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Role of particle masses in the magnetic field generation driven by the parity violating interaction



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ABSTRACT

Recently the new model for the generation of strong large scale magnetic fields in neutron stars, driven by the parity violating interaction, was proposed. In this model, the magnetic field instability results from the modification of the chiral magnetic effect in presence of the electroweak interaction between ultrarelativistic electrons and nucleons. In the present work we study how a nonzero mass of charged particles, which are degenerate relativistic electrons and nonrelativistic protons, influences the generation of the magnetic field in frames of this approach. For this purpose we calculate the induced electric current of these charged particles, electroweakly interacting with background neutrons and an external magnetic field, exactly accounting for the particle mass. This current is calculated by two methods: using the exact solution of the Dirac equation for a charged particle in external fields and computing the polarization operator of a photon in matter composed of background neutrons. We show that the induced current is vanishing in both approaches leading to the zero contribution of massive particles to the generated magnetic field. We discuss the implication of our results for the problem of the magnetic field generation in compact stars.

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The origin of extremely strong magnetic fields $B \gtrsim 10^{15}$ G in some neutron stars, called magnetars, is a puzzle for modern physics and astrophysics. Some models accounting for the generation of such magnetic fields based on, e.g., the turbulent dynamo and strong fossil fields, are reviewed in Ref. [1]. However none of these models adequately describes all the observed characteristics of magnetars. Recently, several models for the explanation of magnetic fields in magnetars, involving elementary particle physics approaches, such as the chiral magnetic effect (CME) [2] and the parity violating electroweak interaction [3], were put forward in Refs. [4,5].

In Refs. [6–9] we proposed the model for the magnetic field generation in magnetars based on the instability of the magnetic field in matter of a neutron star (NS) composed of electrons and neutrons interacting by the electroweak forces. We could predict the growth of the seed magnetic field $B_0 \sim 10^{12}$ G, typical for a pulsar, to values expected in magnetars during the time intervals comparable with magnetars ages. As shown in Refs. [8,9], the mag-

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netic field growth can be powered by the energy of the thermal motion of background fermions in the NS matter.

In Refs. [6–9] we accounted for the interaction between ultrarelativistic electrons and nonrelativistic neutrons, which are both highly degenerate, inside NS. It should be mentioned that the fact that electrons are ultrarelativistic allowed us to neglect the electron mass and approximately consider the separate evolution of right and left chiral components of the electron–positron field. Such an approximation was also used in Refs. [10,11] where the generation of toroidal magnetic fields in NSs was discussed.

Despite an electron in NS is ultrarelativistic, it has a nonzero mass. Any nonzero electron mass will diminish the manifestation of CME. The helicity flip rate Γ_f of relativistic electrons in NS matter was recently computed in Refs. [9,13]. The computed $\Gamma_f \sim m_e^2$, where m_e is the electron mass, mixes the chiral projections of ultrarelativistic electrons reducing the initial chiral imbalance. Moreover, as mentioned in Ref. [12], any nonzero mass of charged particle can make vanishing the induced anomalous electric current resulting in CME.

The main feature of the model in Refs. [6–9] is the existence of a nonzero electric current along the magnetic field direction: $J = \Pi B$. Such a current is effective, i.e. it exists only in matter.

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As shown in Ref. [14], the Maxwell equations, modified by adding this current, have an unstable solution leading to the exponential growth of a seed magnetic field. In the present Letter we will carefully analyze the role the mass of charged particles on the generation of the induced anomalous current in the presence of the parity violating electroweak interaction. It should be mentioned that, besides ultrarelativistic electrons, the NS matter should also contain the same amount of degenerate and nonrelativistic protons for the whole NS to be electrically neutral. The contribution of these protons to the electric current should be also analyzed.

In this Letter we shall study the generation of an electric current of charged particles (electrons or protons) electroweakly interacting with background neutrons under the influence of an external magnetic field. We shall account for the particle mass exactly. For instance, as was mentioned above, unlike electrons, protons are nonrelativistic in NS. To compute the current we shall use two methods: the exact solution of the Dirac equation in external fields [6,7,12,15] and the calculation of the antisymmetric contribution to the photon polarization operator in matter [4,14]. In both cases we will show that the induced current along the magnetic field is vanishing provided a nonzero particle mass is accounted for. Then we discuss the implication of the obtained result for the generation of magnetic fields in NS.

We start with the brief discussion of the electroweak interaction between a charged particle, a proton or an electron, and neutrons. This interaction is accounted for in the elastic forward scattering approximation based on the Fermi model. We consider the background matter of NS composed mainly of neutrons, which are taken to be unpolarized and nonmoving macroscopically. Accounting for both neutral and charged currents contributions [16], we derive the effective Lagrangian for the interaction of a test charged particle, described by the bispinor ψ , with this nuclear matter,

$$\mathcal{L}_{\text{int}} = -\bar{\psi}\gamma^0 \left(V_{\text{L}}P_{\text{L}} + V_{\text{R}}P_{\text{R}} \right)\psi,\tag{1}$$

where $P_{L,R} = (1 \mp \gamma^5)/2$ are the chiral projectors, $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$, and $\gamma^{\mu} = (\gamma^0, \gamma)$ are the Dirac matrices. The effective potentials $V_{L,R}$ in Eq. (1) are

$$V_{\rm L} = \sqrt{2}G_{\rm F}n_n \left(\frac{1}{2} - \sin^2\theta_{\rm W}\right), \quad V_{\rm R} = -\sqrt{2}G_{\rm F}n_n \sin^2\theta_{\rm W}, \qquad (2)$$

for electrons and

$$V_{\rm L} = \sqrt{2}G_{\rm F}n_n \left(2|V_{ud}|^2 + \sin^2\theta_{\rm W} - \frac{1}{2}\right), \quad V_{\rm R} = \sqrt{2}G_{\rm F}n_n \sin^2\theta_{\rm W},$$
(3)

for protons. In Eqs. (2) and (3), $G_{\rm F} = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant, n_n is the neutron density, $V_{ud} \approx 0.97$ is the element of the Cabibbo–Kobayashi–Maskawa matrix, and $\sin^2 \theta_{\rm W} \approx 0.23$ is the Weinberg parameter.

Now we compute the induced electric current with help of the exact solution of the Dirac equation in external fields. The Dirac equation for a charged particle, accounting for the electroweak interaction with nuclear matter in Eqs. (1)–(3) under the influence of the external magnetic field $\mathbf{B} = (0, 0, B)$, has the form,

$$\left[\gamma^{\mu}\left(\mathrm{i}\partial_{\mu}-eA_{\mu}\right)-m-\gamma^{0}\left(V_{\mathrm{L}}P_{\mathrm{L}}+V_{\mathrm{R}}P_{\mathrm{R}}\right)\right]\psi=0,\tag{4}$$

where $A^{\mu} = (0, 0, Bx, 0)$ is the four vector potential in the Landau gauge, *e* is the electric charge (*e* < 0 for an electron and *e* > 0 for a proton), and *m* is the particle mass.

We start solving Eq. (4) for positively charged particles with e > 0, i.e. for protons. We separate the variables in Eq. (4) in the usual way: $\psi = \exp(-iEt + ip_y y + ip_z z)\psi_x$, where $\psi_x = \psi_x(x)$ is

the bispinor depending on x coordinate only. It is convenient to choose the Dirac matrices in the chiral representation [17]

$$\gamma^{0} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \quad \gamma^{5} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (5)$$

where σ are the Pauli matrices. The bispinor ψ_x can be also represented using the two component chiral projections as $\psi_x^T = (\xi, \eta)$. On the basis of Eqs. (4) and (5), one gets the equations for ξ and η ,

$$\begin{pmatrix} P_0 + V_5 - p_z & i\sqrt{eBD_-} \\ i\sqrt{eBD_+} & P_0 + V_5 + p_z \end{pmatrix} \xi = -m\eta,$$

$$\begin{pmatrix} P_0 - V_5 + p_z & -i\sqrt{eBD_-} \\ -i\sqrt{eBD_+} & P_0 - V_5 - p_z \end{pmatrix} \eta = -m\xi,$$
(6)

where $P_0 = E - \bar{V}$, $\bar{V} = (V_L + V_R)/2$, $V_5 = (V_L - V_R)/2$, $D_{\pm} = \partial_{\chi} \pm \chi$, and $\chi = \sqrt{eBx} - p_y/\sqrt{eB}$. Assuming that $\psi_x \to 0$ at $|x| \to \infty$, we shall look for the solution of Eq. (6) in the form,

$$\xi = \begin{pmatrix} C_1 u_n \\ -iC_2 u_{n-1} \end{pmatrix}, \quad \eta = \begin{pmatrix} C_3 u_n \\ -iC_4 u_{n-1} \end{pmatrix}, \tag{7}$$

where $u_n = u_n(\chi)$ is the Hermite function, $n = 0, 1, 2, ..., and C_i$, i = 1, ..., 4, are the spin coefficients. The explicit form of u_n can be found, e.g., in Ref. [6].

Using the following properties of u_n : $D_+u_n = \sqrt{2nu_{n-1}}$ and $D_-u_{n-1} = -\sqrt{2nu_n}$, as well as Eq. (7) we get the relations between C_i ,

$$mC_{1,3} + (P_0 \mp V_5 \pm p_z) C_{3,1} \pm \sqrt{2eBnC_{4,2}} = 0,$$

$$mC_{2,4} + (P_0 \mp V_5 \mp p_z) C_{4,2} \pm \sqrt{2eBnC_{3,1}} = 0.$$
(8)

We shall normalize the total proton wave function by the condition

$$\int d^{3}x \psi_{n,p_{y},p_{z}}^{\dagger} \psi_{n',p'_{y},p'_{z}} = \delta_{nn'} \delta\left(p_{y} - p'_{y}\right) \delta\left(p_{z} - p'_{z}\right).$$
(9)

Therefore, C_i obey the relation

$$\sum_{i=1}^{4} |C_i|^2 = \frac{1}{(2\pi)^2}.$$
(10)

The energy levels can be obtained from Eq. (8) in the form,

$$\left(E - \bar{V}\right)^2 = (E_0 + sV_5)^2 + m^2, \tag{11}$$

where $E_0 = \sqrt{2eBn + p_z^2}$ is the energy of a massless charged particle in the constant uniform magnetic field and $s = \pm 1$. The energy levels in Eq. (11) coincide with those found in Ref. [15], where an electron interacting with neutrons and an external magnetic field was considered. The symmetric gauge for the vector potential **A** was used in Ref. [15].

First, let us study the case n > 0. The explicit form of C_i can be found if we use the following expressions:

$$C_{3,4} (P_0 - V_5 - sE_0) + mC_{1,2} = 0,$$

$$C_{2,4} (sE_0 - p_z) + \sqrt{2eBn}C_{1,3} = 0,$$
(12)

which result from Eq. (8). Note that Eq. (12) is a consequence of the existence of the additional spin integral of Eq. (4) found in Ref. [15]. Using Eqs. (10)–(12), we get C_i as

$$|C_1|^2 = \frac{s}{(2\pi)^2 4E_0 P_0} \left(sE_0 - p_z \right) \left(P_0 - V_5 - sE_0 \right),$$

$$|C_2|^2 = \frac{s}{(2\pi)^2 4E_0 P_0} \frac{2eBn\left(P_0 - V_5 - sE_0 \right)}{(sE_0 - p_z)},$$

$$|C_{3}|^{2} = \frac{s}{(2\pi)^{2} 4 E_{0} P_{0}} \frac{m^{2} (s E_{0} - p_{z})}{(P_{0} - V_{5} - s E_{0})},$$

$$|C_{4}|^{2} = \frac{s}{(2\pi)^{2} 4 E_{0} P_{0}} \frac{m^{2} 2e B n}{(s E_{0} - p_{z}) (P_{0} - V_{5} - s E_{0})}.$$
 (13)

If n = 0, we can use Eq. (8) directly. In this case we get that s = -1, $C_2 = C_4 = 0$, and

$$|C_1|^2 = \frac{1}{(2\pi)^2} \frac{(P_0 - V_5 + p_z)^2}{(P_0 - V_5 + p_z)^2 + m^2},$$

$$|C_3|^2 = \frac{1}{(2\pi)^2} \frac{m^2}{(P_0 - V_5 + p_z)^2 + m^2}.$$
(14)

The energy of the lowest Landau level can be found from (E - E) $(\bar{V})^2 = m^2 + (p_z - V_5)^2.$

Now, when we have the proton wave function in the explicit form, we are ready to compute the averaged electric current of these particles along the magnetic field. It has the form,

$$J_z = e \sum_{n=0}^{\infty} \sum_{s} \int_{-\infty}^{+\infty} dp_y dp_z \psi^{\dagger} \gamma^0 \gamma^3 \psi f \left(E - \mu \right), \qquad (15)$$

where $f(E) = [\exp(E/T) + 1]^{-1}$ is the Fermi-Dirac distribution function, *T* is the temperature, and μ is the chemical potential. Using Eqs. (5) and (7), we obtain the quantum mechanical average

$$j_{z}^{(n)} = e \int_{-\infty}^{+\infty} dp_{y} \psi^{\dagger} \gamma^{0} \gamma^{3} \psi$$

= $e^{2} B \left(|C_{2}|^{2} + |C_{3}|^{2} - |C_{1}|^{2} - |C_{4}|^{2} \right).$ (16)

On the basis of Eqs. (13) and (14) we get that

$$j_z^{(n>0)} = -\frac{e^2 B}{(2\pi)^2} \frac{s p_z (V_5 + s E_0)}{E_0 P_0},$$
(17)

for n > 0, and

$$j_z^{(0)} = -\frac{e^2 B}{(2\pi)^2} \frac{p_z - V_5}{P_0},$$
(18)

for n = 0. Using Eqs. (11), (17) and (18), one obtains that after the statistical averaging,

$$J_{z} = \sum_{n=0}^{\infty} \sum_{s} \langle j_{z}^{(n)} \rangle = 0, \quad \langle j_{z}^{(n)} \rangle = \int_{-\infty}^{+\infty} dp_{z} j_{z}^{(n)} f(E-\mu) = 0,$$
(19)

for any n.

The fact that $\langle j_z^{(n>0)} \rangle = 0$ is obvious. Indeed one can see in Eq. (11) that, at n > 0, both P_0 and E_0 are even in p_z making $j_z^{(n>0)}$ in Eq. (17) odd in p_z . Thus the integration over p_z in Eq. (19) gives $\langle j_z^{(n>0)} \rangle = 0$. To demonstrate that $\langle j_z^{(0)} \rangle = 0$ we recall that, at n = 0, $E = \overline{V} + \sqrt{m^2 + (p_z - V_5)^2}$ for particles. Then, changing the integration variable $p_z \rightarrow p'_z = p_z - V_5$, one obtains that $j_z^{(0)}$ in Eq. (18) is odd in p'_z . Integrating over p'_z in Eq. (19) from $-\infty$ to $+\infty$ (see below), one gets that $\langle j_z^{(0)} \rangle = 0$. Restoring vector notations in Eq. (19), we obtain that the electric current along magnetic field is vanishing: $\mathbf{J} = \Pi \mathbf{B} = 0$. Analogously one can show the absence of the contribution of massive antiparticles to this electric current.

One can demonstrate that the induced anomalous current of electrons along the magnetic field is also vanishing. For this purpose one should either find the electron wave function in the

Landau gauge analogously to Eqs. (6)–(14), and then compute the current as in Eqs. (15)-(19); or use the wave function of an electron, interacting with background neutrons under the influence of the magnetic field, found in Ref. [15] in the symmetric gauge. We shall omit these computations for brevity.

The reason for the disappearance of the electric current for massive particles is the following. It is well known that, in case of massless particles, the nonzero electric current along the external magnetic field is due to the polarization effects of charged particles at zero Landau level [12]. It is actually the manifestation of CME [2]. The momentum of massless particles is correlated with the particle spin. The particle spin, in its turn, is correlated with the magnetic field direction at n = 0. Therefore, for massless charged particles at zero Landau level, the particle momentum will have a certain direction with respect to the magnetic field, i.e. p_7 will vary either from 0 to $+\infty$ or from $-\infty$ to 0 depending on the particle charge [6,7]. Therefore, if we consider the analogue of Eq. (19) for massless particles, the integration over p_z will give a nonzero result. On the contrary, for massive particles, p_z is no longer correlated with the magnetic field, changing from $-\infty$ to $+\infty$. It happens even at n = 0 and makes J_z to vanish.

We can also demonstrate the cancellation of the induced current along the magnetic field direction in case of massive particles using the results of the one loop calculation of the polarization operator $\Pi_{\mu\nu}$ in Ref. [14]. A nonzero antisymmetric part $\Pi_{ij} = i\varepsilon_{ijn}k^n \Pi$ of the polarization operator can induce the current along the magnetic field: $J^i = -\prod_{ij} A^j = \prod B_i$ or $\mathbf{J} = \prod \mathbf{B}$. Performing analogous one loop computation of the polarization operator of a photon in a medium composed of electrons, protons and neutrons as in Ref. [14], one gets the new form factor Π in the limit $k^2 \ll m^2$ as

$$\Pi = -\frac{7}{3}e^{2}V_{5}\int \frac{\mathrm{d}^{3}p}{(2\pi)^{3}}\frac{1}{\mathcal{E}_{\mathbf{p}}^{3}}$$

$$\times \left\{\frac{m^{2}}{\mathcal{E}_{\mathbf{p}}^{2}}\left[\frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1} + \frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]\right\}$$

$$+\frac{m^{2}\beta}{2\mathcal{E}_{\mathbf{p}}}\left[\frac{1}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1} + \frac{1}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]$$

$$-\frac{\beta^{2}\mathbf{p}^{2}}{6}\left[\frac{\tanh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)/2]}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1} + \frac{\tanh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)/2]}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]\right\},$$
(20)

where $\mathcal{E}_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, $\beta = 1/T$ is the reciprocal temperature, and k^{μ} is the photon momentum. In Eq. (20) we consider neutrons as background fermions.

In Refs. [6-9], we are mainly interested in the generation of magnetic field in NS, when it is in a thermal equilibrium, which is reached after $\sim 10^2$ yr after the supernova collapse. At this stage of the NS evolution, charged particles, i.e. electrons and protons, are highly degenerate. Hence we should consider the limit $\mu/T \gg 1$ in Eq. (20). Taking into account the identities,

$$\lim_{\beta \to \infty} \frac{\beta}{\cosh(\beta x) + 1} = 2\delta(x), \quad \lim_{\beta \to \infty} \frac{\beta^2 \tanh(\beta x/2)}{\cosh(\beta x) + 1} = -2\delta'(x),$$
(21)

which were derived in Ref. [14], and recalling that for degenerate fermions one has $\mu_{e,p} = \sqrt{(3\pi^2 n_{e,p})^{2/3} + m^2} > 0$, where $n_{e,p}$ are the densities of electrons and protons, one obtains that $\Pi = 0$ in Eq. (20). Thus we again get that there is no electric current of massive charged particles along the magnetic field.

Note that, the main reason to get $\Pi = 0$ in Eq. (20) is to consider $k^{\mu} = 0$ in the computation of the polarization tensor. It corresponds to the zero momentum of a photon/plasmon in NS matter or the static external magnetic field. It is worth to mention that, in contrast to the present work, in Ref. [14], we assumed that $k^2 = \omega_p^2 > 0$, where ω_p is the plasma frequency, i.e. we considered the propagation of an electromagnetic wave there.

In conclusion we mention that we have shown that the induced electric current along the external magnetic field of massive charged particles, electroweakly interacting with background neutrons, is vanishing. We have studied one particular implementation of this problem: the parity violating electroweak interaction in the Fermi approximation; cf. Eqs. (1)–(3). We have demonstrated the current cancellation using two methods: the exact solution of the Dirac equation in external fields and the analysis of the photon polarization operator. The former approach is beyond the perturbation theory whereas, in the later method, one demonstrates the washing out of the current linear in $G_{\rm F}$ and the fine structure constant $\alpha_{\rm em} = e^2/4\pi$.

Note that, for the first time, the cancellation of the induced current of electroweakly interacting massive particles was mentioned in Ref. [12], whereas for massless particles a nonzero current may well exist [6,7,12]. However, this observation was made in Ref. [12] on the basis of the perturbative computation of the one loop contribution to the photon polarization tensor. The novelty of the present work compared to the result of Ref. [12], is that we have demonstrated the disappearance of the current using the exact solution of the Dirac equation in all orders in $G_{\rm F}$ and $\alpha_{\rm em}$, i.e. nonperturbatively.

Such an unusual dependence of the induced current on the charged particle masses is related to the breaking of the chiral symmetry for massive particles. Massive and massless particles belong to different phases in which the chiral symmetry is broken and restored. The restoration of the chiral symmetry can take place in the presence of background matter having high temperature and/or density. The size of "bubbles", containing matter in the symmetric phase, will depend smoothly on the temperature T and/or the density ρ of background matter. The nonzero anomalous current $\mathbf{J} = \Pi \mathbf{B}$, which results in the magnetic field instability, will exist only in "bubbles" with restored chiral symmetry. Therefore, if one studies the generation of a magnetic field driven by CME in a realistic cosmological/astrophysical media accounting for the chiral phase transition, the scale and the strength of this magnetic field will be smooth functions of T and/or ρ .

It should be noted that, at the absence of the electroweak interaction, the disappearance of CME [12], i.e. the cancellation of the induced current $\mathbf{J} = 2(\alpha_{em}/\pi)\mu_5 \mathbf{B} = 0$ for massive particles can be foreseen. Here $\mu_5 = (\mu_R - \mu_L)/2$ and $\mu_{R,L}$ are the chemical potentials of right and left particles. Indeed, if $m \neq 0$, the decomposition to the left and right chiral projections is impossible and we should set $\mu_5 = 0$ since for massive particles there should be only one chemical potential $\mu = \mu_{\rm R} = \mu_{\rm L}$. However, if the electroweak interaction with background fermions is present, the induced anomalous current for massless particles was found in Refs. [6,7] in the form $\mathbf{J} = 2(\alpha_{\rm em}/\pi)(\mu_5 + V_5)\mathbf{B}$. The washing out of this current for massive particles is not obvious since $V_{\rm L} \neq 0$ and $V_{\rm R} \neq 0$ in Eqs. (1)–(3), giving one $V_5 \neq 0$ for both massless and massive particles. Thus the demonstration that CME is vanishing for massive particles in the presence of the electroweak interaction requires a special analysis which, in fact, was carried out in the present Letter.

The results of our work are equally applied for the currents of massive electrons and protons. As mentioned above, despite electrons are ultrarelativistic in NS they possess nonzero masses. Therefore, basing on our results, the generation of magnetic fields in magnetars driven by the electron–nucleon interaction, proposed in Refs. [6–9], is questionable unless there is a mechanism restoring the chiral symmetry for electrons in NS. As found in Ref. [18], the electroweak phase transition in dense matter can happen if the matter density exceeds $n_{\rm cr} \sim M_{\rm W}^3 \approx 6.6 \times 10^{46}$ cm⁻³, where $M_{\rm W} \approx 80$ GeV is the W-boson mass. This value is far beyond the density in NS. The same disappointing arguments are valid with respect to the findings of Refs. [5,10].

Nevertheless we can still use the approach of Refs. [6-9] for the generation of magnetic fields in compact stars considering the quark-quark electroweak interaction [16]. The chiral symmetry was shown in Ref. [19] to be restored for the lightest *u* and *d* quarks for a specific equation of state of nuclear matter in a hybrid star, i.e. in NS having quark matter core, or in a hypothetical quark star [20]. Moreover, as shown in Ref. [21], the effective masses of baryons can be significantly reduced if QCD radiative corrections are taken into account. It is the indication to the fact that the chiral symmetry can be restored in dense matter. The results of Refs. [6–9] can be straightforwardly applied to describe the magnetic field generation in a quark/hybrid star. The consideration of the details of the magnetic field generation driven by the electroweak quark-quark interaction will be done in our forthcoming work.

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