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Abstract. Verlinde's proposal on the entropic origin of gravity is based strongly on the assumption that the equipartition law of energy holds on the holographic screen induced by the mass distribution of the system. However, from the theory of statistical mechanics we know that the equipartition law of energy does not hold in the limit of very low temperature. Inspired by the Debye model for the equipartition law of energy in statistical thermodynamics and adopting the viewpoint that gravitational systems can be regarded as a thermodynamical system, we modify Einstein field equations. We also perform the study for Poisson equation and modified Newtonian dynamics (MOND). Interestingly enough, we find that the origin of the MOND theory can be understood from Debye entropic gravity perspective. Thus our study may fill in the gap existing in the literature understanding the theoretical origin of MOND theory. In the limit of high temperature our results reduce to their respective standard gravitational equations.

Contents

1 Introduction

Thermodynamics of black holes reveals that geometrical quantities such as horizon area and surface gravity are related to the thermodynamic quantities such as entropy and temperature. The first law of black hole thermodynamics implies that the entropy and the temperature together with the energy (mass) of the black hole satisfy $dE = TdS$ [1]. In 1995 Jacobson [2] put forwarded a new step and suggested that the hyperbolic second order partial differential Einstein equation for the spacetime metric has a predisposition to thermodynamic behavior. He disclosed that the Einstein field equation is just an equation of state for the spacetime and in particular it can be derived from the proportionality of entropy and the horizon area together with the fundamental relation $\delta Q = TdS$. Following Jacobson, however, several recent investigations have shown that there is indeed a deeper connection between gravitational dynamics and horizon thermodynamics. The deep connection between horizon thermodynamics and gravitational dynamics, help to understand why the field equations should encode information about horizon thermodynamics. These results prompt people to take a statistical physics point of view on gravity.

A next great step put forwarded by Verlinde [3] who claimed that the laws of gravity are not fundamental and in particular they emerge as an entropic force caused by the changes in the information associated with the positions of material bodies. According to Verlinde proposal when a test particle with mass m approaches a holographic screen from a distance $\triangle x$, the change of entropy on the holographic screen is

$$\triangle S = 2\pi \frac{m}{\hbar} \triangle x, \tag{1.1}$$

where we have set $k_B = c = 1$ for simplicity, through this paper. The entropic force can arise in the direction of increasing entropy and is proportional to the temperature,

$$F = T \frac{\triangle S}{\triangle x}. \tag{1.2}$$

Verlinde's derivation of Newton's law of gravitation at the very least offers a strong analogy with a well understood statistical mechanism. Therefore, this derivation opens a new window to understand gravity from the first principles. The study on the entropic force has raised a lot of attention recently (see [4–10] and references therein).

Verlinde's proposal on the entropic origin of gravity is based strongly on the assumption that the equipartition law of energy holds on the holographic screen induced by the mass

distribution of the system, namely, $E = \frac{1}{2}NT$. However, from the theory of statistical mechanics we know that the equipartition law of energy does not hold in the limit of very low temperature. By low temperature, we mean that the temperature of the system is much smaller than Debye temperature, i.e. $T \ll T_D$. It was demonstrated that the Debye model is very successful in interpreting the physics at the very low temperature. Hence, it is expected that the equipartition law of energy for the gravitational systems should be modified in the limit of very low temperature (or very weak gravitational field).

It is important to note that Verlinde got the Newton's law of gravitation, Einstein equations and Poisson equation with the assumption that each bit on holographic screen is free of interaction. It should be more general that the bits on holographic screen interact each others. In such case, one could anticipate that the Newton's law of gravitation, Einstein equation and Poisson equation must be modified. For example, Gao [11] studied three dimensional Debye model and modified the entropic force and henece Friedmann equations. Such modification can interpret the current acceleration of the universe without invoking any kind of dark energy [11]. In this paper we use the Debye model to modify the entropic gravity. We find that this modified entropic force affects on the law of gravitations and modify them accordingly.

This paper is structured as follows. In the next section we derive Einstein field equations from Debye entropic gravity. The theoretical origin of MOND theory is discussed in the framework of Debye entropic gravity in section III. Section IV is devoted to the derivation of the Poisson equation from Debye entropic force scenario. We finish our paper with conclusions which appear in section V.

2 Einstein equations from Debye entropic gravity

Following Verlinde's scenario, gravity may have a statistical thermodynamics origin. Thus, any modification of statistical mechanics should modify the laws of gravity accordingly. In this section we use the modified equipartition law of energy to obtain the modified Einstein equations.

We consider a system that its boundary is not infinitely extended and forms a closed surface. We can take the boundary as a storage device for information, i.e. a holographic screen. Assuming that the holographic principle holds, the maximal storage space, or total number of bits N, is proportional to the area A,

$$N = \frac{A}{G\hbar}. \tag{2.1}$$

Suppose there is a total energy E present in the system. Let us now just make the simple assumption that the energy is divided evenly over the bits. Each bit on the holographic screen has one dimensional degree of freedom, hence we can use the one dimensional equipartition law of energy. The equipartition law of energy which is valid in all range of temperatures is

$$E = \frac{1}{2}NTD(x), \tag{2.2}$$

where T is the temperature of the screen and $D(x)$ is the one dimensional Debye function defined as

$$D(x) \equiv \frac{1}{x} \int_0^x \frac{y}{e^y - 1} dy, \tag{2.3}$$

and x is related to the temperature

$$x \equiv \frac{T_D}{T}, \tag{2.4}$$

where T_D is the Debye temperature. Using the equivalence between mass and energy, $E = M$, as well as eq. (2.1), we can rewrite eq. (2.2) in a more general form,

$$M = \frac{1}{2G\hbar} \oint_S TD(x)dA, \tag{2.5}$$

where the integration is over the holographic screen. For temperature, we use the Unruh temperature formula on the holographic screen,

$$T = \frac{\hbar a}{2\pi}, \tag{2.6}$$

where a denotes the acceleration. The acceleration has relation with the Newton's potential and in general relativity it may be written as

$$a^b = -\nabla^b \phi, \tag{2.7}$$

where ϕ is the natural generalization of Newton's potential in general relativity and for it we have [12],

$$\phi = \frac{1}{2} Ln(-\xi^a \xi_a), \tag{2.8}$$

where ξ^a is a global time like Killing vector. The exponent e^ϕ represents the redshift factor that relates the local time coordinate to that at a reference point with $\phi = 0$, which we will take to be at infinity. We choose the holographic screen S as a closed equipotential surface or in other words, a closed surface of constant redshit ϕ. Therefore eq. (2.6) may be written as [3]

$$T = \frac{\hbar}{2\pi} e^\phi N^a \nabla_a \phi, \tag{2.9}$$

where N^a is the unit outward pointing vector that is normal to the equipotential holographic screen S and time like Killing vector ξ^b. We inserted a redshift factor e^ϕ, because the temperature T is measured with respect to the reference point at infinity. Because N^a is normal to the equipotential holographic screen, for it we have

$$N^a = \frac{\nabla^a \phi}{(\nabla^b \phi \nabla_b \phi)^{1/2}}. \tag{2.10}$$

Therefore we can rewrite eq. (2.9) as

$$T = \frac{\hbar}{2\pi} e^\phi (\nabla^a \phi \nabla_a \phi)^{1/2}. \tag{2.11}$$

Substituting eq. (2.9) in eq. (2.5), we get

$$M = \frac{1}{4\pi G} \oint_S e^\phi N^a \nabla_a \phi D(x)dA. \tag{2.12}$$

Following the same logic of [12], we can obtain

$$M = -\frac{1}{8\pi G} \oint_S \nabla^a \xi^b D(x)dS_{ab}, \tag{2.13}$$

where dS_{ab} is the two-surface element [13]. On the other hand, according to the Stokes theorem, we have [13]

$$\oint_S B^{ab} dS_{ab} = 2 \int_\Sigma \nabla_b B^{ab} d\Sigma_a, \tag{2.14}$$

where B^{ab} is an antisymmetric tensor field and S is the two dimensional boundary of the hypersurface Σ. $d\Sigma_a$ is a directed surface element on Σ and for it we have

$$d\Sigma_a = \varepsilon n_a d\Sigma, \tag{2.15}$$

where n^a is the unit normal of the hypersurface Σ and ε is equal to -1 or 1 if the hypersurface is spacelike or timelike, respectively. Now we apply the Stokes theorem (2.14) for eq. (2.13) and get

$$
\begin{aligned}
M &= -\frac{1}{4\pi G} \int_\Sigma \nabla_b [\nabla^a \xi^b D(x)] d\Sigma_a \\
&= -\frac{1}{4\pi G} \int_\Sigma [D(x)\nabla_b \nabla^a \xi^b + \nabla^a \xi^b \nabla_b D(x)] d\Sigma_a \\
&= -\frac{1}{4\pi G} \int_\Sigma [-D(x)\nabla_b \nabla^b \xi^a + \nabla^a \xi^b \nabla_b D(x)] d\Sigma_a,
\end{aligned}
\tag{2.16}
$$

where in the last step we have used the Killing equation,

$$\nabla^a \xi^b + \nabla^b \xi^a = 0. \tag{2.17}$$

Now we use the relation [12]

$$\nabla^a \nabla_a \xi^b = -R_a^b \xi^a, \tag{2.18}$$

which is implied by the Killing equation for ξ^a, and get

$$
\begin{aligned}
M &= -\frac{1}{4\pi G} \int_\Sigma [R_{ab}\xi^b D(x) + \nabla_a \xi^c \nabla_c D(x)] d\Sigma^a \\
&= -\frac{1}{4\pi G} \int_\Sigma [R_{ab}\xi^b D(x) + e^{-2\phi}(-\xi^b \xi_b)\nabla_a \xi^c \nabla_c D(x)] d\Sigma^a \\
&= \frac{1}{4\pi G} \int_\Sigma [R_{ab}D(x) - e^{-2\phi}\xi_b \nabla_a \xi^c \nabla_c D(x)] n^a \xi^b d\Sigma,
\end{aligned}
\tag{2.19}
$$

where in the second line we have used eq. (2.8). In the last line we have used $d\Sigma^a = -n^a d\Sigma$, because the hypersurface Σ is spacelike.

On the other hand, M can be expressed as an integral over the enclosed volume of certain components of stress energy tensor $\mathcal{T}_{ab}$ [12],

$$M = 2 \int \left(\mathcal{T}_{ab} - \frac{1}{2}\mathcal{T} g_{ab} \right) n^a \xi^b d\Sigma. \tag{2.20}$$

Equating eqs. (2.19) and (2.20), we find

$$D(x)R_{ab} - e^{-2\phi}\xi_b \nabla_a \xi^c \nabla_c D(x) = 8\pi G \left(\mathcal{T}_{ab} - \frac{1}{2}\mathcal{T} g_{ab} \right). \tag{2.21}$$

The above equation is the modified Einstein equations resulting from considering the Debye correction to the equipartition law of energy in the framework of entropic gravity scenario.

This equation is now valid for all range of temperature, since we have assumed the general equipartition law of energy. Therefore, we see that in Verlinde's approach, any modification of first principles such as equipartition law of energy will modify the gravitational field equations. The question whether the modified term in Einstein equation can be detectable practically or not needs more investigations in the future. One needs to first specify the Debye function $D(x)$ and then try to solve the field equations (2.21). The resulting solutions should be checked with experiments or observations. It is clear that the correction term only plays role in very low temperature, in which the curvature of spacetime tends to zero and it becomes flat.

It is instructive to examine the modified Einstein equations in the high temperatures limit. According to the Unruh temperature formula we have

$$g = \frac{2\pi}{\hbar}T, \tag{2.22}$$

where g is the norm of the gravitational acceleration. Therefore, the strength of the gravitational field is proportional to the temperature. Also, we can define the Debye acceleration relating to the Debye temperature as

$$g_D = \frac{2\pi}{\hbar}T_D. \tag{2.23}$$

Therefore, if the temperature is larger than the Debye temperature, i.e. $T > T_D$, then the norm of the gravitational acceleration is larger than the Debye acceleration, i.e. $g > g_D$. In other words, the limit of high temperatures compared to the Debye temperature, is corresponding to the strong gravitational fields. In this case we have $T \gg T_D$, thus for x and y in the definition of the Debye function (2.3), we have $x \ll 1$ and consequently $y \ll 1$. Therefore we can use the approximation $e^y \approx 1 + y$ in the integral of eq. (2.3) and as a result, the one dimensional Debye function reduces to

$$D(x) \approx \frac{1}{x}\int_0^x dy = 1. \tag{2.24}$$

Substituting this result ($D(x) = 1$) in the modified Einstein equations (2.21), leads to

$$R_{ab} = 8\pi G \left(\mathcal{T}_{ab} - \frac{1}{2}\mathcal{T}g_{ab} \right). \tag{2.25}$$

Therefore, in the temperatures extremely larger than the Debye temperature (very strong gravitational fields), one obtains the standard Einstein field equations as expected.

3 MOND theory from Debye entropic gravity

Modified Newtonian dynamics (MOND) was proposed to explain the flat rotational curves of spiral galaxies. A great variety of observations indicate that the rotational velocity curves of all spiral galaxies tend to some constant value [14]. Among them are the Oort discrepancy in the disk of Milky Way [15], the velocity dispersions of dwarf Spheroidal galaxies [16] and the flat rotation curves of spiral galaxies [17]. These observations are in contradiction with the prediction of Newtonian theory because Newtonian theory predicts that objects that are far from the galaxy center have lower velocities.

The most widely adopted way to resolve these difficulties is the dark matter hypothesis. It is assumed that all visible stars are surrounded by massive nonluminous matters. Another approach is the MOND theory which was suggested by M. Milgrom in 1983 [18]. This theory appears to be highly successful for explaining the observed anomalous rotational-velocity. In fact, the MOND theory is (empirical) modification of Newtonian dynamics through modification in the kinematical acceleration term 'a' (which is normally taken as $a = v^2/r$) as effective kinematic acceleration $a_{\text{eff}} = a\mu(\frac{a}{a_0})$,

$$a\mu\left(\frac{a}{a_0}\right) = \frac{GM}{R^2}, \tag{3.1}$$

where $\mu = 1$ for usual-values of accelerations and $\mu = \frac{a}{a_0}(\ll 1)$ if the acceleration 'a' is extremely low, lower than a critical value $a_0 = 10^{-10} \ m/s^2$. At large distance, at the galaxy out skirt, the kinematical acceleration 'a' is extremely small, smaller than $10^{-10} \ m/s^2$, i.e., $a \ll a_0$, hence the function $\mu(\frac{a}{a_0}) = \frac{a}{a_0}$. Consequently, the velocity of star on circular orbit from the galaxy-center is constant and does not depend on the distance; the rotational-curve is flat, as it observed.

Although MOND theory can explain the flat rotational curve, however its theoretical origin remains un-known. Thus, it is well motivated to establish a gravitational theory which can results MOND theory naturally. In this section, we are able to show that the MOND theory can be extracted completely from the Debye entropic gravity. This derivation further support the viability of Debye entropic gravity formalism.

Again, we consider a spherical holographic screen with radius R as the boundary of the system. Combining eqs. (3) and (4), and using the equivalence between mass and energy as well as relation $A = 4\pi R^2$, we obtain

$$\frac{2\pi}{\hbar} T D(x) = \frac{GM}{R^2}. \tag{3.2}$$

Using the Unruh temperature formula (2.6), the above equation may be written as

$$a D(x) = \frac{GM}{R^2}. \tag{3.3}$$

Also, if we use the Unruh temperature formula in the definition of x, i.e. eq. (2.4), and define a_0 as

$$a_0 \equiv \frac{12 T_D}{\pi \hbar}, \tag{3.4}$$

then we obtain

$$x = \frac{\pi^2 a_0}{6a}. \tag{3.5}$$

Using the above result in eq. (3.3) gives

$$a D\left(\frac{\pi^2 a_0}{6a}\right) = \frac{GM}{R^2}. \tag{3.6}$$

This is the MOND theory resulting from Debye entropic gravity. If we compare this equation with well-known eq. (3.1), we see that we can define μ function as

$$\mu\left(\frac{a}{a_0}\right) \equiv D\left(\frac{\pi^2 a_0}{6a}\right). \tag{3.7}$$

In what follows we show that this function satisfies the conditions similar to those of μ function in eq. (3.1). Let us examine eq. (3.6) in two limits of temperatures. First, we consider the limit corresponding to the temperatures large relative to the Debye temperature. In this case $x \ll 1$ ($a \gg a_0$) we have $D(x) = 1$. Thus eq. (3.6) reduces to

$$a = \frac{GM}{R^2}.$$ (3.8)

Therefore, for strong gravitational fields, eq. (3.6) turns into the standard Newtonian dynamics. As we discussed, for $a \gg a_0$ we have also $\mu(\frac{a}{a_0}) = 1$. We conclude that in the limit of $a \gg a_0$ both $D(x)$ and $\mu(x)$ have the same behavior and become equal to 1.

The second limit corresponds to the temperatures extremely smaller than the Debye temperature, $T \ll T_D$, that is to say in the weak gravitational fields. In this limit, we have $x \gg 1$ ($a \ll a_0$), and the Debye function can be expanded as

$$D(x) = \frac{1}{x} \int_0^\infty \frac{y}{e^y - 1} dy \approx \frac{\pi^2}{6x}.$$ (3.9)

If we use the approximation (3.9) in eq. (3.6), we obtain

$$a \left(\frac{a}{a_0} \right) = \frac{GM}{R^2}.$$ (3.10)

Therefore, the Newtonian dynamics is modified for weak gravitational fields, e.g. at large distance from the galaxy center, namely at the galaxy out skirt. Thus the origin of the MOND theory can be understood completely in the framework of Debye entropic gravity. In this way we fill in the gap existing in the literature understanding the theoretical origin of MOND theory.

4 Poisson equation from Debye entropic force

Finally, we obtain the modified Poisson equation by taking into account the Debye correction to the equipartition law of energy. We choose a holographic screen S corresponding to an equipotential surface with fixed Newtonian potential ϕ_0. We assume that the entire mass distribution given by $\rho(\vec{x})$ is contained inside the volume enclosed by the screen and there are some test particles outside this volume. To identify the temperature of the holographic screen, we take a test particle and move it close to the screen and measure its local acceleration. The local acceleration is related to the Newton potential as

$$\vec{a} = -\vec{\nabla}\phi.$$ (4.1)

Substituting this relation into Unruh temperature formula, we get

$$T = \frac{\hbar|\vec{\nabla}\phi|}{2\pi}.$$ (4.2)

Using the above equation in the definition of x, we have

$$x \equiv \frac{T_D}{T} = \frac{2\pi T_D}{\hbar|\vec{\nabla}\phi|}.$$ (4.3)

Inserting (4.2) in eq. (2.5), after using eq. (2.1) for the number of bits on the holographic screen, we obtain

$$M = \frac{1}{4\pi G} \oint_S D(x)\vec{\nabla}\phi.d\vec{A}. \tag{4.4}$$

Using the divergence theorem we can rewrite eq. (4.4) as

$$M = \frac{1}{4\pi G} \int_V \vec{\nabla}.[D(x)\vec{\nabla}\phi]dV. \tag{4.5}$$

On the other hand, for the mass distribution M inside the closed surface S, we have the relation

$$M = \int_V \rho(\vec{x})dV. \tag{4.6}$$

Equating eqs. (4.5) and (4.6), we get

$$\vec{\nabla}.[D(x)\vec{\nabla}\phi] = 4\pi G\rho(\vec{x}). \tag{4.7}$$

This is the modified Poisson equation which is valid in all range of temperatures. For high temperatures. i.e. strong gravitational field ($x \ll 1$) and hence $D(x) = 1$. In this case eq. (4.7) reduces to the standard Poisson equation,

$$\nabla^2\phi = 4\pi G\rho(\vec{x}). \tag{4.8}$$

Thus, considering the gravitational system as a thermodynamical system and taking into account the Debye model for the modified equipartition law of energy, we see that not only Einstein equation and MOND theory but also the Poisson equation is modified accordingly. Clearly the modification of Poisson equation leads to modified Newton's law of gravitation.

5 Conclusions

In his work, Verlinde applied the equipartition law of energy as $E = \frac{1}{2}NT$ on the holographic screen induced by the mass distribution of the system, and obtained the Einstein equations, Newton's law of gravitation and the Poisson equation. But we know from statistical mechanics that the equipartition law of energy does not hold at very low temperatures and it should be corrected. In this paper, we considered the Debye correction to the equipartition law of energy as $E = \frac{1}{2}NTD(x)$, where $D(x)$ is the Debye function. Following Verlinde's strategy on the entropic origin of gravity, we obtained the modified form of the Einstein equations, MOND theory and the modified Poisson equation. Interestingly enough, we found that the origin of MOND theory can be understood from the Debye entropic gravity scenario. Since the MOND theory is an acceptable theory for explanation of the galaxy flat rotation curves, thus the studies on its theoretical origin is of great importance. This result is impressive and show that the approach here is powerful enough for deriving the modified gravitational field equations from Debye model. We also showed that in the temperatures extremely larger than the Debye temperature (very strong gravitational fields), the obtained modified equations turn into their respective well-known standard equations. The results obtained here further support the viability of Verlinde's formalism.

Acknowledgments

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