Exclusive Nonleptonic Decays

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Abstract:

Constraints on quark diagrams for exclusive decays from group theory are investigated. In contrast to previous statements, we find a one-to-one correspondence between reduced matrix elements of flavor symmetry (SU_3) and the "naive" quark diagrams of the spectator and annihilation type. In Cabibbo suppressed decays however, group theory relates penguin and annihilation (W-exchange) diagrams, consequently ruling out models with spectator dominance and large penguins. These results are expected to be more general, having as yet been illustrated for two-body decays of charmed mesons.

Motivation: An open problem in particle physics is the calculation of nonleptonic decay rates from the weak Hamiltonian

$$H_W^{\Delta c} = \frac{G}{\sqrt{2}} \begin{cases} V_{ud}\, V_{cs}^{*}(\bar{u}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu c_L) + h.c. & \text{(Cabibbo allowed)} \\ +V_{us}\, V_{cs}^{*}(\bar{u}_L\gamma_\mu s_L)(\bar{s}_L\gamma_\mu c_L) + h.c. & \\ +V_{ud}\, V_{cd}^{*}(\bar{u}_L\gamma_\mu d_L)(\bar{d}_L\gamma_\mu c_L) + h.c. & \text{(Cabibbo suppressed)} \end{cases} \quad (1)$$

and QCD. The practical use of the latter seems for the moment to be limited to the asymptotic region of a large scale involved, being not the case in K and hyperon decays and most probably also not in charm decays.

What can we do?

Remember the good old rather successful phenomenological approaches, under names like "$\Delta I = 1/2$ rule", "spurion", "Lee-Sugawara" and "octet-enhancement"[1]. They were all based on group theory using flavor symmetry (SU_2 or SU_3) and the dominance of some irreducible tensor operator in the weak Hamiltonian, many of its predictions being beautifully borne out by experiment. This motivated many people[2] to generalize this idea to "20-plet enhancement" in H_W for charm decays, because in terms of SU_4 tensors: $H_W = O_{\alpha\beta}O^{+}_{\sigma 4}((\underline{20})^{\alpha 4}_{\sigma\beta} + (\underline{84})^{\alpha 4}_{\sigma\beta}) + h.c.$, where the matrices $O_{\alpha\beta}$ etc. in front contain the Cabibbo like couplings V_{ud} etc. (see refs. 2). If now transition rates, e.g. of the form ($|c\bar{q}_\xi\rangle$ = charmed meson; $\xi = 1,2,3$; $P_\rho^{\rho'} = SU_3$ octet, e.g. $\pi^\pm$ etc.)

$$\langle P_\rho^{\rho'} P_\phi^{\phi'} | H_W | c\bar{q}_\xi \rangle = \sum_i C_i \langle || O_i || \rangle \quad (2)$$

are decomposed by a straightforward procedure into reduced matrix elements ("RME") $\langle || O_i || \rangle$ and Clebsch-Gordan coefficients C_i, the assumption that the $\underline{20}$ tensor in H_W dominates, will yield certain predictions on exclusive branching fractions.

This group theoretical procedure has a great advantage: Even if the multiplet enhancement were not exact (as the $\Delta I = 1/2$ rule is not; note also that perturbative QCD predicts[3] only $C_{84}/C_{20} = 0.3$) this decomposition is the most general one and independent of any details of strong dynamics, besides flavor symmetry. It tells us how many parameters we need in order to parametrize a decay.

But this leads us to the main question: Why do certain tensors dominate and why do certain RME's have certain values (which cannot be calculated from the flavor symmetry). Therefore we are confronted with the problem of calculating RME's from strong interaction dynamics. Somebody might object: Why consider RME's at all, calculate transition rates!

So what one usually does, is to draw all possible diagrams for valence quarks (assuming at most only them to participate in the weak interaction) to flow from the initial to the final state. However, the number of such diagrams is infinite, because of strong interaction corrections. Nevertheless, if we assume that we could sum up all these corrections, we would end up with a certain class of diagrams which do not show all the details of strong interactions, indicating only the flavor flow of the valence quarks (or the "flavor topology"[4]). In the PP-case (charmed mes. → two pseudoscalars) there are four such generic diagrams (called "spectator" and "annihilation") whereas in the VP-case (e.g. $D^0 \to \bar{K}^0 \rho^0$) there are

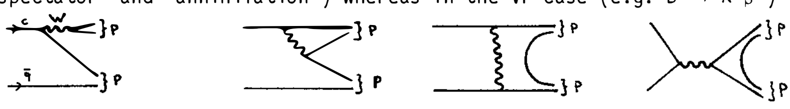


Fig. 1

s_1 s_2 a a'

eight of them, etc., using flavor SU_3 (its breaking could be considered by taking more diagrams). Now the decomposition of all octet-octet amplitudes is a simple exercise, in terms of the above generic graphs, given below on the l.h.s. in eqs. 3. With either of the assumptions: (a) spectator dominance (b) annihilation dom., one can fit one or the other exp. result or get predictions. As to be learnt from the VP-case, no universal simple picture seems to exist for the moment however[5].

Now recalling group theory we ask ourselves: Are there some different predictions or are some of the diagrams constrained? The answer of the usual procedure, to be extracted from the literature[2)5)], in terms of RME's, is written on the r.h.s. in eqs. 3:

$$\begin{aligned}
\langle K^+\pi^-|D^0\rangle &= \cos^2\theta_c(s_1+a) = \cos^2\theta_c\,(\langle 8,20\rangle + \tfrac{4}{5}\langle 27,84\rangle + \tfrac{1}{5}\langle 8,84\rangle)\\
\langle \pi^+\bar{K}^0|D^+\rangle &= \cos^2\theta_c(s_1+s_2) = \cos^2\theta_c\,\langle 27,84\rangle \qquad (3)\\
\langle K^0K^+|F^+\rangle &= \cos^2\theta_c(s_2+a') = \cos^2\theta_c(-\langle 8,20\rangle + \tfrac{4}{5}\langle 27,84\rangle + \tfrac{1}{5}\langle 8,84\rangle)
\end{aligned}$$

etc. (simularly for the VP case, see ref. 5)

But this is very strange and appears to be a big problem! Four diagrams but only three RME's (eight and seven resp. in the VP-case); is therefore one of the diagrams redundant? The answer of a more careful investigation [5] is: No! This is because one RME, called <0,20>, drops out in the decay amplitudes. In order to understand this, look at the group theoretical decomposition (eq. 2) in more detail, where there appear several lines of simular structure: RME's with expressions in Kronecker deltas in front.

$$\begin{aligned}
\langle P^{\rho'}_{\rho} P^{\phi'}_{\phi}|H_W|c\bar{q}_{\xi}\rangle = O_{\alpha\beta}\, O^{+}_{\sigma 4}\, [\ldots &+ (\delta^{\alpha\phi',\sigma\rho'}_{\beta\phi,\xi\rho} + \ldots\ \delta^{\sigma}_{\xi}\delta^{\phi'}_{\rho}\,\delta^{\alpha\rho'}_{\beta\phi} + \ldots)\cdot\langle||RME||\rangle\\
&+ \underbrace{(\delta^{\alpha\phi',\sigma\rho'}_{\beta\phi,\xi\rho} + \ldots - \delta^{\sigma}_{\xi}\delta^{\phi'}_{\rho}\,\delta^{\alpha\rho}_{\beta\phi} + \ldots)}_{=\,0}\cdot\langle||0,20||\rangle\ ]. \qquad (4)
\end{aligned}$$

In the last line of eq. 4, the delta expression in brackets vanishes identically for all α, β, ... = 1,2,3, therefore <||0,20||> not appearing in eqs. 3. But now it is important to observe that these Kronecker deltas are of different kinds, indicating exactly the flavor flow mentioned above in the context of diagrams:

$$\delta^{\alpha\phi'}_{\beta\phi},\ \delta^{\sigma\rho'}_{\xi\rho} \text{ etc.} \rightarrow \text{"spectator"}, \quad \delta^{\sigma}_{\xi}\delta^{\phi'}_{\rho}\delta^{\alpha\rho'}_{\beta\phi} \text{ etc.} \rightarrow \text{"annihilation"}$$

Consequently these diagrams are linear sums of RME's, and spectator or annihilation graphs, if considered seperately, do depend on <0,20>, which need not be zero by itself (this would lead to contradictions discussed in ref. 5).

$$s_1 = \langle 0,20\rangle + \langle 27,84\rangle, \quad s_2 = -\langle 0,20\rangle + \langle 27,84\rangle$$

$$a = -\langle 0,20\rangle + \langle 8,20\rangle - \frac{1}{5}\langle 27,84\rangle + \frac{1}{5}\langle 8,84\rangle$$

$$a' = \langle 0,20\rangle - \langle 8,20\rangle - \frac{1}{5}\langle 27,84\rangle + \frac{1}{5}\langle 8,84\rangle$$

(Similarly in the VP-case).

It is also interesting to invert these relations,

$$\langle 0,20\rangle = \frac{1}{2}(s_1 - s_2) \qquad \langle 27,84\rangle = \frac{1}{2}(s_1 + s_2)$$
$$\langle 8,20\rangle = \frac{1}{2}(s_1 - s_2 + a - a') \qquad \langle 8,84\rangle = \frac{1}{2}(s_1 + s_2 + 5(a + a')) \qquad (6)$$

because we now see, for instance, how we could calculate RME's from diagrams,fig.1

1. Result: The "naive" decomposition of decay amplitudes in terms of generic graphs is completely equivalent to the group theoretical decomposition in terms of RME's (Wigner-Eckhardt theorem) for Cabibbo allowed transitions.

Since either approach uses the valence content of particles only, this was to be expected somehow, though not obvious at first sight.

The usefulness of the diagrammatical language shows up also in decays of the type: → octet - singlet (e.g. $D^0 \rightarrow \phi\bar{K}^0$) or → singlet - singlet (e.g. $\eta'\eta'$,$\eta'\phi$ etc.), because in principle there is a new amplitude, not related to the octet-octet one. In group theory usually new assumptions are made (refs. 2) in order to relate them, and again the question arises for their reason. One can find a group theoretical decomposition of these singlet amplitudes[6], again to be interpreted as a sum of simple quark diagrams. The new graphs involved are of the type

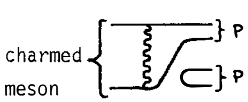


Fig. 2

where a $q\bar{q}$ pair, out of the vacuum, produces a final state. Making an assumption on the importance of these new graphs, e.g. putting them to zero (OZI rule), we obtain many relations to the octet-octet decays, because they depend on the same graphs then (Fig. 1). For reasons of space we have to dismiss further discussion here.

A real constraint from group theory follows however in the Cabibbo suppressed decays for the so - called "penguin" diagrams[7].

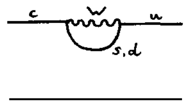
Ps,d = 

Fig. 3

which, in charm decays, enter proportional to the factor in mixing angles (e.g. Kobayashi-Maskawa) $\Delta \sim V_{ud}V^{*}_{cd}+V_{us}V^{+}_{cs}$, whereas most of the decays go via the larger parameter $\Sigma \sim V_{ud}V^{*}_{cd} - V_{us}V^{*}_{cs}$. Note that in four quark GIM: $\Sigma \sim \cos\theta_c \cdot \sin\theta_c$ and $\Delta = 0$ and in Kob.-Mask. $|\Delta/\Sigma| < 1/15$[8].

Looking in the group theoretical tensor decomposition we find that[7]

$$Ps,d = a \tag{7}$$

where "a" is the above annihilation (W-exchange) diagram. This surprising result comes from some more general property of SU(N) irreducible representations, having to be either symmetric or antisymmetric in the interchange of two particles. By construction,the tensor decomposition of the decay amplitudes is always symmetric under the interchange of all the particles. This simple property implies the penguin - annihilation equality (7). From the weak Hamiltonian H_W one can see also that both of the graphs indeed lead to similar form factors. It is very interesting to note that this only works, considering QCD for instance, if annihilation graphs have a gluon in the initial state.

One correction has to be made. The $p_{s,d}$ graph does not contain the b-quark loop: p_b. For exact flavor symmetry ($m_b = m_{s,d}$) the GIM mechanism tells us: $p^{tot} = p_{s,d} - p_b = o$ or $p_{s,d} = p_b$. By breaking effects $p_b = p_{s,d} \cdot f(m_b, m_{s,d})$, with $f = 1$ for $m_b = m_{s,d}$.
Therefore

$$p^{tot} = (1 - f) \cdot a \tag{8}$$

our final result, expected to hold for other nonleptonic decays similarly, f could be specified from QCD and probably one will find $|p^{tot}| < |a|$.
Eq. (8) also says, that the previous group theoretical treatments[2], having implicitly the uncorrected equality p = a, eq. (7), are incorrect, due to the necessary modification by the b-loop. Needless to say however, that SU(3) breaking effects are probably even larger. Because of the above mentioned general property of SU(N) representations, eqs. (7) and (8) are expected to hold similarly in K and hyperon decays, b decays etc. They imply that one cannot neglect annihilation graphs (spectator model) and at the same time assume large penguin contributions. Such a picture is inconsistend with the penguin-annihilation relations (7) or (8), which probably can also be derived directly from H_W and QCD[7]. This should be investigated in more detail.

Conclusions:

It turned out to be worth comparing group theory with the diagrammatical analysis of exclusive nonleptonic decays, because

i) nonleptonic dynamics can presumably be easier understood in terms of diagrams, which show up to make sense also for exclusive channels, since their use is equivalent to the application of group theoretical methods

ii) penguin graphs are related to W-exchange graphs (inconsistency of spectator dominance + large penguins).

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