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Theory perspective on rare $B_{d,s} \to \bar{\ell}\ell$ and $B \to K^{(*)}\bar{\ell}\ell$

CHRISTOPH BOBETH

Technische Universität München, Institute of Advanced Study, Garching, Germany

bobeth@ph.tum.de

Abstract. The current status is reviewed for theoretical predictions of rare B meson decays proceeding via $b \to s\bar{\ell}\ell$ transitions. The leptonic decays $B_{d,s} \to \bar{\ell}\ell$ are under excellent theoretical control, whereas for semi-leptonic decays $B \to K^{(*)}\bar{\ell}\ell$ hadronic contributions need to be further scrutinised in order to test the standard model and tighten constraints on nonstandard effects.

INTRODUCTION

In the course of the last decade the field of b-quark physics advanced hugely in the exploration of rare decays with measurements by Belle I and Babar at the B-factories, CDF at the Tevatron and recently with LHCb, CMS and ATLAS from Run I of the LHC (2011-2012). Especially LHCb provides first precise measurements of decays with branching fractions in the range of $O(10^{-6})$, such as flavour-changing neutral current (FCNC) decays mediated at the parton level by $b \to s\bar{\ell}\ell$: $B \to K^{(*)}\bar{\mu}\mu$ [1, 2, 3, 4, 5, 6], $B_s \to \phi\bar{\mu}\mu$ [7] and $\Lambda_b \to \Lambda\bar{\mu}\mu$ [8]. As an example of the reach, the very rare leptonic decay $B_s \to \bar{\mu}\mu$ presents the current edge of the advances, discovered by LHCb and CMS with a branching fraction of $O(10^{-9})$ [9]. For the first time data sets are sufficiently large in order to explore the wealth of phenomenological proposals. In principle, the unprecedented experimental precision enables us to perform novel tests of the standard model (SM) of particle physics at an advanced quantitative level as well as to strengthen constraints on its extensions, provided theoretical predictions are under control.

The control of theoretical predictions of $b \to s\bar{\ell}\ell$ decays depends usually on our capability to control the effects of the strong interaction and varies depending on the final state. In this respect, decays with only leptons, such as the rare leptonic decay $B_s \to \bar{\mu}\mu$, can be predicted most precisely, whereas decays with three- or four-body final states, as for example $B \to K\bar{\ell}\ell$ and $B \to K^*(\to K\pi)\bar{\ell}\ell$, are on less solid grounds. On the other hand, the latter decays are phenomenologically more interesting as they provide more observables in angular distributions of the final state.

The first experimental results from LHCb brought about also some first $(2-3)\,\sigma$ deviations from the SM expectations, which gave rise to many interpretations within extensions of the SM, but demand also for a critical assessment of theoretical uncertainties. In view of the experimental program for the next decade with the current Run 2 of LHCb and the future Run 3, as well as the run of Belle II, which will continue to increase the precision of experimental results, it will be important to revise the applied theoretical methods. Further, an improvement seems mandatory to test for tinier effects beyond the SM and to exploit the full potential of FCNC $b \to s\bar{\ell}\ell$ decays, such as

- test the SM and its inherent quark-mixing mechanism at the loop-level,
- constraints on effective right-handed, (pseudo-)scalar and tensorial $|\Delta B| = 1$ couplings $[\bar{s}\,\Gamma_{sb}\,b][\bar{\ell}\,\Gamma_{\ell\ell}\,\ell]$,
- search for non-standard CP-violation in $b \to s$, since in the SM they are $\propto V_{ub}$ and CP asymmetries $\lesssim 0.1\%$,
- test of lepton flavour universality (LFU) among $\ell = e, \mu, \tau$.

After an introductory section to the effective theory (EFT) of $|\Delta B| = 1$ decays, the current status will be shortly reviewed for theoretical predictions of $B_s \to \bar{\mu}\mu$ and related decays, and finally the theoretical treatment of $B \to K^{(*)}\bar{\ell}\ell$ will be summarised in the last section.

$|\Delta B| = 1$ EFFECTIVE THEORY

Within the SM, decays of quarks as constituents of some hadronic bound state proceed via the exchange of the rather heavy W bosons, such that a large mass hierarchy is present $m_H \ll m_W$ among the hadron masses $m_H \lesssim 5$ GeV and the W-boson mass $m_W \sim 80$ GeV, apart for top-quark decays. In a first step, this mass hierarchy justifies the decoupling of heavy degrees of freedom, such as W, Z, H-bosons and the top quark, giving rise to an EFT in the spirit of Fermi

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b,\ e,\mu,\tau) + \mathcal{L}_{\rm dim=6} + \mathcal{L}_{\rm dim=8} + \ldots \tag{1}$$

with $SU(3)_{\rm c} \times U(1)_{\rm em}$ gauge interactions of light ($N_f = 5$) quarks and leptons of dim = 4. The second term represents the leading effect of flavour-changing operators O_k of dim = 6 that mediate $|\Delta B| = |\Delta S| = 1$ processes

$$\mathcal{L}_{\rm dim=6} = \frac{4\,G_F}{\sqrt{2}}\left(V_{tb}V_{ts}^* \sum_i C_i(\mu_b)\,O_i + V_{ub}V_{us}^* \sum_j C_i(\mu_b)\,O_j\right) + \text{h.c.}, \tag{2}$$

whereas the higher dimensional operators (dim > 6) are currently neglected due to their suppression by $(m_b/m_W)^2 \sim 0.3\%$. The corresponding short-distance couplings C_i, the so-called Wilson coefficients, are evaluated at the scale $\mu_b \sim m_b$ of the order of the b-quark mass $m_b \sim 4$ GeV. They can be calculated reliably in perturbation theory at the parton level. Their initial conditions at the matching scale $\mu_0 \sim m_W$ are known up to NNLO in QCD [10, 11, 12] and NLO in electroweak (EW) interactions [13, 14, 15, 16]. The renormalisation group (RG) evolution from μ_0 to μ_b is also known to the respective orders [17, 18, 19, 20, 21, 22] and resums the largest logarithmic contributions of radiative corrections between the scales μ_0 and μ_b to all orders in couplings. The part $\propto V_{ub}V_{us}^*$, elements of quark-mixing matrix, gives rise to tiny CP asymmetries in the SM.

In the SM the most interesting operators for $b \to s\bar{\ell}\ell$ are the electric dipole operator $O_{7\gamma}$ and the semileptonic operators $O_{9,10}^{\ell\ell}$

$$O_{7\gamma} = \frac{e}{(4\pi)^2} m_b[\bar{s}\sigma_{\mu\nu}P_R b]F^{\mu\nu}, \qquad\qquad O_{9(10)}^{\ell\ell} = \frac{e^2}{(4\pi)^2}[\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]. \tag{3}$$

Due to operator mixing, also 4-quark current-current operators $O_{2(1)}^p = [\bar{s}\gamma_\mu P_L(\mathbf{T^a})p][\bar{p}\gamma^\mu P_L(\mathbf{T^a})b]$ with $p = (u,c)$ need to be considered. The $O_{1,2}^c$ are numerically the most relevant for $b \to s$ transitions and the evaluation of their contribution to three- and more-body $b \to s\bar{\ell}\ell$ decays constitutes the main theoretical complication. The $O_{1,2}^u$ are suppressed by $V_{ub}V_{us}^* \ll V_{tb}V_{ts}^*$, which is not anymore the case in $b \to d$ transitions as for example $B \to \pi\bar{\ell}\ell$ [23, 24]. Further contributions arise from QCD penguin and the chromo-magnetic dipole operators that are numerically suppressed by small Wilson coefficients for the majority of the observables[1] and at higher order in EW corrections also QED penguin operators have to be taken into account, as for example for the inclusive $B \to X_s\bar{\ell}\ell$ [19, 22].

The effective theory is the starting point for the evaluation of process-specific hadronic matrix elements. Currently all predictions are restricted to the LO in QED, which leads for example to the approximation of matrix elements of semileptonic operators $O_i^{\ell\ell} \propto [\bar{s}\Gamma_{sb}b][\bar{\ell}\Gamma_{\ell\ell}\ell]$ to $B(p_B) \to K^*(p_K)\bar{\ell}\ell$ as follows

$$\mathcal{M} \propto C_i(\mu_b)\left\langle \bar{\ell}\ell K_\lambda^* \middle| [\bar{s}\Gamma_{sb}b][\bar{\ell}\Gamma_{\ell\ell}\ell] \middle| B\right\rangle \overset{\rm LO\,QED}{\approx} C_i(\mu_b)\left\langle \bar{\ell}\ell \middle| \bar{\ell}\Gamma_{\ell\ell}\ell \middle| 0\right\rangle \otimes \left\langle K_\lambda^* \middle| \bar{s}\Gamma_{sb}b \middle| B\right\rangle \;=\; C_i(\mu_b)\, L \otimes F_\lambda(q^2) \tag{4}$$

where λ denotes the polarisation of the K^*, $q^2 = (p_B - p_K)^2$ is the dilepton invariant mass and $L \otimes F_\lambda$ stands for a Lorentz contraction of the leptonic tensor L with the $B \to K^*$ form factor F_λ, involving 4-vectors p_B, q and the polarisation vector of the K^*. Note that the μ_b dependence of the Wilson coefficient is cancelled by $L \otimes F_\lambda$, such that $\mathcal{M}$ is independent of μ_b to the considered order.

$B_{d,s} \to \bar{\ell}\ell$

The hadronic matrix element of leptonic decays $B_q \to \bar{\ell}\ell$, with $q = d,\ s$ and $\ell = e,\ \mu,\ \tau$, has an even simpler structure, which is obtained from (4) by the replacement $K_\lambda^* \to$ *vacuum*. As such, only operators with appropriate Γ_{sb} give

[1] These operators determine for example isospin asymmetries in $B \to K^*\gamma$ [25] and $B \to K^*\bar{\ell}\ell$ [26].

nonvanishing contributions via the decay constant of the B_q meson, f_{B_q}, defined as $\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B_q\rangle = i f_{B_q} p_B^\mu$ and only if their leptonic tensor $L_\mu \neq [\bar{\ell}\gamma_\mu\ell]$, which would otherwise vanish upon contraction with p_B^μ. For this reason, in the SM only $O_{10}^{\ell\ell}$ contributes to $B_{d,s} \to \bar{\ell}\ell$ at LO in QED, whereas $O_9^{\ell\ell}$ or current-current $O_{1,2}^{u,c}$ do not, but additional (pseudo-)scalar operators $O_{S(P)}^{\ell\ell} \propto [\bar{s}\gamma_5 b][\bar{\ell}1(\gamma_5)\ell]$ could contribute beyond the SM. In consequence

$$\mathcal{M} \overset{\text{LO QED}}{\approx} C_{10}(\mu_b)\, f_{B_q}\, p^\mu\, [\bar{\ell}\gamma_\mu\gamma_5\ell] = C_{10}(\mu_b)\, f_{B_q}\, 2m_\ell\, [\bar{\ell}\gamma_5\ell] \qquad (5)$$

where the equation of motion has been applied, giving rise to a helicity suppression by the lepton mass m_ℓ. The decay constant f_{B_q} is a nonperturbative quantity in QCD and can be calculated nowadays on the lattice with an accuracy of about 2% [27], however to leading order in QED only. On the level of the branching fraction $\mathcal{B} \propto |\mathcal{M}|^2$ this amounts to about 4% uncertainty. A reduction to 2% in the future is very likely, whereas further advances in accuracy require the consideration of higher order QED effects as well — some first steps towards this direction are discussed in [28]. Despite this, $\mathcal{B}(B_q \to \bar{\ell}\ell)$ is one of the most precisely predictable observables in b physics — as will become more clear below when discussing perturbative uncertainties — together with the mass difference of neutral B_q mesons, Δm_q, and the leptonic decay of charged $B^- \to \ell^-\bar{\nu}_\ell$. Prospects to extract the top-quark mass in the $\overline{\text{MS}}$ scheme or elements of the quark-mixing matrix are discussed in [29].

Nowadays, the NNLO QCD [12] and NLO EW [16] corrections to $C_{10}(\mu_b)$ have been included. The NNLO QCD corrections reduce the renormalisation scheme dependences from 1.8% to 0.2% at the level of $\mathcal{B}$. Note that NLO EW corrections are of two-fold origin: *i)* NLO corrections present in the SM at the scale μ_0 and *ii)* pure NLO QED corrections within the EFT between the scales μ_0 and μ_b. The renormalisation scheme dependences for EW contributions at the scale μ_0 cancel appropriately in $C_{10}(\mu_b)$. In fact they constituted the largest perturbative uncertainty of about ±8% at LO at the level of $\mathcal{B}$ and are reduced to 0.8% at NLO, where the final scheme dependence has been estimated from using three different renormalisation schemes [16]. The remaining NLO QED scheme dependences of $C_{10}(\mu_b)$ have been estimated to be about 0.3% by scale variation of $\mu_b \in [m_b/2, 2m_b]$. These will cancel upon inclusion of NLO QED corrections in (5) for scales below μ_b, which is complicated by their nonperturbative nature. Some of these corrections, namely soft final-state radiation [30] is accounted for on the experimental side [9, 31] and initial-state radiation can be safely neglected within the experimental signal windows [32]. The lacking NLO QED corrections are helicity suppressed and not enhanced by collinear logaritms.

The most recent SM predictions of the CP-averaged time-integrated branching ratio [33] of the leptonic FCNC decay $B_q \to \bar{\ell}\ell$, including corrections at NNLO in QCD and NLO in EW [34]

$$\begin{aligned}
\overline{\mathcal{B}}(B_s \to \bar{e}e) &= (8.54 \pm 0.55) \times 10^{-14}, & \overline{\mathcal{B}}(B_d \to \bar{e}e) &= (2.48 \pm 0.21) \times 10^{-15},\\
\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu) &= (3.65 \pm 0.23) \times 10^{-9}, & \overline{\mathcal{B}}(B_d \to \bar{\mu}\mu) &= (1.06 \pm 0.09) \times 10^{-10},\\
\overline{\mathcal{B}}(B_s \to \bar{\tau}\tau) &= (7.73 \pm 0.49) \times 10^{-7}, & \overline{\mathcal{B}}(B_d \to \bar{\tau}\tau) &= (2.22 \pm 0.19) \times 10^{-8},
\end{aligned} \qquad (6)$$

have uncertainties of 7% for $q = s$ and 9% for $q = d$, which are due to f_{B_q} and elements of the quark-mixing matrix. The latter are determined using the inclusive determination of $|V_{cb}|$ [35], which is larger than exclusive determinations. As $\mathcal{B}(B_s \to \bar{\ell}\ell) \propto |V_{cb}|^2$ the above values of the branching fractions can be simply rescaled for other values of $|V_{cb}|$. For $\ell = \mu$, the experimental averages of CMS and LHCb of their full LHC Run 1 data sets [9] are $\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ and $\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$ with a statistical significance of $6.2\,\sigma$ and $3.2\,\sigma$, respectively. Only upper bounds exist for $\ell = e, \tau$.

$B \to K^{(*)}\bar{\ell}\ell$

Compared to the leptonic decay $B_q \to \bar{\ell}\ell$, the three- and four-body exclusive decays $B \to K^{(*)}\bar{\ell}\ell$ offer angular distributions with many observables that allow to test various combinations of Wilson coefficients. These are

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{4}(1 - F_H)\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\rm FB}\cos\theta_\ell\,. \qquad (7)$$

$$\begin{aligned}
\frac{8\pi}{3}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} &= (J_{1s} + J_{2s}\cos 2\theta_\ell + J_{6s}\cos\theta_\ell)\sin^2\theta_K + (J_{1c} + J_{2c}\cos 2\theta_\ell + J_{6c}\cos\theta_\ell)\cos^2\theta_K \\
&+ (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_K\sin^2\theta_\ell + (J_4\cos\phi + J_8\sin\phi)\sin 2\theta_K\sin 2\theta_\ell + (J_5\cos\phi + J_7\sin\phi)\sin 2\theta_K\sin\theta_\ell\,,
\end{aligned} \qquad (8)$$

with q^2-dependent F_H, $A_{\rm FB}$ in $B \to K\bar{\ell}\ell$ and the $J_i(q^2)$ in $B \to K^*(\to K\pi)\bar{\ell}\ell$. In combination with the corresponding measurement of the CP-conjugated decays, CP-averages and CP-asymmetries can be formed for each observable. In general, the twelve J_i are independent observables, but for example J_{6c} vanishes in the absence of scalar and tensorial $b \to s\bar{\ell}\ell$ couplings and the relations $J_{1s} = 3J_{2s}$ and $J_{1c} = -J_{2c}$ hold upon neglecting in addition also the lepton mass. For the moment all measurements of angular observables in $B \to K^*\bar{\ell}\ell$ are based on these two assumptions, which prevents in principle tests of scalar and tensorial couplings [36].

As outlined in (4), the matrix elements of semileptonic operators $O_{9,10}^{\ell\ell}$ and the electric dipole operator $O_{7\gamma}$ give rise to $B \to K^{(*)}$ form factors (FF) at LO in QED. The FF's are nonperturbative quantities, which can be calculated on the lattice (LQCD) [37, 38, 39, 40] for q^2 sufficiently large, i.e. in the so-called *high-q^2 region*, or with light-cone sum rules (LCSR) [41, 42, 43, 44] in the *low-q^2 region*. The current uncertainties of the FF's reach (6 – 9)% for LQCD and (10 – 15)% for LCSR predictions, which translate to twice on the branching fractions. There are approximate FF relations at low-q^2 [45, 46] and high-q^2 [47, 48] that relate certain FF's up to corrections of order $\Lambda_{\rm QCD}/m_b \sim 15\%$. These relations allowed to identify combinations of angular observables in $B \to K^*(\to K\pi)\bar{\ell}\ell$, which are to leading order in $\Lambda_{\rm QCD}/m_b$ free of FF's at low-q^2 [49, 50, 51] and high-q^2 [52], the so-called *optimised observables*

$$A_T^{(2)} \equiv P_1 \equiv \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\rm re)} \equiv 2\,P_2 \equiv \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\rm im)} \equiv -2\,P_3 \equiv \frac{J_9}{2\,J_{2s}},$$

$$P_4' \equiv \frac{J_4}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_5' \equiv \frac{J_5/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_6' \equiv \frac{-J_7/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_8' \equiv \frac{-J_8}{\sqrt{-J_{2c}J_{2s}}}. \tag{9}$$

The most prominent is the P_5' at low-q^2 where measurements show deviations from SM predictions with $(2-3)\,\sigma$ from the 1 fb^{-1} data set of LHCb [1] for most theory predictions [53, 54, 55, 56]. Below we will comment more on the details of theoretical uncertainties, since very conservative error estimates [57, 58] reach agreement with the measurements, observable by observable and q^2-bin by q^2-bin.

The nonleptonic (4-quark and chromomagnetic dipole) $|\Delta B| = 1$ operators contribute to $b \to s\bar{\ell}\ell$ at lowest order in QED via the matrix element of the time-ordered product of the electromagnetic current of quarks, $j_\mu^{\rm em}$, and the nonleptonic part of the weak Hamiltonian

$$\mathcal{M}_{\rm hadr} = \frac{\alpha_e}{4\pi}\frac{[\bar{\ell}\gamma^\mu\ell]}{q^2}\int {\rm d}^4x\, e^{i\,q\cdot x}\left\langle K_\lambda^{(*)}\right| {\rm T}\Big\{ j_\mu^{\rm em}(x),\, \sum_i C_i(\mu_b)\, O_i(0)\Big\}\left| B(p)\right\rangle. \tag{10}$$

Different approaches have been used to evaluate this matrix element, depending on the kinematic regime of q^2.

In the high-q^2 region, starting at the open charm threshold $q^2 \gtrsim 15$ GeV^2, i.e. above the narrow resonances J/ψ and ψ', the dilepton mass $q^2 \sim m_b^2$ allows for a local OPE [59, 60, 61], corresponding to the limit $x \to 0$. The leading term is of dim = 3 and involves only $B \to K^{(*)}$ FF's, amenable to a calculation via LQCD. A dim = 4 term is further suppressed by m_s/m_b and numerically at the order of dim = 5 terms. Only the latter involve new FF's, which can be calculated in principle also via LQCD, however they are already suppressed by $(\Lambda_{\rm QCD}/m_b) \sim 2\%$. Apart from these systematically improvable terms, duality violating contributions beyond the OPE are not under theoretical control. They have been estimated with a model and their effect on the integrated rate was found to be about ±2% [61]. Duality violation is usually accounted for in SM predictions and global fits of rare B decays [55, 56, 62] by assigning an additional uncertainty of a few percent to the amplitudes. Moreover it was checked that allowing for large duality violation does not improve the goodness of global fits [56].

In the low-q^2 region with $q^2 \lesssim 6$ GeV^2, i.e. sufficiently below the narrow resonances, the recoil of the $K^{(*)}$ is large $E_{K^{(*)}} \sim m_b$. In this case QCD factorisation (QCDF) has been used to show that at leading order in $\Lambda_{\rm QCD}/m_b$ only FF- and hard-scattering contributions arise [63, 64]. Factorisation attempts failed so far at subleading order, where endpoint divergences are regulated model-dependently [26, 64], however, numerically the considered corrections are small for CP-averaged quantities compared to uncertainties. Further, soft-gluon emission is not accessible to QCDF and was considered in a light-cone OPE at $q^2 \ll 4m_c^2$ for the most important current-current operators $O_{1,2}^c$ [43]. The complete set of nonleptonic operators has been considered in [65] for $B \to K\bar{\ell}\ell$, but not yet to $B \to K^*\bar{\ell}\ell$. There the underlying idea consists of the use of a dispersion relation that relates the hadronic correlation function (10) in the physical region to the unphysical region $q^2 < 0$, where it is approximated by a partonic calculation under the assumption of local quark-hadron duality, justified by the large mass of the initial B meson and the large recoil of the final hadron. In this approach the leading QCDF corrections are incorporated together with soft-gluon contributions, which can be expressed as shifts in the Wilson coefficients C_7 (only for $B \to K^* + (\gamma, \bar{\ell}\ell)$) and C_9. The effect of nonlocal hadronic corrections is smaller than uncertainties due to FF's for the rates of $B \to K^{(*)}\bar{\ell}\ell$ in $m_\phi^2 \lesssim q^2 \lesssim 6$ GeV^2. Soft-gluon effects beyond QCDF have been also considered within a LCSR calculation [66, 67], but are numerically also too small [56] to play a role in global fits.

Schematically the amplitudes with polarisation λ receive various contributions at low q^2 due to semileptonic and nonleptonic operators

$$\mathcal{M}_\lambda \propto \left(\xi_i + \Delta F_9^{\alpha_s} + \Delta F_9^{1/m_b}\right) \left[C_9(\mu_b) \pm C_{10}(\mu_b)\right] + (9 \to 7) + \Delta^{non-fac} + \Delta^{\rm soft}. \tag{11}$$

The use of FF relations introduces two universal FF's ξ_i ($i = \perp, \|$), with known order-α_s corrections $\Delta F_{7,9}^{\alpha_s}$ [46] and lacking subleading corrections $\Delta F_{7,9}^{1/m_b}$. The latter are ad-hoc parameterised $\propto a_i + b_i\,(q^2/m_B^2) + \ldots$ [57], assuming they are power corrections. The choice of the central values of a_i, b_i is the main origin of differences for SM predictions of optimised observables in [55, 56] versus [57, 58]. Whereas former groups[2] use central values given by LCSR calculations of the QCD FF's [44], the latter group fixes them to the heavy quark limit, i.e. setting them to zero. The comparison of the results shows that subleading corrections to FF relations are important for P_5' in the q^2-region of the zero crossing, where the differences are most pronounced. So-called *nonfactorisable* corrections, $\Delta^{non-fac}$ have been calculated in QCDF to leading order [26, 64] and soft-gluon effects $\Delta^{\rm soft}$ in [43, 65]. For the purpose of SM predictions and in global fits, the lacking subleading nonfactorisable corrections (partially known [26, 64]) as well as the soft-gluon effects $\Delta^{\rm soft}$ are parameterised in a similar fashion $\propto A_i + B_i\,(q^2/m_B^2) + C_i\,(q^4/m_B^4) + \ldots$. The parameters A_i, B_i, C_i are complex-valued and their size is chosen either to be of the order of a $\Lambda_{\rm QCD}/m_b$ subleading effect or to reproduce the size (but with arbitrary sign) of the calculation by [43, 65].

The SM predictions of $B \to K^{(*)}\bar{\ell}\ell$ observables in the literature use various procedures for estimates of uncertainties. The most conservative approach [58] determines the spread in each observable (and q^2-bin) by a simultaneous scan over all parameters within some ranges. In less conservative approaches parameters are divided into groups, where parameters belonging to each group are varied simultaneously and the spreads in the observables from the groups are added in quadrature [57, 55]. A bayesian motivated approach [53, 56, 62], assumes prior distributions (mostly gaussian) for the parameters, such that observables are calculated from a large sample of parameter values drawn according to these priors. The uncertainties are then determined from the 68% probability intervals of each observable. Further, a covariance matrix can be derived [56, 62], neglecting nongaussianities, which is used in global fits in order to include the correlations of observables due to the theory parameters, which is computationally much easier than a full bayesian fit [36, 53].

SUMMARY

The leptonic decays $B_q \to \bar{\ell}\ell$ are under excellent theoretical control. The CP-averaged and time-integrated branching fraction $\propto f_{B_q}^2/\Gamma_H^q \times (G_F m_W)^4\,|V_{tb}V_{tq}^*\,C_{10}(\mu_b)|^2$ suffers currently from 4 (7)% hadronic uncertainty from the $B_s\,(B_d)$ decay constants f_{B_q} and 6 (9)% from $V_{ts}\,(V_{td})$. It can be expected that lattice progress will further decrease the uncertainties from decay constants below 2%. The only remaining uncertainty stems from quark-mixing parameters. Once the experimental precision suffices, $\mathcal{B}(B_q \to \bar{\ell}\ell)$ can be used in global CKM fits to get a better handle on $V_{ts}\,(V_{td})$ in the framework of the SM.

The three- and four-body decays $B \to K^{(*)}\bar{\ell}\ell$ are under less theoretical control. At high dilepton invariant mass q^2, the main uncertainties due to form factors (FF) are reducible in the future by lattice calculations. Duality violating corrections to the OPE at high q^2 are not accessible to theory and probably prevent the reach of a precision as in $B_q \to \bar{\ell}\ell$. Their size could be tested with data by search of large violations of relations between angular observables predicted by the OPE. At lowq^2, the factorisability of subleading corrections in QCDF has to be investigated as well as subleading corrections to FF relations. Further, soft-gluon effects have to be studied in more detail for the complete set of angular observables, where the refinement of the use of dispersion relations could be a next step to acquire more inside into such effects.

[2]Note that [56] does not use FF relations, i.e. the issue of $\Delta F_9^{1/m_b}$ does not arise at all, since it is assumed that LCSR allow to account for them.

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