On the role of strong interaction in understanding nuclear beta stability line and nuclear binding energy

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Introduction

In this paper an attempt is made to fit and understand the nuclear binding energy and proton-neutron beta stability lime in terms of strong interaction. In nuclear physics, generally the semi-empirical mass formula (SEMF) is used to approximate the mass and various other properties of an atomic nucleus [1]. In modern nuclear physics the corresponding semi empirical relation can be expressed as follows. With usual notation,

$$\begin{aligned} B &= a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} \\ &\quad ......- a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \end{aligned} \tag{1}$$

where $a_v \cong 15.78$ MeV, $a_s \cong 18.34$ MeV, $a_c \cong 0.71$ MeV, $a_a \cong 23.21$ MeV and $a_p \cong 12.0$ MeV.

Characteristic relation for beta stability line

Inside an atomic nucleus, 'beta decay' is a type of radioactive decay in which a proton is transformed into a neutron, or vice versa. This process allows the atom to move closer to the optimal proton–neutron ratio. Important point to be noted is that, most naturally occurring isotopes on Earth are beta stable. Considering the strong coupling constant $\alpha_s \cong 0.1185$ to 0.12, proton-neutron beta stability relation can be expressed as follows. If A_s is the stable mass number,

$$A_s \cong 2Z + \left(\frac{2}{3}\alpha_s Z\right)^2 \cong 2Z + \left(\frac{2}{3}\alpha_s\right)^2 Z^2 .... (2)$$

Using this relation, starting from Z=11 to 118, stable mass numbers can be fitted and predicted. Considering even-odd correction, accuracy can be improved. There is minor discrepancy in between Z=51 to 60 and can be understood with further research and analysis.

Characteristic nuclear binding energy potential

In terms of strong interaction, characteristic binding energy potential can be expressed as follows.

$$\left.\begin{aligned} B_0 &\cong -\left(1+\frac{\alpha_s}{\alpha}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \\ &\cong -(19.5 \text{ to } 19.7) \text{ MeV} \end{aligned}\right\} \tag{3}$$

$$\text{where} \begin{cases} \alpha_s \cong 0.116 \text{ to } 0.12; \\ \alpha \cong 7.297352533\times 10^{-3}; \\ R_0 \cong 1.20 \text{ to } 1.25 \text{ fm}; \end{cases}$$

Characteristic relation for binding energy at the stable mass number

Starting from Z=2 to 100, nuclear binding energy at the stable mass number can be expressed as:

$$\left.\begin{aligned} &\overline{\underline{B_{A_s} \cong k_Z * Z * B_0}} \\ &\text{where} \begin{cases} k_Z \cong \left(\dfrac{Z}{30}\right)^{1/6} \text{ for } Z < 30; \\ k_Z \cong 1.0 \text{ for } Z \geq 30; \end{cases} \end{aligned}\right\} \tag{4}$$

Considering the semi empirical mass formula, for Z>=30, at the stable mass numbers, it is possible to show that, ratio of binding energy and proton number is close to 19.5 to 19.7 MeV and is practically independent of the stable mass number. It is a very surprising and interesting result [2]. With reference to Fermi gas model it is also possible to say that, at the stable mass numbers of Z>=30, ratio of nuclear binding energy and nucleons kinetic energy seems to be close to the ratio of proton number and stable mass number.

Characteristic relation for binding energy below and above the stable mass number

Starting from Z=2 to 100, nuclear binding energy below and above the stable mass number can be approximately expressed as:

$$B_A \cong (A/A_s)^p * (k_Z * Z * B_0) \cong (A/A_s)^p * B_{A_s} \text{......(5)}$$

$$\begin{cases} k_Z \cong \left(\dfrac{Z}{30}\right)^{1/6} \text{ for } Z < 30,\ k_Z \cong 1.0 \text{ for } Z \geq 30; \\ p \cong 4/3 \text{ if } A < A_s \text{ and } p \cong 2/3 \text{ if } A > A_s\,; \end{cases}$$

It is for further study and analysis.

Tables and Figures

To fit the nuclear data, in this paper we consider

$$\left\{ \left(\frac{2}{3}\alpha\right)^2 \cong 0.00624; B_0 \cong 19.7 \text{ MeV} \right\} \qquad (6)$$

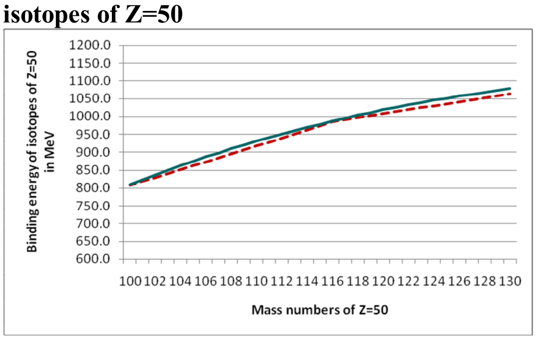
See table -1 for fitting the data at the stable mass numbers of Z=6 to 98 connected with relations (2) and (4). With further study and analysis, accuracy can be improved.

See figure-1 for fitting the binding energy of isotopes of Z=50 with mass numbers A=100 to 130. In the figure, solid green curve represents the binding energy calculated with SEMF and dotted red curve represents the data calculated with relation (5). Minimum error is 0.14% and maximum error is 1.64 %.

Table 1: To fit the nuclear binding energy at stable mass numbers of Z=2 to 98

Proton number	Estimated stable mass number	Even odd correction for the estimated stable mass number	Estimated binding energy at stable mass number in MeV
2	4	4	25.1
5	10	11	73.1
8	16	16	126.4
11	23	23	183.3
14	29	28/30	242.9
17	36	35/37	304.7
20	42	42	368.3
23	49	49	433.5
26	56	56	500.1
29	63	63	568.1
32	70	70	630.4
35	78	77/79	689.5
38	85	84/86	748.6
41	92	91/93	807.7
44	100	100	866.8
47	108	107/109	925.9
50	116	116	985.0
53	124	123/125	1044.1
56	132	132	1103.2
59	140	139/141	1162.3
62	148	148	1221.4
65	156	155/157	1280.5
68	165	164/166	1339.6
71	173	173	1398.7
74	182	182	1457.8
77	191	191	1516.9
80	200	200	1576.0
83	209	209	1635.1
86	218	218	1694.2
89	227	227	1753.3
92	237	236/238	1812.4
95	246	245/247	1871.5
98	256	256	1930.6

Figure 1:To fit the nuclear binding energy of isotopes of Z=50



References

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