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Erratum: Relic density computations at NLO: infrared finiteness and thermal correction

Martin Beneke, Francesco Dighera and Andrzej Hryczuk

Physik Department T31, Technische Universität München,
James-Franck-Str. 1, 85748 Garching, Germany

E-mail: francesco.dighera@tum.de, andrzej.hryczuk@tum.de

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In ref. [1] a systematic approach starting from non-equilibrium quantum field theory to relic density computations at next-to-leading order (NLO) was presented. The primary purpose of this work was to demonstrate the cancellation of the temperature-dependent infrared divergences. In addition, the leading finite temperature-dependent correction in a model, where dark matter annihilation into Standard Model (SM) fermions is mediated by an electrically charged scalar, was computed and found to be of $\mathcal{O}(T^2)$. It was also noted that this correction exhibited a surprisingly simple structure. This result is incorrect, and the $\mathcal{O}(T^2)$ correction actually vanishes altogether. Below we list the corrections to the original manuscript and provide the leading finite-temperature contribution, which is of order $\mathcal{O}(T^4)$. An explanation of the temperature-dependence of the correction and the absence of the $\mathcal{O}(T^2)$ term can be given in terms of an operator product expansion [2].

1. The diagram from the photon-tadpole contribution to the scalar self-energy was missed in ref. [1]. This diagram is infrared-finite and vanishes at $T = 0$, but contributes at finite temperature at order $\tau^2 = T^2/m_\chi^2$. Tables 4 and 5 of ref. [1] must be amended by tables 1 and 2, respectively, below. (An analogous amendment is necessary for tables 1 and 2 of ref. [1].) Once the (purely virtual) terms from these diagrams are added, the "Total" $\mathcal{O}(\tau^2)$ correction given at the bottom of tables 4 and 5 of ref. [1] is exactly zero.

2. It follows that eqs. (5.1) and (5.2) of ref. [1] must be corrected. Δ_a in eq. (5.1) is zero, and the $\mathcal{O}(\tau^2)$ term in eq. (5.2) is absent. The surprisingly simple form of the finite-temperature correction in eq. (5.2) is a consequence of the simplicity of the missed photon self-energy tadpole. The diagrams shown in tables 1 and 2 factorize

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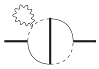
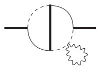
The finite part J_1		
Type A	Real	Virtual
		$\frac{4(1-2\epsilon^2)}{D_\xi^2}$
		— " —

Table 1. The self-energy diagrams of type A, with corresponding coefficients of the finite $\mathcal{O}(\tau^2)$ correction, to be added to table 4 of [1].

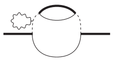
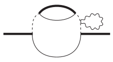
The finite part J_1		
Type B	Real	Virtual
		$-\frac{4}{D_\xi^2}$
		— " —

Table 2. The self-energy diagrams of type B, with corresponding coefficients of the finite $\mathcal{O}(\tau^2)$ correction, to be added to table 5 of [1].

into the tadpole times the tree diagram with an additional scalar propagator, which gives rise to the derivative in ξ^2 in eq. (5.2). Since the $\mathcal{O}(\tau^2)$ correction must vanish on general grounds as shown in ref. [2], the incorrect $\mathcal{O}(\tau^2)$ correction found in ref. [1] is simply the negative of the contribution from the tadpole diagram.

3. The leading temperature-dependent correction from the thermal bath of photons is of order $\mathcal{O}(\tau^4)$. Because the photon tadpole diagrams do not contribute at this order, the expression in the s-wave limit and for massless SM fermions given in eq. (5.3) of [1],
$$\Delta a_{\tau^4}^{\epsilon=0} = \frac{8\pi^2\lambda^4\alpha\tau^4}{45}\frac{1}{(1+\xi^2)^4} = \frac{4\pi}{45}\alpha\tau^4\frac{1}{(1+\xi^2)^2}\frac{a_{\text{tree}}}{\epsilon^2}\bigg|_{\epsilon=0}, \tag{1}$$
is correct. Since this is now the leading correction, we also give the full result without restriction to the s-wave limit ($e_\chi = 1$), and for finite SM fermion mass, but expanded in the mass of the mediator ($\xi = m_\phi/m_\chi \gg 1$):
$$\begin{aligned} s\,\sigma_{\text{ann}}v_{\text{rel}}\,\big|_{\tau^4,\ \text{thermal photons}} \\ = \frac{4\pi^2\lambda^4\alpha\tau^4}{135\,e_\chi^3\,(e_\chi^2-4\epsilon^2)^{5/2}\,\xi^4}\Big(&-2e_\chi^6(e_\chi^2-1)+\epsilon^2e_\chi^4(22e_\chi^2-25) \\ &-\epsilon^4e_\chi^2(80e_\chi^2-101)+3\epsilon^6(38e_\chi^2-47)\Big)+\mathcal{O}(\xi^{-6})\,. \end{aligned} \tag{2}$$

4. The finite-temperature correction from the SM fermions in the thermal bath given in eq. (5.4) of [1] is also incorrect. There is no $\mathcal{O}(\tau^2)$ contribution in the massless fermion limit. (Here the error arose due to an inconsistent Fierz transformation in

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dimensional regularization applied to the collinear divergences.) The leading $\mathcal{O}(\tau^4)$ correction for massless fermions is

$$\Delta a^{\epsilon=0}_{\tau^4,\text{ thermal fermions}} = \frac{7\pi^2\lambda^4\alpha\tau^4}{45}\,\frac{3\xi^4+4\xi^2+5}{(\xi^4-1)^3}\,, \tag{3}$$

in the s-wave limit for arbitrary mediator mass, and

$$s\,\sigma_{\text{ann}}v_{\text{rel}}\,|^{\epsilon=0}_{\tau^4,\text{ thermal fermions}} = \frac{28\pi^2\lambda^4\alpha\tau^4\left(e_\chi^2-1\right)}{135\,e_\chi^2\,\xi^4}\,. \tag{4}$$

without restriction to the s-wave limit, but at leading order in an expansion in the heavy mediator mass.

References

[1] M. Beneke, F. Dighera and A. Hryczuk, *Relic density computations at NLO: infrared finiteness and thermal correction*, *JHEP* **10** (2014) 045 [arXiv:1409.3049] [INSPIRE].

[2] M. Beneke, F. Dighera and A. Hryczuk, *Finite-temperature modification of heavy particle decay and dark matter annihilation*, arXiv:1607.03910 [INSPIRE].

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