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Analytical Approach for the Solution of the Nonlinear GLR-MQ Equation

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The present paper reports the effect of nonlinear corrections to the behaviour of gluon distribution in the Gribov-Levin-Ryskin-Mueller-Qiu (GLR-MQ) approach. Here the nonlinear GLR-MQ evolution equation is solved analytically in order to determine the x and Q^2 dependence of gluon distribution function. We observe that, the gluon distribution increases with increasing Q^2 and decreasing x, which is in agreement with the perturbative QCD fits at small-x, however this particular behaviour of the gluon distribution function is tamed by the shadowing effects of gluon recombination. The obtained results of the nonlinear gluon distribution at the hot spot are compared with different global parton analysis as well as with those obtained from the solution of the modified-DGLAP (MD-DGLAP) equation and are found to be quite compatible.

Key Words : GLR-MQ Equation; Gluon Distribution Function; Shadowing Corrections

Introduction

At small-x the linear QCD evolution equation, viz. the Dokshitzer-Gribov -Lipatov-Altarelli-Parisi (DGLAP) equation (Dokshitzer, 1977; Altarelli and Parisi 1977; Gribov and Lipatov, 1972) predicts a rapid increase in the density of gluons. This behaviour eventually violates unitarity. But, of course, it is expected that the growth of gluon densities saturates at a given time due to Froissart bound (Froissart, 1961), which states that the total hadronic cross sections cannot grow faster than the logarithm squared of the energy, $\ln s^2$, as $s \to \infty$. At small-x the produced gluons start to overlap spatially in the transverse area and thus unitarity is restored. Therefore at small-x apart from the gluon splitting functions, the gluon recombination processes, $gg \to g$, also become significantly important. This phenomenon of gluon recombination, also referred to as nonlinear effects or shadowing corrections, leads to parton saturation at small enough x. This subject has been studied in great detail for the last few years (Levin and Rezaeian, 2010; Kutak, 2011; Giannini and Duraes, 2013). The shadowing corrections of gluon recombination to the integrated parton distributions were

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mainly studied by adding nonlinear terms in the DGLAP evolution equation in the collinear factorization scheme. Pioneering work in this aspect was done by Gribov, Levin, and Ryskin (GLR) and by Mueller and Qiu (MQ) at the twist-4 level and we call their result as GLR-MQ evolution equation (Gribov *et al.*, 1983; Muller and Qin, 1986).

The solution of the GLR-MQ equation attracts considerable attention for the investigation of nonlinear effects as a consequence of gluon recombination in the region of high gluon density at small-x. Immense studies based on GLR-MQ approach have been done in the last few years (Prytz, 2001; Boroun, 2009; Boroun and Zarrin, 2013). The present authors have also pursued such an approach with reasonable phenomenological success (Devee *et al.*, 2014). In the present paper we intend to study the effect of nonlinear corrections to the behavior of gluon distributions in the GLR-MQ approach. Here we estimate the x and Q^2 dependence of gluon distribution considering the influence of gluon recombination at small-x. Our predictions are compared with different parametrizations viz. GJR2008LO (Gluck *et al.*, 2008). MSTW2008LO (Martin *et al.*, 2009), HERAPDF0.1 (Kretzschmar *et al.*, 2009), CT10 (Lai *et al.*, 2010) as well as with EHKQS model (Eskola *et al.*, 2003). We further perform the check of our calculations by comparing the predicted nonlinear gluon density with the solution of the more precise and more complicated modified DGLAP (MD-DGLAP) equation (Zhu, 1999). The MD-DGLAP evolution equation was derived by W. Zhu taking into account the effect of antishadowing corrections in addition to the shadowing corrections.

The plan of the paper is as follows. In Sec. 2 we briefly describe the formalism for the solution of nonlinear GLR-MQ equation to study the effect of gluon recombination on the behavior of gluon density at small-x. We present the results and discussions of our predictions in Sec. 3. We summarize in Sec. 4.

Formalism

The nonlinear corrections to the QCD evolution arise from the recombination of two gluon ladders into one and they modify the evolution equation of gluon distribution as (Gribov *et al.*, 1983),

$$\frac{\partial G(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z} G\Big(\frac{x}{z},Q^2\Big) P_{gg}(z) - \frac{81}{16}\frac{\alpha_s^2(Q^2)}{R^2Q^2}\int_x^1 \frac{dz}{z}\Big[G\Big(\frac{x}{z},Q^2\Big)\Big]^2, \tag{1}$$

which is known as the GLR-MQ evolution equation. In leading twist approximation, the strong coupling constant $\alpha_s(Q^2) = \frac{4\pi}{\beta_0 ln(Q^2/\Lambda^2)}$, where $\beta_0 = 11 - \frac{2}{3}N_f$, with N_f being the number of active quark flavors. The first term is the standard DGLAP result, linear in the gluon distribution function. The quark-gluon emission diagrams are not considered here as they have very little significance in the gluon-dominated small-x region. The minus sign in front of the non-linear term is accountable for gluon recombination that describes shadowing corrections. If the gluons are spread throughout the entire proton then R, the correlation radius between two interacting gluons, will be of the order of proton radius ($R \simeq 5$ GeV^{-1}). On the other hand R is much smaller $(R \simeq 2 \text{ GeV}^{-1})$ if the gluons are concentrated in a hot spot within the proton.

Substituting the leading order gluon-gluon splitting function P_{gg} in Eq.(1) (Abott *et al.*, 1980) and performing the integrations in z for $x \leq z \leq 1$, Eq.(1) can be simplified as

$$\frac{\partial G(x,t)}{\partial t} = A_1(x)\frac{G(x,t)}{t} - A_2(x)\frac{G^2(x,t)}{t^2 e^t}, \tag{2}$$

where, $t = \ln(\frac{Q^2}{\Lambda^2})$, with Λ being the QCD cut off parameter. The explicit forms of the functions $A_1(x)$ and $A_2(x)$ are

$$A_1(x) = 3A_f\Big[\frac{11}{12} - \frac{N_f}{18} + \ln(1-x) + \int_x^1 dz\Big\{\frac{z^{\lambda_G+1}-1}{1-z} + \Big(z(1-z) + \frac{1-z}{z}\Big)z^{\lambda_G}\Big\}\Big] \tag{3}$$

and

$$A_2(x) = \frac{81}{16}\frac{A_f^2\pi^2}{R^2}\int_x^1 z^{2\lambda_G-1}dz, \tag{4}$$

where, $A_f = 4/\beta_0$. Eq.(2) is obtained by employing a simple form of Regge ansatz for the determination of the gluon distribution function at small-x given as

$$G(x,Q^2) = M(Q^2)x^{-\lambda_G}, \tag{5}$$

where $M(Q^2)$ is a function of Q^2 and λ_G is the Regge intercept for gluon distribution function. We accomplish our calculations taking the value of λ_G in the range $0.35 \leq \lambda \leq 0.5$.

From Eq.(2), which is a partial differential equation in x and Q^2, one can obtain the solution of the gluon distribution function as

$$G(x,t) = \frac{t^{A_1(x)}}{C - A_2(x)\Gamma[-1 + A_1(x), t]}, \tag{6}$$

where C is a constant to be determined from initial boundary conditions. Eq. (6) assist us to study how the nonlinear or shadowing corrections due to gluon recombinations influence the behavior of gluons at small-x.

We now use the following two physically plausible boundary conditions

$$G(x,t) = G(x,t_0) \tag{7}$$

at some low $t = t_0$, and

$$G(x,t) = G(x_0,t) \tag{8}$$

at some initial higher value $x = x_0$.

Using the physically plausible boundary condition defined in Eq.(7) along with the use of Eq.(6) we obtain

$$G(x,t) = \frac{t^{A_1(x)} G(x,t_0)}{t_0^{A_1(x)} + A_2(x)\Big\{\Gamma[-1+A_1(x),t_0] - \Gamma[-1+A_1(x),t]\Big\} G(x,t_0)}. \tag{9}$$

Using Eq.(9) we can easily estimate the gluon distribution as a function of Q^2 for a particular value of x by taking a suitable input distribution $G(x,t_0)$. Similarly applying the boundary condition given by Eq.(8), Eq.(6) reads as

$$G(x,t) = \frac{t^{A_1(x)} G(x_0,t)}{t^{A_1(x_0)} + \Big\{A_2(x_0)\Gamma[-1+A_1(x_0),t] - A_2(x)\Gamma[-1+A_1(x),t]\Big\} G(x_0,t)}. \tag{10}$$

from which one can determine the small-x dependence of the gluon distribution function for fixed Q^2 by taking an appropiate input distribution at an initial value of $x = x_0$.

Result and Discussion

The nonlinear GLR-MQ evolution equation for gluon distribution is solved semi-numerically in order to predict the influence of shadowing corrections as a consequence of gluon recombination processes at small-x and moderate Q^2. The input gluon parameterization has been taken from MRST2001LO (Martin *et al*., 2002) to study the effect of nonlinear corrections on the x and Q^2 dependence of $G(x,Q^2)$. In Fig. 1 we plot our best fit results of Q^2-dependence of gluon distribution function computed from Eq.(9) for $x = 10^{-3}$ and 10^{-4} respectively and compared with the GJR2008LO (Gluck *et al*., 2008) and MSTW2008LO (Martin *et al*., 2009) global fits as well as with the EHKQS model (Eskola *et al*., 2003).

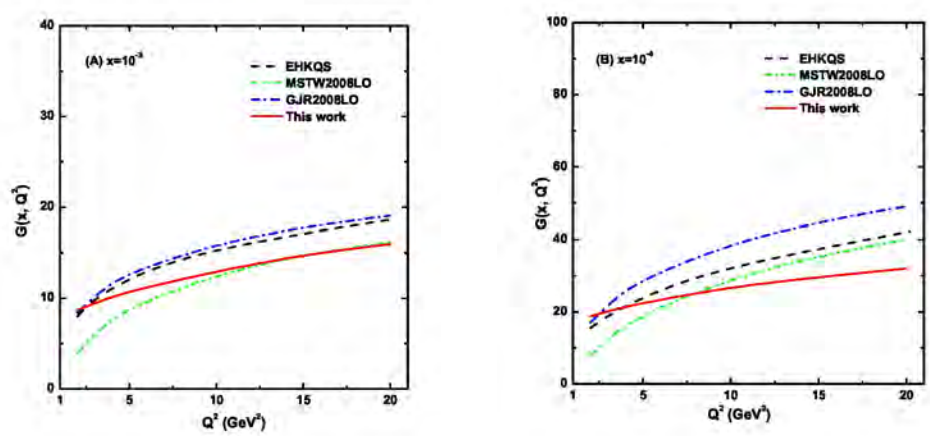

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Fig. 1: (A) Plots of gluon distribution function $G(x,Q^2)$ versus Q^2 according to Eq.(9) at the hot spot R =2GeV^{-1} f or two representative x, viz. x = 10^{-3} a nd x = 10^{-4}. (B) The solid curves are our result, dash curves are the results of EHKQS model, dash-dot curves are from GJR208LO and dash-dot-dot curves are from MSTW2008LO global fit

On the other hand, in Fig. 2 $G(x, Q^2)$ is plotted as a function of x calculated from Eq.(10) for two different values of Q^2, namely Q^2 = 5 and 15 GeV^2 respectively. Our results are compared with HERAPDF0.1 (Kretzschmar *et al.*, 2009) and CT10 (Lal *et al.*, 2010) parametrizations. Our predictions obtained from Eq.(10) are also compared with those obtained in similar analysis by using MD-DGLAP equation (Zhu, 1999) as shown in Fig. 2.

It is interesting to observe as well that the shape of the MD-DGLAP curve is almost similar to our results in the region $x < 10^{-3}$. The gluon distribution function increases with increasing Q^2 and decreasing x as expected, which is also in agreement with perturbative QCD fits at small-x. But the effect of gluon recombination at twist-4 level slows down the rapid increase of gluon densities. We perform our analysis in the kinematic region $2 < Q^2 < 20$ GeV^2 and $10^{-5} < x < 10^{-2}$ at the hot spot with $R = 2$ GeV^{-1} where the nonlinear corrections due to gluon recombination become influential.

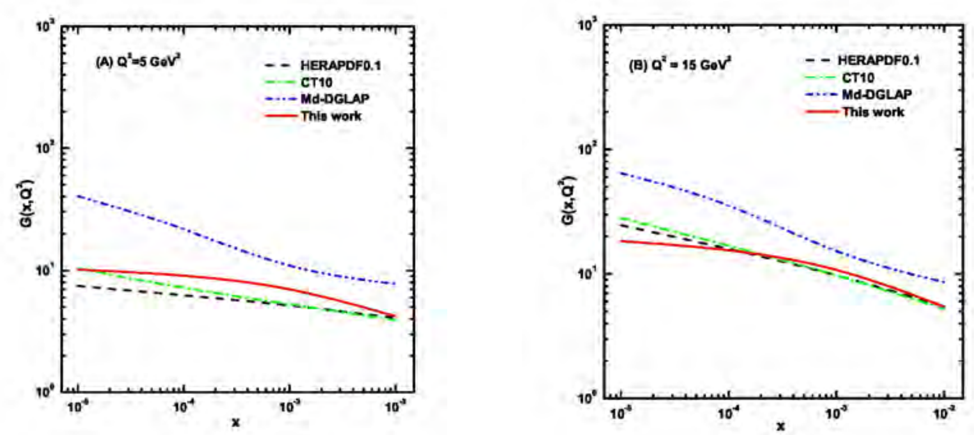

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Fig. 2: (A) Computed values of gluon distribution function $G(x, Q^2)$ plotted against x obtained from Eq.(10) at the hot spot $R = 2$ GeV^{-1} for two fixed $Q^2 = 5$ and 15 GeV^2. (B) The solid curves represent our result, dash curves represent HERAPDF0.1, dash-dot curves represent CT10 and dash-dot-dot curves are the results of MD-DGLAP equation

Conclusion

The nonlinear GLR-MQ equation is solved to study the effect of shadowing corrections to the behaviour of gluon distributions in the region of small-x and moderate-Q^2. Here the x and Q^2 dependence of gluon distribution function are estimated at the hot spot where the gluon recombination effects are expected to play a significant role. We observe that, although the obtained results of gluon distribution complement the perturbative QCD fits, but in the region of small-x and Q^2 this behaviour is tamed due to the nonlinear terms in the GLR-MQ equation. That being so, the Froissart bound is restored in the small-x region where density of gluons becomes very high.

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