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# AXIAL GAUGE PROPAGATORS FOR QUARKS AND GLUONS ON THE POLYAKOV-WILSON LATTICE

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ABSTRACT

We discuss problems encountered in defining gauge-dependent propagators in a confining theory. For precision we use a finite Polyakov-Wilson lattice to define the Yang-Mills theory and to provide the ultraviolet and infrared regularization. Gauge fixing in a class of superaxial gauges is natural in this framework. A variety of approaches for defining the propagators for quarks and gluons is discussed and the propagators are evaluated explicitly in the strong-coupling limit. We speculate upon the infrared behavior of these propagators in the weak-coupling limit and upon the utility and validity of the Schwinger-Dyson equations for these propagators. In conclusion we propose that the leading infrared behavior is strongly gauge dependent and governed by the masses of low-lying color singlet states in the hadron spectrum. In the ultraviolet limit, however, with a properly constructed propagator, we find no reason to question the conventional wisdom derived from perturbation theory. Our conclusions should not depend in any fundamental way on the lattice formulation of the gauge theory, except insofar as that formulation serves to give precision to the continuum functional integration.

## 1. Introduction

In a confining theory such as quantum chromodynamics (QCD), physical states are composite states in the singlet representation of the gauge group. Thus it seems unnatural and academic to attempt to discuss the propagation of non-singlet excitations. Nonetheless, our experience with non-confining theories, such as quantum electrodynamics (QED) and our confidence in the use of perturbation theory for QCD at short distances tempts us to proceed to a treatment of quark and gluon propagation at large distances. Thus, there have been many efforts to determine the long-distance behavior of these propagators for a variety of reasons [1-5]. Of course, even if we found the correct behavior of the propagators, it is not clear what we could do with them, since perturbation theory is not to be trusted at long distances. Nevertheless, in view of continuing interest in these propagators we felt it worthwhile to study them in the context of a lattice version of the gauge theory.

That there are subtleties in the definition of the propagators in axial gauge has been emphasized by Mandelstam [6]. In a companion work we discuss the appearance of spurious source currents in axial gauge propagators induced by gauge dependent operators [7]. In this work we add another chapter to the pedagogical discussion by examining the effect of these spurious sources upon quark and gluon propagators in confining theories. The lattice provides a framework for a precise definition of the propagators and exposes points that are often glossed over in the continuum treatments. The strong coupling lattice results are readily obtained and are suggestive of the weak coupling continuum results. The weak coupling version of the theory, with some qualifications, could indeed be what we intend to mean by the "continuum" version of QCD.

The plan of this paper is as follows. In Section 2 we review the lattice gauge theory and discuss the process of gauge fixing in a class of superaxial gauges. In Section 3 we discuss various definitions of the propagators in a class of superaxial gauges. In Section 4 we find the strong coupling limit of the propagators [8] and speculate upon the weak coupling results both at long and short distances. Concluding remarks are given in Section 5. In the Appendix we discuss the gluon propagator induced by the vector potential.

Our main conclusion, that quark and gluon propagation to long distances is governed by the low-lying color-singlet states, can be understood from one trivial observation: in a confining theory, defined through a Euclidean functional integral, the propagation of any disturbance whatsoever is mediated by finite energy, i.e. physical, intermediate states. The manner in which a non-singlet operator gives rise to singlet excitations is amusing and is described at length below.

## 2. Review of Gauge Fixing on the Lattice

### a. Notation

Let us begin by discussing the pure Yang-Mills theory on a lattice [9]. For definiteness we use the gauge group SU(2). The generalization to other groups is straightforward. On a four-dimensional Euclidean space-time lattice of spacing a and side N the gauge link variables $U_{x\mu} \in SU(2)$ are associated with each site x and direction $\mu = 0,1,2,3$. In the continuum limit the corresponding vector potential $A^{\alpha}_{\mu}$ is obtained from

$$U_{x\mu} = \exp\,[ig\,\frac{a}{2}\,A_{\mu}] \qquad (2.1)$$

where g is the gauge coupling, $A_{\mu} = A^{\alpha}{}_{\mu}\,\sigma^{\alpha}$, and $\sigma^{\alpha}$ are the Pauli matrices. The lattice is assumed to be periodic

$$U_{x+N\hat{\mu},\mu} = U_{x,\mu} \qquad (2.2)$$

where $\hat{\mu}$ is a unit vector in the direction $\mu$. We require periodicity in complex time in order to be able to treat averages over thermal ensembles and to guarantee that zero temperature expectation values are taken on finite energy states. Periodicity in space is optional.

Operator averages are obtained by carrying out the usual functional integration

$$\langle\mathcal{O}\rangle = \frac{\int[dU]\,(\exp S_G)\,\mathcal{O}(U)}{\int[dU]\,\exp S_G} \qquad (2.3)$$

where

$$S_G = 2/g^2 \sum_P \mathrm{Tr}\,U_P\ . \qquad (2.4)$$

The sum is taken over all plaquettes in the lattice, where, as usual, for a plaquette at site x in the $\mu\nu$ plane,

$$U_P \equiv U_{x\mu\nu} = U_{x\mu}\, U_{x+\hat{\mu},\nu}\, U^{\dagger}_{x+\hat{\nu},\mu}\, U^{\dagger}_{x\nu} \; . \tag{2.5}$$

The integration is carried out over each link variable in the sense of the Haar measure. For convenience we choose the normalization

$$\int [dU] = 1 \; . \tag{2.6}$$

The action $S_G$ and integration volume is invariant with respect to gauge transformations $G_x \in SU(2)$ such that

$$U^{G}_{x,\mu} = G_x\, U_{x,\mu}\, G^{\dagger}_{x+\hat{\mu}} \; . \tag{2.7}$$

To preserve periodicity the gauge transformation must itself satisfy

$$G_{x+N\hat{\mu}} = z_\mu\, G_x \tag{2.8}$$

where the $z_\mu$ are elements in the center of the gauge group [10].

b. Hamiltonian Formulation-A Synopsis

We summarize here some useful results relating the Euclidean action formulation and the hamiltonian formulation of the lattice gauge theory [11]. The relationship is most readily established in temporal axial gauge in which the lattice hamiltonian is

$$H = \sum_{\vec{x},i,\alpha} \frac{g^2}{2a} (E^{\alpha}_{\vec{x}i})^2 - \frac{2}{ag^2} \sum_{P_{ss}} \mathrm{tr}\, U_{P_{ss}} \tag{2.9}$$

where i = 1, 2, 3, and where $P_{ss}$ denotes the set of plaquettes constructed with space-like links on all sides. The electric flux $E^{\alpha}_{\vec{x}i}$ emanating from site $\vec{x}$ in the direction i on a given link and the link variable $U_{\vec{y}j}$ satisfy the commutation relation

$$[E^{\alpha}_{\vec{x}i} , U_{\vec{y}j}] = \frac{1}{2} \sigma^{\alpha} U_{\vec{y}j} \delta_{ij} \delta_{\vec{x},\vec{y}} \qquad (2.10a)$$

$$[E^{\alpha}_{\vec{x}i} , E^{\beta}_{\vec{y}j}] = i\epsilon^{\alpha\beta\gamma} E^{\gamma}_{\vec{y}j} \delta_{ij} \delta_{\vec{x}\vec{y}} \quad ; \qquad (2.10b)$$

i.e. the $E^{\alpha}_{\vec{x}i}$ are infinitesimal generators of the left transformation

$$U_{\vec{y}j} \rightarrow G_{\vec{y}} U_{\vec{y}j} \quad . \qquad (2.11)$$

The right transformations are generated by $E'^{\alpha}_{\vec{x}i}$, which have the property

$$[E'^{\alpha}_{\vec{x}i} , U_{\vec{y}j}] = - \frac{1}{2} U_{\vec{y}j} \delta_{ij} \delta_{\vec{x},\vec{y}} \sigma^{\alpha} \quad . \qquad (2.10c)$$

They are related to the other variables through

$$E'^{\alpha}_{\vec{x}i} = - R_{\alpha\beta} (U_{\vec{x}i}) E^{\beta}_{\vec{x}i} \qquad (2.12)$$

where R(U) is the O(3) transformation representing U. These generators are sometimes denoted

$$E^{\alpha}_{\vec{x},-i} \equiv E'^{\alpha}_{\vec{x}-\hat{i},i} \qquad (2.13)$$

and interpreted as the electric flux emanating from site $\vec{x}$ in the direction $-\hat{i}$. The operators

$$Q^{\alpha}_{\vec{x}} = \sum_{i=1}^{3} (E^{\alpha}_{\vec{x},i} + E^{\alpha}_{\vec{x},-i}) \qquad (2.14)$$

are the infinitesimal generators of time-independent gauge transformations. Physical states must satisfy the subsidiary condition

$$Q^{\alpha}_{\vec{x}} \, |\text{phys}\rangle = 0 \ , \qquad (2.15)$$

which is equivalent to requiring that the total color electric flux leaving a site must form a color singlet. This is the lattice version of Gauss' law. In the continuum limit the condition reads

$$\vec{D}_{\alpha\beta} \cdot \vec{E}^{\beta} \, |\text{phys}\rangle = 0 \qquad (2.16)$$

where $\vec{D}_{\alpha\beta}$ is the gauge covariant derivative.

The partition function for the hamiltonian H at inverse temperature $\beta_t$ is

$$Z(\beta_t) = \text{Tr} \exp(-\beta_t H) = \text{Tr}[\exp(-\tau H)]^{N_t} \qquad (2.17)$$

where $\beta_t = \tau N_t$. The trace is taken over the physical Hilbert space. Since it is convenient to use the basis in which the $U_{\vec{x}i}$ are diagonal in carrying out the trace it is necessary to introduce a projection operator P onto the physical states satisfying Gauss' law (2.15) [12]. The projection operator is merely a functional integral over all time-independent gauge transformations

$$P|\{U_{\vec{x}i}\}\rangle = \int [dG] \, | \, \{U^{G}_{\vec{x},i}\}\rangle \qquad (2.18)$$

where $\{U^{G}_{\vec{x}i}\}$ are given by (2.7). Thus

$$Z(\beta_t) = \text{Tr}\,[\exp(-\tau H)P]^{N_t} = \text{Tr}\,\{[\exp(-\tau H)]^{N_t} P\} \qquad (2.19)$$

(Note that [P,H] = 0). The partition function has been expressed as the trace of a power of the transfer matrix. The transfer matrix element in the limit of small $\tau$ has the form

$$<\{U'_{\vec{x},i}\}|\exp(-\tau H)\, P|\{U_{\vec{x},i}\}>$$

$$\propto \int [dG] \exp \{\frac{2a}{\tau g^2} \sum_{P_{st}} \text{tr}\, U_{P_{st}} + \frac{2\tau}{ag^2} \sum_{P_{ss}} \text{tr}\, U_{P_{ss}}\} \tag{2.20}$$

where $P_{st}$ denotes the set of plaquettes formed on each space-link of the form

$$U_{P_{st}} = U^G_{\vec{x}i} (U'_{\vec{x}i})^\dagger = G_{\vec{x}} U_{\vec{x}i} G^\dagger_{\vec{x}+\hat{i}} U'^\dagger_{\vec{x}i} \tag{2.21}$$

and $P_{ss}$ denotes the set of space-space plaquettes appearing in H. The connections with the action formulation is now straightforward. There is an integration over a set of gauge transformations $\{G_{\vec{x}}\}$ in each of the $N_t$ terms in the product and an integration over $N_t$ basis set $\{U_{\vec{x}i}\}$. The G's become the time-like gauge links and the $U_{\vec{x}i}$'s become the space-like gauge links $U_{xi}$. The resulting partition function is that of an anisotropic lattice:

$$Z(\beta_t) \propto \int [dU] \exp S_G (U, \tau/a) \tag{2.22}$$

where

$$S_G (U, \tau/a) = \frac{2a}{\tau g^2} \sum_{P_{st}} \text{tr}\, U_{P_{st}} + \frac{2\tau}{ag^2} \sum_{P_{ss}} \text{tr}\, U_{P_{ss}} \tag{2.23}$$

for a lattice with lattice constant $\tau$ in imaginary time and a in space.

c. Gauge Fixing

Creutz [13] described a method for carrying out the integration over the U's in a class of gauges. Because of the gauge invariance of the functional integration many of the link variables are redundant and can be eliminated from the integrand by a change of variable. Thus, for example, it is possible to eliminate one of the $U_{y\mu}$ by carrying out a gauge transformation with

$$G_x = \begin{cases} U^\dagger_{y\mu} & \text{for } x = y \\ 1 & \text{for } x \neq y \ . \end{cases} \tag{2.24}$$

The result is to replace that particular link variable by the identity. The elimination of further variables can be effected by a sequence of such gauge transformations. Once a variable is eliminated, it is, of course, necessary to take care that further elimination does not restore the variable. It is easily shown [13] that the largest set that can be eliminated in this way is the set of link variables that form a maximally connected tree structure on the lattice (see Fig. 1). The tree has no branches that form a closed loop. Loops that close by virtue of the periodicity of the lattice are also excluded. Every point on the lattice is on a branch of the tree. Furthermore, the tree defines a unique path connecting any pair of points on the lattice. This is, of course, the essence of local gauge fixing: the internal symmetry space on each site is fixed uniquely in relationship to that on any other site.

Some of the gauges obtained in this way have an obvious continuum limit. For example, a continuum temporal axial gauge $A_0 = 0$ is obtained by setting to one as many links as possible in the time direction. For other axial gauges $n_\mu A^\mu = 0$ the correspondence is most natural if the primary gauge fixing direction $n_\mu$ is one of the axes of the lattice. For other directions, paths along the lattice approximating the rays $n_\mu$ would have to be constructed.

Of course to require $A_0 = 0$ does not fix the gauge completely. There is a residual gauge freedom. A gauge transformation

$$A_\mu \to \chi A_\mu \chi^\dagger + \frac{1}{g} \chi \partial_\mu \chi^\dagger \tag{2.25}$$

is still permitted if $\chi$ is time-independent. The gauge can be fixed further, for example, by choosing $A_1 = 0$ at one particular time, say $x_0 = t_0$. Such an operation is analogous to setting to one links in the 1-direction for one time slice as in Fig. 1. After fixing the gauge in this way it is still possible to carry out gauge transformations that have no dependence on $x_1$ and $x_0$. Thus further subsidiary conditions are required. For example one may set $A_2 = 0$ for fixed $x_0 = t_0$ and $x_1 = z_1$, corresponding on the lattice to establishing connections between $(x_0, x_1)$ planes. Then one may set $A_3 = 0$ for fixed $x_0 = t_0$, $x_1 = z_1$, and $x_2 = z_2$. Finally, the only remaining gauge freedom is through a global gauge transformation. With such a transformation it is possible to arrange so that one of the non-fixed components of the vector potential at one space-time point is aligned along in a particular direction in the color space, or correspondingly, one of the remaining link variables is replaced by a specific element of the equivalence class to which it belongs, but we shall instead omit this final step. We call the gauge thus defined up to a global gauge transformation a "superaxial gauge."

In temporal gauge hamiltonian language the subsidiary gauge fixing conditions do not correspond to imposing further restrictions on the Hilbert space, which might be objectionable [14]. Rather, they correspond to introducing a projection operator in the Green's functions that acts at a specific time $t_0$. The projection operator is derived from the completeness relation for the eigenstates of the space-like link matrices (equivalently, the three-vector potential):

$$P_{SG} = \int_{SG} [d\, U_{\vec{x},i}]\, |\{U_{\vec{x},i}\}\rangle\langle\{U_{\vec{x},i}\}| . \qquad (2.26)$$

where the integration is restricted to a subset of configurations satisfying the subsidiary gauge condition. A typical Green's function

$$\langle phys|TO_i(t,\vec{x})\ O_j(t',\vec{y})|\ phys\rangle/\langle phys|phys\rangle \qquad (2.27)$$

is then rewritten as the equivalent expression

$$\langle phys|TP_{SG}(t_o)\ O_i(t,\vec{x})\ O_j(t',\vec{y})|phys\rangle/\langle phys|P_{SG}\ (t_o)|phys\rangle\ . \qquad (2.28)$$

Since the matrix element is evaluated on a physical state the effect of the projection is to remove a common factor of a group volume of time-independent (redundant) gauge transformations from numerator and denominator. This type of projection can be done at only one time without altering the Green's function.

It is apparent that in such a super temporal gauge there is a set of time-like gauge links that can not be set to one, as shown in Fig. 1. The integration over these links has a special function that can be understood in the transfer-matrix language. As we have noted in Sec. IIb above this integration is associated with the projection onto states satisfying Gauss' law and is necessary to define the partition function. This stipulation is often omitted in continuum formulations of the axial gauge function integral. The required integration over a set of time-like links is, in fact, associated with the required periodicity of the lattice in complex time. This periodicity excludes branches in the tree that close from $\tau = 0$ to $\tau = \beta_t$. Thus the integration over these links is common to all axial gauges. There is also a complete set of space-like links for a given value of x in Fig. 1 that is not set to one. This feature is peculiar to the choice of periodic boundary conditions in space. If we had chosen a non-periodic, finite spatial volume, then it would have been possible to enlarge the connected tree so as to include connections from one spatial boundary to the opposite spatial boundary.

If we do not wish to fix the residual gauge, then the cross-connections between the rays in the primary gauge direction should not be set to one.

Thus in temporal axial gauge without residual gauge fixing, Fig. 1 should be drawn without the cross links in the x-direction along the lower part of the figure.

3. Gauge-Dependent Propagators on the Lattice

a. The Naive Axial Gauge Propagators on the Lattice

For simplicity we begin by discussing the gluon propagator defined through the correlation of the field tensor

$$G^{\alpha\beta}_{F_{\mu\nu\rho\sigma}} = \langle F^{\alpha}_{\mu\nu}(x)\ F^{\beta}_{\rho\sigma}(y)\rangle_{SA} \ , \tag{3.1}$$

where $\langle\ \rangle_{SA}$ denotes an expectation value in a superaxial gauge as described in Sec. IIc. The expectation value may be an average over a thermal statistical ensemble or a vacuum expectation value, as desired. (Time ordering is omitted, since we are concerned here with problems associated with the definition of the correlation product. The appropriate time-ordering can always be inserted to give the propagator without altering our conclusions.) The corresponding lattice expression is obtained by using

$$U_{x\mu\nu} \underset{a \to 0}{\sim} \exp\left(ig \frac{a^2}{2} F^{\beta}_{\mu\nu}(x)\ \sigma^{\beta}\right) . \tag{3.2}$$

Thus on the lattice

$$G^{\alpha\beta}_{F_{\mu\nu\rho\sigma}}(x,y) = \frac{1}{g^2 a^4} \langle \mathrm{Tr}\ (U_{x\mu\nu}\ \sigma^{\alpha})\ \mathrm{Tr}\ (U^{\dagger}_{y\rho\sigma}\ \sigma^{\beta})\rangle_{SA} \ . \tag{3.3}$$

Since the gauge restriction still permits a global gauge transformation, the propagator is diagonal and can be written

$$G^{\alpha\beta}_{F_{\mu\nu\rho\sigma}}(x,y) = \frac{\delta_{ab}}{3g^2 a^4} \langle\ \mathrm{Tr}\ (U_{x\mu\nu}\ \sigma^{\gamma})\ \mathrm{Tr}(U_{y\rho\sigma}\ \sigma^{\gamma})\rangle_{SA} \ . \tag{3.4}$$

The traces can be restored to gauge-invariant form by introducing a string operator in the adjoint representation

$$C_A(x,x', \text{path}) = \prod_{z\hat{\rho}\varepsilon \text{path}} U^A_{z\rho} \tag{3.5}$$

where the path is defined by a sequence of links $z\hat{\rho}$ connecting x and x'. If the path follows the connected tree for the superaxial gauge in question, the string operator is the identity. Thus with the expression

$$G^{\alpha\beta}_{F\mu\nu\rho\sigma}(x,y) = \frac{\delta_{ab}}{3g^2a^4} \langle Tr(U_{x\mu\nu}\,\sigma^\gamma)\; C_A(x,y,\text{ gauge})_{\gamma\delta}\; Tr(U^\dagger_{y\rho\sigma}\,\sigma^\delta)\rangle \tag{3.6}$$

we make no change in the gauge in question, but because the expression is gauge-invariant it can be evaluated in that form in any other gauge and it gives the same result. Therefore we can remove the gauge restriction by introducing into the definition of the propagators an appropriate set of string operators. For the gauge of Fig. 1 the result is represented by the diagram in Fig. 2.

If we had not fixed the residual gauge, then it would not have been possible to connect x to y with an "invisible" string unless x and y happened to lie on a ray along the primary gauge fixing direction. The expectation value (3.4) with x and y on different rays would then give zero [6], because the functional integration would be carried out over the residual gauge group, allowing independent gauge transformations of the two non-singlet terms in the product. It is because we wanted to allow propagation between rays in the primary gauge direction that we have chosen to fix the residual gauge.

Although it would seem that the strings are irrelevant, because they are invisible in the gauge of interest, in fact they play a dynamical role that is more apparent in a different gauge. They correspond to the introduction of a source current for the vector potential. For example, consider the evaluation of (3.6) using (2.3). The integration is over all gauges, but the string paths are fixed. In the transfer matrix formalism there are time slices $\tau$ with extra factors of time-like links coming from the strings. These extra factors correspond to a modification of the projection operator (2.18) by the inclusion of an extra factor, say $G^A_{\vec{y}} \equiv U^A_{\vec{y}\tau,0}$, thus:

$$P^A_{\vec{y}} \,|\{U_{xi}\}\rangle = \int [dG]\, G^A_{\vec{y}} \,|\{U^G_{\vec{x},i}\}\rangle \qquad (3.7)$$

The resulting state forms a multiplet in the adjoint representation that satisfies

$$Q^\alpha_{\vec{x}}|c\rangle = \delta_{\vec{x},\vec{y}}\, \Lambda^\alpha_A{}_{cd}|d\rangle \qquad (3.8)$$

where $\Lambda^\alpha_A$ is the color generator for the adjoint representation, i.e. Gauss' law (2.15) is modified by the inclusion of an adjoint point charge at the coordinate $\vec{y}$ of the corresponding time-like link of the string. This interpretation of the time-like string is, of course, well known [15]. The space-like string corresponds in hamiltonian language to an operator that removes or adds a line of flux, at the same time removing or adding a pair of fixed sources at the end points or displacing a fixed source from end to end.

The string sources necessarily have an important effect upon the propagator [7]. To make this point most emphatically, we consider the simpler case of the corresponding electron propagator in temporal axial gauge in quantum electrodynamics:

$$S_{\alpha\beta}(x,y) = \langle\psi_\alpha(x) \exp\left(ie \int_x^y A_\mu(x')dx^{\mu'}\right)\psi^\dagger_\beta(y)\rangle \qquad (3.9)$$

where the path of the line integral again conforms to the tree for the super temporal gauge in question. Here, the subscripts $\alpha$ and $\beta$ label spin degrees of freedom. As with the gluon propagator, the string corresponds to a point source that is fixed in space over most of its trajectory. The spectrum of the propagator is the same as the spectrum of the hamiltonian in the one-electron sector in the presence of a fixed source of the opposite charge. This spectrum is that of the hydrogen atom (with the proton replaced by a spinless source of infinite mass) and not that of a free mass renormalized electron! The Euclidean propagator is dominated by the Rydberg levels at large imaginary time-like separation of x and y. This result can also be demonstrated in perturbation theory. It was shown in Ref. 7 that the lowest order electron self-energy in temporal axial gauge corresponds in Coulomb gauge to the self-energy of both the electron and spurious source and a photon exchange between the electron and spurious source. The result can be generalized to higher orders to produce the traditional ladder graphs, among others, that give rise to the hydrogenic bound states and scattering states. The fixed source is point-like, just like the field operator. Therefore, there will be the usual ultraviolet divergences associated with its self-energy.

The origin of the fixed source can also be understood in a slightly different language. The propagator is to be evaluated on the physical states. Thus it should change nothing if we introduce the projection onto states satisfying Gauss' law:

$$S_{\alpha\beta}(x,y) = \langle \psi_\alpha(x)\ \psi^\dagger_\beta(y) P \rangle_{ST} \quad . \tag{3.10}$$

where ST denotes a super temporal gauge. For this Abelian case

$$P = \int [d\phi] \exp \left\{ i \int (\nabla \cdot \vec{E} - \rho)\ \phi(x)\ d^3x \right\} \quad , \tag{3.11}$$

which enforces

$$\nabla \cdot \vec{E} = \rho = -\ e :\bar{\psi}\ \gamma^0 \psi: \tag{3.12}$$

everywhere. Note, however, that

$$\psi^\dagger_\beta(\vec{y})\ P = P_{\vec{y}}\ \psi^\dagger_\beta(\vec{y}) \tag{3.13}$$

where

$$P_{\vec{y}} = \int [d\phi] \exp \{ i \int [\nabla \cdot \vec{E} - \rho - e\delta^3(x-y)] \, \phi(x) d^3x \} \quad , \qquad (3.14)$$

which enforces

$$\nabla \cdot \vec{E} = \rho + e\delta^3(\vec{x}-\vec{y}) \qquad (3.15)$$

everywhere.

Thus the propagator (3.10) is equivalent to

$$S_{\alpha\beta}(x,y) = \langle \psi_\alpha(x) \, P_{\vec{y}} \, \psi_\beta^\dagger(y) \rangle \qquad (3.16)$$

The intermediate states must therefore be states that satisfy Gauss' law with the addition of a fixed charge opposite that of the fermion. This is the string source.

The alert reader will notice that moving the projection operator to the left of $\psi_\alpha(x)$ gives

$$S_{\alpha\beta}(x,y) = \langle P_{\overline{xy}} \, \psi_\alpha(x) \, \psi_\beta^\dagger(y) \rangle \qquad (3.17)$$

indicating that the bra states should include two fixed sources of opposite charge, one at x and one at y. These sources are removed by making use of the residual gauge fixing condition. (Without residual gauge fixing and with $x \neq y$ this modified projection operator would force the expectation value to vanish since the bra and ket states would be forced to lie in orthogonal Hilbert spaces.) For example, if, as in Fig. 1, the residual gauge fixing takes place at a fixed time, say $t_0$, an invisible space-like string can be constructed at the same fixed time which has the property

$$C(\vec{x},\vec{y}) \, P_{\overline{xy}} = \dot{P} \, C(\vec{x},\vec{y}) \qquad (3.18)$$

i.e. it removes the pair of fixed sources, leaving states that satisfy the original, unadorned version of Gauss' law (3.12). Thus the propagator can be written in the equivalent manner:

$$S_{\alpha\beta}(x,y) = < C(x,y)\ \psi_\alpha(x)\ \psi^\dagger_\beta(y)P> \qquad (3.19)$$

where the bra and ket states are now physical states satisfying (3.12) and the expectation value is taken in the manner of (2.28).

Of course, the naive temporal axial gauge propagator (3.10) is not the familiar propagator of QED. Certainly, we would prefer to discuss propagation without introducing spurious sources. Consider the Coulomb gauge propagator for the electron. In that gauge the scalar potential is determined by the instantaneous charge density, so that Gauss' law is immediately satisfied without the need for a subsidiary condition on the states. At the moment an electron is created from the vacuum, an electric flux is also automatically created, extending to an image charge at infinity. To produce the same result in temporal axial gauge, it is necessary to create the accompanying electric flux by hand, thus:

$$S^{Coul}_{\alpha\beta} = <\Phi(x)\ \psi_\alpha(x)\quad \psi^\dagger_\beta(y)\ \Phi^\dagger(y)>_{ST} \qquad (3.20)$$

where the operator $\Phi^\dagger(y)$ creates the desired longitudinal electric flux. Of course the projection operation P (3.11) commutes with the combinations $\psi^\dagger\ \Phi^\dagger$ and $\Phi\psi$, because they are gauge invariant. One representation for $\Phi^\dagger$ is

$$\Phi^\dagger(y) = \int_{paths} C(\vec{y}, \infty)$$

where the integration is over all paths connecting $\vec{y}$ to $\infty$. Since the flux lines are not explicitly created in (3.10), it is obvious why the spurious source occurs. Because of Gauss' law, an operator cannot create charge without creating flux. Since flux is not created in (3.10), the operator that creates a fermion also creates the canceling spurious charge, whether we wanted it or not.

Why not use Coulomb gauge, then? There are many reasons that Coulomb gauge is unsuitable for QCD [16], not to mention that an operator that produces flux radiating isotropically to infinity creates a state of infinite energy.

Are other axial gauges immune from the spurious sources? In axial gauges other than temporal axial gauge, there is a scalar potential. However, if the theory is defined through a Euclidean path integral on a periodic space-time volume, O(4) symmetry of the theory leads us immediately to the same conclusion for gauges of the type $n_\mu A^\mu = 0$ with fixed $n_\mu$, namely, that fixed sources accompany the creation of charged particles. The sources always follow trajectories defined by the tree. Thus the problem of the spurious source is, in fact, common to all axial gauges, independent of the spatial boundary conditions.

e. An Improved Axial Gauge Propagator

If the naive axial gauge propagator gives trouble with spurious sources, can we define a propagator that is closer to what is desired? Of course, the answer depends on what we intend to do with the propagator. Suppose we are merely interested in defining a propagator that has intermediate states free of spurious charges. For the electron propagator in QED we could try

$$S'_{\alpha\beta}(x,y) = \langle\psi_\alpha(x)\ P\ \psi^\dagger_\beta(y)\rangle_{ST} \qquad (3.22)$$

However, the bra and ket states must now have the fixed sources, since the intermediate state do not. We could put the source at $\vec{y}$ in the ket and bra, if the residual gauge fixing permits a cancellation of the flux between $\vec{x}$ and

$\vec{y}$ through the introduction of C(x,y), thus:

$$S'_{\alpha\beta}(s,y) = \langle C(x,y)\ \psi_\alpha(x)\ \psi^\dagger_\beta(y)\rangle_{\vec{y}} \qquad (3.23)$$

where the subscript $\vec{y}$ denotes an expectation value on states with a fixed source at $\vec{y}$ with the same charge as the fermion. Now the intermediate states are free electrons.

The corresponding gluon propagator has the form

$$\begin{aligned} G^{\alpha\beta}_{F\mu\nu\rho\sigma} &= {}_{\vec{y}\beta}\langle C^A_{\beta\alpha}(y,x)\ F^\alpha_{\mu\nu}(x)\ F^\beta_{\rho\sigma}(y)\rangle_{\vec{y}\beta} \\ &- {}_{\vec{y}\beta}\langle C^A_{\beta\alpha}(y,x)\ F^\alpha_{\mu\nu}(x)|0\rangle\langle 0|F^\beta_{\rho\sigma}(y)\rangle_{\vec{y}\beta} \end{aligned} \qquad (3.24)$$

where a color label $\beta$ for the fixed source has been included and we have subtracted the disconnected part. The state |0> is the physical vacuum state in that gauge with no extra sources present.

What is the spectrum of this propagator? The intermediate states should, of course, be the physical states of the theory. But in a confining theory, they are glueball states. It may seem paradoxical that they should occur, since we have taken pains to construct a non-singlet propagator. However, if, the propagator is defined through a Euclidean functional integral, then the expectation value in (3.24) is an average over a statistical ensemble of states satisfying Gauss' law, but with a non-singlet source at $\vec{y}$. The states of finite free energy in such an ensemble are states in which the fixed source is screened by at least one gluon in the ensemble. These are the states that survive the zero temperature limit. They are the Yang-Mills analogs of the hydrogen atom, i.e. the lightest glueball that can be made with one of the gluons replaced by a fixed adjoint source. Let us call this a "heavy-light" glueball. Since it undoubtedly has an energy different from that of the physical vacuum, it is necessary to normalize the expectation value to the partition function in the vacuum with the same fixed source to avoid getting zero or infinity in the zero temperature limit. The propagator, then, can be described as the amplitude for a process in which a heavy-light glueball is converted into a normal glueball, allowed to propagate, and then is reconverted to a heavy-light state. In this way the intermediate states are glueballs and not gluons. However, the spectrum of the propagator reflects the normalization in that all energies are compared with the energy of the modified vacuum with the fixed source present. Therefore the frequency spectrum of the propagator (3.24) is the difference between the physical glueball frequency and the heavy-light glueball mass.

Because the spectrum of the propagator is physical, and the vacuum state has been removed explicitly, we expect that the dominant contribution to the propagator at large distances is that of the lowest lying glueball of mass $m_G$--thus

$$G F^{ab}_{\mu\nu\rho\sigma}(x,y) \underset{\Lambda|x-y|>>1}{\sim} \exp\left[-(m_G-m_H)|x-y|\right] . \qquad (3.25)$$

The distance scale $1/\Lambda$ is the confinement scale or size of the glueball.

At short distances we expect that the screening gluons will have only a small effect upon the propagation, owing to asymptotic freedom, and we see no reason why we shouldn't recover the perturbation theoretical form of the propagator, albeit, the specific form of the perturbative propagator that has the fixed sources in the in and out states.

4. Strong Coupling Lattice Results for the Axial Gauge Propagtor

a. Strong Coupling Preliminaries

In the strong-coupling limit ($\beta = 4/g^2 \to 0$) operator expectation values are evaluated by expanding the integrand in (2.3) in a Taylor series in $\beta$ and integrating over the gauge link variables [17]. As a preliminary we review the well-known evaluation of the expectation value of the m by n Wilson loop $\langle W_{mn} \rangle$ and the pair of thermal Wilson lines $\langle L_{\vec{x}} L^{\dagger}_{\vec{y}} \rangle$, illustrated in Figs. 3a and b. Each plaquette contributes a factor

$$\exp\,(\beta/2\ \mathrm{Tr}\ U_P) \approx 1 + \beta/2\ \mathrm{Tr}\ U_P \tag{4.1}$$

to the integrand. For most plaquettes the zeroth order term suffices. However, the Wilson loop or thermal Wilson line introduces a gauge link matrix in the integrand of the numerator of (2.3) that must be compensated by including a higher order term in (4.1), according to the second and third of the following identities

$$\int [dU] = 1; \quad \int [dU]\ U = 0; \quad \int [dU]\ U_{\alpha\beta}\ U^{*}_{\gamma\delta} = \frac{1}{2}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta}\ . \tag{4.2a}$$

$$\int [dU]\ U^{*}_{\alpha\beta}\ U^{*}_{\gamma\delta}\ U_{\mu\nu}\ U_{\rho\sigma} = \frac{1}{3}\,(\delta_{\alpha\mu}\,\delta_{\beta\nu}\,\delta_{\gamma\rho}\,\delta_{\delta\sigma} + \delta_{\alpha\rho}\,\delta_{\beta\sigma}\,\delta_{\gamma\mu}\,\delta_{\delta\nu})$$

$$- \frac{1}{6}\,(\delta_{\alpha\rho}\,\delta_{\beta\nu}\,\delta_{\gamma\mu}\,\delta_{\delta\sigma} + \delta_{\alpha\mu}\,\delta_{\beta\sigma}\,\delta_{\gamma\rho}\,\delta_{\delta\nu})\ . \tag{4.2b}$$

The minimal additional plaquette factors are also shown in Figs. 3a and b. They correspond to a minimal tiling of the diagram. We obtain easily

$$\langle W_{mn} \rangle = (\beta/4)^{mn} \tag{4.3}$$

and

$$\langle L_x L^{\dagger}_y \rangle = \frac{1}{2}\,[\frac{1}{2}\,(\beta/4)^{N_t}]^{|x-y|} \tag{4.4}$$

The string tension $\kappa$ is obtained from $W_{mn} \sim \exp(-\kappa a^2 mn)$ so that $\kappa a^2 = -\ln(\beta/4)$ [18]. The thermal Wilson lines measure the "free energy" of a separated pair of fixed sources in the fundamental representation [19].

$$\langle L_x L_y^\dagger \rangle = \exp(-a N_t F(|x-y|)) \tag{4.5}$$

so that at low temperatures (large $N_t$)

$$F(|x-y|) = \kappa a |x-y| \ . \tag{4.6}$$

The "free energy" of a fixed source in the adjoint representation can be obtained from

$$\langle L_x L_x^\dagger - 1 \rangle = \exp(-a N_t F_A) \ . \tag{4.7}$$

Subtracting 1 removes the color singlet component, and requires the higher order contribution represented by the plaquettes in Fig. 4, giving

$$aF_A \underset{N_t \to \infty}{\sim} am_A = -4 \ln(\beta/4) \ . \tag{4.8}$$

Since the fixed adjoint source is screened by dynamical gluons, this result can be interpreted as giving the mass $m_A$ of a "heavy-light" glueball in the strong-coupling limit. The lowest glueball mass can be obtained in the strong coupling limit from the connected correlation between two plaquette operators. Thus for two facing plaquettes separated along one of the lattice axes by a distance d we have

$$\langle |\square| \ |\square| \rangle_d - \langle |\square| \rangle^2 \underset{d \to \infty}{\sim} \exp(-m_G d) \ . \tag{4.9}$$

The leading order strong coupling contribution again comes from a square cylindrical diagram and gives

$$am_G = -\ 4\ \ell n(\beta/4)\ .$$

Thus the lightest glueball and the lightest heavy-light glueball have the same mass in the strong coupling limit.

b. The Naive Gluon Propagator in Temporal Axial Gauge

We turn now to an evaluation of the naive gluon propagator (3.1) in super-temporal gauge in the strong coupling limit. The leading order strong coupling contribution comes from the plaquettes shown in Fig. 5. The square cylindrical structure along the entire string is required. We find that

$$3g^2a^4\ G^{\alpha\beta}_{F_{\mu\nu\rho\sigma}}(x,y) \underset{\Lambda\ell_A \gg 1}{\sim} \delta_{\alpha\beta}\ \exp\ (-m_A\ \ell_A) \tag{4.10}$$

where $\ell_A$ is the length of the string.

The similarity between the structure in Figs. 4 and 5 suggests that a "heavy-light" glueball moves along the line of the adjoint string. In the continuum limit, we expect that as long as the string segments are very long compared with the confinement scale, the propagator will be of the form

$$G^{\alpha\beta}_{F_{\mu\nu\rho\sigma}}(x,y)\ \alpha\ \ \delta_{ab}\ \exp\ (-\ m_A\ \ell_A) \tag{4.11}$$

where $m_A$ is the mass of the "heavy-light" glueball and $\ell_A$ is the length of the string. Even though there appear to be no infrared divergences, the mass $m_A$ is ultraviolet divergent, because of the point source. We have not attended to the details of removing these divergences. Nevertheless, it is still clear that the form of the naive propagator is far from what is usually expected: In particular our manner of incorporating the residual gauge fixing has severely disrupted the Poincaré invariance beyond what is expected in a temporal axial gauge. Furthermore, the long distance behavior is controlled by the mass of fictitious color-singlet objects.

Similar reasoning shows that the naive quark propagator (in a theory with dynamical quarks, of course) has the corresponding form

$$S(x,y) \underset{\Lambda \ell_H >> 1}{\sim} \exp(-\mu_H \ell_H) \qquad (4.12)$$

where $\mu_H$ is the mass of a color singlet meson with a fixed source in the fundamental representation surrounded by one or more light screening quarks and $\ell_H$ is the length of the string.

c. The Improved Gluon Propagator in Temporal Axial Gauge

We now consider the improved lattice version of the propagator (3.24). There are two terms in (3.24). The diagram for the first term is shown in Fig. 6. The string (shown as a heavy line) is now wrapped around from $\tau = 0$ to $\tau = \beta_t$. As $\beta_t \to \infty$ the functional integration for very early and very late time is dominated by the lowest energy state containing the corresponding fixed point source as required by the matrix elements in Eq. (3.24). To represent the second term, i.e. the disconnected part, we use Fig. 6, but enlarge the lattice by making the interval $|\tau_x - \tau_y|$ grow to infinity. The functional integration in the region between $\tau_x$ and $\tau_y$ will then be dominated by the vacuum state, as required in the matrix element.

In the strong coupling limit the plaquettes contributing to the propagator in leading order are shown in Fig. 7. Subtracting the disconnected part has the effect of eliminating all graphs that do not connect x to y directly: (i.e. those that connect x to y only by wrapping around from $\tau = 0$ to $\tau = \beta_t$ are eliminated.) We see that the structure lying along the string is as before, but the structure connecting x and y directly has the form of the glueball propagator. Thus we expect that in the continuum limit the large distance behavior of this propagator is governed by the lightest glueball in the spectrum. Correspondingly, with dynamical quarks present the asymptotic behavior of the improved quark propagator is expected to be dominated by the lightest meson containing the quark in question.

## 5. Summary and Conclusions

We have shown that in axial gauges, correlation products of charge dependent operators contain hidden, spurious non-dynamical sources that affect the propagation of dynamical fields. Arguing from strong-coupling lattice gauge theory, we propose that in confining non-abelian gauge theories, the behavior of quark and gluon propagators in axial gauges is controlled at long distances by low-lying color singlet states. In the naive form of the propagator these color singlet states contain the spurious source. With an improved form of the propagator, proposed in Sec. 3e, these color singlet states are the physical hadrons. At distances short compared to the confinement scale, we expect, in asymptotically free theories, that a perturbative form of the propagator with spurious sources is, nevertheless, valid.

There have been various efforts to determine the long-distance behavior of the gluon and quark propagators in axial gauges using the Dyson-Schwinger equations, the Ward-Takahashi identities, and additional assumptions [2-5]. The results apparently disagree with ours. Although there may be subtleties in taking the continuum limit, we are confident that the spurious sources are present in the continuum limit and that they affect the propagation. Therefore the additional assumptions of the Dyson-Schwinger approach must be questioned.

Finally, we should reiterate that none of the peculiar effects of spurious sources plagues calculations with gauge independent operators and conserved currents.

## Acknowledgement

We thank Jim Ball, Marshall Baker, and Fred Zachariasen for discussions concerning the Dyson-Schwinger equations. This work is supported in part

## Appendix A. Gluon Propagation Induced by the Vector Potential

### 1. Construction of the Improved Lattice Propagator

Because the vector potential transforms in a slightly more complicated way than the field tensor under a gauge transformation, the spurious sources that it creates and the strings associated with it are slightly more complicated. It is most direct to consider an infinitesimal string element, say in the fundamental representation:

$$C(x,x+dx) = P \exp ig \int_{x}^{x+dx} \sigma^{\alpha} \vec{A}^{\alpha}(x') \, d\vec{x}' , \qquad (A.1)$$

which transforms under a gauge transformation according to

$$C(x,x+dx) \to G_{x} \, C(x,x+dx) \, G^{\dagger}_{x+dx} . \qquad (A.2)$$

The super-temporal gauge gluon propagator induced by the vector potential can then be obtained from

$$G_{A_{\mu\nu}}(x,y) \, dx^{\mu} dy^{\nu} = -\frac{1}{3} < \mathrm{Tr}\, [\sigma^{\beta} \, C(y,y+dy)] \, (1 - |0><0|) \, P$$

$$\mathrm{Tr}\, [\sigma^{\beta} \, C(x,x+dx)]>_{ST} , \qquad (A.3)$$

where the projection operator P enforces Gauss' law on the intermediate states with no spurious sources, and we have projected out the vacuum from the intermediate state. The in and out states must contain spurious sources, because C(x,x+dx) is not gauge invariant. The nature of these sources is revealed through the relation

$$P \, C(x,x+dx)_{ab} = C(x, x + dx) \, P_{\overline{x}a;x+dx,b} \qquad (A.4)$$

in analogy with the abelian case (3.18). The projection operator $P_{\overline{x}a;x+dx,b}$ selects a state with a pair of quark-like spurious states in the fundamental representation with color indices a (antiquark) and b (quark), separated by the distance dx. As before, if the residual gauge fixing condition acts at a time $t_0 > y_0$, the spurious sources appear at x and x+dx in both the bra and ket states, if we introduce the appropriate string operators. To exhibit in detail the dependence upon the labels of the fundamental sources in the in and out states, we use the identity

$$\sigma^\beta{}_{ab}\,\sigma^\beta{}_{cd} = -\,\delta_{ab}\,\delta_{cd} + 2\delta_{ad}\,\delta_{bc} \qquad (A.5)$$

so that

$$G_{A\mu\nu}(x,y)dx^\mu\,dx^\nu = \sum_{a,b,c,d} \frac{1}{3}\,{}_{\overline{x}a;x+dx,b}\!< C_{ba}(\vec{x}+d\vec{x},\vec{x},t_0)\; C_{dc}(\vec{y}+d\vec{y},\vec{y},t_0)$$

$$C_{cd}(\vec{y},\vec{y}+d\vec{y},y_0)\;(1 - |0\rangle\langle 0|)\;C_{ab}(\vec{x},\vec{x}+d\vec{x},x_0)\;>_{\overline{x}a,x+dx,b}$$

$$- \sum_{abcd} \frac{2}{3}\,{}_{\overline{x}a,x+dx,b}\!<C_{da}(\vec{y}+d\vec{y},\vec{x},t_0)\; C_{bc}(\vec{x}+d\vec{x},\vec{y},t_0)\; C_{cd}(\vec{y},\vec{y}+d\vec{y},y_0)$$

$$(1 - |0\rangle\langle 0|)\;C_{ab}(\vec{x},\vec{x}+d\vec{x},x_0)>_{\overline{x}a,x+dx,b}\;. \qquad (A.6)$$

On the lattice the infinitesimal string elements are replaced by the gauge link matrices.

$$C(\vec{x},\vec{x} + \hat{\mu}a,x_0) = U_{x,\mu}\;. \qquad (A.7)$$

The string connections corresponding to the second term in the correlation (A.6) are illustrated in Fig. 8.

The use of strings in the fundamental representation has the consequence of requiring states with spurious sources in that representation. Had we chosen a different string representation, correspondingly different sources would have been necessary.

2. Ultraviolet Singularities in the Continuum Limit

Presumably, the conventional propagator is obtained in the continuum limit $dx, dy \to 0$. We have not studied this limit thoroughly. It is clear that, the limit is subtle. In quantum electrodynamics with or without subsidiary gauge fixing, the vector potential is an ultraviolet singular operator in temporal axial gauge, i.e. all of its matrix elements on nonorthogonal states on the physical sector are infinite in the limit of zero lattice spacing. Since the string-bit operator in temporal gauge maps physical states into states containing fixed sources, which are orthogonal to the physical sector of the Hilbert space, we have, for any physical states $|\alpha\rangle$ and $|\alpha'\rangle$

$$\langle\alpha'|\ C(x,x+dx)\ |\alpha\rangle = 0\ . \tag{A.8}$$

Now if the vector potential is non-singular, we may expand C for infinitesimal dx, giving

$$\langle\alpha'|\ (1 + ie\ \vec{A}\cdot d\vec{x})|\alpha\rangle = 0\ , \tag{A.9}$$

which would imply that $\langle\alpha'|\alpha\rangle = 0$ for all physical states--a contradiction. Therefore $\vec{A}(\vec{x})$ is a singular operator on the physical sector. Mandelstam has noted similar problems with Green's functions involving $\vec{A}(x)$ when the subsidiary gauge is not fixed [6].

Although the QED vector potential is itself singular on the physical sector in temporal axial gauge, in non-abelian theories, matrix elements on the (color singlet) physical sector are trivially zero; moreover, matrix elements of a product of two vector potentials do not seem to suffer the same problem provided the subsidiary gauge is fixed. Therefore, it is possible the vector potential propagator is still well-defined. This point deserves further study.

3. Strong Coupling Limit of the Improved Propagator

Returning to the lattice version, we find that in the strong coupling limit the leading contribution to the propagators (A.4) has a square cylindrical structure connecting x to y. Thus the large distance behavior is given by the glueball propagator, as before. However, a difference is found in the structure surrounding the strings. Whereas with the field tensor correlation product, as shown in Figs. 5 and 7, a square cylindrical structure runs parallel to the adjoint string, here in leading order as shown in Fig. 9, the fundamental dipole source pair is tiled over as in Fig. 3(b) where it is separated by one lattice unit and becomes a square cylindrical structure only where the string pair coincides, as at the bottom of Fig. 8.

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Figure Captions

1. Example of a maximally connected tree on a periodic two-dimensional 5x5 lattice. (Note that periodicity requires links and sites on opposite edges to be equivalent.) The links are set to one in a super axial gauge.

2. Example of the location of plaquettes and the adjoint string (heavy line) for the naive propagator in the gauge of Fig. 1.

3. Bold lines: (a) 2x3 Wilson loop (b) pair of thermal Wilson lines separated by one lattice unit. Thin lines: minimum set of plaquettes for the leading strong coupling contribution.

4. The leading strong coupling contribution to the free energy of a fixed source in the adjoint representation is a square cylindrical array of plaquettes. The string for the adjoint source is the heavy line.

5. Leading order strong coupling contribution to the naive gluon propagator of Fig. 2.

6. Location of plaquettes and the adjoint string (heavy line) for the improved propagator in the gauge of Fig. 1.

7. Leading order strong coupling contribution to the improved gluon propagator of Fig. 6.

8. String connections for the propagator of the vector potential in the gauge of Fig. 1 as described in Appendix A.

9. Leading order strong coupling contribution to the gluon propagator induced by the vector potential corresponding to Fig. 8.

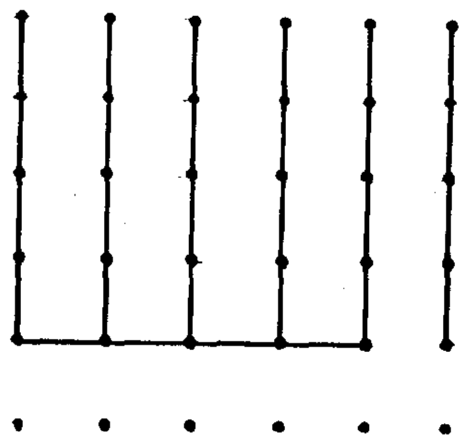

Figure 1

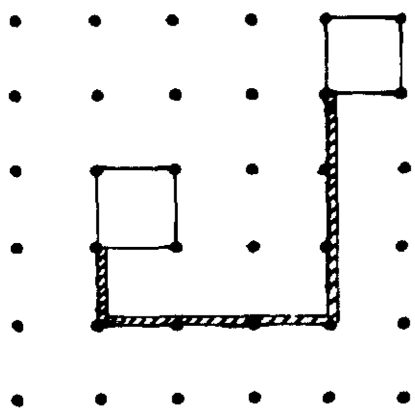

Figure 2

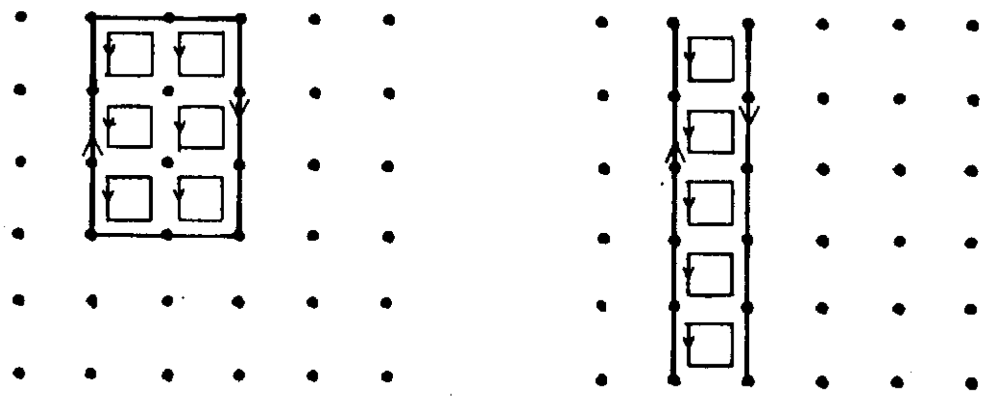

Figure 3

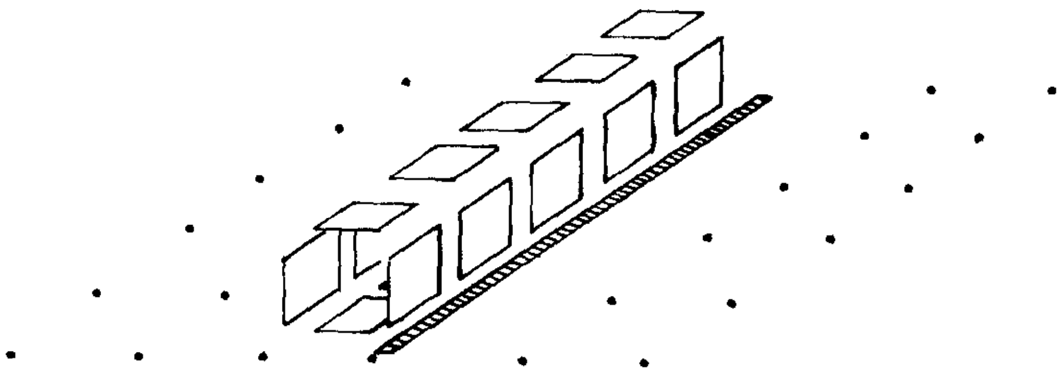

Figure 4

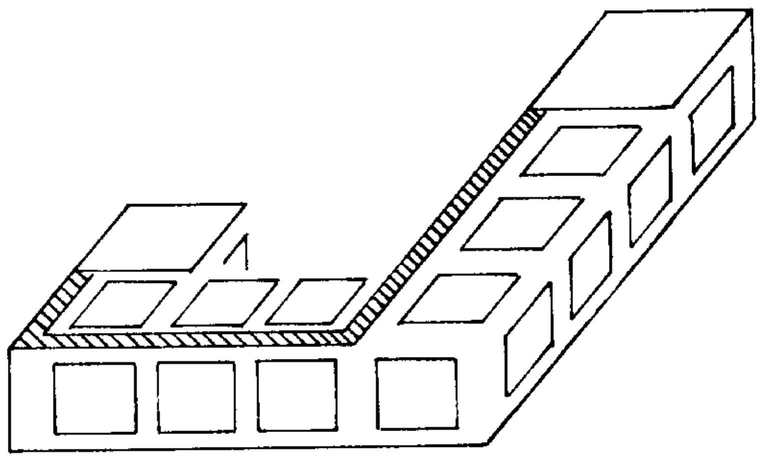

Figure 5

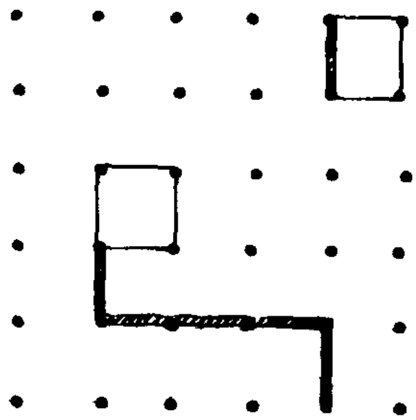

Figure 6

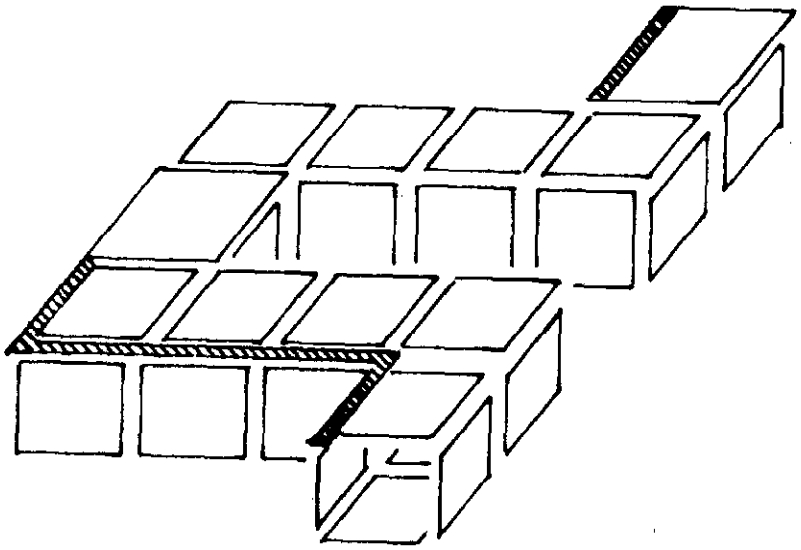

Figure 7

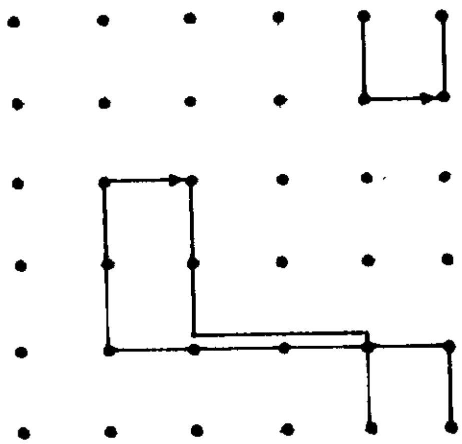

Figure 8

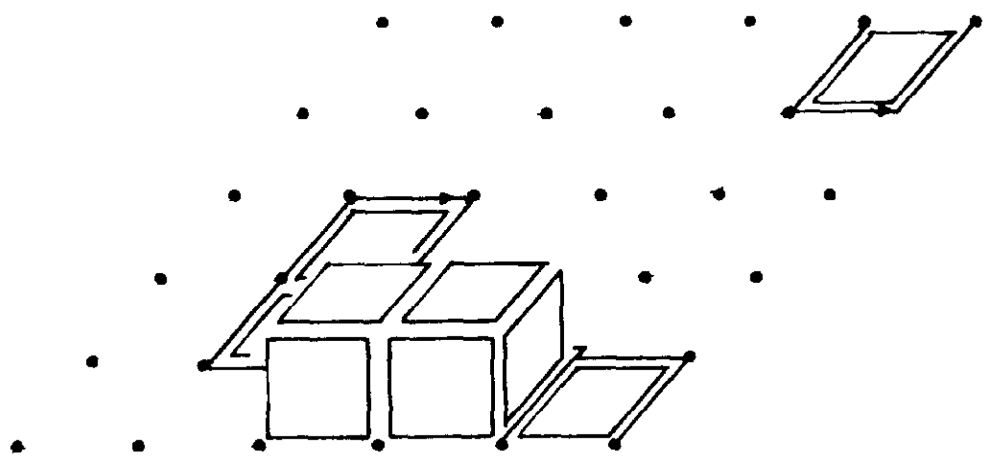

Figure 9