Isobar Analogue States

The discovery[1] in 1961 of isobar analogue states in medium-weight nuclei was a surprise to many nuclear physicists. It had been more or less generally assumed that because of the Coulomb interaction, isobaric spin in all but the light nuclei was a meaningless concept. And so their discovery was delayed, although apparatus capable of their detection had been in operation for many years. In the last few years, several hundred such analogue states have been detected, and it is plain that their presence is a common phenomenon probably occurring throughout the periodic table. Moreover, it is becoming increasingly clear that reactions involving these states provide a new and important method for nuclear spectroscopy; that is, for the determination of various properties of nuclear wave functions or of relations between different nuclear wave functions. In this note, we shall comment on (i) what isobar analogue states are, (ii) how they can be produced, and (iii) how they may prove useful in nuclear spectroscopy.

(i) To describe the isobar analogue states, we compare the energy levels in a nucleus containing Z protons and N neutrons with one containing $(Z + 1)$ protons and $(N - 1)$ neutrons. First we shall consider the relationship between the level structure of these two isobars when the Coulomb interaction is turned off. Then the effect of the Coulomb interaction between the protons will be added. We shall use the term "Coulomb interaction" to include both the Coulomb force between the protons and that part of the nuclear force which violates charge independence.

In the absence of the Coulomb interaction, the isospin T is a good quantum number. The isospin of the ground state of a (Z,N) nucleus is given by

$$T_{>} = |T_3| = (N - Z)/2. \tag{1}$$

Similarly for the ground state of the $(Z + 1, N - 1)$ nucleus, T takes on the value $T_{<}$:

$$T_{<} = (N - Z)/2 - 1. \tag{2}$$

Thus, although the total number of nucleons in both nuclei is the same and although charge independence is taken to be exact, the ground states of the two nuclei are not the same, the ground state of the nucleus with the smaller isotopic spin lying lower as shown in Fig. 1. Generally, the

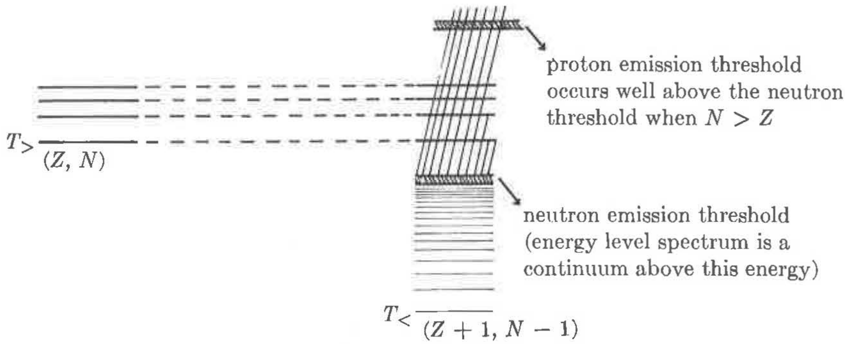

FIG. 1. Schematic energy level diagram in the *absence* of the Coulomb interaction.

system with the smaller value of T will have a wave function which is more symmetric in space and spin; and therefore, more able to take advantage of the short-ranged nucleon-nucleon forces. The difference in energies between the two ground states can be taken from the symmetry term in the semiempirical mass formula. It is of the order of

$$90[(N - Z)/(N + Z)] \text{ MeV}.$$

In a nucleus like ${}_{38}\mathrm{Sr}_{51}{}^{89}$, $T_>$ is 13/2, the isobar with $T_<$ of 11/2 is ${}_{39}\mathrm{Y}_{50}{}^{89}$, the symmetry energy is of the order of 13 MeV. The levels of the (Z,N) nucleus are present in the $(Z + 1, N - 1)$ nucleus and, in the absence of Coulomb forces, with exactly the same energy. These levels are indicated in Fig. 1. They occur in the continuum above the threshold for neutron emission which is roughly 8 MeV above the ground state of the $T_<$ nucleus. However, they remain bound states because their isotopic spin $T_>$ differs from the isotopic spin of the continuum. Isotopic-spin conservation forbids the decay of these $T_>$ bound states into the continuum states.

We now introduce the Coulomb interactions. There are two effects. Because of the Coulomb repulsion, the entire energy level spectrum of the $(Z + 1, N - 1)$ nucleus is displaced upward. Along the stable mass valley, the additional Coulomb energy just about compensates for the symmetry energy. This is shown in Fig. 2 which includes the Coulomb effects absent in Fig. 1. Again because of Coulomb forces, it becomes easier to emit protons; and, in fact, the threshold for this process usually is somewhat below the neutron emission threshold, as depicted in Fig. 2. From Fig. 2, we see that the levels in the nucleus (Z,N) are completely mirrored in the $(Z + 1, N - 1)$ nucleus. These states in the $(Z + 1, N - 1)$ which correspond to the states in the (Z,N), the so-called *parent* states of the *parent* nucleus, are the isobar analogue states. As we can see in Fig. 2, the analogue states occur in the continuum.* Because they lie above the proton threshold, analogue states can decay by proton emission. In addition, because of the presence of the Coulomb interaction, the analogue states can couple to the states with differing T, e.g. $T_<$; and, therefore, neutron emission becomes possible. However, the matrix elements for this process will generally be small because the coupling to the $T_<$ states is weak. This is an expression of the fact that the Coulomb potential is spatially slowly varying so that matrix elements of the Coulomb potential between different and, therefore, orthogonal states are small.

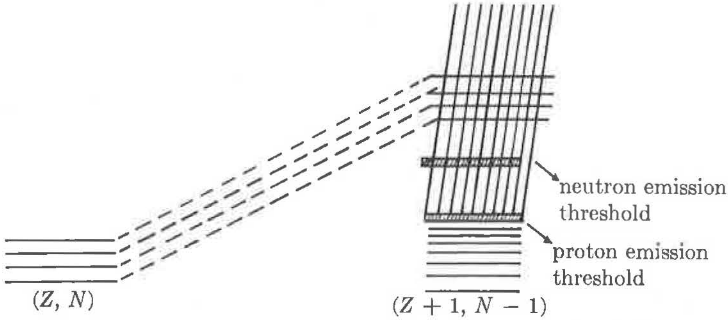

FIG. 2. Schematic energy level diagram in the *presence* of the Coulomb interaction.

The analogue states in virtue of the finite probabilities for proton and neutron decays are no longer bound states. More properly they are "doorway" states as discussed in Comments.[2] As was pointed out there, for reactions in which the doorway system is the analogue nucleus $(Z + 1, N - 1)$, the compound nuclear resonances which couple with the doorway state will exhibit an increased strength. For example, the $T_<$ compound nuclear resonances in the situation pictured in Fig. 1 will broaden because of their coupling to the analogue state when the Coulomb interaction is turned on. This phenomenon is clearly exhibited if these compound nuclear resonances are excited and detected with sufficiently good energy resolution. However, for experiments which have nuclear spectroscopic applications in mind such good resolution is not necessary. It is, in fact, better to average over the compound nuclear resonances, the resultant average reaction or scattering cross sections now exhibiting a resonance structure, the intermediate resonance which can be associated with the analogue state. The width of this intermediate resonance, or more specifically isobar analogue resonance, consists of two parts: $\Gamma^{\uparrow}$ which gives the probability for decay, in this case mostly by proton emission into the proton channel, and $\Gamma^{\downarrow}$ which gives the spreading due to the coupling with the compound nuclear levels.

* For the lightest nuclei the lowest analogue states are bound.

How are isobar analogue states formed and detected? One of the simplest procedures is to knock a neutron out of a target nucleus and replace it by a proton. In this way, one forms states in the $(Z + 1, N - 1)$ nucleus of Fig. 2 from the target and, in this case, parent (Z,N) nucleus. Because transitions to the isobar analogue states do not change the isotopic spin of the target, they are preferentially excited as a direct reaction. The existence of an analogue state may be deduced from examination of the neutron spectrum. In fact, this was the way they were first discovered.[1] Because the isobar analogue state is usually unstable against proton emission, there will be a peak in the proton spectrum corresponding to the energy involved in the transition from the isobar analogue state to the final state, an energy which is independent of the energy of the incident proton once it is above the threshold for exciting the analogue state. This feature is exploited in the experiments of Moore *et al.*[3] and Yavin.[4] In the former, the analogue state is made by a (d,n) reaction in which the target nucleus is of the $(Z, N - 1)$ type, the analogue nucleus of the $(Z + 1, N - 1)$ type, and where the parent is the (Z,N) nucleus. The proton decay spectrum of the $(Z + 1, N - 1)$ nucleus is examined for proton groups whose energy is independent of the incident deuteron energy.

Perhaps the most common method of exciting isobar analogue resonances is by proton bombardment.[1] Resonances in inelastic scattering (p,p') and various reactions such as (p,n), (p,γ) as well as in the elastic scattering indicate the presence of these states. If the target nucleus is $(Z, N - 1)$ and the compound system is $(Z + 1, N - 1)$, the parent nucleus (Z,N) is an isotope of the target. Verification of the analogue character is obtained from the energy differences from their corresponding parent levels which should be calculable from the Coulomb energy, and from their spin and parity which should be identical with that of the parent levels. These quantities, the energy, spin, and parity can be determined by analysis of the resonance cross section and/or from the polarization of the scattered proton. This, in fact, may be the best way of determining the spin and parity of the levels of the *parent* nucleus.

This observation is the simplest and most direct exploitation of the analogue resonance for nuclear spectroscopy. As an example of a more subtle insight gained because of the simple relation between the parent and analogue states, consider the simplest shell-model description of an excited state of the parent nucleus shown in Fig. 3. The shell-model description of the analogue state is a linear superposition of states as illustrated in Fig. 4. We see that in the state in Fig. 4(a) the neutron in Fig. 3 is replaced by a proton in the identical level. The other states are formed by adding a proton particle plus a neutron hole to the neutron of Fig. 3. The analogue state is thus a somewhat more complicated state than the parent state. Its usefulness comes from the fact that it is in the

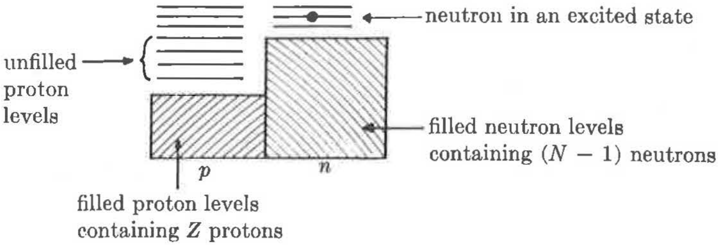

FIG. 3. The parent nucleus in an excited state.

continuum. It can decay to various levels by emission of a proton, going to the target-nucleus ground state or various excited states. The branching ratios, i.e. the relative probabilities for these various possible decays, are indicative of the relation between the final nucleus and the analogue-nucleus wave functions. For example, in the case illustrated in Fig. 4, proton decay from the analogue state would be more favored if the level in the residual nucleus is either the ground state or involves an excited neutron and a neutron hole.

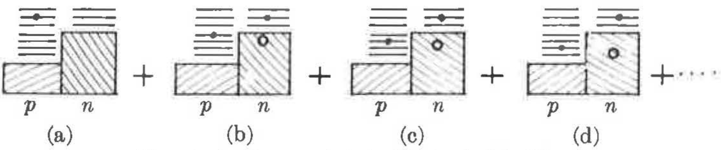

FIG. 4. Analogue of state illustrated in Fig. 3.

As an example of these arguments, consider the scattering of a proton on Pb^{208}.[5] The parent nucleus is then Pb^{209}, Fig. 3; the analogue nucleus is Bi^{209*}, Fig. 4. Inelastic proton scattering should be enhanced if the emergent proton goes to a particle-hole state in Pb^{208}. When the experiment is done, several analogue resonances are found with a variety of spins directly identifiable with parent states in Pb^{209}. All of these decay by proton emission into the 3^- first excited state of Pb^{208} while the second and third excited states 5^- and 4^-, respectively, are populated by only one of the analogue resonances, the one corresponding to a $g_{9/2}$ parent state. We may immediately conclude that the 3^- is a coherent linear superposition of many neutron particle–neutron hole configurations while the 5^- and 4^- are principally a $g_{9/2}$ neutron and neutron hole. This is in agreement with evidence obtained by inelastic scattering on Pb^{208}.

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6. As general references, we suggest the *Proceedings of the Conference on Isobar Spin in Nuclear Physics, Tallahassee, Florida, 1966* (Academic Press, New York) and the invited paper by von Brentano at the 1966 Gatlinburg Meeting.