where they cannot. The curves for values of ν not too different from 2.33 are qualitatively similar to that shown in FIGURE 2. Actually, the causality condition $dp/d\epsilon \leq 1$ will be violated at very high pressures if $\nu > 2$, and thus the $a(\lambda_F, \nu)$ curve shown in FIGURE 2 cannot be valid for $\lambda_F \gtrsim 0.5$ GeV. At high λ_F the boundary of the region where quark star exist will approach the $a(\lambda_F, \nu)$

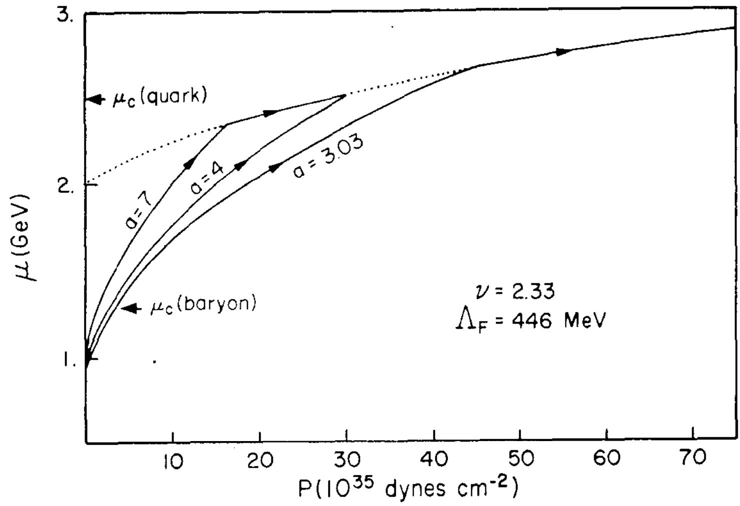

FIGURE 1. Curves labelled $a = 3.03$, 4 and 7 (in units where ϵ is in 10^{35} ergs/cc and n is in fm^{-3}) give μ vs P for baryonic matter, Equation 12, in the cases $\nu = 2.33$ up to the intersection with the dotted curve, which gives μ vs P for quark matter, Equations 4a and 4b, for $\lambda_F = 446$ MeV. The arrows indicate the location of the critical values of the Gibb's energy for quark stars and for neutron stars for these equations of state.

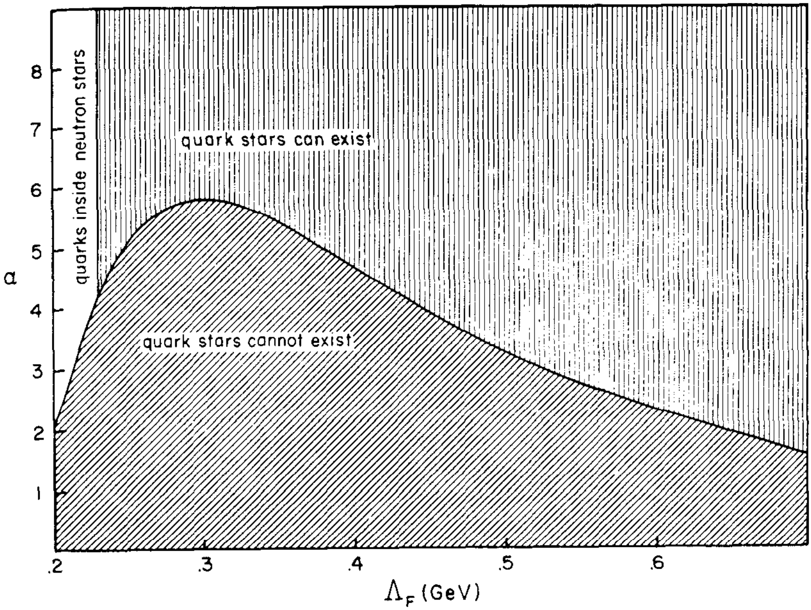

FIGURE 2. Curve $a(\lambda_F, \nu)$ for $\nu = 2.33$, which bounds the region $a > a(\lambda_F, \nu)$ where quark stars can exist. For $\lambda_F < 0.23$ Gev and $a > a(\lambda_F, \nu)$ quark matter can occur inside neutron stars.

curve for $\nu = 2$ which gives values for $a(\lambda_F, \nu)$ about a factor $1\frac{1}{2}$ to 2 times higher than the values shown for $\lambda_F > 0.5$ GeV.

In FIGURE 1 the baryon-quark phase transition pressures were much higher than the critical pressure for a neutron star. Thus, quark stars could only be formed for the ν and λ_F values used in FIGURE 1 if matter were compressed to higher densities than exist in neutron stars; i.e., only in the collapse of an object whose mass exceeded the maximum mass for a neutron star. On the other hand, if λ_F is smaller than a certain value ($\lambda_F < 0.226$ GeV for $\nu = 2.33$) then the critical value μ_c for a quark star, Equation 7b, will be smaller than the critical value of μ for a neutron star. For these values of λ_F it is possible to have quarks inside neutron stars, and one could have a smooth transition from neutron stars to quark stars.

Although it certainly lies within the range of present-day uncertainties in λ_F and the equation of state of dense baryonic matter to have a smooth transition between neutron stars and quark stars, this does not appear likely because for reasonable values of $\nu (\nu \lesssim 3)$ the values of λ_F needed ($\lambda_F \lesssim 0.25$ GeV) are probably unphysical. One can argue[7] that λ_F should be greater than 0.25 GeV, since the Gibb's energy μ for quark matter at zero pressure should be greater than the average of the N and Δ baryon masses. In addition, as mentioned above, high-energy experiments seem to suggest somewhat larger values for λ_F. However, if the equation of state in the baryonic phase of matter is stiff enough, then Q.C.D. and general relativity imply that a new regime of cold stars will exist with densities somewhat higher than those in neutron stars. According to Equation 8, for $\lambda_F > 0.25$ GeV these stars would have masses $\lesssim 1.6\ M_\odot$.

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