QUANTUM FIELD-THEORETICAL APPROACH TO SPONTANEOUSLY BROKEN GAUGE INVARIANCE

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I review my previous work[1] on the Higgs mechanism, adding some new results, and mention its application to the Schwinger model done mainly by K. R. ITO[2].

As is well known, when phase symmetry is spontaneously broken in gauge theory, the gauge field acquires a non-zero mass, while the Goldstone boson disappears according to the Higgs mechanism[3]. It is impossible, however, to invalidiate the Goldstone theorem in the usual framework of local field theory. It is important, therefore, to explain why and how the Higgs mechanism is compatible with the Goldstone theorem in the manifestly covariant quantum field theory.

I consider the Higgs model as an example. Its Lagrangian in the covariant formalism is given by

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + B\partial^{\mu}A_{\mu} + \tfrac{1}{2}\alpha B^2 \\ &+ (\partial^{\mu}+ieA^{\mu})\phi^{\dagger}\cdot(\partial_{\mu}-ieA_{\mu})\phi + \tfrac{1}{2}\mu_0^2\phi^{\dagger}\phi - \tfrac{1}{4}\lambda(\phi^{\dagger}\phi)^2, \end{aligned} \tag{1}$$

where B is an auxiliary scalar filed, α being a gauge parameter. Field equations for the gauge field A_{μ} are

$$\begin{aligned}\Box A_{\mu} - (1-\alpha)\partial_{\mu}B &= j_{\mu}, \qquad & \partial^{\mu}j_{\mu} &= 0, \\ \partial^{\mu}A_{\mu} + \alpha B &= 0, & \Box B &= 0.\end{aligned} \tag{2}$$

Canonical quantization is carried out. Then one can compute all the four-dimensional commutation relations involving B; especially,

$$[B(x),\ \phi(y)] = e\phi(y)D(x-y). \tag{3}$$

The subsidiary condition

$$B^{(+)}(x)\,|\,\mathrm{phys}\rangle = 0 \tag{4}$$

is imposed upon physical states so as to avoid negative probability.

Now, spontaneous symmetry breaking is introduced by setting

$$\sqrt{2}\phi(x) = v + \varphi(x) + i\chi(x),$$
$$v = v^* \neq 0, \quad \langle 0|\varphi(x)|0\rangle = \langle 0|\chi(x)|0\rangle = 0 \tag{5}$$

as usual. Then the commutation relation (3) implies

$$[B(x),\ \chi(y)] = -ie[v + \varphi(y)]D(x-y). \tag{6}$$

Therefore,

$$\langle 0|[B(x),\ \chi(y)]|0\rangle = -iM\cdot D(x-y), \tag{7}$$

where $M = ev$ stands for the zeroth approximation to the mass of the gauge particle.

Let A_μ^{in}, B^{in}, φ^{in}, and χ^{in} be the asymptotic fields of A_μ, B, φ, and χ, respectively. Since $B^{in} = B$, the subsidiary condition (4) holds also for in-states. From (7) one obtains

$$[B^{in}(x),\ \chi^{in}(y)] = -iM\cdot D(x-y). \tag{8}$$

This is a very important relation, which yields the following conclusions.

1. χ^{in} is massless, that is, it should be the Goldstone field. Thus, the Goldstone theorem holds.

2. The Goldstone bosons are unphysical because they do not satisfy the subsidiary condition (4). Hence the Higgs mechanism works for physical states. Here it can be shown that physical fields are $U_\mu^{in} = A_\mu^{in} - Z_\chi M^{-2}\partial_\mu B^{in} - M^{-1}\partial_\mu\chi^{in}$, B^{in}, and φ^{in}.

Thus, the consistency between the Goldstone theorem and the Higgs mechanism is established in a manifestly covariant way.

The Goldstone field χ^{in} can be shown to satisfy

$$[\chi^{in}(x),\ \chi^{in}(y)] = iZ_\chi D(x-y) + i\alpha M^2 E(x-y), \tag{9}$$

where $E(x) \equiv -(\partial/\partial m^2)\Delta(x,m^2)|_{m=0}$, and

$$[Q,\ \chi^{in}(x)] = -iM, \tag{10}$$

where Q denotes the generator of the gauge transformation. Furthermore, the equation for χ^{in} is

$$\Box \chi^{in} = -\alpha M \cdot B^{in}. \tag{11}$$

Thus the Goldstone bosons are dipole ghosts for $\alpha \neq 0$. This may be the first example of dipole-ghost Goldstone bosons. [A similar conclusion has been reached by YOKOYAMA (private communication) independently.]

The formalism developed above has been applied to the Schwinger model[4] (the two-dimensional massless QED) by ITO.[2] This model is known to be exactly solvable, and the gauge field acquires a non-zero mass. According to KOGUT and SUSSKIND,[5] the so-called Schwinger mechanism is different from the Higgs mechanism because there is no spontaneous breakdown of symmetry. An opposite conclusion, however, is deduced on this point: the Schwinger mechanism is nothing but a special case of the Higgs mechanism.

Field equations are again given by (2), but in the Schwinger model, one has $j_\mu = J_\mu - m^2 A_\mu$, where $m^2 = e^2/\pi$ and J_μ is a current composed by free fermion fields. According to KLAIBER,[6] one can construct an "associated boson" field $X(x)$ such that

$$J_\mu = \partial_\mu X. \tag{12}$$

Therefore A_μ satisfies

$$(\Box + m^2) A_\mu - \partial_\mu [(1-\alpha)B + X] = 0. \tag{13}$$

Hence all two-dimensional commutation relations are easily calculated, and one finds that

$$[X(x),\ X(y)] = im^2 D(x-y) + i\alpha m^4 E(x-y), \tag{14}$$

$$\langle 0|[Q,\ X(x)]|0\rangle = [Q,\ X(x)] = -im^2 \neq 0, \tag{15}$$

$$\Box X = -\alpha m^2 B. \tag{16}$$

Eq. (15) shows that there is indeed spontaneous breakdown of symmetry and that χ is the Goldstone field. Furtheremore, I emphasize that there is complete parallelism between the Schwinger model and the Higgs model as is seen by comparing (14)-(16) with (9)-(11), that is, $m^{-1}\chi$ corresponds to χ^{in}. (Detailed accounts will appear elsewhere.)

REFERENCES

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4. J. Schwinger, Phys. Rev. 128, 2425(1962).
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6. B. Klaiber, in Boulder Lectures in Theoretical Physics, Vol. XA (1967), p.141.

DISCUSSIONS

ITO (comment): I comment that the Schwinger model can be solved more rigorously by the generalized Bogolubov transformation. I have found that its bare Hamiltonian cannot be positive without renormalization counterterms and that photons become massive by those counterterms.

SWIECA (question): Is it true that the problem of whether there is or not spontaneous breakdown in the Schwinger model depends on the gauge used?

NAKANISHI (answer): Gauge dependence might be possible; I am not sure. The complete parallelism between the Schwinger model and the Higgs model exists at least in the covariant gauges.

SEKINE (question): By what equations have you defined the in-fields for dipole ghosts? Isn't there any trouble?

NAKANISHI (answer): They can be defined consistently even for dipole-ghost fields by using the Yang-Feldman formalism.