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HIGHER ORDER CORRECTIONS TO SEMILEPTONIC WEAK PROCESSES*

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ABSTRACT

Using a strong formulation of lepton-hadron universality, higher order corrections to semileptonic weak processes are related to the high energy behavior of lepton-lepton weak scattering amplitudes. The "weak cutoff" estimates of ~ 4 GeV from the observed K_L-K_S mass difference and ~ 16 GeV from the absence of the decay $K_L \to \mu^+\mu^-$ are interpreted as upper bounds on the center-of-mass energy at which cross sections for pure leptonic processes deviate significantly from the low energy phenomenological predictions. A phenomenology for describing similar deviations in high energy neutrino-nucleon reactions is developed. A sum rule relating such deviations is derived, as well as estimates of their order of magnitude using the parton model.

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I. INTRODUCTION

It cannot be questioned that the structure of weak interactions as we know them today will be modified at high energy. Modifications will certainly be important at center-of-mass energies of about 600 GeV, for which the present lowest order theory would violate unitarity,[1,2] and it is probable that they are important at considerably lower energies.[3] Most likely, new classes of states, such as intermediate bosons, will be produced at some characteristic center-of-mass energy Λ, which we may hope is within range of the present generation of accelerators. But in any case, the presence of important modifications at higher energy requires, through the dispersion relations, small modifications of the present picture at lower energy. The purpose of this paper is to study such modifications, mainly to semileptonic weak processes. The method of attack generalizes the approach used in a previous paper,[4] hereafter called I, in which pure lepton-lepton scattering amplitudes were studied from an S-matrix point of view. Here, as in I, we assume

1. Electromagnetic effects can be neglected.
2. Lepton masses can be neglected.
3. The low energy limit of the lepton-lepton scattering amplitude takes the conventionally assumed charged current-current form.
4. The full amplitudes exhibit the SU(2) symmetry present in the low energy (i.e., few GeV) region, in which the SU(2) multiplets are (e,μ) and (ν_e,ν_μ) doublets. In addition, the amplitudes are symmetric under the interchange $\begin{pmatrix} e \\ \mu \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$.

As a consequence of these assumptions, it was shown in I that all two-body lepton-lepton scattering amplitudes may be described by three helicity amplitudes, A(s, t), B(s, t), and C(s, t). These amplitudes were parameterized as a double power series in s and t, plus small unitarity corrections, which are easily computed from Mandelstam's iteration method. Thus, for s and t large enough that lepton mass can be neglected, but small enough that the series converges, we may write

$$\begin{aligned} A(s,t) &= \alpha_{10}\,Gs + \alpha_{20}(Gs)^2 + \alpha_{11}\,G^2\,st + \ldots \\ B(s,t) &= \beta_{10}\,Gs + \beta_{20}(Gs)^2 + \beta_{11}\,G^2\,st + \ldots \\ C(s,t) &= \gamma_{10}\,Gs + \gamma_{20}(Gs)^2 + \gamma_{11}\,G^2\,st + \ldots \end{aligned} \tag{1.1}$$

where $\alpha_{10} = 4\ \ 2$. We have neglected the unitarity corrections in (1.1) because at attainable energies they are quite small. However, the corrections proportional to $\alpha_{ij}, \beta_{ij}, \gamma_{ij}$ are not necessarily so small. Other than α_{10}, these coefficients are poorly determined from present limits on lepton-lepton processes.

These amplitudes (1.1) may be represented by an effective Lagrangian of the form

$$\begin{aligned} \mathscr{L}(x) = \frac{G}{\sqrt{2}} \Bigg\{ & \bar{\ell}\gamma_\mu(1-\gamma_5)\,\nu \left[1 - G\left(\frac{4\alpha_{11} - 3\alpha_{20}}{4\alpha_{10}}\right)\frac{\partial^2}{\partial x^2}\right] \bar{\nu}\,\gamma^\mu(1-\gamma_5)\,\ell \\ & + \frac{\beta_{10}}{2\alpha_{10}} \left(\bar{\ell}\gamma_\mu(1-\gamma_5)\ell \left[1 - G\left(\frac{4\beta_{11} - 3\beta_{20}}{4\beta_{10}}\right)\frac{\partial^2}{\partial x^2}\right] \bar{\ell}\gamma^\mu(1-\gamma_5)\,\ell + (\ell \longleftrightarrow \nu)\right) \\ & + \frac{\gamma_{10}}{\alpha_{10}}\,\bar{\ell}\gamma_\mu(1-\gamma_5)\,\ell \left[1 - G\left(\frac{4\gamma_{11} - 3\gamma_{20}}{4\gamma_{20}}\right)\frac{\partial^2}{\partial x^2}\right] \bar{\nu}\gamma^\mu(1-\gamma_5)\,\nu \\ & + 2G\,\frac{\alpha_{20}}{\alpha_{10}}\,\bar{\ell}\,\Gamma_{\mu\nu}(1-\gamma_5)\nu\ \bar{\nu}\Gamma^{\mu\nu}(1-\gamma_5)\ell + G\,\frac{\beta_{20}}{\alpha_{10}}\left(\bar{\ell}\,\Gamma_{\mu\nu}(1-\gamma_5)\ell\ \bar{\ell}\,\Gamma^{\mu\nu}(1-\gamma_5)\ell + (v \longleftrightarrow \ell)\right) \\ & + 2G\,\frac{\gamma_{20}}{\alpha_{10}}\,\bar{\ell}\ \Gamma_{\mu\nu}(1-\gamma_5)\ell\ \bar{\nu}\Gamma^{\mu\nu}(1-\gamma_5)\nu \Bigg\} + \ \ldots \end{aligned} \tag{1.2}$$

where

$$\Gamma_{\mu\nu} = \frac{i}{4}\left(\gamma_\mu \overleftrightarrow{\partial}_\nu + \overleftrightarrow{\partial}_\mu \gamma_\nu\right)$$

and

$$\ell = \begin{pmatrix} e \\ \mu \end{pmatrix}, \qquad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \quad .$$

This Lagrangian describes many lepton-lepton processes in terms of a small number of parameters. Of course, we have paid for this economy of description with the strong assumption that the SU(2) and discrete symmetries which are valid in the few GeV region will continue to hold at higher energy.

One may inquire whether the number of parameters can be reduced further. Since we have only imposed the discrete symmetry $\nu \leftrightarrow \ell$, the effective Lagrangian is not invariant under the full SU(2) associated with the doublets $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ and $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$. To achieve this SU(2) symmetry, it is necessary to impose the additional con constraints

$$a_{mn} + \gamma_{mn} = \beta_{mn} \quad . \tag{1.3}$$

This, however, will force the existence of large neutral currents and will, in general, lead to trouble when we include hadrons.[5] We will return to this question in the next section.

We now proceed to generalize this description to include the hadrons. The idea is to formulate the concept of lepton-hadron universality sufficiently strongly to allow a direct transcription of the effective Lagrangian (1.2) to include hadronic processes. This is done by observing that the basic elements in that Lagrangian are two infinite towers of local operators of increasing tensor rank, one for electrons, and one for muons:

$$j_i^\mu = \frac{1}{2}\bar{\psi}\gamma^\mu \tau_i(1-\gamma_5)\psi \ , \qquad S_i^{\mu\nu} = \frac{1}{2}\bar{\psi}\Gamma^{\mu\nu}\ \tau_i(1-\gamma_5)\psi \ , \ \ldots$$

$$\ldots\ S^i_{\mu_1\ldots\mu_n} = \frac{1}{2}\bar{\psi}\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\ldots\overleftrightarrow{\partial}_{\mu_n}\ \tau_i(1-\gamma_5)\psi \ , \ \ldots \tag{1.4}$$

where ψ equals either $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ or $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$, $\tau_0 = 1$, and τ_i, $i = 1, 2, 3$, are the Pauli matrices. The members of a tower close under equal-time commutation with one another, and a member of one tower commutes with all members of another. Mack[6] has shown that a similar tower of hadronic operators h_i^μ, $T_i^{\mu\nu}$, ... exists as a consequence of the scaling behavior of deep-inelastic lepton-hadron scattering. This tower consists of the coefficients of the expansion about equal times of the light-cone commutator of two hadronic currents. Upon adopting a free-field light-cone algebra, such as that of Fritsch and Gell-Mann,[7] the equal-time commutation relations of the elements of this hadronic tower of operators will likewise be those of free fields. All this suggests a complete isomorphism can be extended to the effective Lagrangian (1.2). Just as (1.2) is invariant under the permutation $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$, we assume it is also invariant under permutation of all the elements of a leptonic tower with corresponding elements of the hadronic tower. This indeed determines the form of the effective Lagrangian, now generalized to hadrons. One simply makes the replacement

$$j_\mu^i \rightarrow J_\mu^i = j_\mu^i + h_\mu^i \ ,$$

$$S_{\mu\nu}^i \rightarrow \theta_{\mu\nu}^i = S_{\mu\nu}^i + T_{\mu\nu}^i \ , \text{ etc.} \tag{1.5}$$

The important feature is that no additional parameters are introduced in this generalization.

We shall explicitly write down this effective Lagrangian through second order in G. We begin by writing down the lepton currents

$$j_+^\mu(x) \equiv \left(j_-^\mu(x)\right)^\dagger \equiv \bar{\nu}(x)\, \gamma^\mu (1-\gamma_5)\, \ell(x) \tag{1.6}$$

which satisfy the equal-time commutation relations

$$\left[j_+^0(\vec{x}, 0),\ j_-^0(0)\right] = 4 j_3^0(\vec{x}, 0)\ \delta(\vec{x}) \tag{1.7}$$

and

$$\left[j_{+}^{i}(\vec{x},0),\ j_{-}^{j}(0)\right] = 4\delta^{ij}\ j_{3}^{0}(\vec{x},0)\ \delta(\vec{x}) - 4i\epsilon^{ijk}\ j_{0}^{k}(\vec{x},0)\ \delta(\vec{x}) \qquad (1.8)$$

where we have the definitions

$$j_{3}^{\mu}(x) \equiv \frac{1}{2}\left[\bar{\nu}(x)\ \gamma^{\mu}(1-\gamma_5)\ \nu(x) - \bar{\ell}(x)\ \gamma^{\mu}(1-\gamma_5)\ \ell(x)\right]$$
$$j_{0}^{\mu}(x) \equiv \frac{1}{2}\left[\bar{\nu}(x)\ \gamma^{\mu}(1-\gamma_5)\ \nu(x) + \bar{\ell}(x)\ \gamma^{\mu}(1-\gamma_5)\ \ell(x)\right] \qquad (1.9)$$

These four currents are each isoscalar in the SU(2) in which $\ell \equiv \begin{pmatrix} e \\ \mu \end{pmatrix}$ and $\nu \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$ transform as doublets, while they form a triplet and a singlet under the SU(2) associated with the doublets $e \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ and $\mu \equiv \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$. The second-order effective Lagrangian (1.2) may be written in terms of these operators and the additional operator $S_{\mu\nu}^{i}$ defined in (1.4). The $S_{\mu\nu}^{i}$ appear in the equal-time commutators of the currents with their time derivatives. The vector part of the tensor $S^{\mu\nu} \equiv \bar{\psi}\ \Gamma^{\mu\nu}\ \psi$ is just the stress-energy tensor for free, massless leptons.

To introduce the hadrons, we begin with the usual Cabibbo current. In the quark mnemonic it is

$$h_{+}^{\mu}(x) \equiv \left[h_{-}^{\mu}(x)\right]^{\dagger} \equiv \bar{p}(x)\ \gamma^{\mu}(1-\gamma_5)\ n'(x) \qquad (1.10)$$

where n' has the Cabibbo mixing, $n' = n \cos\theta_c + \lambda \sin\theta_c$, and p, n, λ are the usual triplet of fractionally charged quarks. Upon postulating that $\left[h_{+}^{\mu}(x), h_{-}^{\nu}(y)\right]_{x_0=y_0}$ satisfies the same algebra as the lepton currents, (1.7) and (1.8), we may define h_{3}^{μ} and h_{0}^{μ}.[8] More generally, when we postulate that these four hadronic currents satisfy the same light-cone algebra as the four lepton currents $j_{\pm}^{\mu}$, j_{3}^{μ}, and j_{0}^{μ}, the remaining members of the tower of hadron operators are defined. In particular, the operators $T_{i}^{\mu\nu}$ corresponding to the lepton operators $S_{i}^{\mu\nu}$ are, in the quark mnemonic, just

$$T_i^{\mu\nu} = \bar{\chi}\, \frac{\tau_i}{2}\, \Gamma^{\mu\nu} (1-\gamma_5)\, \chi \tag{1.11}$$

where $\chi = (p, n')$. Our assumption that the U(2) quartet h_i^μ satisfies a free Fermion light-cone algebra is just a weakened version of the Fritzsch and Gell-Mann proposal for the U(3) × U(3) nonets.[7]

Defining $J_i^\mu \equiv j_i^\mu + h_i^\mu$ and $\theta_i^{\mu\nu} \equiv S_i^{\mu\nu} + T_i^{\mu\nu}$, we state our complete second-order effective Lagrangian:

$$\begin{aligned}\mathscr{L}(x) = \frac{G}{\sqrt{2}} \Bigg\{ & J_-^\mu \left(1 - G\, \frac{4\alpha_{11} - 3\alpha_{20}}{4\alpha_{10}}\, \frac{\partial^2}{\partial x^2}\right) J_{+\mu} \\ & + \frac{\beta_{10}}{\alpha_{10}} \left[J_0^\mu \left(1 - G\, \frac{4\beta_{11} - 3\beta_{20}}{4\alpha_{10}}\, \frac{\partial^2}{\partial x^2}\right) J_{0\mu} + (0 \longleftrightarrow 3)\right] \\ & + \frac{\gamma_{10}}{\alpha_{10}} \left(J_0^\mu - J_3^\mu\right) \left(1 - G\, \frac{4\gamma_{11} - 3\gamma_{20}}{4\alpha_{10}}\, \frac{\partial^2}{\partial x^2}\right) \left(J_{0\mu} + J_{3\mu}\right) + 2G\, \frac{\alpha_{20}}{\alpha_{10}}\, \theta_-^{\mu\nu}\, \theta_{+\mu\nu} \\ & + 2G\, \frac{\beta_{20}}{\alpha_{10}} \left(\theta_0^{\mu\nu}\, \theta_{0\mu\nu} + \theta_3^{\mu\nu}\, \theta_{3\mu\nu}\right) + 2G\, \frac{\gamma_{20}}{\alpha_{10}} \left(\theta_0^{\mu\nu} - \theta_3^{\mu\nu}\right)\left(\theta_{0\mu\nu} + \theta_{3\mu\nu}\right) \Bigg\} \end{aligned} \tag{1.12}$$

It is important to observe that in using the quark mnemonic, one implicitly excludes the possibility that h_μ^i, $T_{\mu\nu}^i$, and the rest of the tower are a sum of independent pieces, each of which is isomorphic to the lepton tower. Our results can be greatly modified if this is the case. An important example is the SU(4) structure advocated by Glashow, Iliopoulos, and Maiani.[5] In that scheme, there are two hadron towers. The first contains the Cabibbo current formed from $\binom{p}{n'}$, and the second is built from $\binom{q}{\lambda'}$, where q is an SU(3) singlet and $\lambda' = \lambda \cos\, \theta_c - n \sin\, \theta_c$. This has important implications for the structure of the neutral-current part of the effective Lagrangian, because anywhere an operator of structure $\bar{n}'\Gamma n'$ appears, our symmetry assumptions require it to appear in the combination

$$\bar{n}'\Gamma n' + \bar{\lambda}'\Gamma\lambda' = \bar{n}\Gamma n + \bar{\lambda}\Gamma\lambda \quad . \qquad (1.13)$$

Thus in such a case, the effective Lagrangian contains no $\Delta S=1$ neutral-current terms.

The Lagrangian (1.12) generalizes the leptonic effective Lagrangian which neglected lepton mass. In the presence of lepton mass, the effective Lagrangian contains additional terms which destroy its V-A structure and μ-e universality. In the hadronic generalization, there are very likely similar additional terms in the effective Lagrangian as a consequence of chiral and SU(3) symmetry breaking. While it is important to recognize the possibility of such terms, this problem is beyond the scope of the present paper.

In the rest of this paper, we shall apply the effective Lagrangian (1.12) to semileptonic processes, generally assuming the "quark" light-cone algebra of Fritsch and Gell-Mann. In Section II, we study the limits on β_{10} and γ_{10} from experiment, in particular the decays $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$ and the K-$\bar{K}$ mass difference. We then study the consequent stringent limits on processes involving $\Delta S=0$ neutral currents, such as $\nu_\mu + p \to \nu_\mu$ + hadrons. By relating β_{10} and γ_{10} to high energy lepton-lepton scattering and using the dispersion relations in I, we are able to estimate the "weak cutoff" Λ. We find $\Lambda \lesssim 4\sim16$ GeV. This low cutoff implies a lower bound on the parameter α_{20} and/or α_{11}.

In Section III, we discuss the modification of high energy neutrino processes, both elastic and deep inelastic, which follow from the nonvanishing of α_{20} and α_{11}. We derive a sum rule which relates the modifications in deep inelastic neutrino scattering due to the α_{20} correction in (1.9) to the equal-time commutator $\left[J^\mu_+, \theta^{\alpha\beta}_-\right]$. There is a large family of related sum rules which can be obtained from the other equal-time and light-cone commutators. The content of these sum rules can also be obtained simply from parton model estimations, which we present.

II. LIMITS ON THE PARAMETERS α, β, γ

The extension of the formalism of I to include hadrons can lead to much stronger constraints on the theory. The success of the conventional theory for $|\Delta Q| = 1$ transitions dictates that $\alpha_{10} = 4\sqrt{2}$, and for these processes we will be interested in the higher order corrections proportional to α_{11} and α_{20}. Upper bounds for β_{10} and γ_{10} are easily obtained from the experimental upper bounds[9,10] for the $\Delta S=1$, $\Delta Q=0$ processes $K_L \to \mu^+\mu^-$ and $K^+ \to \pi^+ \nu \bar{\nu}$, respectively, provided the hadron currents satisfy the "quark" algebra. The small K_L-K_S mass differences provide another, more stringent, upper bound on β_{10}. We will calculate these bounds below and then translate them into an estimate of the cutoff Λ and a lower bound for α_{20} and/or α_{11}.

First consider γ_{10}. The rate for $K^+ \to \pi^+ \nu \bar{\nu}$ is very simply related to the rate for K_{e3} decay:

$$\frac{\Gamma(K^+ \to \pi^+ \nu_\mu \bar{\nu}_\mu)}{\Gamma(K^+ \to \pi^o e^+ \nu_e)} = \frac{\Gamma(K^+ \to \pi^+ \nu_e \bar{\nu}_e)}{\Gamma(K^+ \to \pi^o e^+ \nu_e)} = 2 \cos^2 \theta_c \left|\frac{\gamma_{10}}{\alpha_{10}}\right|^2 \qquad (2.1)$$

where θ_c is the Cabibbo angle. The experimental upper bound[9] on (2.1) is $\sim 8 \times 10^{-6}$, and the upper bound on γ_{10} is then

$$\left|\frac{\gamma_{10}}{\alpha_{10}}\right| \lesssim 2 \times 10^{\;3} \quad .$$

We may similarly bound β_{10} by considering

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+ \nu_\mu)} = 4 \cos^2 \theta_c \left|\frac{\beta_{10}}{\alpha_{10}}\right| \qquad (2.3)$$

The experimental upper bound[10] may be smaller than the bound given by unitarity and TCP. If such a limit is confirmed, clearly something is lacking in the description we give here. Here we take the result of Carithers et al.,[10] $\sim 1 \times 10^{-8}$ for the branching ratio $\Gamma(K_L \rightarrow \mu\mu)/\Gamma(K_L \rightarrow \text{all})$, and we find

$$\left|\frac{\beta_{10}}{\alpha_{10}}\right| \lesssim 3 \times 10^{-5} \tag{2.4}$$

A more stringent bound on β_{10} can be obtained by exploiting the smallness of the $K_L - K_S$ mass difference.[11] We relate the mass difference to the non-leptonic $\Delta I = 3/2$ Hamiltonian responsible for the decay $K^+ \rightarrow \pi^+\pi^o$. Our strategy is to use soft pion techniques[12] to relate the $K^+ \rightarrow \pi^+\pi^o$ amplitude to the matrix element $\langle K | (J_+ \cdot J_-)_{27} | \pi \rangle$. We then assume exact SU(3) symmetry to relate that matrix element to the K^o-$\bar{K}^o$ transition amplitude. We neglect the contributions of the β_{10} and γ_{10} terms in $\mathscr{L}$ to $K^+ \rightarrow \pi^+\pi^o$, since, according to (2.2) and (2.4), these are small compared to the factor of $\sim 1/20$ which characterizes the suppression of $\Delta I = 3/2$ in the $J_+ \cdot J_-$ product. We also neglect possible electromagnetic contributions to $K^+ \rightarrow \pi^+\pi^o$, although they are important in some model calculations.[13] Because of all these approximations, it is clear that the result should not be regarded as anything more than a guide to the order of magnitude of β_{10}.

The K^+ decay amplitude is

$$\mathscr{M}(k, p_0, p_+) = -i \frac{G}{\sqrt{2}} \langle K^+(k) | J_- \cdot J_+ | \pi^o(p_0)\, \pi^+(p_+) \rangle \ . \tag{2.5}$$

The amplitude has $\Delta I = 3/2$ because of the Bose statistics of the pions, so it is important to carry out the soft pion calculation in a way which respects Bose symmetry. By the usual soft pion techniques, we have

$$\mathcal{M}(k,0,p_+) = -G F_\pi^{-1} \langle K^+(k) \left| \left[Q_3^5, J_- \cdot J_+ \right] \right| \pi^+(p_+) \rangle \quad , \tag{2.6}$$

where Q_i^5 and Q_i are axial vector and vector charges, respectively and $F_\pi \approx .95\ m_\pi$ is the $\pi^\pm$ decay constant. Since $Q_3^5 + Q_3$ commutes with $J_- \cdot J_+$, we may replace Q_3^5 by $-Q_3$ and let it act on the states. The result is

$$\mathcal{M}(k,0,p_+) = \frac{-G}{2F_\pi} \langle K^+(k) | J_- \cdot J_+ | \pi^+(p_+) \rangle \tag{2.7}$$

and, similarly,

$$\mathcal{M}(k,p_0,0) = \frac{-G}{\sqrt{2}F_\pi} \left\{ \langle K^o(k) | J_- \cdot J_+ | \pi^o(p_0) \rangle - \sqrt{2} \langle K^+(k) | J_- \cdot J_+ | \pi^+(p_+) \rangle \right\} . \tag{2.8}$$

Now, in order to maintain Bose symmetry and to isolate the $\Delta I = 3\ 2$ contribution to $\langle K | J_+ \cdot J_- | \pi \rangle$, we consider the amplitude

$$\mathcal{M}(k,p) \equiv \frac{1}{2} \left(\mathcal{M}(k,p,0) + \mathcal{M}(k,0,p) \right) \quad . \tag{2.9}$$

Upon performing an SU(2) rotation, followed by a V-spin rotation, we find, using the quark mnemonic for the currents, that

$$\mathcal{M}(k,p) = \frac{G}{4F_\pi} \cos\theta_c \sin\theta_c \langle K^o(k) | \bar{n}\gamma_\mu(1-\gamma_5)\lambda\ \bar{n}\gamma^\mu(1-\gamma_5)p | \pi^+(p) \rangle$$

$$\cong \frac{G}{8F_\pi} \cos\theta_c \sin\theta_c \langle K^o | \bar{n}\gamma_\mu(1-\gamma_5)\lambda\ \bar{n}\gamma^\mu(1-\gamma_5)\lambda | \bar{K}^o \rangle \tag{2.10}$$

We now may relate $\mathcal{M}$ to the K_L-K_S mass difference, which is

$$\Delta m_K = \frac{G}{\sqrt{2}} \frac{\beta_{10}}{\alpha_{10}} \frac{\cos^2\theta_c \sin^2\theta_c}{4m_K} \langle K^o | \bar{n}\gamma_\mu(1-\gamma_5)\lambda\ \bar{n}\gamma^\mu(1-\gamma_5)\lambda | \bar{K}^o \rangle \tag{2.11}$$

Using the experimental values for $\mathcal{M}$ and Δm_K, [14] we find that

$$\left| \frac{\beta_{10}}{\alpha_{10}} \right| \lesssim 2 \times 10^{-6} \quad . \tag{2.12}$$

Sum Rules

We now use the dispersion relations stated in I to relate β_{10} to cross sections for lepton-lepton scattering. The relevant amplitude is for the process $\nu_\mu + \nu_e \to \nu_\mu + \nu_e$, which is described by the amplitude B(s, t). As in I, we evaluate $dB(s,0)/ds|_{s=0}$, assuming an unsubtracted dispersion relation for B(s, 0). The result is the sum rule

$$\beta_{10} G = \frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left[\sigma_{\nu_\mu \nu_e}(s) - \sigma_{\nu_\mu \bar{\nu}_e}(s)\right] \quad . \tag{2.13}$$

where $\sigma_{\nu_\mu \nu_e}$ is the total $\nu_\mu \nu_e$ cross section. The leading contribution to the integral at low and intermediate energies is the allowed first-order process $\nu_\mu \bar{\nu}_e \to \mu^- e^+$, the cross section for which is $2G^2 s/3\pi$ in first order.

Let us define the "cutoff" Λ as the center-of-mass energy at which the correction terms become as important as the first-order term of the Fermi theory. Then the low energy portion of the dispersion integral $s \lesssim \Lambda^2$ can be estimated. We consider it unreasonable that it be >> the total integral. Consequently

$$|\beta_{10} G| \gtrsim \frac{1}{\pi}\int_0^{\Lambda^2} \frac{ds}{s}\left(\frac{2G^2 s}{3\pi}\right) = \frac{2G^2\Lambda^2}{3\pi^2} \tag{2.14}$$

Using the bounds on β_{10}, we estimate that

$$\Lambda \lesssim \begin{cases} 16 \text{ GeV} & K_L \to \mu\mu \\ 5 \text{ GeV} & \Delta m_K \end{cases} \tag{2.15}$$

This result implies that the low energy region does not account for the magnitude of α_{10} calculated according to the sum rule

$$(\alpha_{10} + \gamma_{10})G \cong \alpha_{10} G = \frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left[\sigma_{\nu_e e}(s) - \sigma_{\bar{\nu}_e e}(s)\right] \tag{2.16}$$

It is not too hard to account for the remainder coming from $s \gtrsim \Lambda^2$ if there is e.g., an intermediate-boson resonance in $\bar{\nu}_e e$ scattering.

With an additional "smoothness" assumption about the convergence of the series (1.1), we can show that an upper bound on Λ implies an estimate of α_{20} and/or α_{11}, which increases as Λ decreases. We assume that for $s \cong \Lambda^2$, the lower-order correction terms, such as $\alpha_{20}(Gs)^2$ or $\alpha_{11}(Gs)(Gt)$ are much larger than the higher-order corrections, such as $\alpha_{n0}(Gs)^n$ with $n > 2$. Then in

$$A(s,t) \cong \alpha_{10}(Gs) + \alpha_{20}(Gs)^2 + \alpha_{11}(Gs)(Gt) + \ldots$$

we expect that for $s \cong \Lambda^2$, the second and/or third term is of the same order of magnitude as the first, implying

$$\left|\frac{\alpha_{20}}{\alpha_{10}}\right| \cong \frac{1}{G\Lambda^2} \qquad \text{and/or} \qquad \left|\frac{\alpha_{11}}{\alpha_{10}}\right| \cong \frac{1}{G\Lambda^2} \tag{2.17}$$

Thus, if the Lagrangian (1.9) is correct and if Λ is accurately expressed by Eq. (2.15), it may be feasible to find deviations from the Fermi theory in currently planned high energy neutrino experiments. Notice that this last qualitative conclusion is independent of whether or not (1.1) converges smoothly. That is, if there are pathologies such that $\alpha_{20}G\Lambda^2 \ll \alpha_{10}$ and $\alpha_{11}G\Lambda^2 \ll \alpha_{10}$, then corrections will be present due to the higher-order terms in (1.1). However, we emphasize again that these estimates all depend crucially on the assumption of the "quark" current algebra. With the SU(4) mechanism of Glashow, Iliopoulos, and Maiani,[5] the neutral currents have no $\Delta S = 1$ contribution (apart from symmetry-breaking effects neglected throughout this paper), and this entire section becomes null and void. In addition, our estimates rely on the unsubtracted dispersion relation (2.13), an equally hazardous assumption.[15]

The implications of a large value of α_{20} for high energy neutrino reactions lie in the breakdown of the "locality theorems" used in the analysis of these reactions.[16,17] In particular, one of the methods which has been proposed to determine the neutrino flux at NAL is to use the predicted constancy in E_ν of the cross section for an exclusive process (such as $\bar{\nu}p \rightarrow \bar{\mu}n$ or $\nu p \rightarrow \bar{\mu}\Delta^{++}$) or a group of such processes as a method of normalizing neutrino flux.[18] There will be corrections of the form $\left[1 + K \frac{\alpha_{20}}{\alpha_{10}} (Gs)\right]$ with the constant K of order unity and proportional to the (model-dependent) matrix element of $\Gamma^i_{\mu\nu}$ between the hadron states in question. Thus K will vary from process to process, and a test for the presence of such a locality-breaking term is measurement of the dependence upon neutrino energy of <u>ratios</u> of exclusive cross sections.

III. NEUTRINO-NUCLEON INTERACTIONS

A. Kinematics

We consider the terms in the effective Lagrangian (1.9) responsible for the process

$$\nu_\mu + N \rightarrow \mu^- + \text{hadrons}$$

where N is a nucleon. Those terms are

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}}\, \bar{\mu}\gamma_\mu(1-\gamma_5)\, \nu_\mu \left(1 - G\,\frac{4\alpha_{11} - 3\alpha_{20}}{4\alpha_{10}}\, \frac{\partial^2}{\partial x^2}\right) h^\mu_+$$

$$+ 2G\,\frac{\alpha_{20}}{\alpha_{10}}\, \bar{\mu}\,\Gamma_{\mu\nu}\,(1-\gamma_5)\, \nu_\mu\, T^{\mu\nu}_+ \tag{3.1}$$

where h^μ and $T^{\mu\nu}$ are stated, in the quark mnemonic, in (1.10) and (1.11) and $\Gamma^{\mu\nu}$ is defined following (1.2).

We now proceed from the effective Lagrangian to the differential cross section, using the method of Lee and Yang[19] and the notation and approximation of Bjorken and Paschos.[20] We work in the laboratory frame and choose the z axis along the direction of the momentum transfer q between lepton system and hadron system. We define η to be

$$\eta = -G \frac{4\alpha_{11} - 3\alpha_{20}}{4\alpha_{10}} \frac{\partial^2}{\partial x^2} = -G \left(\frac{4\alpha_{11} - 3\alpha_{20}}{4\alpha_{10}}\right) Q^2$$

in momentum space. Then the amplitude is

$$\mathcal{M}_n = \frac{G}{\sqrt{2}} \left\{ j_\mu(p,p') <n|h_+^\mu(0)|N> (1+\eta) + \frac{1}{2} G \frac{\alpha_{20}}{\alpha_{10}} \left[(p+p')_\nu\, j_\mu(p,p') + (p+p')_\mu\, j_\nu(p,p') \right] <n|T_+^{\mu\nu}|N> \right\} \tag{3.2}$$

where n is the hadronic final state.

We will be interested in the interference term between h_μ and $T_{\mu\nu}$; since h_μ supports only $\Delta J_z = 0, \pm 1$ transitions, only those pieces of $T_{\mu\nu}$ possessing $\Delta J_z = 0, \pm 1$ will contribute. We decompose $(p+p')_\mu$ and the lepton current $j_\mu(p,p')$ into contributions of definite ΔJ_z, following the notation and approximations of Ref. 20.

$$j_\mu(p,p') \cong 4 \frac{\sqrt{EE'Q^2}}{\nu} \left\{ \epsilon_\mu^S + \sqrt{\frac{E'}{2E}}\, \epsilon_\mu^R + \sqrt{\frac{E}{2E'}}\, \epsilon_\mu^L \right\} \tag{3.3}$$

$$(p+p')_\mu \cong \frac{\sqrt{Q^2}}{\nu} \left\{ (E'+E)\, \epsilon_\mu^S + \sqrt{2EE'} \left(\epsilon_\mu^R + \epsilon_\mu^L \right) \right\} \tag{3.4}$$

with

$$\begin{aligned} \epsilon_\mu^S &\cong \frac{\nu}{\sqrt{Q^2}} \left(1, 0, 0, 1 - \frac{Q^2}{2\nu^2} \right) \\ \epsilon_\mu^R &= \frac{1}{\sqrt{2}} (0, 1, i, 0) \\ \epsilon_\mu^L &= \frac{1}{\sqrt{2}} (0, 1, -i, 0) \quad . \end{aligned} \tag{3.5}$$

If the states $|n>$ and $|N>$ are eigenfunctions of J_z (as may be chosen for "elastic" processes without polarization measurements or for inelastic processes without detection of final hadrons), then the contributions of the different helicities may be considered separately:

$$\mathcal{M}_S = 4\frac{G}{\sqrt{2}}\frac{\sqrt{EE'Q^2}}{\nu}\Big\{<n|\epsilon_S\cdot h_+(0)|N>\ (1+\eta)$$

$$+\frac{1}{2}G\frac{\alpha_{20}}{\alpha_{10}}\frac{\sqrt{Q^2}}{\nu}(E+E')\ \epsilon^S_{\mu\nu}<n|T^{\mu\nu}_+(0)|N>\Big\} \tag{3.6a}$$

$$\mathcal{M}_R = 4\frac{G}{\sqrt{2}}\frac{\sqrt{EE'Q^2}}{\nu}\sqrt{\frac{E'}{2E}}\Big\{<n|\epsilon_R\cdot h_+(0)|N>\ (1+\eta)$$

$$+\frac{1}{2}G\frac{\alpha_{20}}{\alpha_{10}}\frac{\sqrt{Q^2}}{\nu}(3E+E')\ \epsilon^R_{\mu\nu}<n|T^{\mu\nu}_+(0)|N>\Big\} \tag{3.6b}$$

$$\mathcal{M}_L = 4\frac{G}{\sqrt{2}}\frac{\sqrt{EE'Q^2}}{\nu}\sqrt{\frac{E}{2E'}}\Big\{<n|\epsilon_L\cdot h_+(0)|N>\ (1+\eta)$$

$$+\frac{1}{2}G\frac{\alpha_{20}}{\alpha_{10}}\frac{\sqrt{Q^2}}{\nu}(3E'+E)\ \epsilon^L_{\mu\nu}<n|T^{\mu\nu}_+(0)|N>\Big\} \tag{3.6c}$$

where

$$\epsilon^S_{\mu\nu} \equiv 2\epsilon^S_\mu\epsilon^S_\nu + \epsilon^R_\mu\epsilon^L_\nu + \epsilon^L_\mu\epsilon^R_\nu$$

$$\epsilon^R_{\mu\nu} \equiv \epsilon^S_\mu\epsilon^R_\nu + \epsilon^R_\mu\epsilon^S_\nu \tag{3.7}$$

$$\epsilon^L_{\mu\nu} \equiv \epsilon^S_\mu\epsilon^L_\nu + \epsilon^L_\mu\epsilon^S_\nu \quad .$$

Thus the cross section for producing hadron system Γ, averaged over azimuthal angles to remove interference terms, is (cf. Eq. (2.8) of Ref. 20):

$$\int\frac{d\phi}{2\pi}\frac{d\sigma}{dQ^2 d\nu\, d\Gamma} \cong \frac{G^2}{2\pi^2}\frac{E'}{E}\frac{Q^2}{\nu}\left(1-\frac{Q^2}{2M\nu}\right)\times\Big\{\frac{d\sigma^S}{d\Gamma}\left[(1+2\,\mathrm{Re}\,\eta)+(E+E')\ \delta_S(Q^2,\nu,\Gamma)\right]$$

$$+\frac{E'}{2E}\frac{d\sigma^R}{d\Gamma}\left[(1+2\,\mathrm{Re}\,\eta)+(3E+E')\ \delta_R(Q^2,\nu,\Gamma)\right]$$

$$+\frac{E}{2E'}\frac{d\sigma^L}{d\Gamma}\left[(1+2\,\mathrm{Re}\,\eta)+(3E'+E)\ \delta_L(Q^2,\nu,\Gamma)\right]\Big\} \tag{3.8}$$

where

$$\delta_i(Q^2,\nu,\Gamma) \equiv \frac{\mathrm{Re}\, G \frac{\alpha_{20}}{\alpha_{10}} \frac{\sqrt{Q^2}}{\nu} \epsilon^i_{\mu\nu} \sum_{n\in\Gamma} <N|\epsilon_i^* \cdot h_-|n><n|T_+^{\mu\nu}|N> \delta^4(p_n-P_N-q)}{\sum_{n\in\Gamma} |<n|\epsilon^i \cdot h_+|N>|^2 \delta^4(p_n-P_N-q)}$$

and

$$\frac{d\sigma^i}{d\Gamma} = \frac{\pi}{2M\nu} \frac{1}{\left[1-\frac{Q^2}{2M\nu}\right]} \sum_{n\in\Gamma} |<n|\epsilon_i \cdot h_+|N>|^2 (2\pi)^3 \delta^4(p_n-P_N-q)$$

Our normalization of states, for both fermions and nucleons, is

$$<p|p'> = 2p^0 (2\pi)^3 \delta^3(p-p')$$

For antineutrino processes, we have a similar formula. Because the lepton current j_μ and the tensor $S_{\mu\nu}$ undergo a relative sign change in crossing from particle to antiparticle, the correction term changes sign:

$$\int \frac{d\phi}{2\pi} \frac{d\bar{\sigma}}{dQ^2 d\nu\, d\Gamma} \cong \frac{G^2}{2\pi^2} \frac{E'}{E} \frac{Q^2}{\nu} \left(1-\frac{Q^2}{2M\nu}\right) \times \Bigg\{ \frac{d\bar{\sigma}^S}{d\Gamma} \left[(1+2\,\mathrm{Re}\,\eta) - (E+E')\, \bar{\delta}_S(Q^2,\nu,\Gamma)\right]$$
$$+ \frac{E'}{2E} \frac{d\bar{\sigma}^L}{d\Gamma} \left[(1+2\,\mathrm{Re}\,\eta) - (3E+E')\, \bar{\delta}_L(Q^2,\nu,\Gamma)\right]$$
$$+ \frac{E}{2E'} \frac{d\bar{\sigma}^R}{d\Gamma} \left[(1+2\,\mathrm{Re}\,\eta) - (3E'+E)\, \bar{\delta}_R(Q^2,\nu,\Gamma)\right]\Bigg\} \qquad (3.9)$$

with

$$\bar{\delta}_i(Q^2,\nu,\Gamma) \equiv \frac{\mathrm{Re}\, G \frac{\alpha_{20}}{\alpha_{10}} \frac{\sqrt{Q^2}}{\nu} \sum_{n\in\Gamma} \epsilon^i_{\mu\nu} <N|\epsilon_i^* \cdot h_+|n><n|T_-^{\mu\nu}|N> \delta^4(p_n-P_N-q)}{\sum_{n\in\Gamma} |<n|\epsilon^i \cdot h_-|N>|^2 \delta^4(p_n-P_N-q)}$$

and a definition of $d\bar{\sigma}^i/d\Gamma$ analogous to that of $d\sigma^i/d\Gamma$.

In the approximation of neglecting $\Delta S = 1$ processes we have, from charge symmetry, relations between δ_i and $\bar{\delta}_i$:

$$\begin{aligned} \delta_i(\nu p) &= \bar{\delta}_i(\bar{\nu} n) \\ \delta_i(\nu n) &= \bar{\delta}_i(\bar{\nu} p) \quad . \end{aligned} \tag{3.10}$$

B. An Analogue of the Adler Sum Rule

All the techniques[21] which have been used to study the processes $eN \to eX$ and $\nu N \to \mu X$ in lowest order in electromagnetic and weak interactions may also be applied to the higher order corrections δ_i. Thus there are analogues of

1. The Adler sum rule,[22]
2. The asymptotic sum rules obtained from the equal-time commutators of the space components of the currents[23]
3. The sum rules of light-cone current algebra,[7] and
4. The simple parton-model calculations.[24]

It would be premature at this time to present all these relations in full detail. We choose to confine our attention to the most important sum rule, the analogue of Adler's. We will also consider the parton-model because it embodies the content of the sum rules, is simple, and may perhaps be a useful, relatively parameter-free guide to the order of magnitude to be expected for the corrections δ_i.

The Adler sum rule is easily obtained by the method of Fubini, Furlan, and Rossetti.[25] We consider the amplitude (spin-averaged)

$$T^{\mu\alpha\beta}(q, P) = -i \int d^4x \, e^{iq\cdot x} \langle P | T^* \left(h_+^\mu(x) \; T_-^{\alpha\beta}(0) \right) | P \rangle \tag{3.11}$$

which may be written in terms of scalar form factors as

$$T^{\mu\alpha\beta}(q, P) = P^\mu P^\alpha P^\beta \, A(q^2, \nu) + q^\mu P^\alpha P^\beta \, B(q^2, \nu) + \ldots \tag{3.12}$$

$T^{\mu\alpha\beta}$ is the covariant T* product, with its divergence condition presumably free of Schwinger terms, so that we have just

$$q_\mu T^{\mu\alpha\beta}(q,P) = \int d^3x \, e^{-i\vec{q}\cdot\vec{x}} \langle P| \left[h_+^0(\vec{x},0),\ T_-^{\alpha\beta}(0)\right] |P\rangle$$

$$= 4 \langle P| T_3^{\alpha\beta}(0) |P\rangle$$

$$\equiv 4\, P^\alpha P^\beta\, C + \ldots \qquad (3.13)$$

In the sum rule, we are only interested in terms proportional to $P^\alpha P^\beta$, so we may ignore the pseudotensor part of $T_3^{\alpha\beta}$. Now the strangeness-conserving part of $T_3^{\alpha\beta}$ has the SU(3) structure

$$t_3' = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2}\cos^2\theta_c & \frac{1}{2}\cos\theta_c \sin\theta_c \\ 0 & \frac{1}{2}\cos\theta_c \sin\theta_c & -\frac{1}{2}\sin^2\theta_c \end{pmatrix}, \qquad (3.14)$$

which, if we neglect terms of order θ_c, is just equal to the isospin matrix,

$$t_3' \cong t_3 = \begin{pmatrix} 1/2 & & \\ & -1/2 & \\ & & 0 \end{pmatrix} \qquad (3.15)$$

In this approximation, $T_3^{\alpha\beta}$ is the first of the tower of tensor operators which cause the inequality of the deep inelastic electron scattering scaling functions for the proton and the neutron. A simple calculation[7] shows that the parameter C in Eq. (3.13) is then given by

$$C = 3\int_0^1 dx \left(F_{2p}(x) - F_{2n}(x)\right) \qquad (3.16)$$

and with the current data[26] (which has sizable errors in the case of $\int F_{2n}$), we have

$$C \sim .17 \quad . \qquad (3.17)$$

Now the divergence condition applied to Eq. (3.12) gives

$$q_\mu T^{\mu\alpha\beta} = A\ M\nu\ P^\alpha P^\beta + B\ q^2 P^\alpha P^\beta + \ldots \tag{3.18}$$

The next step is to equate (3.13) and (3.18). With the assumption (the analogue of which is also necessary for the derivation of Adler's sum rule) that $B(\nu, q^2$ $B(\nu, q^2) \to 0$ as $\nu \to \infty$ at fixed q^2, we find that

$$\lim_{\nu \to \infty} M\nu\ A(\nu, q^2) = 4C \quad . \tag{3.19}$$

According to Regge considerations, we can write an unsubtracted dispersion relation for A,

$$A(\nu, q^2) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\nu' \frac{\mathrm{Im}\, A(\nu', q^2)}{\nu' - \nu} \quad , \tag{3.20}$$

and Eq. (3.19) becomes

$$-\frac{1}{\pi} \int_{-\infty}^{\infty} d\nu\ \mathrm{Im}\ A(\nu, q^2) = \frac{4C}{M} \tag{3.21}$$

Now we must relate Im A to the higher order correction terms, δ_i and $\bar{\delta}_i$. We consider the lepton-nucleon center-of-mass frame and take the energy very large with ν and q^2 fixed. In this approximation, both nucleon and lepton have energy $\mathscr{E}$, the energy of the final lepton is $\mathscr{E}' \cong \mathscr{E}$, the scattering angle is $\theta \cong 0$, and the lepton current is $j^\mu(p', p) \cong 4p^\mu$. The differential cross section for $\nu N \to \mu$ + hadrons is then

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi^2 M}{16\mathscr{E}^4} \sum_n |\mathscr{M}_n|^2\ \delta^4(q+P-p_n) \tag{3.22}$$

where $\mathscr{M}_n$ is given by Eq. (3.2). We shall be interested in the higher order contribution to $|\mathscr{M}_n|^2$ due to the interference term between the two terms in (3.2), i.e.,

$$\delta \left(\frac{d\sigma}{dQ^2 d\nu}\right) \equiv \frac{M\pi^2}{16\mathscr{E}^4} \sum_n \delta |\mathscr{M}_n|^2\ \delta^4(q+P-p_n) \tag{3.23}$$

where

$$\delta\, |\mathscr{M}_n|^2 \equiv \text{Re}\ \frac{G^3}{2}\frac{\alpha_{20}}{\alpha_{10}} <P|h_-^{\mu}|n><n|T_+^{\alpha\beta}|P> \times j_\mu \left(j_\alpha(p+p')_\beta + j_\beta(p+p')_\alpha\right)\Bigg\} \tag{3.24}$$

Similarly for the antineutrino process,

$$\delta\left(\frac{d\bar{\sigma}}{dQ^2 d\nu}\right) \equiv \frac{M\pi^2}{16\mathscr{E}^4} \sum_n \delta\,|\bar{\mathscr{M}}_n|^2\ \delta^4(q+P-p_n) \tag{3.25}$$

and

$$\delta\,|\bar{\mathscr{M}}_n|^2 = -\text{Re}\left\{\frac{G^3}{2}\frac{\alpha_{20}}{\alpha_{10}} <P|h_+^{\mu}|n><n|T_-^{\alpha\beta}|P> \times j_\mu \left(j_\alpha(p+p')_\beta + j_\beta(p+p')_\alpha\right)\right\} \tag{3.26}$$

If we evaluate Eq. (3.26) at $\nu \to -\nu$ and add it to Eq. (3.24), we find

$$\theta(\nu)\,\delta\left(\frac{d\sigma}{dQ^2 d\nu}\right) + \theta(-\nu)\,\delta\left(\frac{d\bar{\sigma}}{dQ^2 d\nu}\right)_{-\nu} = \frac{-G^3 M}{(16\pi)^2\mathscr{E}^4}\ \text{Im}\ \frac{\alpha_{20}}{\alpha_{10}}\ T^{\mu\alpha\beta} \times j_\mu\left(j_\alpha(p+p')_\beta + j_\beta(p+p')_\alpha\right) \tag{3.27}$$

Now with the approximations, valid as $\mathscr{E} \to \infty$, that Im $T^{\mu\alpha\beta} \cong P^\mu P^\alpha P^\beta$ Im A and $j^\mu \cong 4p^\mu$, we find that the sum rule (3.21) may be written in the form

$$\delta\left(\frac{d\sigma}{dQ^2} + \frac{d\bar{\sigma}}{dQ^2}\right) \xrightarrow[E\to\infty]{} \frac{4G^3}{\pi}\ \text{Re}\left(\frac{\alpha_{20}}{\alpha_{10}}\right)\ \text{MEC} \tag{3.28}$$

where E is the laboratory energy.

The left-hand side of Eq. (3.28) may also be evaluated using (3.8) and (3.9). Equating the expression obtained in this way to the right-hand side of

Eq. (3.28), we may express the sum rule in the Lorentz invariant form

$$\int_0^{\infty} d\nu \; \frac{Q^2}{\nu} \left(1 - \frac{Q^2}{2M\nu}\right) \left(\sigma_S\delta_S + \sigma_R\delta_R + \sigma_L\delta_L - \bar{\sigma}_S\bar{\delta}_S - \bar{\sigma}_R\bar{\delta}_R - \bar{\sigma}_L\bar{\delta}_L\right) = 4\pi G \,\mathrm{Re}\left(\frac{\alpha_{20}}{\alpha_{10}}\right) MC \tag{3.29}$$

C. Parton Model

The neutrino-parton cross sections are, for partons with quark quantum numbers, and with the Cabibbo angle taken to be zero for simplicity,

$$\frac{d\sigma}{dQ^2 d(q\cdot P)} = \frac{G^2}{\pi}\frac{1}{q\cdot P}\,\delta\left(1 - \frac{Q^2}{2q\cdot P}\right)\left(1 - 2\,\mathrm{Re}\left(\frac{\alpha_{11}}{\alpha_{10}}\right) GQ^2 + 2\,\mathrm{Re}\left(\frac{\alpha_{20}}{\alpha_{10}}\right) Gs\right) \tag{3.30a}$$

for νn and $\bar{\nu}\bar{n}$ scattering, and

$$\frac{d\sigma}{dQ^2 d(q\cdot P)} = \frac{G^2}{\pi}\frac{1}{q\cdot P}\,\delta\left(1 - \frac{Q^2}{2q\cdot P}\right) y^2 \left(1 - 2\,\mathrm{Re}\left(\frac{\alpha_{11}}{\alpha_{10}}\right) GQ^2 - 2\,\mathrm{Re}\left(\frac{\alpha_{20}}{\alpha_{10}}\right) Gs\,y\right) \tag{3.30b}$$

for $\nu\bar{p}$ and $\bar{\nu}p$ scattering. In the above, $y \equiv E'/E = -u/s$. Now we let $P_\mu \to xP_\mu$ ($s \to xs$, $y \to y$, $Q^2 \to Q^2$) and fold (3.25) over the parton momentum distribution function, $\frac{dx}{x} f_i(x)$, obtaining the cross sections for scattering from a nucleon:

$$\frac{d\sigma^{\nu}}{dQ^2 d\nu} = \frac{G^2}{\pi\nu}\left\{\left(1 - 2\,\mathrm{Re}\,\frac{\alpha_{11}}{\alpha_{10}}\,GQ^2\right)\left(f_n(x) + y^2 f_{\bar{p}}(x)\right) + 2\,\mathrm{Re}\,\frac{\alpha_{20}}{\alpha_{10}}\,Gxs\left(f_n(x) - y^3 f_{\bar{p}}(x)\right)\right\} \tag{3.31a}$$

$$\frac{d\sigma^{\bar{\nu}}}{dQ^2 d\nu} = \frac{G^2}{\pi\nu}\left\{\left(1 - 2\,\mathrm{Re}\,\frac{\alpha_{11}}{\alpha_{10}}\,GQ^2\right)\left(f_{\bar{n}}(x) + y^2 f_p(x)\right) + 2\,\mathrm{Re}\,\frac{\alpha_{20}}{\alpha_{10}}\,G\,xs\left(f_{\bar{n}}(x) - y^3 f_p(x)\right)\right\} \tag{3.31b}$$

With the identities $xs = \frac{3}{4}Q^2 + \frac{Q^2}{4\nu}(3E'+E)$ and $-xsy = \frac{3}{4}Q^2 - \frac{Q^2}{4\nu}(3E+E')$, we find that the structure (3.31) is indeed compatible with (3.8) and (3.9), and we have

$$\delta_L = -\delta_R = \bar{\delta}_L = -\bar{\delta}_R = 2G\,\mathrm{Re}\left(\frac{\alpha_{20}}{\alpha_{10}}\right)\frac{Q^2}{4\nu} \qquad \delta_S = \sigma_S = 0 \tag{3.32}$$

It is interesting that the correction term is independent of the parton momentum distributions.

Conclusions

Our major results are

1. The generalization of the hypothesis of lepton-hadron universality in weak interactions as discussed in the introduction.
2. Given universality, the definition of the "weak cutoff" Λ in terms of high energy behavior of lepton-lepton total cross sections, and the new estimates of Λ from the properties of the K system.
3. Introduction of a phenomenology for higher order corrections to weak processes, at present useful for high energy neutrino reactions.

The first two points are at best of theoretical interest, and re-express in perhaps a more general way concepts which are old and widely discussed. If the universality as we have defined it is correct, and the tower of hadronic current operators we have discussed are those of quarks, then the estimates of the cutoff Λ are so low that we may expect "locality violations" in ν-N processes at accessible laboratory neutrino energies, in the 100 GeV range.

That is, the terms in the generalized neutrino cross sections (3.8) and (3.9) proportional to α_{20} and α_{11} may be of importance.

While the assumption of universality is a very strong one, and the conclusion regarding the low value of Λ fragile, it still remains very probable that in any case the phenomenological description of the deviations of ν-N cross sections from the low energy limit will be very similar to what we have given. Consequently it is of importance to try to bound the magnitude of α_{20} and α_{11} experimentally.

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