Symmetry aspects in emergent quantum mechanics

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Abstract. We discuss an explicit realization of the dissipative dynamics anticipated in the proof of 't Hooft's existence theorem, which states that *"For any quantum system there exists at least one deterministic model that reproduces all its dynamics after prequantization"*. – There is an energy-parity symmetry hidden in the Liouville equation, which mimics the Kaplan-Sundrum protective symmetry for the cosmological constant. This symmetry may be broken by the coarse-graining inherent in physics at scales much larger than the Planck length. We correspondingly modify classical ensemble theory by incorporating dissipative fluctuations (information loss) – which are caused by discrete spacetime continually "measuring" matter. In this way, aspects of quantum mechanics, such as the von Neumann equation, including a Lindblad term, arise dynamically and expectations of observables agree with the Born rule. However, the resulting quantum coherence is accompanied by an intrinsic decoherence and continuous localization mechanism. Our proposal leads towards a theory that is linear and local at the quantum mechanical level, but the relation to the underlying classical degrees of freedom is nonlocal.

1. Introduction

There is a growing number of deterministic models of quantum mechanical objects which are based on conjectured fundamental information loss or dissipation mechanisms [1–7], see also Refs. [8–10].

Such studies are largely motivated by unresolved issues surrounding "quantum gravity", i.e., by the conflict between quantum mechanics necessitating an external time and time reparametrization invariance in general relativity which defies its existence. Furthermore, despite its great success in describing statistical aspects of experiments, quantum theory itself is problematic. This arises foremost from its indeterministic features and is clearly seen, for example, in the unresolved measurement problem (also "wave function collapse" or "objective reduction"). Therefore, there is ample motivation for reconsidering the foundations and trying to understand the *emergence of quantum mechanics* from possibly simpler structures beneath.

The construction of models, so far, proceeded mostly case by case, however, guided by the idea that quantum states may actually represent equivalence classes of deterministically evolving "classical" states – which become indistinguishable when affected by the conjectured information loss or dissipative process [1]. The existence theorem by 't Hooft shows that the evolution of all quantum mechanical objects that are characterized by a finite dimensional Hilbert space can be captured by a dissipative process [11]. The generalization for objects described by a finite set of beables [12] is given in Ref. [13].

However, it has not been shown before how to complement such existence theorem by a dynamical theory. In order to describe the quantum mechanical world, such a theory has to deal with interacting as well as approximately isolated objects that exist throughout a large range

of length and energy scales. Stepping forward, we show here that essential aspects of quantum mechanics can be generated by a generalization of the Liouville equation incorporating suitable fluctuations. Their effect is quite similar to dissipation caused by measurements and described by a Lindblad equation in quantum mechanics. However, the present approach is based on *classical* statistical mechanics. This is motivated by considerations of a fundamentally atomistic spacetime structure, the evolution of which results in information loss affecting all matter.

The approach presented here has been initiated in Ref. [2]. It leads us to the emergence of quantum states in a classical ensemble theory which evolve according to the Schrödinger picture embodied in the von Neumann equation, it reproduces aspects of the Born rule, and, curiously, it may be connected to the breaking of the Kaplan-Sundrum energy-parity symmetry, which could protect the cosmological constant against the far too large corrections determined by particle physics scales [14, 15].

2. A quantum-like formulation of classical Hamiltonian dynamics

We consider objects with a single continuous degree of freedom, in order to keep the notation as simple as possible. Everything that follows in this section, however, is easily repeated for interacting few-body systems and fields.

To begin with, we assume that all forces are conservative and that Hamilton's equations are determined by a generic Hamiltonian function:

$$H(x,p) := \frac{1}{2}p^2 + v(x) \ , \tag{1}$$

in terms of generalized coordinate x and momentum p, and where $v(x)$ denotes the *"true" potential*. In Section 4, we will come back to the notion of the true potential and distinguish it from a related *coarse-grained potential* $V(x)$.

An ensemble of such objects, for example, following trajectories with different initial conditions, is described by a distribution function f in phase space, i.e., by the probability $f(x,p;t)\mathrm{d}x\mathrm{d}p$ to find a member of the ensemble in an infinitesimal volume at point (x,p). This distribution evolves according to the Liouville equation:

$$-\partial_t f = \frac{\partial H}{\partial p} \cdot \frac{\partial f}{\partial_x} - \frac{\partial H}{\partial x} \cdot \frac{\partial f}{\partial_p} = \{p\partial_x - v'(x)\partial_p\}f \ , \tag{2}$$

with $v'(x) := \mathrm{d}v(x)/\mathrm{d}x$. – We recall that the relative minus sign in the Poisson bracket, or between terms here, reflects a symplectic phase space symmetry. This will translate into the familiar commutator structure in Eq. (5).

A Fourier transformation, $f(x,p;t) = \int \mathrm{d}y \ \mathrm{e}^{-ipy} f(x,y;t)$, replaces the Liouville equation by:

$$i\partial_t f = \{-\partial_y\partial_x + yv'(x)\}f \ , \tag{3}$$

without changing the notation for the distribution function, whenever changing variables. Thus, momentum is eliminated and a *doubled number of coordinates* results. Finally, with the transformation:

$$Q := x + y/2 \ , \quad q := x - y/2 \ , \tag{4}$$

we obtain the Liouville equation in the form:

$$i\partial_t f = \{\hat{H}_Q - \hat{H}_q + \mathcal{E}(Q,q)\}f \ , \tag{5}$$

$$\hat{H}_\chi := -\frac{1}{2}\partial_\chi^2 + v(\chi) \ , \quad \text{for} \ \chi = Q,q \ , \tag{6}$$

$$\mathcal{E}(Q,q) := (Q - q)v'(\frac{Q+q}{2}) - v(Q) + v(q) = -\mathcal{E}(q,Q) \ . \tag{7}$$

Several comments are in order here:

- The present reformulation of classical dynamics in phase space can be carried out rather independently of the number of degrees of freedom and is applicable to matrix or Grassmann valued variables as well; see, for example, Refs. [3,15]. Gauge theories or, generally, theories with constraints have to be examined carefully.

- Most importantly, the Eq. (5) closely resembles the *von Neumann equation* for a density operator $\hat{f}(t)$, considering $f(Q, q; t)$ as its matrix elements. We automatically recover the Hamiltonian operator $\hat{H}$ related to the Hamiltonian function, Eq. (1), as in quantum theory. However, an essential difference consists in the interaction $\hat{\mathcal{E}}$ between *bra-* and *ket-* states. [1] The Hilbert space and its dual here are coupled, unlike the case of quantum mechanics.

- Alternatively, the Eq. (5) might be read as the *Schrödinger equation* for two identical (sets of) degrees of freedom. However, their respective Hamilton operators, $\hat{H}_{Q,q}$, contribute with opposite sign, which must be traced to the classical symplectic symmetry. Since their interaction $\hat{\mathcal{E}}$ is antisymmetric under $Q \leftrightarrow q$, the complete (Liouville) operator on the right-hand side of Eq. (5) has a symmetric spectrum with respect to zero and, in general, will not be bounded below. This *Kaplan-Sundrum energy-parity symmetry* has been invoked before to protect a (near) zero cosmological constant which, otherwise, is threatened by many orders of magnitude too large zeropoint energies [14,15].

- Finally, the following observation will turn out to be crucial:

$$\hat{\mathcal{E}} \equiv 0 \quad \Longleftrightarrow \quad \text{true potential } v(x) \text{ is piecewise constant, linear, or harmonic .} \tag{8}$$

  The analogous vanishing of $\hat{\mathcal{E}}$ in a field theory is equivalent with having massive or massless free fields, with or without external sources, and with or without bilinear couplings.

However, with a coupling of the Hilbert space and its dual and without a stable ground state, in the general case with anharmonic interactions, our reformulation of Hamiltonian dynamics does not qualify as a quantum theory yet.

The problem of the missing ground state has been a stumbling block for previous attempts to build deterministic models for quantum objects and has been overcome in some particular examples [1,3,4,7,8]. While it has gone unnoticed in approaches based on classical ensemble theories [5,6,10]. The present work belongs to the latter group and proposes a general solution of the problem.

3. Are quantum features induced by the "atomistic" spacetime?
We have seen that classical ensemble theory with its Liouville equation implies the absence of a stable ground state, when rewritten in the form of a Schrödinger equation. Correspondingly, the analogue of the von Neumann equation obtains the unusual superoperator $\hat{\mathcal{E}}$ which couples the Hilbert space and its dual. – On the other hand, dissipation was of great importance in the proof of existence of deterministic models for quantum mechanical objects [11,13].

In view of these findings, we depart from two working hypotheses, trying to understand the transition <u>from</u> classical <u>to</u> quantum behaviour dynamically:

(A) *The emergence of quantum states with their dynamics originates from a microscopic dissipative process beneath which affects all physical objects.*

(B) *The statistical interpretation of quantum states (Born rule) originates from classical mechanics in an ensemble theory.*

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[1] We will call this a *superoperator*, following the terminology of open quantum systems [16].

The second hypothesis has been illustrated in Ref. [2], to some extent – including some remarks concerning the issue of negative probabilities, which needs further attention. However, in the following, we continue to explore the first hypothesis.

We speculate that the *atomistic structure of spacetime* itself is responsible for effects which are attributed to quantum mechanics, typically operating at length scales much larger than the Planck length. Here "atomistic" refers to a discrete set of elements with the presence, or absence, of a certain order relation between any two elements. Furthermore, this set, in particular the number of its elements changes according to certain dynamical rules, possibly establishing new order relations, or erasing old ones. In this way, time "happens".

Similar ideas about the nature of spacetime have been formulated every now and then throughout the history of natural philosophy [17, 18]. However, only recently such general scenario has been elaborated in considerable detail in the theory of *causal sets*. In mathematical terms, these are locally finite ordered sets. Their evolution by sequential growth through random ("sprinkling") appearance of new set elements together with their order relations has been studied [18–20].

In the absence of an equally elaborate theory of matter in relation to such atomistic structure of spacetime, we can nevertheless state the following, concerning the "situation" of a typical object. Consider an electron, for example, an object that to highest known precision behaves according to the laws of quantum mechanics. We may term its "situation" a hypothetical complete set of its properties that are accessible by experiments.

Now, first of all, there are interactions between such object and its environment (the "rest of the Universe"), last not least gravitational ones. Besides more familiar aspects, however, there must exist a continual *loss of information* about its "situation", since the atomistic spacetime beneath evolves. With respect to the latter, quantum theory at present deals with very coarse grained phenomena, when describing the dynamics of matter. Loosely speaking, in order to fully characterize an electron, the evolution of its causal relations with a continually changing number of spacetime "atoms" has to come into play. Consequently, in a coarse grained picture where this is not explicitly taken into account, information is dissipated into this omnipresent environment of all objects.

Secondly, however, common objects are characterized by a certain persistence, which makes them identifiable in experiments. Therefore, the information loss must be a delicate one. This is an important aspect of the *conservation of probability* to be maintained in ensemble theories – "objects don't get lost".

Contrary to measurement processes in quantum mechanics, where information is transferred from microscopic to macroscopic objects [2], we propose here that microscopic matter degrees of freedom are continually "measured" by the evolution of spacetime.

These heuristic considerations lead us to modify the classical ensemble theory in important ways. We will incorporate dissipation into the Liouville equation, however, in such a way that probability conservation remains intact. This will provide a mechanism which turns the evolution of classical objects, described by an ensemble theory as in Section 2, into the Schrödinger evolution of quantum objects, with a state space based on a stable ground state.

4. From Liouville equation to von Neumann - Lindblad equation
Given that spacetime has an atomistic structure, there must be a characteristic length scale where the continuum description of all phenomena breaks down.

This implies, in particular, that one might overlook important traces of the atomistic structure by employing continuum quantities which allow to arbitrarily extrapolate their functional description to scales $l \approx l_{Pl}$, where l_{Pl} denotes the Planck length.

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[2] In a way which is not understood in all its aspects in theory. For recent discussions, which critically review related ideas, see Refs. [21, 22].

Instead of having such a *coarse grained potential* function $V(x)$, for example, the *true potential* likely becomes a piecewise defined function $v(x)$, when approaching the Planck scale in the continuum picture. Thus, the function $V(x)$ is an approximation to $v(x)$ and the difference between the two must give rise to local *fluctuations* $\delta V(x)$. Therefore, we set:

$$v(x) = V(x) + \delta V(x) \ , \tag{9}$$

in order to map behaviour at small spacetime scales to its coarse grained description.

Furthermore, we speculate that the true potential can be represented as a *piecewise constant, linear, or harmonic* function. This assumption may lead away from the strategy to adapt continuum quantities to a discrete spacetime – for example, by approximating a quartic potential by a quartic expression with support on the elements of a causal set. Yet it seems possible to formulate it entirely based on concepts that are innate to a causal set, such as done for free particles in Refs. [23, 24]. [3]

The fluctuations are treated as white noise with mean and correlation, respectively, given by:

$$\langle \delta V(x) \rangle \ = \ 0 \ , \tag{10}$$

$$\langle \delta V(x) \delta V(y) \rangle \ = \ \nu^2(x)\delta(x-y)/\delta(0) \ , \tag{11}$$

where $\nu(x)$ describes the width of the local distribution of fluctuations. This distribution cannot be assessed without knowing how forces arise near the Planck scale. [4]

Given these assumptions, using the true potential $v(x)$ in Eq. (5), the troubling superoperator $\hat{\mathcal{E}}$ vanishes, in accordance with our observation (8). Next, we employ Eq. (9), to reexpress the remaining terms by coarse grained ones. This yields a von Neumann equation incorporating fluctuating terms:

$$\partial_t \hat{f} = -i[\hat{H}_{eff}, \hat{f}] \ , \tag{12}$$

where the effective Hamilton operator is now given by:

$$\hat{H}_{eff} := \hat{H}_\chi + \delta V(\chi) \ , \tag{13}$$

in the coordinate representation, cf. Eq. (6), and incorporates the coarse grained potential $V(\chi)$.

In order to appreciate the influence of the fluctuations, we formally solve Eq. (13) by $\hat{f}(t) = \exp(-i\hat{H}_{eff}t)\hat{f}(0)\exp(+i\hat{H}_{eff}t)$. Expanding this solution in powers of t and averaging over the fluctuations, with the help of Eqs. (10)–(11), we obtain to first order in the correlation function:

$$\partial_t \hat{f} = -i[\hat{H}_\chi, \hat{f}] - t(\{\hat{\nu}^2, \hat{f}\} - 2\hat{\nu}\hat{f}\hat{\nu}) \ , \tag{14}$$

with $\{\hat{\nu}^2, \hat{f}\}(x,y) := \nu^2(x)f(x,y) + f(x,y)\nu^2(y)$ and $(\hat{\nu}\hat{f}\hat{\nu})(x,y) := \nu^2(x)f(x,y)\delta(x-y)/\delta(0)$. Thus, we arrive at a master equation with a dissipative *"Lindblad term"*, in addition to the leading commutator, which is responsible for unitary quantum evolution in the absence of dissipation.

We observe that the Lindblad term explicitly *breaks* the Kaplan-Sundrum energy-parity symmetry, which we mentioned in Section 2. – However, our master equation preserves the normalization of $\hat{f}$, say $\mathrm{Tr}\hat{f} = 1$, which expresses the conservation of probability.

Another interesting aspect of Eq. (14) is the implied *decoherence and continuous localization mechanism*, which causes the decay of spacelike superpositions ("Schrödinger cat states"). While

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[3] The author is not aware of previous studies to what extent dynamics and, in particular, forces mediated by some correspondent of gauge fields can originate in an atomistic picture of spacetime.

[4] Depending on their origin, the fluctuations might affect also kinetic terms in the Hamiltonian and resulting master equation.

the diagonal matrix elements of $\hat{f}$ are not affected by the Lindblad term, the off-diagonal matrix elements decay:

$$f(x,y;t) = f(x,y;0)\mathrm{e}^{-\frac{1}{2}t^2(\nu^2(x)+\nu^2(y))} \quad , \tag{15}$$

where we neglected the effect of $\hat{H}_\chi$ for simplicity, since it does not influence the decay.

Since there is no theory yet to tell us about the correlation function ν^2, it may suffice to point out that the consideration of stochastic effects to modify quantum mechanics has quite a history, see, for example, Refs. [21,22,25–28] and the literature cited there. This is motivated by attempts to solve the infamous measurement problem and to account for the apparent absence of superposition states of macroscopic objects, besides ideas about quantum gravity leading to secondary stochastic effects [29–31]. In this context, various proposals for the analogue of the "Lindblad operator" $\hat{\nu}$ in Eq. (14) have been made and, usually, decoherence and decay which is exponential in t have been obtained.

While these issues have not been settled, we presently suggest a new perspective: if quantum mechanical behaviour emerges dynamically, as surmised here, this will likely influence how the measurement problem and the problems of "wave function collapse" or "objective reduction" will ultimately be resolved.

Generally, the Lindblad master equations present a large class of linear Markovian master equations, which are usually derived to describe quantum systems that interact with particular environments [16,32,33]. Linearity in the density matrix is an important feature of Eq. (14). It implies here that time evolution is related to a contraction semigroup. We have derived this *quantum mechanical master equation*, beginning with *classical statistical mechanics*, by incorporating a number of assumptions about the nature of atomistic spacetime, which plays the role of the "universal environment".

5. Conclusions

We have presented simple arguments to the effect that quantum mechanics could emerge from classical statistics and concerns dynamics over an atomistic spacetime that is observed with low resolution (i.e., at large distance scales). We were motivated here by the theory of causal sets, where such a description of spacetime does not follow from a quantization of gravity but is assumed as a primary feature.

Several topics for future study remain. First of all, generalizing the approach described here to field theories poses the hard problem to undertand the generation of interactions from the shortest spacetime scale *upwards*. Our approach, paying special attention to the observation (8), seems to require that field theories should become noninteracting in the atomistic limit. Secondly, as we mentioned and discussed previously in Ref. [2], the problem of potentially negative probabilities has to be addressed. Finally, work is in progress making newly use of the formalism adopted here, of using Hilbert space operators to describe classical statistics, which was originally suggested in works of Koopman and von Neumann [34,35].

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