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INTRABEAM SCATTERING FORMULAE FOR ASYMPTOTIC BEAMS WITH UNEQUAL HORIZONTAL AND VERTICAL EMITTANCES

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Abstract

We use the Month-Weng discussion of the Piwinski intrabeam scattering theory to derive formulae which are useful for beams of asymptotically large energies. The results are relatively simple formulae which are applicable to elliptical as well as round beams and which give quite good agreement with the more exact strong-focussing theory as discussed by Bjorken and Mtingwa.

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I. Introduction

An important limitation on the stacking rate of antiprotons in the Pbar Source Accumulator and on the luminosity of the Tevatron is due to the multiple Coulomb scattering of particles within a beam, commonly referred to as intrabeam scattering. The original theory of this phenomenon is due to Piwinski [Ref. 1], where the variation of lattice functions around the accelerator ring was neglected. More recently, Bjorken and Mtingwa [Ref. 2] used a quantum field theory approach to discuss the same problem for strong-focussing accelerators wherein lattice parameters vary around the ring.

At various times, Piwinski, Sacherer and Möhl, and Martini have shown how to extend the original Piwinski formalism to the case of varying lattice parameters. Their results are nicely summarized by Martini in Ref. 3.

In the present work, we will extend the asymptotic expressions given in Month-Weng [Ref. 4] to the case of beams with unequal horizontal (H) and vertical (V) emittances. These formulae involve using average values for the lattice functions around the machine, and the question arises as to the accuracy of such smoothing approximations. To answer this question, we compare the results with the exact theory of Bjorken and Mtingwa [Ref. 2].

A word of explanation as to why one wishes to develop such equations is in order at this point. Intrabeam scattering causes the emittances, both longitudinal and transverse, to change with time. It is necessary to calculate this dependence for colliders in order to predict the luminosity $\mathcal{L}$ as a function of time. An examination of the formalism shows

$$\frac{1}{\varepsilon_T}\frac{d\varepsilon_T}{dt} = \tau_T^{-1}(\varepsilon_H, \varepsilon_V, A_b, \text{machine parameters}) \tag{1}$$

$$\frac{1}{A_b}\frac{dA_b}{dt} = \tau_p^{-1}(\varepsilon_H, \varepsilon_V, A_b, \text{machine parameters}) \tag{2}$$

where T stands for the transverse directions either H or V and τ_T, τ_p are the instantaneous lifetimes for transverse (ε_T) and longitudinal (A_b) emittances. If one could find simple analytical expressions for τ_T^{-1} and τ_p^{-1}, then one could easily integrate the above coupled equations to obtain the beam behavior with time.

The other approach has been to use the Bjorken-Mtingwa formalism to calculate τ_T^{-1} and τ_p^{-1} for values of $\varepsilon_H, \varepsilon_V$, and A_b that are encountered in the beam growth and to paramaterize these growth rates by, for instance, power laws:

$$\tau_H = \varepsilon_H^A A_b^B$$

$$\tau_p = \varepsilon_H^C A_b^D$$

These can be substituted into the differential equations above and integrated. The advantage of the asymptotic expressions over this procedure is that much less computer time is used, and the functional dependence on machine parameters is explicitly displayed. It will turn out that the accuracy of the asymptotic expressions is satisfactory for this use.

A few remarks on the physics of the process are in order. Consider the Tevatron beam at Fermilab with

$$V_{RF} = 1 \text{ MV} \tag{3}$$

$$A_b = 2 \text{ ev-sec (95\% longitudinal emittance)} \tag{4}$$

$$\varepsilon_H = 24\pi \text{ mm-mr (95\% H invariant emittance)} \tag{5}$$

If we calculate the transverse and longitudinal temperature of the beam (with s being the longitudinal variable), we find

$$
\begin{aligned}
< p_{\mathrm{T}} > &= 10 \text{ MeV/c} && (6)\\
< \delta p_s > &= 90 \text{ MeV/c} \quad \text{in the Lab.} \\
&= \frac{90\,\text{MeV/c}}{\gamma} = 90 \text{ Kev} \;\; \text{in the CM of the bunch.} && (7)
\end{aligned}
$$

Thus the beam is much bigger in the transverse direction than in the longitudinal direction. If we accelerate such a beam to higher energy the situation becomes worse since the momenta scale with γ as

$$
\begin{aligned}
< p_{\mathrm{T}} > &\sim \sqrt{\gamma} \\
< \delta p_s >_{\mathrm{cm}} &\sim \gamma^{-\frac{3}{4}}.
\end{aligned}
$$

The process which we consider here is for two protons to undergo Coulomb scattering with a concomitant shift of transverse energy into longitudinal energy. It would seem at first glance that this would result in cooling in the H,V directions and heating in the s direction. This is true for V, but due to dispersion (η) in the machine it is not true for H. Consider the case shown in Fig. 1.

The two particles imagined to be initially performing oscillations around a common equilibrium orbit will have after scattering, in regions of nonzero dispersion, different equilibrium orbits about which they will perform synchrotron and betatron oscillations. These radial jumps of the equilibrium orbits heat up the horizontal betatron oscillations.

It is clear that a mixing parameter exists that depends upon the relative sizes of $\sigma_{\mathrm{H}\beta}$ and $\eta\sigma_\eta$, where σ_η is the rms value of $\frac{\delta p}{p}$. Consider the Tevatron at E = 1000

GeV. We find that $\sigma_{H\beta} \sim 0.5$mm and $\eta\sigma_\eta \sim 0.24$ mm for the example we considered above. Thus since the betatron oscillations are large compared to the dispersion, all of the particles can scatter off each other. On the other hand, if $\eta\sigma_\eta > \sigma_{H\beta}$ then only a fraction of the beam could interact with itself, and the intrabeam scattering rate would slow down. The relevant parameter is defined as

$$\mathrm{T} = \frac{\sigma_{H\beta}^2}{\sigma_{H\beta}^2 + \eta^2\sigma_\eta^2}, \quad \text{where } 0 < T < 1.$$

A picture looks as shown in Fig. 2 for T $<<$ 1. In this case all of the particles are not able to scatter from each other at any given time and the rate of interaction is reduced.

II. Piwinski-Month-Weng Formulae

In the present work, we want to use the Month-Weng [Ref. 4] discussion of the Piwinski theory to derive large γ (ratio of particle energy to rest mass) asymptotic formulae for the intrabeam scattering growth rates. The results are to be applied to the Tevatron where $\gamma = 1000$ and are applicable to both elliptical ($\varepsilon_H \neq \varepsilon_V$) and round ($\varepsilon_H = \varepsilon_V$) beams. Note that Month and Weng calculate the growth rates of the coordinates σ_H, σ_V, and σ_η (rms value of relative momentum spread $\frac{\delta p}{p}$) whereas Bjorken and Mtingwa calculate the emittance growth rates. To compare the two sets of formulae we choose below to include the necessary factor of 2 in all the Month-Weng formulae. Thus, we will always consider emittance growth rates. We also point out that Month and Weng use the normalized 95% emittances which are $6\pi\gamma\beta$ times those defined by Bjorken and Mtingwa.

Keeping the above conventions in mind, the bunched beam emittance growth rates are given by:

$$\frac{1}{\tau_H} \equiv \frac{1}{\sigma^2_{H\beta}}\frac{d\sigma^2_{H\beta}}{dt} = 2A\left\{f(\frac{a}{b},\frac{1}{b},\frac{c}{b}) + (1-T)\,f(\frac{b}{a},\frac{1}{a},\frac{c}{a})\right\} \tag{8}$$

$$\frac{1}{\tau_V} \equiv \frac{1}{\sigma^2_V}\frac{d\sigma^2_V}{dt} = 2A\,f(a,b,c) \tag{9}$$

$$\frac{1}{\tau_p} \equiv \frac{1}{\sigma^2_\eta}\frac{d\sigma^2_\eta}{dt} = 2AT\,f(\frac{b}{a},\frac{1}{a},\frac{c}{a}), \tag{10}$$

where

$$a = \frac{1}{\sqrt{T}\,\sigma_\eta}\left[\frac{\gamma\varepsilon^N_V}{6\pi\beta\bar{\beta}_V}\right]^{1/2} \tag{11}$$

$$b = \left[\frac{\varepsilon^N_V\bar{\beta}_H}{\varepsilon^N_H\bar{\beta}_V}\right]^{1/2} \tag{12}$$

$$c = \left[\frac{\beta\gamma\varepsilon^N_V}{6\pi\bar{\beta}_V\,r_p}\right]^{1/2}\left[\frac{\sqrt{6\pi}\,\sigma_s}{\beta N_b}\right]^{1/6}\left[\frac{\varepsilon^N_V\varepsilon^N_H\bar{\beta}_V\bar{\beta}_H}{T}\right]^{1/12} \tag{13}$$

$$A = \frac{27\pi r_p^2\,m_p\,N_b}{8\,\beta\gamma\varepsilon^N_H\varepsilon^N_V\,A_b} \tag{14}$$

$$T = \frac{\sigma^2_{H\beta}}{\sigma^2_H} = \frac{\sigma^2_{H\beta}}{\sigma^2_{H\beta} + \bar{\eta}^2\sigma^2_\eta} = \frac{\varepsilon^N_H\bar{\beta}_H}{\varepsilon^N_H\bar{\beta}_H + 6\pi\beta\gamma\bar{\eta}^2\sigma^2_\eta} \tag{15}$$

and

$$\sigma_{H\beta} = \left[\frac{\varepsilon^N_H\bar{\beta}_H}{6\pi\beta\gamma}\right]^{1/2} \tag{16}$$

$$\sigma_V = \left[\frac{\varepsilon^N_V\bar{\beta}_V}{6\pi\beta\gamma}\right]^{1/2} \tag{17}$$

$$\sigma_\eta\,\sigma_s = \frac{\beta c}{6\pi\bar{p}}\,A_b. \tag{18}$$

σ_s is the rms bunch length; $\bar{\beta}_H$, $\bar{\beta}_V$, and $\bar{\eta}$ are the average values of the lattice betatron and dispersion functions; m_p and r_p are the proton mass (938 MeV) and classical radius (1.54×10^{-18} m); N_b is the number of particles in a bunch; ε^N_H and ε^N_V are the normalized 95% transverse emittances; T is the fraction of the center-of-mass energy spread participating in the energy exchange; and A_b is the longitudinal 95% invariant emittance. The well known Piwinski scattering function f(a,b,c) is given by:

$$
\begin{aligned}
f(a,b,c) &= 2\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} d\phi d\theta dr \\
&\times \sin\theta e^{-r\left[\cos^2\theta + (a^2\cos^2\phi + b^2\sin^2\phi)\sin^2\theta\right]} \\
&\times \ln(c^2 r)(1 - 3\cos^2\theta), \qquad (19)
\end{aligned}
$$

and satisfies the following relations:

$$f(a,b,c) = f(b,a,c) \qquad (20)$$

$$f(a,b,c) + \frac{1}{a^2} f(\frac{1}{a}, \frac{b}{a}, \frac{c}{a}) + \frac{1}{b^2} f(\frac{1}{b}, \frac{a}{b}, \frac{c}{b}) = 0. \qquad (21)$$

The above formulae are handy in that approximate emittance growth rates can be obtained using only the average lattice functions obtained from:

$$\bar{\beta}_H = \frac{R}{\nu_H} \qquad (22)$$

$$\bar{\beta}_V = \frac{R}{\nu_V} \qquad (23)$$

$$\bar{\eta} = \frac{R}{\nu_H^2}, \qquad (24)$$

where R is the average accelerator radius and ν_H and ν_V are the betatron tunes.

The theory of intrabeam scattering has been verified for a variety of beam parameters at CERN, the first being in the 22.5 GeV/c ISR (Intersecting Storage Ring) [Ref. 5,6]. Subsequently, the theory has been verified in the 270 GeV/c SPS (Super Proton Synchrotron) [Ref. 4,6,7,8], in the AA (Antiproton Accumulator) [Ref. 3,9] with a 3.41 GeV/c intense proton stack, and also in the AA with a 3.41 GeV/c antiproton stack [Ref. 3].

In the next section, we derive the large γ asymptotic formulae for the emittance growth rates due to intrabeam scattering. The numerical results will be given in Section IV.

III. Asymptotic Formulae

To derive the asymptotic formulae, let us first consider the Piwinski scattering function f(a,b,c) defined by Eq. (19). The large γ approximation corresponds to a and c large while b $\sim$ 1. It is straightforward to do the ϕ integration giving

$$\begin{aligned} f(a,b,c) &= 4\pi \int_0^\infty \int_0^\pi e^{-r\left[\cos^2\theta + \frac{a^2+b^2}{2}\sin^2\theta\right]} \\ &\quad \times I_0\left(r\,\frac{a^2-b^2}{2}\sin^2\theta\right)\sin\theta(1-3\cos^2\theta)\ln(c^2 r)\,d\theta dr, \end{aligned} \tag{25}$$

where I_0 is the zeroth order modified Bessel function of the first kind.

Then for large a we can use

$$I_0(x) \underset{\text{large } x}{\simeq} \frac{e^x}{\sqrt{2\pi x}} \tag{26}$$

to get

$$\begin{aligned} f(a,b,c) &\simeq \frac{4\sqrt{\pi}}{a}\int_0^\infty \int_0^\pi \frac{e^{-r(\cos^2\theta + b^2\sin^2\theta)}}{\sqrt{r}} \\ &\quad \times (1-3\cos^2\theta)\ln(c^2 r)\,d\theta\,dr. \end{aligned} \tag{27}$$

Since there is an exponential fall off in r, in the integrand we can make the large c approximation $\ln(c^2 r) \simeq 2\ln c$. Further, by a series of manipulations, the θ integration can be accomplished giving:

$$f(a,b,c) \simeq \frac{-4\pi^{3/2}\ln c}{a}\, g(b), \tag{28}$$

where

$$g(b) = \int_0^\infty \frac{e^{-r(\frac{b^2+1}{2})}}{\sqrt{r}} \left[I_0(\frac{b^2-1}{2}r) + 3I_1(\frac{b^2-1}{2}r) \right] dr. \tag{29}$$

Since

$$\begin{aligned} I_0(0) &= 1 \\ I_1(0) &= 0, \end{aligned} \tag{30}$$

we have $g(1) = \sqrt{\pi}$ and

$$f(a,1,c) \underset{\text{large a,c}}{\simeq} -\frac{4\pi^2 \ln c}{a}, \tag{31}$$

a well-known result for round beams.

Next, we concentrate our efforts on evaluating the function g(b). First take $b > 1$. Set

$$x = \frac{b^2-1}{2} r, \tag{32}$$

so that

$$g(b) = \sqrt{\frac{2}{b^2-1}} \int_0^\infty \frac{e^{-x(\frac{b^2+1}{b^2-1})}}{\sqrt{x}} \left[I_0(x) + 3I_1(x) \right] dx. \tag{33}$$

Then use the formula

$$\int_0^\infty e^{-tz(z^2-1)^{-1/2}} I_\mu(t)\, t^\nu dt = \frac{\Gamma(\nu+\mu+1) P_\nu^{-\mu}(z)}{(z^2-1)^{-\frac{1}{2}(\nu+1)}} \quad [\mathrm{Re}(\nu+\mu) > -1]. \tag{34}$$

Set

$$\alpha \equiv \frac{b^2+1}{b^2-1} = \frac{z}{\sqrt{z^2-1}}, \tag{35}$$

which gives

$$z = \sqrt{\frac{\alpha^2}{\alpha^2 - 1}}. \tag{36}$$

We thus arrive at

$$g(b) = \sqrt{\frac{2\pi}{(b^2-1)\sqrt{\alpha^2-1}}}\left[P^0_{-\frac{1}{2}}\left(\sqrt{\frac{\alpha^2}{\alpha^2-1}}\right) + \frac{3}{2}P^{-1}_{-\frac{1}{2}}\left(\sqrt{\frac{\alpha^2}{\alpha^2-1}}\right)\right], b > 1, \tag{37}$$

where $P^{-\mu}_{\nu}$ are the Associated Legendre Functions.

Next, consider $0 < b < 1$. First, note that $I_0(x)$ and $I_1(x)$ are even and odd functions, respectively, in x. So, rewrite Equation (29) as

$$g(b) = \int_0^\infty \frac{e^{-r(\frac{b^2+1}{2})}\left[I_0\left(\frac{1-b^2}{2}r\right) - 3I_1\left(\frac{1-b^2}{2}r\right)\right] dr}{\sqrt{r}} \tag{38}$$

Then set

$$x = \frac{1 - b^2}{2} r, \tag{39}$$

so that now

$$g(b) = \sqrt{\frac{2}{1 - b^2}} \int_0^\infty \frac{e^{-x(\frac{b^2+1}{1-b^2})}\left[I_0(x) - 3I_1(x)\right] dx}{\sqrt{x}}. \tag{40}$$

We now use Equation (34) again by first setting

$$\tilde{\alpha} \equiv \frac{b^2 + 1}{1 - b^2} = \frac{z}{\sqrt{z^2 - 1}} \tag{41}$$

We arrive at

$$g(b) = \sqrt{\frac{2\pi}{(1-b^2)\sqrt{\tilde{\alpha}^2-1}}}\left[P^0_{-\frac{1}{2}}\left(\sqrt{\frac{\tilde{\alpha}^2}{\tilde{\alpha}^2-1}}\right) - \frac{3}{2}P^{-1}_{-\frac{1}{2}}\left(\sqrt{\frac{\tilde{\alpha}^2}{\tilde{\alpha}^2-1}}\right)\right], b < 1. \tag{42}$$

But

$$\sqrt{\frac{\alpha^2}{\alpha^2 - 1}} \text{ and } \sqrt{\frac{\tilde{\alpha}^2}{\tilde{\alpha}^2 - 1}} \text{ reduce to } \frac{b^2 + 1}{2b}, \tag{43}$$

and

$$\sqrt{\frac{2\pi}{(b^2 - 1)\sqrt{\alpha^2 - 1}}} \text{ and } \sqrt{\frac{2\pi}{(1 - b^2)\sqrt{\tilde{\alpha}^2 - 1}}} \text{ reduce to } \sqrt{\frac{\pi}{b}} \tag{44}$$

in the two domains of b, so that the Master Equation becomes

$$g(b) = \sqrt{\frac{\pi}{b}}\left[P^0_{-\frac{1}{2}}\left(\frac{b^2+1}{2b}\right) \pm \frac{3}{2}P^{-1}_{-\frac{1}{2}}\left(\frac{b^2+1}{2b}\right)\right], \begin{pmatrix} b & \geq & 1 \\ b & \leq & 1 \end{pmatrix}. \tag{45}$$

Note that

$$P^{-1}_{-\frac{1}{2}}(1) = 0 \tag{46}$$

$$P^{0}_{-\frac{1}{2}}(1) = 1, \tag{47}$$

so that g(b) approaches $\sqrt{\pi}$ smoothly as $b \to 1$ from both above and below. Thus the $\begin{pmatrix} b & > & 1 \\ b & < & 1 \end{pmatrix}$ domains were changed to $\begin{pmatrix} b & \geq & 1 \\ b & \leq & 1 \end{pmatrix}$ in Equation (45).

To further evaluate g(b), first note that for $\mu = 0$,

$$P^0_\nu(z) = P_\nu(z) \tag{48}$$

where $P_\nu(z)$ are called simply the Legendre Functions. An important relation is

$$P^{-1}_{-\frac{1}{2}}(z) = (z^2 - 1)^{-\frac{1}{2}} \int_1^z P_{-\frac{1}{2}}(\tilde{z})\, d\tilde{z}. \tag{49}$$

Also, we have the representation

$$P_{-\frac{1}{2}}(z) = \frac{1}{\pi}\int_0^\pi (z + \sqrt{z^2 - 1}\cos\phi)^{-\frac{1}{2}}\, d\phi. \tag{50}$$

Upon substituting Equations (49) and (50) into Equation (45), g(b) can be readily evaluated on the computer, giving the values found in Table I. A graph of g(b) is shown in Fig. 3.

The function g(b) can also be fit with an empirical formula in the range $0.1 < b < 10$,

$$g(b) \sim 2.691 \left(1 - \frac{.2288964}{b}\right) \frac{1}{(1 + 0.16b)(1 + 1.35e^{-b/0.2})} .$$

The fit is good to 2% in the range $0.2 < b < 10$ and $| g(b) |$ is 14% high at $b = 0.1$. The accuracy is probably better than the asymptotic formulas for the lifetimes.

Now that we know how to compute the function f where the first and third arguments are large while the second argument is of order 1, we want to express all the emittance growth rates in terms of f of such arguments in order to obtain the asymptotic formulae. Using the relations given by Equations (20) and (21), we can readily manipulate Equations (8)-(10) to derive

$$\frac{1}{\tau_V} = 2A\, f(a, b, c) \tag{51}$$

$$\frac{1}{\tau_H} = 2A \left\{ f(\frac{a}{b}, \frac{1}{b}, \frac{c}{b}) \left[1 - (1 - T)\frac{a^2}{b^2}\right] - (1 - T)\, a^2 f(a, b, c)\right\} \tag{52}$$

$$\frac{1}{\tau_p} = -2AT\, a^2 \left[f(a, b, c) + \frac{1}{b^2} f(\frac{a}{b}, \frac{1}{b}, \frac{c}{b})\right] . \tag{53}$$

In each case the function f has been manipulated so that the first and third variables are large compared to the second in order that our asymptotic expression for f(a,b,c) can be used. Thus, Equations (28), (51)-(53), and Table I are all that one needs to calculate growth rates for large γ. In the next section, we give the numerical results for the proposed Tevatron upgrade.

IV. Results and Conclusions

It is interesting to examine the implications of the expressions (51) to (53). If we combine them with Equation (28), we obtain

$$\frac{1}{\tau_V} = -\frac{8\pi^{\frac{3}{2}}A}{a} g(b) \ln c \tag{54}$$

$$\frac{1}{\tau_H} = 8\pi^{\frac{3}{2}} A\, a(1-T) \left[g(b) + \frac{1}{b} g(\frac{1}{b}) \right] \ln c \tag{55}$$

$$\frac{1}{\tau_p} = 8\pi^{\frac{3}{2}} A\, a\, T \left[g(b) + \frac{1}{b} g(\frac{1}{b}) \right] \ln c \tag{56}$$

where we have neglected terms involving ln b and assumed that a >> 1.

Note that τ_V is negative, i.e. there is damping in the vertical plane. This effect is also present in the horizontal direction as the first term in the brackets of Eq. (52) but is completely overwhelmed by the heating term from the dispersion.

The dependance of the lifetime on the beam parameters is easy to see from Equations (54)-(56).

$$\tau_H \sim \frac{\gamma^{\frac{1}{2}} \sqrt{\varepsilon_V^N}\, \varepsilon_H^N\, A_b\, \sigma_\eta}{N_b} \frac{\sqrt{T}}{(1-T)} \left[g(b) + \frac{1}{b} g(\frac{1}{b}) \right]^{-1} \tag{57}$$

For fixed A_b,

$$\sigma_\eta \sim \frac{A_b^{\frac{1}{2}}\, V_{RF}^{\frac{1}{4}}\, h^{\frac{1}{4}}}{\gamma^{\frac{3}{4}}} \quad \text{(where h = harmonic number)}$$

which makes

$$\tau_H \sim \frac{\gamma^{-\frac{1}{4}} \sqrt{\varepsilon_V^N}\, \varepsilon_H^N\, A_b^{\frac{3}{2}}}{N_b} (V_{RF}\, h)^{\frac{1}{4}} \frac{\sqrt{T}}{(1-T)} \left[g(b) + \frac{1}{b} g(\frac{1}{b}) \right]^{-1} \tag{58}$$

The function $[g(b) + \frac{1}{b} g(\frac{1}{b})]$ is tabulated in Table I and is graphed in Fig. 4. It is amusing to consider the case of two symmetrical beams colliding when both have limited lifetimes due to intrabeam scattering. Then

$$\int \mathcal{L}dt \sim \frac{\mathcal{L}_o \tau}{2} \sim \frac{N_b^2 \tau}{\sigma_H \sigma_V} \sim \frac{\gamma\, N_b^2 \tau}{\sqrt{\varepsilon_H^N \varepsilon_V^N}}$$

or using (58)

$$\int \mathcal{L}dt \;\sim\; \gamma^{\frac{3}{4}}\, N_b \sqrt{\varepsilon_H^N}\, A_b^{\frac{3}{2}}\, (V_{RF}\, h)^{\frac{1}{4}}\, \frac{\sqrt{T}}{(1-T)} \left[g(b) + \frac{1}{b}\, g(\frac{1}{b}) \right]^{-1} \tag{59}$$

Because the lifetime is proportional to $\frac{1}{N_b}$, one power of N_b in the integrated $\mathcal{L}$ equation is cancelled out. One also notes the strong dependence on A_b which makes short bunch lengths incompatible with large $\int \mathcal{L}dt$. Of course, these conclusions are only qualitative since the time dependence is not exponential as assumed in Equation (59); actually the beams blow up more slowly as they get bigger.

The dependence of τ_p on parameters is given by

$$\tau_p \;\sim\; \frac{\gamma^{-\frac{1}{4}} \sqrt{\varepsilon_V^N}\, \varepsilon_H^N\, A_b^{\frac{3}{2}}\, (V_{RF}\, h)^{\frac{1}{4}}}{N_b \sqrt{T}} \left[g(b) + \frac{1}{b}\, g(\frac{1}{b}) \right]^{-1}$$

We now want to apply our asymptotic formulae to the parameters for the proposed Tevatron upgrade. The beam and lattice parameters we want to consider are as follows:

$$N_b = 10^{11} \tag{60}$$

$$\sigma_s = 0.2\,\text{m},\ 0.4\text{m} \tag{61}$$

$$\sigma_\eta = 4 \times 10^{-5} \text{ to } 1 \times 10^{-3} \tag{62}$$

$$\varepsilon_H^N = 5\pi,\ 12\pi,\ 20\pi \text{ mm} - \text{mrad} \tag{63}$$

$$\varepsilon_V^N = 12\pi,\ 20\pi,\ 80\pi \text{ mm} - \text{rad} \tag{64}$$

$$\gamma = 1000 \tag{65}$$

$$R = 1000\,\text{m} \tag{66}$$

$$\nu_H = 19.82686 \tag{67}$$

$$\nu_V = 19.86333, \tag{68}$$

The results are shown in Tables II-XV. Graphs of τ_p^{M-T}, τ_p^{B-M}, τ_H^{M-T}, τ_H^{B-M}, τ_V^{M-T}, τ_V^{B-M}, and the parameter T [cf. Equation (15)] versus σ_η for $\sigma_s = 0.2$ m, are shown in Figs. 5a-11b, where M-T denotes the results using the average lattice functions together with the asymptotic formulae discussed in this paper, and B-M refers to averaging the full theory of Ref. 2 around the Tevatron lattice. As can be seen, the agreement is remarkably good for both round and elliptical beams, especially for τ_H. The agreement for τ_V is reasonable although the times are too long for one to worry much about how well they agree, but as T is reduced to small values, typically $\tau_p^{M-T} \sim 2\,\tau_p^{B-M}$.

Fig. 12 shows how the lifetime depends on ε_V^N for a fixed ε_H^N and also compares the asymptotic expressions derived here with the exact calculation.

It is useful now to return to our original problem, i.e., the evolution of the beam with time. If we take the ratio of Equations (55) and (56), we obtain

$$\frac{\tau_p}{\tau_H} = \frac{1-T}{T} = \frac{\sigma_\eta^2\,\bar{\eta}^2}{\sigma_{H\beta}^2} \tag{69}$$

But $\varepsilon_{H\beta} \sim \sigma_{H\beta}^2$ and $\sigma_\eta^2 \sim A_b$. So

$$\frac{\tau_p}{\tau_H} \sim \frac{A_b}{\varepsilon_{H\beta}}. \tag{70}$$

Using Equations (8) and (10), we find

$$d\varepsilon_H = k_2\, dA_b \tag{71}$$

where k_2 depends only on machine parameters.

This is a very important result. It says that as the beam evolves with time, it follows a straight line trajectory in ε_H, A_b space.

Equation (69) can be recast in the form

$$\frac{\tau_H}{\tau_p}\frac{\sigma_\eta^2\gamma}{\varepsilon_H^N} = \frac{\bar{\beta}_H}{6\pi\bar{\eta}^2}, \tag{72}$$

a quantity which is dependent only upon lattice parameters and not upon beam parameters, ie. $\varepsilon_{H,V}$, σ_s, σ_η, γ. For the more exact theory of Ref. 2, one has [cf. Equation (4.9) of Ref. 2]

$$\frac{\tau_H}{\tau_p}\frac{\sigma_\eta^2\gamma}{\varepsilon_H^N} = \frac{2}{6\pi[\frac{\eta^2}{\beta_H} + \beta_H(\eta' - \frac{\beta'_H\eta}{2\beta_H})^2]}, \tag{73}$$

which is also dependent only upon lattice parameters. At each point of the lattice, this equation holds. However, a word of caution is in order if one wishes to calculate an average ratio of τ_H to τ_p around the accelerator lattice. If one uses Ref. 2 and calculates the average of the ratio, or $< \frac{\tau_H}{\tau_p} >$ around the Tevatron lattice, one obtains

$$< \frac{\tau_H}{\tau_p} > \frac{\sigma_\eta^2\gamma}{\varepsilon_H^N} = 2.4 \tag{74}$$

for all $\varepsilon_{H,V}^N$, σ_s, σ_η. But this is much larger than 0.413 which one obtains from Equation (72). Both τ_H and τ_p oscillate around the lattice, and thus a more meaningful ratio is obtained by first averaging τ_H and τ_p individually and then taking their ratio to compute $\frac{<\tau_H>}{<\tau_p>}$ instead of $< \frac{\tau_H}{\tau_p} >$.

A comparison of Equation (72) and $\frac{<\tau_H>}{<\tau_p>}\frac{\sigma_\eta^2\gamma}{\varepsilon_H^N}$ as computed according to Ref. 2 is contained in Table XVI for various beam parameters. It is seen that the smaller values of σ_η agree well with the asymptotic formula, but as σ_η increases, the exact calculation disagrees by about a factor of 2. It is also apparent that the trend is only a function of σ_η and not of the transverse emittances. This dependence on σ_η is only a result of how we decided to define the ratio of beam growth times for the overall lattice, namely as the ratio of their averages as opposed to the average of their ratios.

A program is given in the Appendix to follow the beam emittance growth. It is written in BASIC and calculates the horizontal and longitudinal emittances versus time due to the intrabeam scattering blowup. The program can be run interactively on the VAX cluster at Fermilab by the following commands:

```
$ COPY FNALD::USR$ROOT33:[ALVIN.MAC]MAC_CALC.BAS.
$ BASIC
(READY)
OLD MAC_CALC
(READY)
RUN .
```

The program will prompt you for certain inputs, such as changes in the beam parameters. Remember to input all information in uppercase letters.

Fig. 13 shows an example of a set of trajectories all starting from $A_b = 0.5$ ev-sec but with $\varepsilon_H^N = 5\pi, 12\pi, 20\pi$ mm-mrad. Note that the curves do not cross because then the differential equations (1) and (2) would not have unique solutions. The fact that k_2 in Equation (71) is only a function of machine constants guarantees that the curves are parallel. As a result, we see that injecting with a smaller transverse emittance gives a higher integrated luminosity even though the blowup is faster.

Finally we note that the Tevatron has nearly round beams. In the case we considered above, ε_H grows while ε_V stays constant. We should consider the modifications necessary if there is coupling between the H and V phase space. The ansatz proposed here and arrived at independently by D. Finley [Ref. 10] is that the blowup of ε_H should be shared equally between ε_H and ε_V. This seems plausible if the coupling is by particles moving about in phase space by Arnold diffusion. Indeed, the intrabeam scattering may even aid in this process.

In order to modify the program to accommodate this postulate, make the following change:

```
12565 EMIT_X   = EMIT_X + D_EMIT/2
```

and add

```
12566 EMIT_Y   = EMIT_Y + D_EMIT/2  .
```

In conclusion, we have derived large γ asymptotic formulae for emittance blowup rates due to intrabeam scattering. Moreover, for both round and elliptical beams, these asymptotic formulae give good agreement with calculations based upon the more complete theory of Ref. 2. Thus, the formulae should be adequate for predicting the evolution of luminosity with time in the present generation of high energy accelerators.

References

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10. D. Finley, private communication.

Figure Captions

Fig. 1 Sudden jumps in the equilibrium orbits of two protons undergoing Coulomb scattering causing heating of both the horizontal and longitudinal motion.

Fig. 2 Distributions of rms beam sizes due to momentum spread and horizontal betatron oscillations.

Fig. 3 The function $g(b)$ vs. b.

Fig. 4 The function $g(b) + \frac{1}{b} g(\frac{1}{b})$ vs. b.

Fig. 5a-11b Tevatron lattice intrabeam scattering growth times $\tau_p, \tau_{_H}$, and $\tau_{_V}$ calculated using the asymptotic formulae in this work (M-T) and also calculated from the strong-focussing theory of Ref. 2 (B-M) as a function of $\sigma_{_\eta}$, for $N = 10^{11}$ and various emittances and σ_s.

Fig. 12 Variation of $\tau_{_H}$ and τ_p vs. $\varepsilon_{_V}^N$ for $N_p = N_{\bar{p}} = 10^{11}, \varepsilon_{_H}^N = 20\pi$ mm-mrad,

$\sigma_s = 0.2$ m, $\sigma_{_\eta} = 0.2 \times 10^{-4}$. Shown are the intrabeam scattering growth times as calculated from the asymptotic formulae (M-T) and from the strong-focussing theory of Ref. 2 (B-M).

Fig. 13 An example of beam blowup for fixed $\varepsilon_{_V}^N$ and various initial $\varepsilon_{_H}^N$. Time is the parameter along the curves. The very fast blowup for small initial $\varepsilon_{_H}^N$ is evident. However, the integrated luminosity remains greater for the smaller initial ε_H^N since the curves do not cross.

effect of coulomb scattering as seen in the CM system

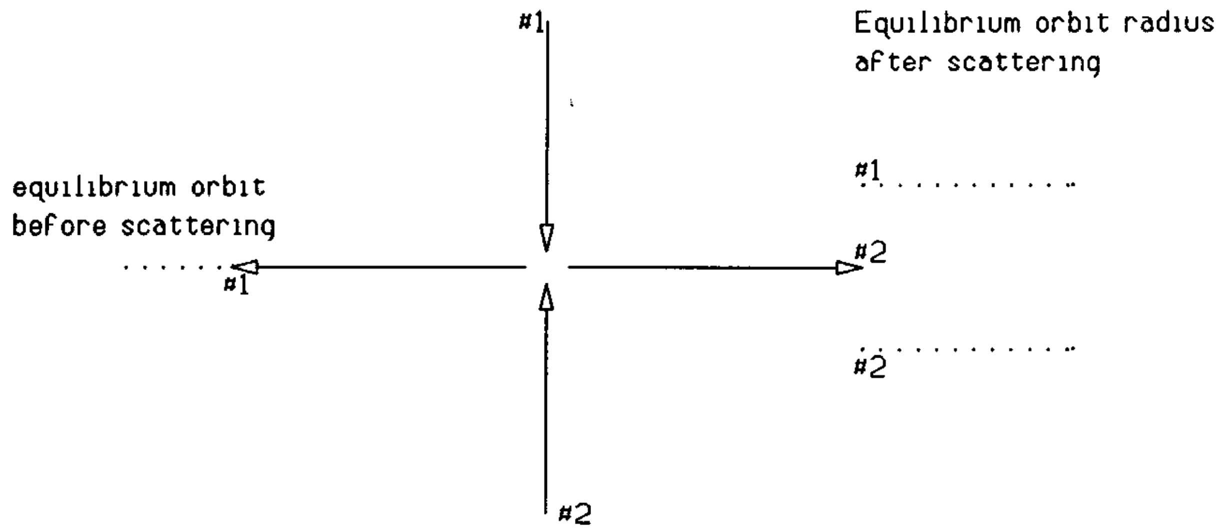
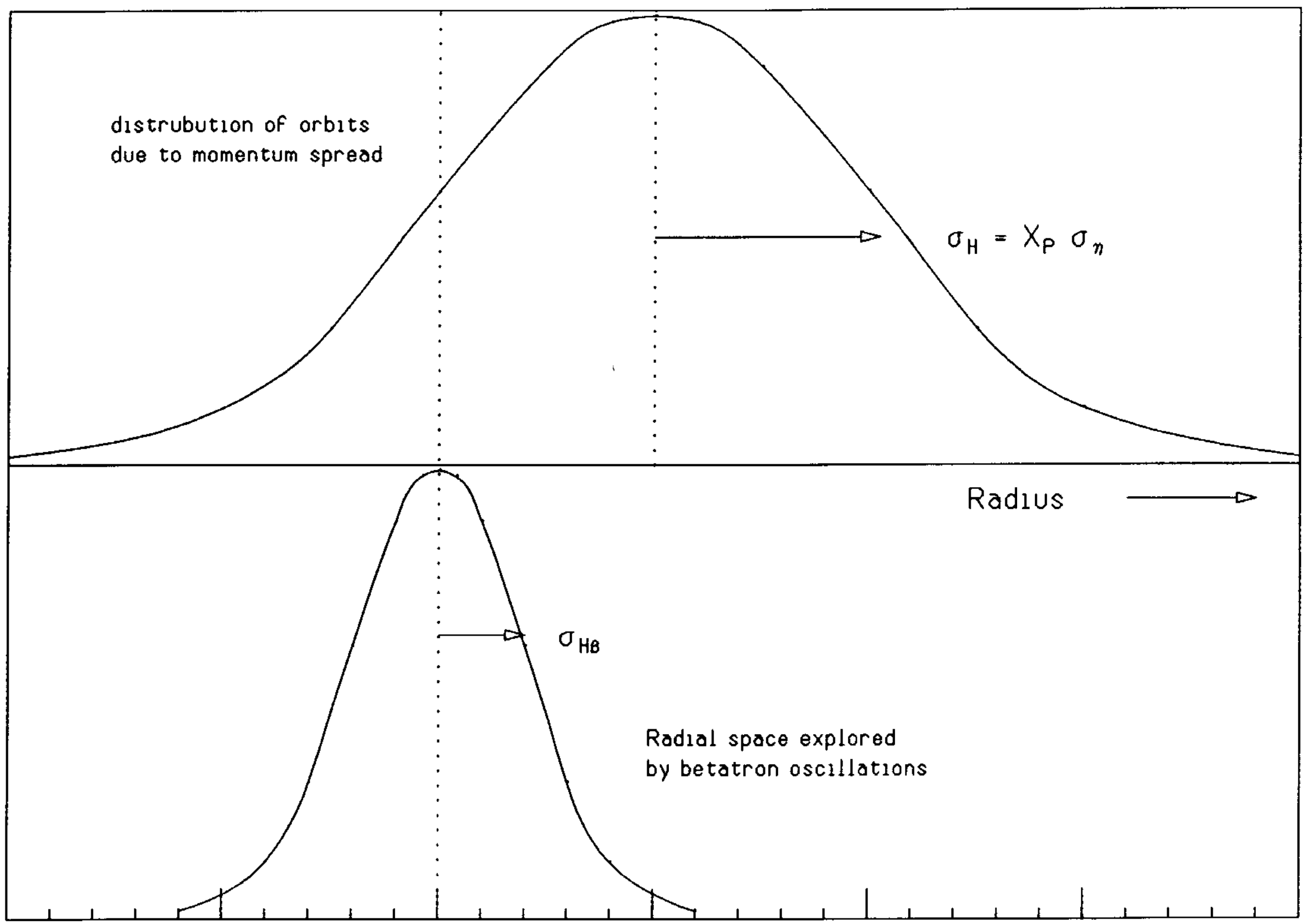
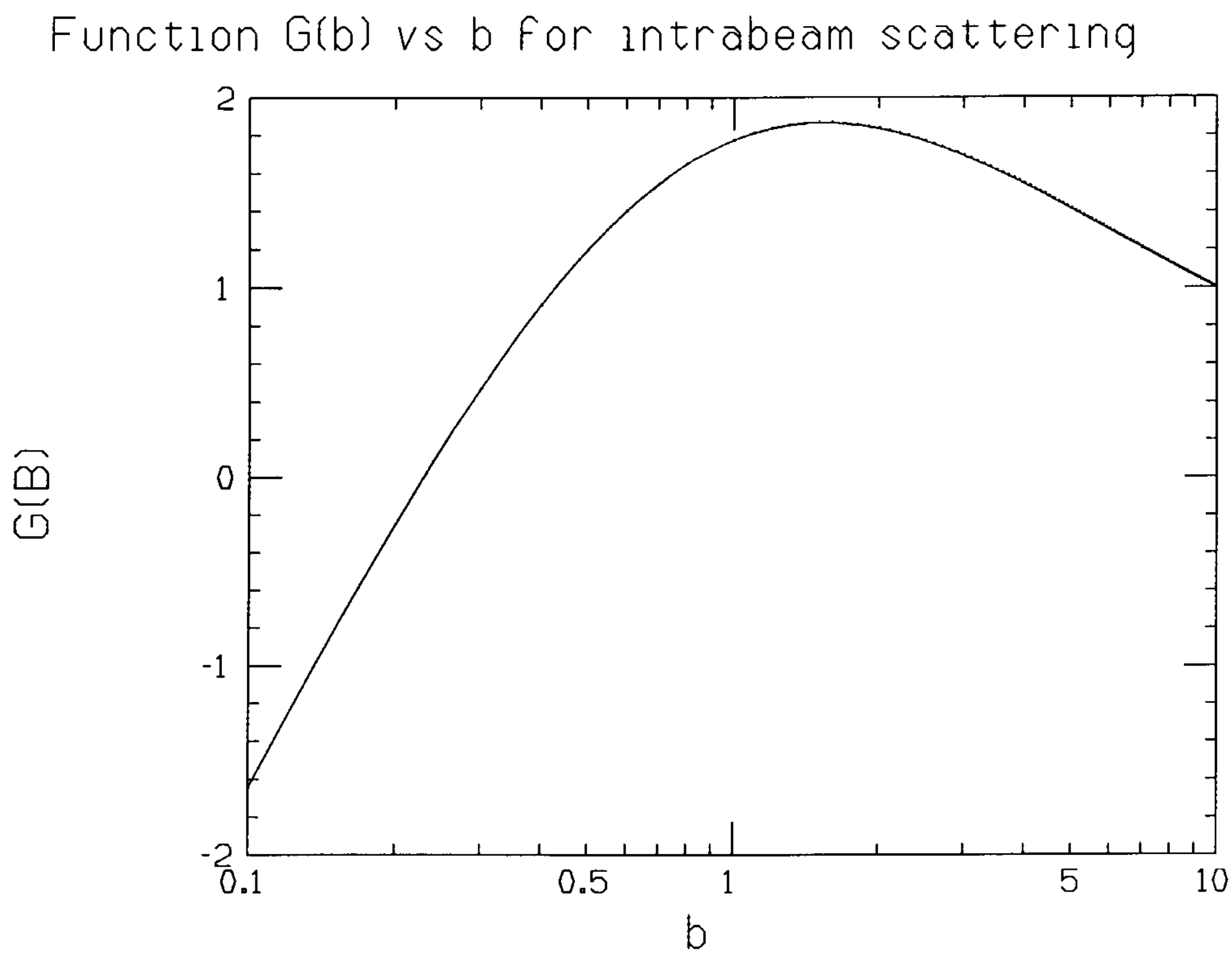


Fig.1



distrubution of orbits
due to momentum spread
$\sigma_H = X_P \sigma_\eta$
Radius
$\sigma_{H\beta}$
Radial space explored
by betatron oscillations

Fig. 2



Function G(b) vs b for intrabeam scattering
G(B)
2
1
0
-1
-2
0.1
0.5
1
5
10
b

FIG. 3

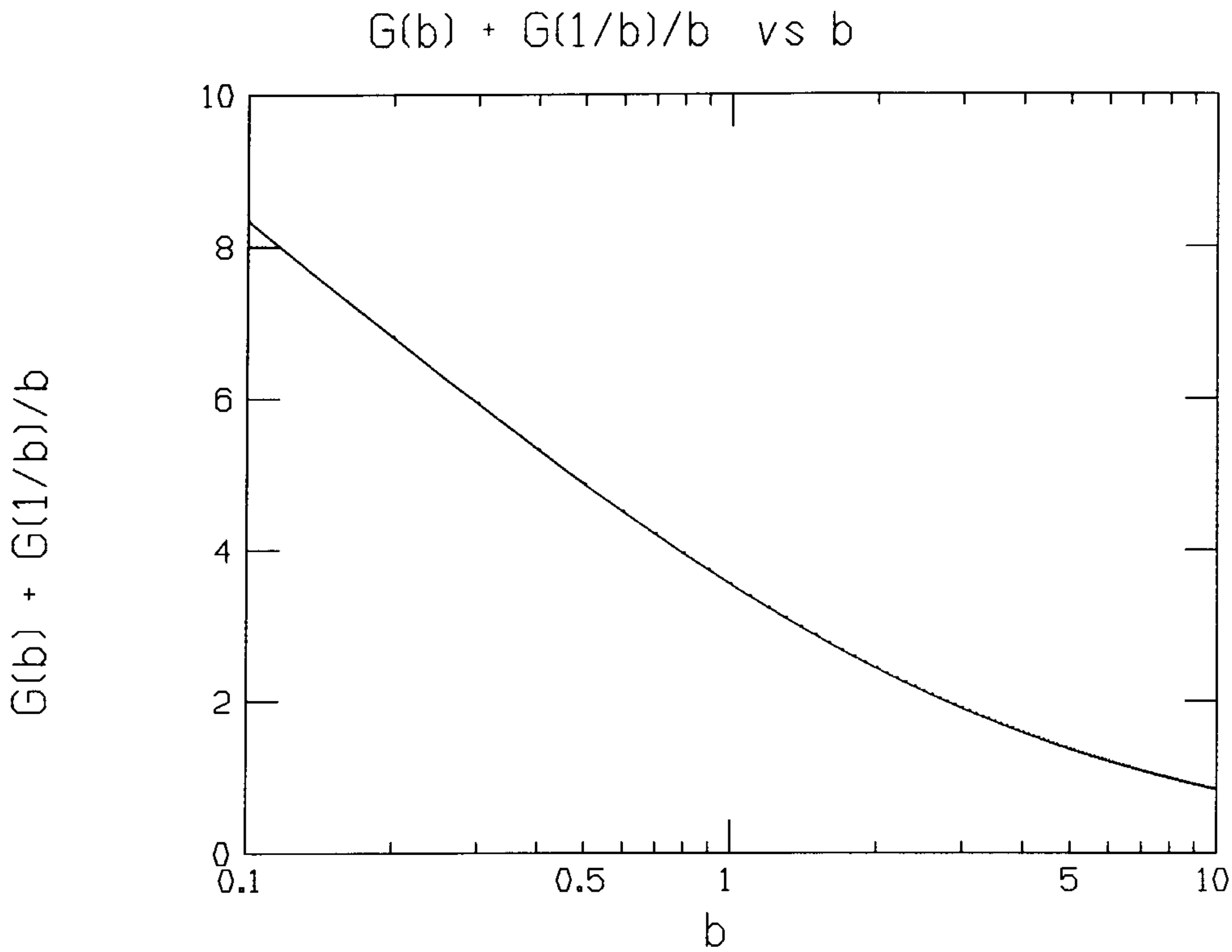


fig. 4

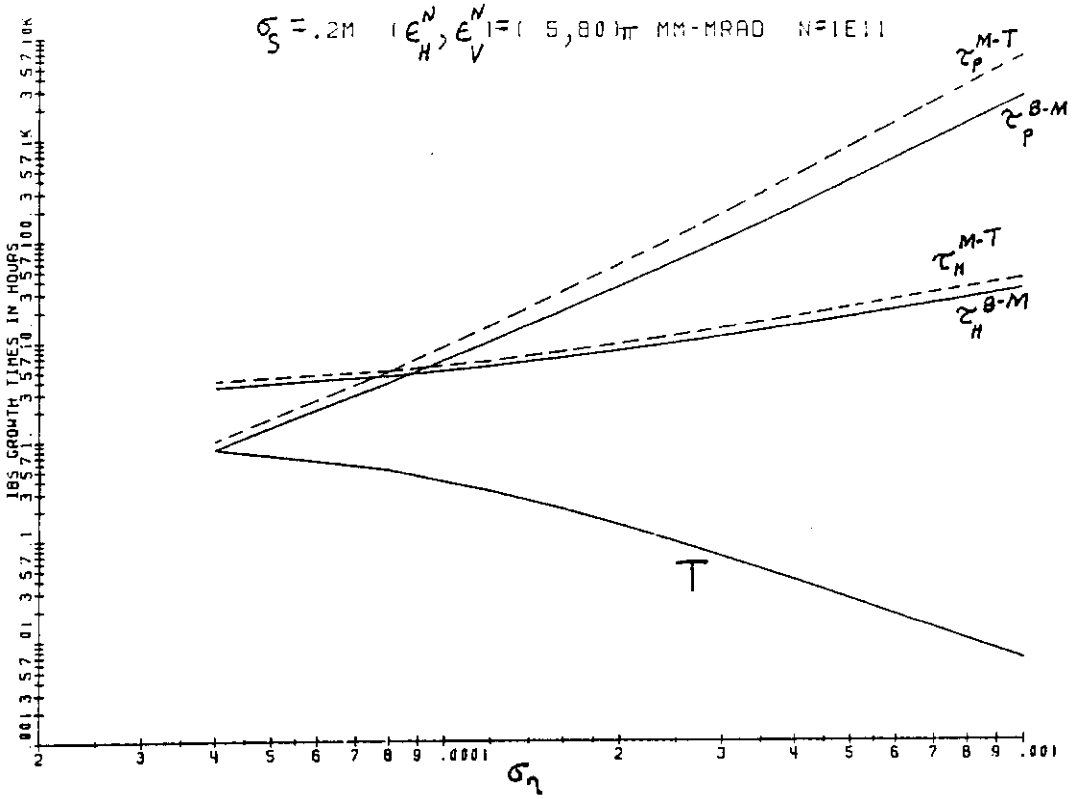

σ_S = .2M (ϵ_H^N, ϵ_V^N)=(5,80)π MM-MRAD N=1E11
IBS GROWTH TIMES IN HOURS
τ_P^{M-T}
τ_P^{B-M}
τ_H^{M-T}
τ_H^{B-M}
T
σ_η

Fig. 5a

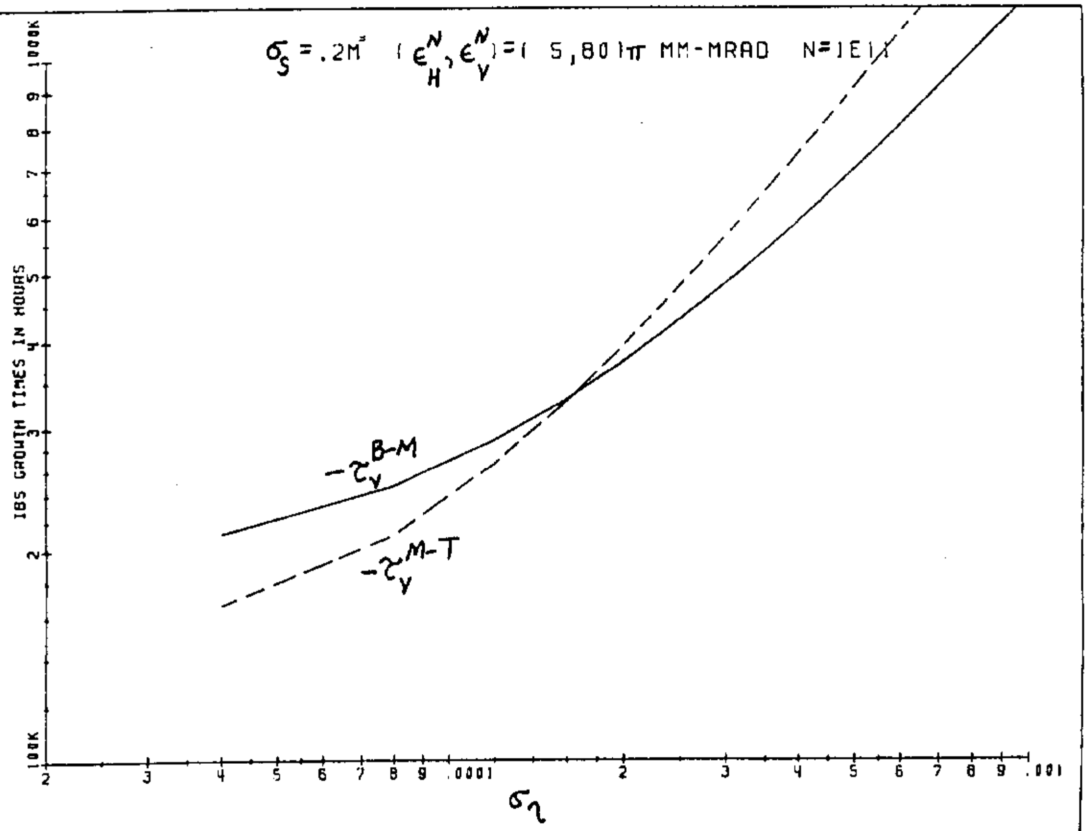

σ_S = .2M (ϵ_H^N, ϵ_V^N)=(5,80)π MM-MRAD N=1E11
IBS GROWTH TIMES IN HOURS
$-\tau_V^{B-M}$
$-\tau_V^{M-T}$
σ_η

Fig. 5b

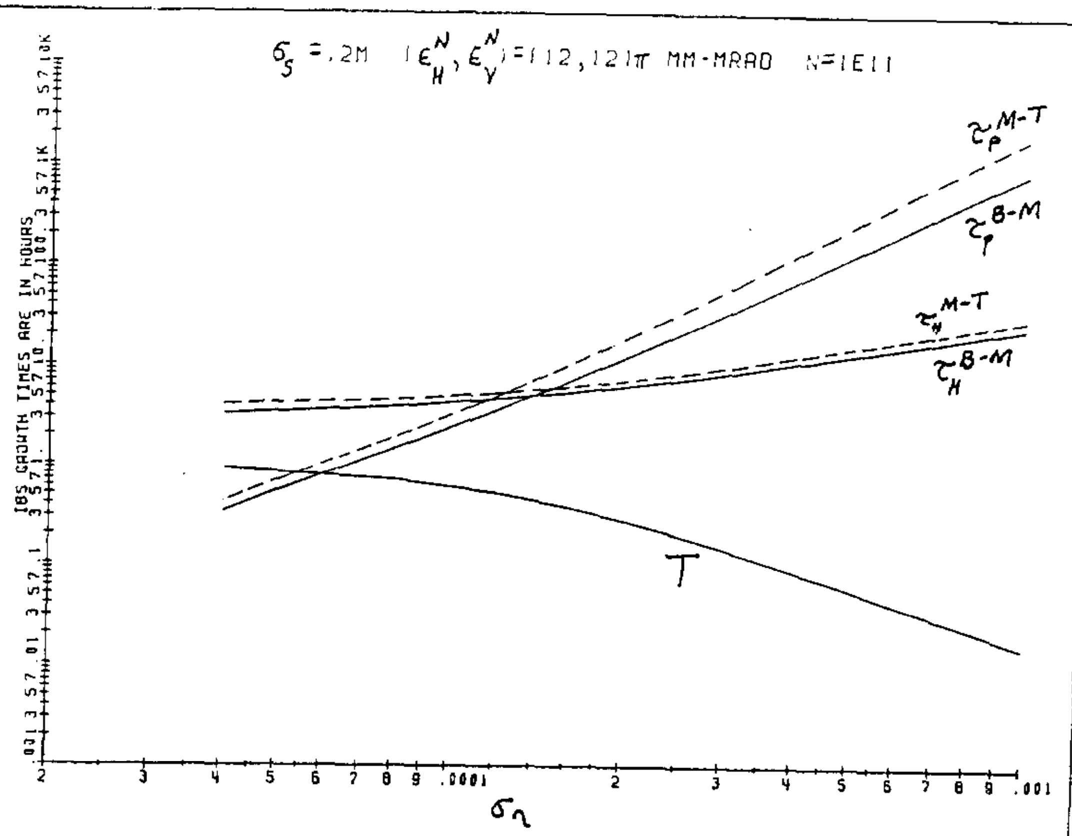

σS =.2M (εHN, εVN)=(12,12)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
τρM-T
τρB-M
τHM-T
τHB-M
T
ση

Fig. 6a

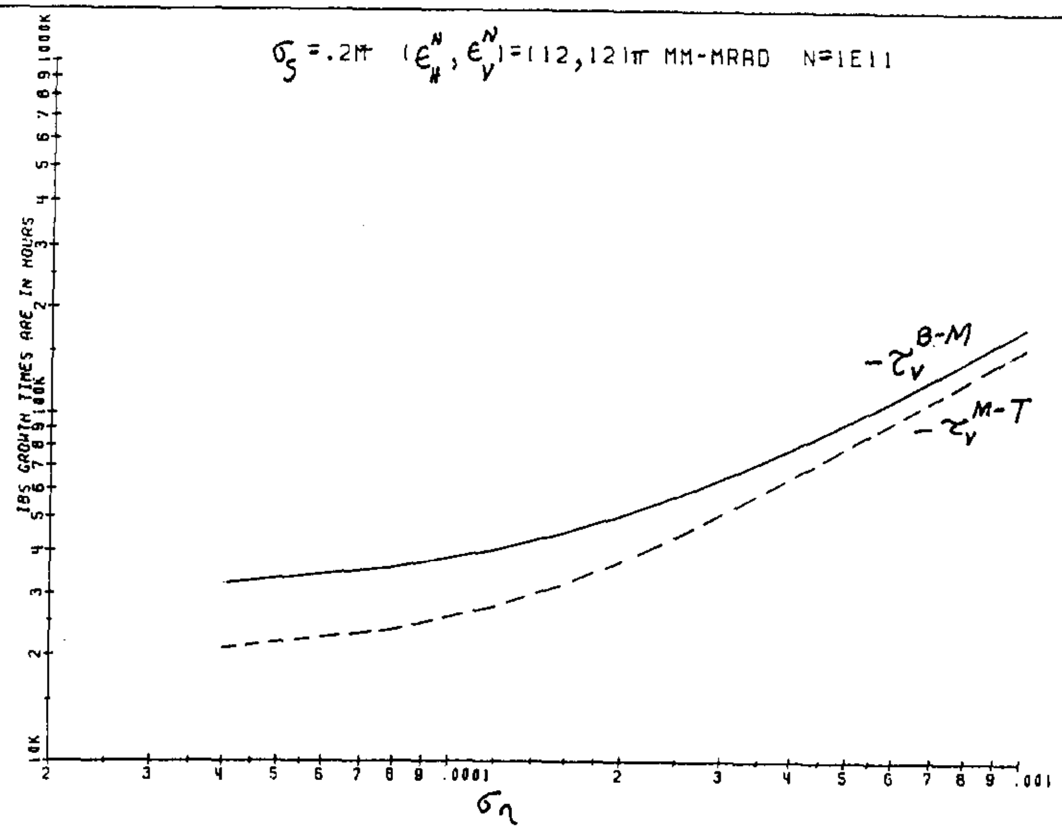

σS =.2M (εHN, εVN)=(12,12)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
-τVB-M
-τVM-T
ση

Fig. 6b

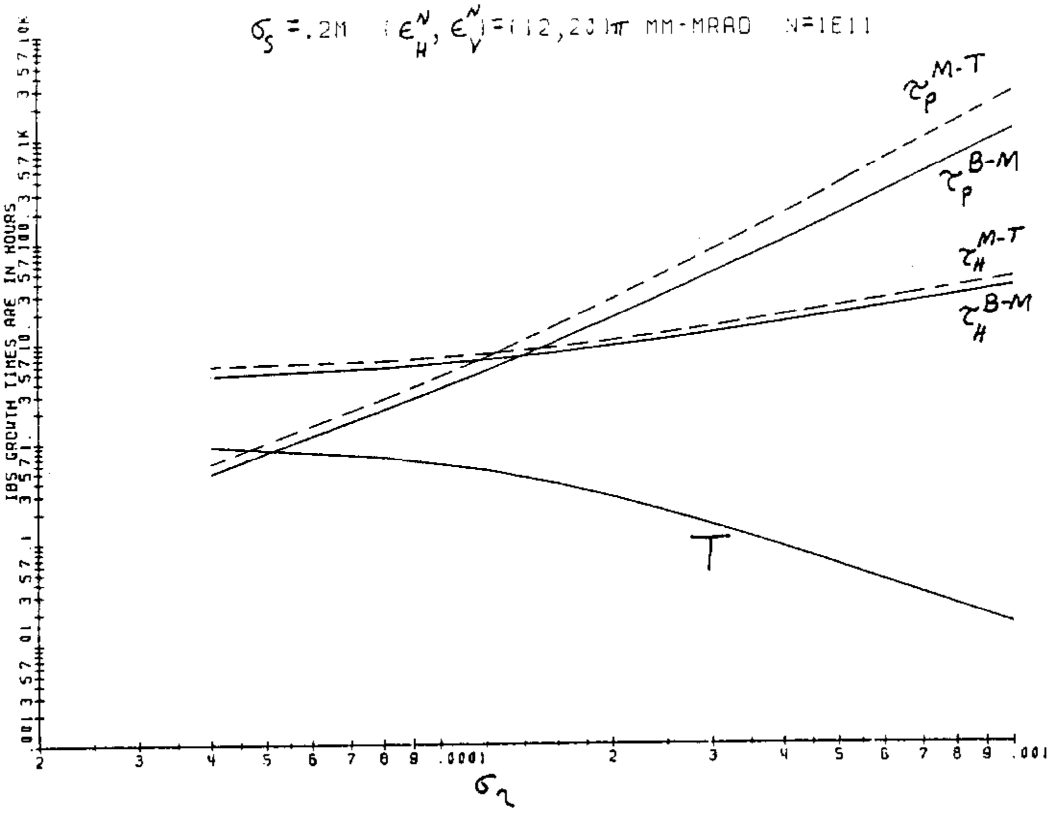

σ_s = .2M (ϵ_H^N, ϵ_V^N) = (12, 20)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
τ_p^{M-T}
τ_p^{B-M}
τ_H^{M-T}
τ_H^{B-M}
T
σ_η

Fig. 7a

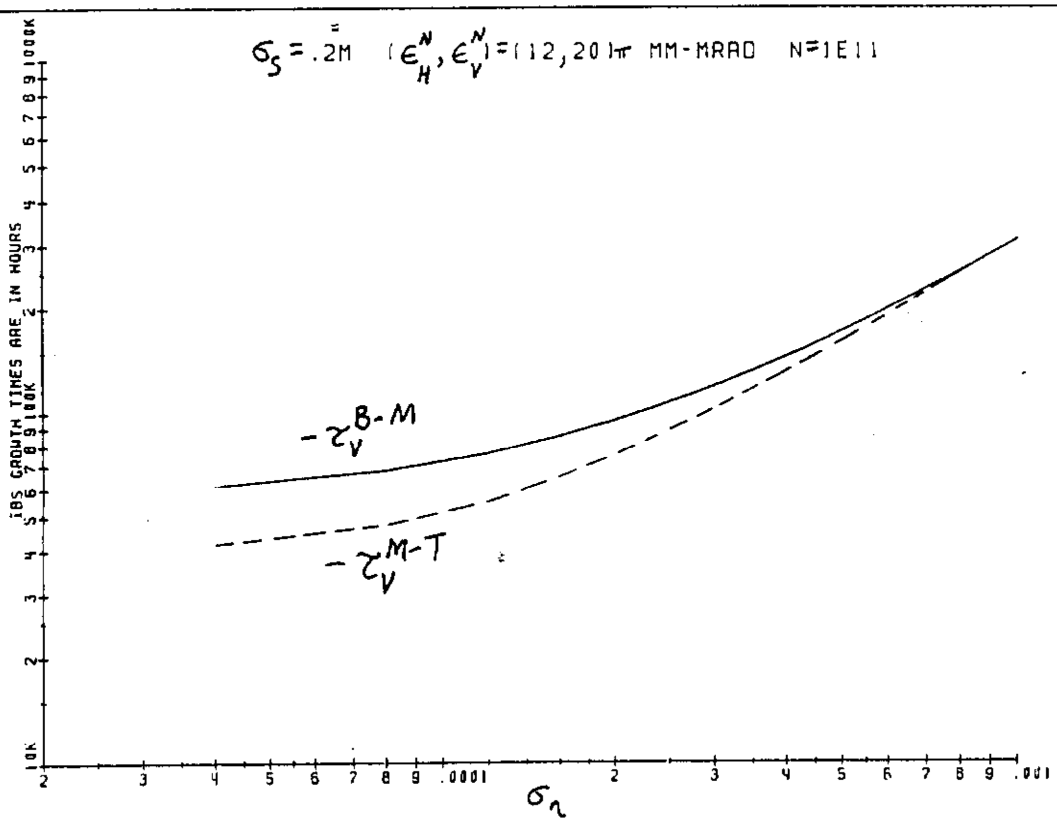

σ_s = .2M (ϵ_H^N, ϵ_V^N) = (12, 20)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
$-\tau_V^{B-M}$
$-\tau_V^{M-T}$
σ_η

Fig. 7b

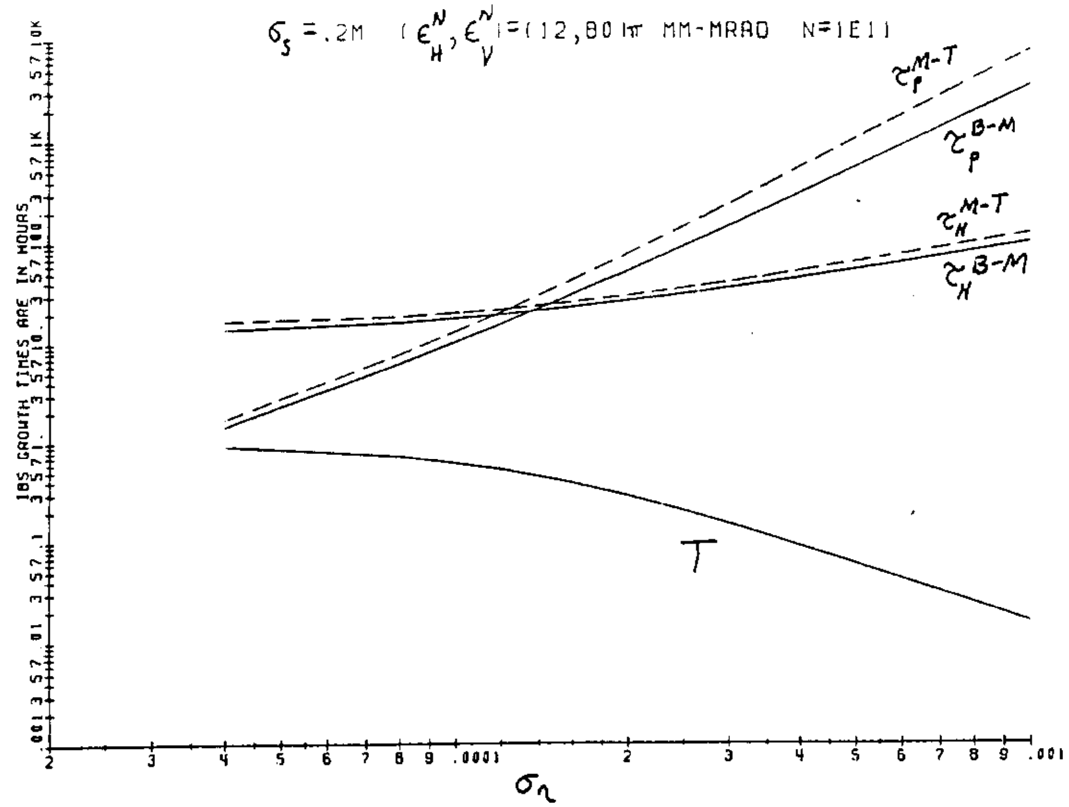

σ_s = .2M ($\epsilon_H^N, \epsilon_V^N$)=(12,80)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
τ_p^{M-T}
τ_p^{B-M}
τ_H^{M-T}
τ_H^{B-M}
T
σ_η

Fig. 8a

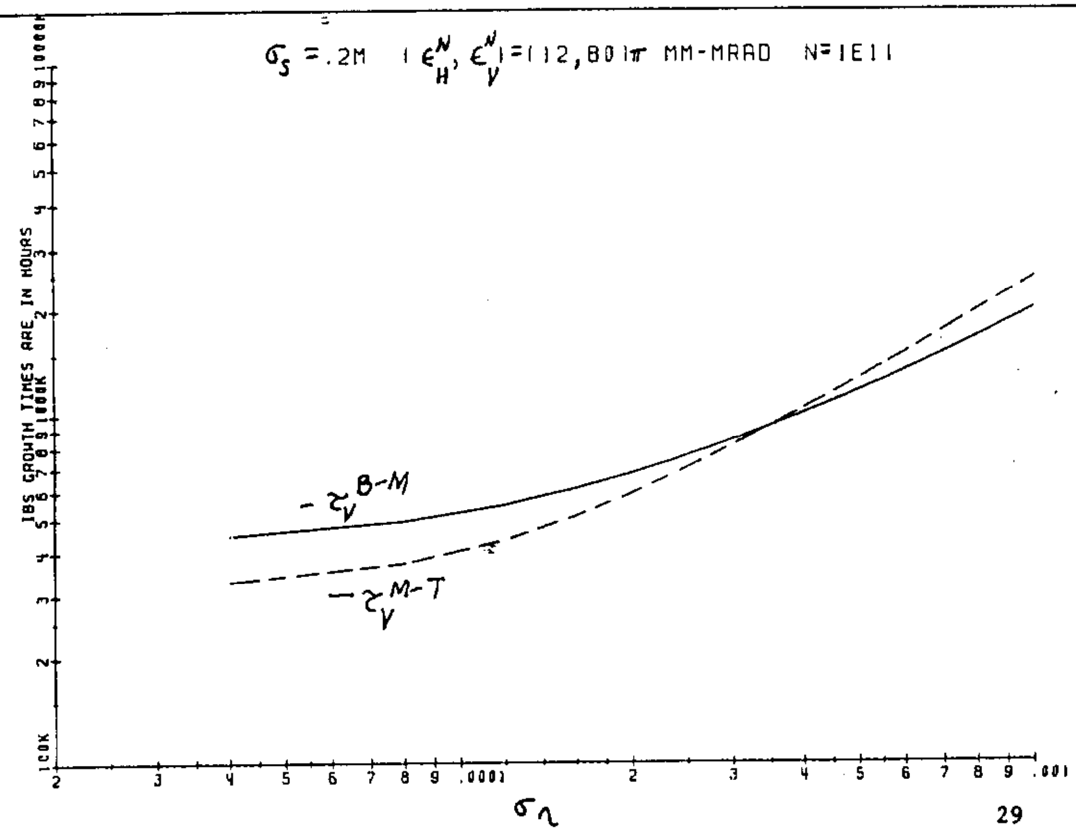

σ_s = .2M ($\epsilon_H^N, \epsilon_V^N$)=(12,80)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
τ_V^{B-M}
τ_V^{M-T}
σ_η

Fig. 8b

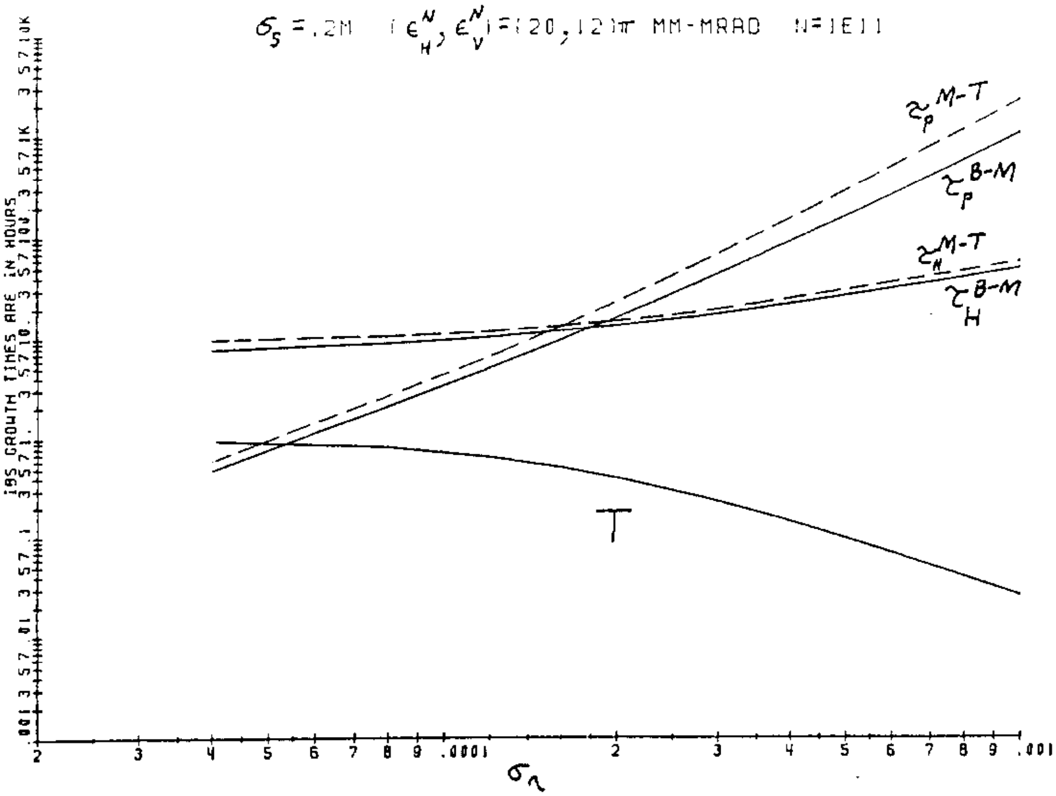

σs = .2M (ε^N_H, ε^N_V)=(20,12)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
τp^M-T
τp^B-M
τH^M-T
τH^B-M
T
.0001
.001
σℓ

Fig. 9a

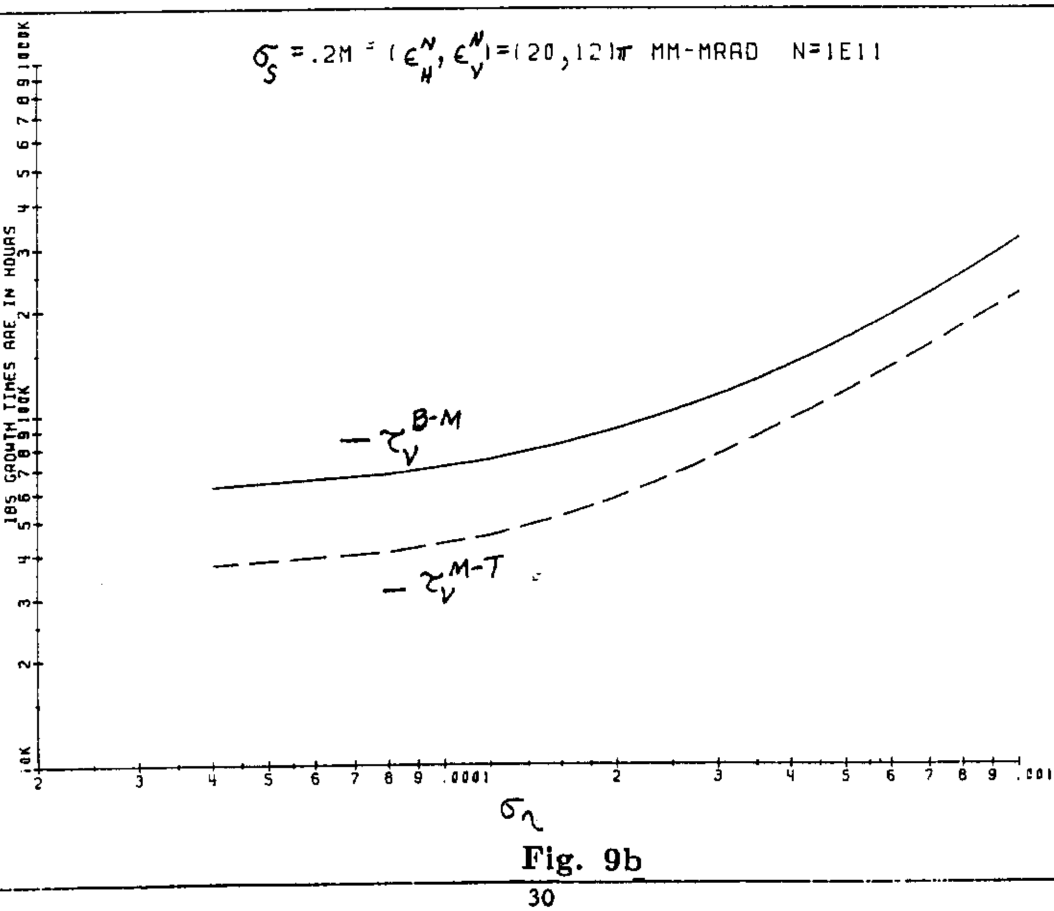

σs = .2M (ε^N_H, ε^N_V)=(20,12)π MM-MRAD N=1E11
IBS GROWTH TIMES ARE IN HOURS
100K
10K
τV^B-M
τV^M-T
.0001
.001
σℓ

Fig. 9b

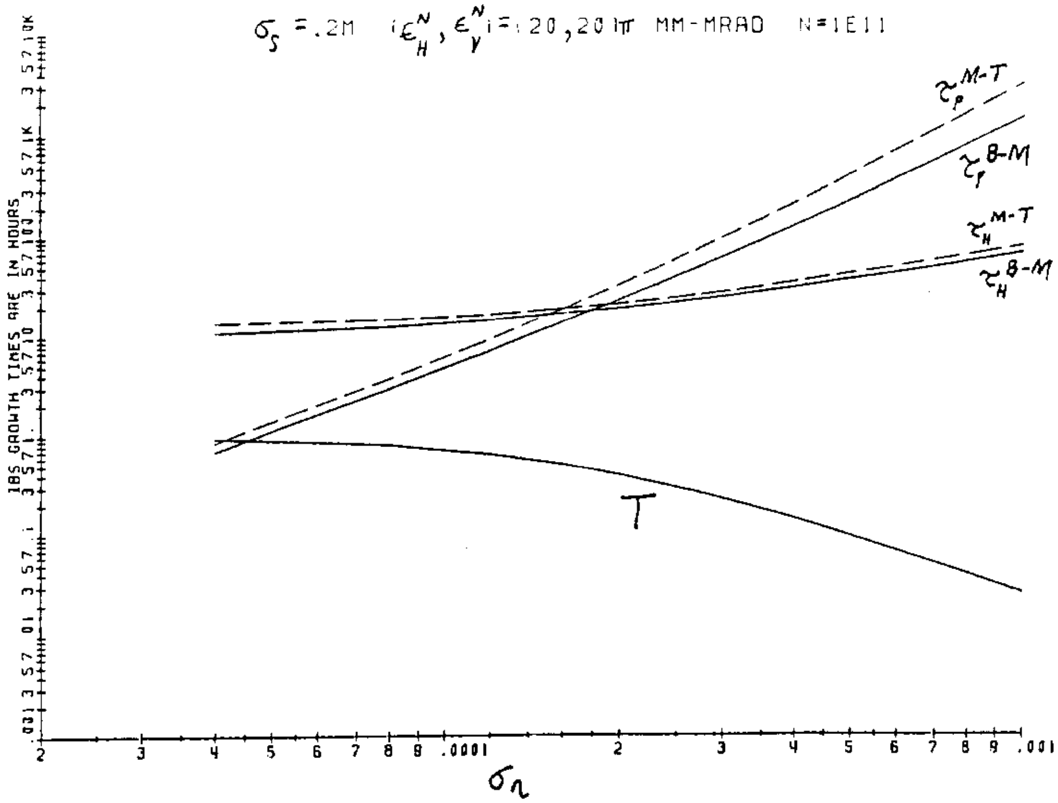


Fig. 10a

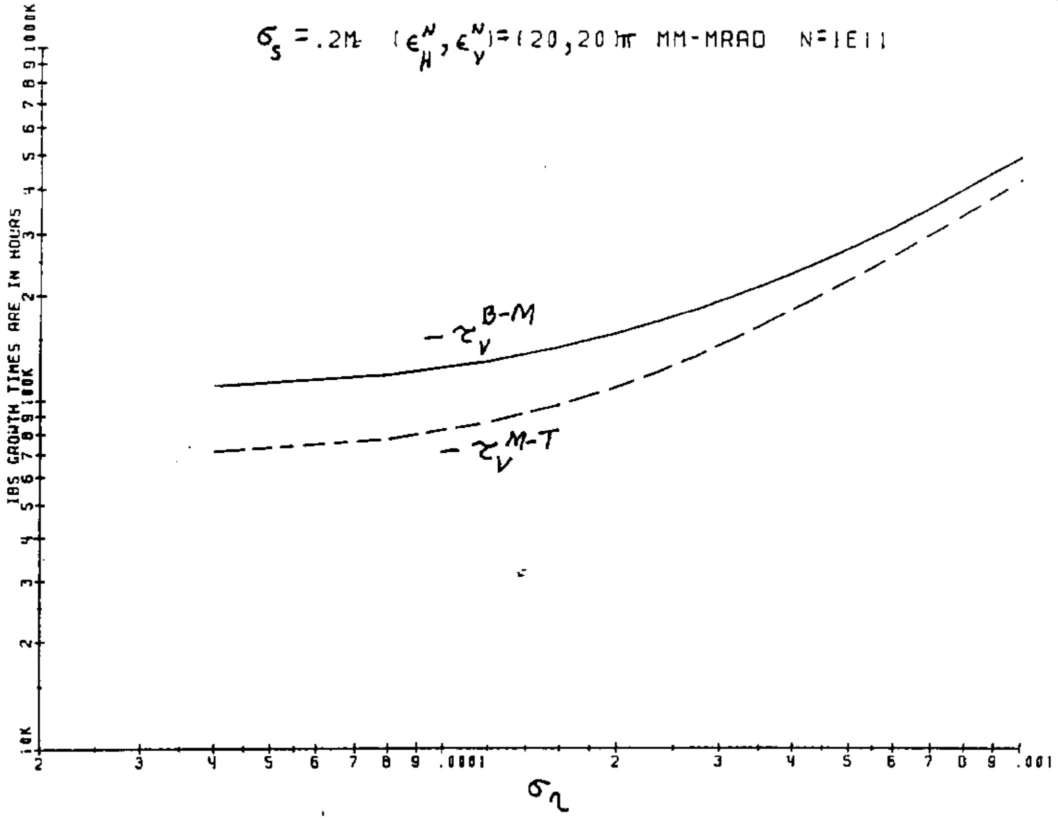


Fig. 10b

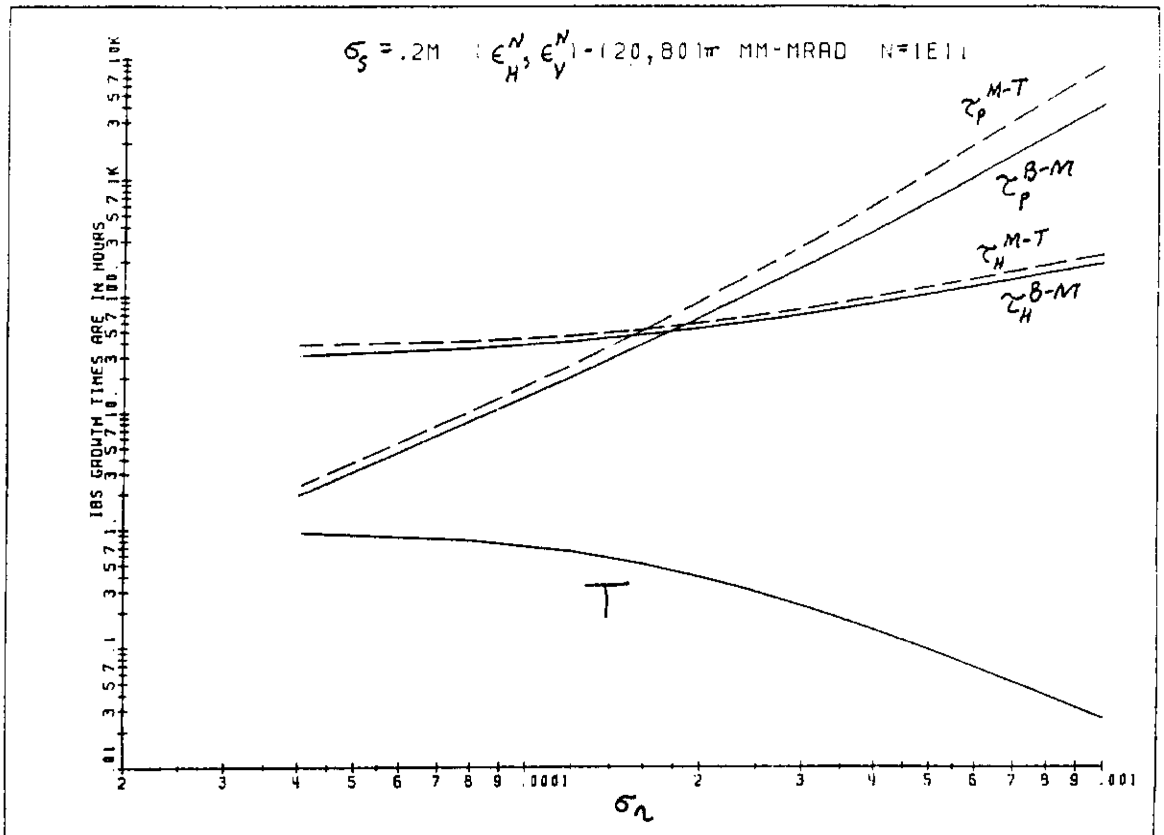


Fig. 11a

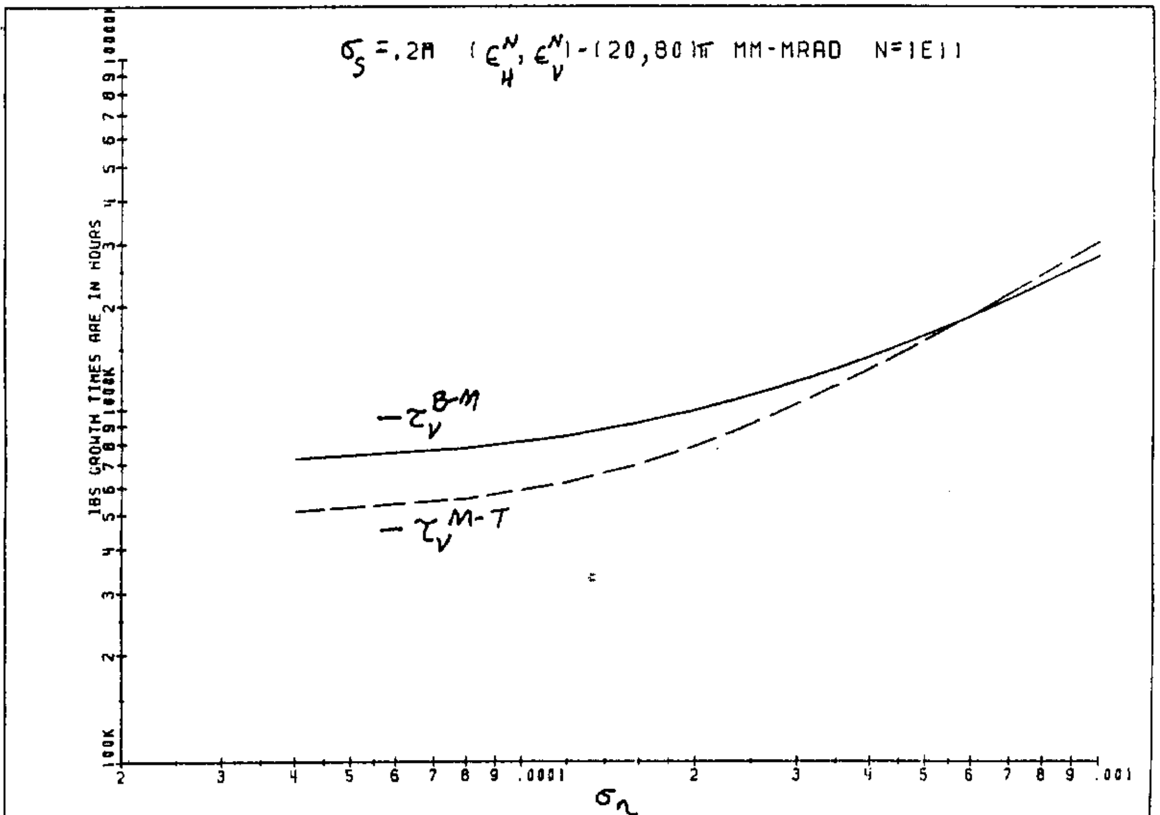


Fig. 11b

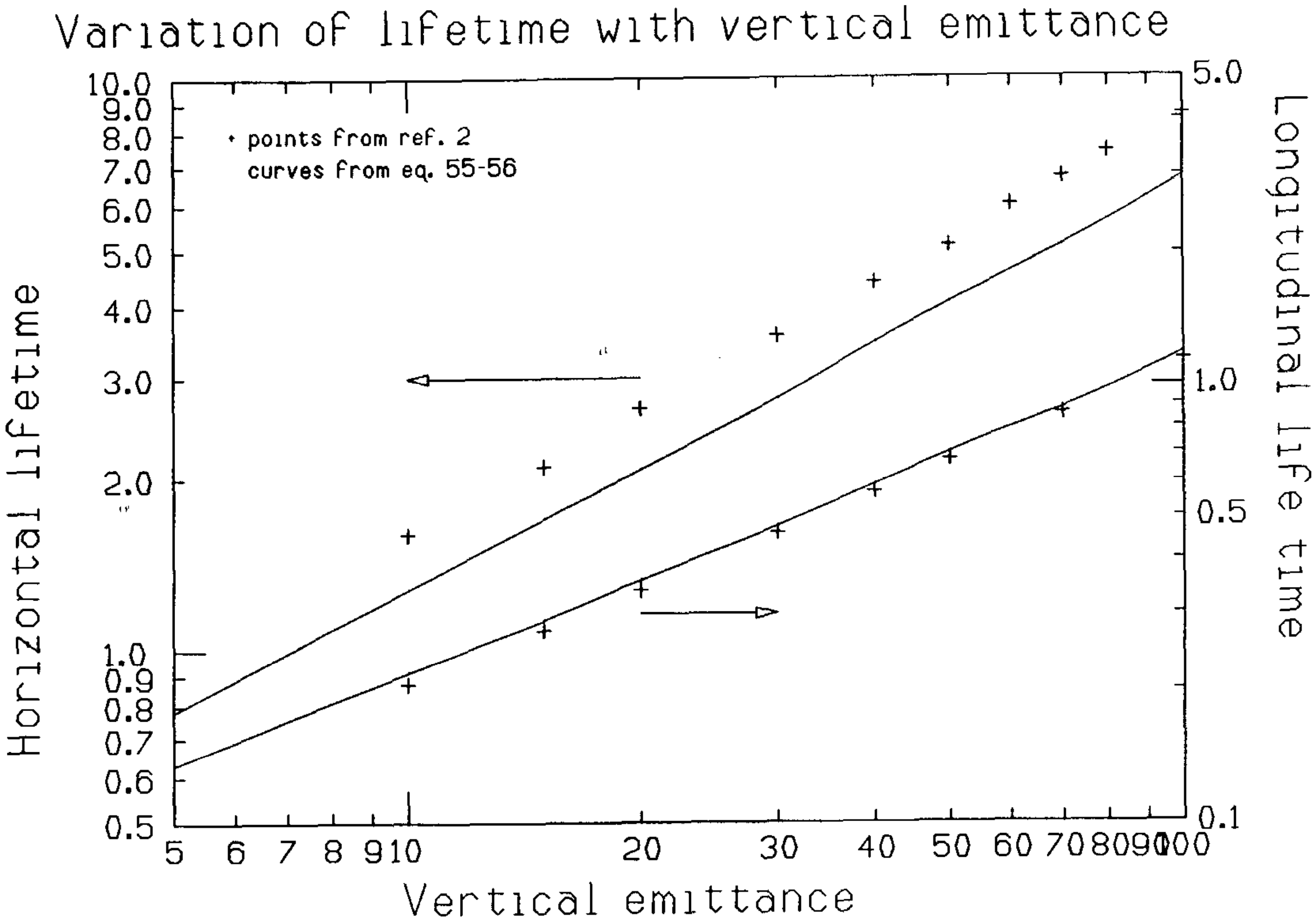

Variation of lifetime with vertical emittance
+ points from ref. 2
curves from eq. 55-56
Horizontal lifetime
Longitudinal life time
Vertical emittance
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.9
0.8
0.7
0.6
0.5
5.0
1.0
0.5
0.1
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100

Fig. 12

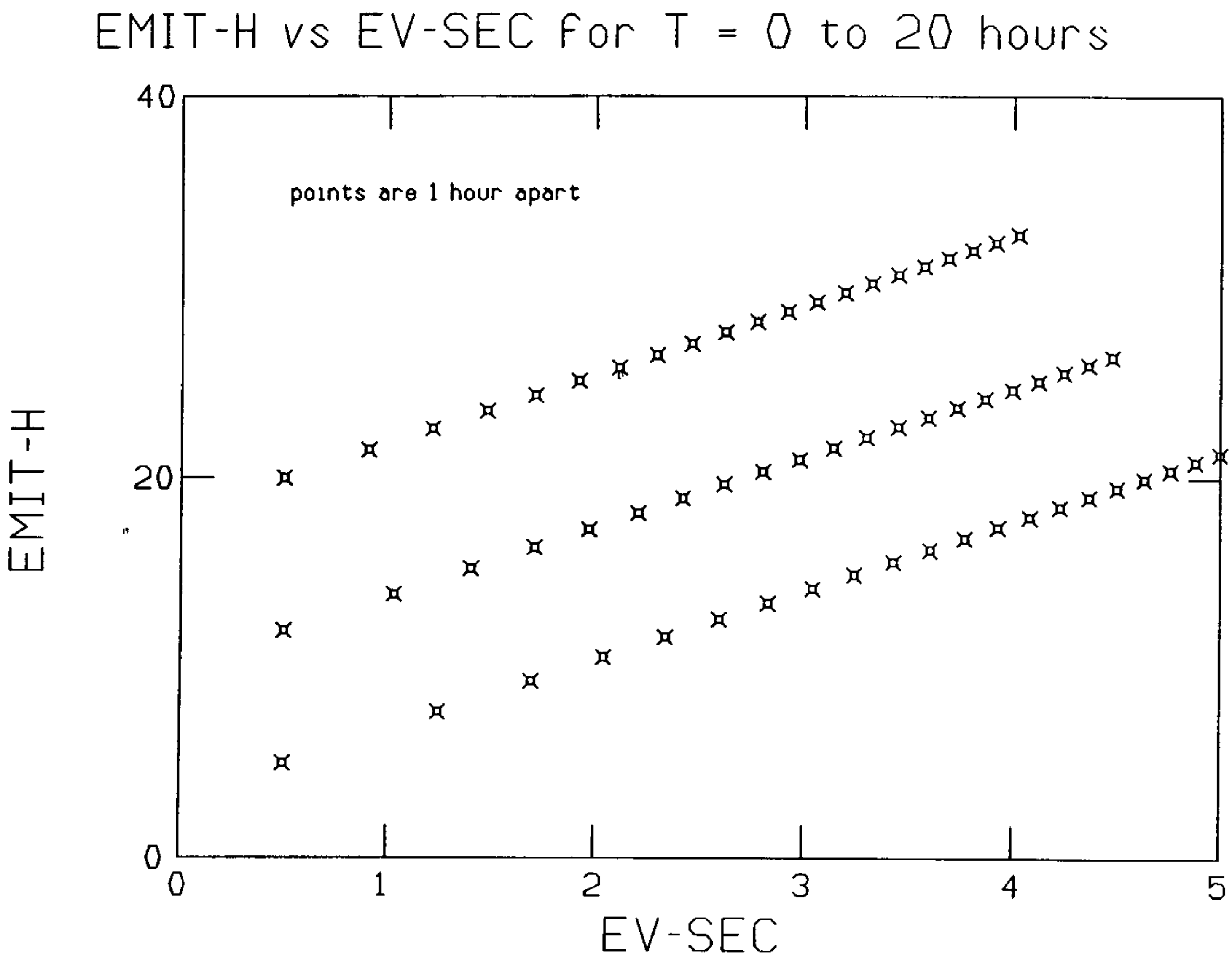

EMIT-H vs EV-SEC for T = 0 to 20 hours
points are 1 hour apart
EMIT-H
EV-SEC
0
20
40
1
2
3
4
5

Fig. 13

Table Captions

Table I

Values of $g(b)$ and $g(b) + \frac{1}{b} g(\frac{1}{b})$ vs. b.

Tables II-XV

Numerical comparison for the Tevatron lattice of the intrabeam scattering growth times as calculated from the asymptotic formulae of this work (M-T) with those calculated from the more exact calculations of Ref. 2 (B-M) as a function of σ_η, for $N = 10^{11}$ and various emittances and σ_s.

Table	σ_s(m)	ε_H^N(mm-mrad)	ε_V^N(mm-mrad)
II	0.2	5π	80π
III	0.2	12π	12π
IV	0.2	12π	20π
V	0.2	12π	80π
VI	0.2	20π	12π
VII	0.2	20π	20π
VIII	0.2	20π	80π
IX	0.4	5π	80π
X	0.4	12π	12π
XI	0.4	12π	20π
XII	0.4	12π	80π
XIII	0.4	20π	12π
XIV	0.4	20π	20π
XV	0.4	20π	80π

Table XVI

Comparison for the Tevatron lattice of $\frac{\tau_H}{\tau_p}\frac{\sigma_\eta^2\gamma}{\epsilon_H^N}$ from the asymptotic formulae of this work and $\frac{<\tau_H>}{<\tau_p>}\frac{\sigma_\eta^2\gamma}{\epsilon_H^N}$ calculated from the more precise strong-focussing theory of Ref. 2. In each case, $N = 10^{11}$, $\sigma_s = 0.2$ m, $\gamma = 1000$.

Table	ε_H^N (mm-mrad)	ε_V^N (mm-mrad)
XVI A	20π	20π
XVI B	12π	12π
XVI C	5π	80π

TABLE I

b	$g(b)$	$g(b)+\frac{1}{b}g(\frac{1}{b})$	b	$g(b)$	$g(b)+\frac{1}{b}g(\frac{1}{b})$
.10	-1.6451	8.3404	5.10	1.3993	1.3431
.20	-.2491	6.8069	5.20	1.3876	1.3255
.30	.4677	5.9305	5.30	1.3761	1.3083
.40	.9072	5.3245	5.40	1.3649	1.2917
.50	1.1987	4.8669	5.50	1.3538	1.2755
.60	1.4002	4.5031	5.60	1.3429	1.2598
.70	1.5431	4.2037	5.70	1.3322	1.2446
.80	1.6457	3.9512	5.80	1.3216	1.2297
.90	1.7195	3.7343	5.90	1.3113	1.2152
1.00	1.7724	3.5451	6.00	1.3011	1.2011
1.10	1.8097	3.3782	6.10	1.2910	1.1874
1.20	1.8351	3.2295	6.20	1.2812	1.1740
1.30	1.8514	3.0959	6.30	1.2715	1.1610
1.40	1.8606	2.9751	6.40	1.2619	1.1483
1.50	1.8644	2.8651	6.50	1.2525	1.1359
1.60	1.8639	2.7644	6.60	1.2432	1.1238
1.70	1.8599	2.6719	6.70	1.2341	1.1120
1.80	1.8532	2.5864	6.80	1.2252	1.1005
1.90	1.8445	2.5072	6.90	1.2164	1.0892
2.00	1.8340	2.4335	7.00	1.2077	1.0782
2.10	1.8222	2.3648	7.10	1.1991	1.0674
2.20	1.8094	2.3004	7.20	1.1907	1.0569
2.30	1.7958	2.2401	7.30	1.1824	1.0466
2.40	1.7815	2.1833	7.40	1.1743	1.0365
2.50	1.7668	2.1298	7.50	1.1662	1.0267
2.60	1.7518	2.0793	7.60	1.1583	1.0170
2.70	1.7365	2.0315	7.70	1.1505	1.0076
2.80	1.7211	1.9862	7.80	1.1428	.9984
2.90	1.7056	1.9432	7.90	1.1352	.9893
3.00	1.6901	1.9023	8.00	1.1278	.9804
3.10	1.6746	1.8633	8.10	1.1204	.9717
3.20	1.6592	1.8262	8.20	1.1132	.9632
3.30	1.6438	1.7907	8.30	1.1060	.9549
3.40	1.6287	1.7567	8.40	1.0990	.9467
3.50	1.6136	1.7243	8.50	1.0921	.9386
3.60	1.5987	1.6931	8.60	1.0852	.9307
3.70	1.5840	1.6633	8.70	1.0785	.9230
3.80	1.5695	1.6346	8.80	1.0718	.9154
3.90	1.5551	1.6070	8.90	1.0652	.9079
4.00	1.5410	1.5805	9.00	1.0588	.9006
4.10	1.5271	1.5550	9.10	1.0524	.8934
4.20	1.5134	1.5304	9.20	1.0461	.8864
4.30	1.4999	1.5067	9.30	1.0398	.8794
4.40	1.4866	1.4838	9.40	1.0337	.8726
4.50	1.4735	1.4616	9.50	1.0277	.8659
4.60	1.4606	1.4403	9.60	1.0217	.8593
4.70	1.4479	1.4196	9.70	1.0158	.8528
4.80	1.4355	1.3996	9.80	1.0100	.8465
4.90	1.4232	1.3802	9.90	1.0042	.8402
5.00	1.4112	1.3614	10.00	.9985	.8340

TABLE II (τ in hrs.)

σ_S = 2M $(\epsilon_H^N, \epsilon_V^N)$=(5, 80)$\pi$ MM-MRAD N=1E11

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.3419E+01	.3991E+01	-.2105E+06	-.1664E+06	.8197E+00	.9832E+00	.8023E+00
.8000E-04	.4450E+01	.5020E+01	-.2464E+06	-.2092E+06	.3797E+01	.4947E+01	.5037E+00
.1200E-03	.5614E+01	.6367E+01	-.2874E+06	-.2654E+06	.9882E+01	.1412E+02	.3108E+00
.1600E-03	.6831E+01	.7867E+01	-.3303E+06	-.3279E+06	.2003E+02	.3101E+02	.2024E+00
.2000E-03	.8073E+01	.9443E+01	-.3737E+06	-.3936E+06	.3515E+02	.5815E+02	.1397E+00
.2400E-03	.9328E+01	.1106E+02	-.4173E+06	-.4610E+06	.5615E+02	.9809E+02	.1013E+00
.2800E-03	.1059E+02	.1270E+02	-.4608E+06	-.5295E+06	.8388E+02	.1533E+03	.7651E-01
.3200E-03	.1186E+02	.1436E+02	-.5042E+06	-.5986E+06	.1192E+03	.2264E+03	.5964E-01
.3600E-03	.1313E+02	.1603E+02	-.5473E+06	-.6681E+06	.1629E+03	.3198E+03	.4772E-01
.4000E-03	.1440E+02	.1770E+02	-.5904E+06	-.7379E+06	.2159E+03	.4361E+03	.3901E-01
.4400E-03	.1567E+02	.1938E+02	-.6333E+06	-.8079E+06	.2790E+03	.5777E+03	.3246E-01
.4800E-03	.1694E+02	.2106E+02	-.6761E+06	-.8780E+06	.3530E+03	.7471E+03	.2742E-01
.5200E-03	.1822E+02	.2275E+02	-.7189E+06	-.9481E+06	.4386E+03	.9469E+03	.2346E-01
.5600E-03	.1949E+02	.2443E+02	-.7616E+06	-.1018E+07	.5368E+03	.1180E+04	.2029E-01
.6000E-03	.2077E+02	.2612E+02	-.8043E+06	-.1089E+07	.6483E+03	.1447E+04	.1772E-01
.6400E-03	.2205E+02	.2780E+02	-.8470E+06	-.1159E+07	.7739E+03	.1753E+04	.1561E-01
.6800E-03	.2333E+02	.2949E+02	-.8897E+06	-.1229E+07	.9145E+03	.2099E+04	.1385E-01
.7200E-03	.2460E+02	.3117E+02	-.9325E+06	-.1299E+07	.1071E+04	.2488E+04	.1237E-01
.7600E-03	.2588E+02	.3286E+02	-.9753E+06	-.1370E+07	.1244E+04	.2922E+04	.1112E-01
.8000E-03	.2716E+02	.3454E+02	-.1018E+07	-.1440E+07	.1434E+04	.3404E+04	.1005E-01
.8400E-03	.2845E+02	.3623E+02	-.1061E+07	-.1510E+07	.1642E+04	.3936E+04	.9121E-02
.8800E-03	.2973E+02	.3791E+02	-.1104E+07	-.1580E+07	.1870E+04	.4520E+04	.8317E-02
.9200E-03	.3101E+02	.3960E+02	-.1147E+07	-.1651E+07	.2117E+04	.5160E+04	.7615E-02
.9600E-03	.3229E+02	.4128E+02	-.1190E+07	-.1721E+07	.2385E+04	.5857E+04	.6998E-02
.1000E-02	.3358E+02	.4296E+02	-.1233E+07	-.1791E+07	.2674E+04	.6615E+04	.6453E-02

TABLE III

σ_s = 2M $(\epsilon_H^N, \epsilon_V^N)$=(12, 12)$\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.3284E+01	.4103E+01	-.3211E+05	-.2090E+03	.3340E+00	.4211E+00	.9069E+00
.8000E-04	.3945E+01	.4632E+01	-.3568E+05	-.2359E+05	.1457E+01	.1901E+01	.7089E+00
.1200E-03	.4714E+01	.5395E+01	-.4016E+05	-.2748E+05	.3624E+01	.4983E+01	.5198E+00
.1600E-03	.5547E+01	.6307E+01	-.4513E+05	-.3212E+05	.7117E+01	.1036E+02	.3785E+00
.2000E-03	.6417E+01	.7309E+01	-.5041E+05	-.3722E+05	.1222E+02	.1875E+02	.2804E+00
.2400E-03	.7310E+01	.8368E+01	-.5591E+05	-.4261E+05	.1920E+02	.3092E+02	.2130E+00
.2800E-03	.8219E+01	.9464E+01	-.6157E+05	-.4819E+05	.2835E+02	.4759E+02	.1659E+00
.3200E-03	.9139E+01	.1058E+02	-.6737E+05	-.5390E+05	.3993E+02	.6952E+02	.1321E+00
.3600E-03	.1007E+02	.1172E+02	-.7329E+05	-.5969E+05	.5421E+02	.9743E+02	.1074E+00
.4000E-03	.1100E+02	.1287E+02	-.7931E+05	-.6554E+05	.7145E+02	.1321E+03	.8878E-01
.4400E-03	.1194E+02	.1403E+02	-.8542E+05	-.7144E+05	.9191E+02	.1742E+03	.7452E-01
.4800E-03	.1288E+02	.1519E+02	-.9162E+05	-.7737E+05	.1159E+03	.2245E+03	.6337E-01
.5200E-03	.1382E+02	.1636E+02	-.9789E+05	-.8332E+05	.1436E+03	.2838E+03	.5451E-01
.5600E-03	.1477E+02	.1753E+02	-.1042E+06	-.8929E+05	.1753E+03	.3527E+03	.4735E-01
.6000E-03	.1571E+02	.1871E+02	-.1107E+06	-.9528E+05	.2112E+03	.4320E+03	.4150E-01
.6400E-03	.1666E+02	.1989E+02	-.1171E+06	-.1013E+06	.2517E+03	.5225E+03	.3666E-01
.6800E-03	.1761E+02	.2107E+02	-.1237E+06	-.1073E+06	.2969E+03	.6248E+03	.3261E-01
.7200E-03	.1857E+02	.2225E+02	-.1303E+06	-.1133E+06	.3472E+03	.7397E+03	.2919E-01
.7600E-03	.1952E+02	.2343E+02	-.1370E+06	-.1193E+06	.4027E+03	.8679E+03	.2628E-01
.8000E-03	.2047E+02	.2461E+02	-.1437E+06	-.1253E+06	.4638E+03	.1010E+04	.2378E-01
.8400E-03	.2143E+02	.2579E+02	-.1504E+06	-.1313E+06	.5307E+03	.1167E+04	.2161E-01
.8800E-03	.2238E+02	.2698E+02	-.1573E+06	-.1374E+06	.6037E+03	.1340E+04	.1973E-01
.9200E-03	.2334E+02	.2816E+02	-.1641E+06	-.1434E+06	.6829E+03	.1529E+04	.1808E-01
.9600E-03	.2429E+02	.2934E+02	-.1710E+06	-.1494E+06	.7687E+03	.1734E+04	.1663E-01
.1000E-02	.2525E+02	.3052E+02	-.1780E+06	-.1554E+06	.8613E+03	.1958E+04	.1535E-01

TABLE IV

$\sigma_s = .2M \quad (\epsilon_H^N, \epsilon_V^N) = (12, 20)\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.4758E+01	.5930E+01	-.6110E+05	-.4185E+05	.4868E+00	.6086E+00	.9069E+00
.8000E-04	.5706E+01	.6694E+01	-.6759E+05	-.4725E+05	.2122E+01	.2748E+01	.7089E+00
.1200E-03	.6811E+01	.7798E+01	-.7573E+05	-.5504E+05	.5275E+01	.7203E+01	.5198E+00
.1600E-03	.8008E+01	.9116E+01	-.8471E+05	-.6434E+05	.1035E+02	.1497E+02	.3785E+00
.2000E-03	.9258E+01	.1057E+02	-.9418E+05	-.7457E+05	.1776E+02	.2711E+02	.2804E+00
.2400E-03	.1054E+02	.1210E+02	-.1040E+06	-.8539E+05	.2790E+02	.4469E+02	.2130E+00
.2800E-03	.1184E+02	.1368E+02	-.1140E+06	-.9657E+05	.4117E+02	.6880E+02	.1659E+00
.3200E-03	.1316E+02	.1530E+02	-.1242E+06	-.1080E+06	.5795E+02	.1005E+03	.1321E+00
.3600E-03	.1449E+02	.1695E+02	-.1345E+06	-.1196E+06	.7862E+02	.1409E+03	.1074E+00
.4000E-03	.1583E+02	.1861E+02	-.1450E+06	-.1314E+06	.1036E+03	.1910E+03	.8878E-01
.4400E-03	.1717E+02	.2028E+02	-.1555E+06	-.1432E+06	.1332E+03	.2519E+03	.7452E-01
.4800E-03	.1852E+02	.2197E+02	-.1662E+06	-.1551E+06	.1678E+03	.3246E+03	.6337E-01
.5200E-03	.1987E+02	.2366E+02	-.1769E+06	-.1670E+06	.2078E+03	.4103E+03	.5451E-01
.5600E-03	.2122E+02	.2535E+02	-.1876E+06	-.1790E+06	.2536E+03	.5100E+03	.4735E-01
.6000E-03	.2258E+02	.2705E+02	-.1985E+06	-.1910E+06	.3054E+03	.6247E+03	.4150E-01
.6400E-03	.2394E+02	.2876E+02	-.2093E+06	-.2030E+06	.3638E+03	.7555E+03	.3666E-01
.6800E-03	.2530E+02	.3046E+02	-.2203E+06	-.2150E+06	.4290E+03	.9035E+03	.3261E-01
.7200E-03	.2666E+02	.3217E+02	-.2313E+06	-.2271E+06	.5015E+03	.1070E+04	.2919E-01
.7600E-03	.2802E+02	.3388E+02	-.2424E+06	-.2392E+06	.5815E+03	.1255E+04	.2628E-01
.8000E-03	.2939E+02	.3559E+02	-.2535E+06	-.2512E+06	.6694E+03	.1461E+04	.2378E-01
.8400E-03	.3075E+02	.3730E+02	-.2646E+06	-.2633E+06	.7657E+03	.1688E+04	.2161E-01
.8800E-03	.3212E+02	.3901E+02	-.2758E+06	-.2754E+06	.8706E+03	.1938E+04	.1973E-01
.9200E-03	.3349E+02	.4072E+02	-.2871E+06	-.2875E+06	.9846E+03	.2211E+04	.1808E-01
.9600E-03	.3486E+02	.4243E+02	-.2983E+06	-.2996E+06	.1108E+04	.2508E+04	.1663E-01
.1000E-02	.3623E+02	.4414E+02	-.3097E+06	-.3116E+06	.1241E+04	.2831E+04	.1535E-01

TABLE V

$\sigma_s = .2M \quad (\epsilon_H^N, \epsilon_V^N) = (12, 80)\pi$ MM-MRAD N=1E11

(τ in hours)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.1360E+02	.1645E+02	-.4462E+06	-.3280E+06	.1414E+01	.1688E+01	.9069E+00
.8000E-04	.1621E+02	.1857E+02	-.4902E+06	-.3703E+06	.6193E+01	.7623E+01	.7089E+00
.1200E-03	.1929E+02	.2163E+02	-.5453E+06	-.4314E+06	.1527E+02	.1998E+02	.5198E+00
.1600E-03	.2261E+02	.2529E+02	-.6056E+06	-.5045E+06	.2993E+02	.4154E+02	.3785E+00
.2000E-03	.2607E+02	.2932E+02	-.6686E+06	-.5848E+06	.5126E+02	.7523E+02	.2804E+00
.2400E-03	.2962E+02	.3357E+02	-.7331E+06	-.6696E+06	.8040E+02	.1241E+03	.2130E+00
.2800E-03	.3322E+02	.3798E+02	-.7985E+06	-.7575E+06	.1185E+03	.1910E+03	.1659E+00
.3200E-03	.3686E+02	.4248E+02	-.8644E+06	-.8473E+06	.1665E+03	.2790E+03	.1321E+00
.3600E-03	.4052E+02	.4705E+02	-.9307E+06	-.9385E+06	.2256E+03	.3911E+03	.1074E+00
.4000E-03	.4420E+02	.5167E+02	-.9971E+06	-.1031E+07	.2968E+03	.5303E+03	.8878E-01
.4400E-03	.4789E+02	.5632E+02	-.1064E+07	-.1123E+07	.3812E+03	.6995E+03	.7452E-01
.4800E-03	.5160E+02	.6100E+02	-.1130E+07	-.1217E+07	.4797E+03	.9016E+03	.6337E-01
.5200E-03	.5531E+02	.6570E+02	-.1197E+07	-.1311E+07	.5934E+03	.1140E+04	.5451E-01
.5600E-03	.5903E+02	.7042E+02	-.1263E+07	-.1405E+07	.7233E+03	.1417E+04	.4735E-01
.6000E-03	.6275E+02	.7515E+02	-.1330E+07	-.1499E+07	.8704E+03	.1735E+04	.4150E-01
.6400E-03	.6648E+02	.7988E+02	-.1396E+07	-.1594E+07	.1036E+04	.2099E+04	.3666E-01
.6800E-03	.7021E+02	.8463E+02	-.1463E+07	-.1688E+07	.1220E+04	.2510E+04	.3261E-01
.7200E-03	.7394E+02	.8937E+02	-.1529E+07	-.1783E+07	.1425E+04	.2972E+04	.2919E-01
.7600E-03	.7768E+02	.9413E+02	-.1596E+07	-.1878E+07	.1651E+04	.3488E+04	.2628E-01
.8000E-03	.8142E+02	.9888E+02	-.1662E+07	-.1973E+07	.1899E+04	.4060E+04	.2378E-01
.8400E-03	.8516E+02	.1036E+03	-.1729E+07	-.2068E+07	.2170E+04	.4691E+04	.2161E-01
.8800E-03	.8890E+02	.1084E+03	-.1795E+07	-.2162E+07	.2465E+04	.5385E+04	.1973E-01
.9200E-03	.9265E+02	.1132E+03	-.1862E+07	-.2257E+07	.2785E+04	.6144E+04	.1808E-01
.9600E-03	.9640E+02	.1179E+03	-.1929E+07	-.2352E+07	.3132E+04	.6971E+04	.1663E-01
.1000E-02	.1001E+03	.1227E+03	-.1995E+07	-.2447E+07	.3506E+04	.7870E+04	.1535E-01

TABLE VI

σ_S = .2M $(\epsilon_H^N, \epsilon_V^N)$=(20, 12)$\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.7710E+01	.9701E+01	-.6196E+05	-.3717E+05	.4810E+00	.5973E+00	.9420E+00
.8000E-04	.8905E+01	.1050E+02	-.6704E+05	-.4022E+05	.2042E+01	.2586E+01	.8023E+00
.1200E-03	.1028E+02	.1170E+02	-.7368E+05	-.4484E+05	.4954E+01	.6485E+01	.6434E+00
.1600E-03	.1179E+02	.1320E+02	-.8131E+05	-.5058E+05	.9536E+01	.1300E+02	.5037E+00
.2000E-03	.1339E+02	.1490E+02	-.8961E+05	-.5709E+05	.1612E+02	.2294E+02	.3938E+00
.2400E-03	.1505E+02	.1674E+02	-.9841E+05	-.6413E+05	.2502E+02	.3710E+02	.3108E+00
.2800E-03	.1675E+02	.1867E+02	-.1076E+06	-.7154E+05	.3657E+02	.5634E+02	.2489E+00
.3200E-03	.1848E+02	.2068E+02	-.1172E+06	-.7921E+05	.5110E+02	.8148E+02	.2024E+00
.3600E-03	.2023E+02	.2273E+02	-.1270E+06	-.8706E+05	.6891E+02	.1133E+03	.1670E+00
.4000E-03	.2200E+02	.2481E+02	-.1372E+06	-.9505E+05	.9034E+02	.1528E+03	.1397E+00
.4400E-03	.2378E+02	.2693E+02	-.1476E+06	-.1031E+06	.1157E+03	.2006E+03	.1183E+00
.4800E-03	.2557E+02	.2906E+02	-.1583E+06	-.1113E+06	.1452E+03	.2576E+03	.1013E+00
.5200E-03	.2737E+02	.3121E+02	-.1692E+06	-.1195E+06	.1794E+03	.3247E+03	.8766E-01
.5600E-03	.2918E+02	.3337E+02	-.1803E+06	-.1278E+06	.2183E+03	.4027E+03	.7651E-01
.6000E-03	.3099E+02	.3554E+02	-.1917E+06	-.1361E+06	.2624E+03	.4924E+03	.6731E-01
.6400E-03	.3281E+02	.3772E+02	-.2032E+06	-.1445E+06	.3120E+03	.5946E+03	.5964E-01
.6800E-03	.3463E+02	.3991E+02	-.2150E+06	-.1528E+06	.3673E+03	.7101E+03	.5320E-01
.7200E-03	.3645E+02	.4210E+02	-.2270E+06	-.1612E+06	.4287E+03	.8398E+03	.4772E-01
.7600E-03	.3828E+02	.4429E+02	-.2391E+06	-.1696E+06	.4964E+03	.9844E+03	.4304E-01
.8000E-03	.4011E+02	.4649E+02	-.2514E+06	-.1781E+06	.5708E+03	.1145E+04	.3901E-01
.8400E-03	.4194E+02	.4869E+02	-.2640E+06	-.1865E+06	.6522E+03	.1322E+04	.3551E-01
.8800E-03	.4377E+02	.5089E+02	-.2767E+06	-.1949E+06	.7408E+03	.1516E+04	.3246E-01
.9200E-03	.4561E+02	.5309E+02	-.2896E+06	-.2033E+06	.8370E+03	.1729E+04	.2978E-01
.9600E-03	.4745E+02	.5530E+02	-.3026E+06	-.2118E+06	.9410E+03	.1961E+04	.2742E-01
.1000E-02	.4929E+02	.5750E+02	-.3158E+06	-.2202E+06	.1053E+04	.2213E+04	.2532E-01

TABLE VII

σ_S = .2M $(\epsilon_H^N, \epsilon_V^N)$=(20, 20)$\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.1110E+02	.1398E+02	-.1103E+06	-.7117E+05	.6964E+00	.8606E+00	.9420E+00
.8000E-04	.1280E+02	.1512E+02	-.1187E+06	-.7702E+05	.2959E+01	.3725E+01	.8023E+00
.1200E-03	.1477E+02	.1686E+02	-.1298E+06	-.8586E+05	.7164E+01	.9343E+01	.6434E+00
.1600E-03	.1693E+02	.1902E+02	-.1424E+06	-.9685E+05	.1378E+02	.1874E+02	.5037E+00
.2000E-03	.1921E+02	.2147E+02	-.1559E+06	-.1093E+06	.2328E+02	.3305E+02	.3938E+00
.2400E-03	.2158E+02	.2412E+02	-.1702E+06	-.1228E+06	.3612E+02	.5346E+02	.3108E+00
.2800E-03	.2401E+02	.2691E+02	-.1850E+06	-.1370E+06	.5277E+02	.8118E+02	.2489E+00
.3200E-03	.2647E+02	.2979E+02	-.2002E+06	-.1517E+06	.7368E+02	.1174E+03	.2024E+00
.3600E-03	.2897E+02	.3275E+02	-.2157E+06	-.1668E+06	.9932E+02	.1633E+03	.1670E+00
.4000E-03	.3149E+02	.3575E+02	-.2316E+06	-.1821E+06	.1301E+03	.2202E+03	.1397E+00
.4400E-03	.3403E+02	.3880E+02	-.2477E+06	-.1976E+06	.1665E+03	.2891E+03	.1183E+00
.4800E-03	.3658E+02	.4187E+02	-.2640E+06	-.2132E+06	.2090E+03	.3713E+03	.1013E+00
.5200E-03	.3914E+02	.4497E+02	-.2805E+06	-.2290E+06	.2580E+03	.4680E+03	.8766E-01
.5600E-03	.4171E+02	.4809E+02	-.2973E+06	-.2449E+06	.3138E+03	.5803E+03	.7651E-01
.6000E-03	.4429E+02	.5122E+02	-.3142E+06	-.2608E+06	.3771E+03	.7096E+03	.6731E-01
.6400E-03	.4688E+02	.5436E+02	-.3313E+06	-.2768E+06	.4481E+03	.8568E+03	.5964E-01
.6800E-03	.4947E+02	.5751E+02	-.3485E+06	-.2928E+06	.5273E+03	.1023E+04	.5320E-01
.7200E-03	.5207E+02	.6066E+02	-.3659E+06	-.3089E+06	.6152E+03	.1210E+04	.4772E-01
.7600E-03	.5467E+02	.6383E+02	-.3835E+06	-.3250E+06	.7121E+03	.1419E+04	.4304E-01
.8000E-03	.5727E+02	.6699E+02	-.4012E+06	-.3412E+06	.8185E+03	.1650E+04	.3901E-01
.8400E-03	.5988E+02	.7016E+02	-.4190E+06	-.3573E+06	.9348E+03	.1905E+04	.3551E-01
.8800E-03	.6249E+02	.7334E+02	-.4369E+06	-.3735E+06	.1061E+04	.2186E+04	.3246E-01
.9200E-03	.6510E+02	.7652E+02	-.4550E+06	-.3896E+06	.1199E+04	.2492E+04	.2978E-01
.9600E-03	.6771E+02	.7969E+02	-.4732E+06	-.4058E+06	.1347E+04	.2826E+04	.2742E-01
.1000E-02	.7033E+02	.8288E+02	-.4915E+06	-.4220E+06	.1508E+04	.3189E+04	.2532E-01

TABLE VIII

σ_s = .2M $(\epsilon_H^N, \epsilon_V^N)$=(20, 80)$\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_P^{B-M}	τ_P^{M-T}	T
.4000E-04	.3122E+02	.3840E+02	-.7248E+06	-.5113E+06	.1986E+01	.2364E+01	.7420E+00
.8000E-04	.3581E+02	.4155E+02	-.7750E+06	-.5533E+06	.8417E+01	.1024E+02	.8023E+00
.1200E-03	.4118E+02	.4633E+02	-.8408E+06	-.6169E+06	.2038E+02	.2568E+02	.6434E+00
.1600E-03	.4708E+02	.5226E+02	-.9152E+06	-.6960E+06	.3915E+02	.5149E+02	.5037E+00
.2000E-03	.5332E+02	.5900E+02	-.9946E+06	-.7857E+06	.6603E+02	.7084E+02	.3938E+00
.2400E-03	.5978E+02	.6629E+02	-.1077E+07	-.8828E+06	.1023E+03	.1470E+03	.3108E+00
.2800E-03	.6639E+02	.7396E+02	-.1162E+07	-.9850E+06	.1492E+03	.2232E+03	.2489E+00
.3200E-03	.7311E+02	.8190E+02	-.1248E+07	-.1091E+07	.2081E+03	.3228E+03	.2024E+00
.3600E-03	.7990E+02	.9003E+02	-.1335E+07	-.1199E+07	.2801E+03	.4491E+03	.1670E+00
.4000E-03	.8676E+02	.9831E+02	-.1423E+07	-.1309E+07	.3666E+03	.6054E+03	.1397E+00
.4400E-03	.9363E+02	.1067E+03	-.1511E+07	-.1421E+07	.4686E+03	.7949E+03	.1183E+00
.4800E-03	.1006E+03	.1152E+03	-.1600E+07	-.1534E+07	.5874E+03	.1021E+04	.1013E+00
.5200E-03	.1075E+03	.1237E+03	-.1689E+07	-.1647E+07	.7241E+03	.1287E+04	.8766E-01
.5600E-03	.1145E+03	.1323E+03	-.1779E+07	-.1761E+07	.8800E+03	.1596E+04	.7651E-01
.6000E-03	.1215E+03	.1409E+03	-.1868E+07	-.1876E+07	.1056E+04	.1952E+04	.6731E-01
.6400E-03	.1285E+03	.1495E+03	-.1958E+07	-.1991E+07	.1254E+04	.2357E+04	.5964E-01
.6800E-03	.1355E+03	.1582E+03	-.2047E+07	-.2107E+07	.1474E+04	.2815E+04	.5320E-01
.7200E-03	.1425E+03	.1669E+03	-.2137E+07	-.2223E+07	.1718E+04	.3329E+04	.4772E-01
.7600E-03	.1495E+03	.1756E+03	-.2227E+07	-.2339E+07	.1987E+04	.3903E+04	.4304E-01
.8000E-03	.1566E+03	.1843E+03	-.2317E+07	-.2455E+07	.2282E+04	.4540E+04	.3901E-01
.8400E-03	.1636E+03	.1930E+03	-.2408E+07	-.2571E+07	.2604E+04	.5242E+04	.3551E-01
.8800E-03	.1707E+03	.2018E+03	-.2498E+07	-.2688E+07	.2954E+04	.6014E+04	.3246E-01
.9200E-03	.1777E+03	.2105E+03	-.2588E+07	-.2804E+07	.3334E+04	.6858E+04	.2978E-01
.9600E-03	.1848E+03	.2193E+03	-.2679E+07	-.2921E+07	.3744E+04	.7778E+04	.2742E-01
.1000E-02	.1918E+03	.2280E+03	-.2769E+07	-.3038E+07	.4186E+04	.8776E+04	.2532E-01

TABLE IX

σ_s = .4M $(\epsilon_H^N, \epsilon_V^N)$=(5, 80)$\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_P^{B-M}	τ_P^{M-T}	T
.4000E-04	.6838E+01	.7901E+01	-.4210E+06	-.3293E+06	.1639E+01	.1946E+01	.8023E+00
.8000E-04	.8900E+01	.9938E+01	-.4927E+06	-.4142E+06	.7595E+01	.9792E+01	.5037E+00
.1200E-03	.1123E+02	.1261E+02	-.5749E+06	-.5255E+06	.1976E+02	.2795E+02	.3108E+00
.1600E-03	.1366E+02	.1557E+02	-.6606E+06	-.6492E+06	.4005E+02	.6139E+02	.2024E+00
.2000E-03	.1615E+02	.1870E+02	-.7475E+06	-.7793E+06	.7031E+02	.1151E+03	.1397E+00
.2400E-03	.1866E+02	.2190E+02	-.8346E+06	-.9129E+06	.1123E+03	.1942E+03	.1013E+00
.2800E-03	.2118E+02	.2515E+02	-.9216E+06	-.1048E+07	.1678E+03	.3036E+03	.7651E-01
.3200E-03	.2371E+02	.2844E+02	-.1008E+07	-.1185E+07	.2384E+03	.4483E+03	.5964E-01
.3600E-03	.2625E+02	.3174E+02	-.1095E+07	-.1323E+07	.3259E+03	.6333E+03	.4772E-01
.4000E-03	.2880E+02	.3505E+02	-.1181E+07	-.1461E+07	.4319E+03	.8635E+03	.3901E-01
.4400E-03	.3134E+02	.3838E+02	-.1267E+07	-.1600E+07	.5580E+03	.1144E+04	.3246E-01
.4800E-03	.3389E+02	.4171E+02	-.1352E+07	-.1739E+07	.7059E+03	.1479E+04	.2742E-01
.5200E-03	.3644E+02	.4504E+02	-.1438E+07	-.1878E+07	.8772E+03	.1875E+04	.2346E-01
.5600E-03	.3899E+02	.4838E+02	-.1523E+07	-.2017E+07	.1074E+04	.2336E+04	.2029E-01
.6000E-03	.4154E+02	.5171E+02	-.1609E+07	-.2156E+07	.1297E+04	.2866E+04	.1772E-01
.6400E-03	.4410E+02	.5505E+02	-.1694E+07	-.2295E+07	.1548E+04	.3472E+04	.1561E-01
.6800E-03	.4665E+02	.5839E+02	-.1779E+07	-.2434E+07	.1829E+04	.4157E+04	.1385E-01
.7200E-03	.4921E+02	.6173E+02	-.1865E+07	-.2573E+07	.2142E+04	.4927E+04	.1237E-01
.7600E-03	.5177E+02	.6507E+02	-.1951E+07	-.2712E+07	.2487E+04	.5786E+04	.1112E-01
.8000E-03	.5433E+02	.6840E+02	-.2036E+07	-.2852E+07	.2868E+04	.6740E+04	.1005E-01
.8400E-03	.5689E+02	.7174E+02	-.2122E+07	-.2991E+07	.3285E+04	.7794E+04	.9121E-02
.8800E-03	.5945E+02	.7508E+02	-.2208E+07	-.3130E+07	.3740E+04	.8952E+04	.8317E-02
.9200E-03	.6202E+02	.7841E+02	-.2294E+07	-.3269E+07	.4234E+04	.1022E+05	.7615E-02
.9600E-03	.6459E+02	.8175E+02	-.2380E+07	-.3408E+07	.4769E+04	.1160E+05	.6998E-02
.1000E-02	.6715E+02	.8508E+02	-.2467E+07	-.3547E+07	.5348E+04	.1310E+05	.6453E-02

TABLE X

$\sigma_s = .4M \quad (\epsilon_H^N, \epsilon_V^N) = (12, 12)\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_P^{B-M}	τ_P^{M-T}	T
.4000E-04	.6568E+01	.8115E+01	-.6423E+05	-.4132E+05	.6680E+00	.8328E+00	.9069E+00
.8000E-04	.7890E+01	.9160E+01	-.7135E+05	-.4665E+05	.2913E+01	.3760E+01	.7089E+00
.1200E-03	.9427E+01	.1067E+02	-.8031E+05	-.5434E+05	.7247E+01	.9855E+01	.5198E+00
.1600E-03	.1109E+02	.1247E+02	-.9026E+05	-.6352E+05	.1423E+02	.2048E+02	.3785E+00
.2000E-03	.1283E+02	.1446E+02	-.1008E+06	-.7362E+05	.2443E+02	.3709E+02	.2804E+00
.2400E-03	.1462E+02	.1655E+02	-.1118E+06	-.8428E+05	.3841E+02	.6115E+02	.2130E+00
.2800E-03	.1644E+02	.1872E+02	-.1231E+06	-.9532E+05	.5670E+02	.9413E+02	.1659E+00
.3200E-03	.1828E+02	.2094E+02	-.1347E+06	-.1066E+06	.7986E+02	.1375E+03	.1321E+00
.3600E-03	.2013E+02	.2319E+02	-.1466E+06	-.1181E+06	.1084E+03	.1927E+03	.1074E+00
.4000E-03	.2200E+02	.2546E+02	-.1586E+06	-.1296E+06	.1429E+03	.2613E+03	.8878E-01
.4400E-03	.2387E+02	.2775E+02	-.1708E+06	-.1413E+06	.1838E+03	.3446E+03	.7452E-01
.4800E-03	.2575E+02	.3005E+02	-.1832E+06	-.1530E+06	.2317E+03	.4441E+03	.6337E-01
.5200E-03	.2764E+02	.3237E+02	-.1958E+06	-.1648E+06	.2871E+03	.5613E+03	.5451E-01
.5600E-03	.2953E+02	.3469E+02	-.2085E+06	-.1766E+06	.3505E+03	.6977E+03	.4735E-01
.6000E-03	.3143E+02	.3701E+02	-.2213E+06	-.1885E+06	.4224E+03	.8546E+03	.4150E-01
.6400E-03	.3333E+02	.3934E+02	-.2343E+06	-.2003E+06	.5034E+03	.1034E+04	.3666E-01
.6800E-03	.3523E+02	.4167E+02	-.2474E+06	-.2122E+06	.5939E+03	.1236E+04	.3261E-01
.7200E-03	.3713E+02	.4401E+02	-.2606E+06	-.2241E+06	.6944E+03	.1463E+04	.2919E-01
.7600E-03	.3904E+02	.4635E+02	-.2739E+06	-.2360E+06	.8055E+03	.1717E+04	.2628E-01
.8000E-03	.4094E+02	.4869E+02	-.2873E+06	-.2479E+06	.9277E+03	.1998E+04	.2373E-01
.8400E-03	.4285E+02	.5102E+02	-.3009E+06	-.2598E+06	.1061E+04	.2309E+04	.2161E-01
.8800E-03	.4476E+02	.5336E+02	-.3145E+06	-.2718E+06	.1207E+04	.2651E+04	.1973E-01
.9200E-03	.4668E+02	.5571E+02	-.3283E+06	-.2837E+06	.1366E+04	.3024E+04	.1808E-01
.9600E-03	.4859E+02	.5805E+02	-.3421E+06	-.2956E+06	.1537E+04	.3431E+04	.1663E-01
.1000E-02	.5050E+02	.6039E+02	-.3560E+06	-.3075E+06	.1723E+04	.3873E+04	.1535E-01

TABLE XI

$\sigma_s = .4M \quad (\epsilon_H^N, \epsilon_V^N) = (12, 20)\pi$ MM-MRAD N=1E11

(τ in hrs.)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_P^{B-M}	τ_P^{M-T}	T
.4000E-04	.9515E+01	.1173E+02	-.1222E+06	-.8280E+05	.9736E+00	.1204E+01	.9069E+00
.8000E-04	.1141E+02	.1324E+02	-.1352E+06	-.9347E+05	.4243E+01	.5436E+01	.7089E+00
.1200E-03	.1362E+02	.1543E+02	-.1515E+06	-.1089E+06	.1055E+02	.1425E+02	.5198E+00
.1600E-03	.1602E+02	.1803E+02	-.1694E+06	-.1273E+06	.2071E+02	.2961E+02	.3785E+00
.2000E-03	.1852E+02	.2090E+02	-.1884E+06	-.1475E+06	.3552E+02	.5362E+02	.2804E+00
.2400E-03	.2108E+02	.2393E+02	-.2080E+06	-.1689E+06	.5580E+02	.8841E+02	.2130E+00
.2800E-03	.2369E+02	.2707E+02	-.2280E+06	-.1911E+06	.8233E+02	.1361E+03	.1659E+00
.3200E-03	.2632E+02	.3027E+02	-.2484E+06	-.2137E+06	.1159E+03	.1988E+03	.1321E+00
.3600E-03	.2898E+02	.3353E+02	-.2691E+06	-.2367E+06	.1572E+03	.2787E+03	.1074E+00
.4000E-03	.3165E+02	.3681E+02	-.2900E+06	-.2599E+06	.2071E+03	.3778E+03	.8878E-01
.4400E-03	.3434E+02	.4013E+02	-.3111E+06	-.2833E+06	.2663E+03	.4983E+03	.7452E-01
.4800E-03	.3704E+02	.4346E+02	-.3323E+06	-.3068E+06	.3356E+03	.6423E+03	.6337E-01
.5200E-03	.3974E+02	.4681E+02	-.3537E+06	-.3304E+06	.4156E+03	.8118E+03	.5451E-01
.5600E-03	.4245E+02	.5016E+02	-.3752E+06	-.3541E+06	.5071E+03	.1009E+04	.4735E-01
.6000E-03	.4516E+02	.5353E+02	-.3969E+06	-.3779E+06	.6109E+03	.1236E+04	.4150E-01
.6400E-03	.4788E+02	.5690E+02	-.4187E+06	-.4017E+06	.7276E+03	.1495E+04	.3666E-01
.6800E-03	.5060E+02	.6027E+02	-.4406E+06	-.4255E+06	.8581E+03	.1788E+04	.3261E-01
.7200E-03	.5332E+02	.6365E+02	-.4626E+06	-.4494E+06	.1003E+04	.2116E+04	.2919E-01
.7600E-03	.5605E+02	.6703E+02	-.4847E+06	-.4733E+06	.1163E+04	.2483E+04	.2628E-01
.8000E-03	.5878E+02	.7041E+02	-.5069E+06	-.4972E+06	.1339E+04	.2891E+04	.2378E-01
.8400E-03	.6151E+02	.7380E+02	-.5292E+06	-.5211E+06	.1531E+04	.3340E+04	.2161E-01
.8800E-03	.6424E+02	.7718E+02	-.5516E+06	-.5450E+06	.1741E+04	.3834E+04	.1973E-01
.9200E-03	.6698E+02	.8057E+02	-.5741E+06	-.5689E+06	.1969E+04	.4374E+04	.1808E-01
.9600E-03	.6971E+02	.8396E+02	-.5967E+06	-.5928E+06	.2216E+04	.4963E+04	.1663E-01
.1000E-02	.7245E+02	.8734E+02	-.6193E+06	-.6167E+06	.2482E+04	.5602E+04	.1535E-01

TABLE XII

σ_S =.4M $(\epsilon_H^N, \epsilon_V^N)$=(12, 80)π MM-MRAD N=1E11

(τ in hrs.)

σ_η	$\tau_H^{B\text{-}M}$	$\tau_H^{M\text{-}T}$	$\tau_V^{B\text{-}M}$	$\tau_V^{M\text{-}T}$	$\tau_p^{B\text{-}M}$	$\tau_p^{M\text{-}T}$	T
.4000E-04	.2719E+02	.3255E+02	-.8924E+06	-.6493E+06	.2827E+01	.3341E+01	.9069E+00
.8000E-04	.3243E+02	.3675E+02	-.9805E+06	-.7331E+06	.1231E+02	.1509E+02	.7089E+00
.1200E-03	.3857E+02	.4282E+02	-.1091E+07	-.8542E+06	.3054E+02	.3956E+02	.5198E+00
.1600E-03	.4521E+02	.5007E+02	-.1211E+07	-.9988E+06	.5985E+02	.8223E+02	.3785E+00
.2000E-03	.5214E+02	.5804E+02	-.1337E+07	-.1158E+07	.1025E+03	.1489E+03	.2804E+00
.2400E-03	.5923E+02	.6647E+02	-.1466E+07	-.1326E+07	.1608E+03	.2456E+03	.2130E+00
.2800E-03	.6644E+02	.7518E+02	-.1597E+07	-.1500E+07	.2369E+03	.3781E+03	.1659E+00
.3200E-03	.7371E+02	.8410E+02	-.1729E+07	-.1678E+07	.3330E+03	.5524E+03	.1321E+00
.3600E-03	.8104E+02	.9315E+02	-.1861E+07	-.1858E+07	.4512E+03	.7744E+03	.1074E+00
.4000E-03	.8840E+02	.1023E+03	-.1994E+07	-.2041E+07	.5937E+03	.1050E+04	.8878E-01
.4400E-03	.9579E+02	.1115E+03	-.2127E+07	-.2225E+07	.7624E+03	.1385E+04	.7452E-01
.4800E-03	.1032E+03	.1208E+03	-.2260E+07	-.2410E+07	.9594E+03	.1785E+04	.6337E-01
.5200E-03	.1106E+03	.1301E+03	-.2393E+07	-.2595E+07	.1187E+04	.2256E+04	.5451E-01
.5600E-03	.1181E+03	.1394E+03	-.2526E+07	-.2782E+07	.1447E+04	.2805E+04	.4735E-01
.6000E-03	.1255E+03	.1488E+03	-.2659E+07	-.2968E+07	.1741E+04	.3436E+04	.4150E-01
.6400E-03	.1330E+03	.1582E+03	-.2792E+07	-.3156E+07	.2071E+04	.4156E+04	.3666E-01
.6800E-03	.1404E+03	.1676E+03	-.2925E+07	-.3343E+07	.2440E+04	.4970E+04	.3261E-01
.7200E-03	.1479E+03	.1770E+03	-.3058E+07	-.3531E+07	.2850E+04	.5885E+04	.2919E-01
.7600E-03	.1554E+03	.1864E+03	-.3191E+07	-.3718E+07	.3301E+04	.6905E+04	.2628E-01
.8000E-03	.1628E+03	.1958E+03	-.3324E+07	-.3906E+07	.3797E+04	.8038E+04	.2378E-01
.8400E-03	.1703E+03	.2052E+03	-.3457E+07	-.4094E+07	.4339E+04	.9288E+04	.2161E-01
.8800E-03	.1778E+03	.2146E+03	-.3590E+07	-.4282E+07	.4930E+04	.1066E+05	.1973E-01
.9200E-03	.1853E+03	.2241E+03	-.3724E+07	-.4471E+07	.5571E+04	.1217E+05	.1808E-01
.9600E-03	.1928E+03	.2335E+03	-.3857E+07	-.4659E+07	.6264E+04	.1380E+05	.1663E-01
.1000E-02	.2003E+03	.2429E+03	-.3991E+07	-.4847E+07	.7011E+04	.1558E+05	.1535E-01

TABLE XIII

σ_S =.4M $(\epsilon_H^N, \epsilon_V^N)$=(20, 12)π MM-MRAD N=1E11

(τ in hrs.)

σ_η	$\tau_H^{B\text{-}M}$	$\tau_H^{M\text{-}T}$	$\tau_V^{B\text{-}M}$	$\tau_V^{M\text{-}T}$	$\tau_p^{B\text{-}M}$	$\tau_p^{M\text{-}T}$	T
.4000E-04	.1542E+02	.1919E+02	-.1239E+06	-.7352E+05	.9620E+00	.1182E+01	.9420E+00
.8000E-04	.1781E+02	.2077E+02	-.1341E+06	-.7955E+05	.4084E+01	.5114E+01	.8023E+00
.1200E-03	.2056E+02	.2315E+02	-.1474E+06	-.8868E+05	.9907E+01	.1283E+02	.6434E+00
.1600E-03	.2358E+02	.2611E+02	-.1626E+06	-.1000E+06	.1907E+02	.2573E+02	.5037E+00
.2000E-03	.2678E+02	.2948E+02	-.1792E+06	-.1129E+06	.3223E+02	.4537E+02	.3938E+00
.2400E-03	.3010E+02	.3311E+02	-.1968E+06	-.1268E+06	.5004E+02	.7340E+02	.3108E+00
.2800E-03	.3350E+02	.3694E+02	-.2152E+06	-.1415E+06	.7315E+02	.1115E+03	.2489E+00
.3200E-03	.3698E+02	.4090E+02	-.2344E+06	-.1567E+06	.1022E+03	.1612E+03	.2024E+00
.3600E-03	.4046E+02	.4496E+02	-.2541E+06	-.1722E+06	.1378E+03	.2242E+03	.1670E+00
.4000E-03	.4400E+02	.4909E+02	-.2744E+06	-.1880E+06	.1807E+03	.3022E+03	.1397E+00
.4400E-03	.4756E+02	.5327E+02	-.2953E+06	-.2040E+06	.2314E+03	.3969E+03	.1183E+00
.4800E-03	.5114E+02	.5749E+02	-.3166E+06	-.2202E+06	.2905E+03	.5097E+03	.1013E+00
.5200E-03	.5474E+02	.6174E+02	-.3384E+06	-.2365E+06	.3587E+03	.6425E+03	.8766E-01
.5600E-03	.5836E+02	.6602E+02	-.3607E+06	-.2529E+06	.4366E+03	.7967E+03	.7651E-01
.6000E-03	.6198E+02	.7032E+02	-.3833E+06	-.2693E+06	.5248E+03	.9741E+03	.6731E-01
.6400E-03	.6561E+02	.7463E+02	-.4065E+06	-.2858E+06	.6239E+03	.1176E+04	.5964E-01
.6800E-03	.6926E+02	.7895E+02	-.4300E+06	-.3024E+06	.7346E+03	.1405E+04	.5320E-01
.7200E-03	.7290E+02	.8329E+02	-.4539E+06	-.3190E+06	.8573E+03	.1661E+04	.4772E-01
.7600E-03	.7656E+02	.8763E+02	-.4782E+06	-.3356E+06	.9928E+03	.1948E+04	.4304E-01
.8000E-03	.8022E+02	.9198E+02	-.5029E+06	-.3522E+06	.1142E+04	.2265E+04	.3901E-01
.8400E-03	.8388E+02	.9633E+02	-.5279E+06	-.3689E+06	.1304E+04	.2616E+04	.3551E-01
.8800E-03	.8755E+02	.1007E+03	-.5534E+06	-.3856E+06	.1482E+04	.3001E+04	.3246E-01
.9200E-03	.9122E+02	.1050E+03	-.5791E+06	-.4023E+06	.1674E+04	.3422E+04	.2978E-01
.9600E-03	.9489E+02	.1094E+03	-.6052E+06	-.4190E+06	.1882E+04	.3880E+04	.2742E-01
.1000E-02	.9857E+02	.1138E+03	-.6317E+06	-.4357E+06	.2107E+04	.4378E+04	.2532E-01

TABLE XIV

$\delta_s = .4M \quad (\epsilon_H^N, \epsilon_V^N) = (20, 20)\pi$ MM-MRAD N=1E11

(τ in hours)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.2221E+02	.2765E+02	-.2206E+06	-.1408E+06	.1393E+01	.1702E+01	.9420E+00
.8000E-04	.2561E+02	.2992E+02	-.2375E+06	-.1524E+06	.5909E+01	.7370E+01	.8023E+00
.1200E-03	.2953E+02	.3336E+02	-.2596E+06	-.1699E+06	.1433E+02	.1849E+02	.6434E+00
.1600E-03	.3385E+02	.3763E+02	-.2847E+06	-.1916E+06	.2757E+02	.3707E+02	.5037E+00
.2000E-03	.3842E+02	.4248E+02	-.3119E+06	-.2163E+06	.4656E+02	.6539E+02	.3938E+00
.2400E-03	.4316E+02	.4772E+02	-.3404E+06	-.2430E+06	.7224E+02	.1058E+03	.3108E+00
.2800E-03	.4801E+02	.5324E+02	-.3699E+06	-.2711E+06	.1055E+03	.1606E+03	.2489E+00
.3200E-03	.5294E+02	.5895E+02	-.4003E+06	-.3002E+06	.1474E+03	.2323E+03	.2024E+00
.3600E-03	.5794E+02	.6479E+02	-.4314E+06	-.3300E+06	.1986E+03	.3232E+03	.1670E+00
.4000E-03	.6298E+02	.7074E+02	-.4631E+06	-.3603E+06	.2602E+03	.4356E+03	.1397E+00
.4400E-03	.6805E+02	.7677E+02	-.4953E+06	-.3909E+06	.3331E+03	.5720E+03	.1183E+00
.4800E-03	.7316E+02	.8286E+02	-.5280E+06	-.4219E+06	.4180E+03	.7347E+03	.1013E+00
.5200E-03	.7828E+02	.8899E+02	-.5611E+06	-.4532E+06	.5159E+03	.7260E+03	.8766E-01
.5600E-03	.8343E+02	.9515E+02	-.5945E+06	-.4846E+06	.6277E+03	.1148E+04	.7651E-01
.6000E-03	.8859E+02	.1013E+03	-.6284E+06	-.5161E+06	.7541E+03	.1404E+04	.6731E-01
.6400E-03	.9376E+02	.1076E+03	-.6625E+06	-.5477E+06	.8961E+03	.1695E+04	.5964E-01
.6800E-03	.9894E+02	.1138E+03	-.6970E+06	-.5795E+06	.1055E+04	.2025E+04	.5320E-01
.7200E-03	.1041E+03	.1200E+03	-.7318E+06	-.6113E+06	.1230E+04	.2395E+04	.4772E-01
.7600E-03	.1093E+03	.1263E+03	-.7669E+06	-.6432E+06	.1424E+04	.2807E+04	.4304E-01
.8000E-03	.1145E+03	.1326E+03	-.8023E+06	-.6751E+06	.1637E+04	.3265E+04	.3901E-01
.8400E-03	.1198E+03	.1388E+03	-.8380E+06	-.7071E+06	.1870E+04	.3770E+04	.3551E-01
.8800E-03	.1250E+03	.1451E+03	-.8739E+06	-.7390E+06	.2123E+04	.4325E+04	.3246E-01
.9200E-03	.1302E+03	.1514E+03	-.9100E+06	-.7711E+06	.2398E+04	.4932E+04	.2978E-01
.9600E-03	.1354E+03	.1577E+03	-.9464E+06	-.8031E+06	.2695E+04	.5593E+04	.2742E-01
.1000E-02	.1407E+03	.1640E+03	-.9830E+06	-.8352E+06	.3015E+04	.6311E+04	.2532E-01

TABLE XV

$\delta_s = .4M \quad (\epsilon_H^N, \epsilon_V^N) = (20, 80)\pi$ MM-MRAD N=1E11

(τ in hours)

σ_η	τ_H^{B-M}	τ_H^{M-T}	τ_V^{B-M}	τ_V^{M-T}	τ_p^{B-M}	τ_p^{M-T}	T
.4000E-04	.6244E+02	.7601E+02	-.1450E+07	-.1012E+07	.3972E+01	.4680E+01	.9420E+00
.8000E-04	.7163E+02	.8226E+02	-.1550E+07	-.1095E+07	.1683E+02	.2026E+02	.8023E+00
.1200E-03	.8235E+02	.9171E+02	-.1682E+07	-.1221E+07	.4076E+02	.5083E+02	.6434E+00
.1600E-03	.9416E+02	.1035E+03	-.1830E+07	-.1378E+07	.7831E+02	.1019E+03	.5037E+00
.2000E-03	.1066E+03	.1168E+03	-.1989E+07	-.1556E+07	.1321E+03	.1798E+03	.3938E+00
.2400E-03	.1196E+03	.1312E+03	-.2154E+07	-.1748E+07	.2046E+03	.2909E+03	.3108E+00
.2800E-03	.1328E+03	.1464E+03	-.2324E+07	-.1950E+07	.2985E+03	.4418E+03	.2489E+00
.3200E-03	.1462E+03	.1621E+03	-.2496E+07	-.2160E+07	.4162E+03	.6390E+03	.2024E+00
.3600E-03	.1598E+03	.1782E+03	-.2670E+07	-.2374E+07	.5603E+03	.8891E+03	.1670E+00
.4000E-03	.1735E+03	.1946E+03	-.2846E+07	-.2593E+07	.7331E+03	.1199E+04	.1397E+00
.4400E-03	.1873E+03	.2112E+03	-.3023E+07	-.2814E+07	.9372E+03	.1574E+04	.1183E+00
.4800E-03	.2011E+03	.2280E+03	-.3200E+07	-.3037E+07	.1175E+04	.2022E+04	.1013E+00
.5200E-03	.2150E+03	.2449E+03	-.3379E+07	-.3262E+07	.1448E+04	.2548E+04	.8766E-01
.5600E-03	.2289E+03	.2618E+03	-.3557E+07	-.3488E+07	.1760E+04	.3160E+04	.7651E-01
.6000E-03	.2429E+03	.2789E+03	-.3736E+07	-.3715E+07	.2112E+04	.3864E+04	.6731E-01
.6400E-03	.2569E+03	.2960E+03	-.3915E+07	-.3944E+07	.2508E+04	.4667E+04	.5964E-01
.6800E-03	.2709E+03	.3132E+03	-.4095E+07	-.4172E+07	.2948E+04	.5574E+04	.5320E-01
.7200E-03	.2850E+03	.3304E+03	-.4275E+07	-.4402E+07	.3436E+04	.6592E+04	.4772E-01
.7600E-03	.2990E+03	.3476E+03	-.4455E+07	-.4631E+07	.3974E+04	.7728E+04	.4304E-01
.8000E-03	.3131E+03	.3649E+03	-.4635E+07	-.4861E+07	.4563E+04	.8989E+04	.3901E-01
.8400E-03	.3272E+03	.3822E+03	-.4815E+07	-.5092E+07	.5207E+04	.1038E+05	.3551E-01
.8800E-03	.3413E+03	.3995E+03	-.4996E+07	-.5323E+07	.5908E+04	.1191E+05	.3246E-01
.9200E-03	.3554E+03	.4168E+03	-.5176E+07	-.5553E+07	.6667E+04	.1358E+05	.2978E-01
.9600E-03	.3695E+03	.4342E+03	-.5357E+07	-.5784E+07	.7487E+04	.1540E+05	.2742E-01
.1000E-02	.3837E+03	.4515E+03	-.5538E+07	-.6016E+07	.8371E+04	.1738E+05	.2532E-01

TABLE XVI

A.

$N = 10^{11}$ $\varepsilon_H^N = \varepsilon_V^N = 20\pi \times 10^{-6}$ m

$\sigma_s = 0.2$ m $\gamma = 1000$

	Ref. 2		Eq. 69	
σ_η	$\frac{\tau_H}{\tau_p}$	$\frac{<\tau_H>\sigma_\eta^2\gamma}{<\tau_p>\varepsilon_H^N}$ (m^{-1})	$\frac{\tau_H}{\tau_p}$	$\frac{\tau_H\sigma_\eta^2\gamma}{\tau_p\varepsilon_H^N}$ (m^{-1})
0.40×10^{-4}	15.9400	0.405	16.24	0.413
0.80×10^{-4}	4.3300	0.441	4.06	0.413
0.12×10^{-3}	2.0600	0.472	1.81	0.413
0.16×10^{-3}	1.2290	0.500	1.02	0.413
0.20×10^{-3}	0.8250	0.525	0.650	0.413
0.40×10^{-3}	0.2420	0.616	0.162	0.413
0.60×10^{-3}	0.1170	0.673	0.072	0.413
0.10×10^{-2}	0.0466	0.742	0.026	0.413

B.

$N = 10^{11}$ $\varepsilon_H^N = \varepsilon_V^N = 12\pi \times 10^{-6}$ m

$\sigma_s = 0.2$ m $\gamma = 1000$

	Ref. 2		Eq. 69	
σ_η	$\frac{\tau_H}{\tau_p}$	$\frac{<\tau_H>\sigma_\eta^2\gamma}{<\tau_p>\varepsilon_H^N}$ (m^{-1})	$\frac{\tau_H}{\tau_p}$	$\frac{\tau_H\sigma_\eta^2\gamma}{\tau_p\varepsilon_H^N}$ (m^{-1})
0.40×10^{-4}	9.830	0.417	9.740	0.413
0.80×10^{-4}	2.710	0.460	2.440	0.413
0.12×10^{-3}	1.300	0.497	1.080	0.413
0.16×10^{-3}	0.779	0.529	0.609	0.413
0.20×10^{-3}	0.525	0.557	0.390	0.413
0.40×10^{-3}	0.154	0.653	0.097	0.413
0.60×10^{-3}	0.074	0.710	0.043	0.413
0.10×10^{-2}	0.029	0.777	0.016	0.413

C.

$N = 10^{11}$ $\varepsilon_H^N = 5\pi \times 10^{-6}$ m $\varepsilon_V^N = 80\pi \times 10^{-6}$ m

$\sigma_s = 0.2$ m $\gamma = 1000$

	Ref. 2		Eq. 69	
σ_η	$\frac{\tau_H}{\tau_p}$	$\frac{<\tau_H>\sigma_\eta^2\gamma}{<\tau_p>\varepsilon_H^N}$ (m^{-1})	$\frac{\tau_H}{\tau_p}$	$\frac{\tau_H\sigma_\eta^2\gamma}{\tau_p\varepsilon_H^N}$ (m^{-1})
0.40×10^{-4}	4.170	0.424	4.06	0.413
0.80×10^{-4}	1.170	0.478	1.02	0.413
0.12×10^{-3}	0.568	0.521	0.451	0.413
0.16×10^{-3}	0.341	0.556	0.254	0.413
0.20×10^{-3}	0.230	0.585	0.162	0.413
0.40×10^{-3}	0.067	0.679	0.041	0.413
0.60×10^{-3}	0.032	0.734	0.018	0.413
0.10×10^{-2}	0.013	0.799	0.006	0.413

APPENDIX A

MAC_CALC.BAS 1/20/88

A.V. TOLLESTRUP

FNAL

```
1 REM THIS PROGRAM CALCULATES VARIOUS MACHINE PARAMETERS FROM INPUT DATA
2 REM    LISTED IN THE FIRST 100 LINES
3 PRINT "VERSION 1/20/88  BETTER APPROX FOR G(B)"
4 PRINT\PRINT\PRINT
8 DECLARE REAL FUNCTION GB(REAL)
9 DECLARE REAL FUNCTION IO(REAL)
10 DECLARE REAL FUNCTION LUM
11 DECLARE REAL FUNCTION SIGMA
12 DECLARE REAL FUNCTION ETA(REAL)
13 DECLARE REAL FUNCTION SYNC_FREQ(REAL)
14 DECLARE REAL FUNCTION THETA_MAX(REAL)
15 DECLARE REAL FUNCTION DELTA_E(REAL)
16 DECLARE REAL FUNCTION DELTA_NU    !TUNE SHIFT
17 DECLARE REAL FUNCTION TAU_P(REAL)
18 DECLARE STRING FOR_STRING
19 FOR_STRING&
="##.#       ##.##     ##.##     .###^^^^  ##.##     ###.#     ###.#"
92 OPEN "MAC.OUT" FOR OUTPUT AS FILE#1
98 !
99          !*****    CONSTANTS           *******
100 EMIT_X = 20\ EMIT_Y = 20
101 NB = 3      ! NUMBER OF BUNCHES
102 NPBAR = .30     ! NUMBER OF PBAR IN THE MACHINE IN UNITS OF E12
103 NP = .3
104 BETA_STAR = 1.0   ! BETA AT INTERACTIOMN POINT
105 VRF = .13536
106 EV_SEC = .23576
107 FO = 47619
108 HARM = 1113
109 E=938
110 R = 1000
111 TUNE = 19.82686
112 XP = R/(TUNE)**2
210 GAMT = TUNE
215 PRINT "USE APPROX CALC FOR G(B)";\INPUT APPROX$
250 PRINT "SPS PARAMETERS [SPS] OR SSC [SSC]";\INPUT A$
260 IF A$ = "N" THEN 300
270 READ EMIT_X,EMIT_Y,NB,NPBAR,NP,VRF,EV_SEC,FO,HARM,E,GAMT,TUNE,XP,R
280 IF A$ = "SPS" THEN 300
290 READ EMIT_X,EMIT_Y,NB,NPBAR,NP,VRF,EV_SEC,FO,HARM,E,GAMT,TUNE,XP,R
300          !****************  CHANGE INPUT   ****************
302 PRINT "CHANGE CONSTANTS";
305 INPUT A$
306 IF A$ = "N" THEN 600 ELSE PRINT "VARIABLE =";
310 INPUT A$
320 IF A$ = "EMIT_X" THEN PRINT EMIT_X;
                        INPUT EMIT_X
321 IF A$ = "EMIT_Y" THEN PRINT EMIT_Y;
                        INPUT EMIT_Y
330 IF A$ = "NB" THEN PRINT NB;
                    INPUT NB
340 IF A$ = "NPBAR" THEN PRINT NPBAR;
                       INPUT NPBAR
```

```
350 IF A$ = "NP" THEN PRINT NP;
                    INPUT NP
360 IF A$ = "BETA_STAR" THEN PRINT BETA_STAR;
                           INPUT BETA_STAR
370 IF A$ = "VRF" THEN PRINT VRF;
                     INPUT VRF
380 IF A$ = "EV_SEC" THEN PRINT EV_SEC;
                        INPUT EV_SEC
390 IF A$ = "FO" THEN PRINT FO;
                    INPUT FO
400 IF A$ = "HARM" THEN PRINT HARM;
                      INPUT HARM
410 IF A$ = "E" THEN PRINT E;
                   INPUT E
420 IF A$ = "GAMT" THEN PRINT GAMT;
                      INPUT GAMT
430 IF A$ = "TUNE" THEN PRINT TUNE;
                      INPUT TUNE
440 IF A$ = "XP" THEN PRINT XP;
                    INPUT XP
590 IF A$ = "N" THEN 600
595 PRINT "MORE"
       INPUT A$
       GOTO 320
600  ! END CHANGE DATA
1000 PRINT DATE$(0),TIME$(0)
1010             !**********   CALCUATED CONSTANTS    **********
1020 GAMMA = E/.938
1030 BETAX = R/TUNE
5000 Y = TAU_P(VRF)
7998 !
7999        !*****    PRINT STATEMENTS    *******
8000 PRINT "LUM PER CROSSING  ";LUM,"TOTAL LUM   ";NB*FO*LUM
8001 PRINT "SIGMA_X MM  ";10*SIGMA_X, "SIGMA_Y MM ";10*SIGMA_Y
8002 PRINT "ETA   "; ETA(E)
8003 PRINT "SYNC FREQ   ";SYNC_FREQ(VRF)
8004 PRINT "THETA RMS NS=  "; THETA_MAX(VRF),"LENGTH,CM  :";30*THETA_MAX(VRF)
8005 PRINT "DELTA_E RMS IN MEV"; DELTA_E(VRF) ,"DELTA P/P:";DELTA_E(VRF)/E*.001
8006 PRINT "DELTA_NU,SINGLE CROSSING TUNE SHIFT   ";DELTA_NU
8010 PRINT\PRINT "TAU_P HOURS: ";TAU_P(VRF)
8011 PRINT       "TAU_X HOURS: ";TAU_X
8012 PRINT       "TAU_Y HOURS: ";TAU_Y
8013 PRINT "A1  ";A1;"  C1  ";C1;"  T  ";T;"  A, (HOURS)**-1  ";A*3600
8014 PRINT "B= ";B, "APPROX USED FOR G(B)?";APPROX$,"G(B) =";GB(B)
8100 PRINT#1, "LUM PER CROSSING  ";LUM,"TOTAL LUM   ";NB*FO*LUM
8101 PRINT#1, "SIGMA_X MM  ";10*SIGMA_X, "SIGMA_Y MM ";10*SIGMA_Y
8102 PRINT#1, "ETA   "; ETA(E)
8103 PRINT#1, "SYNC FREQ   ";SYNC_FREQ(VRF)
8104 PRINT#1, "THETA RMS NS=  "; THETA_MAX(VRF),"LENGTH,CM  :";30*THETA_MAX(VRF)
8105 PRINT#1, "DELTA_E RMS IN MEV"; DELTA_E(VRF) ,"DELTA P/P:";DELTA_E(VRF)/E*.001
8106 PRINT#1, "DELTA_NU,SINGLE CROSSING TUNE SHIFT   ";DELTA_NU
8110 PRINT#1,\PRINT#1, "TAU_P HOURS: ";TAU_P(VRF)
8111 PRINT#1,       "TAU_X HOURS: ";TAU_X
8112 PRINT#1,       "TAU_Y HOURS: ";TAU_Y
8113 PRINT#1, "A1  ";A1;"  C1  ";C1;"  T  ";T;"  A, (HOURS)**-1  ";A*3600
8114 PRINT#1,"B= ";B, "APPROX USED FOR G(B)?";APPROX$,"G(B)=";GB(B)
8900   !   PRINT OUT CONSTANTS USED
8901 PRINT \PRINT\PRINT\PRINT"                 *********** CONSTANTS USED  *************"
8910 PRINT "EMIT_X",EMIT_X,, "EMIT_Y",EMIT_Y
```

```
8911 PRINT "NO. OF BUNCHES: ";NB
8920 PRINT "NPBAR",NPBAR*1E12,,"NP",NP*1E12
8921 PRINT "NPBAR/BUNCH  ",1E12*NPBAR/NB,,"NP/BUNCH  ",1E12*NP/NB
8930 PRINT "BETA_STAR",BETA_STAR,,"VRF",VRF
8940 PRINT "EV_SEC",EV_SEC,,"FO",FO
8950 PRINT "HARM",HARM,,"E",E
8960 PRINT "GAMT", GAMT,,"TUNE",TUNE
8970 PRINT "XP",XP,,"BETAX",BETAX
8980 PRINT "RADIUS R",R
8981 PRINT#1\PRINT#1\PRINT#1,"                  ************* CONSTANTS USED **************"
8982 PRINT#1, "EMIT_X",EMIT_X,, "EMIT_Y",EMIT_Y
8983 PRINT#1, "NO. OF BUNCHES: ";NB
8984 PRINT#1, "NPBAR",NPBAR*1E12,,"NP",NP*1E12
8985 PRINT#1, "NPBAR/BUNCH  ",1E12*NPBAR/NB,,"NP/BUNCH  ",1E12*NP/NB
8986 PRINT#1, "BETA_STAR",BETA_STAR,,"VRF",VRF
8987 PRINT#1, "EV_SEC",EV_SEC,,"FO",FO
8989 PRINT#1, "HARM",HARM,,"E",E
8990 PRINT#1, "GAMT", GAMT,,"TUNE",TUNE
8991 PRINT#1, "XP",XP,,"BETAX",BETAX
8992 PRINT#1, "RADIUS R",R
9998!

9999          !**** DEFINE LUM  ******
10000 DEF REAL LUM
10002 ZZ = SIGMA
10010 LUM = NP*NPBAR/(4*PI*SIGMA_X * SIGMA_Y) * (1E24/NB**2)
10020 END DEF
10098 !
10099          !**** DEFINE SIGMA_X  UNITS CM  ********
10100 DEF REAL SIGMA
10110 SIG2_X = EMIT_X * BETA_STAR/(6*GAMMA)
10120 SIG2_Y = EMIT_Y * BETA_STAR/(6*GAMMA)
10130 SIGMA_X = SQRT(SIG2_X) * .1    ! SIGMA IN MM, CHANGE TO CM
10140 SIGMA_Y = SQRT(SIG2_Y) * .1
10145 SIGMA = SIGMA_X
10150 END DEF
10198 !
10199          !**** DEFINE ETA(E)  ***
10200 DEF REAL ETA(E1)
10210 ETA = (1/GAMT**2) - (.938/E1)**2
10220 END DEF
10298 !
10299          !**** DEFINE SYNC_FREQ(V)  *****
10300 DEF REAL SYNC_FREQ(V)
10310 SYNC_FREQ = SQRT(V*ETA(E)*HARM/(2*PI*E*1000))* FO
10320 END DEF
10398 !
10399          !**** DEFINE THETA_MAX(V)  (UNITS NS)   *****
10400 DEF REAL THETA_MAX(V)
10410 THETA = SQRT(.3333*EV_SEC*FO*FO*ETA(E)/(E*SYNC_FREQ(V)*1E9))
10420 THETA_MAX = THETA/(2*PI*FO) * 1E9
10430 END DEF
10498 !
10499          !**** DEFINE DELTA_E(V) (UNITS MEV)  ****
10500 DEF REAL DELTA_E(V)
10510 DELTA_E = SQRT(EV_SEC * E * SYNC_FREQ(V) * 1E9/(3*ETA(E))) * 1E-6
10520 END DEF
10598 !
10599          !**** DEFINE DELTA_NU      *****
```

```
10600 DEF REAL DELTA_NU
10610 DELTA_NU = 1.54 * NP * BETA_STAR/(4*PI*GAMMA*SIGMA**2 * NB*100)
10620 END DEF
10698 !
10699         !*****      DEFINE TAU_P      ******
10700 DEF REAL TAU_P(V)
10710 T = 6 * PI * GAMMA * XP**2
10720 T = T * (DELTA_E(V)/E * .001)**2
10730 T = T /(EMIT_X *  PI *1E-6* BETAX)
10740 T = 1/(1+T)
10760 A1 = GAMMA * EMIT_Y * PI * 1E-6
10770 A1 = A1/(6 * PI * BETAX)
10780 A1 = SQRT(A1) /SQRT(T)
10790 A1 = A1/(DELTA_E(V)/E * .001)
10810 C1 = (EMIT_X * EMIT_Y * (PI*1E-6)**2 * BETAX**2/T)**(1/12)
10820 C1 = C1 * ((6*PI)**.5 * THETA_MAX(V) * .3 * NB/NP*1E-12)**(1/6)
10830 C1 = C1*(GAMMA*EMIT_Y * PI * 1E-6/( 6*PI*BETAX*1.54E-18) )**.5
10840 AA = 27 * PI * (1.54E-18)**2 * .938E9 * NP*1E12/NB
10850 AA  =  AA/(8 * GAMMA * PI**2 * 1E-12)
10860 A = AA/( EMIT_X  * EMIT_Y * EV_SEC )
10865 K1 = 4 * A * PI**1.5
10870 B = ( EMIT_Y/EMIT_X)**.5
10875 G1 = GB(B)
10880 G2 = GB(1/B)
10885 TAUP = LOG(C1) * G1 + LOG(C1/B)*G2/B
10890 TAUP = TAUP * A1 * K1 * T
10895 TAUP = TAUP*3600
10900 TAUX =(1-T)*(A1/B)**2
10905 TAUX = (1-TAUX)*B*LOG(C1/B)*G2/A1 - (1-T)*A1*G1*LOG(C1)
10910 TAUX = K1*TAUX
10915 TAUX = -3600*TAUX
10920 TAU_Y =K1*LOG(C1) * G1/A1
10925 TAU_Y = 1/TAU_Y
10996 TAU_X = 1/TAUX
10997 TAUP = 1/TAUP
10998 TAU_P = TAUP
11000 END DEF
11010!
11020       !**********    DEFINE G(B)    ******************
11030 DEF REAL GB(X)
11040 GBB = 0
11041 if APPROX$ = "Y" THEN 11170
11045 FB = ( X**2 - 1)/(X**2 + 1)
11050 YMAX = 20
11060 KMAX = 1000
11070 FOR K=1 TO KMAX-1
11080 Y = K*YMAX/KMAX
11090 Y = Y*Y
11100 GBB= GBB+ EXP((ABS(FB)-1)*Y)*(I0(FB*Y) + 3*I1)
11110 NEXT K
11120 GBB= GBB+.5
11130 DELTA_GB = EXP((ABS(FB)-1)*YMAX**2)*(I0(FB*YMAX**2) + 3*I1)
11140 GBB= GBB+ .5*DELTA_GB
11150 GB = GBB* 2 * SQRT(2/(X**2 + 1))* YMAX/KMAX
11160 GOTO 11180
11170 GB = 2.691*(1 - .2288964/X)/((1 + .16*X)*(1 + 1.35*EXP(-X/.2)))
11180 !
11190 END DEF
11200!
```

```
11210!       ***********   DEF I0(X) AND I1(X)  ****************
11220 DEF REAL I0(X) ! AND I1
11230 XS = SGN(X)
11240 XA = ABS(X)
11250 IF XA>4 GOTO 11390
11260 Y1 = .5*XA
11270 I00= 1\ U=1
11280 I11= Y1 \ V=Y1
11290 FOR M=1 TO 20
11300 U = U * Y1**2/M**2
11310 V = V * Y1**2/(M*(M+1))
11320 I00= I00+ U
11330 I11= I11+ V
11340 NEXT M
11350 I00= I00*EXP(-XA) \ I11 = EXP(-XA)*I11  ! REMOVE EXP DEPENDANCE
11360 GOTO 11430
11390 I00= 1/SQRT(2*PI*XA)
11400 I11= I00
11410 I00= I00*(1 + 1/(8*XA) + 4.5/(8*XA)**2 + .07324/XA**3)
11420 I11= I11*(1 - 3/(8*XA) - 15/(2*(8*XA)**2) - 105/(2*(8*XA)**3) )
11430 I0 = I00
11440 I1 = I11* XS
11450 END DEF
11800! *** *** *** *** LIFE TIME VS DELTA-P/P  *** *** *** *** *** ***
11880!  LOOP FOR PRINT OUT LIFE TIME ALA Bj/M
11881 GOTO 12200 !REMOVE THIS LINE FOR LIFETIME VS DELTA_P/P
11890 PRINT\PRINT "******   LIFETIME VS DELTA_P/P   ********"\PRINT\PRINT
11900 PRINT "DELTA P/P","TAU_P/2","TAU_X/2","SIGMA_Z","T","A1"
12000 FOR I= 1 TO 10
12020 PRINT DELTA_E(VRF)/E*.001,TAU_P(VRF)/2,TAU_X/2,30*THETA_MAX(VRF),T,A1
12030 IF TAU_X > 10000 THEN GO TO 16000
12040 EV_SEC = 2* EV_SEC
12050 VRF = 4*VRF
12060 A = A/2
12080 NEXT I
12200 !CONTINUE
12480!
12481!
12482!
12490!       ***********  EMITANCE VS TIME CALC    *****************
12491 PRINT\PRINT\PRINT "          ******************************"\PRINT
12492 PRINT#1\PRINT#1\PRINT#1,"         *******************************"\PRINT#1
12495 PRINT &
 "T        EMIT_X    EV_SEC    DELTA_P/P SIGMA_Z   TAU_X     TAU_P"
12496 PRINT#1, &
 "T        EMIT_X    EV_SEC    DELTA_P/P SIGMA_Z   TAU_X     TAU_P"
12500 FOR I=0 TO 400
12502 IF I/20 - INT(I/20) > .01 THEN GOTO 12510
12505 PRINT USING FOR_STRING,&
 .05*I , EMIT_X , EV_SEC , DELTA_E(VRF)/E*.001 , 30*THETA_MAX(VRF) ,&
 TAU_X , TAUP
12506 PRINT#1 USING FOR_STRING,&
 .05*I , EMIT_X , EV_SEC , DELTA_E(VRF)/E*.001 , 30*THETA_MAX(VRF) ,&
 TAU_X , TAUP
12510!
12520 Y = TAU_P(VRF)
12530 DT = .05
12540 D_EV_SEC = EV_SEC * 2 * DT/Y
12550 D_EMIT = EMIT_X * 2 * DT/TAU_X
```

```
12560 EV_SEC = EV_SEC + D_EV_SEC
12565 EMIT_X = EMIT_X + D_EMIT
12600 NEXT I
13000 PRINT "EMIT_X= ";EMIT_X;"    EMIT_Y=";EMIT_Y;"  T= ";T;"   A= ";A
13001 PRINT#1, "EMIT_X= ";EMIT_X;"    EMIT_Y=";EMIT_Y;"  T= ";T;"   A= ";A
15000 DATA 22.5,22.5,3,.255,.255,3.9,.9522,43406,4620,270,23.3,26.7,1.78,1100
15010 DATA 6,6,17397,127,127,10,4.39,3614,103680,20000,67,78,3.14,13200
16000 END
```