

BACKGROUND INDEPENDENCE OF STRING FIELD THEORY

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ABSTRACT

We formulate closed string field theories in background described by arbitrary two dimensional conformally invariant field theories, study the classical equations of motion in these theories, and show that for every solution of these equations of motion, we can construct a nilpotent BRST charge. We also identify specific solutions of the string field theory equations of motion which correspond to specific conformal field theories.

In this talk I shall discuss the relationship between string field theory and two dimensional conformally invariant field theories. The motivation for the formulation of string field theory is twofold. First of all, one hopes that string field theory may help us understand non-perturbative effects in string theory. Secondly, one expects to get a proper understanding of the classical configuration space of string fields from string field theory. It is the second point that I shall concentrate on in this talk. The work reported here is based on refs.[1][2][3][4].

We begin with the formulation of closed string field theory around a background which corresponds to an arbitrary two dimensional conformal field theory with $c = 26$. The states of first quantized string theory are given by the states in the Hilbert space $\mathcal{H}$ of combined matter-ghost conformal field theory, with the matter CFT having central charge 26, and the ghost CFT having central charge -26 . The ghost number of a state is defined as $+1$ for the ghost fields c and $\bar{c}$, -1 for the fields b and $\bar{b}$, and 0 for the vacuum state $|0\rangle$. This uniquely fixes the ghost number of all the states. We define $|\Phi_{n,r}\rangle$ to be a basis of states with ghost number n . Then, according to ref.[5], there is a one to one correspondence between any state $|\Phi\rangle$ and a local field $\Phi(z, \bar{z})$, such that, $|\Phi\rangle = \Phi(0)|0\rangle$. We also define the state $\langle\Phi| = \langle 0|I \circ \Phi(0)$ for any field Φ where $I \circ \Phi$ is the conformal transform of the field Φ under the $SL(2, \mathbb{R})$ map $I(z) = -1/z$. Also, we define $c_0^\pm = (c_0 \pm \bar{c}_0)/\sqrt{2}$ and $b_0^\pm = (b_0 \pm \bar{b}_0)/\sqrt{2}$, $P = \delta_{L_0, \bar{L}_0}$. Here c_0 etc. denote the zero modes

of the corresponding ghost fields, and $L_n, \bar{L}_n$ denote the Virasoro generators.

For two dimensional matter ghost system, we can define a BRST charge Q_B in the combined matter-ghost CFT with the property that, $(Q_B)^2 = 0$. On shell states (physical states) in string theory are characterized by the fact that they are annihilated by the BRST operator. Furthermore, if two physical states differ by a state of the form $Q_B|s\rangle$ for some state $|s\rangle$, then they are considered to be gauge equivalent. In order to formulate string field theory, however, we need to enlarge our space of states so as to include 'off-shell' string states. It turns out that for closed strings, the set of off-shell states is identified with the set of states with ghost number 2, which are annihilated by b_0^- and $L_0 - \bar{L}_0$. If $|\Phi_{3,r}\rangle$ denote a basis of states of ghost number 3, annihilated by c_0^- and $L_0 - \bar{L}_0$, then a general off-shell closed string state may be taken to be,

$$b_0^- P \sum_r \psi_r |\Phi_{3,r}\rangle \equiv b_0^- P |\Psi\rangle \quad (1)$$

ψ_r may be regarded as wave-function of the first quantized string. Thus they become the dynamical variables of the second quantized string theory. We now need to write down an action $S(\{\psi_r\}) \equiv S(|\Psi\rangle)$ which reproduces the correct S-matrix of the string theory as given by Polyakov formulation.

Gauge invariant closed string field theory action in flat space-time was written down in ref.[6] based on non-polynomial interactions [7]. We

now discuss the corresponding construction for string theory in arbitrary background. It turns out that in order to write down the string field theory action, we need a map $[[A_1 \dots A_{N-1}]]$ from the $(N-1)$ fold tensor product of $\mathcal{H}$ to $\mathcal{H}$ satisfying the conditions that $[A_1 \dots A_N]$ is annihilated by b_0^- and L_0^- and,

$$\begin{aligned} & Q_B[A_1 \dots A_{N-1}] \\ & - \sum_{i=1}^{N-1} (-1)^{\sum_{j=1}^{i-1} (\pi_j+1)} [A_1 \dots Q_B A_i \dots A_{N-1}] \\ & + \sum_{\substack{m, n \geq 3 \\ m+n=N+2}} \sum_{\{j_k\}} \sum_{\{i_k\}} \\ & [A_{j_1} \dots A_{j_{m-2}} c_0^- [A_{i_1} \dots A_{i_{n-1}}]] (-1)^{\sigma(\{i_k, j_k\})} \\ & = 0 \end{aligned} \quad (2)$$

where $(-1)^{\sigma(\{i_k, j_k\})}$ takes into account the extra $-$ sign that appears due to the rearrangement of the terms inside the correlator. Finally, we demand that $[A_1 \dots A_N]$ depend on A_i only through the combination $b_0^- P|A_i\rangle$ and that it picks up a factor of $(-1)^{(\pi_i+1)(\pi_{i+1}+1)}$ under the interchange of A_i and A_{i+1} where π_i is the ghost number carried by the operator A_i . With these axioms, the action,

$$S(\Psi) = \frac{1}{2} \langle \Psi | Q_B b_0^- P | \Psi \rangle + \sum_{N=3}^{\infty} \frac{g^N}{N!} \{ \Psi^N \} \quad (3)$$

can be shown to be invariant under the gauge transformation:

$$\delta(b_0^- P | \Psi) = Q_B b_0^- P | \Lambda \rangle + \sum_{N=3}^{\infty} \frac{g^{N-2}}{(N-2)!} [\Psi^{N-2} \Lambda] \quad (4)$$

where,

$$\{ \Psi^N \} = \langle \Psi | [\Psi^{N-1}] \rangle \quad (5)$$

where $|\Lambda\rangle$ is an arbitrary state of ghost number 2, annihilated by c_0^- and $L_0 - \bar{L}_0$. Thus the problem is to find an expression for $[A_1 \dots A_{N-1}]$ satisfying the above axioms. It turns out that it is possible to write down explicit expressions for $[A_1 \dots A_{N-1}]$ satisfying the above axioms, in terms of correlation functions in the conformal field theory. Since these expressions look quite complicated, I shall not write them down here (see, for example, ref.[1]). Also, the tree level S-matrix elements calculated from this action agree with the answer given by Polyakov prescription.

We now ask the following question. Do the solutions of the classical equations of motion derived from the action (3) correspond to other two dimensional conformally invariant field theories? To answer this question we start from some arbitrary solution Ψ_{cl} of the classical equations of motion, and define shifted fields $\tilde{\Psi}$ through the equation $\Psi = \Psi_{cl} + \tilde{\Psi}$. It turns out that in terms of $\tilde{\Psi}$ we may express the action $S(\Psi)$ as,

$$\begin{aligned} S(\Psi) &= S(\Psi_{cl}) + \frac{1}{2} \langle \tilde{\Psi} | \hat{Q}_B b_0^- P | \tilde{\Psi} \rangle \\ &+ \sum_{N=3}^{\infty} \frac{g^{N-2}}{N!} \langle \tilde{\Psi} | [\tilde{\Psi}^{N-1}]' \rangle \\ &\equiv S(\Psi_{cl}) + \hat{S}(\tilde{\Psi}) \end{aligned} \quad (6)$$

where $\hat{Q}_B$ is a new operator, and $[A_1 \dots A_{N-1}]'$ is a new map, defined as,

$$\hat{Q}_B b_0^- P | A \rangle = Q_B b_0^- P | A \rangle + \sum_{N=3}^{\infty} \frac{g^{N-2}}{(N-2)!} [\Psi_{cl}^{N-2} A] \quad (7)$$

$$\begin{aligned} [A_1 \dots A_{N-1}]' &= [A_1 \dots A_{N-1}] \\ &+ \sum_{M=3}^{\infty} \frac{g^{M-2}}{(M-2)!} [\Psi_{cl}^{M-2} A_1 \dots A_{N-1}] \end{aligned} \quad (8)$$

In terms of these new operations, the gauge transformation takes the same form as eq.(4) with Ψ replaced by $\tilde{\Psi}$, Q_B replaced by $\hat{Q}_B$, and $[]$ replaced by $[]'$. One can show that if Ψ_{cl} satisfies the classical equations of motion, then $\hat{Q}_B$ is a nilpotent operator. Furthermore, the operator $\hat{Q}_B$ and $[A_1 \dots A_{N-1}]'$ satisfy identities exactly similar to those satisfied by Q_B and $[A_1 \dots A_{N-1}]$, with Q_B replaced by $\hat{Q}_B$ and $[A_1 \dots A_{N-1}]$ replaced by $[A_1 \dots A_{N-1}]'$. This shows that the shifted action looks very much like the original action formulated in a new conformal field theory background, with $\hat{Q}_B$ being the BRST charge of this new conformal field theory. Similar results for open string theory in flat space-time was established by Witten [8].

These results strongly indicate that there is a deep connection between the solutions of classical equations of motion in both, open and closed string field theories, and conformally invariant field theories in two dimensions. A more interesting question would be, given two 'nearby' conformal field theories with $c = 26$, and the corresponding string field theories, can we explicitly

find a classical solution of the equations of motion of the string field theory formulated around one of the conformal field theories, which represents the other conformal field theory? We shall address this question for two different cases. First let us consider the case where the conformal field theory associated with the matter sector has an exactly marginal operator φ of dimension $(1,1)$. Then it is well known that we can define a nearby conformal field theory by perturbing the original conformal field theory by a term of the form $-\lambda d^2 z \varphi(z, \bar{z})$, where λ is a small parameter. The new two dimensional field theory CFT', defined this way, can be shown to satisfy the axioms of conformal field theory to first order in λ , and consequently we can define the Virasoro generators for this theory and the nilpotent BRST charge $\tilde{Q}_B$ in the combined matter-ghost system. The question we want to raise now is, is there a classical solution of the string field theory equations of motion which represents this theory? It turns out that there is, indeed, such a solution, given by,

$$\Psi_{cl} = \frac{\sqrt{2}\lambda}{g} c_0^- c(0) \bar{c}(0) \varphi(0) |0\rangle \quad (9)$$

This is shown in two stages. First we construct the charge $\hat{Q}_B$ corresponding to this classical solution according to eq.(7). It is then shown that $\hat{Q}_B$ and $\tilde{Q}_B$ are identical after a similarity transformation, i.e. there is an operator S such that $\hat{Q}_B = S \tilde{Q}_B S^{-1}$ [2]. Secondly, one can calculate the on-shell S-matrix elements in the string theory based on the conformal field theory CFT', and compare them with those calculated from the action $\hat{S}(\tilde{\Psi})$ defined in eq.(6). It is shown that these results are identical for arbitrary n -point amplitude involving tachyonic external states, and also three point amplitude involving arbitrary external states [3]. This is a strong indication that these two theories are indeed equivalent.

The next example we consider is the case when the matter conformal field theory contains a nearly marginal operator φ of dimension $(1-h, 1-h)$, where h is a small number. In this case, if we construct a new two dimensional field theory by perturbing the original conformal field theory by an operator of the form $-\lambda \int \varphi(z, \bar{z})$, then it can be shown that for $\lambda = 2h/C$ where C is the coefficient of φ in the operator product

of φ with φ , we get a new conformal field theory CFT'' [9]. It turns out that it is indeed possible to construct a solution Ψ_{cl} explicitly which satisfies string field theory equations of motion to order λ^2 and corresponds to this new conformal field theory [4]. The solution is given by,

$$\begin{aligned} |\Psi_{cl}\rangle = & \frac{\sqrt{2}\lambda}{g} c_0^- \left[\left(1 + \frac{1}{2}\lambda K\right) \varphi(0) \right. \\ & \left. - \frac{1}{2}\lambda \sum_{\phi \neq \varphi} \frac{C_{\phi, \varphi \varphi}}{h_\phi - 1} \right. \\ & \left. |\epsilon|^{2h_\phi - 2} \phi_\tau(0) \right] c(0) \bar{c}(0) |0\rangle \end{aligned} \quad (10)$$

where ϵ is a small number and K is a known operator dependent on ϵ such that the net ϵ dependence is absent from the right hand side of eq.(10).

To summarize, what we have shown here is that different conformal field theories which are related to each other through perturbation by marginal or nearly marginal operators may be considered as different classical solutions of the same underlying string field theory. Hopefully this result can be extended in the future to cases where the two conformal field theories are not necessarily nearby.

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