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Precise Calculation of the Lightest CP-Even Higgs Boson Mass in the MSSM with FlexibleSUSY

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Kurzdarstellung

Seit der Entdeckung des Higgs-Bosons dient dessen Masse als eine wesentliche Ausschlussbedingung für physikalische Theorien jenseits des Standardmodells welche die Vorhersage dieser Masse erlauben. Für präzise Berechnungen dieser Observable wurde eine Methode FlexibleEFTHiggs innerhalb des Programms FlexibleSUSY entwickelt, die es ermöglicht einen Ansatz der effektiven Feldtheorie mit einer diagrammatischen Rechnung zu vereinen. Diese Methode wird in dieser Arbeit erweitert um konsistent Beiträge aus höheren Ordnungen in der Störungstheorie einzubeziehen um somit eine Verbesserung in der Vorhersage der Pol-Masse des leichtesten CP-geraden Higgs-Bosons zu erreichen. Für dieses Ziel wird die Äquivalenz zur Bestimmung der Kopplung λ im Standardmodell auf Zweischleifenordnung zwischen der angewendeten Vorgehensweise und anderer Ansätze, die in diversen Programmen Anwendung finden, analytisch bewiesen. Weiterhin wird die Vorhersage der Higgs-Boson Pol-Masse mit Ergebnissen anderer Spektrumgeneratoren ausführlich verglichen. Abschließend wird eine Fehlerabschätzung vorgenommen und mit den Ergebnissen der ursprünglichen Version von FlexibleEFTHiggs verglichen.

Abstract

Since the discovery of the Higgs boson its mass became a crucial constraint for physical theories beyond the Standard Model, which predict a value for it. For precise calculations of this observable the method FlexibleEFTHiggs was developed in the program FlexibleSUSY, which unites the effective field theory approach with a diagrammatic calculation. In this thesis the method FlexibleEFTHiggs is extended to incorporate consistently higher order contributions in perturbation theory and to improve the prediction for the pole mass of the lightest CP-even Higgs boson in the MSSM. For this purpose, the applied matching condition for the Standard Model coupling λ at the two-loop order is analytically proven to be equivalent to other choices of the matching conditions implemented in various public codes. A detailed comparison of the Higgs boson mass prediction is performed with the results of other mass spectrum generators. An estimation for the uncertainty of the extended version of FlexibleEFTHiggs is performed and results are compared with the former version of FlexibleEFTHiggs.

Contents

1 Introduction

The Standard Model (SM) of particle physics provides a profoundly confirmed description of all observed elementary particles and their fundamental interactions, except for gravity. A substantial building block of the SM is the gauge principle: If a physical system is symmetric with respect to a spacetime independent group of continuous transformations, the symmetry remains if the transformations are considered to be spacetime dependent. This symmetry-based argument gives the SM its outstanding beauty. The enormous success has been accentuated by the discovery of the predicted Higgs boson with a mass of $M_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\,\text{GeV}$ [1,2] in 2012. From a theoretical point of view this discovery, however, manifests simultaneously a loophole in the SM – the so-called hierarchy problem. This sort of fine-tuning problem is not a difficulty of the SM itself. Instead embedded together with gravity, the observation of a scalar field at $\sim 125\,\text{GeV}$ in the presence of a much larger scale would cause questions regarding naturalness if no other mechanism is involved. This fact is expressed in the large radiative corrections the mass of the scalar field acquires. Thus, it is desirable to find a way to cancel large contributions and allow the observation of the light scalar field. Hence, the SM has to be enlarged in order to resolve its problems and to draw a coherent picture of all elementary processes in nature.

Motivated by extending the symmetry of spacetime, supersymmetry (SUSY) provides at present the most elaborated guide to new physics. Yielding a promising step towards to theory of everything, SUSY completes the SM in many ways under the assumption of a fundamental relation between fermions and bosons. In particular the Minimal Supersymmetric Standard Model (MSSM) has the potential to resolve the puzzles the SM left us with. However, since SUSY predicts the same masses for the observed particles and their superpartners, it would have to be realized in a broken fashion.

The attractiveness of the MSSM and SUSY in general emerges from the conceptual elegance it contains. This is expressed rather in the principles this theory is based on than in the extensive calculations in perturbation theory. An example of this fact is obtained by the Higgs boson mass parameter m_h. In contrast to the SM, this is not an additional parameter in supersymmetric models but a prediction, which depends on couplings of the SUSY theory. This circumstance allows to calculate the pole mass of the Higgs boson in dependence of other model parameters. Or stated otherwise, the comparison of the theory prediction and the experimentally observed value enables restriction of model parameters beyond the SM. Especially, this includes the mass spectrum of the superpartners, which is affected by a breaking mechanism of SUSY.

Indeed, much effort has been invested since more than 20 years in order to perform calculations of the pole mass of the lightest CP-even Higgs boson in the MSSM. Nevertheless, the theoretical estimation of the uncertainty is with several GeV much larger than the experimental uncertainty. In particular the large mass gap between the Higgs boson mass and the potentially large SUSY scale affects the prediction by fixed-order mass spectrum generators strongly, since higher order logarithms are missing. This work is dedicated to improve the correct resummation of these large logarithms in the framework of the mass spectrum generator generator FlexibleSUSY.

This thesis will start off with an introduction of the theoretical background. It covers the SM in chapter 2 , the concept of SUSY in chapter 3 and especially the MSSM in sec. 3.2. After the introduction of FlexibleSUSY in sec. 4.1 the implemented method is present in sec. 4.2 with which the Higgs boson mass calculation is performed . In chapter 5 the elaborated method is presented in detail. This includes also the comparison to other approaches. Furthermore, the performed matching condition is explained by analytical formulas and the equivalence to other approaches is proven. After the theoretical part, the numerical effects for the SM coupling λ are presented and discussed in chapter 6. Chapter 7 provides the obtained results for the lightest CP-even Higgs boson pole mass and compares them with the ones from spectrum generators. Finally, in chapter 8 the theoretical uncertainties are discussed and illustrated.

2 Standard Model of Particle Physics

In order to describe all known elementary particles and their interactions the relativistic quantum field theory (QFT) the Standard Model of particle physics (SM) was developed and accomplished in the 1970s by Steven Weinberg, Abdus Salam and Lee Glashow [3]. Through the second half of the 20th century its predictions were tested and verified to high precision in observables like particle masses and cross sections [4]. The tremendous success of this theory continued with the discovery of the predicted Higgs boson at the LHC in 2012 [1]. Despite its weaknesses [5] , the mentioned arguments emphasize the validity of the SM as a suitable low energy approximation of the fundamental theory of nature. Two main advantages of the SM are on the one hand the unification of the electroweak (EW) theory and on the other hand the inclusion of Quantumchromodynamics (QCD), the theory of the strong interaction. These sectors of the SM are based on the principle of symmetries, which will be explained in this chapter.

2.1 Symmetries

The structure of the SM is given by the symmetries it is based on. Due to Noether's theorem every continuous symmetry of the action[1] S is associated with a conserved current $j^\mu(x) = (j^0(x), \vec{j}(x))$, $\mu, \in \{1,2,3,4\}$, which satisfies the continuity equation,

$$\partial_\mu j^\mu(x) = 0. \tag{2.1}$$

Therefore, the conserved current implies, by integrating the equation (2.1) the existence of a conserved quantity Q of this symmetry, also known as Noether charge [2],

$$Q := \int_V j^0(x) dV, \tag{2.2}$$

$$\frac{\partial}{\partial t} Q = \frac{\partial}{\partial t} \int_V \nabla \cdot \vec{j}\, dV = 0. \tag{2.3}$$

Subsequently, the conservation of e.g. energy and charges in the SM can be formulated as a consequence of symmetry transformation acting on the SM Lagrangian.[3] The symmetry of the SM is determined by the global Poincaré symmetry and internal symmetries.

[1] The action is defined by the spacetime integral over the Lagrange density $S = \int \mathcal{L}\, d^4x$.

[2] By using Gauß law, the space integral in (2.3) translates into a surface integral at infinity, where the current components $\vec{j}(x)$ vanish.

[3] Due to the Weinberg-Witten theorem every non-Abelian gauge theory with charged massless spin 1 particles cannot admit a gauge invariant conserved current [6]. However, considering a current, coming from the charged vector potential e.g. gluon field in $SU(3)_C$, a conserved quantity can be constructed.

2.1.1 Poincaré Symmetry

The global Poincaré symmetry is the full symmetry of special theory of relativity and a (special) relativistic QFT has to contain it. Accordingly, the SM action S_{SM} is required to be Poincaré invariant. This invariance leads i.a. to the conservation of the four momentum. Within the language of Lie groups, the Poincaré group is a semi-direct product of Lorentz group $O(3,1)$ and translations group $\mathbb{R}^{3,1}$ on the affine Minkowski space $\mathbb{M}^4$. Hence, the building blocks of the Poincaré group are the generators $J^{\mu\nu}$ and P^{μ} which correspond to Lie groups $O(3,1)$ and $\mathbb{R}^{3,1}$. Those satisfy the commutation relations:

$$[P_\mu, P_\nu] = 0, \tag{2.4}$$

$$[P_\mu, J_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho), \tag{2.5}$$

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(g_{\nu\rho}J_{\mu\sigma} - g_{\mu\rho}J_{\nu\sigma} + g_{\mu\sigma}J_{\nu\rho} - g_{\nu\sigma}J_{\mu\rho}). \tag{2.6}$$

Beside translations the Poincaré algebra describe generalized rotations on the Minkowski space generated by $J_{\mu\nu}$, i.e. for $i,j \in \{1,2,3\}$. The J_{ij} build up spatial rotations and J_{0i} are associated with boosts.

2.1.2 Gauge Symmetry

The SM describes gauge fields and their interaction among themselves, with fermion- and scalar fields. These interaction terms in the SM Lagrange density $\mathcal{L}_{\mathrm{SM}}$ are restricted by the local symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. For the $\mathcal{L}_{\mathrm{SM}}$, this means in particular all terms are omitted that are not invariant under the symmetry transformation at every event in spacetime.

To illustrate the formalism one consider a massless fermion $SU(N)$ multiplet Ψ of N Dirac spinors $\Psi = (\psi_1, ..., \psi_N)^T$ and a gauge field

$$G_\mu \equiv G^a_\mu T^a, \tag{2.7}$$

with $a \in \{0, .., N^2 - 1\}$. The $(N \times N)$ hermitian matrices T_a gather the $N^2 - 1$ generators of the $SU(N)$ symmetry group and satisfy the Lie Algebra

$$[T^a, T^b] = i f^{abc} T^c, \tag{2.8}$$

where f^{abc} denote the structure constants. If $f^{abc} = 0$, the Lie group is Abelian e.g. $U(1)$ and non-Abelian in the case of non-vanishing structure constants e.g. $SU(N)$, $N \geq 2$. The infinitesimal gauge transformation $U(x) \in SU(N)$ acts then on the fermion multiplet and the gauge field,

$$U(x) = e^{ig\theta(x)_a T_a}, \tag{2.9}$$

$$\Psi \to U(x)\Psi, \tag{2.10}$$

$$G^\mu_a \to G^\mu_a + \frac{1}{g}\partial_\mu \theta^a(x) - f^{abc}\theta^b(x) G^c_\mu. \tag{2.11}$$

Furthermore, we introduced in eq. (2.9) the dimensionless gauge coupling g. Since the group element $U(x)$ depends on functions θ^a, which map from the Minkowski spacetime into the real numbers

$$\theta^a : \mathbb{M}^4 \to \mathbb{R}, \tag{2.12}$$

the parameter $\theta^a(x)$ clarify the spatial dependence of a group element. From this specific transformation one obtains that the kinetic term $\bar{\Psi}\gamma^\mu\partial_\mu\Psi$ preserves global gauge symmetry but breaks the local gauge symmetry.[4] The replacement of the partial derivative ∂_μ with the covariant derivative D_μ conserves the gauge symmetry at the local level

$$D_\mu := \partial_\mu - igT^a G^a_\mu. \tag{2.13}$$

Indeed, the fermion fields obeys the following $SU(N)$ invariant Lagrangian

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu D_\mu \Psi \tag{2.14}$$

$$= i \sum_{m,n=1}^{N} \bar{\psi}_n(\delta_{nm}\gamma^\mu\partial_\mu - ig\gamma^\mu G^a_\mu T^a_{nm})\psi_m, \tag{2.15}$$

where the bar notion about the Dirac spinor is defined as the adjoint spinor

$$\bar{\psi} \equiv \psi^\dagger\gamma^0. \tag{2.16}$$

Moreover, this Lagrangian also introduces the interaction between the fermion and the gauge field. A physical interpretation of the gauge boson G^μ requires also dynamic terms in the Lagrangian.

For the kinetic part of gauge fields one introduces the field strength tensor $G_{\mu\nu}$ and its components $G^a_{\mu\nu}$

$$G_{\mu\nu} := \frac{i}{g}[D_\mu, D_\nu] \equiv G^a_{\mu\nu}T^a, \tag{2.17}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu. \tag{2.18}$$

The transformation of $G^a_{\mu\nu}$ under gauge group $SU(N)$ is given by

$$G^a_{\mu\nu} \to G^a_{\mu\nu} - f^{abc}\theta(x)^b G^c_{\mu\nu}. \tag{2.19}$$

Therefore, a quadratic term in $G^a_{\mu\nu}$ gives rise to the kinetic part and ensures the gauge symmetry of this kinetic term. In addition, this dynamic term covers also interactions between the non-Abelian G^a_μ -fields. On the whole, one can now obtain the $SU(N)$ invariant Lagrangian including the fermion multiplet and gauge bosons as

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + i\bar{\Psi}\gamma^\mu D_\mu\Psi. \tag{2.20}$$

[4] If one consider a constant map θ in (2.2).

gauge field	coupling	Lie group	structure constants	generator
G_μ^a	g_s	$SU(3)_C$	f^{abc}	$\frac{\lambda^a}{2}$
W_μ^i	g_w	$SU(2)_L$	d^{ijk}	$\frac{\sigma^i}{2}$
B_μ	g_y	$U(1)_Y$	0	$\frac{\hat{Y}}{2}$

Table 2.1: This table provides all gauge bosons in the SM before symmetry breaking, where the coupling refers to the interaction either among themselves or to other fields. In the matrix representation, the $SU(3)_C$ group is generated by the eight Gell-Mann-Low matrices λ^a $(a \in \{1, ..., 8\})$ the $SU(2)_L$ by the Pauli matrices σ^i $(i \in \{1, 2, 3\})$ and $U(1)_Y$ by the weak hypercharge operator $\hat{Y}$.

2.2 Gauge Bosons

It has already been mentioned that the gauge symmetry of the SM is composed of three Lie groups, $SU(3)_C, SU(2)_L, U(1)_Y$, where the indices are referencing to the associated charges: C color, L weak isospin and Y weak hypercharge. In summary, the kinetic part of all gauge bosons reads

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{2.21}$$

where the field strength tensors have the similar form as in (2.18) and are constructed with the boson fields from the table 2.1

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\mu, \tag{2.22}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_w d^{ijk} W^j_\mu W^k_\mu, \tag{2.23}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.24}$$

Likewise, in eq. (2.14), the fermion Lagrangian of the SM provides the fermion-gauge boson interaction within the covariant derivative according to the SM gauge group

$$D_\mu = \partial_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu - i g_w \frac{\sigma^i}{2} W^i_\mu - i g_y \frac{\hat{Y}}{2} B_\mu. \tag{2.25}$$

The proper quantization of gauge theories requires a replacement of the local gauge invariants with the more general BRST symmetry. In the SM, this method introduces three arbitrary gauge fixing parameters (ξ_s, ξ_w, ξ_y) for each gauge group [5]

$$\mathcal{L}_{\text{Fixing}} = -\frac{1}{2\xi_s}(\partial^\mu G^a_\mu)^2 - \frac{1}{2\xi_w}(\partial^\mu W^i_\mu)^2 - \frac{1}{2\xi_y}(\partial^\mu B_\mu)^2. \tag{2.26}$$

Although the introduced fixing parameters are in principle arbitrary, the requirement is satisfied, that every predictive observable within this BRST- invariant theory is independent

[5] After symmetry breaking, new mass eigenstates for the $SU(2)_L$ and $U(1)_Y$ vector bosons occur. This leads into a kinetic mixing term of a Goldstone boson field with a gauge boson. However, in the R^ξ-gauge $\mathcal{L}_{\text{Fixing}}$ has a different form and ensures a diagonal propagator for massive vector bosons.

of them. In the non-Abelian case one has to include new unphysical fields c_s^a, c_w^i, the Fadeev-Poppov ghosts

$$\mathcal{L}_{\text{Ghost}} = -\bar{c}_s^a \partial_\mu \partial^\mu c_s^a + g_s f^{abc} (\partial^\mu c_s^a) G_\mu^a c_s^a \tag{2.27}$$

$$- \bar{c}_w^i \partial_\mu \partial^\mu c_w^i + g_w d^{ijk} (\partial^\mu c_w^i) W_\mu^i c_w^i, \tag{2.28}$$

where $\bar{c}_s^a$ and $\bar{c}_w^i$ denote the anti-ghosts with respect to $SU(3)_C$ and $SU(2)_L$.[6] For a detailed explanation the interested reader is invited to explore the topic in ref. [7].

2.3 Fermions

A suitable way to summarize the fermions of the SM is to classify them according to the value of their Noether charges. Table 2.2 lists all spin $s = 1/2$ (matter) fields of the SM and subdivide them in three generations. However, fermions in this chapter, are introduced as Dirac spinors and are therefore composed of a left-handed(lh) and a right-handed(rh) part. Since the $SU(2)_L$ gauge group interacts only with lh particles one needs to distinguish between the lh and the rh part of a generic Dirac spinor Ψ. The lh and rh part of Ψ are defined as the eigenstate of the chirality operator γ^5 with the correspondent eigenvalues $+1$ and (-1) respectively. Hence we define the projection as follows

$$\Psi_{L/R} := \frac{1}{2} \, (\mathbb{1} \mp \gamma^5) \Psi, \tag{2.29}$$

where L/R denote the lh/rh part of an arbitrary Dirac spinor.

If the spinor of a SM fermion couples to a gauge boson, it transforms non-trivially under gauge symmetries, as shown in (2.10), with the correspondent generator out of table 2.1. The fermions with no color charge are called leptons and are therefore not participating the strong interaction. Whereas quarks are strong interacting fermions.

As listed in table 2.3, all lh fermions can be assembled in $SU(2)_L$ doublets L_L^j and Q_L^j, where j labels the generation. The lh fields within a $SU(2)_L$ doublet are then characterized by their eigenvalue $I_3 = \pm\frac{1}{2}$ of the operator $\sigma^3/2$, a matrix representation of the weak isospin. On the contrary rh fermions e_R^j, u_R^j and d_R^j are not participating on the interactions with $SU(2)_L$ gauge bosons. This is characterized by a weak isospin $I_3 = 0$. Especially rh leptons transform therefore non-trivial only under the $U(1)_Y$ gauge group, i.e. the gauge transformation of a group element $g \in SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ on this fields is: $g \cdot e_R = \mathbb{1} e^{i g_y \theta(x) \hat{Y}} e_R$. Due to empirical fact, that no rh neutrinos have been discovered so far, they are not included in the SM. Moreover, all fermions have a weak hypercharge and are considered as singlet representations of $U(1)_Y$.

Because strong interacting fields incorporate the so-called color charge r, b and g all quarks occur as $SU(3)_C$ triplets

$$q_i = \begin{pmatrix} q_i^r \\ q_i^b \\ q_i^g \end{pmatrix}. \tag{2.30}$$

Where the index i in eq. (2.30) denotes handedness of the up- or down-type quark multiplet,

[6] Although this ghosts are scalar fields, they obey fermionic commutation relations and therefore violating the Spin-Statistic theorem of QFT.

gen.	fermion		C (lh,rh)	I_3 (lh)	I_3 (rh)	Y (lh)	Y (rh)
1st	e^-	electron	0	$-\frac{1}{2}$	0	-1	-2
	ν_e	electron neutrino	0	$\frac{1}{2}$	none	-1	none
	u	up quark	(r,b,g)	$\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{4}{3}$
	d	down quark	(r,b,g)	$-\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{2}{3}$
2nd	μ^-	muon	0	$-\frac{1}{2}$	0	-1	-2
	ν_μ	muon neutrino	0	$\frac{1}{2}$	none	-1	none
	c	charm quark	(r,b,g)	$\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{4}{3}$
	s	strange quark	(r,b,g)	$-\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{2}{3}$
3rd	τ^-	tau	0	$-\frac{1}{2}$	0	-1	-2
	ν_τ	tau neutrino	0	$\frac{1}{2}$	none	-1	none
	t	top quark	(r,b,g)	$\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{4}{3}$
	b	bottom quark	(r,b,g)	$-\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{2}{3}$

Table 2.2: This table summarizes all particles in the SM with spin $s = 1/2$. The quantum numbers C, I_3 and Y denote the eigenvalue of the generator according to the SM gauge group and are called color, the third component of the weak isospin and the weak hypercharge, respectively. Except for the neutrinos, all fermions of the SM contain left- and right-handed (lh,rh) fields.

symbol	name	generation j: 1	2	3
L_L^j	lh neutrino lh charged lepton	$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$
e_R^j	rh charged lepton	e_R	μ_R	τ_R
Q_L^j	lh up-type quarks lh down-type quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$
u_R^j	rh up-type quarks	u_R	c_R	t_R
d_R^j	rh down-type quarks	d_R	s_R	b_R

Table 2.3: This table explains the notion behind the symbols used in eq. (2.40).

$q_i \in \{Q_L^j, u_R^j, d_R^j\}$. In summary, the fermion Lagrange density then reads, with the notation of table 2.3,

$$\begin{aligned}\mathcal{L}_{\text{Fermion}} =& i\bar{L}_L^j\gamma^\mu D_\mu L_L^j + i\bar{e}_R^j\gamma^\mu D_\mu e_R^j \\ &+ i\bar{Q}_L^j\gamma^\mu D_\mu Q_L^j + i\bar{u}_R^j\gamma^\mu D_\mu u_R^j + i\bar{d}_R^j\gamma^\mu D_\mu d_R^j.\end{aligned} \tag{2.31}$$

2.4 Electroweak Symmetry Breaking

Until now, no mass terms have been considered, for matter fields and for gauge bosons. Because of the doublet structure of the $SU(2)_L$, it is not possible to introduce fermionic mass terms as in the Dirac theory of quantum mechanics and maintain gauge invariance.[7] Including mass terms for gauge bosons, the Lagrangian would violate the gauge invariance, performed by the transformation in (2.10).[8] Nevertheless, there is a way for fermions and gauge bosons to become massive provided by mechanism of spontaneous symmetry breaking [8–11]. Therefore, one postulates a scalar $SU(2)_L$ doublet $\Phi = (\phi^+, \phi^0)^T$ together with a potential $V(\Phi)$. It is in general gauge invariant, whereas its ground state, the vacuum, breaks the $SU(2)_L \otimes U(1)_Y$ symmetry

$$\mathcal{L}_{\text{Higgs}} = (D^\mu\Phi)^\dagger(D_\mu\Phi) - V_{\text{Higgs}}(\Phi), \tag{2.32}$$

$$V_{\text{Higgs}}(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4. \tag{2.33}$$

If the mass parameter satisfy the condition $\mu^2 > 0$, a non-zero minimum of the potential $V(\Phi)$ occur. Therefore the field Φ receives a vacuum expectation value (VEV) $v = \sqrt{\frac{2\mu^2}{\lambda}} + \mathcal{O}(\hbar)$. Without the loss of generality, the expansion of field around the minimum of the potential reads

$$\Phi = \frac{1}{\sqrt{2}}e^{i\frac{\pi^a\sigma^a}{v}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}, \qquad \Phi_0 \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{2.34}$$

Since the Goldstone modes π^a are gauge dependent and therefore unphysical, they can be gauged to zero in the unitary gauge.[9] The Higgs boson h in eq. (2.34) is a massive real scalar mode, which is the only physical remnant of the Higgs mechanism. Inserting the expansion (2.34) in the Higgs Lagrangian, the covariant derivative in the kinetic part induces a quadratic term in the gauge bosons

$$(D^\mu\Phi_0)^\dagger(D_\mu\Phi_0) = \frac{v^2}{8}\left[g_w^2((W_\mu^1)^2 + (W_\mu^2)^2) + \left(g'B_\mu - g_w W_\mu^3\right)^2\right]. \tag{2.35}$$

The mass eigenstate of this mass matrix are three massive ones $Z_\mu, W_\mu^\pm$ and a massless one A^μ

$$\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}B_\mu\\ W_\mu^3\end{pmatrix}, \qquad W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \tag{2.36}$$

[7] $\mathcal{L}_{Dirac} = \bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi$ where m is the mass parameter.
[8] This means a quadratic term in the gauge fields: $W^\mu W_\mu$.
[9] I.e. $\xi \to \infty$

where the Weinberg angle is given as $\theta_W = \arctan \frac{g_y}{g_w}$. The existence of a massless gauge boson is justified by the remaining of a gauge symmetry after $SU(2)_L \otimes U(1)_Y$ symmetry breaking, the gauge symmetry of Quantumelectrodynamics (QED). It can be obtained by the invariance of the ground state Φ_0 under the gauge transformation generated by $\hat{Q}$

$$\hat{Q} := \frac{1}{2}(\sigma_3 + \hat{Y}), \tag{2.37}$$

$$\Phi_0 \to e^{i\theta(x)\hat{Q}}\Phi_0 = \begin{pmatrix} e^{i\theta(x)} & 0 \\ 0 & 1 \end{pmatrix} \Phi_0 = \Phi_0. \tag{2.38}$$

Hence, the remaining $U(1)_Q$ gauge symmetry of the broken theory, where the associated charge is connected to the 3rd component of the weak isospin I_3 and the weak hypercharge Y by the Gell-Mann-Nishijima relation

$$Q = I_3 + \frac{Y}{2}. \tag{2.39}$$

In the same manner, mass terms for fermions appear from an interaction between the Higgs field and the fermion spinors. Therefore, it is indispensable to introduce, within the SM, new kind of interaction in a gauge invariant way. The so-called Yukawa interaction provides new dimensionless undetermined coupling constants $y^{ij}_{l,u,d}$

$$\mathcal{L}_{\text{Yukawa}} = -y_l^{ij}\bar{L}_L^i\Phi e_R^j - y_u^{ij}\bar{Q}_L^i\Phi^c u_R^j - y_d^{ij}\bar{Q}_L^i\Phi d_R^j + h.c, \tag{2.40}$$

where $\Phi^c = i\sigma^2\Phi^*$ denotes the charge conjugated Higgs doublet. An expansion as in eq. (2.34) leads to mass matrices

$$m_l^{ij} = \frac{v}{\sqrt{2}}y_l^{ij}, \qquad m_u^{ij} = \frac{v}{\sqrt{2}}y_u^{ij}, \qquad m_d^{ij} = \frac{v}{\sqrt{2}}y_d^{ij}. \tag{2.41}$$

From (2.41), one can obtain that mass eigenstates and the interaction eigenstates of the EW theory are different. This originates from the non-diagonal $y^{ij}_{l,u,d}$ matrices and for quarks it is addressed by the V_{CKM} mixing matrix [12]. Unless no right handed neutrinos were included, it is viable to choose the y_l^{ij} diagonal. All contributions considered, the full Lagrange density of th SM is given by

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Ghost}} + \mathcal{L}_{\text{Fixing}}. \tag{2.42}$$

3 Supersymmetry

Although some predictions of the SM are in a astonishing accurate agreement with experimental data, it contains some weaknesses. For example it is deficient in describing the CP-violation that is required to give rise to the large imbalance between the amount of matter and antimatter within the observable universe. Moreover, it does not comprise gravity, even though for higher energies e.g. $M_{Pl} = 1/\sqrt{8\pi/G} = 10^{18}$GeV corrections from quantum gravity become relevant for particle physics [5]. Associated with new states at that high energies, the masses of scalar fields, e.g. the Higgs boson h, would acquire large quantum corrections. This argument apparently contradicts the discovery of the Higgs boson 15 orders of magnitude below the more natural scale M_{Pl}. Supersymmetry provides a promising solution for this puzzle and protects scalar fields against large quantum corrections. The main idea behind SUSY is to extend the Poincaré symmetry by fermionic generators and relate bosons and fermions with each other. The Haag-Loposzanski-Sohnius theorem states the uniqueness of this non-trivial extension. This chapter outlines the basic structure of SUSY and the language in which the MSSM is formulated.

3.1 General Concepts

For this purpose, this section starts off with an introduction in the general concepts of SUSY.

3.1.1 Super-Poincaré Algebra

In order to extend the Poincaré algebra by fermionic generators, one introduces them as a Majorana spinor $Q = (Q^\alpha, \bar{Q}_{\dot{\alpha}})^T$ which is composed of the Weyl spinors (Q^α) and $(\bar{Q}_{\dot{\alpha}})$, with $\alpha \in \{1, 2\}$. Similarly to (2.16) the bar notion on a generic Weyl spinor χ means

$$(\bar{\chi}^{\dot{\alpha}}) \equiv i\sigma^2(\chi^*_\alpha). \tag{3.1}$$

For more detailed explanations on this topic the reader is invited to look in the appendix A. These operators satisfy certain anti-commuting relations and the algebra of (2.4) is extended by

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, \tag{3.2}$$

$$[Q_\alpha, J^{\mu\nu}] = \frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta, \tag{3.3}$$

$$[\bar{Q}^{\dot{\alpha}}, J^{\mu\nu}] = \frac{1}{2}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Q}^{\dot{\beta}}, \tag{3.4}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = [Q_\alpha, P^\mu] = [\bar{Q}_{\dot{\alpha}}, P^\mu] = [P^\mu, P^\nu] = 0. \tag{3.5}$$

The relations (3.2)-(3.4) express the key point of the non-trivial extension. Since the Coleman-Mandula theorem shows that every extension of the Poincaré-group provided by bosonic generators T^i, which obey $[T^i, T^j] = f^{ijk}T^k$, would result in a trivial Lie product

$$[T^i, P]_{Lie} = 0 \tag{3.6}$$

for any Poincaré generator $P \in \{P_\mu, J_{\mu\nu}\}$. The graded Lie product $[.,.]_{Lie}$ denotes in general $\{.,.\}$ or $[.,.]$, but as P and T^i are considered to be bosonic generators the Lie product reduces to the usual commutator (3.6), see appendix B.2.

Furthermore, it is in general allowed to include more than one Majorana operator Q. In 4-dimensional flat spacetime, for example, the maximum number of $N = 4$ fermionic generators is determined by the argument of renormalizability of the theory. This is because for $N > 4$ fields with spin $> 3/2$ are present and interactions among them would require coupling constants with negative mass dimensions [13].[1] Henceforth, this thesis considers only $N = 1$ SUSY. Due to the fact that the operator Q is fermionic, it transforms bosons into fermions and vice versa

$$|\text{boson}\rangle \xleftrightarrow{\;Q\;} |\text{fermion}\rangle\,. \tag{3.7}$$

If two or more fields are related by the transformation in (3.7), they are then assembled in one supermultiplet. All fields within a supermultiplet share the same quantum numbers, provided by gauge symmetries, and are therefore labeled as superpartners.

3.1.2 Superspace Formalism

A manifest supersymmetric way to formulate SUSY is achieved within the superspace formalism. The superspace in $N = 1$ SUSY is in general an extension of the Minkowski space with four additional anticommuting spinorial coordinates θ_α, θ_α $\alpha, \dot{\alpha} \in \{1, 2\}$, see appendix B.1. The additional coordinates of two spinorial spaces can be therefore regarded as Weyl components. In this sense, θ_α and $\bar{\theta}^{\dot{\alpha}}$ are interpreted as left-handed and right-handed, respectively. This enlargement of the Minkowski space allows by design an equal treatment of bosonic and fermionic degrees of freedom of the SUSY-transformation. A global SUSY transformation within the superspace is characterized by translation u^μ and two spinorial parameters ξ_α and $\bar{\xi}^{\dot{\alpha}}$

$$\begin{pmatrix} x^\mu \\ \theta_\alpha \\ \bar{\theta}^{\dot{\alpha}} \end{pmatrix} \rightarrow \begin{pmatrix} x'^\mu \\ \theta'_\alpha \\ \bar{\theta}'^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} x^\mu + u^\mu + i\xi\sigma^\mu\bar{\theta} + i\bar{\xi}\bar{\sigma}^\mu\theta \\ \theta_\alpha + \xi_\alpha \\ \bar{\theta}^{\dot{\alpha}} + \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \tag{3.8}$$

Furthermore, the aim of this formulation is to unify fields that are related by the transformation in one irreducible supermultiplet provided by superfields $\mathcal{F}$. For this reason, superfields are considered as holomorphic functions [16] on the superspace which, at first, are allowed to depend on all coordinates $\mathcal{F}(x^\mu, \theta_\alpha, \theta^{\dot{\alpha}})$. In order to define a general SUSY-transformation on the function space of superfields one finds representations of the generators of the Super-Poincaré-Algebra given by differential operators

$$P^\mu = i\partial^\mu, \qquad J^{\mu\nu} = i(x^\mu\partial^\nu - x^\nu\partial^\mu), \tag{3.9}$$

$$Q_\alpha = i(\partial_\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu), \qquad \bar{Q}_{\dot{\alpha}} = -i(\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu). \tag{3.10}$$

[1] In supergravity are eight generators allowed. Although $N = 8$ Supergravity contains negative dimensional couplings, until now all UV divergences for scattering processes canceled at all evaluated loop orders [14] [15].

The pure SUSY transformation is associated to the group element $e^{i(uP+\xi Q+\bar{\xi}\bar{Q})}$, where u^μ parametrizes the translations generated by P^μ. A SUSY transformation then affects the superfield $\mathcal{F}$ as follows

$$e^{i(uP+\xi Q+\bar{\xi}\bar{Q})}\mathcal{F}(x^\mu,\theta_\alpha,\theta^{\dot{\alpha}}) = \mathcal{F}(x-u-i\xi\sigma^\mu\bar{\theta}-i\bar{\xi}\bar{\sigma}^\mu\theta,\theta_\alpha-\xi_\alpha,\theta^{\dot{\alpha}}-\bar{\xi}^{\dot{\alpha}}) \tag{3.11}$$

with the differential operators from (3.9) and (3.10). Henceforth, this thesis addresses superfields which transform as scalar, this means

$$e^{i(uP+\xi Q+\bar{\xi}\bar{Q})}\mathcal{F}(x'^\mu,\theta'_\alpha,\theta'^{\dot{\alpha}}) = \mathcal{F}(x^\mu,\theta_\alpha,\theta^{\dot{\alpha}}). \tag{3.12}$$

Due to the nilpotent spinorial coordinates $\theta^\alpha,\bar{\theta}_{\dot{\alpha}}$, the expansion series of a general superfield truncates early. Therefore all superfields can be expressed as

$$\begin{aligned}\mathcal{F}(x,\theta,\bar{\theta}) =& A(x)+\sqrt{2}\theta\psi(x)+\theta\theta F(x)+\sqrt{2}\bar{\theta}\bar{\chi}(x)+\bar{\theta}\bar{\theta}E(x)\\ &+\theta\sigma^\mu\bar{\theta}v_\mu(x)+\bar{\theta}\bar{\theta}\,\theta\lambda(x)+\theta\theta\,\bar{\theta}\bar{\xi}(x)+\frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta},\end{aligned} \tag{3.13}$$

with the complex scalar component fields A, F, E and D. There are also two left-handed and two right-handed Weyl spinors ψ, λ, $\bar{\chi}$ and $\bar{\xi}$. Additionally, in (3.13) a vector field v_μ is introduced by contracting it with the four vector $\theta\sigma^\mu\bar{\theta}$. All this fields appearing in (3.13) belong to one supermultiplet, and are therefore superpartners. Further restrictions are necessary to reduce the number of component fields in $\mathcal{F}$.

Chiral Superfields

The goal is now to construct a chiral field Φ which does not depend on a right-handed spinor $\bar{\chi}$. For this purpose one introduces the covariant supersymmetric derivative

$$\mathcal{D}_\alpha := \partial_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \tag{3.14}$$

$$\overline{\mathcal{D}}_{\dot{\alpha}} := -\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \tag{3.15}$$

A chiral superfield Φ is introduced by satisfying the relation

$$\bar{D}_{\dot{\alpha}}\Phi = 0. \tag{3.16}$$

One obtains that a chiral field is supersymmetric, since the derivative anticommutes with the SUSY-generator $\{Q_\beta,\bar{D}_{\dot{\alpha}}\} = \{\bar{Q}_{\dot{\beta}},\overline{\mathcal{D}}_{\dot{\alpha}}\} = 0$. If the spacetime coordinate x^μ is modified

$$x^\mu \to y^\mu := x^\mu - i\theta\sigma^\mu\bar{\theta}, \tag{3.17}$$

it is possible to decompose a chiral field as:

$$\Phi(y,\theta) = A(y)+\sqrt{2}\theta\psi(y)+\theta\theta F(y) \tag{3.18}$$

$$\Phi(y,\theta) = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}\Phi(x,\theta) \tag{3.19}$$

Comparing this result with (3.13), the components of chiral supermultiplet are two complex scalar and one left-handed Weyl spinor. The F-field has auxiliary character, since it is zero on the classical level. But on the quantum level[3] the F field ensures the equal treatment of bosonic and fermionic degrees of freedom. In the same manner the antichiral field $\Phi^\dagger$ can be introduced by the restriction

$$\mathcal{D}_\alpha \Phi^\dagger = 0. \tag{3.20}$$

A similar shift of the spacetime component $x^\mu \to \bar{y}^\mu := x^\mu + i\theta\sigma^\mu\bar{\theta}$ leads to the following decomposition of the anti chiral multiplet

$$\Phi(\bar{y},\theta)^\dagger = A(\bar{y})^\dagger + \sqrt{2}\bar{\theta}\bar{\chi}(\bar{y}) + \bar{\theta}\bar{\theta}F(\bar{y})^\dagger, \tag{3.21}$$

$$\Phi(\bar{y},\theta)^\dagger = e^{+i\theta\sigma^\mu\bar{\theta}\partial_\mu}\Phi(x,\theta)^\dagger, \tag{3.22}$$

with the components $A(\bar{y})^\dagger, F(\bar{y})^\dagger$ and the right-handed $\bar{\chi}(y)$.

Vector Superfields

A vector superfield is defined to be real i.e. such a function depends on all superspace coordinates which satisfy $V = V^\dagger$. However, supergauge transformation can be introduced by a chiral superfield Λ

$$V \to V' := V + i(\Lambda - \Lambda^\dagger). \tag{3.23}$$

Because $i(\Lambda - \Lambda^\dagger)$ is real and therefore a vector superfield, one obtains that the transformed field V' is it too. The field components of Λ are independent and for practical reasons it is useful to choose them in order to eliminate the first three terms in (3.13). The realness condition for V implies that $i\Lambda - i\Lambda^\dagger$ cancels the first line in (3.13). This special choice is called Wess-Zumino gauge. After applying the gauge transformation $v_\mu \to v_\mu - \partial_\mu(A - A^\dagger)$, the remaining vector superfield has the simple form

$$V(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu(x) + \bar{\theta}\bar{\theta}\,\theta\lambda(x) + \theta\theta\,\bar{\theta}\bar{\lambda}(x) + \frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}, \tag{3.24}$$

with a real vector field V_μ, its superpartner left-handed and a right-handed Weyl spinor λ $\bar{\lambda}$ and an auxiliary scalar field D. The real $D-$field can be eliminated in the same way as F for chiral fields.

[3]The fields do not have to satisfy the equation of motion.

3.1.3 Superspace Lagrangian

In the SM, the principle of gauge invariance is a restrictive criteria for interactions with matter fields. Hence, it is justified to demand this gauge invariance in the context of SUSY, where the role of matter fields is now replaced by chiral fields.[1] The general gauge transformation law for superfields can be defined as

$$\Phi \to \Phi e^{-ig2\Lambda^\dagger}, \tag{3.25}$$

$$\Phi^\dagger \to \Phi^\dagger e^{-ig2\Lambda^\dagger}, \tag{3.26}$$

$$e^{2gV} \to e^{-i2g\bar{\Lambda}^\dagger} e^{2gV} e^{2ig\Lambda}, \tag{3.27}$$

$$e^{-2gV} \to e^{-i2g\bar{\Lambda}} e^{2gV} e^{2ig\Lambda^\dagger}, \tag{3.28}$$

where $\Lambda \equiv \Lambda^a(x,\theta,\bar{\theta})T^a$ and $V \equiv V^a T^a$. T^a represents the generators of the gauge group, Λ^a denotes a complex chiral superfield, and V^a vector superfield. A gauge invariant interaction can be introduced as $\Phi^\dagger e^{2gV}\Phi$ by replacing the partial derivative ∂_μ with the familiar gauge covariant derivative D_μ defined in (2.13). The evaluation of the $\mathrm{d}^4\theta$ integral leads, i.a. to the kinetic terms of the scalar A and fermionic field ψ

$$\begin{aligned}\int \mathrm{d}^4\theta\, \Phi^\dagger e^{2gV}\Phi =& F^\dagger F + (D_\mu A)(D^\mu)A^\dagger + i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi \\ &+ i\sqrt{2}g\left(A^\dagger T^a \psi \lambda^a - \bar{\lambda}^a \bar{\psi} T^a A\right) + g(A^\dagger T^a A)D^a.\end{aligned} \tag{3.29}$$

As in the SM case, the covariant derivative leads to an interaction between the chiral components ψ, A and the gauge fields v^a_μ. The kinetic terms for gauge bosons are included via the chiral field strength tensor

$$W_\alpha := -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\left(e^{-2gV}\mathcal{D}_\alpha e^{2gV}\right) \equiv 2gW^a_\alpha T^a, \tag{3.30}$$

$$\bar{W}_{\dot{\alpha}} := -\frac{1}{4}\mathcal{D}\mathcal{D}\left(\left(\mathcal{D}_{\dot{\alpha}} e^{2gV}\right)e^{2gV}\right) \equiv 2gW^a_{\dot{\alpha}} T^a. \tag{3.31}$$

A gauge invariant expression can be constructed in the following way

$$\begin{aligned}\int \mathrm{d}^4\theta\, \frac{1}{16g^2} W^{\alpha,a}W^a_\alpha \delta^2(\bar{\theta}) + \text{h.c.} =& -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \frac{1}{2}D^a D^a \\ &+ \frac{i}{2}\lambda^a \sigma^\mu (D_\mu \bar{\lambda})^a + \frac{i}{2}\bar{\lambda}^a \sigma^\mu (D_\mu \lambda)^a.\end{aligned} \tag{3.32}$$

Comparing this result (3.32) with the SM Lagrangian, the kinetic term for gauge bosons is recovered in the superspace language. Furthermore, its superpartners, λ and $\bar{\lambda}$, also obtain dynamics provided by the covariant derivative D_μ, where the generators act in the adjoint representation $(T^{ad})^{abc} = -if^{abc}$ on λ.

Further supersymmetric and gauge invariant interactions arise from an polynomial in chiral fields. [6] For reasons of renormalizability, the polynomial expansion truncates at powers three in the chiral fields. Otherwise couplings with negative mass dimension would

[1] Because a SUSY extension of the SM has to contain it in the low energy regime.

[6] Supersymmetric denotes the property of the action and holds therefore in the Lagrangian up to total derivatives.

be required. In this sense, the expression

$$\int \mathrm{d}^4\theta\, \delta^2(\bar{\theta})\mathcal{W}(\Phi) \tag{3.33}$$

fulfills the requirement of SUSY and gauge transformation invariance with the holomorphic function, the so-called superpotential $\mathcal{W}(\Phi)$

$$\mathcal{W}(\Phi) = a_i\Phi_i + \frac{1}{2}m_{ij}^2\Phi_i\Phi_j + \frac{y_{ijk}}{3!}\Phi_i\Phi_j\Phi_k. \tag{3.34}$$

The couplings a_i, m_{ij}^2, and y_{ijk}, are new parameters, which give rise to new interactions between the fields. As described in the appendix B.1, the functional $\int \mathrm{d}^4\theta\, \delta^2(\bar{\theta})$ projects the θ^2-component of a function. The evaluation of both, the bilinear and trilinear part in $\mathcal{W}$, leads to the following decomposition

$$\int \mathrm{d}^4\theta\, \delta^2(\bar{\theta})\Phi_1\Phi_2 = A_1F_2 + F_1A_1 - \psi_1\psi_2, \tag{3.35}$$

$$\int \mathrm{d}^4\theta\, \delta^2(\bar{\theta})\Phi_1\Phi_2\Phi_3 = A_1F_2A_3 + F_1A_2A_3 + A_1A_2F_3 - \psi_1\psi_2A_2 - A_1\psi_2\psi_3 - \psi_1A_2\psi_3. \tag{3.36}$$

Similarly, the inclusion of a superpotential, which addresses right-handed fermions, can be done by considering $\mathcal{W}(\bar{\Phi})$. The general form of a supersymmetric action S_{SUSY}, which contain a gauge multiplet Φ_i, can be found by integrating superfields and their interaction over the whole superspace

$$S_{\mathrm{SUSY}} = \int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{SUSY}}, \tag{3.37}$$

$$\mathcal{L}_{\mathrm{SUSY}} = \int \mathrm{d}^4\theta \left\{\Phi_i^\dagger e^{2gV}\Phi_i + \left[\left(\frac{1}{16g^2}W^{\alpha,a}W_\alpha^a + \mathcal{W}(\Phi)\right)\delta^2(\bar{\theta}) + \text{h.c.}\right]\right\}. \tag{3.38}$$

As mentioned earlier, the introduced auxiliary fields can be eliminated by using the Euler-Lagrange equation.[8] The so-called F- and D-terms originate from this replacement procedure out of the quadratic expressions in eq. (3.29) and (3.32).

$$\mathcal{L}_F := -F_i^\dagger F_i = -\left|\frac{\partial\mathcal{W}(A)}{\partial A_i}\right|^2, \tag{3.39}$$

$$\mathcal{L}_D := -\frac{1}{2}D^aD^a = -\frac{g^2}{2}\left(A^\dagger T^a A\right)^2. \tag{3.40}$$

These two contributions build up the complete scalar potential of a supersymmetric theory. A remarkable result, is that all four point functions are given by gauge couplings and three point functions at tree level.

[8] $\frac{\partial\mathcal{L}}{\partial\phi} = 0$, $\phi = F, D$

Soft Supersymmetry Breaking

The super-Poincaré algebra contains a Casimir operator $P^\mu P_\mu$, i.e. this operator commutes with all generators of this algebra and especially with Q

$$[P^\mu P_\mu, Q_\alpha] = [P^\mu P_\mu, \bar{Q}_{\dot{\alpha}}] = 0. \tag{3.41}$$

By assuming exact SUSY this implies; a field and its superpartner share the same mass. Until now, no supersymmetric partner have been found, consequently this infers SUSY has to be broken in the low energy regime. In analogy to the SM, it is possible to include dynamical breaking of SUSY to resolve this disagreement between theory and experiment. This can be achieved by introducing further manifest supersymmetric interactions with a new chiral field $\eta(y) = a(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$

$$\begin{aligned}\mathcal{L}_{\text{Soft}} = \int \mathrm{d}^4\theta \Bigg\{ &\eta^\dagger\eta\, m_{ij}\Phi_i^\dagger(e^{2gV})_{ij}\Phi_j \\ &+ \Big[\eta\Big(\frac{M}{2}W^{\alpha,a}W_\alpha^a + \mathcal{W}(\Phi)\Big)\delta^2(\bar{\theta}) + \text{h.c.}\Big]\Bigg\},\end{aligned} \tag{3.42}$$

where M, m_{ij} and the new parameters in $\mathcal{W}$ denote the coupling to the spurion field η [17]. Assigning a VEV f_0 to the θ^2 component in η

$$F(y) = f_0 + f(y), \tag{3.43}$$

the theory will break SUSY softly at the low energy scale, where the heavy component field f is integrated out. For a detailed explanation for the origin of such a symmetry breaking the reader is referred to [5]. This method gives rise to mass terms for the scalar fields in a chiral multiplet Φ_i and for fermions within a vector multiplet. Due to new coupling constants between the scalar F field, it introduces contributions to couplings between scalar fields within the superpotential $\mathcal{W}$. An advantageous feature of this method is that it avoids new interactions which lead to quadratic divergences. Hence, "softly broken" denotes the property of including couplings with non-negative mass power and therefore guarantees the renormalizability of the broken theory.

3.2 Minimal Supersymmetric Standard Model

When imposing SUSY on the SM its particle content has to be enlarged not at least by the set of superpartners. The MSSM introduces new fields in a minimal fashion. For motivating this model, this section starts with some noteworthy results [18]

i The mass spectrum of all experimentally observed particles is in agreement with the prediction of the MSSM with superpartner masses at a few TeV. As explained later, this is in particular the case for the SM Higgs boson , where, unlike in the SM, the Higgs boson coupling λ is in the MSSM no free parameter but a function of the gauge couplings. Moreover, all quartic scalar couplings are fixed by Yukawa and gauge couplings.

ii The gauge couplings, introduced in 2.2, regarded as $\overline{\text{MS}}$ parameters obey a scale dependent running $\alpha_i(Q) = g_i^2(Q)/4\pi$. Although all three couplings do not meet exactly, they have the tendency to unite at $Q \approx 10^{16}$ GeV. However, by including superpartners with masses in the range of 100 GeV-10 TeV the $\overline{\text{DR}}$-gauge couplings will meet in the MSSM. Thus in supersymmetry all gauge couplings may unify at a certain scale $M_{\text{GUT}} \approx 10^{16}$ GeV, which is referred to as grand unification (GUT).

iii The origin of electroweak symmetry breaking arises in a more natural way in this model. Since mass parameters exhibit a scale dependency in the $\overline{\text{DR}}$ scheme, the soft breaking term $m_{H_u}^2$ increases at larger scales. Considering the framework of minimal supergravity or gauge mediated symmetry breaking as the mechanism for SUSY breaking, the soft breaking parameters $m_{H_u}^2$ and $m_{H_d}^2$ are supposed to unify at a certain large scale. Thus, the parameter $m_{H_u}^2$ is allowed to get small enough values at the electroweak scale $Q \approx 100$ GeV and can then trigger the electroweak symmetry breaking. A complete derivation of this argument for radiative electroweak symmetry breaking is given in ref. [18].

Besides these robust advantages, the minimal supersymmetric extension of the SM is capable of explaining the anomalous magnetic dipole moment of the muon[2] and provides a suitable candidate for an explanation of dark matter. Nevertheless most arguments hold only if some superpartners do not exceed a mass range of a few TeV.

In the same manner as for the SM, the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group assembles chiral fields in an additional gauge multiplet structure. Tab. 3.1 provides a complete overview of the chiral fields in the MSSM. The number of generations of quarks and leptons stays the same in the MSSM. Whereas in contrast to the SM, the MSSM requires a second Higgs $SU(2)_L$ doublet. Otherwise, the weak hypercharge of a fermionic partner of a single Higgs would yield a non-vanishing gauge anomaly [5]. Avoiding the destruction of anomaly cancellation, a second fermionic Higgs is introduced with the opposite weak hypercharge $Y = \pm 1$ respectively. In the language of superspace, this is implemented in the MSSM by the inclusion of two chiral superfields, as shown in tab. 3.1.

By convention, the scalar superpartners of a fermion (e.g. the left-handed top quark u_{L3}) are labeled by a prefixed letter s (e.g. stop quark $\tilde{u}_{L3}$). The index L or R of an scalar component denotes the handedness of the fermionic partner and has no further meaning. The only exceptions of this rule are the two Higgs chiral fields, where the scalar component is introduced as usual (e.g. charged Higgs) and the name of the fermionic superpartner ends with ino (e.g. charged higgsino).

The number of gauge fields stays in the MSSM the same. Introduced through supervector fields, the gauge bosons of the SM posses fermionic superpartners, the gauginos. The complete list of the MSSM vector superfields and their components is presented in tab. 3.2.

[2] The experimental result for the value of the magnetic dipole moment of the muon deviates from the theoretic SM prediction.

	chiral superfield	components spin 0	components spin 1/2	$SU(3)_C$	$SU(2)_L$	Y
Higgs bosons, higgsinos	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	$\psi_{H_u} = \begin{pmatrix} \psi_{H_u}^+ \\ \psi_{H_u}^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	1
	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	$\psi_{H_d} = \begin{pmatrix} \psi_{H_d}^0 \\ \psi_{H_d}^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	-1
squarks, quarks	$Q_L^j = \begin{pmatrix} Q_u^j \\ Q_d^j \end{pmatrix}$	$\tilde{q}_L^j = \begin{pmatrix} \tilde{u}_L^j \\ \tilde{d}_L^j \end{pmatrix}$	$q_L^j = \begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$
	U^j	$\tilde{u}_R^{j\,\dagger}$	u_R^j	$\mathbf{3^*}$	$\mathbf{1}$	$-\frac{4}{3}$
	D^j	$\tilde{d}_R^{j\,\dagger}$	d_R^j	$\mathbf{3^*}$	$\mathbf{1}$	$\frac{2}{3}$
sleptons, leptons	$L_L^j = \begin{pmatrix} L_\nu^j \\ L_e^j \end{pmatrix}$	$\tilde{l}_L^j = \begin{pmatrix} \tilde{\nu}_L^j \\ \tilde{e}_L^j \end{pmatrix}$	$l_L^j = \begin{pmatrix} \nu_L^j \\ e_L^j \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	-1
	E^j	$\tilde{e}_R^{j\,\dagger}$	e_R^j	$\mathbf{1}$	$\mathbf{1}$	2

Table 3.1: This table lists all chiral superfields that occur in the MSSM, together with their field decomposition. Since $s = 0$ and $s = 1/2$ components are assembled in a single supermultiplet, they transform in the same representation of the gauge group. The $SU(3)_C$ column refers to the representation occurring in the covariant derivative (2.13). If the fields within a supermultiplet do not participate in strong interactions, the symbol **1** labels that they possess no color charge. The notation of $\mathbf{3}^*$ and $\mathbf{3}$ indicate the antifundamental representation $(T^a)^{\text{anti}} = -\frac{(\lambda^a)^T}{2}$ and fundamental representation $T^a = \frac{\lambda^a}{2}$ respectively. Likewise, the $SU(2)_L$ singlets **1** do not interact weakly and transform trivially under gauge transformation. The notion **2** indicates the fact that the covariant derivative in (2.13) contains the fundamental representations of the generators $T^i = \frac{\sigma^i}{2}$. The classification in three generations is indicated by the label $j \in \{1, 2, 3\}$.

vector superfield	components		chiral field	$SU(3)_C$	$SU(2)_L$	Y
	spin 1/2	spin 1	field strength			
	gaugino	gauge boson				
V_s^a	$\tilde{\lambda}_s^a$	G_μ^a	W_s	**8**	**1**	0
V_w^j	$\tilde{\lambda}_w^i$	W_μ^i	W_w	**1**	**3**	0
V_y	$\tilde{\lambda}_y$	B_μ	W_y	**1**	**1**	0

Table 3.2: This table summarizes all vector superfields of the MSSM, together with their field decomposition. The gauge fields, with $a \in \{1, .., 8\}$ and $i \in \{1, 2, 3\}$, transform under the adjoint representation (2.11), whereas **1** refers to the trivial representation.

3.2.1 R-Parity

The minimal supersymmetric extension of the SM, given by the MSSM, is capable of reproducing the phenomenology of all known particles and their interactions. However, the restrictions of renormalizability, supersymmetry, gauge invariance under the SM symmetry group and holomorphy of the superpotential allow further interactions. One infamous example for such an interaction would imply a dramatically fast proton decay, if the coupling constants of these lepton and baryon number (L and B respectively) violating interactions were unsuppressed [5]. Obviously, this contradicts the experimental data, where the lifetime of the proton is at least $\tau \approx 10^{32}$ years [5]. The SM avoids these interactions rather by coincidence than by a fundamental symmetry. This justifies, the assumption of an additional $\mathbb{Z}_2$ symmetry, which forbids combined B and L violating interaction terms in the MSSM. In particular a term is only allowed if the product of the quantum numbers P_M of the involved superfields is $+1$, with matter parity defined as

$$P_M = (-1)^{3(B-L)}. \tag{3.44}$$

It is of advantage to supersede this matter parity by $R-$parity, defined for each field as

$$P_R = (-1)^{3(B-L)+2s}. \tag{3.45}$$

Thus, R-parity distinguishes between particles and their superpartners and therefore does not commute with SUSY. In contrast, matter parity commutes with SUSY, but for interactions which preserve angular momentum both, P_M and P_R are equivalent [5].

3.2.2 MSSM Lagrangian

Like the SM, the supersymmetric extension of it is specified by a Lagrangian density which can be split into two parts; in a supersymmetric one and in a softly broken part.

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}. \tag{3.46}$$

This Lagrangian consists of superfields listed in tab. 3.1 and 3.2. The first part is invariant under supersymmetry transformation and given as

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = \int d^4\theta \Big\{ & \bar{Q}_j e^{2g_w V + 2g_y V' + 2g_s V_s} Q_j + \bar{U}_j e^{2g_w V + 2g_y V' + 2g_s V_s^T} U_j \\ & + \bar{D}_j e^{2g_w V + 2g_y V' + 2g_s V_s^T} D_j + \bar{L}_j e^{2g_w V + 2g_y V'} L_j \\ & + \bar{E}_j e^{2g_w V + 2g_y V'} E_j + \bar{H}_d e^{2g_w V + 2g_y V'} H_d + \bar{H}_u e^{2g_w V + 2g_y V'} H_u \Big\} \\ & + \int d^2\theta \, [W^{a\alpha} W^a_\alpha + W'^\alpha W'_\alpha + W_s^{a\alpha} W^a_{s\alpha}] + h.c. \\ & + \int d^2\theta \, \mathcal{W}_{\text{MSSM}} + h.c. \end{aligned} \tag{3.47}$$

A further notion was introduced in (3.47) by $V' \equiv V_y \frac{Y}{2}$, $V \equiv V_w^i \frac{\sigma^i}{2}$ and $V_s = V_s^a \frac{\lambda^a}{2}$. The MSSM superpotential is defined as

$$\mathcal{W}_{\text{MSSM}} = y_d H_d Q_j D_j + y_u H_u Q_j U_j + y_e H_d L_j E_j - \mu H_d H_u, \tag{3.48}$$

Indeed, this is the only gauge invariant possibility for the superpotential which respects R-parity, renormalizability and gives rise to the discussed field content. The $SU(2)_L$-product is carried out in the form

$$H_u Q = H_u^1 Q^2 - H_u^2 Q^1, \tag{3.49}$$

in order to respect gauge invariance. The superpotential introduces the Yukawa-couplings $y_{u,d,e}$ and a dimensional parameter μ for the bilinear term in the Higgs superfields. In summary, the Lagrangian density $\mathcal{L}_{\text{SUSY}}$ describes the dynamics and interactions of a much richer field content as the SM Lagrangian. In contrast to the SM, the $\mathcal{L}_{\text{SUSY}}$ needs one parameter less, namely the SM coupling λ. Because of the arguments mentioned before, a SUSY breaking part is introduced in eq. (3.47)

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & - M^2_{\tilde{Q}_j} |\tilde{q}_{L_j}|^2 - M^2_{\tilde{U}_j} |\tilde{u}_{R_j}|^2 - M^2_{\tilde{D}_j} |\tilde{d}_{R_j}|^2 - M^2_{\tilde{L}_j} |\tilde{l}_{L_j}|^2 - M^2_{\tilde{E}_j} |\tilde{e}_{R_j}|^2 \\ & - M^2_{H_d} |h_d|^2 - M^2_{H_u} |h_u|^2 \\ & + \frac{1}{2} (M_1 \lambda_y \lambda_y + M_2 \lambda_w^i \lambda_w^i + M_3 \lambda_s^a \lambda_s^a + h.c.) \\ & - (A_{d_j} y_{d_j} h_d \tilde{q}_{L_j} \tilde{d}^\dagger_{R_j} + A_{u_j} y_{u_j} h_u \tilde{q}_{L_j} \tilde{u}^\dagger_{R_j} + A_{e_j} y_{e_j} h_d \tilde{l}_{L_j} \tilde{e}^\dagger_{R_j} - B\mu h_u h_d + h.c.). \end{aligned} \tag{3.50}$$

As discussed before, $\mathcal{L}_{\text{soft}}$ introduces mass terms for all scalar fields and gauginos that are present in the MSSM. Consequently the so-called $B\mu$- and A-terms occur which modify the mass mixing and trilinear couplings between sfermions and Higgs scalars [5].

3.2.3 Higgs Sector and Symmetry Breaking

In order to evaluate the Higgs-potential integrals it is sufficient to use the introduced rules for the superspace formalism from this chapter. The Higgs potential gathers all scalar contributions

$$V_{\text{Higgs}} = -\mathcal{L}^{\text{soft}}_{\text{Higgs}} - \mathcal{L}^{F}_{\text{Higgs}} - \mathcal{L}^{D}_{\text{Higgs}}, \tag{3.51}$$

$$\mathcal{L}^{F}_{\text{Higgs}} = -|\mu|^2(h_d^\dagger h_d + h_u^\dagger h_u), \tag{3.52}$$

$$\mathcal{L}^{D}_{\text{Higgs}} = -\frac{g^2}{2}(h_d^\dagger T^a h_d + h_u^\dagger T^a h_u)^2 - \frac{g'^2}{2}(Y_{h_d} h_d^\dagger h_d + Y_{h_u} h_u^\dagger h_u)^2, \tag{3.53}$$

$$\mathcal{L}^{\text{soft}}_{\text{Higgs}} = -M^2_{H_d} h_d^\dagger h_d - M^2_{H_u} h_u^\dagger h_u + (B\mu h_d h_u + h.c.). \tag{3.54}$$

The F-terms can be computed according to (3.39). A second contribution to the Higgs potential comes from D-terms (3.40). Furthermore, terms from the soft breaking part in $\mathcal{L}_{\text{soft}}$ form the last part of the Higgs-potential.

In order to describe, massive vector bosons and fermions also the MSSM-potential must feature spontaneous symmetry breaking $SU(2)_L \otimes U(1)_Y \to U(1)_{QED}$. In general scalar fields like sfermions can give rise to a VEV and this can imply gauge boson masses. However, this is phenomenologically excluded and only the scalar component h_u^2 and h_d^1 of the Higgs superfield achieve a VEV. A possible parameterization of scalar component of the Higgs $SU(2)_L$ doublet reads

$$h_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \phi_d^0 - i\chi_d^0) \\ -\phi^- \end{pmatrix}, \qquad h_u = \begin{pmatrix} \phi_u^+ \\ \frac{1}{\sqrt{2}}(v_u + \phi_u^0 + i\chi_u^0) \end{pmatrix}. \tag{3.55}$$

Here $\phi^0_{u,d}$, $\chi^0_{u,d}$ denote real scalar fields and $\phi^\pm_{u,d}$ represent complex scalar fields. The two real and positive values v_u and v_d are the two VEVs [3]

$$\langle h_d \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle h_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}. \tag{3.56}$$

It is convenient to introduce a further parameter for the ratio of the VEVs

$$\tan\beta = \frac{v_u}{v_d}. \tag{3.57}$$

To be consistent with positive VEVs, the angle has to fulfill $0 < \beta < \pi/2$. Because the gauge invariance of electromagnetism is not broken, the charged components vanish at the minimum of the potential $\langle \phi^- \rangle = \langle \phi^+ \rangle = 0$. The two equations $\partial V/\partial \phi_u^0 = 0$ and $\partial V/\partial \phi_d^0 = 0$ are the conditions of the minimum and can be used to eliminate $M^2_{H_u}$ and $M^2_{H_d}$ in the potential:

$$M^2_{H_d} = \frac{(g_y^2 + g_w^2)}{8}(v_u^2 - v_d^2) + B\mu\frac{v_u}{v_d} - |\mu|^2, \tag{3.58}$$

$$M^2_{H_u} = \frac{(g_y^2 + g_w^2)}{8}(v_d^2 - v_u^2) + B\mu\frac{v_d}{v_u} - |\mu|^2. \tag{3.59}$$

Next, one computes the eigenvalues of the bilinear part in the potential to find the associated

[3] The SM VEV is related to the MSSM VEVs by $v^2 = v_u^2 + v_d^2$.

mass eigenstates

$$m_A^2 = \frac{2B\mu}{\sin 2\beta}, \tag{3.60}$$

$$m_{h,H}^2 = \frac{1}{2}[m_Z^2 + m_A^2 \pm \sqrt{(m_Z^2 + m_A^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}], \tag{3.61}$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2. \tag{3.62}$$

where the masses of the vector bosons $W_\mu^\pm$ and Z_μ are given by

$$m_W^2 = \frac{g_w^2}{4}(v_u^2 + v_d^2), \tag{3.63}$$

$$m_Z^2 = \frac{g_w^2 + g_y^2}{4}(v_u^2 + v_d^2). \tag{3.64}$$

The derived mass eigenstates in eq.(3.60) correspond to two CP-even neutral (h, H) fields, two CP-even charged $H^\pm$ fields and one CP-odd neutral A field. Except for $m_{h^0}^2$, all masses in eq. (3.60) are in general allowed to be arbitrary large. Considering a large value for $m_A^2 \gg M_Z^2$, the tree level mass for h has an upper bound:

$$m_h^2 = m_Z^2 \cos^2 2\beta \leq m_Z^2. \tag{3.65}$$

In this work the lightest Higgs boson out of both CP-even neutral mass eigenstates is considered as the SM Higgs boson. Since the discovery in 2012, its known that the SM Higgs-Boson possess a higher mass than the Z-Boson. Nevertheless, eq. (3.65) is not a contradiction to the experiment, since quantum corrections were not taken into account. Indeed, those correction are of order $\mathcal{O}(m_Z^2)$ in the MSSM if SUSY-parameters are in a certain range, which is so far in agreement with experimental data.

Mass terms for fermions and sfermions

In analogy to the SM the MSSM mass terms for fermions origin from the interactions in the MSSM superpotential. After EW symmetry breaking the Dirac fermions Ψ, constructed from the Weyl spinors ψ occurring in the chiral fields Φ, possess the following mass terms

$$\mathcal{L}_{f\text{mass}} = -\sum_{j=1}^{3} \left[m_{u_j} \bar{\Psi}_{u_j} \Psi_{u_j} + m_{d_j} \bar{\Psi}_{d_j} \Psi_{d_j} + m_{e_j} \bar{\Psi}_{e_j} \Psi_{e_j} \right], \tag{3.66}$$

where the label u, d and e correspond to the up-type quark, down-type quark and charged lepton respectively. The mass parameters depend on the Yukawa coupling and the VEV v_u or v_d [4]

$$m_{u_j} = \frac{v_u}{\sqrt{2}} y_{u_j}, \qquad m_{d_j} = \frac{v_d}{\sqrt{2}} y_{d_j}, \qquad m_{l_j} = \frac{v_u}{\sqrt{2}} y_{l_j}. \tag{3.67}$$

[4] The MSSM can be extended in a straightforward way by the mixing of mass eigenstates provided by the V_{CKM}-matrix.

For the MSSM sfermions mass terms the Lagrange density has the form

$$\mathcal{L}_{\tilde{f},\text{mass}} = \sum_{j=1}^{3} \Bigg[-\begin{pmatrix} \tilde{u}_{L_j}^\dagger & \tilde{u}_{R_j}^\dagger \end{pmatrix} M_{\tilde{u}_j}^2 \begin{pmatrix} \tilde{u}_{L_j} \\ \tilde{u}_{R_j} \end{pmatrix} - \begin{pmatrix} \tilde{d}_{L_j}^\dagger & \tilde{d}_{R_j}^\dagger \end{pmatrix} M_{\tilde{d}_j}^2 \begin{pmatrix} \tilde{d}_{L_j} \\ \tilde{d}_{R_j} \end{pmatrix} - \begin{pmatrix} \tilde{e}_{L_j}^\dagger & \tilde{e}_{R_j}^\dagger \end{pmatrix} M_{\tilde{e}_j}^2 \begin{pmatrix} \tilde{e}_{L_j} \\ \tilde{e}_{R_j} \end{pmatrix} - \tilde{\nu}_{L_j}^\dagger m_{\tilde{\nu}_j}^2 \tilde{\nu}_{R_j}^\dagger \Bigg], \tag{3.68}$$

where the mass matrices are

$$M_{\tilde{u}_j}^2 = \begin{pmatrix} m_{u_j}^2 + M_{\tilde{Q}_j}^2 + m_Z^2 c_{2\beta} \left(T_3 - Q s_{\theta_W}^2 \right) & m_{u_j} (A_{u_j}^* - \mu \cot\beta) \\ m_{u_j} (A_{u_j} - \mu^* \cot\beta) & m_{u_j}^2 + M_{\tilde{U}_j}^2 + m_Z^2 c_{2\beta} Q s_{\theta_W}^2 \end{pmatrix}, \tag{3.69}$$

$$M_{\tilde{d}_j}^2 = \begin{pmatrix} m_{d_j}^2 + M_{\tilde{Q}_j}^2 + m_Z^2 c_{2\beta} \left(T_3 - Q s_{\theta_W}^2 \right) & m_{d_j} (A_{d_j}^* - \mu \tan\beta) \\ m_{d_j} (A_{d_j} - \mu^* \tan\beta) & m_{d_j}^2 + M_{\tilde{D}_j}^2 + m_Z^2 c_{2\beta} Q s_{\theta_W}^2 \end{pmatrix}, \tag{3.70}$$

$$M_{\tilde{e}_j}^2 = \begin{pmatrix} m_{e_j}^2 + M_{\tilde{L}_j}^2 + m_Z^2 c_{2\beta} \left(T_3 - Q s_{\theta_W}^2 \right) & m_{e_j} (A_{e_j}^* - \mu \tan\beta) \\ m_{e_j} (A_{e_j} - \mu^* \tan\beta) & m_{e_j}^2 + M_{\tilde{E}_j}^2 + m_Z^2 c_{2\beta} Q s_{\theta_W}^2 \end{pmatrix}, \tag{3.71}$$

and the sneutrino mass obeys

$$m_{\tilde{\nu}_j}^2 = M_{\tilde{L}_j}^2 + \frac{1}{2} m_Z^2 c_{2\beta} \,. \tag{3.72}$$

Further, it is convenient to introduce an abbreviation for the off-diagonal matrix elements, especially, for the parameterization for the up-type squarks

$$X_{u_j} \equiv A_{u_j} - \mu^* \cot\beta \,. \tag{3.73}$$

Unless stated otherwise, the important stop mixing parameter is written as $X_t \equiv X_{u_3}$ or $\hat{X}_t \equiv X_t/m_s$, where m_s denotes the typical SUSY scale. This is often considered if the massive soft breaking parameters are degenerated. These mixing angle θ of the mass matrix $(M_{\tilde{u}_3}^2)_{ij}$ is defined as

$$\tan 2\theta := \frac{(M_{\tilde{u}_3}^2)_{12}}{(M_{\tilde{u}_3}^2)_{11} - (M_{\tilde{u}_3}^2)_{22}}. \tag{3.74}$$

4 FlexibleSUSY

4.1 FlexibleSUSY the Spectrum Generator Generator

In order to determine the mass spectrum and additional observables in a wide range of extension of the SM, FlexibleSUSY [19] provides a suitable framework to perform fast and reliable calculations. Thus, FlexibleSUSY is regarded as a generator for spectrum generators, constructed for precise phenomenological studies. The implementation is achieved by a C++ and Mathematica package, which is by design flexible and can therefore accommodate a variety of models beyond the SM. The model is specified in an external SARAH [20] model file. Based on the properties of the model i.e. field content, gauge symmetries, super potential[1] and mass mixings, SARAH creates expressions for vertices, tadpole diagrams, self energies and renormalization group equations (RGE). These algebraic expressions are then converted by the FlexibleSUSY meta code into C++ code. Simultaneously, FlexibleSUSY incorporates model boundary conditions for all parameters. In general, these conditions can be defined by the user for three different scales.

low scale: A suitable scale for these constraints is the pole mass of the Z-Boson M_Z. At this scale model parameters like gauge couplings and masses are calculated from SM parameters like masses and couplings. This parameters can be fixed numerically in order to meet the experimental values. In this work the pole masses of the Z-boson, top-quark, tau-lepton are set to $M_Z = 91,1846\,\text{GeV}$, $M_t = 173,34\,\text{GeV}$ and $M_\tau = 1,777\,\text{GeV}$. The value for the fine structure constant is set to $\alpha_{e.m.}^{\overline{\text{MS}}(5)}(M_Z) = \frac{1}{127,944}$ and is introduced as the value in the five-flavor SM [2].

SUSY scale: For supersymmetric models the SUSY scale M_S provides the proper scale for the mass spectrum of supersymmetric partners. After the model parameters have been adjusted to the constraint values at this scale, FlexibleSUSY solves the electroweak symmetry breaking (EWSB) equations. If not stated otherwise, in this work the massive non-SM parameters are set to the SUSY scale.

high scale: Some models favor gauge-coupling unification at a high scale M_{GUT}, where some other parameters are specified. Thus FlexibleSUSY provides the possibility to impose boundary conditions for parameters at this scale M_{GUT}.

After the C++ code for the spectrum generator has been generated and compiled, the input parameter values have to be fixed in a file of SLHA[3] format [21], [22]. All parameters are defined in the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ scheme. Thus the spectrum generator solves the RGEs in order to evaluate the $\overline{\text{MS}}/\overline{\text{DR}}$ parameter at a different scale, where the constraints are imposed. After the parameters are determined at the scale, FlexibleSUSY respects the scale

[1] This is a necessary information about the fundamental interactions in supersymmetric theories as it is shown in eq. (3.34).

[2] This model considers all SM fields with the exception of the top quark.

[3] SUSY Les Houches Accord

constraints and finds a consistent set of values for the model parameters. With these values for the model parameters the spectrum is calculated and is then written in a SLHA output file.

4.2 Tower of Effective Theories and FlexibleEFTHiggs

Due to the modular structure of the generated C++ code the user is enabled to stress one or more scales and impose constraints on model parameters. This allows the user to define matching conditions and combine different models. In particular, FlexibleSUSY provides the framework to combine a model with a low energy effective theory. In the light of this advantage, this work assumes that the SM is the valid low energy approximation of the MSSM and FlexibleSUSY is extended for a more precise Higgs-mass calculation. Thus the considered EFT Tower consists of two models, the MSSM above the SUSY scale $Q > m_s$ and the SM below $Q < m_s$.[4] This idea has become more manifest with the work of Tom Steudtner et al. [23], where the so-called method FlexibleEFTHiggs has been elaborated. It is now explained how the calculations are performed at highest precision. The code is based on operations at four scales.

M_Z *scale:* At the low scale (M_Z = Z boson mass pole mass) the SM parameters are obtained from numerical values of the input parameter by imposing the low scale constraints described in *low scale*.

M_t *scale:* At the pole mass of the top quark M_t the one loop EWSB conditions are imposed and the mass spectrum of the SM particles is evaluated. After the one loop EWSB conditions are applied the pole mass of the Higgs Boson is calculated at one loop order. If convergence is achieved, the particle masses are written in the output SLHA file after the final iteration at this scale.

SUSY-SM matching scale: After the running with three loop RGEs from the EW scale to the SUSY-SM matching scale, the SM parameters are matched to the SUSY model parameters. For this purpose also the SUSY model parameters run this scale by solving two-loop RGEs. After invoking the EWSB conditions at one loop order the spectrum is calculated in the SM and in the SUSY model. By imposing a pole mass matching condition, the SUSY fermions tree level masses are computed

$$M_{f,\mathrm{MSSM}} = M_{f,\mathrm{SM}}. \tag{4.1}$$

In particular, in both models the pole mass is calculated by

$$M_{f,\mathrm{model}} = m_{f,\mathrm{model}} - \mathrm{Re}\,\hat{\Sigma}_{f,\mathrm{model}}, \tag{4.2}$$

where a small letter m_f denotes the tree level mass and $\hat{\Sigma}_f$ denotes the renormalized correction beyond LO. This condition determines uniquely the value of the SUSY model Yukawa couplings. Thus, it is straightforward to include more than one loop order corrections and indeed, for top quark pole mass also two loop corrections from scheme conversion are considered. Since the top Yukawa coupling is very important

[4] In FlexibleSUSY the SUSY scale m_s is defined as [19] $m_s = \sqrt{\prod_{i=1}^{6} m_{\tilde{u}_i}^{|(Z_u)_{i3}|^2+|(Z_u)_{i6}|^2}}$, where the $\overline{\mathrm{DR}}$ parameter $m_{\tilde{u}_i}$ denotes the mass of the up-type quark i and Z_u is the up-type squark mixing matrix.

for the Higgs mass calculation the two-loop contributions are included if the FlexibleEFTHiggs is set to high precision. Thus the matching condition is modified by two-loop QCD contributions in the SM [24] and MSSM [25]

$$\begin{aligned}\mathrm{Re}\,\hat{\Sigma}^{2L,\,\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}} = -\frac{(g^{\overline{\mathrm{DR}}}_{s,\mathrm{MSSM}})^4\, m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}}}{4608\pi^4}\Bigg[&396\ln^2\frac{(m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}})^2}{Q^2} - 1476\ln\frac{(m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}})^2}{Q^2}\\ &+ 2011 + 48\zeta(3) + 16\pi^2(1+\ln 4)\Bigg],\end{aligned} \tag{4.3}$$

$$\begin{aligned}\mathrm{Re}\,\hat{\Sigma}^{2L,\,\overline{\mathrm{MS}}}_{t,\mathrm{SM}} = -\frac{(g^{\overline{\mathrm{DR}}}_{s,\mathrm{MSSM}})^4\, m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}}}{4608\pi^4}\Bigg[&396\ln^2\frac{(m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}})^2}{Q^2} - 1452\ln\frac{(m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}})^2}{Q^2}\\ &+ 2053 - 48\zeta(3) + 16\pi^2(1+\ln 4)\Bigg],\end{aligned} \tag{4.4}$$

where ζ denotes the zeta function. This expressions will be important for discussions in chapter 5.

By including a renormalization scheme conversion and threshold corrections, the gauge couplings $\{g_s,\, g_w,\, g_y\}$ of the SUSY model are obtained from the SM gauge couplings. The two VEVs v_u and v_d are calculated from the SM VEV v and the input value $\tan\beta$

$$v_u = \frac{v\tan\beta}{\sqrt{1+\tan^2\beta}}, \qquad v_d = \frac{v}{\sqrt{1+\tan^2\beta}}. \tag{4.5}$$

A more detailed description for the corrections which have been included can be found in [23].

Furthermore, when RG running is performed in the SUSY model between the *SUSY scale* and EW scale two-loop beta functions are used. Thereafter, the code of FlexibleEFTHiggs solves the EWSB conditions and performs a pole mass matching of the Higgs boson mass in order to fix the SM parameter λ. Both steps are done at one-loop order precision.

SUSY scale: At this scale the SUSY scale constraints are applied as described earlier. If not mentioned otherwise, the *SUSY-SM matching scale* is considered to be the *SUSY scale*. However, beside the *SUSY-SM matching scale* it is useful to introduce this scale. For the performed method of estimating the theoretical uncertainty these two scales are considered to differ.

This procedure is repeated until a set of fitting parameters is found and then the calculated mass spectrum is written in the output file. This profound and efficient logic of pole mass matching allows FlexibleEFTHiggs to be applicable for phenomenological studies of many supersymmetric models. Thus, the new method FlexibleEFTHiggs has been validated for different SUSY models in ref. [26] with a λ matching at next to leading order (NLO).

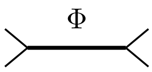
This work is considered as an extension of FlexibleEFTHiggs in order to refine the Higgs mass calculation in the MSSM. To achieve this, the FlexibleEFTHiggs Tower code has been extended in this work by an NNLO matching for λ and a consistent pole mass calculation. This means that the algorithm has been generalized at the discussed point M_t *scale* such that the calculation of the Higgs boson mass is done at two-loop order in the SM. As it is described in chapter 5, the SM self energy at $\mathcal{O}(\alpha_s\alpha_t)$ was in this work derived from the SM effective potential and the MSSM self energy was adopted from the implemented code in FlexibleSUSY. Furthermore the λ matching, described in *SUSY scale*, has been extended in this work to consider leading effects from two-loop order. In particular,the consideration of two-loop EWSB equations is implemented and the pole mass matching is performed at two loop. Henceforth, a new notation is used in this thesis. The method of FlexibleEFTHiggs with NLO and NNLO λ matching is denoted as FlexibleEFTHiggsNLO and FlexibleEFTHiggsNNLO, respectively.

5 Higgs Boson Mass Calculation

This chapter starts off with an introduction in the theoretical background of the method FlexibleEFTHiggs and explains how the resummation of large logarithms is combined with diagrammatic calculations.

5.1 Effective Field Theory Technique

If a fundamental QFT exhibits a large mass gap between two or more characteristic scales, a low energy approximation can be found. In particular one can find a QFT which only involves light particles, if the in- and outgoing momenta are considered to be much smaller in comparison to the typical high mass scale of the theory. A heuristic explanation is given in the case of two scalar fields ϕ (light) and Φ (heavy) associated with two scales m_L and M_H, respectively. If the heavy scale M_H is much higher compared to the momenta Q of the scattering process $\phi + \phi \to \phi + \phi$, the propagator of heavy particles can be expanded as



$$\propto -\frac{i}{Q^2 - M_H^2} = \frac{i}{M_H^2}\left[1 + \frac{Q^2}{M_H^2} + \mathcal{O}\left(\frac{Q^4}{M_H^4}\right)\right]. \tag{5.1}$$

Truncating the Taylor series at finite order yields a local interaction term for the effective Lagrangian e.g. in the first order gives the term $\mathcal{L}_{\mathrm{EFT}\phi^4} \propto \frac{1}{M_H^2}\phi^4$. As shown above these new interactions are suppressed by a high mass scale and, therefore, non-renormalizable if the couplings posses negative mass dimension. In this work, the SM is regarded as the EFT of the MSSM without suppressed interaction terms.

Taking loop corrections into account, the theorem by Appelquist and Carazzone [27] states that the contribution of heavy particles in loops is suppressed and thus the heavy scale is decoupled. The existence of the EFT can be proven conveniently in the path integral formalism, where Green functions are generated by functional derivatives of the partition function $\mathcal{Z}(j_l, J_H)$. Since only small momenta are considered, the current of heavy particles is neglected, $J_H = 0$, leading to

$$\mathcal{Z}(j_l, 0) = \int \mathcal{D}\phi \mathcal{D}\Phi \, e^{i \int \mathrm{d}^4x \mathcal{L}(\phi,\Phi) + j_l \phi} \tag{5.2}$$

$$= \int \mathcal{D}\phi_l \, e^{i \int \mathrm{d}^4x \mathcal{L}_{\mathrm{EFT}}(\phi_l) + j_l \phi_l}. \tag{5.3}$$

The assumption was made that the light field, regarded as function of the momentum, can be decomposed into fields with low and high momentum $\phi = \phi_l + \phi_h$. After the integral over the field configuration $\mathcal{D}\phi_h \mathcal{D}\Phi$ has been carried out, the partition function for the EFT remains. Hence, the effective field theory has to exist. This is characterized by the derived Lagrange density $\mathcal{L}_{\mathrm{EFT}}(\phi_l)$, which describes light fields ϕ_l with small values of the in-and outgoing momenta only.

5.2 Fixed order calculations in the MSSM

In the first approaches to study radiative corrections to the mass of the lightest CP-even Higgs Boson of the MSSM much effort has been invested in calculating the perturbative expansion of the self energy in a fixed order. Hence, the Feynman diagrammatic computations are characterized by the power of the involved couplings. FlexibleSUSY is based on SARAH, which calculates one loop self energies, and therefore provides a framework to determine the pole mass with diagrammatic tools. Generated by FlexibleSUSY, some spectrum generators perform mass calculations at a fixed order. If not mentioned otherwise a spectrum generator is denoted as FlexibleSUSY, it is assumed that the provided calculation for the pole mass of the Higgs boson is done at fixed order [1] in the MSSM. As mentioned before, in the calculation of the Higgs boson pole mass FlexibleSUSY will impose conditions to the model parameters at the SUSY scale. By using two-loop RGEs, the $\overline{\text{DR}}$ model parameters run to the low scale $Q = m_Z$. At the EW scale some parameters are fixed by the EWSB relations. At tree level they are obtained by eq. (3.58). In this work the parameters μ and $B\mu$ are fixed by these equations including two loop corrections [2]. Furthermore, the gauge and Yukawa couplings can be obtained from the experimental data e.g. Weinberg angle, pole masses, coupling constants. A very important quantity, especially for Higgs boson mass calculations, is the top Yukawa coupling. This is determined by the pole mass M_t at NNLO. After finding the best fitting set of $\overline{\text{DR}}$ parameters for both high and low scale constraints, FlexibleSUSY will determine the mass spectrum. Especially, for the uncharged and CP-even Higgs boson mass matrix the spectrum generator FlexibleSUSY will calculate the eigenvalues of the loop corrected mass matrix

$$\det[p^2\mathbb{1} - M^2_{\phi_u\phi_d,\text{tree}} - \operatorname{Re}\Pi^{1L}_{\phi_u\phi_d,\text{MSSM}}(p) - \operatorname{Re}\Pi^{2L}_{\phi_u\phi_d,\text{MSSM}}(p=0)] = 0, \tag{5.4}$$

where $M^2_{\phi_u\phi_d,\text{tree}}$ represents the tree level mass and $\Pi(p)$ the renormalized negative self energy and tadpoles. The included radiative corrections are complete at one-loop order. At two-loop order only leading momentum independent contributions are taken into account in FlexibleSUSY $\mathcal{O}(\alpha_s(\alpha_b + \alpha_t) + (\alpha_b + \alpha_t)^2 + \alpha_\tau^2)$ [28–31].

5.3 Resummation of Large Logarithms

In fixed order calculations, loop corrections from supersymmetric partners lead to logarithms of the ratio of the typical SUSY scale m_s and the renormalization scale Q: $\ln^i(m_s/Q)$. Independently of the renormalization scale, logarithms of the form $\ln(m_s/v)$ will always appear in the calculation of the Higgs boson pole mass. Since experimental evidence for SUSY partners are missing so far, it is reasonable to assume that the SUSY scale lies far above the electroweak scale M_{EW}. Thus, the expansion series will have high powers in the large logarithms $\ln(m_s/Q)$. Hence, very high orders in perturbation theory are necessary to guarantee a reliable value for the Higgs boson pole mass. Because this method does not yield a fast convergence, new ways must be found to take radiative corrections into account. By utilizing EFT and RG techniques, large logarithms can be resummed to all orders in perturbation theory, which requires all SM parameters to be defined in the $\overline{\text{MS}}$ scheme. This method is based on three steps:

[1] To ensure a high precision, it will always be at two loop order.

[2] This is the case for a high precision calculation. For a lower precision the EWSB conditions can be applied at one loop, but this is not considered in this thesis

1) Matching at high scale: The basic assumption is that all SUSY fields and extra Higgs bosons have a common high scale m_s. [3] For all SM parameters a matching condition relates them to model parameters at the large scale m_s. For example, in the MSSM the pole mass matching condition together with eq. (3.65) for the coupling λ out of eq.(2.33) evaluated at the high scale and leading order reads

$$\lambda(m_s) = \frac{1}{4}(g_w^2 + g_y^2)\cos^2 2\beta\,. \tag{5.5}$$

The gauge couplings of the $SU(2)_L$ and $U(1)_Y$ agree between the SM and MSSM at LO. For the correct use of RG running the SM top Yukawa coupling has to be found. Once more pole mass matching at LO provides the simple relation

$$(y_t \equiv) y_{t,\mathrm{SM}} = y_{t,\mathrm{MSSM}} \sin\beta\,, \tag{5.6}$$

where $y_{t,\mathrm{SM}}$ is the $\overline{\mathrm{MS}}$ coupling in the SM and $y_{t,\mathrm{MSSM}}$ represents the $\overline{\mathrm{DR}}$ coupling in the MSSM.

2) Running to the lower scale: As mentioned before, the SM parameters are determined in the $\overline{\mathrm{MS}}$ scheme. Thus, beta functions are used to run between the scales

$$\frac{d}{d\ln Q}\lambda = \sum_{i=1}^{\infty}\frac{1}{(4\pi)^i}\beta_\lambda^{(i)}, \tag{5.7}$$

$$\beta_\lambda^{(1)} = 12(\lambda^2 + \lambda y_t^2 - y_t^4)\,. \tag{5.8}$$

Because beta functions $\beta_\lambda^{(i)}$ depend also on other couplings, which for themselves obey renormalization group equations, eq. (5.7) cannot be solved analytically in general. However, numerical algorithms are suitable for solving the set of highly coupled differential equations and guarantee a high precision for the EFT approach [32]. For the purpose of encountering the principles behind the resummation of logarithms, the analytic approach is continued. This can be done by expanding the beta function in powers of large logarithms

$$\beta_\lambda(t) = \sum_{n=1}^{\infty}\frac{1}{(4\pi)^{2n}}\sum_{i=0}^{\infty}\frac{\beta_\lambda^{(n,i)}(\tilde{t})}{i!}(t-\tilde{t})^i, \tag{5.9}$$

where

$$t = \ln\frac{m_{\mathrm{scale}}}{Q}, \qquad \tilde{t} = \ln\frac{m_s}{Q}, \qquad \beta_\lambda^{(n,i)}(t) := \frac{d^i}{dt^i}\beta_\lambda^{(n)}(t). \tag{5.10}$$

By inserting the expansion (5.9) in eq. (5.7), the encountered differential equation can be solved conveniently

$$\lambda(t) = \lambda(\tilde{t}) - \sum_{n=1}^{\infty}\frac{1}{(4\pi)^{2n}}\sum_{i=1}^{\infty}(-1)^{i+1}\frac{\beta_{\lambda_t}^{(n,i-1)}(\tilde{t})}{i!}(t-\tilde{t})^i. \tag{5.11}$$

[3] It is not a conceptual difficult to work with a non-degenerated mass spectrum for the fields beyond the SM, but without loss of generality this is not considered.

Thus, eq. (5.11) demonstrates the resummation of large logarithms explicitly if the tree-level mass is calculated at the low scale m_t:

$$m_h^2(m_t) = v^2\lambda(m_t), \tag{5.12}$$

$$\lambda(m_t) = \lambda(m_s) + \frac{12}{(4\pi)^2}(y_t^4 - \frac{1}{2}\lambda y^2 - \frac{1}{4}\lambda^2)\ln\frac{m_s}{m_t} + \ldots . \tag{5.13}$$

The resummed logarithms depend on the considered loop order n of the β functions in eq. (5.11). Hence, for every loop order k one can obtain the resummed logarithms of powers ranging from $k-n+1$ to n.

3) Calculating the pole mass of the Higgs boson: In the last step the pole mass of the lightest CP-even Higgs boson is obtained by

$$M_h^2 = m_h^2(M_t) - \hat{\Sigma}_h^{\text{SM}}(p^2 = M_h^2, Q = M_t) + \hat{t}_h(Q = M_t), \tag{5.14}$$

where the renormalized radiative correction $\hat{\Sigma}_h + \hat{t}_h$ is calculated in the SM. At the EW scale M_t, the values of SM parameters are extracted from experimental data of pole masses and couplings, which fixes all parameters needed for the Higgs mass calculation.

Thus, the pole mass contains implicitly all leading logarithms beyond any finite loop order in perturbation theory. This method can be improved by considering higher orders in the MSSM-SM matching of the parameters. This means in particular that the SM Higgs boson coupling λ becomes threshold corrections from higher orders

$$\lambda(m_s) = \frac{1}{4}(g_w^2 + g_y^2)\cos^2 2\beta + \Delta\lambda^{(1)} + \Delta\lambda^{(2)}. \tag{5.15}$$

By taking higher corrections $\Delta\lambda^{(m)}$ into account, the occurring N^m leading logarithms are resummed correctly, when the pole mass is calculated at the low scale $Q = M_t$ only by the inclusion of renormalization group equations at $m+1$ loop order. This approach is implemented in FlexibleSUSYs HSSUSY and SUSYHD [33]. In particular HSSUSY is regarded as the FlexibleSUSY version of SUSYHD, since both follow the same strategy and use the same corrections for $\Delta\lambda^{(1)}$ and $\Delta\lambda^{(2)}$. These can be obtained from ref. [34] and [33], respectively. As mentioned before the loop order of beta functions plays an important role. Hence, HSSUSY and SUSYHD are using RG running for λ at three-loop order [35,36]. FlexibleSUSYs HSSUSY calculates the gauge and Yukawa couplings at one-loop and leading two-loop corrections. The Higgs boson pole mass is determined with one loop and leading two-loop contributions at $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$. In contrast SUSYHD uses a fit formula to include all NNLO corrections at the low scale [33].

5.4 Pole Mass Matching in λ

In general, the matching of parameters ensures the equivalence of two theories. In the low energy regime the heavy scale ought to decouple from the effective description of the full theory. To fix the parameters in the effective theory a matching condition can be defined by demanding the equality of Green functions in both theories at a given scale. The matching corrections for the spectrum generator HSSUSY, and likewise SUSYHD, do not include suppressed terms of order $\Delta\lambda \propto (m_t^2/m_s^2)$ in eq. (5.15). Since such terms increase their influence with SUSY scale in the sub-TeV range, HSSUSY and SUSYHD produce a large theoretical uncertainties in the low-M_{SUSY} regime. This can be avoided by the inclusion of higher-order operators in the EFT below the SUSY scale. However, for HSSUSY/SUSYHD this is not considered because the spectrum generator assumes the SM without higher dimensional operators as the valid EFT. In order to include these terms in an EFT-based approach, a new computation method has been developed and examined in ref. [26]. By using a different matching procedure, FlexibleEFTHiggs combines diagrammatic calculations with EFT-based resummation of logarithms in the computation of the lightest Higgs boson mass.
For the SM coupling λ one has the freedom to choose between different matching conditions e.g. by comparing four point functions or two point functions. In an expansion series it is not well defined, whether a matching condition has to be preferred. The equivalence matching condition between equating pole masses and four point functions has been proven in ref. [26] at one-loop order. The conceptual motivation why this also hold at two-loop order is: the fixing of one parameter is done by using the same physical quantity in both models.

This chapter elaborates the matching of the SM coupling λ by imposing the equality of the lightest CP-even Higgs boson pole mass in the MSSM and the Higgs boson mass in the SM at the SUSY scale

$$M_{h,\text{MSSM}}^2 = M_{h,\text{SM}}^2, \qquad Q = M_{SUSY}. \tag{5.16}$$

In general, the pole mass at two-loop order can be written as

$$M_{h,\text{pole}}^2 = m_{h,\text{model}}^2 - \hat{\Sigma}_{h,1L,\text{model}} + \hat{t}_{h,1L,\text{model}} - \hat{\Sigma}_{h,2L,\text{model}} + \hat{t}_{h,2L,\text{model}}. \tag{5.17}$$

In eq. (5.20) the $\hat{\Sigma}$ and $\hat{t}$ denote the one- and two-loop renormalized self energy and tadpole, respectively. In the MSSM, corrections to the mass of the lightest CP-even Higgs boson are obtained as the eigenvalues of the mass matrix with loop corrections. This differs from the SM, hence one can define the quantities in eq. (5.17) as

$$\hat{\Sigma}_{h,\text{MSSM}} \equiv c_\alpha^2 \hat{\Sigma}_{\phi_u\phi_u,\text{MSSM}} + s_\alpha^2 \hat{\Sigma}_{\phi_d\phi_d,\text{MSSM}} + 2 s_\alpha^2 c_\alpha^2 \hat{\Sigma}_{\phi_u\phi_d,\text{MSSM}}\,, \tag{5.18}$$

$$\hat{t}_{h,\text{MSSM}} \equiv c_\beta^2 \hat{t}_{\phi_d,\text{MSSM}} + s_\beta^2 \hat{t}_{\phi_u,\text{MSSM}}\,. \tag{5.19}$$

It is convenient to consider the decoupling limit, where the mass of the CP-odd Higgs boson is much larger than the EW scale, $m_A^2 \gg M_Z^2$. In this practical limit the mixing angle obeys $\alpha = \beta - \pi/2$, which infers $\cos\alpha = \sin\beta$. In general the mixing angle α at two loop is different from the mixing angle at tree level. Since this difference in the mixing angle is small for the examined contributions to the Higgs boson mass, one can approximate α by the mixing angle at tree level [37]. Henceforth, this limit is considered in the entire chapter.

5.4.1 Momentum Iteration

In this section, the implications of the momentum iteration of the self energy, which affect the matching condition at two-loop order are studied. Determining the pole mas needs an evaluation of the self energy at the pole momentum $p_{\text{pole}} = M_h$. Since this work considers only leading two loop effects, the expansion of the one loop self energy around the tree-level mass $p = m_h$ is truncated after one iteration and therefore yields a two loop contribution. This correction is elaborated in this work and an important point for the comparison with other methods for pole mass matching. In general, one determines the MSSM pole mass momentum p^2_{pole} which satisfies the condition

$$p^2_{\text{pole}} - m^2_{h,\text{MSSM}} + \hat{\Sigma}_{h,\text{MSSM}}(p^2 = p^2_{\text{pole}}) - \hat{t}_{h,\text{MSSM}} = 0. \tag{5.20}$$

To ensure compact formulas the argument of the self energy is suppressed. Henceforth, the notation without an argument implies that the self energy $\hat{\Sigma}$, or its derivative $\hat{\Sigma}'$ with respect to p^2, is evaluated at $p^2 = m^2_{h,\text{MSSM}}$.

$$\hat{\Sigma}'_{h,\text{model}} := \frac{d}{d(p^2)} \hat{\Sigma}_{h,\text{model}}(p^2)|_{p^2=m^2_{h,\text{MSSM}}}. \tag{5.21}$$

Moreover, a second abbreviation is introduced as

$$\Delta m^2_{h,\text{model}} := -\hat{\Sigma}_{h,1L,\text{model}} + \hat{t}_{h,1L,\text{model}} \,. \tag{5.22}$$

An expansion of $\hat{\Sigma}(p^2 = p^2_{\text{pole}})$ around the tree-level mass $m^2_{h,\text{MSSM}}$ introduces a two loop correction in eq. (5.20)

$$p^2_{\text{pole}} = m^2_{h,\text{MSSM}} + \Delta m^2_{h,\text{MSSM}} - \hat{\Sigma}'_{h,\text{MSSM}} \Delta m^2_{h,\text{MSSM}}. \tag{5.23}$$

For later calculations, the gaugeless limit is considered often, i.e. the gauge couplings obey $g_y = g_w = 0$. In this limit the MSSM tree level mass vanishes $m_{h,\text{MSSM}} = 0$.[4] Subsequently, the pole mass matching condition requires that the same momentum p^2_{pole} has to satisfy the same condition (5.20) in the SM

$$0 = p^2_{\text{pole}} - m^2_{h,\text{SM}} + \hat{\Sigma}_{\text{SM}}(p^2 = p^2_{\text{pole}}) - \hat{t}_{h,\text{SM}}, \tag{5.24}$$

$$p^2_{\text{pole}} = m^2_{h,\text{SM}} + \Delta m^2_{h,\text{SM}} - \hat{\Sigma}'_{h,\text{SM}} \Delta m^2_{h,\text{MSSM}}. \tag{5.25}$$

The eq. (5.24) contains $m^2_{h,\text{SM}} = \lambda v^2$ and can therefore be regarded as a condition for the additional SM parameter λ. Using (5.22) and (5.23), this equation can now be solved for λ

$$\lambda = \frac{1}{v^2} \left[m^2_{h,\text{MSSM}} + \Delta m^2_{h,\text{MSSM}} - \Delta m^2_{h,\text{SM}} - \hat{\Sigma}'_{\text{MSSM}} \Delta m^2_{h,\text{MSSM}} + \hat{\Sigma}'_{\text{SM}} \Delta m^2_{h,\text{MSSM}} \right]. \tag{5.26}$$

[4] In the gaugeless limit the matched tree-level Higgs boson mass is also vanishing in the SM at the high scale. Through effects of RG running, the tree-level mass is shifted by beta functions yielding a non-vanishing quantity at the EW scale.

From eq. (5.26) one can identify the two-loop contribution coming from momentum iteration easily

$$\Delta m^2_{h,p-\text{iter}} := \Delta m^2_{h,\text{MSSM}} \left[\hat{\Sigma}'_{h,1L,\text{SM}} - \hat{\Sigma}'_{h,1L,\text{MSSM}}\right], \tag{5.27}$$

where both self energies are evaluated at $p^2 = m_{h,\text{MSSM}} \propto g_y^2 + g_w^2 = 0$.

5.4.2 Corrections to the Higgs Boson Pole Mass at $\mathcal{O}(\alpha_t)$

The leading contributions to the Higgs boson mass result from top Yukawa contributions at one loop level. For practical reasons, only these corrections in the case of degenerated soft breaking masses are taken into account, as described in eq. (5.28). It is convenient to choose a common limit. To keep the analytical expression short one considers the following form for the two eigenvalues of the stop mass matrix

$$\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}, \qquad m^2_{\tilde{t},1} = m^2_{\tilde{t}} - m_t X_t, \qquad m^2_{\tilde{t},2} = m^2_{\tilde{t}} + m_t X_t, \tag{5.28}$$

where θ denotes the mixing angle of stop mass matrix. This is achieved in the gaugeless limit for a degenerated soft mass spectrum [5]

$$m_s^2 \equiv M_3^2 = m^2_{Q_3} = m^2_{U_3}, \tag{5.29}$$

and the soft breaking term is the dominating part in the diagonal elements of the stop mass matrix in eq. (3.69) parameterized by the mean value

$$m^2_{\tilde{t}} \equiv (M^2_{\tilde{t}_3})_{11} = (M^2_{\tilde{t}_3})_{22} = m_s^2 + m_t^2\,. \tag{5.30}$$

Thus, contributions from EW gauge couplings are consistently neglected. In this convenient limit the stop mixing is still arbitrary and allows the conclusion that the obtained results also hold for a more general case. If not stated otherwise, this notation is used in the this chapter henceforth. Because of the decoupling limit $c_\alpha^2 \to s_\beta^2$ the one-loop top Yukawa contribution $\Delta m^2_{h,\text{MSSM}}$ possesses a convenient form at the SUSY scale, where $m_{h,\text{MSSM}} = 0$

$$\begin{aligned}
-\hat{\Sigma}_{h,\text{MSSM}}(p^2) + \hat{t}_{h,\text{MSSM}} = s_\beta^2 \Bigg\{ & -\hat{\Sigma}^{m_t^{\text{MSSM}}}_{h,\text{SM}}(p^2) + \hat{t}^{\,m_t^{\text{MSSM}}}_{h,\text{SM}} \\
& - \frac{3(y^{\overline{\text{DR}}}_{t,\text{MSSM}})^2}{2(4\pi)^2} \Big\{ (2m_t + X_t)^2 B_0\left(p^2, m^2_{\tilde{t},1}, m^2_{\tilde{t},1}\right) \\
& + (2m^{\overline{\text{DR}}}_{t,\text{MSSM}} - X_t)^2 B_0\left(p^2, m^2_{\tilde{t},2}, m^2_{\tilde{t},2}\right) \\
& - \frac{X_t}{m^{\overline{\text{DR}}}_{t,\text{MSSM}}} \left[A_0\left(m^2_{\tilde{t},1}\right) - A_0\left(m^2_{\tilde{t},2}\right)\right] \Big\} \Bigg\},
\end{aligned} \tag{5.31}$$

[5]The notation for M_{SUSY}, that was introduced in previous sections, reduces to m_s in the considered case.

$$-\hat{\Sigma}^{m_t^{\text{mod}}}_{h,\text{SM}}(p^2) + \hat{t}^{\,m_t^{\text{mod}}}_{h,\text{SM}} := \frac{6(y_{t,\text{mod}})^2}{(4\pi)^2}\left(\frac{4(m_{t,\text{mod}})^2 - p^2}{2}\right) B_0(p, m_{t,\text{mod}}, m_{t,\text{mod}}), \tag{5.32}$$

$$\Delta m^2_{h,\text{MSSM}} = s^2_\beta \frac{12(m^{\overline{\text{DR}}}_{t,\text{MSSM}})^2 (y^{\overline{\text{DR}}}_{t,\text{MSSM}})^2}{(4\pi)^2}\left[\ln\frac{m_s^2}{(m^{\overline{\text{DR}}}_{t,\text{MSSM}})^2} + \frac{X_t^2}{m_s^2} - \frac{X_t^4}{12 m_s^4}\right]. \tag{5.33}$$

In the calculation of the Higgs boson self energy Passarino-Veltman integrals $\tilde{B}_0(x, y, z)$ and $\tilde{A}_0(x)$ occur through one-loop calculations. These integrals are carried out in $D = 4 - 2\epsilon$ dimensions. As a consequence of regularization $1/\epsilon$-poles are obtained in the evaluation of loop integrals. The divergent quantity Δ contains this pole. In the considered regularization scheme, the poles are eliminated by counter terms, leaving a finite part as function of the involved masses. The functions $B_0(x, y, z)$ and $A_0(x)$ represent this finite part of the one loop integrals. It is sufficient for this work to consider the finite part in dependence of two different masses M and m with $M \gg m$

$$\tilde{A}_0(m) = m^2\left[\Delta - \ln\frac{m^2}{Q^2} + 1\right] = A_0(m) + \Delta, \tag{5.34}$$

$$\tilde{B}_0(m, M, M) = \Delta - \ln\frac{M^2}{Q^2} + \frac{m^2}{6M^2} + \frac{m^4}{60M^4} + \mathcal{O}\left(\frac{m^6}{M^6}\right) = B_0(m, M, M) + \Delta. \tag{5.35}$$

Those relations are used to obtain eq. (5.33).

5.4.3 Two Loop Corrections from the Effective Potential

In this section, it is explained how the two-loop contributions to the two-loop self energy and two-loop tadpoles are obtained for the leading orders $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$. The interesting momentum independent part $\Pi^{2L}(0)$ is derived from the effective potential at the considered loop order

$$\Pi^{2L}_{h,\text{model}}(M_{h,pole}) \equiv -\hat{\Sigma}_{h,2L,\text{model}}(p^2 = M^2_{h,pole}) + \hat{t}_{h,2L,\text{model}}\,, \tag{5.36}$$

$$\Pi^{2L}_{h,\text{model}}(M_{h,pole}) = \Pi^{2L}_h(0) + [\Pi^{2L}_h(M_{h,pole}) - \Pi^{2L}_h(0)]\,. \tag{5.37}$$

The last bracket yields corrections proportional to λ in the SM

$$\Pi^{2L}_h(m_h) - \Pi^{2L}_h(0) \simeq m_h^2 \left.\frac{d}{dp^2}\Pi^{2L}_h\right|_{p^2=0}, \tag{5.38}$$

where m_h denotes the tree-level mass in the MSSM and the SM. Without loss of generality the practical gaugeless limit is considered. Hence, two-loop terms in combination with the tree-level mass are neglected. Since the matching condition is only evaluated for the leading two loop contributions, the momentum iteration would lead to higher order contributions. Thus, it is reasonable to neglect these kinds of corrections. The momentum independent part in eq. (5.37) is determined by the effective potential at two-loop order

$$V_{\text{Eff}} = V_{\text{Higgs}} + V^{1L}_{\text{Higgs}} + V^{2L}_{\text{Higgs}}, \tag{5.39}$$

$$\Pi^{2L}_{ij,\text{MSSM}}(0) = \left[-\frac{1}{\phi_i}\frac{\partial}{\partial\phi_i}\delta_{ij} + \frac{\partial^2}{\partial\phi_i\partial\phi_j}\right]\Bigg|_{\phi_u=v_u,\phi_d=v_d} V^{2L}_{\text{Higgs,MSSM}}, \tag{5.40}$$

$$\Pi^{2L}_{h,\text{SM}}(0) = \left[-\frac{1}{\phi}\frac{\partial}{\partial\phi} + \frac{\partial^2}{(\partial\phi)^2}\right]\Bigg|_{\phi=v} V^{2L}_{\text{Higgs,SM}}. \tag{5.41}$$

Under the considered assumptions one may identify the leading two loop contributions to the self energy of the Higgs boson with $\Pi^{2L}_h(0)$ in both models. The self energy correction in the MSSM can be obtained in the considered decoupling limit as

$$\Pi^{2L}_{h,\text{MSSM}} = c^2_\beta\Pi^{2L}_{11,\text{MSSM}}(0) + s^2_\beta\Pi^{2L}_{22,\text{MSSM}}(0) + 2s_\beta c_\beta\Pi^{2L}_{12,\text{MSSM}}(0) + \mathcal{O}\left(\frac{m_Z^4}{m_A^2}\right). \tag{5.42}$$

5.5 Pole Mass Matching EFT-Tower at $\mathcal{O}(\alpha_s\alpha_t)$ in λ

In order to compare the matching corrections from FlexibleEFTHiggsNNLO with the MSSM threshold corrections derived from matching of the four point functions in ref. [34], this section examines the $\mathcal{O}(\alpha_s\alpha_t)$ contributions to the Higgs boson pole mass. The $\mathcal{O}(\alpha_S\alpha_t)$ contributions in the MSSM result from $\Pi^{2L}_{h,\text{MSSM}}$. Furthermore, the one loop correction to the Higgs boson pole mass $\Delta m^2_{h,\text{MSSM}}$ is expressed in the MSSM by the SUSY parameter $m^{\overline{\text{DR}}}_{t,\text{MSSM}}$ and $\Delta m^2_{h,\text{SM}}$ by $m^{\overline{\text{MS}}}_{t,\text{SM}}$. The difference of both one-loop contributions occurs in the λ matching and has to be calculated with quantities defined in the $\overline{\text{MS}}$-scheme. The MSSM top mass can be matched by considering scheme conversion and threshold effects to a SM parameter in the $\overline{\text{MS}}$ scheme. The general case, including $SU(2)_L \otimes U(1)_Y$ gauge coupling contributions, is listed in ref. [34]. Since only α_s contributions in the top mass matching yield a $\mathcal{O}(\alpha_s\alpha_t)$ contribution in $\Delta m^2_{h\text{MSSM}}$ the eq. (22) in ref. [34] reduces to

$$m^{\overline{\text{DR}}}_{t,\text{MSSM}}(Q) = m^{\overline{\text{MS}}}_{t,\text{SM}}(Q)\left[1 - \frac{4g_s^2}{3(4\pi)^2}\left(1 + \ln\frac{m_s^2}{Q^2} - \frac{X_t}{m_s}\right)\right]. \tag{5.43}$$

It is emphasized that the top Yukawa coupling transforms with the same contributions as the top mass at $\mathcal{O}(\alpha_s)$. This equivalence for the m_t and y_t matching corrections is not manifest at a every order, since the corrections form the matching of the VEV have to be included.

Correction in λ at $\mathcal{O}(\alpha_t\alpha_s)$

Expressed in SM parameters in the $\overline{\text{MS}}$ scheme, the Higgs boson self energy in the order $\mathcal{O}(\alpha_s\alpha_t)$ in the MSSM is obtained from ref. [37]

$$\begin{aligned}\Pi^{2L}_{h,\text{MSSM}}|_{\alpha_t\alpha_s} = \frac{32y_t^2g_s^2m_t^2}{(4\pi)^4}\Bigg\{&3\ln^2\frac{m_t^2}{m_s^2} + \ln\frac{m_t^2}{m_s^2} - 1\\ &+\frac{X_t}{m_s}\left[-1 + 2\ln\frac{m_t^2}{m_s^2}\right] + \frac{X_t^2}{m_s^2} + \frac{X_t^3}{3m_s^3} - \frac{X_t^4}{12m_s^4}\Bigg\},\end{aligned} \tag{5.44}$$

where the renormalization scale is set to $Q = m_s$. The NLO conversion at $\mathcal{O}(\alpha_t)$ of the

MSSM parameters m_t and y_t in $\Delta m^2_{h,\mathrm{MSSM}}$ yields contributions of $\mathcal{O}(\alpha_t^2)$

$$\Delta m^{2,\mathrm{con}}_{h,\mathrm{MSSM}}|_{\alpha_t\alpha_s} = \frac{32 y_t^2 g_s^2 m_t^2}{(4\pi)^4}\left[2\left(1-\frac{X_t}{m_s}\right)\ln\frac{m_t^2}{m_s^2}+1-\frac{X_t}{m_s}-2\frac{X_t^2}{m_s^2}+\frac{X_t^4}{6m_s^4}\right], \tag{5.45}$$

where $\ln(m_t/m_s) \equiv \ln(m^{\overline{\mathrm{MS}}}_{t,\mathrm{SM}}/m_s)$ and $y_t^2 g_s^2 m_t^2 \equiv (g^{\overline{\mathrm{MS}}}_{s,\mathrm{SM}})^2 (y^{\overline{\mathrm{MS}}}_{t,\mathrm{SM}})^2 (m^{\overline{\mathrm{MS}}}_{t,\mathrm{SM}})^2$. For the full two loop correction at $\mathcal{O}(\alpha_s\alpha_t)$ it is not necessary to consider NLO m_t threshold corrections in the self energy Π^{2L}_{MSSM} of the Higgs boson, because they would yield a contribution of three loop $\mathcal{O}(\alpha_s^2\alpha_t)$. Furthermore, the renormalization scale was fixed to the SUSY scale m_s. The SM two loop correction to the Higgs boson pole mass has been calculated in this work by the evaluation of eq. (5.41) with the effective potential in ref. [38]. Setting the renormalization scale $Q = m_s$ one finds

$$\Pi^{2L}_{h,\mathrm{SM}}|_{\alpha_t\alpha_s} = \frac{32 g_s^2 y_t^2 m_t^2}{(4\pi)^4}\left[3\ln^2\frac{m_t^2}{m_s^2}+\ln\frac{m_t^2}{m_s^2}\right]. \tag{5.46}$$

At this order no contributions arise from momentum iteration. Using eq. (5.16), the two loop $\mathcal{O}(\alpha_s\alpha_t)$ corrections reads

$$\Delta\lambda|_{\alpha_s\alpha_t} = \frac{1}{v^2}\left[\Pi^{2L}_{h,\mathrm{MSSM}} + \Delta m^{2,\mathrm{con}}_{h,\mathrm{MSSM}} - \Pi^{2L}_{h,\mathrm{SM}}\right], \tag{5.47}$$

$$= \frac{16 g_s^2 y_t^2 m_t^2}{3(4\pi)^4 v^2}\left[-12\frac{X_t}{m_s} - 6\frac{X_t^2}{m_s^2} + 14\frac{X_t^3}{m_s^3} - \frac{X_t^4}{2m_s^4} - \frac{X_t^5}{m_s^5}\right]. \tag{5.48}$$

Therefore, the same matching correction for λ was recovered by pole mass matching as in ref. [34].[6] As one can obtain from eq. (5.48) easily, the $\mathcal{O}(\alpha_s\alpha_t)$ correction vanishes if the mixing parameter satisfies $X_t = 0$.

5.6 Pole Mass Matching EFT-Tower at $\mathcal{O}(\alpha_t^2)$ in λ

For deriving the two loop $\mathcal{O}(\alpha_t^2)$ contributions to the λ matching , SUSYHD imposes a slightly different matching condition than FlexibleEFTHiggsNNLO. Although this matching condition involves two point functions, both approaches differ by a subtlety. Nevertheless, the equivalence between both approaches is obtained and the difference in the chosen matching condition is discussed in this section. For deriving the λ-matching condition at $\mathcal{O}(\alpha_t^2)$ one can follow the same logic as for the $\mathcal{O}(\alpha_s\alpha_t)$ contributions in eq. (5.47). The limit, shown in the eq. (5.28), is considered throughout this section, since the two loop contributions, derived in ref. [33], are presented only in that case.[7] In the light of eq. (5.16), the matching condition has a similar form as in eq. (5.47)

$$\Delta\lambda|_{\alpha_t^2} = \frac{1}{v^2}\left[\Pi^{2L}_{h,\mathrm{MSSM}}|_{\alpha_t^2} + \Delta m^{2\mathrm{con}}_{h,\mathrm{MSSM}}|_{\alpha_t^2} - \Pi^{2L}_{h,\mathrm{SM}}|_{\alpha_t^2} + \Delta m^2_{h,p-\mathrm{iter}}|_{\alpha_t^2}\right], \tag{5.49}$$

where all two loop contributions are of $\mathcal{O}(\alpha_t^2)$. The additional contribution $\Delta m^2_{h,p-\mathrm{iter}}$ comes from the momentum iteration as described earlier. As it is shown in eq. (5.31) the

[6] In ref. [34] a different definition for the VEV v is used. But the matching correction $\Delta\lambda|_{\alpha_s\alpha_t}$ is independent of v and therefore the same.

[7] The MSSM two loop corrections in FlexibleEFTHiggsNNLO are implemented in a more general case.

momentum dependence is contained only in the B_0 expressions. The top Yukawa coupling is matched at tree level, in the considered eq. (5.31). Otherwise three loop terms would occur. Neglecting terms of $\mathcal{O}(m_h^2/m_t^2, v^2/m_s^2)$ the two loop effect of $\mathcal{O}(\alpha_s\alpha_t)$ in the momentum iteration reads

$$\Delta m^2_{h,p-\text{iter}}|_{\alpha_t^2} = -\frac{6y_t^4 m_t^2}{(4\pi)^4}\frac{X_t^2}{m_s^2}\left[\ln\frac{m_s^2}{m_t^2} + \frac{X_t^2}{m_s^2} - \frac{1}{12}\frac{X_t^4}{m_s^4}\right]. \tag{5.50}$$

As before the parameters without a label are considered to be SM parameters renormalized in the $\overline{\text{MS}}$-scheme .

Corrections for the Higgs boson pole mass from the effective potential $\mathcal{O}(\alpha_t^2)$

As in the case before, the considered self energy at two loop level is evaluated at zero momentum. In this work the SM momentum independent self energy was derived at (α_t^2). This is done by inserting the effective potential from ref. [38] in the eq. (5.41). Since the matching condition is applied at the scale $Q = m_s$, the SM result reads

$$\Pi^{2L}_{h,\text{SM}}|_{\alpha_t^2} = \frac{6y_t^4 m_t^2}{(4\pi)^4}\left[-3\ln^2\frac{m_t^2}{m_s^2} + 7\ln\frac{m_t^2}{m_s^2} - 2 - \frac{\pi^2}{3}\right]. \tag{5.51}$$

In ref. [39] the MSSM two loop self energy was derived by the effective potential at two loop $\mathcal{O}(\alpha_t^2)$. The correction is expressed in terms of MSSM parameters in the $\overline{\text{DR}}$ scheme. In ref. [33] these contributions are also presented in terms of SM parameters in the $\overline{\text{MS}}$ scheme. Because λ is expressed in terms of SM parameters , y_t and m_t are considered as SM parameters in the $\overline{\text{MS}}$ scheme. Since the matching condition is applied at the high scale, the correction is evaluated at $Q = m_s$

$$\begin{aligned}\Pi^{2L}_{h,\text{MSSM}}|_{\alpha_t^2} =&\frac{6y_t^4 m_t^2}{(4\pi)^4 s_\beta^2}\Bigg\{-3\ln\frac{m_s^2}{m_t^2} + 2[3f_2(\hat\mu) - 3f_1(\hat\mu) - 8]\ln\frac{m_s^2}{m_t^2} + 6\hat\mu^2 - 4(4-2\mu)f_1(\hat\mu)\\
&+ 4f_3(\hat\mu) - \frac{\pi^2}{3} + \frac{X_t^2}{m_s^2}\left[-10 - 6\hat\mu^2 - 4f_2(\hat\mu) + (4 - 6\hat\mu^2 f_1(\hat\mu))\right]\\
&+ \frac{X_t^4}{4m_s^4}\left[23 + 4\hat\mu^2 + 2f_2(\hat\mu) - 2(1-2\hat\mu^2)f_1(\hat\mu)\right] - \frac{1}{2}s_\beta^2\frac{X_t^6}{m_s^6}\\
&+ c_\beta^2\Bigg[3\ln^2\frac{m_s^2}{m_t^2} - 4\ln\frac{m_t^2}{m_s^2} - 3 + 60K + \frac{4\pi^2}{3} + \frac{X_t^2}{m_s^2}(12-24K)\\
&- \left(\frac{4X_tY_t}{m_s^2} + \frac{Y_t^2}{m_s^2}\right)(3+16K) - 6\frac{X_t^4}{m_s^4} + \frac{X_t^3Y_t}{m_s^4}(4+16K)\\
&+ \frac{X_t^2Y_t^2}{m_s^4}\left(\frac{14}{3} + 24K\right) - \frac{X_t^4Y_t^2}{m_s^4}\left(\frac{19}{12} + 8K\right)\Bigg]\Bigg\}.\end{aligned} \tag{5.52}$$

where the new parameter Y_t is given by the couplings $Y_t = A_t + \mu\cot\beta$ and $K = -0.1953256$. The ratio of the SUSY scale and the mass parameter, obtained from the MSSM superpotential, is denoted by $\hat{\mu} = \mu/m_s$. The functions f_i are defined as

$$f_1(x) = \frac{x^2}{1-x^2}\ln x^2, \tag{5.53}$$

$$f_2(x) = \frac{x^2}{1-x^2}\left[1 + \frac{x^2}{1-x^2}\ln x^2\right], \tag{5.54}$$

$$f_3(x) = \frac{2(x^2+x^4)-1}{(1-x^2)^2}\left[\ln x^2 \ln(1-x^2) + Li(x^2) - \frac{\pi^2}{6} - x^2\ln x^2\right], \tag{5.55}$$

where Li is the dilogarithm function.

Corrections for the Higgs boson pole mass from parameter conversion at $\mathcal{O}(\alpha_t^2)$

In the same manner as for the $\mathcal{O}(\alpha_s\alpha_t)$ contributions, a MSSM/SM conversion in the one loop correction $\Delta m^2_{h,\mathrm{MSSM}}$ yields two loop contributions in the λ matching. Unlike for the α_s correction, the conversion of the Yukawa coupling y_t and m_t differs at the NLO [40]

$$m^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}}(m_s) = m^{\overline{\mathrm{MS}}}_{t,\mathrm{SM}}(m_s)\left[1 + \frac{1}{(4\pi)^2}\frac{3y_t^2}{4s_\beta^2}\left(\frac{1}{2}(1+c_\beta^2) - \hat{\mu}^2 f_2(\mu)\right)\right], \tag{5.56}$$

$$y^{\overline{\mathrm{DR}}}_{t,\mathrm{MSSM}}(m_s) = \frac{y^{\overline{\mathrm{MS}}}_{t,\mathrm{SM}}}{s_\beta}(m_s)\left[1 + \frac{1}{(4\pi)^2}\frac{3y_t^2}{4s_\beta^2}\left(\frac{1}{2}(1+c_\beta^2) + \frac{1}{3}X_t^2 s_\beta^2 - \hat{\mu}^2 f_2(\mu)\right)\right], \tag{5.57}$$

where the parameters on the r.h.s are defined in the SM [34]. This difference is due to the VEV matching between the SM and MSSM, which provides a $\mathcal{O}(\alpha_t)$ contribution at NLO [40]. Inserting the equations (5.56) and (5.57) in (5.33) yields a two loop correction of $\mathcal{O}(\alpha_t^2)$

$$\begin{aligned}\Delta m^{2\mathrm{con}}_{h,\mathrm{MSSM}}|_{\alpha_t^2} =& \frac{6y_t^4 m_t^2}{(4\pi)^4 s_\beta^2}\Bigg\{\left(3\ln\frac{m_s^2}{m_t^2} - \frac{3}{2}\right)(c_\beta^2+1) - 6\hat{\mu} f_2(\hat{\mu})\ln\frac{m_s^2}{m_t^2} \\ &+ 3\hat{\mu}^2 f_2(\hat{\mu}) + \frac{X_t^2}{m_s^2}\left[(1+c_\beta^2)3 + s_\beta^2\ln\frac{m_s^2}{m_t^2} - 6\hat{\mu}^2 f_2(\hat{\mu})\right] \\ &+ \frac{X_t^4}{m_s^4}\left[\frac{3}{4} - \frac{5}{4}c_\beta^2 + \frac{1}{2}\hat{\mu}^2 f_2(\hat{\mu})\right] - s_\beta^2\frac{X_t^6}{12 m_s^6}\Bigg\}.\end{aligned} \tag{5.58}$$

Correction in λ at $\mathcal{O}(\alpha_t^2)$

Comparing the λ matching relations, the origin of all contributions in eq. (5.47) agrees with the one in the matching condition chosen by the authors of SUSYHD [33] with the exception of $\Delta m^2_{h,p-\mathrm{iter}}$. SUSYHD performs a general matching of two point functions which receives contributions from the difference between the renormalization of the Higgs boson wave function in the MSSM and in the SM. In the pole mass matching, these contributions are automatically included by the momentum iteration of the one loop self energy of the Higgs boson. This work therefore provides a proof of equivalence between the momentum iteration contributions and corrections from wave function renormalization. This is a non-trivial statement and holds even beyond the considered matching order $\mathcal{O}(\alpha_t^2)$. The proof shows that the strategy used in SUSYHD and pole mass matching are considering the same contributions with different origins. Evaluating equation (5.49) in the limit $\tan\beta \to \infty$, $\hat{\mu} =$

1 and $X_t = 0$ leads to a rather compact analytical formula

$$\Delta\lambda|_{\alpha_t^2} = \frac{3y_t^6}{(4\pi)^4}[12 + 16K]. \tag{5.59}$$

This formula agrees with the $\Delta\lambda$ correction in ref. [33] in the considered case. In summary, this chapter is simulating a pole mass matching for λ at $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$ in an equivalent way to the FlexibleEFTHiggsNNLO code. The obtained analytical formulas agree with the results derived by different strategies at the same order. Furthermore, the $\mathcal{O}(\alpha_t^2)$ corrections in public codes are only implemented for degenerated masses of SUSY partners and a special mixing angle as shown in eq.(5.28). Since the implemented two loop self energies for the examined code include a non-degenerated mass spectrum at the order $\mathcal{O}(\alpha_t^2)$, FlexibleEFTHiggsNNLO provides a generalized extension of the matching condition at two loop order.

In addition, it is mentioned that FlexibleEFTHiggsNLO/NNLO generates some logarithmic terms of three loop order in the matching condition for λ. They originate from the momentum iteration of the momentum dependent one loop self energy and from the MSSM/SM conversion of the model parameters in the two loop self energy contribution. Nevertheless, in perturbation theory they are assumed to have very small coefficients.

This issue also plays an important role for FlexibleEFTHiggsNLO, since the generated two loop logarithms are not vanishing at the SUSY-scale. Hence, the need for a more precise calculation infers inevitably the inclusion of a two loop matching with the consideration of all contributions at a certain order. This has been achieved in this work for FlexibleEFTHiggsNNLO at the order $\mathcal{O}(\alpha_t\alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$. Although, the two loop self energy contributions from bottom and tau Yukawa coupling are small in the SM and therefore neglected, they contribute non-negligibly to the correction to the self energy for large $\tan\beta$ in the MSSM [31,41].

5.7 Corrections to λ in FlexibleEFTHiggsNNLO at $\mathcal{O}(\alpha_s^2\alpha_t)$

As mentioned before, the pole mass matching procedure for λ gives rise to higher order contributions even if the considered self energy is included at one-loop order. For a correct matching all contributions of the considered order have to be included. However, this is not the general case in the extended FlexibleEFTHiggsNNLO spectrum generator, because some contributions at the three loop order are missing e.g. the three loop self energies are not included so far. The missing corrections yield inevitably large logarithms $\ln(m_s/m_t)$. This is obviously a deficiency and is examined in this section with analytical formulas.

For the purpose of compact formulas a convenient limit is considered as in (5.28) with degenerated soft SUSY masses $m_s^2 = M_3^2 = m_{\tilde{t}}^2 = m_{Q_i}^2 = m_{U_i}^2 = m_{D_i}^2$. Similarly to the calculation in 5.4, the λ matching condition at three loop order with the contributions implemented in FlexibleEFTHiggsNNLO is obtained by analytical pole mass iteration and conversion in SM parameters in the $\overline{\text{MS}}$ scheme

$$\begin{aligned}\Delta\lambda|_{\alpha_s^2\alpha_t,\alpha_s\alpha_t^2,\alpha_t^3} =&\frac{1}{v^2}\Bigg\{\Delta m^{2,\text{con}}_{h,\text{MSSM}} + \Pi^{2L,\text{con}}_{h,\text{MSSM}} + \left[\hat{\Sigma}'_{h,1L,\text{SM}} - \hat{\Sigma}'_{h,1L,\text{MSSM}}\right]\cdot \\ &\left[\Delta m^{2,\text{con}}_{h,\text{MSSM}} + \Pi^{2L}_{h,\text{MSSM}} - \Delta m^{2}_{h,\text{MSSM}}\hat{\Sigma}'_{h,1L,\text{MSSM}}\right] \\ &+ \left[\hat{\Sigma}'_{h,1L,\text{SM}} - \hat{\Sigma}'_{h,1L,\text{MSSM}}\right]^{\text{con}}\Delta m^{2}_{h,\text{MSSM}} \\ &+\frac{1}{2}\left[\frac{d^2}{(dp^2)^2}\left(\hat{\Sigma}_{h,1L,\text{SM}}(p^2) - \hat{\Sigma}_{h,1L,\text{MSSM}}(p^2)\right)|_{p^2=0}\right](\Delta m^2_{h,\text{MSSM}})^2\Bigg\},\end{aligned} \tag{5.60}$$

where the superscript con denotes that the MSSM parameter have to be converted into a SM parameter. However, the leading NNNLO contributions the eq. (5.60) is simplified, since momentum iteration does not contribute at the order $\mathcal{O}(\alpha_s^2\alpha_t)$

$$\Delta\lambda|_{\alpha_s^2\alpha_t} =\frac{1}{v^2}\left[\Delta m^{2,\text{con}}_{h,\text{MSSM}}|_{\alpha_s^2\alpha_t} + \Pi^{2L,\text{con}}_{h,\text{MSSM}}|_{\alpha_s^2\alpha_t}\right]. \tag{5.61}$$

In the following, the contributions in eq. (5.61) are discussed and only the logarithmic structure is obtained. For the conversion of the $\overline{\text{DR}}$ parameters in the MSSM into $\overline{\text{MS}}$ parameters in the SM, FlexibleEFTHiggsNLO/NNLO uses NLO corrections for g_s and NNLO corrections in y_t and m_t. Considering $Q = m_s$, the evaluation of the matching conditions at $\mathcal{O}(\alpha_s + \alpha_s^2)$ with the contributions, which are implemented in FlexibleEFTHiggsNNLO, leads to

$$m^{\overline{\text{DR}}}_{t,\text{MSSM}} = m^{\overline{\text{MS}}}_{t,\text{SM}}\left[1 - \frac{4g_s^2}{3(4\pi)^2}(1-\hat{X}_t) - \frac{55g_s^4}{2304\pi^4}\right], \tag{5.62}$$

$$g^{\overline{\text{DR}}}_{s,\text{MSSM}} = g^{\overline{\text{MS}}}_{s,\text{SM}}\left[1 + \frac{4g_s^2}{3(4\pi)^2}\right], \tag{5.63}$$

where the parameters on the r.h.s. of eq. (5.62) and (5.63) are defined as SM quantities in the $\overline{\text{MS}}$ scheme. Since the VEV conversion does not give rise to QCD contributions at NLO, the Yukawa coupling transforms with the same contributions as m_t

$$y^{\overline{\text{DR}}}_{t,\text{MSSM}} = \frac{y^{\overline{\text{MS}}}_{t,\text{SM}}}{s_\beta^2}\left[1 - \frac{4g_s^2}{3(4\pi)^2}(1-\hat{X}_t) - \frac{55g_s^4}{2304\pi^4}\right]. \tag{5.64}$$

By inserting the eq.s (5.62), (5.63) and (5.64) in (5.44) and (5.45) one finds expression for λ in eq. (5.61)

$$\begin{aligned}\Delta\lambda|_{\alpha_s^2\alpha_t} = \frac{1}{v^2}\frac{4y_t^2m_t^2g_s^4}{3(4\pi)^6}\Bigg[&384\left(\hat{X}_t - \frac{1}{2}\right)\ln^2\frac{m_s^2}{m_t^2} + \ln\frac{m_s^2}{m_t^2}\left(-\hat{X}_t^2 160 - \hat{X}_t 576 + 324\right) \\ &+\mathcal{O}\left(\ln^0\frac{m_s^2}{m_t^2}\right)\Bigg].\end{aligned} \tag{5.65}$$

This eq. (5.65) shows explicitly the logarithms which occur in the λ matching in FlexibleEFTHiggsNNLO through parameter conversion in the self energy contributions. It is presumed that the leading three-loop contributions in eq. (5.60) contain a similar structure of logarithmic contributions. In order to avoid the incorrect inclusion of large logarithms at NNNLO it is possible to identify the contributions from eq. (5.60) and subtract them explicitly in the matching condition for λ. But this idea is limited to the considered case of eq. (5.28). However, this procedure destroys the flexibility of FlexibleEFTHiggsNNLO which can hold for a more general case of the soft mass spectrum. A more recommended way is obtained by the inclusion of three-loop self energies, which by design will cancel the logarithmic contributions.

6 Comparison of Threshold Corrections to λ

As described before, the code of FlexibleEFTHiggsNLO has been extended in order to include leading two loop corrections for the λ matching. For Higgs boson mass calculations this quantity is important, as explained in section 5.3, in order to include large logarithms correctly. In this chapter, the numerical influence from these corrections is studied in more detail and a comparison is given to HSSUSY.

In figure 6.1 the SM parameter λ is shown for various SUSY scales, where the value of the non-SM dimensional parameters are chosen to satisfy

$$\mu = M_{\text{gauginos}} = M_{\text{sfermions}} = m_A = m_s \equiv M_{\text{SUSY}} \tag{6.1}$$

at the SUSY scale. The low energy constraints are applied as it is described in 4.2. Comparing FlexibleEFTHiggsNLO (solid magenta line) with FlexibleEFTHiggsNNLO (solid red line), the largest influence comes from the inclusion of the two loop $\mathcal{O}(\alpha_s\alpha_t)$ corrections. This observation, however, does not contradict the obtained formula (5.48), which predicts a vanishing contribution to the λ matching at the SUSY scale if only the $\mathcal{O}(\alpha_s\alpha_t)$ contributions are used. This is because FlexibleEFTHiggsNLO also includes incomplete two loop contributions of the considered order. Since this code performs a matching for SUSY parameters m_t in an equivalent way to eq. (5.43), inevitably the one loop corrections for the pole mass of the Higgs boson yields a two loop contribution of order $\mathcal{O}(\alpha_s\alpha_t)$. For vanishing squark mixing X_t the eq. (5.45) reduces to

$$(\Delta m^{\text{con}}_{h,\text{MSSM}})^2|_{\alpha_t\alpha_s} = \frac{32 y_t^2 g_s^2 m_t^2}{(4\pi)^4}\left[2\ln\frac{m_t^2}{m_s^2}+1\right]. \tag{6.2}$$

Because of the pole mass matching, this correction translates in a two loop correction for λ at the same order. This contribution is only treated correctly if the matching procedure includes the two loop self energy corrections, as it is the case for FlexibleEFTHiggsNNLO. The green line in figure 6.1 represents this case. Thus, the difference between FlexibleEFTHiggsNLO and FlexibleEFTHiggsNNLO is not characterized by the considered self energy contributions. Hence, corrections from matching other parameters can result in two loop contributions for λ in FlexibleEFTHiggsNLO with one loop λ matching.

Furthermore, momentum iteration in the one loop self energy leads also to higher order corrections in the pole mass of the Higgs boson. In the case of vanishing squark mixing, the following logarithmic structure of order $\mathcal{O}(\alpha_t^2)$ is present in FlexibleEFTHiggsNLO unless the two loop self energy is not included

$$(\Delta m^{\text{con}}_{h,\text{MSSM}})^2|_{\alpha_t^2} + \Delta m^2_{h,p-\text{iter}}|_{\alpha_t^2} = \frac{6 y_t^4 m_t^2}{(4\pi)^4}\left(3\ln\frac{m_s^2}{m_t^2}-\frac{3}{2}\right)\cot^2\beta. \tag{6.3}$$

Therefore, it is reasonable to conclude that the inclusion of the SM and MSSM loop self energy at $\mathcal{O}(\alpha_t^2)$ [1] leads to the difference between the magenta and cyan colored lines in figure 6.1.

Additionally, one can obtain from figure 6.1 (the red solid line with blue marks), that the two loop threshold corrections $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_b)^2)$ are the dominant contribution in FlexibleEFTHiggsNNLO. Only for large values in $\tan\beta$ the correction $\mathcal{O}(\alpha_s\alpha_b + \alpha_\tau^2)$ become more influential. It is emphasized that the implemented SM self energy for the Higgs boson in FlexibleEFTHiggsNNLO does not take contributions from tau and bottom Yukawa couplings at two loop into account. Nevertheless, the inclusion of MSSM two loop contributions $\mathcal{O}(\alpha_b^2 + \alpha_b\alpha_s + \alpha_b\alpha_t + \alpha_\tau^2)$ does not lead to any inconsistencies, because of the relative small value of the SM bottom and tau Yukawa coupling.

By comparing the numerical value for FlexibleEFTHiggsNNLO (red line) and HSSUSY (black pluses) for low SUSY scales, one can see that the prediction of both matching procedures in λ deviates. HSSUSY and SUSYHD perform the λ matching according to eq. (5.15) while neglecting terms suppressed by powers of v^2/M_{SUSY}. This difference originates from these neglected terms of order v/M_S in HSSUSY, which are of $\mathcal{O}(1)$ if the SUSY scale is considered to be low. FlexibleEFTHiggsNLO/NNLO includes them and hence does not give rise to this so-called EFT uncertainty at the considered matching order.

For higher SUSY scales the predictions for the parameter $\lambda(M_{SUSY})$ of HSSUSY and FlexibleEFTHiggsNNLO are more closely. Nevertheless, the pole mass matching for λ results in higher order corrections in FlexibleEFTHiggsNNLO. In analogy to FlexibleEFTHiggsNLO, the conversion of the SUSY parameters in the self energy $\Pi^{2L}_{h,\text{MSSM}}$ and momentum iteration in the one loop self energy $\hat{\Sigma}_{h,\text{model}}(p^2 = p^2_{pol})$ yields three loop logarithms $\ln(m_s/m_t)$ in the matching condition for λ. Assumedly, this can cause the deviation between HSSUSY and FlexibleEFTHiggsNLO/NNLO.

The figure 6.2 presents the predicted values for λ for the discussed versions of FlexibleEFTHiggsNNLO and HSSUSY. In comparison, the inclusion of the two loop self energy of order $\mathcal{O}(\alpha_s\alpha_t)$ in FlexibleEFTHiggsNNLO influences λ most of all. The effect of the $\mathcal{O}((\alpha_b + \alpha_t)^2)$ contributions is smaller than corrections of $\mathcal{O}(\alpha_s\alpha_t)$. Nonetheless, it is non-negligible and has numerically a larger influence than the $\mathcal{O}(\alpha_s\alpha_b + \alpha_\tau^2)$ contributions. In comparison, FlexibleEFTHiggsNNLO predicts for non-vanishing squark mixing X_t a higher value for λ than HSSUSY. Although both models agree for a certain range $|X_t/M_{SUSY}| < 1$ with each other. The consideration of the two loop self energy for the Higgs boson leads to a noticeable numerical impact in the matching for λ and improves the agreement of the HSSUSY and FlexibleEFTHiggsNNLO in their prediction for λ at the SUSY scale.

For higher squark mixing $|X_t/M_{SUSY}| > 1$ FlexibleEFTHiggsNNLO predicts a slightly higher value for λ than HSSUSY does. Since the blue pluses and the red solid line agree numerically to very high precision, the additional two loop contributions in FlexibleEFTHiggs $\mathcal{O}(\alpha_s\alpha_b + \alpha_\tau^2)$ cannot be responsible for this effect. Although FlexibleEFTHiggsNNLO reproduces the correct matching equation for λ , which is implemented in HSSUSY, the pole mass matching procedure gives rise to contributions at NNNLO e.g. $\mathcal{O}(\alpha_s^2\alpha_t + \alpha_s\alpha_t^2 + \alpha_t)$. These corrections contain powers of large logarithms $\ln M_{SUSY}/m_t$ and are capable to shift the prediction for λ. This effect can increase for $|X_t| > 1$ since powers of X_t are involved as it can be obtained from eq. (5.65). This indicates that the momentum iteration or a conversion of model parameters, as it is performed in FlexibleEFTHiggsNNLO, can effect

[1] Due to the implemented two loop self energy it is only possible to include $(\alpha_t + \alpha_b)^2$ corrections simultaneously. For small $\tan\beta$ the additional contributions from the bottom Yukawa coupling are sufficiently small [31].

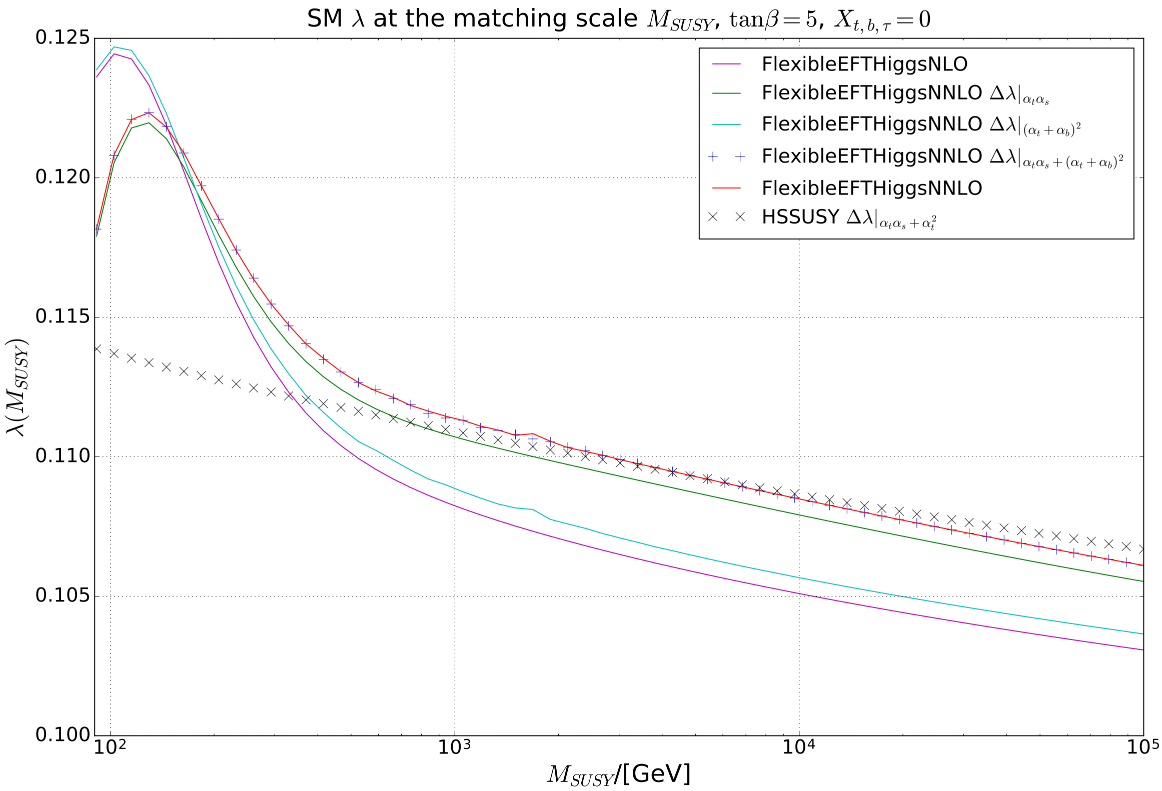


Figure 6.1: In this figure, the SM parameter λ is shown at different SUSY scales for $\tan\beta = 5$ with absent sfermion mixing $X_{t,b,\tau} = 0$. The solid lines represent the FlexibleEFTHiggsNLO/NNLO with different threshold corrections for λ. Considering the same order of threshold corrections $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$, HSSUSY and the correspondent FlexibleEFTHiggsNNLO version are represented by the black and blue markers, respectively. The red solid line includes further corrections at two loop order $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$.

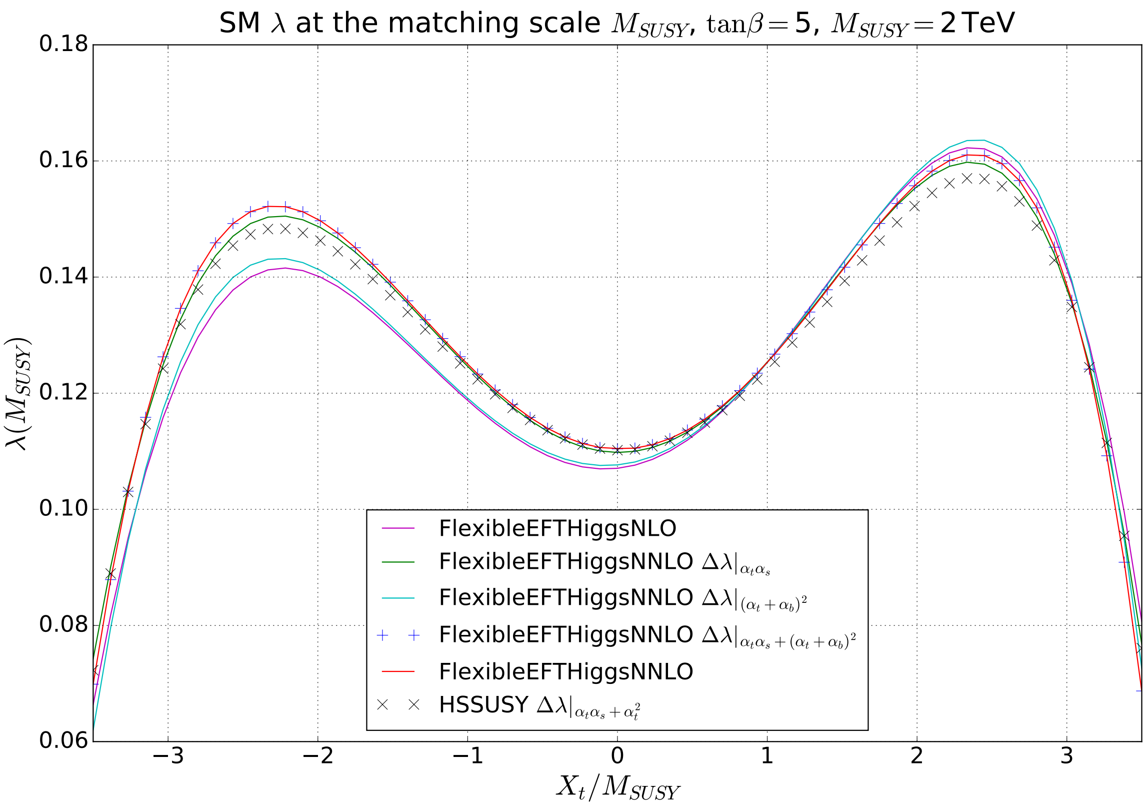


Figure 6.2: Comparison of prediction for the SM parameter λ at different values for X_t and $\tan\beta = 5$. The solid lines represent the FlexibleEFTHiggsNLO/NNLO with different threshold corrections for λ. Considering the same threshold corrections $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$, HSSUSY and the correspondent FlexibleEFTHiggsNNLO version are represented by the black and blue markers, respectively. The red solid line includes further corrections at two loop order $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$.

the Higgs mass. But since this contribution originates from three loop order this correction is assumed to be numerically rather small. In summary one can obtain that the correct λ matching at NNLO is an improvement, which not at least provide a better agreement between the spectrum generator HSSUSY and FlexibleEFTHiggs for high SUSY scales. Thus, the inclusion of two loop self energy contributions allows more reliable prediction for λ at low SUSY scales, where suppressed corrections become higher.

7 Comparison of FlexibleEFTHiggs with other Spectrum Generators

This chapter provides a detailed comparison of the Higgs boson pole mass prediction of various spectrum generator with FlexibleEFTHiggsNNLO. For this purpose this work focuses on the models: FlexibleEFTHiggsNLO, HSSUSY, FeynHiggs 2.12.0 [42] and FlexibleSUSY 1.5.1. Hence, a brief documentation of all considered spectrum generators is given in the following.

HSSUSY: As mentioned in section 5.3 before HSSUSY(HDSUSY) can be regarded as a pure EFT approach for the Higgs boson mass calculation. In this chapter, a small review is given. By considering threshold corrections for λ at NNLO, this approach will resum large logarithms $\ln m_s/m_t$ if the $\overline{\text{MS}}$ parameters are evaluated at the EW scale. However, for SUSY scales near the EW scale, this strategy exhibit a large EFT uncertainty, because the corrections in the λ matching of $\mathcal{O}(v/m_s)$ become important and are strictly neglected in the pure EFT approach. The NNLO contributions for λ comprise the $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$.

FlexibleSUSY: As described in section 5.2, this approach is a fixed order calculation for the lightest uncharged CP-even Higgs boson mass in the SUSY model. By design, this strategy includes power suppressed terms v/m_s in the considered order of the pole mass correction. However, for a large mass gap this approach will yield a rather large error through truncating the perturbative expansion at a finite order. If not stated otherwise the considered self energy contributions are of $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$.

FlexibleEFTHiggs: For a SUSY scale in the $m_s \approx 1\,\text{TeV}$ range, a combination of both techniques; EFT based resummation of large logarithms and the inclusion of power suppressed terms would yield a reliable prediction for the pole mass of the Higgs boson in the MSSM. As mentioned in section 5.4, this symbiosis was achieved for the spectrum generator FlexibleEFTHiggs. Before this work only NLO corrections in λ were included correctly. However, higher order contributions are involved incorrectly, see eq. (6.2) and (6.3). FlexibleEFTHiggsNNLO is proved to include the correct NNLO contributions at $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$. Henceforth, FlexibleEFTHiggsNNLO is considered with maximal precision i.e. NNLO corrections for λ are included at $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$.

FeynHiggs 2.12.0: This approach results from a combination of a fixed order calculation and an EFT based resummation of large logarithms. At the NNLO level in the determination of λ FeynHiggs 2.12.0 includes two loop contributions of the order $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$. Though, this calculation differs from the FlexibleEFTHiggsNNLO approach in some aspects i.e. the matching condition for λ is chosen differently [42]. In this thesis the result for the λ matching at the order $\mathcal{O}(\alpha_s\alpha_t)$ and $\mathcal{O}(\alpha_t^2)$ is proved to be equivalent to the FlexibleEFTHiggsNNLO two loop corrections in the considered limit (5.28). This is because FeynHiggs 2.12.0 uses corrections out of ref. [33, 34, 40], which are equivalent to λ corrections in SUSYHD/HSSUSY at NNLO. It is emphasized that FeynHiggs 2.12.0 performs a fixed order calculation for the Higgs boson pole mass in a mixed OS/$\overline{\text{DR}}$-scheme after the RGE running to the low scale.

7.1 Comparison for $X_t = 0$

In figure 7.1 predictions of the pole mass of the lightest Higgs boson are shown for different SUSY scales, where SUSY mass parameters are considered as in the case (6.1) with vanishing sfermion mixing $X_{t,b,\tau} = 0$. It can be obtained that for low SUSY scales the prediction of the fixed order approach, represented by the blue solid line, is completely in agreement with the one from FlexibleEFTHiggsNLO/NNLO and FeynHiggs. This behavior is expected, because the latter are known to predict the right value for the Higgs boson mass also in the sub TeV range.

Furthermore, it is noticed that the agreement between FlexibleEFTHiggsNNLO and the fixed order generator FlexibleSUSY is remarkably well for SUSY scales next to the EW scale. This indicates that the terms suppressed in powers of v/m_s at two loop are included correctly in FlexibleEFTHiggsNNLO at NNLO. On the contrary, HSSUSY gives rise to a large EFT uncertainty in the low M_{SUSY} region, hence it is not considered to give reliable predictions below $M_{SUSY} = 500\,\text{GeV}$. As discussed before, for higher SUSY scales the fixed order approach neglects large logarithms beyond the fixed order calculation in the Higgs boson pole mass. This can be obtained from figure 7.1, where the blue solid line deviates from the prediction of the spectrum generators which include a resummation of large logarithms. The inclusion of the two loop self energy contributions in the Higgs boson pole mass improves the agreement between FlexibleEFTHiggs and the spectrum generators HSSUSY and FeynHiggs 2.12.0 in the range $1\,\text{TeV} < M_{SUSY} < 10\,\text{TeV}$. For larger SUSY scales FeynHiggs 2.12.0 exhibit numerical instabilities and its predictions become unreliable for $M_{SUSY} \approx 100\,\text{TeV}$.

Comparing the red and magenta solid lines reveals that the inclusion of the two loop thresholds in λ has an effect on the pole mass of $\Delta M_h^{pole} \approx 500\,\text{MeV}$. The correct λ matching at NNLO improves the agreement between HSSUSY and FlexibleEFTHiggs up to $50\,\text{MeV} - 200\,\text{MeV}$, which is an astonishingly precise result. For high SUSY scales, where power suppressed terms do not contribute noticeably, a very small gap remains in figure 7.1.

The tendency for a lower value in FlexibleEFTHiggsNNLO than in HSSUSY can be also obtained in figure 6.1, where the predicted value for λ at the SUSY scale is lower than the one from HSSUSY. Large logarithms $\ln(m_s/m_t)$ from higher order contributions can affect the λ matching at the high scale and, after the evaluation of RGEs, also have an influence the Higgs boson pole mass in FlexibleEFTHiggsNNLO.

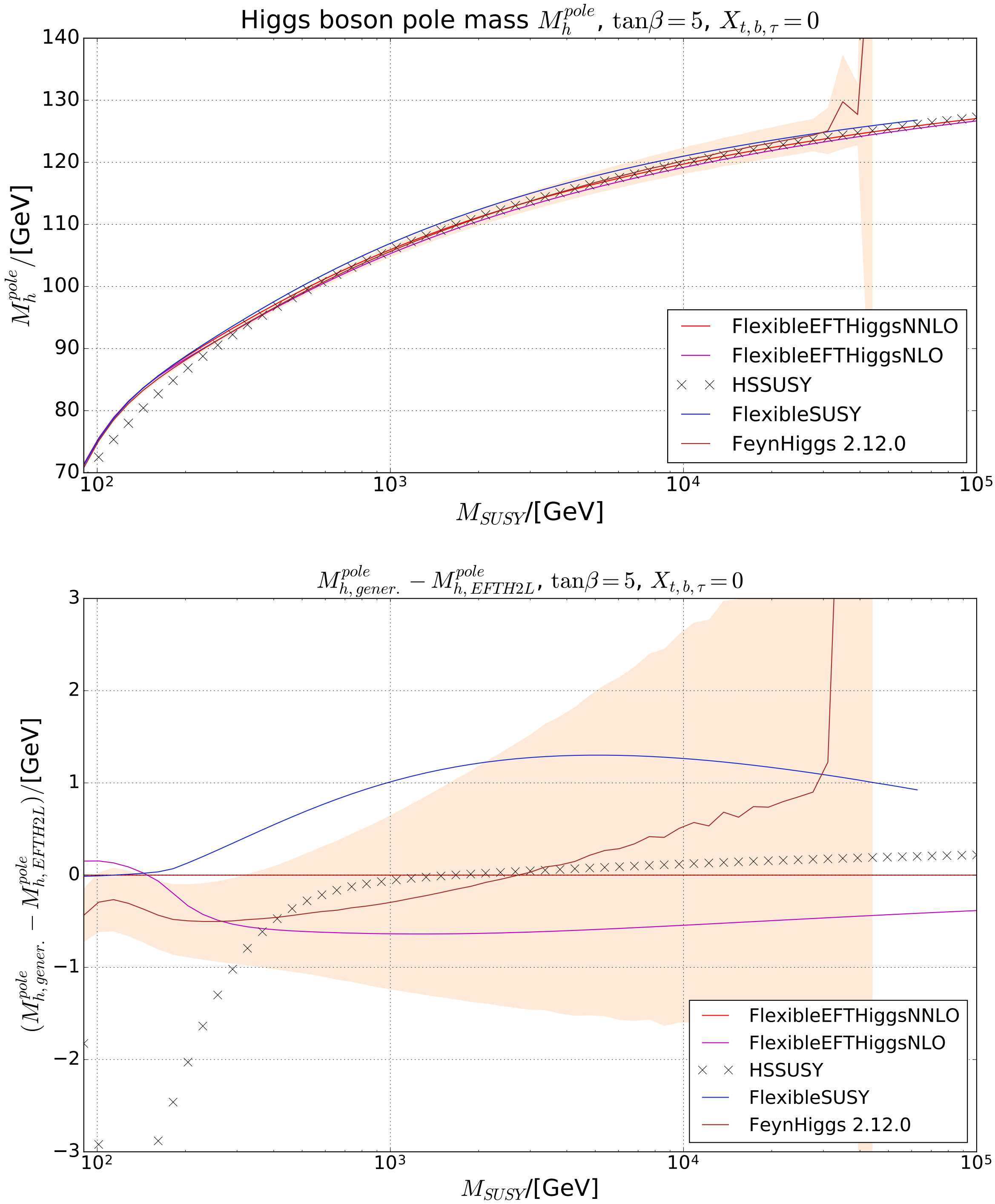


Figure 7.1: Comparison of predictions for the pole mass of the lightest CP-even uncharged Higgs boson in the MSSM at different values for the SUSY scale M_{SUSY}, $\tan\beta = 5$ and vanishing mixing for sfermions, where FlexibleSUSY is considered as the fixed order generator, HSSUSY determines the Higgs boson mass by the pure EFT approach and FlexibleEFTHiggsNLO/NNLO uses a combined technique as well as FeynHiggs 2.12.0. The brown band shows the uncertainty for the prediction of the spectrum generator FeynHiggs 2.12.0 (brown solid line). By the virtue of a higher resolution, the lower figure shows the difference between the prediction of a considered spectrum generator and the value obtained from FlexibleEFTHiggsNNLO.

7.2 Comparison for $X_t \neq 0$

In this section, the effects of the non-vanishing stop mixing parameter X_t are studied. Figure 7.2 shows the Higgs boson pole mass as it is predicted by the spectrum generators discussed in the beginning of this chapter. For low squark mixing $|X_t| < 1$ the inclusion of two loop self energy contributions in FlexibleEFTHiggsNNLO improves the agreement with HSSUSY and FeynHiggs. In the same X_t range the fixed order generator FlexibleSUSY gives the same numerically results as FlexibleEFTHiggsNNLO. However, for larger $|X_t|$ values the prediction for the Higgs boson pole mass from HSSUSY and FlexibleEFTHiggsNNLO deviates. A similar observation is made in figure 6.2, where the prediction for λ from these two spectrum generators differs slightly. Nevertheless, the inclusion of the two loop self energy in the λ matching condition improves the agreement between the pure EFT approach HSSUSY and FlexibleEFTHiggsNNLO. For the range $|X_t| < 2$ FlexibleEFTHiggsNNLO and FeynHiggs 2.12.0 excellently agree.

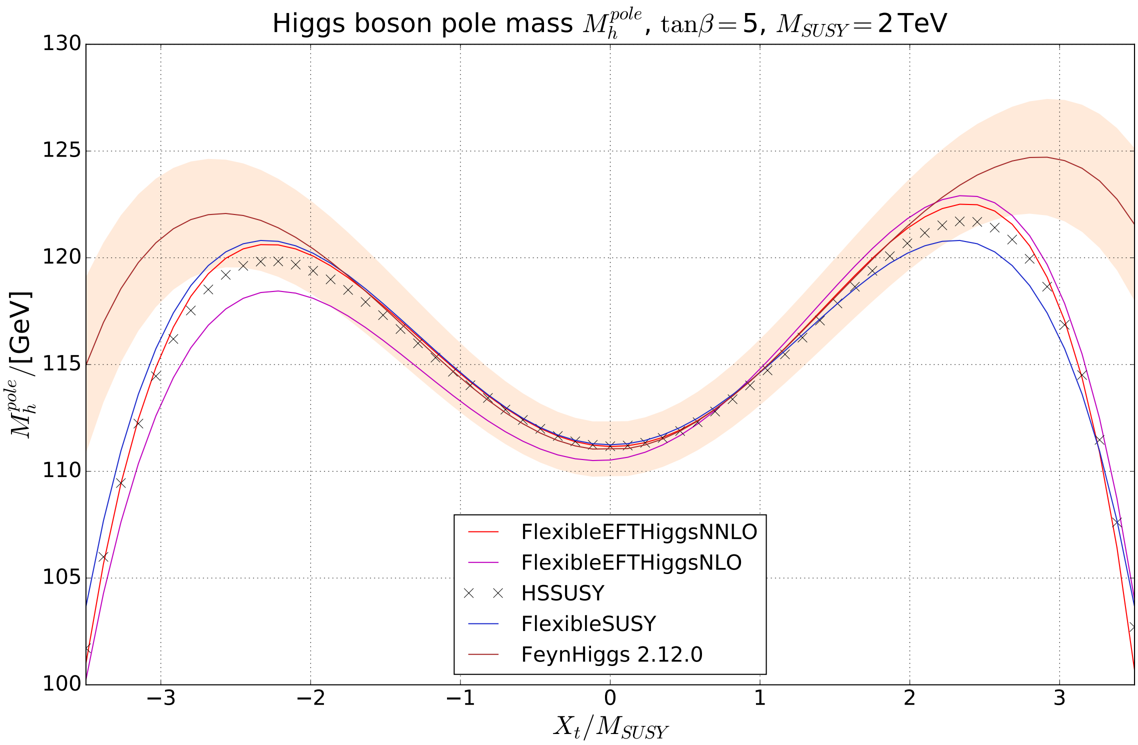


Figure 7.2: Comparison of prediction for the pole mass of the Higgs boson at different values for X_t, $M_{SUSY} = 2$ TeV and the $\tan\beta = 5$. The brown band represents the uncertainty estimated for FeynHiggs 2.12.0.

Figure 7.3 illustrates the pole mass as predicted by the discussed generators for a higher value for the squark mixing $X_t = -2M_{SUSY}$ and $\tan\beta = 5$. Because of numerical instabilities only the range $0.2 < M_{SUSY}/\text{TeV} < 100$ is examined. For low SUSY scales near to the EW scale FlexibleEFTHiggsNNLO (red line) achieves better agreement with the fixed order calculation provided by FlexibleSUSY (blue line). This verifies the spectrum generator FlexibleEFTHiggsNNLO, since for SUSY scales M_{SUSY} lower than 500 GeV fixed

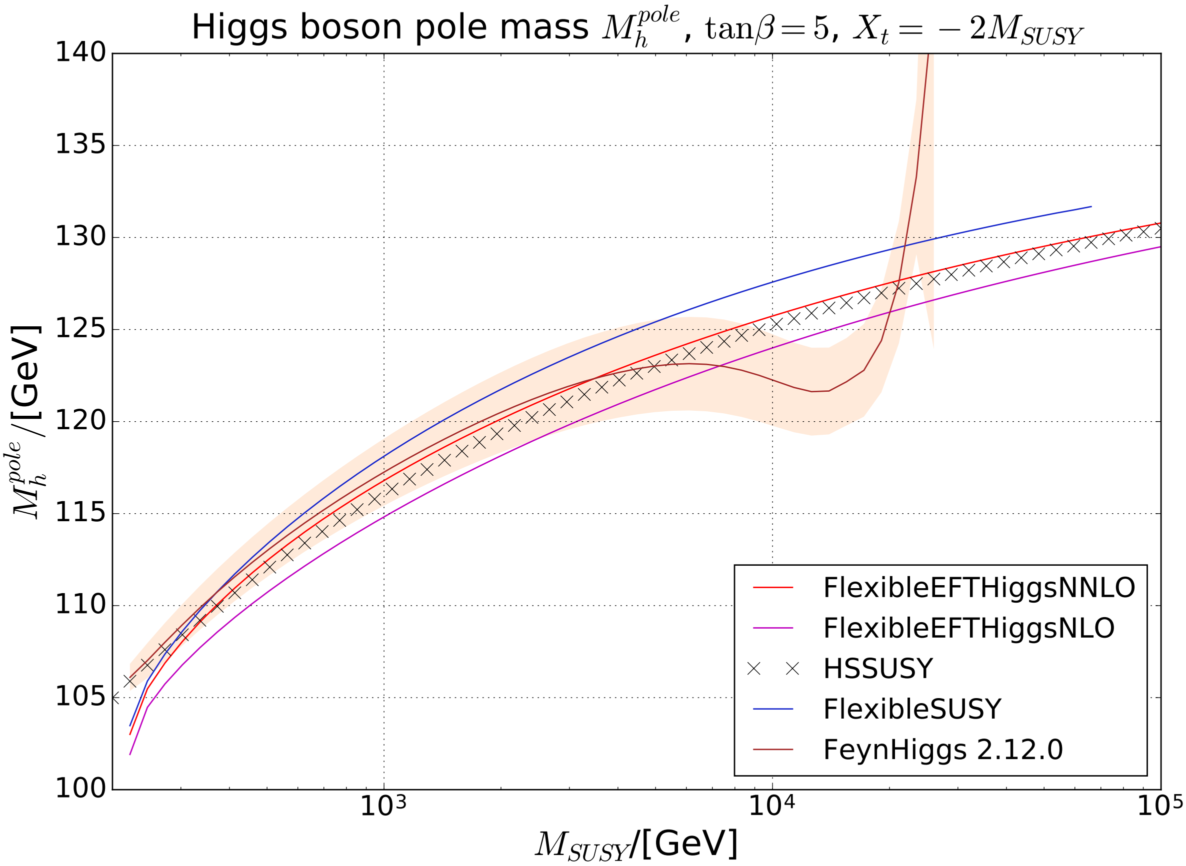


Figure 7.3: Comparison of predictions for the pole mass of the lightest CP-even uncharged Higgs boson in the MSSM at different values for the SUSY scale M_{SUSY}, $\tan\beta = 5$ and squark mixing $X_t = -2M_{SUSY}$.

order calculations give a more reliable value for Higgs boson mass calculations. It is emphasized that this agreement indicates that the discussed suppressed terms of two loop order $\mathcal{O}((\text{two loop})(v^2/M_{SUSY}^2))$ are included correctly in the Higgs boson mass calculation of FlexibleEFTHiggsNNLO.

In the range $1 < M_{SUSY}/\text{TeV} < 10$ the predictions for spectrum generators, which incorporate EFT based resummation of logarithms (red line, magenta line and brown line), differs by a few GeV. The result for this parameter point
($X_t = -2M_{SUSY}$, $\tan\beta = 5$, $M_{SUSY} = 2$ TeV) is also obtained in figure 7.2.

As it is shown in relation (5.65), FlexibleEFTHiggsNNLO does not include the leading logarithms at three loop order correctly. These logarithms can give rise to a deviation between the prediction of HSSUSY and FlexibleEFTHiggsNNLO, which increases for higher SUSY scales. However, by design the suppressed logarithms of
$\mathcal{O}((\text{two loop})v^2/M_{SUSY}^2 \ln(M_{SUSY}^2/v^2))$ are treated correctly in FlexibleEFTHiggsNNLO. Indeed, this is a crucial point in the discussion, because in the pure EFT approach terms $\propto v^2/M_{SUSY}^2 \ln(M_{SUSY}^2/v^2)$ are missing. These contributions vanish for very low and very high SUSY scales, but they can become noticeable effects in the SUSY scale range $1 < M_{SUSY}/\text{TeV} < 10$. Additionally, the spectrum generator FeynHiggs 2.12.0 is considered to contain these contributions too and its predictions for the pole mass of the Higgs boson agree with FlexibleEFTHiggsNNLO very good in the lower SUSY-scale region. This indicates that the observed gap between the between HSSUSY and FlexibleEFTHiggsNNLO in the range $1 < M_{SUSY}/\text{TeV} < 10$ can also correspond to the correct treated contributions of logarithms suppressed by the factor v^2/M_{SUSY} instead of uncontrolled higher order contributions.

Moreover, the inclusion of two loop self energy contributions in the λ matching improves the agreement between HSSUSY (black marks) and FlexibleEFTHiggsNNLO for vanishing squark mixing and non-vanishing mixing at large SUSY scales. At this high SUSY scales the pure EFT approach is the more reliable strategy for Higgs boson mass calculations than fixed order approaches. Therefore, FlexibleEFTHiggsNNLO is able to reproduce the prediction of both calculations, fixed order and EFT approach, in the regimes where they are considered as the more precise and hence favored method. The spectrum generator FeynHiggs 2.12.0 is based on the same idea as FlexibleEFTHiggsNNLO. The predictions of FlexibleEFTHiggsNNLO for the pole mass of the Higgs boson are contained in the error band until the numerical instabilities become significant at $M_{SUSY} > 10\,\text{TeV}$. This is an achievement of FlexibleEFTHiggsNNLO, since both codes consider similar contributions but work differently.

8 Uncertainty Estimation in FlexibleEFTHiggsNNLO

In this chapter, the theoretical uncertainties for the pole mass of the Higgs boson in FlexibleEFTHiggsNNLO are estimated. These uncertainties originate from neglecting higher order contributions in the calculation. Although FlexibleEFTHiggs provides a resummation of logarithms beyond any fixed order not all contributions in perturbation theory are included neither in the matching of λ nor in the pole mass calculation within the EFT. As discussed in section 5.4, in the pure EFT approach suppressed contributions of $\mathcal{O}(v/M_{SUSY})$ are absent in λ at all orders. However, for the method FlexibleEFTHiggs it is shown in ref. [26] that at NLO the right power suppressed terms are recovered. By construction FlexibleEFTHiggsNNLO is considered to include all suppressed terms of order $\mathcal{O}\left((\text{one loop})v^2/M_{SUSY}^2 \ln(M_{SUSY}/v) + (\text{two loop})v^2/M_{SUSY}^2\right)$. Moreover, using RGEs of a finite order also induces theoretical uncertainties, but they are assumed to be much smaller than the higher order contributions in the λ matching and in the Higgs boson pole mass. Hence, in this section the focus is set on the truncation error of the latter calculations.

8.1 Estimation of the Low Scale Uncertainty

In this section, the uncertainty which occurs due to the finite order calculation of the Higgs boson pole mass in the EFT at the low scale. A suitable way to simulate the effects of higher logarithmic contributions has been introduced in ref. [26]. As presented in this reference the low energy scale, where EWSB conditions are imposed and the Higgs boson mass is calculated, is varied in a certain range. The default value for the renormalization scale is the top quark pole mass $Q_0 = M_t$ is estimated as discussed in chapter 4. The missing three loop contributions contain corrections, which can be simulated by choosing a different scale for the calculation of the Higgs boson pole mass. Varying the renormalization scale in the range $Q \in [M_t/2, 2M_t]$ yields a conservative estimation of the missing higher order corrections. The uncertainty is then defined in the same way as in ref. [26]

$$\Delta M_h^Q := \max_{Q\in[M_t/2,2M_t]} |M_h^{pole}(M_t) - M_h^{pole}(Q)|. \tag{8.1}$$

In figure 8.1, the red line shows the prediction for the Higgs boson pole mass for a fixed renormalization scale $Q_0 = M_t$ and input parameters $\tan\beta = 5$, $X_t = 0$, $M_{SUSY} = 2$ TeV. Furthermore, black line corresponds to the prediction by varying the scale, at which the Higgs pole mass is determined. One obtains that, M_h^{pol} behaves non-monotonous for different scales. Thus, the estimation of the uncertainty is sensitive to the bounds $Q = M_t/2$ and $Q = 2M_t$.

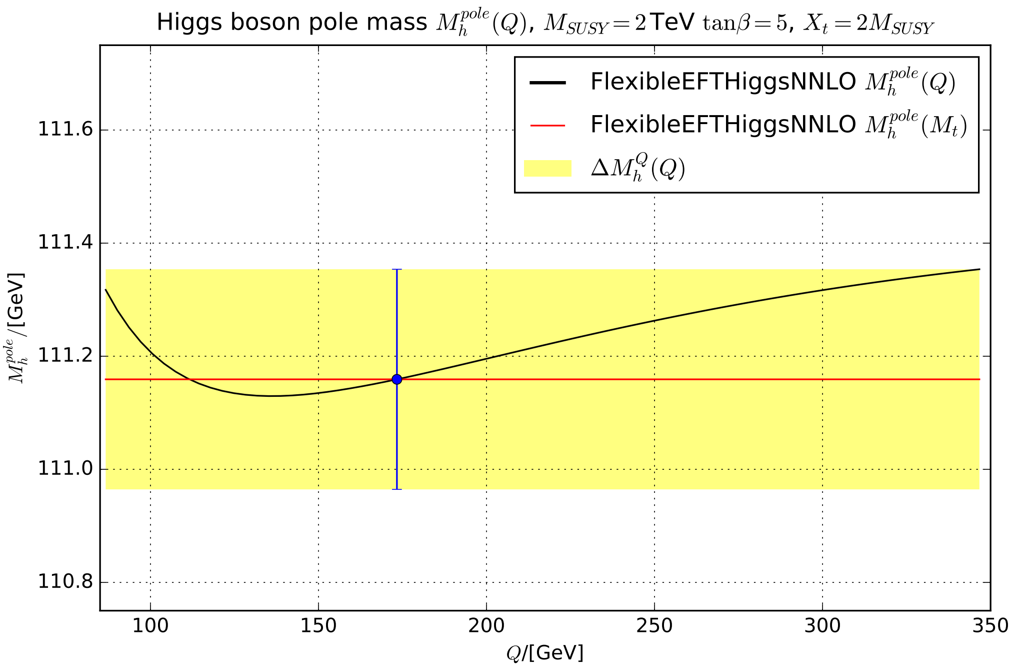


Figure 8.1: Visualization of the estimation of the low scale uncertainty ΔM_h^Q for the specified parameters $\tan\beta = 5$, $X_t = 0$, $M_{SUSY} = 2$ TeV. The obtained uncertainty band covers the range $[M_h^{pole}(M_t)+\Delta M_h^Q, M_h^{pole}(M_t)-\Delta M_h^Q]$ The red line is associated with the prediction of the Higgs boson pole mass in FlexibleEFTHiggsNNLO at the scale $Q = M_t$. Whereas the black line is the prediction of the Higgs boson pole mass for different low scales Q. The yellow band represents the obtained uncertainty for $M_h^{pole}(Q = M_t)$.

8.2 Estimation of the High Scale Uncertainty

In this section, the aim is now to estimate the uncertainty of the higher order contributions, which are neglected in the matching condition for λ in FlexibleEFTHiggsNNLO. For this purpose one can apply two strategies which generate higher order terms as it is defined in [26]. This section introduces both ways and discusses the strategies.

1 Variation of the matching scale

In analogy to section 8.1 a possible way is to evaluate the pole mass matching condition at different scales than the SUSY scale M_{SUSY}. As described in section 4.2, by default the matching scale and the SUSY scale are considered to be equal for the Higgs boson mass calculation in FlexibleEFTHiggsNNLO. Varying the two scales will provide RG running of the model parameters between the M_{SUSY} and the matching scale Q_{match}. By construction the matching condition for λ obtained in this way contains the logarithms of the ratio $Q_{\text{match}}/M_{SUSY}$. Therefore, higher order contributions can be estimated conservatively if the variation of the matching scale is performed at scales $Q \in [M_{SUSY}/2, 2M_{SUSY}]$. Similarly, the obtained value for the uncertainty can be introduced as in eq. (8.1)

$$\Delta M_h^{Q_{\text{match}}} := \max_{Q\in[M_{SUSY}/2,2M_{SUSY}]} |M_h^{pole}(M_t) - M_h^{pole}(Q)|. \tag{8.2}$$

Thus, the estimated uncertainty can be obtained from figure 8.2 for the predicted value of the Higgs boson pole mass for the parameters $\tan\beta = 5$, $X_t = 0$, $M_{SUSY} = 2$ TeV. Furthermore, the value for the uncertainty is obtained to be very sensitive for the bounds $Q = M_{SUSY}/2$ and $Q = 2M_{SUSY}$.

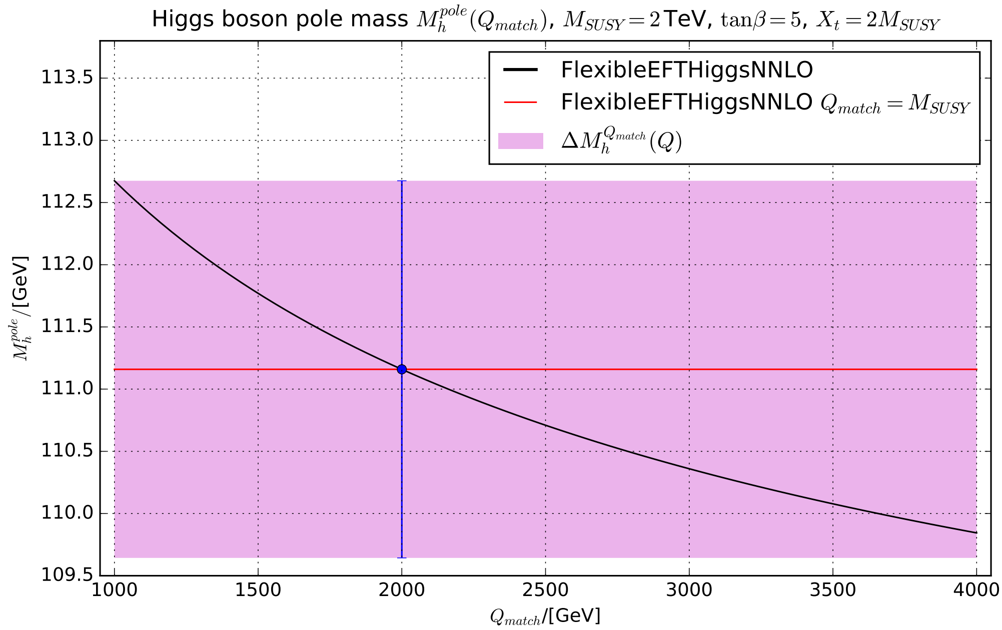


Figure 8.2: Illustration of the estimation of the high scale uncertainty $\Delta M_h^{Q_{\text{match}}}$ for the parameters $\tan\beta = 5$, $X_t = 0$, $M_{SUSY} = 2$ TeV. The red line represents the Higgs boson pole mass as predicted by FlexibleEFTHiggsNNLO for the λ matching scale $Q_{\text{match}} = M_{SUSY}$. The black line is the prediction of the Higgs boson pole mass for different renormalization scales. The purple band shows the estimated uncertainty for M_h^{pole} for $Q_{\text{match}} = M_{SUSY}$.

2 Variation of the matching of the top Yukawa coupling

The one loop Higgs boson mass is affected by the leading Yukawa coupling contributions, as presented in eq. (5.33). Because of the large coupling and the color factor, this contribution alone is capable of shifting the pole mass of the Higgs boson significantly. The variation of the top Yukawa coupling by few percent leads simultaneously to changes in the Higgs boson pole mass in the 1 GeV range. Inevitably, this behavior causes a noticeable change in λ through the pole mass matching condition. In FlexibleEFTHiggsNNLO the maximal precision is used by default e.g. the top Yukawa coupling is matched at two loop order, see section 4.2. As discussed in section 5.7 this introduces three loop terms in the matching condition, which contain logarithmic as well as non-logarithmic terms. Hence, changing the order of the matching of the top Yukawa coupling estimates the magnitude of higher order contributions. The uncertainty for the pole mass of the Higgs boson, introduced in ref. [26], was motivated by the missing two loop contributions in FlexibleEFTHiggsNLO. Since they are included in FlexibleEFTHiggsNNLO the corresponding uncertainty is defined by

$$\Delta M_h^{y_t \text{ 1L vs. 2L}} = |M_h^{pole}(y_t^{\text{MSSM},(2)}) - M_h^{pole}(y_t^{\text{MSSM},(1)})|. \tag{8.3}$$

For FlexibleEFTHiggsNLO a slightly different estimation has been used in ref. [26]. This is because for an NLO λ matching the missing NNLO contributions are estimated by varying the top Yukawa coupling matching between LO and NLO.

8.3 NLO-NNLO Uncertainty Comparison within FlexibleEFTHiggs

The different sources of uncertainties for FlexibleEFTHiggsNLO and FlexibleEFTHiggsNNLO are presented in figure 8.3. From this figure it is obtained that the uncertainty ΔM_h^Q (yellow), which results from varying the low scale, decreases if the λ matching is performed at NNLO. Thus the estimation of this uncertainty is reduced to $\Delta M_h^Q \approx 200\,\text{MeV}$. Furthermore, the uncertainty $\Delta M_h^{y_t}$ (turquoise) is reduced to even lower values than ΔM_h^Q in FlexibleEFTHiggsNNLO. The purple band shows the uncertainty for the high scale in $\Delta M_h^{Q_\text{match}}$, which originates from varying the matching scale as described in 8.2. This source causes a uncertainty up to 2 GeV in FlexibleEFTHiggsNNLO. This is obtained for different squark mixing with a SUSY scale of 2 TeV and for different SUSY scales with a stop quark mixing $X_T = 2M_{SUSY}$. Comparing the figures (c) and (d) one observes that the magnitude of the induced error increases substantially for the NNLO calculation. A similar behavior is also obtained in the figures (a) and (b) , where the uncertainty due to varying the matching scale rises for $|X_t| < 1$. This behavior of FlexibleEFTHiggsNNLO is not expected, since it possesses higher order contributions which are missing in FlexibleEFTHiggsNLO at the matching scale. However, the uncertainties in FlexibleEFTHiggsNLO are in general underestimated if only the discussed contributions are considered. A relevant uncertainty, as presented in ref. [26], is the estimation of two loop contributions to the coupling λ. Since these contributions are correctly reproduced by the pole mass matching in FlexibleEFTHiggsNNLO, they have not been included in the discussion.

8.4 Combined Uncertainty in FlexibleEFTHiggsNNLO

The two contributions to the uncertainty in FlexibleEFTHiggsNNLO are now combined in order to provide a reliable estimation for the theoretical uncertainty. The low scale uncertainties discussed in 8.1 are partially induced by the same higher order contributions in the MSSM. A possible way to combine these two uncertainties has been presented in ref. [26]. By taking the maximum of both low scale uncertainties for every examined parameter point the procedure ensures that no underestimation occurs. Furthermore, the contributions from varying the scale contain logarithms, thus the correct treatment is done by taking the uncertainty to the second power [26]

$$\Delta M_h = \sqrt{\left(\Delta M_h^Q\right)^2 + \left(\max\left\{\Delta M_h^{y_t\,\text{1L}\,vs.\,\text{2L}}, \Delta M_h^Q\right\}\right)^2}. \tag{8.4}$$

This treatment is consistent with th usual formula for the propagation of uncertainties for uncorrelated errors.

The combined uncertainty for different SUSY scales is shown in figure 8.4. It is obtained that this value for the uncertainty is of order $2-3\,\text{GeV}$ for high SUSY scales. Due to correct treatment of the power suppressed contributions at the considered order, the “EFT“ uncertainty does not need to be included. The missing suppressed terms are of three-loop order in FlexibleEFTHiggsNNLO and their influence to the Higgs boson mass is much

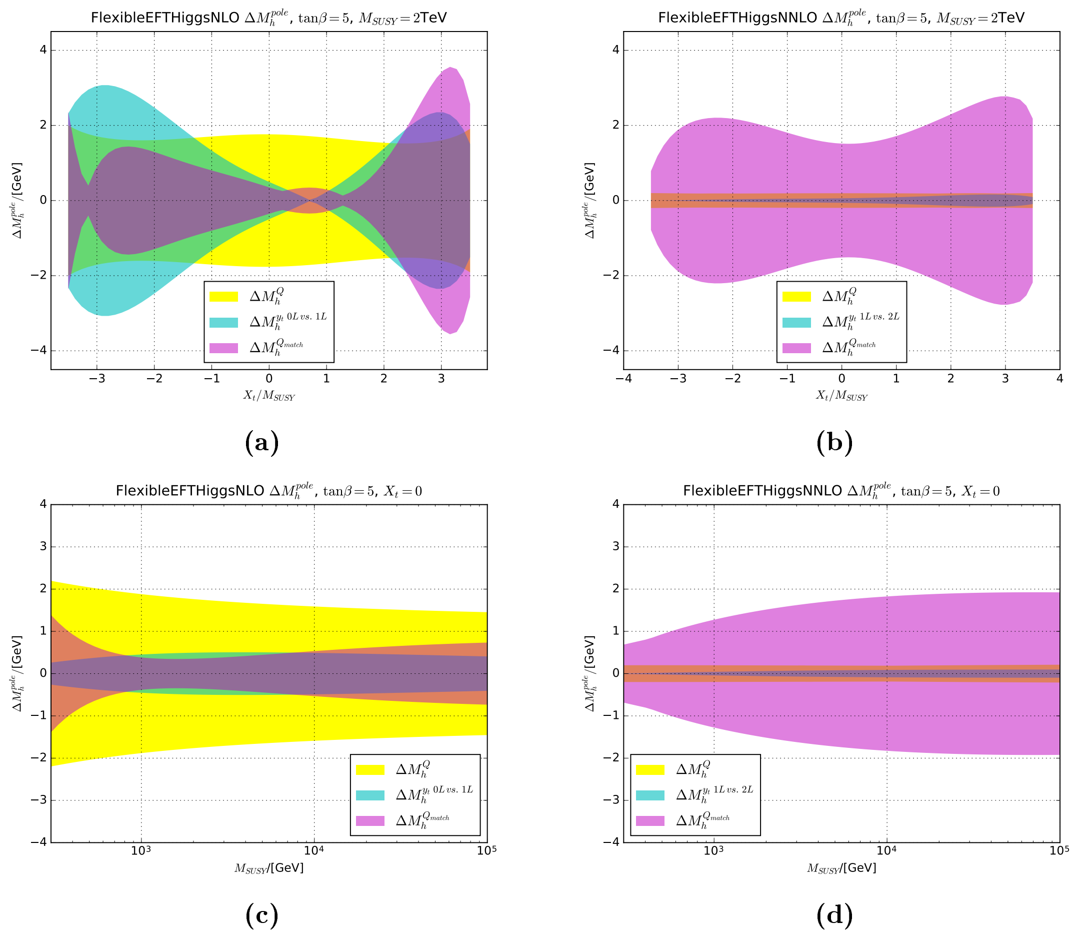


Figure 8.3: Comparison of different sources of uncertainties for FlexibleEFTHiggsNLO and FlexibleEFTHiggsNNLO. The right panels visualize the uncertainties in FlexibleEFTHiggsNNLO of the low scale: ΔM_h^Q in yellow and $\Delta M_h^{y_t\,\mathrm{1L}\,vs.\,\mathrm{2L}}$ in turquoise and the high scale: $\Delta M_h^{Q_\mathrm{match}}$ in purple. Figures (a) and (c) illustrate the same uncertainties with the low scale estimation $\Delta M_h^{y_t\,0L\,vs.\,1L}$ in turquoise. In all presented plots the value is generated for $\tan\beta = 5$. For the top row the SUSY scale is set to $M_{SUSY} = 2\,\mathrm{TeV}$ and in the bottom row considered stop mixing is set to $X_t = -2M_{SUSY}$.

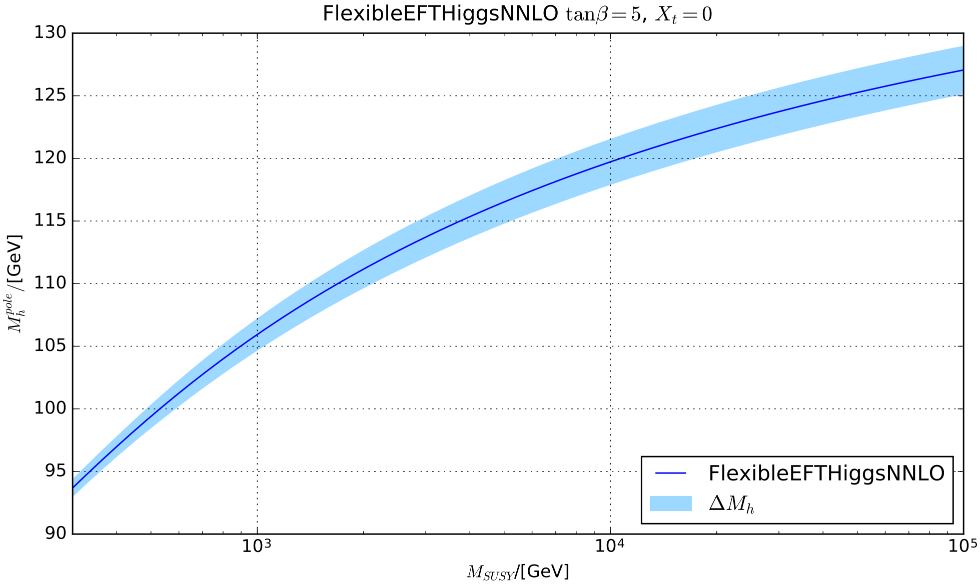


Figure 8.4: Prediction for the pole mass of the Higgs boson and for theory uncertainty in FlexibleEFTHiggsNNLO for different SUSY scales. The stop mixing is vanishing $X_t = 0$ and $\tan\beta = 5$.

smaller than higher unsuppressed contributions. Thus, at low SUSY scales the uncertainty is in general smaller in FlexibleEFTHiggs than in pure-EFT calculations. The uncertainty for different values in X_t and $M_{SUSY} = 2\,\text{TeV}$ is visualized in figure 8.5. One obtains that for a small stop mixing range $X_t < 1$ the uncertainty is about $\Delta M_h \approx 2\,\text{GeV}$. However for larger stop mixing e.g. $X_t \approx 2.5 M_{SUSY}$ the uncertainty increases to $\Delta M_h \approx 5\,\text{GeV}$, which is mainly caused by the estimation of the high scale uncertainty $\Delta M_h^{Q_{\text{match}}}$ as it is shown in figure 8.3b.

The obtained theoretical uncertainty for the Higgs boson pole mass is still much higher in FlexibleEFTHiggsNNLO than the experimental uncertainty. This is mainly caused by the high scale uncertainty $\Delta M_h^{Q_{\text{match}}}$. There is no reason to believe, that the NNLO λ matching produces a higher uncertainty than the NLO matching at the high scale. Hence, it is presumed that this is a too conservative approach for the uncertainty at the high scale. This motivates to search for other strategies which estimate a lower uncertainty in the λ matching at NNLO than at NLO.

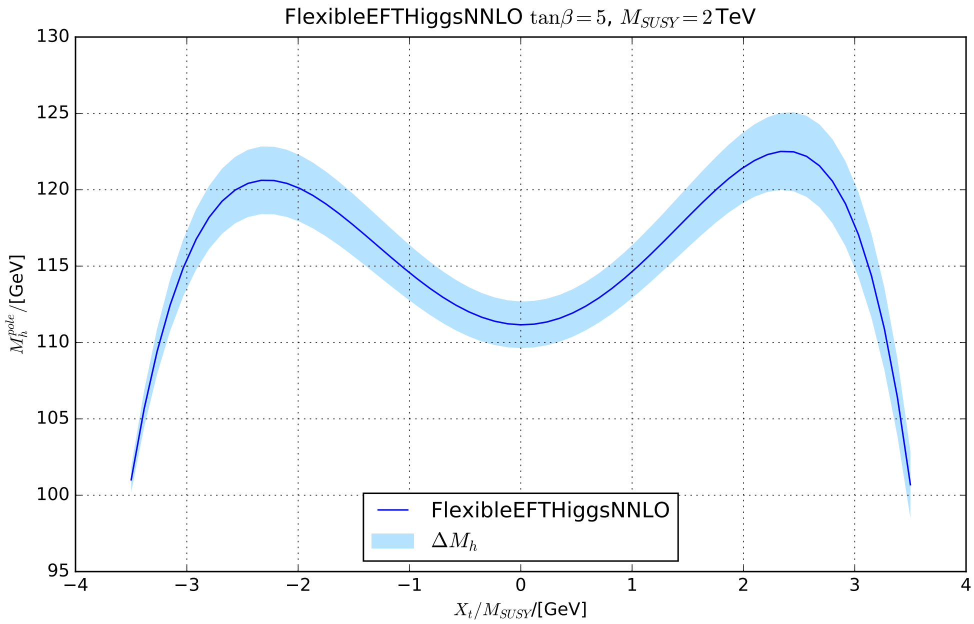


Figure 8.5: Illustration of the pole mass of the Higgs boson and the combined uncertainty ΔM_h calculated with eq. (8.4). for different squark mixing parameter X_t. The SUSY scale is set to $M_{SUSY} = 2\,\text{TeV}$ and $\tan\beta = 5$.

9 Summary

The purpose of this thesis was the improvement of precise calculations of the Higgs boson pole mass in the MSSM within the framework of FlexibleSUSY. In particular, the already existing method FlexibleEFTHiggsNLO has been extended in order to include leading NNLO corrections at $\mathcal{O}(\alpha_s(\alpha_t+\alpha_b)+(\alpha_t+\alpha_b)^2+\alpha_\tau^2)$ in the matching condition in λ and, thus, allowing the correct resummation of further large logarithms while ensuring that the pole mass of the Higgs boson is correct at two loop. This was achieved and the procedure was examined by using analytical calculations and numerical studies.

The equivalence of the matching condition between FlexibleEFTHiggsNNLO and other public codes has been proven at $\mathcal{O}(\alpha_s\alpha_t+\alpha_t^2)$. This is a non-trivial statement since the choice of the matching condition is ambiguous if contributions from finite order are considered. Furthermore, it has been shown that the matching procedure performed by FlexibleEFTHiggsNNLO gives rise to three loop logarithms in the matching condition for λ. In particular, the logarithmic contributions to λ at $\mathcal{O}(\alpha_s^2\alpha_t)$ have been determined. Nevertheless the pole mass matching condition for λ , applied in FlexibleEFTHiggsNNLO, is an elegant method which reproduces by construction the correct matching procedure at NNLO for a nearly arbitrary choice of a soft mass spectrum. This fact emphasizes the advantage of FlexibleEFTHiggsNNLO with regard to the discussed spectrum generators, since at present the public codes are not capable of handling the resummation of large logarithms with an $\mathcal{O}(\alpha_t^2)$ matching condition in λ for two different stop masses $m_{\tilde{t}1}\neq m_{\tilde{t}2}$. Thus, FlexibleEFTHiggsNNLO generalizes the matching in an elegant way. The effects of the enlarged matching condition in λ were tested numerically and it has been observed that the inclusion of self energies at $\mathcal{O}(\alpha_s\alpha_t+\alpha_t^2)$ is important for the agreement with other spectrum generators.

In this thesis, the prediction for the pole mass was compared to a fixed order calculation and EFT based calculations. The central result of this thesis is that FlexibleEFTHiggsNNLO is capable of reproducing the numerical results of both methods, which are considered to give reliable predictions in their respective SUSY scales. This agreement is very good for arbitrary stop mixing and excellent for vanishing X_t. It is emphasized that also FlexibleEFTHiggsNLO agrees with the extended version and HSSUSY for vanishing squark mixing, where the two loop contributions in the λ matching are rather small. However, for a larger amount of squark mixing, where FlexibleEFTHiggsNNLO agrees with HSSUSY, FlexibleEFTHiggsNLO results deviate from both predictions by 1-2 GeV at high SUSY scales. Nevertheless, all results from HSSUSY and FlexibleEFTHiggsNLO/NNLO lie in the estimated uncertainty at these SUSY scales. For a large magnitude of stop mixing and SUSY scales in the $1 < M_{SUSY}/\text{TeV} < 10$ range FlexibleEFTHiggsNNLO predicts a higher value for the pole mass of the Higgs boson than HSSUSY. It has been discussed that this effect does not seem to be convincingly caused by incorrect logarithmic contributions at NNNLO in the λ matching procedure. Based on the same method as FlexibleEFTHiggsNNLO, the spectrum generator FeynHiggs 2.12 produces the same effect in this parameter region. Hence, it is presumed that this gap of 1 GeV may also correspond to correctly included suppressed logarithms, which are missing in HSSUSY. This is an issue of high interest and has to be investigated further because a large magnitude of stop mixing and SUSY scales at few TeV are consistent with a Higgs boson mass of 125 GeV in the MSSM. This interesting scenario is in accessible range for current experiments at the LHC.

Finally, an elaborated method for the estimation of the uncertainty was applied to FlexibleEFTHiggsNNLO. The uncertainty is mainly dominated by the SUSY scale uncertainty, which is sensitive to higher order contributions in the pole mass matching condition. This effect causes the uncertainty to take value of 1–5 GeV. From the comparison of FlexibleEFTHiggsNLO/NNLO it has been obtained, that the low scale uncertainty reduces substantially if two loop contributions are consistently included. In contrast, the high scale uncertainty remains very large if the two loop contributions are treated correctly. This has been not expected to happen and therefore there is a reason to believe that the applied approach for estimating the high scale uncertainty is too conservative for FlexibleEFTHiggsNNLO.

Conclusively, the combined methods of a fixed order calculation and EFT based resummation of large logarithms was successfully embedded in FlexibleEFTHiggs framework with λ matching at NNLO. This calculation makes reliable predictions for low and high scales of soft mass spectra in the MSSM. All spectrum generators, which are considered to reproduce reliable predictions, are consistent with the results obtained from FlexibleEFTHiggsNNLO. Due to its judicious matching procedure it is possible to extend this method on non-minimal extensions of the SM in the near future.

A Weyl Spinors

Dirac Spinors
The Dirac spinor $\Psi = (\psi_1, \psi_2, \psi_3, \psi_4)$ is an element in a four dimensional complex vector space, the spinor space on whom a certain Lorentz transformation (LT) $S(\Lambda)$ can be defined. Furthermore, the gamma matrices γ^μ are defined on this so-called spinor space and are restricted by the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_{4\times4}. \tag{A.1}$$

In the Weyl representation they have the concrete form

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \qquad \gamma^k = \begin{pmatrix} 0 & \bar{\sigma}^k \\ -\sigma^k & 0 \end{pmatrix}. \tag{A.2}$$

Note that the Weyl representation is used throughout all of the following section. The chirality operator γ^5 completes together with γ^μ the Clifford algebra and is defined as

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}. \tag{A.3}$$

This means also that the anticommutation relation (A.1) can be extended for γ^5: $\{\gamma^5, \gamma^\mu\} = 0$ for $\mu \in \{0, 1, 2, 3\}$ together with the condition $\{\gamma^5, \gamma^5\} = 2\mathbb{1}_{4\times4}$. However, both the left-handed and right-handed part of an arbitrary Dirac spinor, are eigenvectors of the chirality operator with eigenvalue $+1$ and -1 respectively. With the aim of defining Weyl spinors, we introduce the projection operators

$$P_L := \frac{1}{2}(\mathbb{1} - \gamma^5) = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}, \tag{A.4}$$

$$P_L := \frac{1}{2}(\mathbb{1} + \gamma^5) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix}. \tag{A.5}$$

Hence, we can decompose one Dirac spinor Ψ into left- and right-handed part, which correspond to the so-called Weyl spinors ψ and $\bar{\chi}$

$$P_L\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ 0 \\ 0 \end{pmatrix}, \qquad P_R\Psi = \begin{pmatrix} 0 \\ 0 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \tag{A.6}$$

$$\psi := \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \equiv (\psi_\alpha), \qquad \bar{\chi} := \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \equiv (\bar{\chi}^{\dot{\alpha}}). \tag{A.7}$$

As mentioned above, a unitary representation of the Lorentz group on the spinor space is defined such that the spacetime LT Λ affects the spinor as

$$\Psi \to S(\Lambda)\Psi. \tag{A.8}$$

An infinitesimal transformation $\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$ leads to an expansion in the representation

$$S(\Lambda = \delta + \omega) = \mathbb{1}_{4\times 4} - \frac{i}{2}\omega^{\rho\sigma} S_{\rho\sigma}, \qquad \text{with} \tag{A.9}$$

$$S_{\rho\sigma} := \frac{i}{4}[\gamma_\rho, \gamma_\sigma] = \begin{pmatrix} s_{\rho\sigma} & 0 \\ 0 & \bar{s}_{\rho\sigma} \end{pmatrix}. \tag{A.10}$$

Since the Matrix $S_{\rho\sigma}$ in (A.10) has a block form, the transformation is reducible. This means that the four dimensional complex spinor space decompose into two invariant subspaces, where the Lorentz group is acting on. Indeed, a Weyl spinor can therefore be regarded as a element of a two dimensional complex vector space on whom the simplest, non-trivial and irreducible representations of the Lorentz algebra is defined on.

B Grassmann Numbers and Graded Lie Algebras

B.1 Grassmann Numbers

Grassmann numbers $\mathbb{G}$ are defined by satisfying a fermionic relation

$$\{\theta_i, \theta_j\} = 0, \qquad \theta^i, \theta^j \in \mathbb{G}. \tag{B.1}$$

In the case $i, j \in \{1, 2\}$ they are therefore characterized as components of Weyl spinors. This additional structure implies an mathematically interesting point. Since $(\theta_i)^2 = 0$, Grassmann numbers are non-zero square-roots of zero. As one would expect from the vector space structure of Weyl spinors, Grassmann numbers incorporate complex numbers $z \in \mathbb{C}$ in this way

$$z \cdot \theta_i \in \mathbb{G}, \qquad [z; \theta_i] = 0. \tag{B.2}$$

Henceforth, the case $i, j \in \{1, 2\}$ is considered. One can go one step further and define a function on this numbers. The fact that such a function is holomorphic makes it easy to express $f(\theta)$ in general. The Taylor series of $f(\theta)$ truncates early and the function has the general form

$$f(\theta) = a + \psi^\alpha \theta_\alpha + d\theta\theta. \tag{B.3}$$

Derivatives w.r.t. Grassmann numbers can be defined as

$$\frac{\partial}{\partial \theta^\alpha}\theta^\beta := \delta^\beta_\alpha, \qquad \frac{\partial}{\partial \theta_\alpha}\theta_\beta := \delta^\alpha_\beta, \tag{B.4}$$

$$\frac{\partial}{\partial \bar\theta^{\dot\alpha}}\bar\theta^{\dot\beta} := \delta^{\dot\beta}_{\dot\alpha}, \qquad \frac{\partial}{\partial \bar\theta_{\dot\alpha}}\bar\theta_{\dot\beta} := \delta^{\dot\alpha}_{\dot\beta}. \tag{B.5}$$

This derivatives are fermionic and obey

$$\frac{\partial}{\partial \theta_\alpha} = -\epsilon^{\alpha\beta}\frac{\partial}{\partial \theta^\beta}, \qquad \frac{\partial}{\partial \bar\theta_{\dot\alpha}} = -\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial \bar\theta^{\dot\beta}}. \tag{B.6}$$

By using the principles of translation invariance and linearity, integrals over θ can be defined as functionals:

$$\int \mathrm{d}^2\theta (a + \psi^\alpha\theta_\alpha + d\theta\theta) := d, \tag{B.7}$$

$$\int \mathrm{d}^2\bar\theta (a + \bar\psi_{\dot\alpha}\bar\theta^{\dot\alpha} + d\bar\theta\bar\theta) := d, \tag{B.8}$$

$$\int \mathrm{d}^4\theta := \int \mathrm{d}^2\theta \mathrm{d}^2\bar\theta. \tag{B.9}$$

B.2 Graded Lie Algebras

The concept of Lie algebra can be extended with the idea of including fermionic generators, i.e. operators which satisfy anticommuting relations. The structure of a graded algebra captures this feature by introducing the Lie product in a generalized way. Let O_i, O_j be operators of a graded Lie algebra, then

$$[O_i, O_j]_{Lie} := O_i O_j - (-1)^{\eta_i \eta_j} O_j O_i = h_{ijk} O_k, \tag{B.10}$$

introduces new structure constants h_{ijk} that no longer have to be totally antisymmetric. The graded Lie product is defined by the use of the so-called gradings η_i.

$$\eta_i = \begin{cases} 0 & : O_i \text{ bosonic generator} \\ 1 & : O_i \text{ fermionic generator} \end{cases} \tag{B.11}$$

Since η_i can only take two values $i \in \{0, 1\}$, this algebra is called $\mathbb{Z}_2$-graded. If O_i, O_j are both fermionic the graded Lie bracket $[.,.]_{Lie}$ become, as required, the anticommuting brackets. Otherwise the graded Lie bracket is still the commutator with total antisymmetric structure constants.

Bibliography

[1] CMS Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. **B716** (2012) 30–61, `arXiv:1207.7235 [hep-ex]`.

[2] ATLAS, CMS Collaboration, G. Aad et al., *Combined Measurement of the Higgs Boson Mass in pp Collisions at* $\sqrt{s} = 7$ *and 8 TeV with the ATLAS and CMS Experiments*, Phys. Rev. Lett. **114** (2015) 191803, `arXiv:1503.07589 [hep-ex]`.

[3] S. Weinberg, *The Making of the standard model*, Eur. Phys. J. **C34** (2004) 5–13, `arXiv:hep-ph/0401010 [hep-ph]`. [,99(2005)].

[4] SLD Electroweak Group, SLD Heavy Flavor Group, DELPHI, LEP, ALEPH, OPAL, LEP Electroweak Working Group, L3 Collaboration, t. S. Electroweak, *A Combination of preliminary electroweak measurements and constraints on the standard model*, `arXiv:hep-ex/0312023 [hep-ex]`.

[5] S. P. Martin, *A Supersymmetry primer*, `arXiv:hep-ph/9709356 [hep-ph]`. [Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[6] M. Schwartz, *Quantum Field Theory and the Standard Model*. Quantum Field Theory and the Standard Model. Cambridge University Press, 2014.

[7] K.-i. Shizuya, *Quantization of the Massive Yang-Mills Field in Arbitrary Gauges*, Nucl. Phys. **B94** (1975) 260–268.

[8] F. Englert and R. Brout, *Broken Symmetry and the Mass of Gauge Vector Mesons*, Phys. Rev. Lett. **13** (1964) 321–323.

[9] P. W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, Phys. Rev. Lett. **13** (1964) 508–509.

[10] P. W. Higgs, *Broken symmetries, massless particles and gauge fields*, Phys. Lett. **12** (1964) 132–133.

[11] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *Global Conservation Laws and Massless Particles*, Phys. Rev. Lett. **13** (1964) 585–587.

[12] M. Kobayashi and T. Maskawa, *CP Violation in the Renormalizable Theory of Weak Interaction*, Prog. Theor. Phys. **49** (1973) 652–657.

[13] M. F. Sohnius, *Introducing Supersymmetry*, Phys. Rept. **128** (1985) 39–204.

[14] R. Kallosh, *An Update on Perturbative N=8 Supergravity*, `arXiv:1412.7117 [hep-th]`.

[15] Z. Bern, S. Davies, and T. Dennen, *The Ultraviolet Critical Dimension of Half-Maximal Supergravity at Three Loops*, `arXiv:1412.2441 [hep-th]`.

[16] S. Ferrara, J. Wess, and B. Zumino, *Supergauge Multiplets and Superfields*, Phys. Lett. **B51** (1974) 239.

[17] L. Girardello and M. T. Grisaru, *Soft Breaking of Supersymmetry*, Nucl. Phys. **B194** (1982) 65.

[18] I. Aitchison, *Supersymmetry in Particle Physics: An Elementary Introduction.* Cambridge University Press, 2007.

[19] P. Athron, J.-h. Park, D. Stöckinger, and A. Voigt, *FlexibleSUSY—A spectrum generator generator for supersymmetric models*, Comput. Phys. Commun. **190** (2015) 139–172, `arXiv:1406.2319 [hep-ph]`.

[20] F. Staub, *SARAH 4 : A tool for (not only SUSY) model builders*, Comput. Phys. Commun. **185** (2014) 1773–1790, `arXiv:1309.7223 [hep-ph]`.

[21] B. C. Allanach et al., *SUSY Les Houches Accord 2*, Comput. Phys. Commun. **180** (2009) 8–25, `arXiv:0801.0045 [hep-ph]`.

[22] P. Z. Skands et al., *SUSY Les Houches accord: Interfacing SUSY spectrum calculators, decay packages, and event generators*, JHEP **07** (2004) 036, `arXiv:hep-ph/0311123 [hep-ph]`.

[23] T. Steudtner, *Automatic Resummation of Logarithmic Enhanced Multiloop Contributions to the lightest CP-even Higgs Boson Mass in FlexibleSUSY*, master thesis, Technische Universität Dresden, 2015.

[24] J. Fleischer, F. Jegerlehner, O. V. Tarasov, and O. L. Veretin, *Two loop QCD corrections of the massive fermion propagator*, Nucl. Phys. **B539** (1999) 671–690, `arXiv:hep-ph/9803493 [hep-ph]`. [Erratum: Nucl. Phys.B571,511(2000)].

[25] A. Bednyakov, A. Onishchenko, V. Velizhanin, and O. Veretin, *Two loop O(alpha-s**2) MSSM corrections to the pole masses of heavy quarks*, Eur. Phys. J. **C29** (2003) 87–101, `arXiv:hep-ph/0210258 [hep-ph]`.

[26] P. Athron, J.-h. Park, T. Steudtner, D. Stöckinger, and A. Voigt, *Precise Higgs mass calculations in (non-)minimal supersymmetry at both high and low scales*, `arXiv:1609.00371 [hep-ph]`.

[27] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields*, Phys. Rev. D **11** (1975) 2856–2861.

[28] G. Degrassi, P. Slavich, and F. Zwirner, *On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing*, Nucl. Phys. **B611** (2001) 403–422, `arXiv:hep-ph/0105096 [hep-ph]`.

[29] A. Brignole, G. Degrassi, P. Slavich, and F. Zwirner, *On the O(alpha(t)**2) two loop corrections to the neutral Higgs boson masses in the MSSM*, Nucl. Phys. **B631** (2002) 195–218, `arXiv:hep-ph/0112177 [hep-ph]`.

[30] A. Dedes and P. Slavich, *Two loop corrections to radiative electroweak symmetry breaking in the MSSM*, Nucl. Phys. **B657** (2003) 333–354, `arXiv:hep-ph/0212132 [hep-ph]`.

[31] A. Dedes, G. Degrassi, and P. Slavich, *On the two loop Yukawa corrections to the MSSM Higgs boson masses at large tan beta*, Nucl. Phys. **B672** (2003) 144–162, `arXiv:hep-ph/0305127 [hep-ph]`.

[32] P. Draper and H. Rzehak, *A Review of Higgs Mass Calculations in Supersymmetric Models*, Phys. Rept. **619** (2016) 1–24, `arXiv:1601.01890 [hep-ph]`.

[33] J. Pardo Vega and G. Villadoro, *SusyHD: Higgs mass Determination in Supersymmetry*, JHEP **07** (2015) 159, `arXiv:1504.05200 [hep-ph]`.

[34] E. Bagnaschi, G. F. Giudice, P. Slavich, and A. Strumia, *Higgs Mass and Unnatural Supersymmetry*, JHEP **09** (2014) 092, `arXiv:1407.4081 [hep-ph]`.

[35] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, *Investigating the near-criticality of the Higgs boson*, JHEP **12** (2013) 089, `arXiv:1307.3536 [hep-ph]`.

[36] A. V. Bednyakov, A. F. Pikelner, and V. N. Velizhanin, *Higgs self-coupling beta-function in the Standard Model at three loops*, Nucl. Phys. **B875** (2013) 552–565, `arXiv:1303.4364 [hep-ph]`.

[37] J. R. Espinosa and R.-J. Zhang, *MSSM lightest CP even Higgs boson mass to O(alpha(s) alpha(t)): The Effective potential approach*, JHEP **03** (2000) 026, `arXiv:hep-ph/9912236 [hep-ph]`.

[38] C. Ford, I. Jack, and D. R. T. Jones, *The Standard model effective potential at two loops*, Nucl. Phys. **B387** (1992) 373–390, `arXiv:hep-ph/0111190 [hep-ph]`. [Erratum: Nucl. Phys.B504,551(1997)].

[39] J. R. Espinosa and R.-J. Zhang, *Complete two loop dominant corrections to the mass of the lightest CP even Higgs boson in the minimal supersymmetric standard model*, Nucl. Phys. **B586** (2000) 3–38, `arXiv:hep-ph/0003246 [hep-ph]`.

[40] P. Draper, G. Lee, and C. E. M. Wagner, *Precise estimates of the Higgs mass in heavy supersymmetry*, Phys. Rev. **D89** no. 5, (2014) 055023, `arXiv:1312.5743 [hep-ph]`.

[41] B. C. Allanach, A. Djouadi, J. L. Kneur, W. Porod, and P. Slavich, *Precise determination of the neutral Higgs boson masses in the MSSM*, JHEP **09** (2004) 044, `arXiv:hep-ph/0406166 [hep-ph]`.

[42] H. Bahl and W. Hollik, *Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations*, Eur. Phys. J. **C76** no. 9, (2016) 499, `arXiv:1608.01880 [hep-ph]`.

[43] S. Dimopoulos and H. Georgi, *Softly Broken Supersymmetry and SU(5)*, Nucl. Phys. **B193** (1981) 150–162.

[44] S. Heinemeyer, W. Hollik, and G. Weiglein, *The Mass of the lightest MSSM Higgs boson: A Compact analytical expression at the two loop level*, Phys. Lett. **B455** (1999) 179–191, `arXiv:hep-ph/9903404 [hep-ph]`.

[45] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, *Higgs mass and vacuum stability in the Standard Model at NNLO*, JHEP **08** (2012) 098, `arXiv:1205.6497 [hep-ph]`.

[46] J. Unger, *Supersymmetry Invariance of Dimensional Reduction Applied to Higgs Boson Mass Calculations*, master thesis, Technische Universität Dresden, 2016.

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