

BETA-DELAYED NEUTRON ENERGY SPECTRUM  
CALCULATED IN EFFECTIVE DENSITY MODEL\*

KRZYSZTOF MIERNIK

University of Warsaw, Faculty of Physics  
Pasteura 5, 02-093 Warszawa, Poland*(Received March 23, 2015)*

Effective density model is used to calculate delayed neutron spectra. Comparison is made with some of the available experimental results. The calculated spectra show good agreement, on statistical level, with the experimental results.

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**1. Introduction**

Beta-delayed neutron emission ( $\beta n$ ) is a decay path opened in neutron-rich nuclides whenever beta-decay energy ( $Q_\beta$ ) is larger than decay daughter neutron separation energy ( $S_n$ ). The importance of this decay mode is growing with the excess of neutrons, and eventually it becomes a dominant decay mode for currently known most exotic neutron-rich isotopes.

Many fission fragments from the actinides exhibit this property, thus  $\beta n$  is one of the most crucial processes of nuclear reactor operation or nuclear fuel post-processing [1]. Predictions based on mass models show that about 270 of  $^{235}\text{U}$  fission fragments should be delayed neutron emitters [2]. Yet, even though the  $\beta n$  was discovered already in 1939, the most basic property — total delayed neutron emission probability ( $P_n$ ) — was experimentally established (with a definite value) so far for less than 200 isotopes. Moreover, not only the neutron emission probability is important, but also the delayed neutrons energy spectrum. Depending on the type of the reactor, and their spectrum, the delayed neutrons may be more (in thermal reactors) or less effective than prompt neutrons (in fast reactors) [3]. However, the experimental knowledge of delayed neutrons energy spectra is even more scarce, limited to about 20 isotopes of largest importance in reactor physics.

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Delayed neutrons play also an important role in calculations of the astrophysical r-process nucleosynthesis, particularly in the so-called “freeze-out” phase, changing the final mass abundance pattern [4]. In this case, there are very few measured  $P_n$  values for isotopes that are on or near the proposed r-process path, and almost no information on delayed neutron energy spectra.

The limited experimental knowledge of delayed-neutron emission is mainly due to low production cross section for exotic neutron-rich isotopes and experimental challenges of neutron detection. Therefore, nuclear physics input for applications like r-process modeling comes largely from theoretical models.

In this contribution, calculations of delayed neutrons energy spectra, based on effective density model, are presented. The details of this model are described elsewhere [5, 6] and only a brief introduction will be given.

## 2. Model description

The most crucial part in modeling the delayed neutron emission is the  $\beta$ -strength function ( $S_\beta$ ), which holds the nuclear structure description in  $\beta$ -decay theory. The effective density model [5] is based on assumption that  $S_\beta$  is proportional to the total level density

$$S_\beta(E) \sim \rho(E) = \frac{\exp(a_d \sqrt{E})}{E^{3/2}}, \quad (1)$$

where  $a_d$  is an effective parameter. The systematics of this parameter were built based on experimentally available  $P_n$  values and modeled with a phenomenological 12 parameters function [5]. It was shown that this method reproduces experimental  $P_n$  values better than other models, and allows for extrapolations. In Ref. [6], this model was extended by dividing the total neutron emission probability into one, two, and three delayed neutron probabilities. This was calculated with a sequential neutron emission model, based on statistical level densities in daughter nuclei. Using the same method (details are given in [6]), one can also retrieve the delayed neutron spectra, presented in the next section.

## 3. Results

The results of calculations of delayed neutron spectra for selected isotopes are presented in Fig. 1. Since the model is based on statistical description of the nuclei, one cannot expect that the details of the spectrum, like discrete lines, will be reproduced. However, in all presented cases, the general character of the spectrum is on par with the experimental one. Table I presents comparison of the first and second moment of the distribution, showing good compatibility on quantitative level as well.

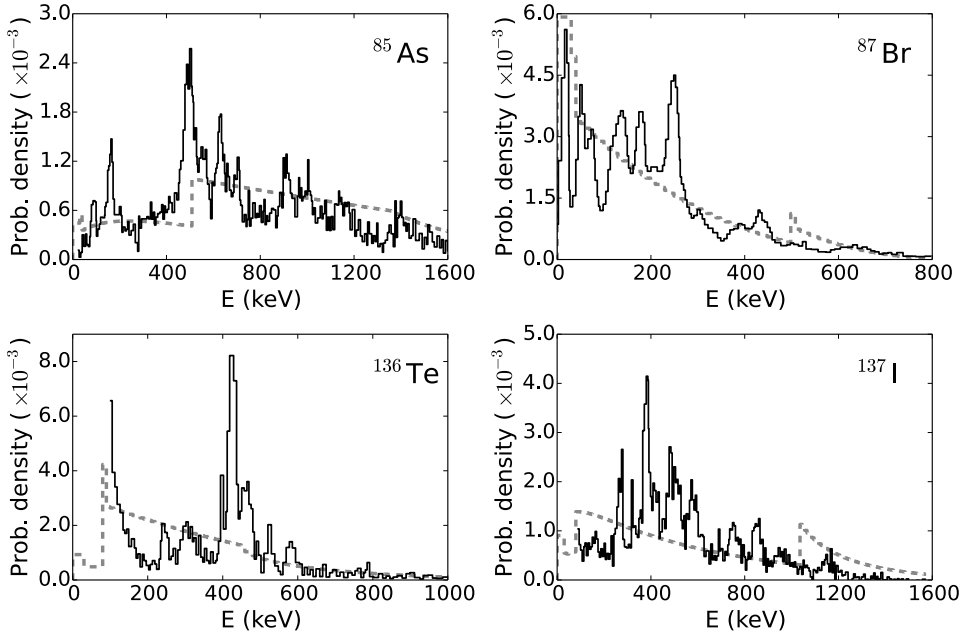


Fig. 1. Comparison of experimental (black solid lines) and calculated (dashed gray lines) delayed neutron spectra. The experimental data for  $^{85}\text{As}$ ,  $^{87}\text{Br}$  are taken from Ref. [7, 8], while those for  $^{136}\text{Te}$ ,  $^{137}\text{I}$  from Ref. [9].

TABLE I

Average neutron energy ( $\bar{E}_n$ ) and standard deviation ( $\sigma$ ) calculated for experimental and theoretical neutron spectra. Experimental data are taken from: [7, 8] ( $^{85}\text{As}$ ,  $^{87}\text{Br}$ ,  $^{135}\text{Sb}$ ), [9] ( $^{134}\text{Sn}$ ,  $^{136}\text{Te}$ ,  $^{137}\text{I}$ ), and [10] ( $^{94,97}\text{Rb}$ ).

| Nuclide           | $\bar{E}_n^{\text{exp}}$ [MeV] | $\sigma^{\text{exp}}$ [MeV] | $\bar{E}_n^{\text{calc}}$ [MeV] | $\sigma^{\text{calc}}$ [MeV] |
|-------------------|--------------------------------|-----------------------------|---------------------------------|------------------------------|
| $^{85}\text{As}$  | 726                            | 9.4                         | 830                             | 10.1                         |
| $^{87}\text{Br}$  | 212                            | 5.6                         | 213                             | 5.9                          |
| $^{94}\text{Rb}$  | 399                            | 13.7                        | 440                             | 7.5                          |
| $^{97}\text{Rb}$  | 437                            | 11.8                        | 607                             | 9.7                          |
| $^{134}\text{Sn}$ | 478                            | 9.4                         | 520                             | 8.4                          |
| $^{135}\text{Sb}$ | 843                            | 12.0                        | 680                             | 11.0                         |
| $^{136}\text{Te}$ | 381                            | 5.9                         | 325                             | 6.5                          |
| $^{137}\text{I}$  | 538                            | 7.0                         | 604                             | 10.4                         |

It is worth to notice that it was shown by Hardy *et al.* [11] that experimental delayed neutron spectra can be characterized as noise described by a Porter–Thomas statistics and the observed peaks are due to random fluctuations in level density. This means that  $\beta$ -strength function is essentially featureless and consistent with a statistical model. This conclusion further justifies the use of statistical modeling in calculations of delayed neutron properties.

One must notice that there is a special class of isotopes, located two and three neutrons above magic numbers (*i.e.*  $N = 30, 31, 52, 53, 84, 85$ ) that reveal different nature than other isotopes [5]. In the effective density model, the decay of these isotopes is described by an enhanced density parameter value, that properly reproduces  $P_n$ , but might be otherwise unphysical. Presently, there are no experimental data available for these cases, however, hopefully a future possibility to compare spectra for these special, “non-statistical” cases might shed some more light on their properties.

#### 4. Summary

Delayed neutron spectra were calculated with the effective density model. The results show good compatibility with experimental spectra on statistical level. Comparison was made for 8 selected isotopes, including both heavy and light fission fragments, and even–even, odd-mass, and odd–odd isotopes. This suggests that statistical model is capable of predicting of basic features of delayed neutron spectra (*e.g.* average neutron energy), that might be useful for applications like r-process modeling.

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