INTERREGIONAL CENTRE FOR ADVANCED STUDIES

PROBLEMS OF FUNDAMENTAL PHYSICS

Proceedings
7th Lomonosov Conference on
Elementary Particle Physics

(24-30 August 1995, Moscow, Russia)

Edited by

Alexander I. Studenikin

URSS
Moscow 1997

This volume is published by the Publishing Division of Interregional Centre for Advanced Studies (ICAS).

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ISBN 5-88417-108-0

CONTENTS

Editorial vi

Organizers & Sponsors viii

List of Participants x

Programme of the 7th Lomonosov Conference xii

Quantum Field Perturbation Theory with Convergent Series Does Exist 1
V.V. Belokurov, E.T. Shavgulidze, Yu.P. Solovyov

The Problem of Quark and Lepton Masses 12
C.D. Froggatt

Relativistic Description of Exclusive Weak Decays of Heavy Mesons Using Heavy Quark Expansion 20
R.N. Faustov, V.O. Galkin, A.Yu. Mishurov

The Extended Nambu-Jona-Lasinio Model with Separable Interaction: Low Energy Pion Physics 31
I.V. Anikin, M.A. Ivanov, V.E. Lubovitskij

Measurements of CP and T Symmetry Violation Parameters and Tests of CPT Invariance in the Neutral Kaon System 37
P. Deby

Testing Bell's Inequality in Neutral Kaon System at a ϕ - Factory 42
A. Di Domenico

Experimental Problems for Testing Macroscopic Quantum Coherence with SQUIDs 48
C. Cosmelli

Time Dependent Supersymmetry in Quantum Mechanics 54
V.G. Bagrov, B.F. Samsonov

Constraints on Neutrino Magnetic Moment and Strength of Magnetic Field from Neutrino Spin-Flavour Oscillations in Supernova Explosion and Neutron Star 62
A.I. Studenikin

Radiative Decay of a Massive Neutrino in the Weinberg-Salam Model with Mixing in a Constant Uniform Magnetic Field 67
V.Ch. Zhukovsky, P.A. Eminov, A.E. Grigoruk

Heisenberg Representation for Creation-Annihilation Operatos in Nonstationary Background 71
A.A. Lobashov, V.M. Mostepanenko

A Deterministic Subset of Maxwell-Dirac Electrodynamics 81
A.M. Akhmeteli

Photon Polarization Operator and the Photon Elastic Scattering Amplitude in (2+1) QED in a Constant Magnetic Field 85
K.V. Zhukovskii, P.A. Eminov

On Manifold of Exact Solutions of the Problem of Bosonisation of a Pair of 2D-Electrons in a Quantinizing Magnetic Field 89
B.A. Lysov, O.F. Dorofeyev

A Pair Production of Pions by a Constant Electromagnetic Field 93
S.I. Kruglov

Radiation of Relativistic Dipoles 102
V.A. Bordovitsyn, V.S. Gushchina

Electron Impact Double Ionization Like a Method for Atomic Spectroscopy 110
Yu.V. Popov, A. Lahmam-Bennani, M. Coplan, I.V. Farnakeev

Photon-Like Resonance Decay $\gamma^* \to \nu_l \nu_l$ in a Strong Magnetic Field 117
V.V. Skobelev, A.A. Kuznetsov

Semiclassically Concentrated States of Charged Particles in a Curved Space-Time 122
V.G. Bagrov, V.V. Belov, A.Yu. Trifonov

Synchrotron and Compton Mechanisms of Axion Emission by Electrons in a Magnetized Neutron Star 129
V.Yu. Grishina

Effects of Anisotropy in Supernovae 133
O.F. Dorofeyev

Adelic Wave Function of the De Sitter Universe 137
B. Dragovich

On the History of the Special Relativity Concept 141
A.A. Tyapkin

Maxwell Form of the Einstein Equations and Quasi-Friedmannian Cosmology 160
R.F. Polishuk

On Weyl Equations 165
S.V. Kopylov

Hydrogen-Like Energy Spectrum of the Early Universe 169
M.L. Fil'chenkov

On Parity Conservation in Weak Interactions and Reason for Originating Spontaneous β - Decay 174
I.M. Dmitrievskiy

EDITORIAL

The Conference on "PROBLEMS OF FUNDAMENTAL PHYSICS" (the 7th Lomonosov Conference on Elementary Particle Physics) was held from 24 to 30 August, 1995 at Moscow State University, Moscow, Russia.

The conference was organized by the Interregional Centre for Advanced Studies in cooperation with the Skobeltsyn Institute of Nuclear Physics and the Faculty of Physics of the Moscow State University and supported by the Joint Institute for Nuclear Research (Dubna), the Institute for High Energy Physics (Protvino) and the Institute for Nuclear Research (Moscow), and was also sponsored by the Ministry of Science and Technical Policy of Russia.

It was more than fourteen years ago when the first of the series of conferences, now called the "Lomonosov Conferences on Elementary Particle Physics", was held at the Department of Theoretical Physics of the Moscow State University (June 1983, Moscow). The second conference was held in Kishinev, Republic of Moldavia, USSR (May 1985).

After the four years break this series was resumed on a new conceptual basis for the conference programme focus. During the preparation of the third conference (held in September 1989, Maykop, Russia) a desire to broaden the programme to include more general issues in particle physics became apparent. At subsequent meetings in this series (August 1990, Minsk, Republic of Belorussia, USSR; April 1992, Yaroslavl, Russia; August 1993, Moscow, Russia) a wide variety of interesting things both in theory and experiment of particle physics, field theory, gravitation and astrophysics were included into the programmes. During the conference of 1992 in Yaroslavl it was proposed by myself and approved by numerous participants that these irregulary held meetings should be transformed into regular events under the title "Lomonosov Conferences on Elementary Particle Physics". It was also decided to enlarge the number of organizations that would take part in preparation of future conferences.

Mikhail Lomonosov (1711-1765), a brilliant Russian encyclopaedist of the era of the Russian Empress Catherine the 2nd, was world renowned for his distinguished contributions in the fields of science and art. He also helped establish the high school educational system in Russia. Moscow State University was founded in 1755 based on his plan and initiative, and the University now bears the name of Lomonosov.

The Sixth Lomonosov Conference on Elementary Particle Physics "Cosmomicrophysics and Gauge Fields" was held at Moscow State University (August, 1993) and was sponsored by the Interregional Centre for Advanced Studies. The publication of the volume containing articles written on the basis of presentations at the 5th and 6th Lomonosov Conferences was supported by the Accademia Nazionale dei Lincei (Italy).

The idea to devote the 7th Lomonosov Conference to "Problems of Fundamental Physics" appeared because the year 1995 marked the ninetieth anniversary of the special theory of relativity (1905), the eightieth anniversary of the general theory of relativity (1915) and also seventy years after the foundations of quantum mechanics had been formulated (1925 - 1926). That was the reason to included the following set of items into the programme of the 7th Lomonosov Conference:

1) Quantum mechanics and paradoxes,
2) Foundations and developments of theory of space-time,
3) Frontiers of particle physics (the Standard Model and beyond, strings, particle astrophysics, neutrino mass and oscillations).

The aim of the Conference was to review the present situation and results so far obtained to the end of the twentieth century and discus perspectives for the future.

On behalf of the Organizing Committee I should like to warmly thank the speakers and all of the participants of the 7th Lomonosov Conference. We are grateful to the Directors of the Skobeltsyn Institute of Nuclear Physics, Mikhail Panasyuk and the Joint Institute for Nuclear Research, Vladimir Kadyshevsky, and the Dean, Vladimir Trukhin, and Vice Dean, Yury Pirogov, of the Faculty of Physics of the Moscow State University for the support in the organization of the Conference. I should like to thank the Secretaries of the Conference, Vladimir Galkin, Gennady Likhachev and Artem Mishurov.

These Proceedings were published by the Publishing Division of the Interigional Centre for Advanced Studies. Special thanks are due to Vladimir Galkin for his contribution in preparation of this volume.

Alexander Studenikin

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Programme of the 7th Lomonosov Conference

24 August, THU

PLENARY SESSION

V.A.Matveev, *Fundamental Laws of Physics and the Dimensionality of Space--Time.*

E.P.Shabalin, *What's the News on CP and CPT.*

V.G.Kadyshevsky, *Fermion Generations and Mass Scale Limit.*

M.Vysotsky, *Radiative Corrections in the Electroweak Theory.*

E.P.Shabalin, *What's the News on CP and CPT.*

AFTERNOON SESSION

D.V.Shirkov, *Renormalization Group Simmetry and Lie Group Analysis.*

G.V.Bagrov, V.A.Bordovitsyn & I.M.Ternov, *Synchrotron Radiation and Spin-Light.*

P.I.Pronin & K.V.Stepanyandz, *New Algorithm for Calculation of One-Loop Corrections in the Theories with Higher Derivations.*

K.V.Shokikiu, *Canonical Quantization of Degenerate Anomalous Yang-Mills Model.*

I.V.Anikin, M.A.Ivanov, V.E.Lyubovitsky, *The Extended Nambu-Jona-Lasinio Model with Separable Interaction: Low Energy Pion and Nucleon Physics.*

V.S.Yarunin & L.A.Siurakshina, *Quantum Excitations of the Classical Bose-Condensate.*

25 August, FRI

PLENARY SESSION

V.B.Braginsky, *1)How to Isolate the Mirrors of Gravitational Wave Antenna; 2)How to Measure the Fase of Quantum oscillator.*

R.N.Faustov, V.O.Galkin & A.Yu.Mishurov, *Relativistic Description of Heavy Mesons.*

O.A.Khrustalev & M.V.Chichikina, *Quantum Field in the Neighbour of the Nonstationary Classical Field.*

M.Polikarpov, *Monopoles, Strings, and Confinement in Gluodynamics.*

P.Debu, *Measurments of CP and CPT Violation Parameters in the Neutral Kaon System.*

A. Di Domenico, *Testing Bell's Inequality in the Neutral Kaon System at Phi-Factory.*

AFTERNOON SESSION

S.N.Mayburov, *Quantum Measurments and Nonperturbative Quantum Field Theory.*

G.Chizov, V.Khalilov and V.Rodionov, A Colored -Fermion Triplet in the Field of a Polarized Yang-Mills Wave.

V.V.Belokurov, Yu.P.Soloviev & E.T.Shavbulidze, *Perturbation Theory in Quantum Field Theory Does Exist!*

V.A.Lysov & O.F.Dorofeev, *On Manifold of Exact Solutions of the Problem of Bozonization of a Pair of 2D-Electrons in a Quantinizing Magnetic Field.*

V.G.Bagrov & V.V.Obukhov, *Exact Solutions of Einstein-Dirac Equations.*

O.F.Dorofeev, *Effects of Anisotropy in Supernovae.*

S.V.Kopylov, *On Weyl Equations.*

S.Kruglov, *A pair production of pions by a constant electromagnetic field.*

S.Y.Sadov, *Variations of Tubes Volumes and the Einstain Equations.*

26 August, SAT

PLENARY SESSION

G.Diambrini-Palazzi, *A Method for Testing the Macroscopic Quantum Coherence.*

C.Cosmelli, *Experimental Problems for Testing Macroscopic Quantum Coherence with SQUIDs.*

C.Froggatt, *The Problem of Quark and Lepton Masses.*

P.Spillantini, *Search for Antimatter Component in Universe.*

D.N.Klyshko & A.V.Masalov, *Photon Noise in Optical Systems with Feedback.*

AFTERNOON SESSION

I.V.Anikin, M.A.Ivanov & V.E.Lyubovitsky, *Test of the Bjorken-Xu Inequality for Baryonic Isgur-Wise Functions.*

R.N.Faustov, V.O.Galkin & A.Yu.Mishurov, *Semileptonic Decays of Beauty and Charmed Mesons.*

G.S.Iroshnikov, *Effective String Dynamics in Large N QCD Taking into Account Quark's Spin Degrees of Freedom.*

V.P.Bykov, *Localization in Quantum Mechanics.*

I.Dmitrievskiy, *On Parity Conservation in Weak Interactions and Reason for Originating Spontaneous β - Decay.*

28 August, MON

PLENARY SESSION

A.M.Egorov, G.G.Likhachev & A.I.Studenikin, *Neutrino Oscillations in Matter and Magnetic Fields.*

V.Ivashchuk & V.N.Melnikov, *Multidimensional Classical and Quantum Gravity.*

B.C.Pal, *Conceptual Development of Quantum Mechanics.*

N.P.Klepikov, *Principles of Special Theory of Relativity of Particle Systems.*

V.M.Lipunov, *High Energy Sources in Astrophysics.*

AFTERNOON SESSION

P.A.Eminov, A.E.Grigoruk & V.P.Zhukovsky, *Radiative Decay of Massive Neutrino in the Weinberg-Salam Model with Mixing.*

V.V.Skobelev, *Massive Neutrino Decay in a Magnetic Field.*

V.G.Bagrov & B.F.Samsonov, *Supersymmetry of Nonstationary Schroedinger Equation.*

A.Akhmeteli, *Deterministic Subset of Maxwell-Dirac Electrodynamics.*

B.N.Zakhariev, *New Rules of Quantum Intuitions.*

E.A.Tagirov, *Quantum-Mechanical Operators of Operators of Observables in Curved Space-Times.*

M.L.Filchenkov, *Hydrogen-Like Energy Spectrum of the Early Universe.*

R.Polyschuk, *Maxwell Form of Einstein Equations and Conservation Laws.*

G.V.Ryazanov, *Title to be announced.*

29 August, TUE

PLENARY SESSION

A.Kataev, *Status of the QCD Predictions for Z-decay Width and Deep-Inelastic Neutrino-Nucleon Scattereing.*

V.Rubakov, *Relativistic Strings and Gravitation.*

V.I.Ritus & A.I.Nikishov, *Moving Mirrors Radiation.*

B.K.Kerimov, M.Ya.Safin, *Properties of Neutrino and Structure of Hadronic Neutral Current beyond the Standard Model.*

A.A.Tyapkin, *On the Story of the Spacial Relativity Concept.*

AFTERNOON SESSION

Yu.S.Vladimirov, *Binary Geometrophysics and Kaluza-Klein Theory.*

V.M.Lipunov, K.A.Postnov, M.E.Prokhorov, S.N.Nazin & I.E.Panchenko, *Astrophysical Sources of Gravitational Waves.*
Yu.A.Kukharenko & P.Yu.Kukharenko, *Quantum Mechanics as a Classical Random Proccess.*
V.G.Bagrov, V.V.Belov & A.Yu.Trifonov, *Semiclassically Concentrated States of Charged Particle in Curved Space-Time.*
Yu. Kukharenko, P.Polishchuk, *Non-Equlibrium States of Scalar Field in Quantum Gravity.*
V.T.Anikushin, *Discreteness and a solution of Fundamental ems of Modern Physics.*
A.Deriglazov & A.V.Galajinsky, *Algebraic Motivations in Formulating the Superparticle in a Curved Superspace and Supergravity.*
V.V.Belov, M.F.Kondratieva, *Dirac Brakcets and Equation for a Quantum Average.*
B.Dragovich, *Adelic Wave Function of the Universe.*

30 August, WED

PLENARY SESSION

D.V.Gal'tsov & O.V.Kechkin, *Exact Solutions to Dilaton-Axion Gravity.*
M.Yu.Khlopov, *Cosmoparticle Physics: A Way to True Theory of the Universe and Microworld?.*
Yu.V.Popov, *Many Body Problem in Atomic Physics.*
A.A.Lobashov & V.M.Mostepanenko, *Heisenberg Representation for Creation-Annihilation Operators in Non-Stationary Background.*
A.A.Slavnov, *Fermions in the Lattice Models of Quantum Field Theory.*

Quantum field perturbation theory with convergent series does exist.

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Abstract

Asymptotic expansions, employed in quantum physics as series of perturbation theory, appear as a result of the representation of functional integrals by power series with respect to coupling constant. To derive these series one has to change the order of functional integration and infinite summation. In general case, this procedure is incorrect and is responsible for the divergence of the asymptotic expansions.

In the present work, we suggest a method of construction of a new perturbation theory. In the framework of this perturbation theory, a convergent series corresponds to any physical quantity represented by a functional integral. The relations between the coefficients of these series and those of the asymptotic expansions are established.

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1. Nowadays, perturbative quantum field theory is a very well developed theory that can be considered as the most realistic theory of fundamental interactions [1]. In the framework of this theory, the coupling constant g is supposed to be a small parameter and any physical quantity is represented by the expansion over powers of this parameter [2].

However, it is well known [3] that power series in quantum field theory diverge and are nothing but asymptotic expansions of functions under study in the region of sufficiently small values of g. A sum of finite number of terms gives an approximation to the function. But, for g fixed, irrespective of the number of terms in the sum, the accuracy of approximation can not be made arbitrary.

The power series divergence is due to nonanaliticity in coupling constant of the studied functions. In particular, they are known to have the essential singularity at $g = 0$ (see e.g. [4]).

In this paper, we propose a new perturbation theory with convergent series for any quantity represented by a functional integral. Here, we formulate the basic ideas of the method and apply it to obtain convergent series for ordinary integral that can be considered as a toy model for functional integrals in quantum field theory. In the following papers, convergent series will be obtained for functional integrals appearing in quantum mechanics and quantum field theory.

2. Consider the "zero-dimensional" analog of functional integral in quantum field theory with the interaction $V = g\phi^4$ $(g > 0)$, that is, the ordinary integral

$$I(g) = \int\limits_{-\infty}^{+\infty} e^{-x^2-gx^4}\,dx\,. \tag{1}$$

If we expand the integrand into power series with respect to g and rearrange the operations of integration and infinite summation

$$\int\limits_{-\infty}^{+\infty} e^{-x^2} \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} x^{4n} dx \to \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} \int\limits_{-\infty}^{+\infty} e^{-x^2} x^{4n} dx\,, \tag{2}$$

then we obtain the asymptotic expansion for $g \to 0$)

$$\sum_{n=0}^{\infty} F_n g^n\,,$$

$$F_n = \frac{(-1)^n}{n!}\Gamma\left(\frac{4n+1}{2}\right) = \frac{(-1)^n}{n!}\frac{\sqrt{\pi}}{2^{4n}}\frac{(4n)!}{(2n)!}\,. \tag{3}$$

The series is obviously divergent.

At the same time, for $g > 0$, the integral (1) is finite. And the result is

$$I(g) = \exp(\frac{1}{8g})K_{\frac{1}{4}}\left(\frac{1}{8g}\right)\frac{1}{\sqrt{4g}} = \frac{\pi}{(2g)^{\frac{1}{4}}}\exp(\frac{1}{8g})D_{-\frac{1}{2}}\left(\frac{1}{\sqrt{2g}}\right). \quad (4)$$

Here, $K_{\frac{1}{4}}$ is the Mcdonald function and $D_{-\frac{1}{2}}$ is the function of the parabolic cylinder.

To understand why the divergent series (3) appears for the convergent integral (1), notice that the procedure (2) is incorrect. Actually, the conditions for it to be correct are given by the following theorem ([5]):

Let (a, b) be a finite or infinite segment and $u_n(x)$ be a sequence of real or complex functions satisfying the following conditions:

(1) all $u_n(x)$ are continuous in (a, d);

(2) the series $\sum_{n=0}^{\infty} u_n(x)$ converges uniformly on every finite segment in (a, b),

(3) at least one of the expressions

$$\int_a^b \left(\sum_{n=0}^{\infty} |u_n(x)|\right) dx\,, \quad \sum_{n=0}^{\infty}\int_a^b |u_n(x)|\, dx\,.$$

is finite. Then

$$\int_a^b \left(\sum_{n=0}^{\infty} u_n(x)\right) dx = \sum_{n=0}^{\infty}\int_a^b u_n(x)dx\,.$$

It is easy to see that for the transformation (2) these conditions are not fulfilled.

3. Now we are ready to formulate a method that gives us the possibility to construct a new perturbation theory with convergent series. Let us consider the integral (1). Denote by $\tilde{\varphi}(\rho)$ the Fourier transformation of the function $\exp(-r^4)$:

$$\tilde{\varphi}(\rho) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\rho r}e^{-r^4}\, dr\,. \quad (5)$$

For large values of $|\rho|$ the following inequality takes place [6]

$$|\tilde{\varphi}(\rho)| \leq C\exp\left(-\frac{1}{5}|\rho|^{1+\frac{1}{3}}\right). \quad (6)$$

Rewrite the integral (1) as follows:

$$I(g) = \int_{-\infty}^{+\infty} e^{-x^2} \left(\int_{-\infty}^{+\infty} \tilde{\varphi}(\rho) e^{ig^{\frac{1}{4}}\rho x}\, d\rho \right) dx\,. \tag{7}$$

In view of the equation (6) we obtain

$$\left| e^{-x^2} \tilde{\varphi}(\rho) e^{ig^{\frac{1}{4}}\rho x} \right| \le C \exp\left(-\frac{1}{5} |\rho|^{1+\frac{1}{3}} - x^2 \right). \tag{8}$$

Therefore,the following Lebesgue integral

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-x^2} \tilde{\varphi}(\rho) e^{ig^{\frac{1}{4}}\rho x}\, d\rho dx \tag{9}$$

converges absolutely. Hence, the conditions for the Fubini theorem are fulfilled and we may change the orders of the integrations with respect to ρ and x :

$$I(g) = \int_{-\infty}^{+\infty} \tilde{\varphi}(\rho) \left(\int_{-\infty}^{+\infty} e^{-x^2} e^{ig^{\frac{1}{4}}\rho x}\, dx \right) d\rho\,. \tag{10}$$

The integral $I(g)$ can be represented as a limit of proper integrals with respect to ρ :

$$I(g) = \lim_{R\to\infty} \int_{-R}^{+R} \tilde{\varphi}(\rho) \left(\int_{-\infty}^{+\infty} e^{-x^2} e^{ig^{\frac{1}{4}}\rho x}\, dx \right) d\rho\,. \tag{11}$$

If R is sufficiently large $(R \sim -\ln\varepsilon)$ then the integral

$$J(g,R) = \int_{-R}^{+R} \tilde{\varphi}(\rho) \left(\int_{-\infty}^{+\infty} e^{-x^2} e^{ig^{\frac{1}{4}}\rho x}\, dx \right) d\rho \tag{12}$$

approximates the integral (1) with an accuracy ε :

$$|J(g,R) - I(g)| \le \varepsilon\,.$$

We will show that for any finite R, $J(g,R)$, that is the regularization of the integral (1), can be expanded into absolutely convergent series.

Expanding the function $\exp(ig^{\frac{1}{4}}\rho x)$ into power series and substituting it into the integrand of (12) we obtain

$$J(g,R)=\int_{-R}^{+R}\tilde{\varphi}(\rho)\left(\int_{-\infty}^{+\infty}e^{-x^2}\left(\sum_{n=0}^{\infty}\frac{i^n g^{\frac{n}{4}}\rho^n x^n}{n!}\right)dx\right)d\rho\,. \tag{13}$$

Let us prove that here we can change the order of operations $\sum_{n=0}^{\infty}$ and $\int_{-\infty}^{+\infty}$. Consider the function $\exp\left(-\frac{1}{2}x^2\right)\frac{x^n}{n!}$. It has the maximum at the point $x=\sqrt{n}$. So, taking into account the Stirling formula we get the inequality

$$\left|\exp(-\frac{1}{2}x^2)\frac{x^n}{n!}\right|\leq\frac{e^{-\frac{n}{2}}n^{\frac{n}{2}}}{n!}\leq\frac{e^{-\frac{n}{2}}n^{\frac{n}{2}}}{\sqrt{2\pi n}n^n e^{-n}}=\frac{1}{\sqrt{2\pi}}\frac{e^{\frac{n}{2}}}{n^{\frac{n+1}{2}}}\,. \tag{14}$$

From here, it follows that

$$J(g,R)=\int_{-R}^{+R}\tilde{\varphi}(\rho)\left(\sum_{n=0}^{\infty}\frac{i^n g^{\frac{n}{4}}\rho^n}{n!}\left(\int_{-\infty}^{+\infty}e^{-x^2}x^n dx\right)\right)d\rho\,. \tag{15}$$

Using the inequality (6) we also prove that

$$J(g,R)=\sum_{n=0}^{\infty}\frac{i^n g^{\frac{n}{4}}}{n!}\int_{-R}^{+R}\tilde{\varphi}(\rho)\rho^n d\rho\int_{-\infty}^{+\infty}e^{-x^2}x^n dx\,. \tag{16}$$

Now let us estimate the terms of this series and prove that the series obtained converges absolutely. The series (16) looks as follows

$$J(g,R)=\sum_{n=0}^{\infty}u_n(g,R)\,. \tag{17}$$

Since

$$|\tilde{\varphi}(\rho)|\leq\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-r^4}\,dr=\frac{1}{4\pi}\Gamma\left(\frac{1}{4}\right), \tag{18}$$

we have

$$\left|\int_{-R}^{+R}\tilde{\varphi}(\rho)\rho^n d\rho\right|\leq C\int_{-R}^{+R}|\rho|^n d\rho=2C\frac{R^{n+1}}{n+1}\,. \tag{19}$$

Taking into account the equation

$$\int_{-\infty}^{+\infty} e^{-x^2} x^n dx = \begin{cases} \Gamma(\frac{n+1}{2}), & n = 2k \\ 0, & n = 2k+1 \end{cases}, \tag{20}$$

and using the Stirling formula we get

$$|u_n(g,R)| \le \frac{1}{n!} 2C\, g^{\frac{n}{4}} \frac{R^{n+1}}{n+1} \Gamma\left(\frac{n+1}{2}\right) \le C_1\, g^{\frac{n}{4}} \frac{R^{n+1}}{(n+1)^{\frac{1}{2}} \Gamma\left(\frac{n+1}{2}\right)}. \tag{21}$$

Therefore, the series (17) converges absolutely.

Let us examine in more detail the structure of the terms of this series. Rewrite the series (16) as

$$J(g,R) = \sum_{k=0}^{\infty} \frac{1}{(2k)!} g^{\frac{2k}{4}} \Gamma\left(\frac{2k+1}{2}\right) A_{2k}(R), \tag{22}$$

where

$$A_n(R) = i^n \int_{-R}^{+R} \tilde{\varphi}(\rho)\rho^n d\rho = \frac{i^n}{2\pi} \int_{-R}^{+R} \rho^n \left(\int_{-\infty}^{+\infty} e^{-r^4} e^{-i\rho r}\, dr \right) d\rho =$$

$$\frac{1}{2\pi} \int_{-R}^{+R} \int_{-\infty}^{+\infty} \frac{d^n}{dr^n} e^{-r^4} e^{-i\rho r}\, dr d\rho. \tag{23}$$

We see that the Fubini theorem is applicable here and

$$A_n(R) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d^n}{dr^n} e^{-r^4} \frac{\sin Rr}{r} dr. \tag{24}$$

In view of the equation (20), the series (22) contains the even-numbered terms only (it follows directly also from the formula (24) which gives $A_{2k+1}(R) = 0$).

Consider the coefficients $A_n(R)$ at R large. First, notice that if we substitute the limiting values

$$A_n(\infty) = \lim_{R \to \infty} A_n(R)$$

into the series (22) we get the divergent series (3). Actually,

$$A_{4k}(\infty) = \int_{-\infty}^{+\infty} \frac{d^{4k}}{dr^{4k}} e^{-r^4} \delta(r)\, dr = \frac{d^{4k}}{dr^{4k}} \left(\sum_{m=0}^{\infty} \frac{(-r^4)^m}{m!} \right)_{r=0} = \frac{(-1)^k}{k!}(4k)!\,, \tag{25}$$

so,

$$F_n = \frac{1}{(4n)!} \Gamma\left(\frac{4n+1}{2}\right) A_{4n}(\infty)\,.$$

$$A_{4k+2}(\infty) = \frac{d^{4k+2}}{dr^{4k+2}} \left(\sum_{m=0}^{\infty} \frac{(-r^4)^m}{m!} \right)_{r=0} = 0\,. \tag{26}$$

For large R, we also have

$$|A_{4k}(R) - A_{4k}(\infty)| \leq \frac{C(4k)}{R^4}, \quad |A_{4k+2}(R)| \leq \frac{C(4k+2)}{R^2}, \tag{27}$$

and

$$\left| F_n - \frac{1}{(4n)!} \Gamma\left(\frac{4n+1}{2}\right) A_{4n}(R) \right| \leq \frac{D(4n)}{R^4}\,.$$

The method is also applicable for the integrals that lead to the so-called nonperturbative contributions. The typical example is the integral

$$I(\gamma) = \int_{-\infty}^{+\infty} e^{-x^2\left(1-\frac{\sqrt{\gamma}}{2}x\right)^2} dx \tag{28}$$

(see, e.g. [4]). The terms of the corresponding divergent series in the common perturbation theory have the same sign.

If we rewrite the integral (28) as

$$I(\gamma) = \exp\left(-\frac{27}{4\gamma}\right) \int_{-\infty}^{+\infty} e^{-x^2 - P_\gamma(x)} dx \tag{29}$$

where $P_\gamma(x) = \frac{\gamma}{4}x^4 - \sqrt{\gamma}x^3 + \frac{27}{4\gamma}$, all the above proof is valid and we get

$$J = J_1 + J_2\,,$$

$$J_1 = \exp\left(-\frac{27}{4\gamma}\right) \sum_{k=0}^{\infty} \frac{1}{(4k)!} A_{4k}(R) \int_{-\infty}^{+\infty} P_\gamma^k(x) e^{-x^2} dx\,, \tag{30}$$

$$J_2 = \exp\left(-\frac{27}{4\gamma}\right) \sum_{k=0}^{\infty} \frac{1}{(4k+2)!} A_{4k+2}(R) \int_{-\infty}^{+\infty} P_\gamma^{k+\frac{1}{2}}(x) e^{-\|x\|^2} dx \,. \tag{31}$$

The convergent series that approximate integrals

$$I(m,g) = \int_{-\infty}^{+\infty} e^{-x^2 - gx^{2m}} dx \tag{32}$$

can be obtained in the similar way. The result is

$$I(m,g) = \lim_{R\to\infty} J(m,g,R) \,, \tag{33}$$

$$J(m,g,R) = \sum_{k=0}^{\infty} \frac{1}{(2k)!} g^{\frac{k}{m}} \Gamma\left(\frac{2k+1}{2}\right) A_{2k}(m,R) \,, \tag{34}$$

$$A_n(m,R) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d^n}{dr^n} \exp(-r^{2m}) \frac{\sin Rr}{r} dr \,, \tag{35}$$

$$\tilde{\varphi}(m,\rho) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\rho r} e^{-r^{2m}} dr \,. \tag{36}$$

In this case, the parameter of power expansion is $g^{\frac{1}{m}}$, and the absolutely convergent series for $J(m,g,R)$ can be divided into m convergent series with the following powers of g : $g^{\frac{1}{m}+n}, g^{\frac{2}{m}+n}, \ldots, g^{\frac{m-1}{m}+n}, g^{1+n}$ $(n = 0,1,\ldots)$.

4. Now we generalize the suggested method for the multiple integral

$$I(g) = \int_{\mathbf{R}^N} e^{-\|x\|^2 - gP(x)} dx \,. \tag{37}$$

Here, $P(x)$ is a nonnegative homogeneous polynomial of the fourth power ($P(x) \geq 0$, $\forall x \in \mathbf{R}^N$). Let $Q(x) = P^{\frac{1}{4}}(x)$. $Q(x)$ is a homogeneous function of the first order.

Then

$$e^{-gP(x)} = \int_{-\infty}^{+\infty} \tilde{\varphi}(\rho) e^{ig^{\frac{1}{4}}\rho Q(x)} \, d\rho dx \,. \tag{38}$$

where $\tilde{\varphi}(\rho)$ is defined by the formula (5).

The integral (37) satisfies the requirements of the Fubini theorem.

Hence,

$$I(g) = \lim_{R\to\infty} \int_{-R}^{+R} \tilde{\varphi}(\rho) \left(\int_{\mathbf{R}^N} e^{-\|x\|^2} e^{ig^{\frac{1}{4}}\rho Q(x)}\, dx \right) d\rho\,. \tag{39}$$

Consider the integral

$$J(g,R) = \int_{-R}^{+R} \tilde{\varphi}(\rho) \left(\int_{\mathbf{R}^N} e^{-\|x\|^2} e^{ig^{\frac{1}{4}}\rho Q(x)}\, dx \right) d\rho \tag{40}$$

Expanding the function $\exp\left(ig^{\frac{1}{4}}\rho Q(x)\right)$ into the series and changing the order of the summation and the integration we obtain

$$J(g,R) = \sum_{n=0}^{\infty} \frac{1}{n!} g^{\frac{n}{4}} A_n(R) \int_{\mathbf{R}^N} Q^n(x) e^{-\|x\|^2} dx\,, \tag{41}$$

where $A_n(R)$ is given by equation (24).

Similarly to one-dimensional case, the series (41) can be divided into the sum of two convergent series

$$J = J_1 + J_2\,,$$

$$J_1 = \sum_{k=0}^{\infty} \frac{1}{(4k)!} g^k A_{4k}(R) \int_{\mathbf{R}^N} P^k(x) e^{-\|x\|^2} dx\,, \tag{42}$$

$$J_2 = \sum_{k=0}^{\infty} \frac{1}{(4k+2)!} g^k \sqrt{g} A_{4k+2}(R) \int_{\mathbf{R}^N} P^{k+\frac{1}{2}}(x) e^{-\|x\|^2} dx \tag{43}$$

The generalization to polynomials of the $2m$ power is obvious. In that case, we have

$$J(m,g,R) = \sum_{k=0}^{\infty} \frac{1}{n!} g^{\frac{n}{2m}} A_n(m,R) \int_{\mathbf{R}^N} Q^n(x) e^{-\|x\|^2} dx\,, \tag{44}$$

where $Q(x) = P^{\frac{1}{2m}}(x)$ and $A_n(m,R)$ is given by equation (35).

All these results are valid for the functional integrals in Euclidean quantum field theory if the exponent in Caussian measure is defined by means of a nuclear operator. In that case, the above proof can be generalized directly [7].

5. Now let us discuss in more detail the main points where the suggested approach differs from the standard one.

In the standard or common perturbation theory any quantum field function (written as a functional integral) is represented by a power series in coupling constant g. The series is divergent and can be considered as an asymptotic expansion valid in the region of sufficiently small values of g only. A sum of finite number of terms gives an approximation to the function. But, for g fixed, irrespective of the number of terms in the sum, the accuracy of approximation can not be made arbitrary.

The essence of the method suggested in this paper is in the following. First, the initial functional integral $I(g)$ is approximated by some other functional integral $J(g,R)$ that depends on an auxiliary parameter R. An arbitrary accuracy of the approximation can be achieved by the appropriate choice of the auxiliary parameter. Then, in some special way, the integral $J(g,R)$ is expanded into absolutely convergent series. Now, to calculate the initial integral $I(g)$ with an arbitrary accuracy for every value of g it is possible to take proper but finite number of terms in this series.

In the common approach, there are nonperturbative terms that can not be calculated in the framework of standard perturbation theory in any way. In the suggested approach, due to an arbitrary accuracy of calculation there are no incalculable terms.

The convergent series of new perturbation theory have an unusual property. Besides the terms with integer powers of the coupling constant g, there are terms with fractional powers of g. Although, for some first orders of perturbation theory, the coefficients at these "shadowy" terms are relatively small, their contribution becomes significant for large orders.

We are grateful to D.I.Kazakov, V.A.Rubakov, D.V.Shirkov and O.G. Smolianov for useful discussions and to S.O.Krivonos for the help in numerical calculations.

Research is supported in part by Russian Fund for Fundamental Researches.

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The Problem of Quark and Lepton Masses

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Abstract

Different approaches to the fermion mass problem are reviewed. We illustrate these approaches by summarizing recent developments in models of quark and lepton mass matrices. Dynamical calculations of the top quark mass are discussed, based on (a) infrared quasi-fixed points of the renormalisation group equations, and (b) the multiple point criticality principle in the pure Standard Model. We also consider Yukawa unification and mass matrix texture. Models with approximately conserved gauged chiral flavour charges beyond the Standard Model are shown to naturally give a fermion mass hierarchy.

1 Introduction

The explanation of the fermion mass and mixing hierarchies and the three generation structure of the Standard Model (SM) constitutes the most important unresolved problem in particle physics. We shall discuss recent developments in three of the approaches to this problem:

1. The dynamical determination of the top quark mass.
2. Mass matrix ansätze and texture zeroes.
3. Chiral flavour symmetries and the fermion mass hierarchy..

Neutrino masses, if non-zero, have a different origin to those of the quarks and charged leptons; we do not have time here to discuss recent applications of the so-called see-saw mechanism, which seems the most natural way to generate neutrino masses.

2 Dynamical Top Quark Mass

There is presently a lively interest [1, 2, 3, 4] in determining the top quark mass m_t (or more generally third generation masses) dynamically. Most of the discussed models lead to the top quark running Yukawa coupling constant $g_t(\mu)$ being attracted to its infra-red quasi-fixed point value. We have very recently pointed out [4] that the top quark (and Higgs) mass can be calculated within the pure SM, assuming the multiple point criticality principle. We now discuss these two possibilities.

2.1 Top Mass as a Renormalisation Group Fixed Point

The idea that some of the properties of the quark-lepton mass spectrum might be determined dynamically as infrared fixed point values of the renormalisation group equations (RGE) is quite old [5, 6, 7]. In practice one finds an effective infrared stable quasifixed point behaviour for the SM quark running Yukawa coupling constant RGE at the scale $\mu \simeq m_t$, where the QCD gauge coupling constant $g_3(\mu)$ is slowly varying. The quasifixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative SM is valid up to some high (e.g. GUT or Planck) energy scale $M_X \simeq 10^{15} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(M_X) \gtrsim 1$. The nonlinearity of the RGE then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasifixed point value. We note that while there is a rapid convergence to the top Yukawa coupling fixed point value from above, the approach from below is much more gradual. The RGE for the Higgs self-coupling $\lambda(\mu)$ similarly focuses $\lambda(\mu)$ towards a quasifixed point value, leading to the SM fixed point predictions [7] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV} \tag{1}$$

Unfortunately these predictions are inconsistent with the CDF and D0 results [8], which require a running top mass $m_t \simeq 170 \pm 12$ GeV.

However the fixed point top Yukawa coupling is reduced by 15% in the Minimal Supersymmetric Standard model (MSSM), with supersymmetry breaking at the electroweak scale or TeV scale, due to the contribution of the supersymmetric partners to the RGE. Also the top quark couples to just one of the two Higgs doublets in the MSSM, which has a VEV of $v_2 = (174 \text{ Gev}) \sin\beta$, leading to the MSSM fixed point prediction for the running top quark mass [9]:

$$m_t(m_t) \simeq (190 \text{ Gev}) \sin\beta \tag{2}$$

which is remarkably close to the CDF and D0 results for $\tan\beta > 1$.

For large $\tan\beta$ it is possible to have a bottom quark Yukawa coupling satisfying $g_b(M_X) \gtrsim 1$ which then approaches an infrared quasifixed point and is no longer negligible in the RGE for $g_t(\mu)$. Indeed with $\tan\beta \simeq m_t(m_t)/m_b(m_t) \simeq 60$ we can trade the mystery of the top to bottom quark mass ratio for that of a hierarchy of vacuum expectation values, $v_2/v_1 \simeq m_t(m_t)/m_b(m_t)$, and have all the third generation Yukawa coupling constants large:

$$g_t(M_X) \gtrsim 1 \quad g_b(M_X) \gtrsim 1 \quad g_\tau(M_X) \gtrsim 1 \tag{3}$$

Then m_t, m_b and $R = m_b/m_\tau$ all approach infrared quasifixed point values compatible with experiment [10]. This large $\tan\beta$ scenario is consistent with the idea of Yukawa unification [11]:

$$g_t(M_X) = g_b(M_X) = g_\tau(M_X) = g_G \tag{4}$$

as occurs in the SO(10) SUSY-GUT model with the two MSSM Higgs doublets in a single **10** irreducible representation and $g_G \gtrsim 1$ ensures fixed point behaviour. However it should be noted that the equality in Eq. (4) is not necessary, since the weaker assumption of large third generation Yukawa couplings, Eq. (3), is sufficient for the fixed point dynamics to predict [10] the running masses $m_t \simeq 180$ GeV, $m_b \simeq 4.1$ GeV and $m_\tau \simeq 1.8$ GeV in the large $\tan\beta$ scenario. Also the lightest Higgs particle mass is predicted to be $m_{h^0} \simeq 120$ GeV (for a top squark mass of order 1 TeV).

The origin of the large value of $\tan\beta$ is of course a puzzle, which must be solved before the large $\tan\beta$ scenario can be said to explain the large m_t/m_b ratio. It is possible to introduce approximate symmetries [12, 13] of the Higgs potential which ensure a hierarchy of vacuum expectation values - a Peccei-Quinn symmetry and a continuous $\mathcal{R}$ symmetry have been used. However these symmetries are inconsistent with the popular scenario of universal soft SUSY breaking mass parameters at the unification scale and radiative electroweak symmetry breaking [14]. Also, in the large $\tan\beta$ scenario, SUSY radiative corrections to m_b are generically large: the bottom quark mass gets a contribution proportional to v_2 from some one-loop diagrams with internal superpartners, such as top squark-charged Higgsino exchange , whereas its tree level mass is proportional to $v_1 = v_2/\tan\beta$. Consequently these loop diagrams give a fractional correction $\delta m_b/m_b$ to the bottom quark mass proportional to $\tan\beta$ and generically of order unity [13, 14]. The presence of the above-mentioned Peccei-Quinn and $\mathcal{R}$ symmetries and the associated hierarchical SUSY spectrum (with the squarks much heavier than the gauginos and Higgsinos) would protect m_b from large radiative corrections, by providing a suppression factor in the loop diagrams and giving $\delta m_b/m_b \ll 1$. However, in the absence of experimental information on the superpartner spectrum, the predictions of the third generation quark-lepton masses in the large $\tan\beta$ scenario must, unfortunately, be considered unreliable.

2.2 Criticality and the Standard Model

Here we consider the idea [15] that Nature should choose coupling constant values such that several "phases" can coexist, in a very similar way to the stable coexistence of ice, water and vapour (in a thermos flask for example) in a mixture with fixed energy and number of molecules. The application of this so-called multiple point criticality principle to the determination of the top quark Yukawa coupling constant requires the SM (renormalisation group improved) effective Higgs potential to have coexisting vacua, which means degenerate minima: $V_{eff}(\phi_{min\,1}) = V_{eff}(\phi_{min\,2})$. The important point for us, in the analogy of the ice, water and vapour system, is that the choice of the fixed extensive variables, such as energy, the number of moles and the volume, can very easily be such that a mixture must occur. In that case then the temperature and pressure (i. e. the intensive quantities) take very specific values, namely the values at the triple point, without any finetuning. We stress that this phenomenon of thus getting specific intensive quantitities is only *likely* to happen for stongly first order phase transitions, for which the interval of values for the extensive variables that can only be realised as an inhomogeneous mixture of phases is rather large.

In the SM, the top quark Yukawa coupling and the Higgs self coupling correspond to intensive quantities like temperature and pressure. If these couplings are to be determined by the criticality condition, the two phases corresponding to the two effective Higgs field potential minima should have some "extensive quantity", such as $\int d^4x|\phi(x)|^2$, deviating "strongly" from phase to phase. If, as we shall assume, Planck units reflect the fundamental physics it would be natural to interpret this strongly first order transition requirement to mean that, in Planck units, the extensive variable densities $\frac{\int d^4x|\phi(x)|^2}{\int d^4x} = <|\phi|^2>$ for the two vacua should differ by a quantity of order unity. Phenomenologically we know that for the vacuum 1 in which we live, $<\phi>_{vacuum\,1} = 246$ GeV and thus we should really expect $<\phi>_{vacuum\,2}$ in the other phase just to be of Planck order of magnitude. In vacuum 2 the ϕ^4 term will a priori strongly dominate the ϕ^2 term. So we basically get the degeneracy to mean that, at the vacuum 2 minimum, the effective coefficient $\lambda(\phi_{vacuum\,2})$ must be zero with high accuracy. At the same ϕ-value the derivative of the renormalisation group improved effective potential $V_{eff}(\phi)$ should be zero because it has a minimum there. Thus at the second minimum the beta-function β_λ vanishes as well as $\lambda(\phi)$.

We use the renormalisation group to relate the couplings at the scale of vacuum 2, i.e. at $\mu = \phi_{vacuum\,2}$, to their values at the scale of the masses themselves, or roughly at the electroweak scale $\mu \approx \phi_{vacuum\,1}$. Figure 1 shows the running $\lambda(\phi)$ as a function of $\log(\phi)$ computed for two values of $\phi_{vacuum\,2}$ (where we impose the conditions $\beta_\lambda = \lambda = 0$) near the Planck scale $M_{Planck} \simeq 2 \times 10^{19}$ GeV.

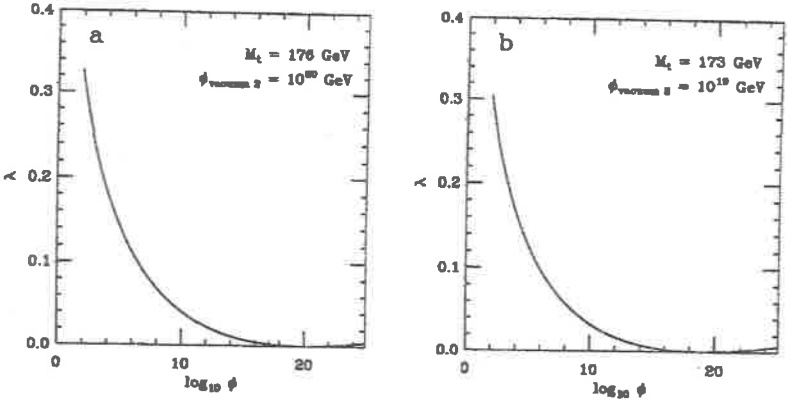

Figure 1: Plot of λ as a function of the scale of the Higgs field ϕ for degenerate vacua with the second Higgs VEV at the scale (a) $\phi_{vacuum\,2} = 10^{20}$ GeV and (b) $\phi_{vacuum\,2} = 10^{19}$ GeV. We formally apply the second order SM renormalisation group equations up to a scale of 10^{25} GeV.

Combining the uncertainty from the Planck scale only being known in order of magnitude and the $\alpha_{QCD}(M_Z) = 0.117 \pm 0.006$ uncertainty with the calculational uncertainty , we get our predicted combination of top and Higgs pole masses:

$$M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}. \tag{5}$$

3 Ansätze and Mass Matrix Texture

The best known ansatz for the quark mass matrices is due to Fritzsch [16]:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \qquad M_D = \begin{pmatrix} 0 & C' & 0 \\ C' & 0 & B' \\ 0 & B' & A' \end{pmatrix} \tag{6}$$

where it is necessary to *assume*: $|A| \gg |B| \gg |C|$, $|A'| \gg |B'| \gg |C'|$ in order to obtain a good fermion mass hierarchy. However, in addition to predicting a generalised version of the relation $\theta_c \simeq \sqrt{\frac{m_d}{m_s}}$ for the Cabibbo angle, which originally motivated the ansatz, it predicts the relationship:

$$|V_{cb}| \simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right| \tag{7}$$

which cannot be satisfied with a top quark mass $m_t > 100$ GeV [17]. Consistency with experiment can be restored by, for example, introducing a non-zero 22 mass matrix element [18]. In fact a systematic analysis [19] of *symmetric* quark mass matrices with 5 or 6 "texture" zeros at the SUSY-GUT scale has been made, yielding 5 ansätze consistent with experiment. Recently ansätze incorporating the Georgi-Jarlskog [20] SUSY-GUT mass relations between leptons and quarks, $m_b(M_X) = m_\tau(M_X)$, $m_s(M_X) = m_\mu(M_X)/3$ and $m_d(M_X) = 3m_e(M_X)$, have been studied. In particular a systematic analysis of fermion mass matrices in SO(10) SUSY-GUT models [12, 21] has been made in terms of 4 effective operators. A scan of millions of operators leads to just 9 solutions consistent with experiment of the form:

$$Y_u = \begin{pmatrix} 0 & \frac{-1}{27}C & 0 \\ \frac{-1}{27}C & 0 & x'_u B \\ 0 & x_u B & A \end{pmatrix} \; Y_d = \begin{pmatrix} 0 & C & 0 \\ C & Ee^{i\phi} & x'_d B \\ 0 & x_d B & A \end{pmatrix} \; Y_l = \begin{pmatrix} 0 & C & 0 \\ C & 3Ee^{i\phi} & x'_l B \\ 0 & x_l B & A \end{pmatrix} \tag{8}$$

For each of the 9 models the Clebschs x_i and x'_i have fixed values and the Yukawa coupling matrices Y_i depend on 6 free parameters: A, B, C, E, ϕ and $\tan\beta$. Each solution has Yukawa unification and gives 8 predictions consistent with the data.

4 Chiral Flavour Symmetry and the Mass Hierarchy

It is natural [5] to interpret the fermion mass hierarchy in terms of partially conserved chiral quantum numbers beyond those of the SM gauge group. Mass matrix elements are then suppressed by powers of a symmetry breaking parameter, which may be thought of as the ratio of the new chiral symmetry breaking scale to the fundamental scale of the theory. The degree of forbiddenness of a mass matrix element is then determined by the quantum number difference between the left- and right-handed SM Weyl states under consideration and the assumed superheavy fermion spectrum. For example the four effective operators in the ansatz of Eq. (8) can each be associated with a unique tree diagram, by assigning an approximately conserved global $U(1)_f$ flavour charge appropriately to the quarks, leptons and the superheavy states, which are presumed to belong to vector-like SO(10) $\mathbf{16} + \overline{\mathbf{16}}$ representations. The required parameter hierarchy $A \gg B$, $E \gg C$ is naturally obtained in this way and, in particular, the texture zeros reflect the assumed absence of superheavy fermion states which could mediate the transition between the corresponding Weyl states.

We now turn to models in which the chiral flavour charges are part of the extended gauge group. The values of the chiral charges are then strongly constrained by the anomaly conditions for the gauge theory. It will also be assumed that any superheavy state needed to mediate a symmetry breaking transition exists, so that the results are insensitive to the details of the superheavy spectrum. The aim in these models is to reproduce all quark-lepton masses and mixing angles within a factor of 2 or 3.

Ibanez and Ross [22] have constructed an anomaly free $MSSM \times U(1)_f$ model. The $U(1)_f$ charges assigned to the quarks and leptons generate Yukawa matrices of the following form:

$$Y_u \simeq \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix} \quad Y_d \simeq \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix} \quad Y_l \simeq \begin{pmatrix} \bar{\epsilon}^5 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & \bar{\epsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

which are symmetric up to factors of order unity. The correct order of magnitude for all the masses and mixing angles are obtained by fitting ϵ, $\bar{\epsilon}$ and $\tan\beta$. This is a large $\tan\beta \simeq m_t/m_b$ model, but not necessarily having exact Yukawa unification. The $U(1)_f$ symmetry is spontaneously broken by two Higgs singlets, θ and $\bar{\theta}$, having $U(1)_f$ charges $+1$ and -1 respectively and equal vacuum expectation values. The $U(1)_f^2U(1)_Y$ gauge anomaly vanishes. The $U(1)_f^3$ anomaly and the mixed $U(1)_f$ gravitational anomaly are cancelled against unspecified spectator particles neutral under the SM group. However cancellation of the mixed $SU(3)^2U(1)_f$, $SU(2)^2U(1)_f$ and $U(1)_Y^2U(1)_f$ anomalies is only possible in the context of superstring theories via the Green-Schwarz mechanism [23] with $\sin^2\theta_W = 3/8$. Consequently the $U(1)_f$ symmetry is spontaneously broken slightly below the string scale.

A number of generalisations of this model has been considered during the last year. By using non-symmetric mass matrices an anomaly free model has been constructed [24] without the need for the Green-Schwarz mechanism. Models have also been considered [24, 25], in which the $U(1)_f$ symmetry is broken by just one chiral singlet field θ having a $U(1)_f$ charge, say, -1. It then follows, from the holomorphicity of the superpotential, that only positive $U(1)_f$ charge differences between left and right handed Weyl states can be balanced by θ tadpoles. Consequently mass matrix elements corresponding to negative $U(1)_f$ charge differences have texture zeros [26]. Furthermore if the two Higgs doublet fields carry $U(1)_f$ charges that do not add up to zero, the $\mu H_1 H_2$ term is forbidden in the superpotential [27]. Finally we remark that in effective superstring theories the role of the $U(1)_f$ symmetry can be played by modular symmetry [2], with the $U(1)_f$ charges replaced by the modular weights of the fermion fields.

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Relativistic description of exclusive weak decays of heavy mesons using heavy quark expansion *

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Abstract

The decay matrix elements of exclusive weak decays of heavy mesons are studied in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. It is shown that the heavy quark $1/m_Q$ expansion considerably simplifies the analysis both for heavy-to-heavy and heavy-to-light decays. The comparison is made with the model-independent predictions of heavy quark effective theory and available experimental data.

1 INTRODUCTION

The investigation of weak decays of mesons is important for the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and testing quark dynamics in a meson. Recently a significant progress has been achieved in the theoretical understanding of weak decays of mesons and baryons with heavy quarks. It has been found that in the limit of infinitely heavy quarks new spin-flavour symmetries in the heavy-to-heavy weak transitions arise [1], which considerably simplify their description. All weak decay form factors become related to a single universal form factor – Isgur-Wise function [1]. This allows to get some model-independent predictions and establish relations between different decay processes. However, the corrections in inverse powers of the heavy quark mass m_Q can be substantial. The heavy quark effective theory (HQET) [2] provides a framework for systematic $1/m_Q$ expansion of weak decay amplitudes. The number of independent form factors at each order of heavy quark expansion is considerably

*Supported in part by the Russian Foundation for Fundamental Research under Grant No.94-02-03300-a

reduced due to heavy quark symmetry and QCD. These form factors originate from the infrared region and thus cannot be calculated without model assumptions.

The methods of HQET are less powerful in the case of heavy-to-light decays, because there is no heavy quark in the final state. Only the relations between different decays can be established in the heavy quark limit [2]. However, the ideas of heavy quark expansion can be applied here too. It is easy to see that the final light meson has a large recoil momentum compared to its mass almost in the whole kinematical range. At the point of maximum recoil of the final meson it bears the large relativistic recoil momentum $|\mathbf{\Delta}_{\rm max}|$ of order $m_Q/2$ and the energy of the same order. Thus at this kinematical point it is possible to expand the matrix element of the weak current both in inverse powers of heavy quark mass of the initial meson and in inverse powers of the recoil momentum $\mathbf{\Delta}_{\rm max}$ of the final light meson. As a result the expansion in powers $1/m_Q$ arises.

In this talk we present the heavy quark expansion for heavy-to-heavy and heavy-to-light decays in the framework of relativistic quark model and compare the results with model-independent predictions of HQET. Our relativistic quark model is based on the quasipotential approach in quantum field theory with the specific choice of the $q\bar{q}$ potential. It provides a scheme for calculation of meson properties with the consistent account of relativistic effects.

2 RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [3] of the Schrödinger type [4]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4-(m_a^2-m_b^2)^2}{4M^3}, \tag{2}$$

$$b^2(M) = \frac{[M^2-(m_a+m_b)^2][M^2-(m_a-m_b)^2]}{4M^2}, \tag{3}$$

$m_{a,b}$ are the quark masses; M is the meson mass; $\mathbf{p}$ is the relative momentum of quarks. While constructing the kernel of this equation $V(\mathbf{p},\mathbf{q};M)$ — the quasipotential of quark-antiquark interaction — we have assumed that effective interaction is the sum of the one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials. We have also assumed that at large distances the vector long-range potential contains the Pauli interaction. The quasipotential is defined by [5]:

$$V(\mathbf{p},\mathbf{q},M) = \bar{u}_a(p)\bar{u}_b(-p)\Big\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_a^\mu\gamma_b^\nu + V_{\rm conf}^V(\mathbf{k})\Gamma_a^\mu\Gamma_{b;\mu} + V_{\rm conf}^S(\mathbf{k})\Big\}u_a(q)u_b(-q), \quad (4)$$

where α_S is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator; γ_μ and $u(p)$ are the Dirac matrices and spinors; $\mathbf{k}=\mathbf{p}-\mathbf{q}$; the effective long-range vector vertex is

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \quad (5)$$

κ is the Pauli interaction constant. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$\begin{aligned} V_{\rm conf}^V(r) &= (1-\varepsilon)(Ar+B), \\ V_{\rm conf}^S(r) &= \varepsilon(Ar+B), \end{aligned} \quad (6)$$

reproducing $V_{\rm nonrel}^{\rm conf}(r) = V_{\rm conf}^S + V_{\rm conf}^V = Ar+B$, where ε is the mixing coefficient. The explicit expression for the quasipotential with the account of the relativistic corrections of order v^2/c^2 can be found in ref.[5]. All the parameters of our model: quark masses, parameters of linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ were fixed from the analysis of meson masses [5] and radiative decays [6]. Quark masses: $m_b = 4.88$ GeV; $m_c = 1.55$ GeV; $m_s = 0.50$ GeV; $m_{u,d} = 0.33$ GeV and parameters of linear potential: $A = 0.18$ GeV^2; $B = -0.30$ GeV have standard values for quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been chosen from the consideration of meson radiative decays [6] and of the heavy quark expansion [8] (see below), which are very sensitive to the Lorentz-structure of the confining potential: the resulting leading relativistic corrections coming from vector and scalar potentials have opposite signs for the radiative Ml-decays [6]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [5].

The matrix element of the local current J between bound states in the quasipotential method has the form [7]

$$\langle M'|J_\mu(0)|M\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}_{M'}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_M(\mathbf{q}), \quad (7)$$

where $M(M')$ is the initial (final) meson, $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{M,M'}$ are the meson wave functions projected onto the positive energy states of quarks.

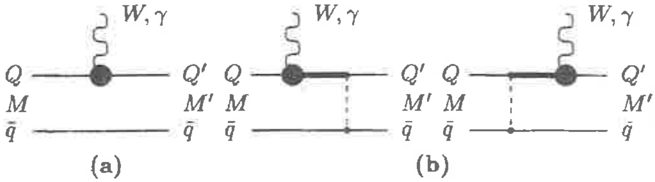

Figure 1: (a) Lowest order vertex function $\Gamma_\mu^{(1)}$. (b) Vertex function $\Gamma_\mu^{(2)}$ with account of the quark interaction. Dashed line corresponds to the effective potential (4). The bold line denotes the negative-energy part of the quark propagator.

This relation is valid for the general structure of the current $J = \bar{Q}'G_\mu Q$, where G_μ can be an arbitrary combination of Dirac matrices. The contributions to Γ come from Figs. 1(a) and 1(b). Note that the contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. The form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz-structure of $q\bar{q}$-interaction.

The general structure of the current matrix element (7) is rather complicated, because it is necessary to integrate both with respect to d^3p and d^3q. The δ-function in the expression for the vertex function $\Gamma^{(1)}$ permits to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has usual structure and can be calculated without any expansion, if the wave functions of initial and final meson are known. The situation with the contribution $\Gamma^{(2)}$ is different. Here instead of δ-function we have a complicated structure, containing the potential of $q\bar{q}$-interaction in meson. Thus in general case we cannot perform one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (7). Therefore, it is necessary to use some additional considerations. The main idea is to expand the vertex function $\Gamma^{(2)}$ in such a way that it will be possible to use the quasipotential equation (1) in order to perform one of the integrations in the current matrix element (7). The realization of such expansion differs for the cases of heavy-to-heavy and heavy-to-light transitions.

3 HEAVY-TO-HEAVY DECAYS

3.1 Decay matrix elements

In the case of the heavy-to-heavy meson decays we have two natural expansion parameters, which are the heavy quark masses in the initial and final meson. The most convenient point for the expansion of vertex function $\Gamma^{(2)}$ in inverse powers of the heavy quark masses for semileptonic decays is the point of zero recoil of the final meson, where $\Delta = 0$ ($\Delta = \mathbf{p}_M - \mathbf{p}_{M'}$). It is easy to see that $\Gamma^{(2)}$ contributes to the current matrix element at first order of $1/m_Q$ expansion. We limit our analysis to the consideration of the terms up to the second order. After the expansion we perform the integrations in the contribution of $\Gamma^{(2)}$ to the decay matrix element. As a result we get the expression for the current matrix element, which contains the ordinary mean values between meson wave functions and can be easily calculated numerically.

3.2 Comparison with heavy quark effective theory

The leading approximation of HQET is the infinitely heavy quark limit. As $m_Q \to \infty$, the properties of hadron become independent of the heavy quark flavour and spin. The arising spin-flavour symmetry relates all the hadronic form factors to a single Isgur-Wise function [1]. In our model the heavy symmetry relations [1, 2] are exactly satisfied [8]. We get for the Isgur-Wise function the expression after approximating the wave functions by Gaussians

$$\xi(w) = \sqrt{\frac{2}{w+1}} \exp\left(-\left(2\rho^2 - \frac{1}{2}\right)\frac{w-1}{w+1}\right), \tag{8}$$

$$w \equiv v \cdot v' = \frac{M_M^2 + M_{M'}^2 - q^2}{2M_M M_{M'}},$$

with the slope parameter $\rho^2 \simeq 1.02$, which is in accordance with recent CLEO II measurement [9] $\rho^2 = 1.01 \pm 0.15 \pm 0.09$.

At first order of $1/m_Q$ expansion only four additional independent form factors arise [2]. One of these subleading form factors $\xi_3(w)$ emerge from the corrections to the current and three functions $\chi_i(w)$ ($i = 1, 2, 3$) - from the corrections to the effective Lagrangian in HQET [2]. For determination of these four functions we get 28 equations. We find that this structure of first order corrections can be reproduced in our model [8] only if we set long-range Pauli interaction constant $\kappa = -1$, which coincides with the value obtained from meson mass spectra [5]. For subleading form factors we find [8]:

$$\xi_3(w) \quad = \quad (\bar{\Lambda} - m_q)\left(1 + \frac{2}{3}\frac{w-1}{w+1}\right)\xi(w),$$

$$
\begin{aligned}
\chi_1(w) &= \bar{\Lambda}\frac{w-1}{w+1}\xi(w), \\
\chi_2(w) &= -\frac{1}{32}\frac{\bar{\Lambda}}{w+1}\xi(w), \\
\chi_3(w) &= \frac{1}{16}\bar{\Lambda}\frac{w+1}{w-1}\xi(w),
\end{aligned} \tag{9}
$$

where the HQET parameter $\bar{\Lambda} = M - m_Q$ in our model is equal to the mean value of light quark energy in heavy meson $\bar{\Lambda} =< \varepsilon_q >\simeq 0.54$ GeV.

The structure of the second order power corrections predicted by HQET at the point of zero recoil of the final meson [2] can be reproduced in our model if the mixing parameter of vector and scalar confining potentials $\varepsilon = -1$ [8]. Therefore we get QCD and heavy quark symmetry motivation for the choice of the main parameters of our potential model. The found values of ε and κ imply that confining quark-antiquark potential has predominantly Lorentz-vector structure, while the scalar potential is anticonfining and helps to reproduce the initial nonrelativistic potential.

Our model predicts that the second order $1/m_Q$ corrections to the decay rate $B \to D^* e\nu$, which is protected from the first order $1/m_Q$ corrections by Luke's theorem [2], are small [8]

$$\delta_{1/m_Q^2} = -(2.0 \pm 0.5)\%.$$

Then for the hadronic form factor of this decay at zero recoil, we obtain

$$\mathcal{F}(1) = \eta_A(1 + \delta_{1/m_Q^2}) = 0.94 \pm 0.03,$$

where $\eta_A = 0.965 \pm 0.020$ accounts for the short distance corrections [2]. Comparing this prediction with the experimental determination [9] of the product $\mathcal{F}(1)|V_{cb}|$, we get for the CKM matrix element

$$|V_{cb}| = (38.2 \pm 1.9 \pm 1.5) \times 10^{-3}, \tag{10}$$

where the first error is experimental and the second is theoretical one. This result agrees with the $|V_{cb}|$ value obtained from the comparison of our model predictions for exclusive $B \to D(D^*)e\nu$ decay rates with experimental data [10].

4 HEAVY-TO-LIGHT DECAYS

4.1 Decay matrix elements

In the case of heavy-to-light decays the final meson contains only light quarks (u, d, s), thus, in contrast to the heavy-to-heavy transitions, we cannot expand matrix elements in inverse powers of the final quark mass. The expansion of $\Gamma^{(2)}$ only in inverse powers of the initial heavy quark mass at $\Delta = 0$ does not solve the problem. However, as it was already mentioned in the introduction, the final light meson has the large recoil momentum almost in the whole kinematical range. At the point of maximum recoil of final light meson the large value of recoil momentum $\Delta_{\max} \sim m_Q/2$ allows for the expansion of decay matrix element in $1/m_Q$. The contributions to this expansion come both from the inverse powers of heavy m_Q from initial meson and from inverse powers of the recoil momentum $|\Delta_{\max}|$ of the final light meson. We carry out this expansion up to the second order and perform one of the integrations in the current matrix element (7) using the quasipotential equation as in the case of heavy final meson. As a result we again get the expression for the current matrix element, which contains only the ordinary mean values between meson wave functions, but in this case at the point of maximum recoil of final light meson.

4.2 Rare radiative decays of B mesons

Rare radiative decays of B mesons are induced by flavour changing neutral currents. These decays are described by one-loop (penguin) diagrams with the main contribution from virtual top quark and W boson. The momentum transfer is fixed at the maximum value for the processes with the emission of real photon, such as $B \to K^*\gamma$. The hadronic matrix element (see *e.g.* [12]) in this case is parameterized by one form factor $F_1(q^2 = 0)$. We have performed the $1/m_b$ expansion for this form factor up to the second order [11]. Here we present only the numerical results. Our values of rare radiative form factors are presented in Table 1 in comparison with recent calculations within the light-cone QCD sum rule [12] and hybrid sum rule [13] approaches. There is an overall agreement between the predictions within errors.

Table 1: Rare radiative decay form factors $F_1(0)$

Decay	our results	[12]	[13]
$B \to K^*\gamma$	0.32 ± 0.03	0.32 ± 0.05	0.308 ± 0.039
$B \to \rho\gamma$	0.26 ± 0.03	0.24 ± 0.04	0.27 ± 0.034
$B_s \to \phi\gamma$	0.27 ± 0.03	0.29 ± 0.05	
$B_s \to K^*\gamma$	0.23 ± 0.02	0.20 ± 0.04	

Our value of form factor $F_1^{B\to K^*\gamma}$ yields for the ratio of exclusive to inclusive decay rates

$$R(B \to K^*\gamma) \equiv \frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)} = (15 \pm 3)\%. \tag{11}$$

Combining this result with the QCD-improved inclusive radiative branching ratio $\mathcal{B}(B \to X_s\gamma) = (3.0 \pm 1.2) \times 10^{-4}$ [12], we find

$$\mathcal{B}^{\rm th}(B \to K^*\gamma) = (4.5 \pm 1.5) \times 10^{-5}. \tag{12}$$

This branching ratio agrees well with experimental measurement by CLEO [14]

$$\mathcal{B}^{\rm exp}(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}.$$

4.3 Semileptonic heavy-to-light decays of B mesons

The heavy quark $1/m_b$ expansion for the form factors of semileptonic decays $B \to \pi(\rho)e\nu$ has been carried out up to the second order at the point of maximum recoil of the final light meson in [15]. The calculated values of form factors, defined in the usual way [16], are presented in Table 2. There we compare our results for the form factors of $B \to \pi(\rho)e\nu$ decays with the predictions of quark models [16, 17], QCD sum rules [18, 19, 12] and lattice calculations [20, 21]. We find an agreement between our value of $f_+^{B\to\pi}(0)$ and QCD sum rule and lattice predictions. Our $B \to \rho e\nu$ form factors agree with lattice and QCD sum rule ones [12], while they are approximately 1.5 times less than QCD sum rule results of refs. [18, 19].

Table 2: Semileptonic $B \to \pi$ and $B \to \rho$ decay form factors at $q^2 = 0$.

Ref.	$f_+^{B\to\pi}(0)$	$A_1^{B\to\rho}(0)$	$A_2^{B\to\rho}(0)$	$V^{B\to\rho}(0)$
our [15]	0.20 ± 0.02	0.26 ± 0.03	0.31 ± 0.03	0.29 ± 0.03
[16]	0.33	0.28	0.28	0.33
[17]	0.09	0.05	0.02	0.27
[18]	0.26 ± 0.02	0.5 ± 0.1	0.4 ± 0.2	0.6 ± 0.2
[19]	0.23 ± 0.02	0.38 ± 0.04	0.45 ± 0.05	0.45 ± 0.05
[12]		0.24 ± 0.04		0.28 ± 0.06
[20]	0.35 ± 0.08	0.24 ± 0.12	0.27 ± 0.80	0.53 ± 0.31
[21]	$0.30 \pm 0.14 \pm 0.05$	0.22 ± 0.05	$0.49 \pm 0.21 \pm 0.05$	0.37 ± 0.11

To calculate the $B \to \pi(\rho)$ semileptonic decay rates it is necessary to determine the q^2-dependence of the form factors. Analysing the $\mathbf{\Delta}^2_{\rm max}$ dependence of the form factors, we find [15] that the q^2-dependence of these form factors near $q^2 = 0$ could be given by

$$
\begin{aligned}
f_+(q^2) &= \frac{M_B+M_\pi}{2\sqrt{M_BM_\pi}}\tilde{\xi}(w)\mathcal{F}_+(\Delta^2_{\rm max}), && (13)\\
A_1(q^2) &= \frac{2\sqrt{M_BM_\rho}}{M_B+M_\rho}\frac{1}{2}(1+w)\tilde{\xi}(w)\mathcal{A}_1(\Delta^2_{\rm max}), && \\
A_2(q^2) &= \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}}\tilde{\xi}(w)\mathcal{A}_2(\Delta^2_{\rm max}), && (14)\\
V(q^2) &= \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}}\tilde{\xi}(w)\mathcal{V}(\Delta^2_{\rm max}). && (15)
\end{aligned}
$$

We have introduced the function [15]

$$
\tilde{\xi}(w) = \sqrt{\frac{2}{w+1}}\exp\left(-\eta\frac{\bar{\Lambda}^2}{\beta_B^2}\frac{w-1}{w+1}\right), \qquad (16)
$$

which reduces to the Isgur-Wise function (8) in the limit of infinitely heavy quarks in the initial and final mesons.

It is important to note that the form factor A_1 in (14) has a different q^2-dependence than the other form factors (13), (14), (15). In the quark models it is usually assumed the pole [16] or exponential [17] q^2-behaviour for all form factors. However, the recent QCD sum rule analysis indicates that the form factor A_1 has q^2-dependence different from other form factors [18, 19, 12].

We have calculated the decay rates of $B \to \pi(\rho)e\nu$ using our form factor values at $q^2 = 0$ and the q^2-dependence (13)–(15) in the whole kinematical region. The results are presented in Table 3 in comparison with the quark model [16, 17], QCD sum rule [18, 19] and lattice [20] predictions. The predictions for the rates with longitudinally and transversely polarized ρ meson differ considerably in these approaches. This is mainly due to different q^2-behaviour of A_1. Thus the measurement of the ratios $\Gamma(B\to\rho e\nu)/\Gamma(B\to\pi e\nu)$ and Γ_L/Γ_T may discriminate between these approaches.

Recently CLEO reported [22] about the experimental measurement of B semileptonic decays to π and ρ:

$$
\begin{aligned}
\mathcal{B}(B^0\to\pi^-l^+\nu) &= (1.34\pm0.35\pm0.28)\times10^{-4},\\
\mathcal{B}(B^0\to\rho^-l^+\nu) &= (2.28\pm0.36\pm0.59^{+0.00}_{-0.46})\times10^{-4},\\
\frac{\Gamma(B^0\to\rho^-l^+\nu)}{\Gamma(B^0\to\pi^-l^+\nu)} &= 1.70^{+0.80}_{-0.50}\pm0.58^{+0.00}_{-0.34}.
\end{aligned}
$$

We see that the experimental ratio of the ρ and π rates supports the models with a specific q^2-behaviour of A_1 form factor. For the CKM matrix element V_{ub}

Table 3: Semileptonic decay rates $\Gamma(B \to \pi e\nu)$, $\Gamma(B \to \rho e\nu)$ $(\times|V_{ub}|^2 \times 10^{12}\mathrm{s}^{-1})$, the ratio of the rates for longitudinally (L) and transversely (T) polarized ρ meson and the ratio of ρ and π rates.

Ref.	$\Gamma(B \to \pi e\nu)$	$\Gamma(B \to \rho e\nu)$	Γ_L/Γ_T	$\Gamma^{B\to\rho}/\Gamma^{B\to\pi}$
our [15]	3.0 ± 0.6	5.4 ± 1.2	0.5 ± 0.3	1.8 ± 0.6
[16]	7.4	26	1.34	3.5
[17]	2.1	8.3	0.75	4.0
[18]	5.1 ± 1.1	12 ± 4	0.06 ± 0.02	2.4 ± 0.9
[19]	3.6 ± 0.6	5.1 ± 1.0	0.13 ± 0.08	1.4 ± 0.2
[20]	8 ± 4			

we find in our model

$$\begin{aligned} |V_{ub}| &= (5.4 \pm 0.9 \pm 0.5) \times 10^{-3} \quad (B \to \pi l\nu) \\ |V_{ub}| &= (5.3^{+0.8}_{-0.9} \pm 0.6) \times 10^{-3} \quad (B \to \rho l\nu) \end{aligned}$$

where the first error is experimental and the second one is theoretical.

5 CONCLUSIONS

We have presented the method of calculating weak decay matrix elements in the framework of relativistic quark model using heavy quark expansion. It has been shown that in the case of heavy-to-heavy meson decays the obtained expansion is in accordance with the model independent predictions of HQET. This allowed to determine the Isgur-Wise function and the first and second order form factors.

In the case of heavy-to-light transitions the expansion in inverse powers of the heavy quark mass from initial heavy meson has been carried out at the point of maximum recoil of the final light meson. The results of application of such expansion for the calculations of the exclusive rare radiative decays as well as semileptonic decays $B \to \pi(\rho)e\nu$ have been discussed. A good agreement with available experimental data has been found.

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The extended Nambu-Jona-Lasinio model with separable interaction: low energy pion physics *

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Abstract

A Lagrangian formulation of the Nambu-Jona-Lasinio model with separable interaction is given. The electromagnetic interaction is introduced in a non-minimal way to the nonlocal quark current. Various choices of the vertex form factors characterizing the composite structure of mesons and baryon are investigated. We find that the physical observables depend very weakly on form factor shapes.

1 INTRODUCTION

The main goal of this paper is to give a Lagrangian formulation of the NJL-model with separable interaction for mesons. We check the Goldstone theorem in this approach which means that a zero-mass pion appears in the chiral limit. Here, we introduce the electromagnetic interactions by means the time-ordering P-exponent in the nonlocal quark currents. This reproduces automatically the Ward-Takahashi identities and electromagnetic gauge invariance in each step of calculation. One of the principal goals of this paper is to investigate the dependence of the physical properties on the choice of the various form factors of the separable interaction. There are two adjustable parameters, a range parameter Λ appearing in the separable interaction and a constituent quark mass m_q. As in the papers [1, 2], the weak decay constant f_π, the two-photon decay width $\Gamma_{\pi^0\to\gamma\gamma}$, as well as the charge form factor $F_\pi(q^2)$ and the $\gamma^*\pi^0 \to \gamma$ transition form factor $F_{\gamma\pi}(q^2)$ are calculated. Here we consider both monopole and dipole, Gaussian, and screened Coulomb form factors.

*Supported in part by the RFFR, Russia under contract 94-02-03463-a

2 THE NJL-MODEL WITH SEPARABLE INTERACTION

The Lagrangian of the NJL-model with separable interaction are given in [3]. The standard way of the bosonization of the NJL-model may be found in many papers (see for instance [4, 5]). We give here the Lagrangians of interaction describing octets of vector (axial), pseudoscalar (scalar) mesons.

$$L_M^{\rm int}(x) = g_M \int dy_1 \int dy_2 f((y_1-y_2)^2)\delta\left(x - \frac{y_1+y_2}{2}\right)\bar{q}(y_1)\Gamma_M M(x) q(y_2) \tag{1}$$

The form factor $f(y^2)$ characterizes a region of a quark-antiquark interaction. Here we would like to suggest to introduce the electromagnetic fields to the interaction Lagrangian using the time-ordering P-exponent. In this case the gauge invariant meson-quark vertex has the form

$$\begin{aligned} L_M^{\rm int}(x) = g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1+y_2}{2}\right) f\left((y_1-y_2)^2\right) q(y_1) \qquad (2)\\ \cdot P\exp\left\{ieQ\int\limits_{y_1}^{x} dz^\mu A^\mu(z)\right\}\Gamma_M M(x) P\exp\left\{ieQ\int\limits_{x}^{y_2} dz^\mu A^\mu(z)\right\} q(y_2) \end{aligned}$$

where $Q = \mathrm{diag}(2/3, -1/3, -1/3)$. For neutral mesons one obtains

$$\begin{aligned} L_{M^0}^{\rm int}(x) &= g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1+y_2}{2}\right) f\left((y_1-y_2)^2\right) \qquad (3)\\ &\cdot\bar{q}(y_1) P\exp\left\{ieQ\int\limits_{y_1}^{y_2} dz^\mu A^\mu(z)\right\}\Gamma_M M^{(0)}(x) q(y_2). \end{aligned}$$

The T-product and the S-matrix is defined in a standard manner. The hadron-quark coupling constants g_M in Eq. (2) and (3) are defined from the compositeness condition [3, 6].

3 MODEL PARAMETERS AND PION DECAY CONSTANTS

We consider four kinds of widely used form factors: monopole, dipole, Gaussian and screened Coulomb. All Feynman diagrams are calculated in the

Euclidean region where the form factors decrease rapidly so that no ultraviolet divergences arise. There are two adjustable parameters, Λ characterizing the region of quark-antiquark interaction, and the constituent quark mass m_q. We shall define these parameters by fitting the experimental pion decay constant f_π ($f_\pi^{\rm expt}$= 132 MeV) and $g_{\pi\gamma\gamma}$ ($g_{\pi\gamma\gamma}^{\rm expt}$= 0.276 GeV^{-1}).

1. Pion-quark coupling constants. The pion-quark coupling constants are defined from the compositeness condition [3].Neglecting the pion mass one has

$$\left(\frac{3g_\pi^2}{4\pi^2}\right)^{-1} = \frac{1}{4}\int\limits_0^\infty du u f^2(-u)\frac{(3m_q^2+2u)}{(m_q^2+u)^3}. \tag{4}$$

2. Pion weak decay. The weak decay of the pion is defined by the diagram of Fig.1. After simple we have

$$f_\pi \simeq \frac{3g_\pi}{4\pi^2}m_q\int\limits_0^\infty du u f(-u)\frac{1}{(m_q^2+u)^2}.$$

3. Pion two-photon decay. The two-photon decay of the pion is defined by the diagram of Fig.2. After similar transformations we have

$$G_{\pi\gamma\gamma}(p^2,q_1^2,q_2^2) = \frac{g_\pi}{2\sqrt{2}\pi^2}\frac{m_q}{\Lambda^2}\cdot\int\frac{d^4k}{\pi^2 i}\frac{f(k^2)}{[m_q^2-(k+p/2)^2][m_q^2-(k-p/2)^2]} \cdot\frac{1}{[m_q^2-(k+(q_1-q_2)/2)^2]}. \tag{5}$$

The two-photon decay coupling constant is obtained from Eq. (5) where both photons are on the mass shell $g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(m_\pi^2,0,0)$ The numerical results for the physical observables for the best fit are shown in Table 1. for different choices of form factors.

4 Pion electromagnetic form factors.

1. The $\gamma^*\pi^0 \to \gamma$ form factor. In our model this form factor is expressed as $F_{\gamma\pi}(Q^2) = e^2 G_{\pi\gamma\gamma}(m_\pi^2,-Q^2,0)$. Results for monopole vertex are shown in Fig.3 (for various form factors see in [3]). The numerical results for the radius $r_{\pi\gamma}$ are given in Table 2. Our results practically do not depend on the choice of vertex form factors $f(k^2)$.

2. The pion charge form factor. The pion charge form factor is defined by the diagrams of Fig.4. These diagrams are not gauge invariant separately

Table 1. The best fit of the physical observables.

Form Factors	Λ(MeV)	m_q(MeV)	f_π(MeV) NJL SI	f_π(MeV) EXP	$g_{\pi^0\gamma\gamma}$(GeV^{-1}) NJL SI	$g_{\pi^0\gamma\gamma}$(GeV^{-1}) EXP [14]
monopole	400	267	132	132	0.251	0.276
dipole	1000	245	132		0.263	
Gaussian	1000	237	132		0.261	
Coulomb	450	250	132		0.262	

Table 2. The radius of the $\gamma^*\pi^0 \to \gamma$ form factor.

Vertex Function	$r_{\pi\gamma}(fm)$ NJL SI	$r_{\pi\gamma}(fm)$ EXP [14]
monopole	0.655	0.65 ± 0.03
dipole	0.658	
Gaussian	0.654	
Coulomb	0.659	

Table 3. The electromagnetic radius of pion.

Form Factor	NJL SI			EXP [14]
	$< r_\pi^2 >^\Delta$	$< r_\pi^2 >^\circ$	total	fm^2
monopole	0.545	-0.012	0.533	0.430
dipole	0.461	-0.005	0.456	
Gaussian	0.409	-0.002	0.407	
Coulomb	0.488	-0.006	0.482	

but the sum of the diagrams are gauge invariant. The analytical expression for the vertex function and the form factor also the Ward-Takahashi identity for our case are given in [3]. The numerical results for the radius are presented in Table 3. One can see that our results are in good agreement with the available experimental data and depend very weakly on the choice of vertex form factors $f(k^2)$. The behavior of charge form factor for monopole vertex is shown in Fig.5

5 SUMMARY

We have formulated the Nambu-Jona-Lasinio model with separable interaction using the Lagrangian with the compositeness condition and non-minimal inclusion of the electromagnetic interaction. On one hand the form factors in the hadron-quark vertices take into account the composite structure of hadrons thereby being related to a quark-antiquark potential, on the other hand, they make the Feynman integrals convergent. We have calculated the pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0 \to \gamma$-transition, and the pion charge form factor. The two adjustable parameters, the range parameter Λ appearing in the separable interaction and the constituent quark mass m_q, have been fixed by fitting the experimental data for the pion decay constants. We have considered the following form factors: monopole, dipole, Gaussian and screened Coulomb, and found that the numerical results depend very little on these shapes.

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Measurements of CP and T symmetry violation parameters and tests of CPT invariance in the neutral kaon system

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1 Direct CP violation

1.1 Introduction

The observation in 1964 of the long lived neutral kaon decay to two charged pions [1] has demonstrated the violation of CP symmetry in natural laws.

It is well known that CP violation can be incorporated in the standard model of electroweak interactions with three families. A non trivial complex phase in the quark mixing matrix can induce this tiny asymmetry between matter and anti-matter [2].

However, this phase is a free parameter just like the masses of fermions, the origin of which is not addressed by the Standard Model and remains one of the fundamental questions of particle physics.

In addition, since its discovery, CP violation has only been observed in the neutral kaon system. Moreover, all observations are consistent with the superweak model of L. Wolfenstein [3]. Indeed, no measurement is significantly inconsistent with the existence of a single mixing parameter $\epsilon_K$in the physical neutral kaon states K_L and K_S : $K_{L,S} = (K_{2,1}+\epsilon_K K_{1,2})/\sqrt{1+|\epsilon_K|^2}$, where K_1 and K_2 are CP even and odd eigenstates. The non zero value of ϵ_K is induced by an asymmetry in the $K^0 \to \overline{K^0}$ and $\overline{K^0} \to K^0$ transitions. In such a model, the relative amplitudes $\eta^{+-} = A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)$ and $\eta^{00} = A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0)$ are identical.

In contrast, direct CP violation in the $K \to 2\pi$ decays is naturally present in the Standard Model and is described by the parameter ϵ' with $|\epsilon'| = \mathrm{Im}(a_2/a_0)$, where $a_{0(2)}$ is the K^0 decay amplitude to the 2π state with isospin 0 (2). The relative amplitudes become [4] $\eta^{+-} = \epsilon + \epsilon'$ and $\eta^{00} = \epsilon - 2\epsilon'$.

A major experimental effort has been made over the past 30 years and is being carried on to search for direct CP violation.

1.2 Calculations and measurements of ϵ'/ϵ

In the Standard Model, direct CP violation is believed to originate from the contribution of so-called "penguin diagrams" in the K to 2π decay amplitude. Significant progress has recently been made in the calculation of ϵ'/ϵ. This is reviewed in [5]. The precision on the theoretical estimate of ϵ'/ϵ is limited by the uncertainty on the measurement of the quark mixing matrix elements, the big cancellation between the strong and electroweak penguin diagrams, the difficulty to compute hadronic matrix elements and, related to these, the uncertainty in the QCD scale and the strange quark mass.

The two most recent predictions [5, 6] are consistent and give ϵ'/ϵ in the range of a few 10^{-4}, negative values not being excluded. The two most precise measurements are $\mathrm{Re}(\epsilon'/\epsilon) = (23 \pm 6.5)\ 10^{-4}$[7] and $\mathrm{Re}(\epsilon'/\epsilon) = (7.4 \pm 5.9)\ 10^{-4}$[8]. These results can be seen as a success of the Standard Model. However, they are not in good agreement. Above all, they do not establish the existence of direct CP violation. Several experiments are in preparation, aiming at a precision of order 10^{-4} on $\mathrm{Re}(\epsilon'/\epsilon)$: E832 (KTEV) at Fermilab [9], NA48 at CERN [10], and KLOE at Frascati [11].

1.3 Prospects

Nowadays, the favorite technique to measure $\mathrm{Re}(\epsilon'/\epsilon)$ is to measure the double ratio R :

$$R \equiv \frac{\Gamma(K_L \to \pi^0\pi^0) \times \Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^0\pi^0) \times \Gamma(K_L \to \pi^+\pi^-)} = \left|\frac{\eta^{00}}{\eta^{+-}}\right|^2 = 1 - 6\mathrm{Re}(\epsilon'/\epsilon)$$

Such a measurement is extremely delicate, and systematic uncertainties are minimised by detecting concurrently the four decay modes $K_{L,S} \to \pi^0\pi^0, \pi^+\pi^-$. We will mention here only one out of many difficult requirements common to all experiments.

The decay rates have to be compared in the rest frame of the kaon. Consequently, the absolute time, and hence energy scale has to be the same for charged and neutral decays within better than a per mil accuracy. This precision is more difficult to achieve in the neutral decay mode, and the electromagnetic energy has to be measured with excellent resolution and linearity. E832 is building a pure CsI crystal array with more than 3000 blocks. The expected resolution is $\sigma_E/E \simeq .006/E \oplus .7\%/E^{.5} \oplus 1\%/E^{.25}$ (E in GeV). NA48 has chosen a quasi-homogeneous liquid krypton calorimeter with 13500 cells. The last prototype tested showed $\sigma_E/E = .04/E \oplus .4\% \oplus 3.5\%/E^{.5}$ [12]. The KLOE collaboration has tested a full size module of its fine sampling lead-scintillating fibers calorimeter. A $5\%/E^{.5}$ resolution has been achieved [13], and improvements are expected for the final setup. All those results meet the design goals.

All other elements of the detectors must also have very good performances so that backgrounds to $K_L \to \pi^+\pi^-, \pi^0\pi^0$ decays remain at the per mil level, and especially K_{e3} and $K_L \to 3\pi^0$ decays.

E832 is expected to start taking data in 1996 while NA48 and KLOE should start in 1997.

We should not end this section without stressing that an experimental program at Fermilab (KAMI) [14], scheduled after the KTEV run, aims at detecting the $K_L \to \pi^0 l\bar{l}$ decays for which direct CP violation is expected to be important : the Standard Model expectation of these decays is in the range 10^{-12}-10^{-11} [15]. The experimental expected sensitivity is at the level of 10^{-13} or below.

2 T and CPT symmetries

2.1 The phase of ϵ

It has been shown since a long time that, under CPT invariance, the phase of ϵ Φ_ϵ should be very close to Φ_{SW} ,the so-called superweak phase defined by $\Phi_{SW} = \arctan(2\Delta m/\Delta\Gamma)$, where Δm is the K_L-K_S mass difference and $\Delta\Gamma$ is the K_S-K_L width difference in the appropriate units [4]. How close ?

A simple calculation shows that one can write : $\Phi_\epsilon = \Phi_{SW} + \mathrm{Im}\gamma_{12}/\sqrt{2}|\epsilon|\Gamma_S$, where $\gamma_{12} = \sum_{f\neq 2\pi} A^*(K^0 \to f)A(\overline{K^0} \to f)$ and Γ_S the K_S decay rate. To a sufficiently good approximation, the summation can be restricted to the $\pi l\nu$ and 3π final states [16]. For the $\pi l\nu$ final state to contribute, the $\Delta S = \Delta Q$ rule must be violated, and for the 3π final state, direct CP violation must be dominant in the $K_S \to 3\pi$ decays. The recent results of the CPLEAR experiment at CERN (see section 2.3) lead to the expectation $\Phi_\epsilon = \Phi_{SW} \pm .4°$.

2.2 Φ^{+-} and Φ^{00}

If CPT symmetry holds, the phase of ϵ' is $\psi = \pi/2 + \delta_2 - \delta_0$, where $\delta_{2(0)}$ is the strong $\pi\pi$ phase shift in the isospin 2 (0) final state. Extrapolation of experimental results [17] and theoretical calculations [18] lead to $\psi \approx 44° \pm 6°$. It follows that the phases Φ^{+-} and Φ^{00} of η^{+-} and η^{00} should be very close : $\Delta\Phi \equiv \Phi^{00} - \Phi^{+-} \simeq 3\mathrm{Im}\,\epsilon'/\epsilon < .05°$, and $\Phi^{+-} = \Phi_\epsilon - \Delta\Phi/3 = \Phi_{SW} \pm .4°$.

The experimental situation follows :

Experiment	$\Delta\Phi(°)$	Reference
NA31(CERN)	$.2 \pm 2.9$	[19]
E731(FNAL)	-1.6 ± 1.2	[20]
E773(FNAL)	$.6 \pm 1.0$	[21]
Average	$-.3 \pm 0.8$	

The measurement of Φ^{+-} is always strongly correlated with the value of the K_L-K_S mass difference, and to a smaller extent with the value of the K_S lifetime. The same is true for the superweak phase. A consistent analysis of experimental data has been published recently [22]. The result is :

$$\begin{aligned}
\Phi^{+-} &= (43.75 \pm .60)° \\
\Delta m &= (530.6 \pm 1.3)\ 10^7\ \hbar/s \\
\Phi_{SW} &= (43.44 \pm .09)°
\end{aligned}$$

Both results on $\Delta\Phi$ and Φ^{+-} support CPT symmetry. Since CPT phenomenology is addressed in another contribution to these proceedings, we only stress here that these two tests only concern two combinations of CPT parameters of the neutral K system, namely direct CPT violation in the 2π decays for $\Delta\Phi$ and a combination of direct CPT violation and CPT violation in the mixing for Φ_ϵ. The latter test reads : $\Phi_\epsilon - \Phi_{SW} = .3° \pm .4° \pm .2°(\Delta m) \pm .3°(\gamma_{12})$. If one neglects CPT violation in the decays, this can be interpreted as the equality between K^0 and $\overline{K^0}$ masses to a precision of about 4 10^{-19} GeV/c^2.

2.3 Other tests

The CPLEAR experiment at CERN measures time dependant asymmetries of decay rates of initially pure K^0 or $\overline{K^0}$ states. Depending on the observed final state, various quantities are extracted. We list here the most significant results [23, 24] not previously mentioned :

$$A_T \equiv \frac{N(K^0 \to \overline{K^0}) - N(\overline{K^0} \to K^0)}{N(K^0 \to \overline{K^0}) + N(\overline{K^0} \to K^0)} = (6.3 \pm 2.1 \pm 1.8)\ 10^{-3}$$

If CPT symmetry and $\Delta S = \Delta Q$ rule hold, $A_T = 4\mathrm{Re}\epsilon$. The result is compatible with other measurements of ϵ.

$$\begin{aligned} \mathrm{Re}\ x &= (12.4 \pm 13.7)\ 10^{-3} \\ \mathrm{Im}\ x &= (4.8 \pm 4.4)\ 10^{-3} \end{aligned}$$

($x \equiv A(K^0 \to l^-\pi^+\bar{\nu})/A(K^0 \to l^+\pi^-\nu)$ measures the violation of the $\Delta S = \Delta Q$ rule)

$$\begin{aligned} \mathrm{Re}\ \eta^{+-0} &= (-4 \pm 18)\ 10^{-3} \\ \mathrm{Im}\ \eta^{+-0} &= (-16 \pm 21)\ 10^{-3} \end{aligned}$$

($\eta^{+-0} \equiv \int A(K_S \to \pi^+\pi^-\pi^0)A^*(K_L \to \pi^+\pi^-\pi^0)\ d\Omega / \int\ |A(K_L \to \pi^+\pi^-\pi^0)|^2\ d\Omega$ is the CP violation parameter in the $\pi^+\pi^-\pi^0$ decays)

$$A_{CPT} \equiv \frac{N(\overline{K^0} \to \overline{K^0}) - N(K^0 \to K^0)}{N(\overline{K^0} \to \overline{K^0}) + N(K^0 \to K^0)} = (.28 \pm 2.12 \pm 1.80)\ 10^{-3}$$

This measurement uses K_{e3} decays to tag the strangeness of the decaying K.

The η^{+-0} parameter is also measured by the E621 experiment at FNAL [25] :
Re $\eta^{+-0} = .019 \pm .027$; Im $\eta^{+-0} = .019 \pm .061$.

No evidence for CP violation in K_S decays is yet found.

3 Conclusion

In the past recent years, significant progress has been made in the K physics sector. Although direct CP violation has not been established, the experimental accuracy is outstanding. CPT symmetry is tested to a precision such that one might expect detecting quantum gravity effects [26].

Very promising experimental programs are under way. At Fermilab and at CERN, the already quoted projects KTEV, KAMI and NA48 will address direct CP violation and rare K decays. At Frascati, the KLOE experiment will measure most CP and CPT parameters at about the 10^{-4} level and will search for CP violation in K_S decays [27]. We also wish to mention the experiment E246 at KEK designed to measure the T violating transverse polarization P_T of the muon in the decay $K^+ \to \pi^0\mu^+\nu$ [28]. This experiment should start in 1996 and aims at improving by more than one order of magnitude the present experimental accuracy.

The K mesons have been discovered more than 50 years ago. They have brought crucial information on weak interactions which helped building the Standard Model. They now offer a tool to search for physics beyond the Standard Model.

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Testing Bell's inequality in the Neutral Kaon System at a ϕ-factory

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Abstract

It is shown that a Bell's inequality can be formulated for the neutral kaon system at a ϕ-factory using a formalism based on a kaon *quasispin* picture and taking into account CP violation. Experimental methods to reveal tiny violations of this inequality by quantum mechanics are discussed. The precision detector of an experiment at a high luminosity ϕ-factory could be successfully exploited to perform such a test.

In 1935, Einstein, Podolsky and Rosen (EPR) advanced a famous argument [1] to raise the question of whether or not quantum mechanics offers a complete description of physical reality. Their conclusion could be summarized as follows: either quantum mechanics is incomplete in the sense that there exist some variables (e.g. hidden variables) not taken into account by the wave-function description, or the locality principle is violated by quantum mechanics (i.e. there exist faster-than-light signals). Assuming the validity of the locality principle (at first physicists were reluctant to abandon this fundamental principle) the EPR conclusion was sometimes interpreted as an argument in favour of local hidden-variables theories, which explain the stochastic nature of quantum mechanics as due to the lack of knowledge of the values that some *hidden* parameters (the exact nature of which remains unspecified) are assuming during the measurement process. In this way hidden variables could restore completeness in the theory, and presumably reconcile quantum mechanics with a deterministic and/or realistic viewpoint[1]. However, as long as quantum mechanics and local hidden-variables theory predictions were believed indistinguishable, the discussion on a scientific ground remained quite sterile.

[1]Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone

Starting in 1965, however, the situation changed dramatically. J.Bell proved an important theorem, concerning the whole class of local hidden-variables theories, stating that such theories cannot reproduce all statistical predictions of quantum mechanics [2].

Let us consider, as usual, a pair of spin 1/2 particles in the singlet state moving in opposite directions:

$$|i\rangle = \frac{1}{\sqrt{2}}\{|A\uparrow(\vec{p})\rangle|A\downarrow(-\vec{p})\rangle - |A\downarrow(\vec{p})\rangle|A\uparrow(-\vec{p})\rangle\} \tag{1}$$

where the state $|A\uparrow(\vec{p})\rangle$ represents a particle moving in the $\vec{p}$ direction with the spin *up* along the $\vec{a}$ direction. Let us consider three different axes $\vec{a}$, $\vec{b}$, $\vec{c}$ and the corresponding spin projection operators $A = \vec{\sigma}\cdot\vec{a}$, $B = \vec{\sigma}\cdot\vec{b}$ and $C = \vec{\sigma}\cdot\vec{c}$. Let us assume that we perform a measurement of A, B or C on the particle moving in the $\vec{p}$ direction, and a measurement of A, B or C on the other particle moving in the opposite direction. Bell showed that, according to local hidden-variables theories, the following inequality can be derived for the system considered [2, 3]:

$$\begin{aligned} P[A\uparrow(\vec{p}); B\uparrow(-\vec{p})] \;\leq\; & P[A\uparrow(\vec{p}); C\uparrow(-\vec{p})] + P[C\uparrow(\vec{p}); B\uparrow(-\vec{p})] + \\ & + P[C\downarrow(\vec{p}); C\downarrow(-\vec{p})] \end{aligned} \tag{2}$$

where $P[A\uparrow(\vec{p}); B\uparrow(-\vec{p})]$ is the probability of finding A=+1 (spin up) for the particle moving in the $\vec{p}$ direction, and B=+1 (spin up) for the particle moving in the opposite direction. Inequality (2) is generally referred as Bell's inequality.

For some axes choices, quantum mechanical predictions violate inequality (2), showing the incompatibility of the quantum theory with the local hidden-variables viewpoint [2]. Hence the surprising conclusion seems that quantum mechanics implies an unavoidable violation of the locality principle, regardless of a possible hidden-variables completion. This statement has a dramatic impact on our concepts of reality and space-time, and the importance of Bell's theorem just lies in having made possible experiments aimed at testing quantum mechanics against local hidden-variables theories.

Up till now the experimental tests of inequality (2) that have been performed can be divided in three main categories:

a) tests using optical photons from atomic-cascades [4];

b) tests using γ-rays from positronium annihilation [5];

c) tests using protons from p-p scattering [6].

Experiments of category (a) were the most precise and significant ones, and most of them yielded results in excellent agreement with quantum mechanics. However the validity of any experimental test of inequality (2) relies on some *ad hoc* additional assumptions that restrict the ensemble of local hidden-variables theories for which inequality (2) holds. Hence, in practice, only a class of these theories may be tested against quantum mechanics. How wide and general this class is, it depends on the *reasonability* of additional assumptions. Here a possible test at a ϕ-factory using neutral kaons and requiring somewhat different and slightly more restrictive additional hypotheses than ones made in atomic-cascade experiments is discussed [7].

At a ϕ-factory a coherent neutral kaon state is produced in ϕ decays:

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0(\vec{p})\rangle|\overline{K^0}(-\vec{p})\rangle - |\overline{K^0}(\vec{p})\rangle|K^0(-\vec{p})\rangle\} \tag{3}$$

A ***quasispin*** picture can be introduced such that the strangeness eigenstates $|K^0\rangle$ and $|\overline{K^0}\rangle$ are regarded as a quasispin doublet and called quasispin ***up*** and ***down*** along the z direction. Then CP eigenstates $|K_1\rangle \propto \left\{|K^0\rangle + |\overline{K^0}\rangle\right\}$ and $|K_2\rangle \propto \left\{|K^0\rangle - |\overline{K^0}\rangle\right\}$ are quasispin eigenstates ***up*** and ***down*** along the x axis. Let us also consider the quasispin eigenstates along a third generic direction x' and call them $|K_\alpha\rangle \propto \{|K_1\rangle - \eta^*|K_2\rangle\}$ and $|K_\beta\rangle \propto \{|K_2\rangle + \eta|K_1\rangle\}$ where η is a complex parameter that determines the x' direction. It is worth reminding that the physical states, i.e. Hamiltonian eigenstates, $|K_S\rangle \propto \{|K_1\rangle + \epsilon|K_2\rangle\}$ and $|K_L\rangle \propto \{|K_2\rangle + \epsilon|K_1\rangle\}$ where ϵ is the usual CP violation parameter, are non-orthogonal because of CP violation[2]. In the following $|\eta|$ is chosen of about the same order of $|\epsilon|$, so that terms of the order $O(|\eta|^2) \sim O(|\epsilon|^2) \sim 10^{-6}$ can always be neglected, while terms $O(|\eta|) \sim O(|\epsilon|) \sim 10^{-3}$ are retained.

In complete analogy to the spin case, a Bell's inequality can be written for the state (3), where spin projections along the $\vec{a}$, $\vec{b}$, $\vec{c}$ axes are substituted by quasispin projections along, for instance, the x', z, x axes (disregard, for the moment, any technical complication about quasispin measurement):

$$\begin{aligned} P[K_\alpha(\vec{p},t);K^0(-\vec{p},t)] \;\leq\; & P[K_\alpha(\vec{p},t);K_1(-\vec{p},t)] + P[K_1(\vec{p},t);K^0(-\vec{p},t)] + \\ & + P[K_2(\vec{p},t);K_2(-\vec{p},t)] \end{aligned} \tag{4}$$

Compared to inequality (2), inequality (4) shows an explicit dependence on time. This does not constitute a difficulty. However here, for simplicity, only the case in which both quasispins are measured at equal times is considered. The more general case of measurements at different times is treated elsewhere [7].

Inequality (4) holds for local hidden-variables theories. For some η values it is violated by quantum mechanical predictions. However in order to perform an experimental test of inequality (4) the following additional assumptions have to be made:

(i) *the decay process is a local random process, namely decay rates are time-independent quantities, e.g.* $\Gamma(K^0 \to \pi^-\ell^+\nu) = const$*;*

[2]Here CPT invariance is assumed for simplicity, even if it is not a necessary assumption

(iia) *quantum mechanical predictions for single kaon state propagation in matter and in vacuum are valid, in particular those related to the four probabilities* $P[K_{\alpha,\beta}(0) \Rightarrow K_{1,2}(\Delta t)]$ *of finding a* $K_{1,2}$ *state at time* Δt *from an initial* $K_{\alpha,\beta}$ *state at time* $t=0$;
(iib) *the four probabilities* $P[K_{\alpha,\beta}(0) \Rightarrow K_{1,2}(\Delta t); \lambda]$ *do not depend on* λ, *for any hidden variable* λ [3] ;
(iii) *the* $\Delta S = \Delta Q$ *rule holds* [4] .

The above assumptions are quite reasonable and allow us to identify quasispin states and to perform measurements of probability distributions in inequality (4). In fact, as a consequence of assumptions (i) and (iii), detection and reconstruction of one of the following neutral kaon decay channels, i.e. $K \rightarrow \pi^-\ell^+\nu$, $K \rightarrow \pi^+\ell^-\overline{\nu}$, $K \rightarrow \pi^+\pi^-, \pi^0\pi^0$ and $K \rightarrow \pi^0\pi^0\pi^0$ [5], unambiguously identifies at the decay vertex position a $|K^0\rangle$, $|\overline{K^0}\rangle$, $|K_1\rangle$ or $|K_2\rangle$ state, respectively. Identification of the $|K_\alpha\rangle$ state is slightly more complicated, and can be performed by means of a regenerator (method (a)) or, more conveniently, by time evolution in vacuum of the state itself (method (b)). Additional assumptions (iia) and (iib) are necessary for the validity of these methods [7]. Here only method (b) will be considered[6]. This method is based on the fact that, as quasispin is not conserved, it *oscillates* in time. A suitable parameter $\eta = \eta(\Delta t)$ can be chosen such that, after a certain time Δt, time evolution in vacuum rotates a $|K_\beta\rangle$ state into a pure $|K_2\rangle$ state, i.e.

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[3] This assumption is quite reasonable because, as shown in the following, the η parameter can be chosen such that:

$$
\begin{aligned}
P[K_\alpha(0) \Rightarrow K_1(\Delta t)] &= \int_\Lambda P[K_\alpha(0) \Rightarrow K_1(\Delta t); \lambda]\rho^\alpha(\lambda)d\lambda = e^{-\Gamma_S \Delta t} \\
P[K_\alpha(0) \Rightarrow K_2(\Delta t)] &= \int_\Lambda P[K_\alpha(0) \Rightarrow K_2(\Delta t); \lambda]\rho^\alpha(\lambda)d\lambda = 0 \\
P[K_\beta(0) \Rightarrow K_1(\Delta t)] &= \int_\Lambda P[K_\beta(0) \Rightarrow K_1(\Delta t); \lambda]\rho^\beta(\lambda)d\lambda = 0 \\
P[K_\beta(0) \Rightarrow K_2(\Delta t)] &= \int_\Lambda P[K_\beta(0) \Rightarrow K_2(\Delta t); \lambda]\rho^\beta(\lambda)d\lambda = e^{-\Gamma_L \Delta t}
\end{aligned}
$$

where $\rho^{\alpha,\beta}(\lambda)$ are the λ distribution functions for the case of one-kaon state, and such that $\int_\Lambda \rho^{\alpha,\beta}(\lambda)d\lambda = 1$. This means that $P[K_\alpha(0) \Rightarrow K_2(\Delta t); \lambda]$ and $P[K_\beta(0) \Rightarrow K_1(\Delta t); \lambda]$ should vanish for $\rho^{\alpha,\beta}(\lambda) > 0$. Then, $P[K_{\alpha,\beta}(0) \Rightarrow K_{1,2}(\Delta t); \lambda]$ do not depend on λ, at least in the λ domain in which $\rho^{\alpha,\beta}(\lambda) > 0$. Even if this domain could be different, in general, from that corresponding to the two-kaons state produced in ϕ decay, it is reasonable to assume that the above probabilities do not depend on λ even in the case of two-kaons state.

[4] Experimentally $r = \frac{A(\overline{K^0} \rightarrow \pi^- \ell^+ \nu)}{A(K^0 \rightarrow \pi^- \ell^+ \nu)} < 2 \cdot 10^{-2}$. Here it is necessary that $r \leq 10^{-4}$, whereas the Standard Model prediction is $r = 10^{-6} \div 10^{-7}$.

[5] Apart from a negligible direct CP violation effect, neutral kaon decays into $\pi^+\pi^-$ or $\pi^0\pi^0$ are allowed for $|K_1\rangle$ and forbidden for $|K_2\rangle$, while the opposite happens for decays into $\pi^0\pi^0\pi^0$.

[6] see Ref.[7] for method (a) using a thin regenerator

$P[K_\beta(0) \Rightarrow K_1(\Delta t)] = 0$. Then a $K_1 \to \pi\pi$ decay at time $t + \Delta t$ unambiguously identifies the presence of a $|K_\alpha\rangle$ state at time t, and the following proportionality relation (and similar ones), holds:

$$P[K_\alpha(\vec{p},t); K^0(-\vec{p},t)] = \kappa P[K_1(\vec{p},t+\Delta t); K^0(-\vec{p},t)] \tag{5}$$

where $\kappa^{-1} = P[K_\alpha(0) \Rightarrow K_1(\Delta t)] = e^{-\Gamma_S \Delta t}$, and $P[K_1(\vec{p},t+\Delta t); K^0(-\vec{p},t)]$ is a measurable probability.

In order to test inequality (4) at a ϕ-factory a two step measurement can be performed:

Step(I): first, one should verify that the following relations hold, as predicted by quantum mechanics:

$$P[K_\alpha(\vec{p},t); K_1(-\vec{p},t)] \ll\sim |\Re(\eta)| e^{-\Gamma t}/2 \tag{6}$$

$$P[K_2(\vec{p},t); K_2(-\vec{p},t)] \ll\sim |\Re(\eta)| e^{-\Gamma t}/2 \tag{7}$$

with $\Gamma = \Gamma_S + \Gamma_L$. Relation (6) can be easily experimentally verified, as shown elsewhere [7]. Verification of relation (7) is much more difficult because it corresponds to a suppressed decay rate measurement. However this verification should be still possible at a high-luminosity ϕ-factory like DAΦNE. Alternatively, one could avoid the experimental verification of relation (7) making the following additional assumption:

(iv) *the singlet state perfect anti-correlation holds,*

that makes $P[K_2(\vec{p},t); K_2(-\vec{p},t)] = 0$, even if it reduces the validity of the test to a less general class of hidden-variables theories.

Step(II): let us consider the quantity:

$$\Sigma = \frac{P[K_\alpha(\vec{p},t); K^0(-\vec{p},t)]/P[K_\alpha(\vec{p},t); \overline{K^0}(-\vec{p},t)]}{P[K_1(\vec{p},t); K^0(-\vec{p},t)]/P[K_1(\vec{p},t); \overline{K^0}(-\vec{p},t)]} \tag{8}$$

If step (I) measurements (and similar ones[7]) do not exhibit any deviation from

[7]Other relations of the kind (6,7) should be verified, in particular one involving the probability $P[K_\beta(\vec{p},t); K_\beta(-\vec{p},t)]$ not directly measurable. However it can be rewritten as:

$$\begin{aligned} P[K_\beta(\vec{p},t); K_\beta(-\vec{p},t)] = {} & P[K_\alpha(\vec{p},t); K_\alpha(-\vec{p},t)] + P_{TOT} \\ & - P[K_\alpha(\vec{p},t); K_1(-\vec{p},t)] - P[K_\alpha(\vec{p},t); K_2(-\vec{p},t)] \\ & - P[K_1(\vec{p},t); K_\alpha(-\vec{p},t)] - P[K_2(\vec{p},t); K_\alpha(-\vec{p},t)] \end{aligned}$$

with

$$\begin{aligned} P_{TOT} = {} & P[K_1(\vec{p},t); K_1(-\vec{p},t)] + P[K_1(\vec{p},t); K_2(-\vec{p},t)] \\ & + P[K_2(\vec{p},t); K_1(-\vec{p},t)] + P[K_2(\vec{p},t); K_2(-\vec{p},t)] \end{aligned}$$

where terms at r.h.s. are all measurable probabilities.

quantum mechanical predictions, it can be easily shown [7] that a family of inequality of the form (4) can be reduced to the equality:

$$\Sigma = 1 \tag{9}$$

while the corresponding quantum mechanical prediction is $\Sigma_{QM} = 1 + 4\Re(\eta)$, independent on time t, and violating (9) by the quantity $4\Re(\eta)$.

From an experimental point of view, it is convenient to measure Σ through the relation:

$$\Sigma(\Delta t) = \frac{N[\pi\pi(\vec{p}, t+\Delta t); \pi^- \ell^+ \nu(-\vec{p}, t)]/N[\pi\pi(\vec{p}, t+\Delta t); \pi^+ \ell^- \overline{\nu}(-\vec{p}, t)]}{N[\pi\pi(\vec{p}, t); \pi^- \ell^+ \nu(-\vec{p}, t)]/N[\pi\pi(\vec{p}, t); \pi^+ \ell^- \overline{\nu}(-\vec{p}, t)]} \tag{10}$$

where $N[f_1(\vec{p}, t'); f_2(-\vec{p}, t'')]$ is the measured decay rate into f_1 and f_2 channels at times t' and t'', respectively.

At DAΦNE, assuming an integrated luminosity $L \sim 10^4\ pb^{-1}$, a statistical error $\Delta\Sigma/\Sigma$ smaller than 0.1 % could be obtained, whereas the quantity $|4\Re(\eta)|$ is of the order of 1% in the case of $\Delta t = 1\ \tau_S$, with τ_S the K_S lifetime. Hence the statistical precision achievable at DAΦNE seems adequate to reveal Bell's inequality violations. However the systematics should be carefully investigated. A general purpose detector like KLOE might successfully perform such a measurement.

I would like to express my gratitude to Profs. M.Cini, P.H. Eberhard and G.C.Ghirardi for fruitful suggestions and very stimulating and clarifying discussions on the subject. I am very grateful to Prof. G.Diambrini-Palazzi for his interest and help. I am taking this opportunity to thank the conference committee for inviting me, in particular Prof. A.Studenikin, and all the organizers for their kind support.

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EXPERIMENTAL PROBLEMS FOR TESTING MACROSCOPIC QUANTUM COHERENCE WITH SQUIDs

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Abstract

The study of phenomena related to the quantum behavior of macroscopic systems is subjected to new efforts both from theoretical and experimental point of view. The validity, in fact, of the description of macroscopic systems given by Quantum Mechanics (QM) is still under test, especially for what concerns the macrorealistic interpretation of the real world.

In 1980 A.J.Leggett[1] proposed a test of QM at macroscopic level made by a system of SQUIDs. Some experimentalists have tried, in the last ten years, to perform such experiment[2,3], but they failed in realizing the proper set-up for the experiment.

We think however that the technology for fabricating SQUIDs and the knowledge of their behavior are now so advanced that is worthwhile try to perform the experiment. The MQC group of Rome started then in 1994 a program to perform a test of the validity of Q.M. description on a macroscopic scale following a modified version of the original one proposed by Leggett.[4].

In this paper we will present all the experimental requirements necessary to realize the real experiment. The system analized is a set of (rf-SQUID/Switch SQUID/Amplifier) described in [4]. An introduction to the theoretical bases on the experimental method is presented by G.Diambrini Palazzi in this Conference. See also ref. 4b.

A. On the applied external flux.

To theoretical obtain a double well potential we must apply an external flux equal to $\Phi_0/2$. This from a point of view; but how much the real flux that will be applied can be different from the above value? It happens in fact that if the external flux is greater or smaller than $\Phi_0/2$ the potential $U(\Phi)$ will have one of the two relative minima lower that the other, resulting in a state where the flux will remain indefinitely in the lower pit. The minimum imbalance is the one that result less than the height of the first level of oscillation in the pit, namely:

$$|U(\Phi^+) - U(\Phi^-)| \leq \hbar\omega_p/2 \tag{1}$$

If we compute the maximum difference of the external flux from $\Phi_0/2$ satisfying the above condition we obtain for a typical frequency in the bottom of the well of about $1\ GH_z$:

$$If \quad \Phi_{ext} = \frac{\Phi_0}{2} + \delta\Phi \qquad then \quad \delta\Phi \leq 1 \times 10^{-8}\Phi_0 \tag{2}$$

This stability is indeed a not easy task. To fulfill this requirement the system must be well isolated from the external fluctuations of the static (earth) magnetic field as well as from the rf interference. This can be accomplished by using a set of mu-metal shields together with rf filters on all the cables entering into the experimental area. A good mechanical isolation must be obtained also from the external vibration. The experimental dewar must then be put on an isolating platform with a sufficient attenuation in the audio frequency region.

B. On the temperature, I.

To have just the tunnel effect driving the motion of the flux between the two equilibrium points, we must avoid thermal transition over the well. So the energy associated to the mean thermal excitation must be lower than the well height, i.e.:

$$k_B T \ll \Delta U \quad \Rightarrow \quad T \ll 10\ K \tag{3}$$

C. On the temperature, II.

We must have the Josephson junction working in the quantum regime, this means that the energy associated to the thermal excitation must be lower than the first oscillation level:

$$k_BT \ll \hbar\omega_p \qquad \Rightarrow \qquad T \ll 1\ K \tag{4}$$

D. On the temperature, III.

The must severe requirement on the temperature is that imposed by the effect of the dissipations on the system. Any dissipation in fact will cause a damping that will bring to the loss of the coherence that is supposed to be observed by our experiment. In the limit of $T = 0K$ and no dissipation the system will have no dumping at all, and the coherent oscillation will be just an indefinite oscillation. As long as the temperature and dissipations will be different from zero , a damping of the oscillation occurs that will damp or wash out completely the effect to be observed. Garg[5] calculated the limit on the temperature necessary to have a coherent oscillation of the system.

For very low damping we can use an approximate expression for this temperature limit:

$$T < T^* \cong \frac{8\hbar^2}{k_B\phi_0^2}\omega_\tau R^* \tag{5}$$

where ω_τ is the tunnelling frequency and R^* the equivalent resistance associated to the overall damping.

For typical values of the experimental parameters we have then:

$$T^*(2\pi \cdot 1MH_z \ \ ; \ \ 1M\Omega) = 10\ mK \tag{6}$$

This requirement is of course the most difficult to realize. The experiment is in fact planned to operate with an $^3He -\ ^4He$ refrigerator that is supposed to cool the system down to a thermodynamic temperature of 5 mK.If this low temperature will not be enought, an adiabatic demagnetization stage will be connected to the diluition refrigerator to reach a temperature of $0.1 - 0.5\ mK$.

E. On the back action from the analyzer.

The signal coming from the rf SQUID must be read by an instrument the takes the same role of the analyzer in an experiment made by photons where the experimental chain is composed by the system (source/analyzer/detector). Now for a photon experiment no problem arise due to the analyzer, since in a coincidence experiment there can be no effect on the source due the measurement apparatus. In our experiment however, any back action from the analyzer on the rf SQUID can destroy the coherence by making an "observation" of the status of the SQUID, and making an "invasive" (in the classical sense) measurement. To avoid this problem the solution can be to use (as analyzer) an hysteretic dc SQUID working as a switch[6].

Briefly, an hysteretic dc SQUID is a device that can have only two states, superconducting or normal, depending on the direction of the flux concatenated with the SQUID. With a proper set up of the device we can set the dc SQUID such to switch from the superconducting to the normal state only if the flux in the rf SQUID is (for instance) in the right well; the dc SQUID will remain in the superconducting state if the flux in the rf SQUID is in the left well. If we "turn on" the dc SQUID for a very small time (of the order of 10ns) and if we see no transition of the SQUID from the superconducting to the normal state, we can infer the direction of the flux in the dc SQUID and hence the position of the flux (left or right) in the rf SQUID . This measurement will be a non-invasive measurement since the dc SQUID will undergo no transition and hence no back action will flow from the dc to the rf SQUID. The measurement where the dc SQUID will have a transition will be an invasive measurement, and cannot be used for a "series" of measurements, but only to detect the state of the SQUID at the time of the transition.

F. Limit on the analyzer efficiency.

The dc SQUID switch will have of course an efficiency lower than one. We can ask then what is the lower limit on this efficiency. The limit will depend on the type of measure that must be done. Suppose that we are analysing the probability $P\pm(t)$ of measuring the versus ($\pm$) of the circulating current in the SQUID. If the experimental procedure is to measure $P(T/4)$ after a measure done at $t = T/8$ (T being the tunnelling period), a measure i.e. aiming to detect if the macroscopic system is described by a superposition of states or by a statistical mixture, the requirement is such to detect a difference between $P(T/4) = 0$ and $P(T/4) = 1/4$. The efficiency must be in this case of the order or better than 80%.

If we want to make a test on the Bell inequalities, the limit on the analyzer efficiency is much more stiff; it can be demonstrated in fact that must be larger than 95% [7].

G. Time resolution on the start time.

Every measurement of the SQUID system is supposed to start with the SQUID prepared in one of the two possible states (circulating current in the SQUID clockwise or counterclokwise). Of course we cannot realize this by "measuring" the state, since we cannot know the possible influence of the measure on the subsequent time evolution. What can be done is to "prepare" the SQUID by tilting the double well potential in such a way to force the "flux" in one of the two wells (the left well for instance). If we restore then the symmetric double potential, the flux will continue the time evolution by starting from the left well. Of course the problem is to realize this "preparation" in a time fast enough respected to the time period of the tunnelling frequency ($1MHz$). This task can be accomplished by using a laser driven superconducting switch. This object consist essentially in a closed superconducting circuit inductively coupled to the rf SQUID. In normal conditions, i.e. when no current is stored in the circuit, the SQUID potential is the standard symmetric double well. When we store a persistent current in to the circuit we apply an external flux to the SQUID, so we cause an imbalance on the SQUID potential. If then a very short laser pulse is applied to a short region of the superconducting circuit, this becomes normal, and the persistent current will die out in a time of the order of $L_{circuit}/R_{normal}$. With this apparatus it has been demonstrated[8] that one can have a transition as short as few nanoseconds, a time short enough for the MQC experiment.

H. Quality of Josephson Junctions.

As we showed in the previous paragraphs great attention must be paid on realizing a very low dissipation system. Once every source of external dissipation has been removed, it remains just the intrinsic dissipation of the Josephson junction. So it is very important to realize a junction with the lowest possible dissipation. One of the must important parameters that characterize the junction construction is the current density; this parameter must therefore be optimized if the optimum junction is desired for the experiment. It has been shown[9] that the best junction with respect to the current density are those realized with a current density as small as possible; a current density lower than few hundred of Ampere/cm^2 is in practice sufficient to maintain the BCS behavior of the junction at temperatures as low as 0.35 K. At lower temperatures it may be possible that lower current densities must be used. However current densities of the order of 1-10 A/cm^2 are currently realized, so this should not be a major problem.

I. Shielding.

A very good shielding from mechanical and e.m. interference must be obtainded. The isolation from mechanical noise should guarantee that no vibration can modulate the magnetic flux trapped into the system (that must be stale at one part in 10^8). The experimental apparatus will be placed on a vibration isolation platform having horizontal and vertical resonances around $1Hz$. This should guarantee many hundred of dB of attenuation in the kHz region of frequency where the external noises are expected to be relevant. A proper design of the SQUID holder should then be realized to avoid relative movement of the holder with respect to the squid inductance.

For what concerns the e.m. interference, a part from the standard superconducting shields at low temperatures, that should guarantee the stability of the trapped field, the shielding from the external noise will be realized with standard multiple shield system made be Aluminum and mu-metal shields, for respectively high and low frequencies shielding.

An additional system of Helmoltz coil arranged in a cube of $1m$ of side has been realized to create o region of low dc magnetic field to reduce the trapped field when the system pass from the normal to the superconducting state. With such a system a field as low as few milligauss has been realized in a volume of few liters around the experimental region, lowering the earth mean field of a two orders of magnitude.

Conclusions.

In this paper we have shown the most important experimental problems that should be solved to realize a Macroscopic Quantum experiment with a system of SQUIDs. We have shown moreover how the MQC group of Rome University wants to approach these problems. We are confident that all the experimental requirements can be solved by using the present technologies and the solutions presented in this paper. We hope therefore that in a few years the experiment will be in operation, and that we will be able to give an answer to the ability of Q.M. to explain the behavior of quantum macroscopic objects.

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Time Dependent Supersymmetry in Quantum Mechanics

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1. The supersymmetric quantum mechanics is in a permanent intensive development since the Witten papers [1]. One can cite the N-extended supersymmetric quantum mechanics [2], parasupersymmetric quantum mechanics [3], and high order derivative supersymmetric quantum mechanics [4]. The field of supersymmetric quantum mechanics is recently reviewed in [5]. We want to point out that all above mentioned constructions are valid for the time independent Hamiltonians and if one restrict oneself by the stationary solutions of the Schrödinger equation. Hence, these constructions can be referred to the stationary supersymmetric quantum mechanics and the nonstationary one needs to be developed. We hope that this report gives a stimulus for the further developments in this domain.

2. The nonstationary supersymmetric quantum mechanics is based on the nonstationary Darboux transformation [6] in just the same way as the stationary one [1, 7] is based on the conventional Darboux transformation [8].

Let us consider two time-dependent Schrödinger equations

$$(i\partial_t - H_0)\psi(x,t) = 0, \quad \partial_t = \partial/\partial t, \quad H_0 = -\partial_x^2 - V_0(x,t), \quad \partial_x^2 = \partial_x\partial_x, \quad (1)$$

$$(i\partial_t - H_1)\varphi(x,t) = 0, \qquad H_1 = -\partial_x^2 - V_1(x,t), \qquad x \in R, \quad t \in \mathbf{R}^1. \quad (2)$$

Here $-V_0(x,t)$ is a potential energy and $R = [a,b]$ is the interval for x variable which can be both finite and infinite. If the Schrödinger operators for Eqs. (1) and (2) are connected by intertwining relation

$$L(i\partial_t - H_0) = (i\partial_t - H_1)L, \quad (3)$$

where L is a linear operator, named transformation operator, the functions ψ and φ are related as follows: $\varphi = L\psi$ if $L\psi \neq 0$.

If $(i\partial_t - H_0)$ and $(i\partial_t - H_1)$ are self-adjoint (in the sense of some scalar product) the equation (3) implies

$$L^+(i\partial_t - H_1) = (i\partial_t - H_0)L^+, \tag{4}$$

where the superscript plus sign $(^+)$ is used to denote the operator adjoint to L, and Eqs. (1) and (2) become "peer". It follows from Eqs. (3) and (4) that $s_0 = L^+L$ commutes with $(i\partial_t - H_0)$ and $s_1 = LL^+$ commutes with $(i\partial_t - H_1)$ and consequently s_0 is a symmetry operator for the initial equation (1) and s_1 is a symmetry operator for the final one (2).

The constructions such as in Eq. (3) are well-known in mathematics and are intensively investigated since the Delsart's paper [9]. The most significant results obtained with the help of the transformation operators concern the backscattering problem in quantum mechanics [10] and its application for the solving of the nonlinear equations [11].

3. We now assume that L is a differential of the first degree in ∂_x operator with smooth coefficients depending on both variables x and t. We should not include in L the derivative ∂_t since it, being found from equation (1), transforms L into the second-order differential operator. In this case the operator L and the real potential difference $A(x,t) = V_1(x,t) - V_0(x,t)$ are completely defined by a function $u(x,t)$ named transformation function [6]:

$$L = L_1(-u_x/u + \partial_x), \tag{5}$$

$$L_1 = L_1(t) = \exp\left(2\int dt\ \mathrm{Im}(\log u)_{xx}\right), \tag{6}$$

$$A = (\log|u|^2)_{xx}. \tag{7}$$

The transformation function u must be subjected to the new potential reality condition [6]

$$(\log u/u^*)_{xxx} = 0, \tag{8}$$

where the asterisk implies the complex conjugation.

In the majority of cases of physical interest we can introduce the Hilbert space structure $L_0^2(R)$ in the space of the solutions of the equation (1) with the scalar product appropriately defined. Symmetry operator $s_0 = L^+L$, being self-adjoint, can have either discrete spectrum in $L_0^2(R)$ or continuous one and since $Lu = 0$ (see Eq. (5)), the function u is its proper function corresponding to zero proper value. It follows that u is the one of the proper functions of operator $L^+L = h - \alpha$. In general case h is a self-adjoint integral of motion for the initial Schrödinger equation (1) which in particular case (if V_0 does not depend on t) can be the Hamiltonian H_0 and the function u is its proper function corresponding to the proper value α.

With the help of the transformation operator L, in just the same way that in the conventional supersymmetric quantum mechanics [1, 7], we can construct the supercharge operators

$$Q = \begin{pmatrix} 0 & 0 \\ L & 0 \end{pmatrix} = (Q^+)^\dagger \tag{9}$$

acting on the two-component wavefunctions $\Psi(x,t) = \begin{pmatrix} \psi(x,t) \\ L\psi(x,t) \end{pmatrix}$, and assemble two Schrödinger equations (1) and (2) into one two-component equation

$$(iI\partial_t - \mathcal{H})\Psi(x,t) = 0, \tag{10}$$

where I is 2×2 identity matrix and $\mathcal{H} = \begin{pmatrix} H_0 & 0 \\ 0 & H_1 \end{pmatrix}$ is a superhamiltonian. Since $s_0 = L^+L$ and $s_1 = LL^+$ are the symmetry operators for equations (1) and (2) respectively, the superoperator $S = \begin{pmatrix} L^+L & 0 \\ 0 & LL^+ \end{pmatrix}$ is the symmetry operator for the equation (10). The operators Q, Q^+, and S form a well-known superalgebra [1, 7] with the single difference that instead of the Hamiltonian we can use any other integral of motion of the equation (1). When h coincides with the initial Hamiltonian, the correspondence becomes exact. This is the reason to name the transformation L (5), (6), *time-independent Darboux transformation* [6].

4. With the help of the other proper functions of operator h we can perform the chain of Darboux transformations and construct the parasuperalgebra in full analogy with papers [3]. If in this chain we eliminate the intermediate operators h and express the final N-degree in ∂_x operator L in terms of the particular solutions u_i of the initial equation (1), we obtain higher-derivative nonstationary quantum mechanics analogous to the stationary one [4]. In this case

$$L \equiv L^{(N)} = L_N(t)w^{-1}(u_1,\ldots,u_N)\begin{vmatrix} u_1 & u_2 & \ldots & 1 \\ u_{1x} & u_{2x} & \ldots & \partial_x \\ \vdots & & & \\ u_{1x}^{(N)} & u_{2x}^{(N)} & \ldots & \partial_x^N \end{vmatrix} \tag{11}$$

where w denotes the conventional symbol for Wronskian of the functions $u_1,\ldots,u_N$, and for the real function $L_N(t)$ we have

$$L_N(t) = \exp\left\{2\int dt\ \mathrm{Im}[\log w(u_1,\ldots,u_N)]_{xx}\right\}. \tag{12}$$

The new potential reality condition (8) transforms into the following relation

$$\left[\log \frac{w(u_1,\ldots,u_N)}{w^*(u_1,\ldots,u_N)}\right]_{xxx} = 0. \tag{13}$$

For the potential difference function we have

$$A_N = \left(\log |w(u_1,\ldots,u_N)|^2\right)_{xx}. \tag{14}$$

We can recognize in formula (11) the generalization of the known Crum-Krein formula [12, 13] obtained for stationary case. Note that the condition (13) is more feeble than the reality condition (8) imposed on every function u_i. Thus, we can construct the higher-derivative supersymmetry with the self-adjoint final Hamiltonian even if the intermediate Hamiltonians are not self-adjoint (so-named irreducible case described for stationary case in Ref. 4). The basic relation for the time-dependent polynomial supersymmetric quantum mechanics is the following factorization property

$$L^+L = \prod_{i=1}^{N}(h_0 - C_i), \qquad LL^+ = \prod_{i=1}^{N}(h_1 - C_i) \tag{15}$$

first obtained for the stationary case in Ref. 4. The C_i entering in Eqs. (15) are the proper values corresponding to the functions u_i for the integral of motion h_0 of the initial Schrödinger equation (1).

5. To obtain the regular potential difference by the single Darboux transformation (5)–(7) the transformation function u should be nodeless. In the space $L_0^2(R)$ a single nodeless proper function of the operator h exists (if operator h has the discrete spectrum). This function is the ground state function of operator h. Beyond the space $L_0^2(R)$ there are many nodeless proper functions of the operator h suitable for the construction of the transformation operator L. They should have the proper value $\alpha < \varepsilon_0$ (ε_0 being the lowest eigenvalue of h corresponding to the bounded states). In this case the discrete spectra of the symmetry operators $h = L^+L + \alpha$ and $\tilde{h} = LL^+ + \alpha$ differ by one level and we have a broken supersymmetry [1]. Every bounded state of the superoperator S, except for its ground state, is double degenerate. We now will describe the unexpected peculiarities in the breakdown of the supersymmetry in the higher-derivative supersymmetric quantum mechanics. These peculiarities (as far as we know) are not discussed in the available literature.

The single Darboux transformation being performed with the discrete spectrum function $u_n(x,t)$ of the integral of motion h having n zeros in interval (a,b) gives a potential difference with n poles and the solutions obtained with the help of transformation operator (5) does not belong to the space $L_1^2(R)$ of functions square integrable in $[a, b]$. Nevertheless, the second transformation with the transformation function $u_{n+1}(x,t)$, having $n+1$ zeros in (a,b) removes all singularities and the transformation operator of the second degree $L^{(2)} = L_2 L_1$, where $L_{1,2}$ are the single Darboux transformation operators, is well defined. This fact reflects the known property of the Wronskian constructed from the functions u_{k_i} belonging to $L^2(R)$ space with k_i zeros [13]: the Wronskian $w(u_{k_1}, \ldots, u_{k_N})$ conserves its sign if for all $k = 0, 1, 2, \ldots$ the unequality $(k-k_1)(k-k_2)\cdots(k-k_N) \geqslant 0$ holds. In particular, the functions u_{k_i} may be two by two juxtaposed functions. The discrete eigenvalues α_{k_i} of the operator h corresponding to the transformation functions u_{k_i} are absent in the spectrum of its superpartner $\tilde{h}$. This signifies that the ground state level of the superoperator S is double degenerate and the excited states constructed with the help of the functions u_{k_i} are nondegenerate. Furthermore, these states are annihilated by the operators Q and Q^+ in contrast to the ground states annihilated only by the one of these operators. It should be noted that this property remains valid for the stationary states, i.e., in the ordinary supersymmetric quantum mechanics.

6. The differential symmetry operators for the stationary Schrödinger equation are Hamiltonian and its polynomial functions. The algebra of symmetry operators for the nonstationary Schrödinger equation is more rich then the stationary one. We can use the whole Lie algebra of the differential symmetry operators of the initial Schrödinger equation to construct the supersymmetric algebra. For this purpose we should define the operator inverse to L.

The equation (5) implies that $Lu = 0$. Choose the transformation function u such that the absolute value of $u^{-1}(x,t)$ be square-integrable in the interval R and the condition (8) be valid. Then for every $\psi \in L_0^2(R)$ we have $\varphi = L\psi \in L_1^2(R)$, but the set $L_{11}^2(R) = \{\varphi : \varphi = L\psi,\ \psi \in L_0^2(R)\}$ does not span the whole space $L_1^2(R)$. The function $\varphi_0(x,t) = [L_1(t)u^*(x,t)]^{-1} \in L_1^2(R)$ [6] can not be obtain by the action of the operator L on any $\psi \in L_0^2(R)$. If we designate by $L_{10}^2(R)$ the linear hull of the function φ_0 then $L_1^2(R) = L_{10}^2(R) \oplus L_{11}^2(R)$.

Choose as the transformation function the function $v = \varphi_0$ for defining the following integral operator acting from $L_{11}^2(R)$ to $L_0^2(R)$

$$M\varphi(x,t) = [L_1(t)v^*(x,t)]^{-1} \int_0^x v^*(y,t)\varphi(y,t)dy. \qquad (16)$$

The straightforward calculations persuade that $LM\varphi = \varphi$ for all $\varphi \in L_{11}^2(R)$ and the condition $v \in L_{10}^2(R)$ implies $ML\psi = \psi$ for all $\psi \in L_0^2(R)$ operator M, hence, is inverse to $L : M = L^{-1}$, and we have one-to-one correspondence between the spaces $L_0^2(R)$ and $L_{11}^2(R)$.

If in the space $L_0^2(R)$ the symmetry operators g_i forming n-dimensional Lie algebra G with the structural constants $f_{ij}^l : [g_i, g_j] = f_{ij}^l g_l$ is defined and this space is invariant under the action of these operators then in the space $L_{11}^2(R)$

we can define the operators $\bar{g}_i = Lg_iM$ and this space will be invariant under the action of all $\bar{g}_i$. Furthermore these operators form a basis for the n-dimensional Lie algebra $\bar{G}$ with the same structural constants f_{ij}^l isomorphic to G. Designate by T_0 the space of two-component wave functions $\Psi(x,t)$ with the basis $\Psi_+(x,t) = \psi(x,t)e_+$ and $\Psi_-(x,t) = L\psi(x,t)e_-$, $\psi \in L_0^2(R)$, and $e_+ = \binom{1}{0}$, $e_- = \binom{0}{1}$. It follows that in the space T_0 we can define the operators

$$G_i = \begin{pmatrix} g_i & 0 \\ 0 & \bar{g}_i \end{pmatrix} \tag{17}$$

forming Lie algebra isomorphic to G and these operators are the symmetry operators for the supersymmetric equation (10). Besides the operators G_i in the space T_0 the following operators can be defined: $P_0 = L\sigma^-$, $Q_i = g_iL^{-1}\sigma^+$ where $\sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. These operators are evidently nilpotent: $P_0^2 = 0$, $G_i^2 = 0$, and $\{Q_i, Q_j\} \equiv Q_iQ_j + Q_jQ_i = 0$. Furthermore, we can find by the direct calculations that $\{P_0, Q_i\} = G_i$ and the generalized Jacobi identities are fulfilled. The operators G_i, Q_i, P_0, $i = 1, 2, \ldots, n$, hence, form a basis for $2n+1$-dimensional Lie superalgebra sG.

We note that since $h = L^+L + \alpha \in G$ we have for operator S introduced in sec. 3, $S \in sG$ and Q, $Q^+ \in sG$.

7. **Examples.** Consider first the simplest case of a free particle: $v_0(x,t) = 0$. Choose the following solutions of the initial Schrödinger equation (1) [14]:

$$\psi_\lambda(x,t) = (1+t^2)^{-1/4}\exp[ix^2t/(4+4t^2) + i\lambda \arctan t]Q_\lambda(z), \tag{18}$$
$$z = x(1+t^2)^{-1/2},$$

where $Q_\lambda(z)$ is the parabolic cylinder functions satisfying the equation $Q_\lambda''(z) - (z^2/4 + \lambda)Q_\lambda(z) = 0$ with λ being an arbitrary parameter (the separation constant). For $\lambda = n + 1/2$, $n \in N^1$, the functions $Q_\lambda(z)$ are expressed via the Hermite polynomials $Q_{n+1/2}(z) = \exp(z^2/4)H_n(iz/\sqrt{2})$. The reality condition (8) is satisfied for all real λ. Functions (18) being nodeless for $\lambda = n + 1/2$ and for even n are suitable for use as transformation functions. Formula (7) gives the new Schrödinger equation (2) potential

$$v_1^{(2k)} = (1+t^2)^{-1}\Big(1 + 4k(2k-1)\frac{q_{2k-2}(z)}{q_{2k}(z)} - 8k^2\Big(\frac{q_{2k-1}(z)}{q_{2k}(z)}\Big)^2\Big),$$

where $q_k(z) = (-i)^k He_k(iz)$, $He_k(z) = 2^{-k/2}H_k(z/\sqrt{2})$.

The same functions for odd n are nodeless in half-interval $(0,\infty)$ and with their help we obtain the following time-dependent exactly solvable potential

$$v_1^{(2k+1)} = (1+t^2)^{-1}\Big(1 + 4k(2k+1)\frac{q_{2k-1}(z)}{q_{2k+1}(z)} - 2(2k+1)^2\Big(\frac{q_{2k}(z)}{q_{2k+1}(z)}\Big)^2\Big),$$

$$x = z\sqrt{1+t^2} \in (0,\infty).$$

The functions (18) for $\lambda = -n - 1/2$ form a discrete basis set in $L_0^2(\mathbf{R}^1)$. The double Darboux transformation with juxtaposed functions $u_n = \psi_{-n-1/2}$ and u_{n+1} gives a regular potential of the form

$$v_2^{(n,n+1)} = 2(1+t^2)[J_n''(z)/J_n(z) - (J_n'(z)/J_n(z))^2 - 1],$$
$$J_k(z) = \Gamma(k+1)\sum \Gamma^{-1}(s+1)He_s^2(z) = kJ_{k-1}(z) + He_k^2(z),$$
$$J_0(z) = 1, \qquad J_1(z) = z^2+1, \qquad J_2(z) = z^4+3, \dots .$$

Just these potentials correspond to the supersymmetric models with double degenerate eigenvalue of the superoperator S except two ones constructed with the help of the functions u_n and u_{n+1}. For $n > 0$ the ground state of S is double degenerate.

We can establish that the Wronskian constructed from two functions (18) with $\lambda_1 = m + 1/2$ and $\lambda_2 = l + 1/2$ for $m = 0, 2, 4, \dots$ and $l = m+1, m+3, \dots$ is nodeless and consequently these functions are suitable for double Darboux transformation. This gives the following exactly solvable potential

$$v_2^{(m,l)} = 2(1+t^2)^{-1}(1 + d^2 \log f_{ml}(z)/dz^2),$$
$$f_{ml}(z) = q_m(z)q_{l+1}(z) - q_l(z)q_{m+1}(z).$$

We will cite as well one example for harmonic oscillator potential: $H_0 = -\partial_x^2 + \omega^2x^2$, $H_0\psi_n = (2n+1)\Psi_n$, $\psi_n = H_n(\sqrt{\omega}x)\exp(-i\omega(2n+1)t - \omega x^2/2)$, $n \in \mathbf{N}^1$. If we choose the following nonstationary solution of the initial Schrödinger equation (1) as a transformation function:

$$u(x,t) = \sin^{-1/2}(2\omega t)\cos h(\lambda x/\sin 2\omega t)\exp[i(\omega x^2 - \lambda^2/\omega)\cot(2\omega t)/2] \notin L_0^2(\mathbf{R}^1),$$
$$\lambda \in \mathbf{R}^1,$$

we obtain the nonstationary anharmonic potential of the form: $v_1(x,t) = \omega^2x^2 - 2\lambda^2\sin^{-1/2}(2\omega t)\text{sech}^2(\lambda x/\sin 2\omega t))$.

First author was supported by the Russian Foundation for Fundamental Research.

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CONSTRAINTS ON NEUTRINO MAGNETIC MOMENT AND STRENGTH OF MAGNETIC FIELD FROM NEUTRINO SPIN-FLAVOUR OSCILLATIONS IN SUPERNOVA EXPLOSION AND NEUTRON STAR

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Abstract

Effects of neutrino conversion and oscillations induced by strong magnetic fields that can exist in supernova explosion and neutron star are discussed. We examine possibilities to get constraints on magnetic moment of neutrino and on strenght of magnetic field from the demand that the loss of active neutrinos νeL due to the magnetic field induced oscillations of the type $\nu_{e_L} \leftrightarrow \nu_{e_R}$ is negligible. The constraint on neutrino magnetic moment on the level of $\mu \leq 10^{-11}\mu_B$ can be obtained from analysis of energy balance of a supernova explosion. More stringent constraint $\mu_\nu \leq 10^{-15}\mu_B$ is received from consideration of neutrino conversion in a neutron star and the limit on the neutron star magnetic field, $B \leq 5 \times 10^{12}\ G$, on the scale $R = 1\ km$ is also obtained.

In our previous studies [1, 2, 3, 4, 5, 6, 7] we discussed neutrino conversion and oscillations among the two neutrino species induced by strong magnetic field. Implications of these phenomena to the case of neutrinos in the Sun, interstellar galactic media, neutron stars and supernova were examined.

Initially these investigations have been stimulated, above all, by the desire for a solution to the solar neutrino puzzle on the base of matter and magnetic field enhancement of spin and flavour neutrino conversion (see, for example, [8, 9, 10, 11, 12, 13]).

Another important motivation of these studies have been provided by the common belief that neutrino conversion and oscillations may play a significant role in supernova bursts and cooling of neutron stars (see, for example,[14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]).

In [1, 2, 3, 4, 5, 6, 7] we determined the value of the the critical strength of magnetic field B_{cr} as a function of characteristics of neutrinos in vacuum (Δm_ν^2, θ), neutrino magnetic (transition) moment $\tilde{\mu}$ and energy E, effective particle density of matter n_{eff} that determins the range of fields ($B \geq B_{cr}$) for which the magnetic field induced neutrino conversion and oscillations become significant. These neutrino conversion and oscillations could result in loss of a significant amount of active ν_{e_L} neutrinos during a supernova explosion and inside or near a neutron star.

As it was pointed out in our previous studies (see, for example, [2, 3, 4, 6]), effects of the magnetic field induced neutrino conversion become important if the following two conditions are satisfied:

1) the magnetic field exeeds the critical value B_{cr} (see eq.(3) bellow)

$$B \geq B_{cr}, \tag{1.1}$$

and

2) the length x of the neutrinos path in the medium must be greater than the effective oscillation length L_{eff} (see eq.(5) bellow)

$$x \geq \frac{L_{eff}}{2}. \tag{1.2}$$

The effect of suppression of amount of active ν_{e_L} electron neutrinos (due to the magnetic field induced oscillations of the type $\nu_{e_L} \leftrightarrow \nu_{e_R}$) was used [2, 3, 4, 6] to constrain the value of μB in the frame of the proposed model [16] of about 60 % increase in the supernova explosion energy. Supposing that the magnetic field induced neutrino oscillations do not destroy the proposed increase of the explosion energy we got an upper limit on the value of the magnetic moment of neutrino from the following arguments. If the magnetic field $B \sim B_0 = 10^{14}\ G$ exists at the radius of $r_0 = 45\ km$ from the centre of the hot proto neutron star (the matter density in this region is $\rho \sim 10^{12}\ g/cm^3$) and decreases with distance from the centre according to the law

$$B(r) = B_0\Big(\frac{r_0}{r}\Big)^3, \tag{2}$$

then on the distances $r \sim 160\ km$ from the centre the magnetic field is $\sim 0.6 \times 10^{13}\ G$. This field is of the order of the B_{cr} determined by

$$B_{cr} = \left|\frac{1}{2\mu}\Big(\frac{\Delta m_\nu^2 A}{2E} - \sqrt{2}G_F n_{eff}\Big)\right| \tag{3}$$

(where $A = \frac{1}{2}(\cos 2\theta - 1), \Delta m_\nu^2 = 10^{-4} eV^2$ and $E = 10\ MeV$) for the density $\rho \sim 6 \times 10^8\ g/cm^3$ and the magnetic moment $\mu \sim 10^{-10}\mu_B$. For this case the probability of finding sterile ν_{eR}'s among the initially emitted ν_{eL}'s

$$\bar{P}_{\nu_{eL}\to\nu_{eR}} = \frac{1}{2}\sin^2 2\theta_{eff} = \frac{(2\mu B)^2}{(\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{eff})^2 + (2\mu B)^2}, \tag{4}$$

is equal to $\bar{P}_{\nu_{eL}\to\nu_{eR}} = 0.25$. The effective length

$$L_{eff} = 2\pi\Big[\Big(\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{eff}\Big)^2 + (2\tilde{\mu}B)^2\Big]^{-1/2} \tag{5}$$

for this effect is $L_{eff} \sim 10\ cm$. Consequently, to avoid the loss of a substantial (25 %) amount of energy that could escape from the region behind the shock together with the sterile neutrinos ν_{eR}, one has to constrain the magnetic moment on the level of $\mu \leq 10^{-11}\mu_B$.

There is another possibility to get constraints on the value of neutrino magnetic moment and also on the strenght of magnetic field of a neutron star based on the assumption that the effects of the convertion of the left neutrinos ν_{eL} into right neutrinos ν_{eR} induced by magnetic field are negligible.

Let us assume that the magnitude of the magnetic moment of electron neutrino is on the level of $\mu = 10^{-18}\mu_B$. This value is in the range of the one-loop contribution

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} = 3 \times 10^{-19}\mu_B\Big(\frac{m_\nu}{1\ eV}\Big) \tag{6}$$

to the neutrino magnetic moment induced in easy models [25] for neutrino magnetic moment based on the standard gauge group $SU(2)_L \times U(1)_Y$ with a singlet right-handed neutrino and the neutrino mass about $m_\nu \sim 10\ eV$. Suppose that very strong magnetic field B exist in a region of a neutron star with characteristic scale of about $R \sim 1\ km$ and condition $B \geq B_{cr}$ is satisfied inside this region. Then in order to avoid the loss of significant amount of ν_{eL} due to the magnetic field induced conversion $\nu_{eL} \to \nu_{eR}$ we have to demand that the second condition (eq.(1.2)) is unsatisfied, i.e., the effective oscillation lenght $L_{eff} = \pi/\mu_\nu B$ has to exeed at least two times R, $L_{eff} > 2R$. It follows that the magnetic field is constraint to the value $B < 5 \times 10^{15}\ G$.

So we can conclude that if there is no important effects of magnetic field induced conversion $\nu_{eL} \to \nu_{eR}$ for the values of the neutrino magnetic moment $\mu_\nu \sim 10^{-18}\mu_B$ the magnetic field of the neutron star on the scale of about $R \sim 1\ km$ have to be smaller than $B = 5 \times 10^{15}\ G$. It must be pointed out that this constraint is derived with out direct use of information about the densty of matter of neutron star in the region of the scale $R = 1\ km$. However, this information is used inderectly in the condition $B \geq B_{cr}$ because the value of B_{cr} is determined by the density of matter.

To get constraint on the neutrino magnetic moment we include into consideration observational date on the value of magnetic field of neutron stars. If we use estimation of the neutron star magnetic field on the scale $R = 1\ km$, $B \leq 5 \times 10^{12}\ G$, and also demand that the effects of the convertion of the left neutrinos ν_{eL} into right neutrinos ν_{eR} induced by magnetic field are negligible then we have to limit the neutrino magnetic moment on the level $\mu_\nu \leq 10^{-15}\mu_B$.

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Radiative decay of a massive neutrino in the Weinberg-Salam model with mixing in a constant uniform magnetic field

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Abstract

Influence of an external magnetic field on the decay process with emission of a linearly polarised photon is considered. It is shown that there is a region of the dynamic parameter of the process where the contribution of the non-zero neutrino mass is essential, and the probability of the process is still higher than in the zero field case. In the opposite limit of negligible neutrino mass contribution the emitted photons are totally linearly polarised, which may help in identification of these photons.

In the WSG model with mixing a massive neutrino that takes part in the weak interactions is a superposition of the states ν_i with fixed masses m_i:

$$\nu_a = U_{ai}\nu_i, \tag{1}$$

where U_{ai} is the mixing matrix. This leads to such interesting phenomena as neutrino oscillations [1], that are closely related to the solution of the problem of the solar neutrino deficit [2]; to rare decays with the lepton number nonconservation such as $f_i \to f_j + \gamma$ or $f_i \to f_j + \gamma + \gamma$ (f_i and f_j are fermions with different flavours, γ is a radiated photon) which are investigated because of their possible astrophysical applications do to the by sensitivity of these processes to masses and mixing angles.

The radiative decay of a massive neutrino $f_i \to f_j + \gamma$ both in a constant magnetic field and in crossed fields without consideration for polarisation effects has been studied in [3,4]. In the present paper we once again turn our attention to the process of radiative decay of a massive neutrino in a constant magnetic field. We calculate the probability of the process with consideration for the contributions of the σ and π-linear components of the emitted photon polarisation and find the asymptotics of the rate of the process with consideration for the polarisation properties of radiation in the limiting case of a relativistic neutrino and comparatively weak magnetic field.

Similar to paper [3] only the contribution of the diagram with a virtual W-boson in the Feynman gauge to the process will be considered. Going over to the contact approximation and employing the Fierz identity [1,5], we present the probability of the process in the form, which is convenient for future calculations:

$$S_{ij} = -i\frac{4eG_F}{\sqrt{2}}\varepsilon^{\alpha}(k)j^{\beta}\langle K^{a}_{\alpha\beta}\rangle\delta(q_i - q_j - k), \tag{2}$$

where

$$j^\beta = \bar{\nu}_j j^\beta \frac{(1+\gamma^5)}{2}\nu_i, \ \langle K^a_{\alpha\beta}\rangle = \sum_a U^+_{ja} U_{ai} K^a_{\alpha\beta},$$

$$K^a_{\alpha\beta} = \int d^4p Sp\left[\gamma_\beta\left(1+\gamma^5\right) G_a(p)\gamma_\alpha G_a(p+k)\right], \tag{3}$$

Here $G_a(p)$ is the propagator of a charged lepton in the magnetic field specified by the potential $A^\mu(x) = (0, 0, x^1 H, 0)$ in the momentum representation; $\varepsilon^\alpha(k)$ is the polarisation vector of the emitted photon with the momentum k; $\nu_i(q)$ is the Dirac bispinor of the emitted photon with the momentum $q(q^2 = m_i^2)$, and $\bar{\nu}_j(q')$ is the Dirac conjugate bispinor of the final neutrino with the momentum $q'(q'^2 = m_j^2)$.

We will perform integration over p^μ in (3) in an invariant form. To this end we will use a covariant representation for $G_a(p)$ in the form of an integral over the proper time [6] and the orthonormalized basis is introduced

$$e^\mu_{(0)} = \frac{1}{\eta\sqrt{a}}\left[\eta^2 k^\mu + (FFk)^\mu\right], \ e^\mu_{(1)} = \frac{(Fk)^\mu}{\sqrt{a}}, \ e^\mu_{(2)} = \frac{(\tilde{F}k)^\mu}{\sqrt{a}},$$
$$e^\mu_{(3)} = -\frac{(FFk)^\mu}{\eta\sqrt{a}}, \ e^\mu_{(\alpha)} e_{\mu(\beta)} = g_{\alpha\beta}, \ a = kFFk. \tag{4}$$

Next we turn from integrating over Cartesian components of the momentum p^μ to integrating over coefficients of its decomposition in the basis (4), where the quadratic form $p\Phi p$ is diagonal. Therefore we obtain for the quantity $K^a_{\alpha\beta}$ in (2) the following expression, which is exact with respect to the field:

$$\begin{aligned}
K^{\alpha\beta}_a = -i(2\pi)^2 \int \frac{ds\,ds'}{s+s'}\frac{b}{\sin b(s+s')} \exp\left[-im_a^2(s+s') + i\,kGk\right] \times \\
\times\Big\{ m_a^2\left[g^{\alpha\beta}\cos b(s+s') - i\tilde{\varphi}^{\alpha\beta}\sin b(s+s') - 2\sin bs \sin bs' \Lambda^{\alpha\beta}\right] - \\
-g^{\alpha\beta}\left[\cos b(s-s')\left(\frac{i}{s+s'} - \frac{ass'}{\eta^2(s+s')^2}\right) + \frac{ib}{\sin b(s+s')} + \frac{a}{\eta^2}\frac{\sin bs \sin bs'}{\sin^2 b(s+s')}\right] - \\
-2i\frac{a}{\eta^2}\left[e^\alpha_{(2)} e^\beta_{(0)}\frac{ss'}{(s+s')^2}\sin bs \cos bs' + e^\alpha_{(2)} e^\beta_{(3)}\frac{s'}{s+s'}\sin^2 bs \frac{1}{\sin b(s+s')}\right] - \\
-i\Lambda^{\alpha\beta}\frac{b}{\sin b(s+s')} + i\bar{\Lambda}^{\alpha\beta}\frac{1}{s+s'}\cos b(s-s') + \\
+2\frac{a}{\eta^2} e^\alpha_{(2)} e^\beta_{(2)}\frac{ss'}{(s+s')^2}\sin bs \sin bs'\Big\}
\end{aligned} \tag{5}$$

where

$$\bar{\Lambda}^{\alpha\beta} = g^{\alpha\beta} + \Lambda^{\alpha\beta}, \ \Lambda^{\alpha\beta} = \frac{(FF)^{\alpha\beta}}{\eta^2}, \ b = e\eta,$$

$$G^{\mu\nu} = \frac{ss'}{s+s'}\tilde{\Lambda}^{\mu\nu} - \frac{\sin bs \sin bs'}{b \sin b(s+s')}, \ \tilde{\varphi}^{\mu\nu} = \frac{\tilde{F}^{\mu\nu}}{\eta}. \tag{6}$$

We note that the result (5) after summation over polarisation agrees with the result of [3], where no polarisation properties have been studied, and as for the vector $e^{\alpha}_{(1)}(k)$ and $e^{\alpha}_{(2)}(k)$, they after the corresponding gauge transformation turn into the well known three dimensional transverse vectors of σ- and π-polarisation.

Let us consider the case of relativistic neutrino energies and a comparatively weak magnetic field, when

$$\varepsilon_{\nu} >> m_{\nu}, \ H << H_0^a = \frac{m_a^2}{e}, \tag{7}$$

where h_0^a is the critical field for the intermediate lepton of the type $a = e, \mu, \tau$. If the invariant dynamical parameter $\chi_a = (\sqrt{qFFq})/(m_a^3) = (H/H_0^a)(q_{\perp}/m_a)$ is introduced we obtain the following asymptotics for the probability of the radiative decay $f_i \to f_j + \gamma$ with emission of a photon with the polarisation $\lambda = \sigma$

$$w(\sigma) = \frac{\alpha}{2\pi}\frac{G_F^2}{(15\pi)^3}\frac{m_{\nu}^2}{q_0}m_e^4\chi_e^4|U_{ie}U_{je}^*|^2, \ \chi_e << 1. \tag{8}$$

We note that $w(\sigma)$ is proportional to the square of the neutrino mass. However, when the photon with the polarisation $\lambda = \pi$ is emitted, the probability of this process, besides the term proportional to the powers of the neutrino mass, has the part independent of the neutrino mass. Indeed, for small values of the dynamic parameter, when $\chi_e << m_{\nu}/m_e$ or, which is the same condition put into another form,

$$\frac{H}{H_0^e}\frac{p_{\perp}}{m_{\nu}} << 1, \tag{9}$$

the probability of the process with emission of a photon with the polarisation $\lambda = \pi$ has the following asymptotics

$$w(\pi) = \frac{\alpha}{2\pi}\frac{G_F^2}{(15\pi)^3}\frac{m_{\nu}^2}{q_0}|U_{ie}U_{je}^*|^2\left[\frac{49}{16}m_e^4\chi_e^4 + \frac{12}{32}m_{\nu}^2 m_e^2\chi_e^2\right], \tag{10}$$

i. e. it has the same behaviour as that of the σ-component for $\chi_a << 1$, while in the intermediate region

$$\frac{m_{\nu}}{m_e} << \chi_e << 1 \tag{11}$$

the main contribution to the probability of the process is described by the formula

$$w(\pi) = \frac{\alpha}{4\pi}\frac{G_F^2}{(15\pi)^3}\frac{m_e^6\chi_e^6}{q_0}|U_{ie}U_{je}^*|^2 \tag{12}$$

which does not contain any dependence on the neutrino mass. The last result corresponds to that of the Ref. [3] where no polarisation has been predicted. The latter can be easily compared to that of Galtsov and Nikitina [7], obtained for the massless neutrino and without any mixing, by putting the mixing matrix equal to unity. At $\chi_e << m_\nu/m_e$ we can neglect it. In the opposite case of large values of the parameter $\chi_e >> m_\nu/m_e$ (though $\chi_e << 1$) only the π-component of polarisation contributes to the probability and the radiated photon is linearly polarised. The detailed consideration performed in the present paper shows that in the fairly large region of the dynamic parameter χ_a the contribution of terms, depending on the neutrino mass, to the probability of the process, that has not been taken into account in [3], is the leading one (see formulas (8)-(10)). No less important is the conclusion, that with growth of the dynamic parameter the radiated photons become to a large degree linearly polarised, and in the limiting case of large values of this parameter the radiation is practically completely polarised. At the same time the conclusion of the previous publications about of the GIM factor suppression in the external field is confirmed by our results.

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HEISENBERG REPRESENTATION FOR CREATION-ANNIHILATION OPERATORS IN NONSTATIONARY BACKGROUND

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Abstract

The Heisenberg formalism for the creation and annihilation operators of quantized spinor field in nonstationary external electromagnetic or gravitational fields is developed. Heisenberg equations of motion are obtained for the creation-annihilation operators. The additional terms which arise in these equations take into account the effects of scattering and particle creation from vacuum by the external field.

1 Introduction

Heisenberg representation is the well-known and commonly used formalism of a standard quantum field theory (see, e.g., [1, 2]). For the case of quantized field interacting with some nonstationary external field, however, the time dependence of the Schrōdinger wave functions cannot be completely transferred to the creation and annihilation operators. This circumstance was established for the special case of a spatially homogeneous nonstationary electric fields [3, 4]. As it was shown in [4], the same situation takes place for the quantized fields interacting with nonstationary space homogeneous gravitational fields.

The theory of quantized fields interacting with nonstationary external fields has a great number of interesting applications in different brunches of physics [4, 5]. Here it is reasonable to mention the effect of particle production from vacuum by the electromagnetic field of high-powered lasers and by the gravitational field near the cosmological singularity. Additional application is the effect of vacuum polarization by strong external field both electromagnetic or gravitational.

The main concern of this paper is to construct the Heisenberg representation for the quantized fields interacting with a nonstationary electromagnetic or gravitational field of a general form. To make this we perform a canonical quantization of a spinor field in nonstationary external fields. In particular we give some reasonable generalization of the Hamiltonian operator which plays the same role in canonical quantization as the usual Hamiltonian in stationary situations which accomplishes translations in physical time.

In Sec.2 the Heisenberg formalism for spinor field in nonstationary background is constructed. The Sec.3 is devoted to the obtaining of the Heisenberg equations for the creation and annihilation operators in nonstationary electromagnetic or gravitational background of general form.

Throughout the paper we use units in which $\hbar = c = 1$. The Greek indices have the values 0, 1, 2, 3, and the Latin ones — 1, 2, 3.

2 Heisenberg formalism for the spinor field in nonstationary background

In this section we develop Heisenberg quantization procedure for the spinor field in an external electromagnetic or gravitational field.

From the Lagrangian of the spinor field $\psi(x)$ in the external electromagnetic field $A_\mu(x)$:

$$\mathcal{L}(x) = \frac{i}{2}[\bar{\psi}(x)\gamma^\mu D_\mu\psi(x) - D^*_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)] - m\bar{\psi}(x)\psi(x) \tag{1}$$

we obtain in the standard manner the canonically conjugate momenta and the Hamiltonian

$$\pi(x) = \frac{i}{2}\psi^+(x), \qquad \pi^+(x) = -\frac{i}{2}\psi(x), \tag{2}$$

$$\begin{aligned} H(t) = \int d^3x[&- \pi(x)\alpha^k D_k(x)\psi(x) - D^*_k(x)\psi^+(x)\alpha^k\pi^+(x) \\ &+ im\psi^+(x)\beta\pi^+(x) - im\pi(x)\beta\psi(x)], \end{aligned} \tag{3}$$

where $\beta = \gamma^0$, $\alpha^k = \gamma^0\gamma^k$, γ^μ are the Dirac matrices.

Now we shall consider Hamiltonian of the spinor field interacting with external gravitational field. The space-time manifolds are supposed to allow decomposition into a set of space-like hypersurfaces $\Sigma(t)$. So it is possible to intorduce the global time t. Such decomposition and the existence of the global time coordinate are necessary for developing the canonical quantization formalism. We shall use also the system of reference for which

$$ds^2 = g_{00}(t, \mathbf{x})dt^2 + g_{kl}(t, \mathbf{x})dx^k dx^l, \tag{4}$$

i.e., in which all the components g_{0k} are equal to zero. This metric is provided by the tetrade $h^{(\rho)}_\mu$ with such orientation that vector $h^{(0)}$ is directed along the time-like coordinate t and the other vectors $h^{(k)}$ lie in the tangent space of the space-like hypersurface $\Sigma(t)$

$$\begin{aligned} h^{(0)}_\mu(x) &= h(x)\delta_{0\mu}, \\ h^{(k)}_l(x) &\neq 0, \qquad h^{(k)}_0(x) = 0. \end{aligned} \tag{5}$$

Covariant differentiation of the spinor field in Riemannian space is defined as follows [4]

$$\nabla_\mu\psi = [\partial_\mu + \frac{1}{4}C_{\alpha\beta\rho}h^{(\rho)}_\mu\gamma^\alpha\gamma^\beta]\psi \equiv [\nabla_\mu + C_\mu(x)]\psi, \tag{6}$$

where $C_{\alpha\beta\rho}$ are the Ricci rotation coefficients which are connected with tetrad $h_{(\rho)\mu}$ by the relation

$$C_{\alpha\beta\rho} = (\nabla_\mu h^\nu_{(\alpha)})h_{(\beta)\nu}h^\mu_{(\rho)}. \tag{7}$$

In the Riemannian space instead of the constant Dirac matrices γ^μ it is necessary to use the matrices $\gamma^\mu(x)$ which are 4-vectors relatively to the general coordinate transformations

$$\gamma^\mu(x) = h^\mu_{(\rho)}(x)\gamma^\rho. \tag{8}$$

Lagrangian of a spinor field in external gravitational field has the form [4]

$$\mathcal{L}(x) = \sqrt{-g}\Big[\frac{i}{2}\big(\bar\psi(x)\gamma^\mu(x)\nabla_\mu\psi(x) - (\nabla_\mu\bar\psi(x))\gamma^\mu(x)\psi(x)\big) - m\bar\psi(x)\psi(x)\Big], \tag{9}$$

where the Dirac conjugate spinor is $\bar\psi(x) = \psi^+(x)\gamma^0$. From the Lagrangian (38) the canonically conjugate momenta are defined as follows

$$\pi(x) = \frac{i}{2}\sqrt{-g}\bar\psi(x)\gamma^0(x), \quad \bar\pi(x) = -\frac{i}{2}\sqrt{-g}\gamma^0(x)\psi(x). \tag{10}$$

For canonical quantization the canonical energy-momentum tensor is necessary. It may be obtained from the Lagrangian (9) with the variation by the variables ψ and $\bar\psi$.

The Hamiltonian is defined by the integration of T_{00} over the space-like hypersurface $\Sigma(t)$ [4]

$$H(t) = \int_{\Sigma(t)} \zeta^0 T_{00}(x)g^{00}(x)\sqrt{-g}d^3x, \tag{11}$$

where ζ^0 is the zero component of vector field orthogonal to $\Sigma(t)$ which provides translations along the time coordinate. We scale t in such a manner that $\zeta^0 = 1$. Hamiltonian of a spinor field is defined by eq. (11) with

$$T_{00} = g_{00}(\pi\gamma^0(x)\partial_0\psi + \partial_0\bar\psi\gamma^0(x)\bar\pi) - g_{00}\mathcal{L}(x), \tag{12}$$

where ζ^0 in (11) is normalized on unity. With the expressions (10) for the momenta and (5) for the tetrad the Hamiltonian has the form

$$H(t) = \int_{\Sigma(t)} d^3x \Big[- \pi(x)\frac{\alpha^k(x)}{h(x)}\nabla_k\psi(x) + \nabla_k\bar{\psi}(x)\frac{\alpha^k(x)}{h(x)}\bar{\pi}(x) - \pi(x)C_0(x)\psi(x) - C_0(x)\bar{\psi}(x)\bar{\pi}(x) - im\pi(x)\frac{\gamma^0}{h(x)}\psi(x) + im\bar{\psi}(x)\frac{\gamma^0}{h(x)}\bar{\pi}(x)\Big], \tag{13}$$

where $\alpha^k(x) = \gamma^0\gamma^k(x)$. The measure in (13) is invariant because the quantity $\sqrt{-g}$ is included into the canonical momenta (10). It is necessary to mention here that the Hamiltonian (13) has some conventional meaning. For the particular cases when ζ^α is the Killing vector field (or at least conformal Killing vector field [4]) this Hamiltonian generates the translations in physical time and has the usual meaning. In the other cases the situation with time transitions is not so simple but, as it will be shown later, the operator (13) also possess all properties which are demanded from the Hamiltonian in the usual canonical formalism.

The canonically conjugated operators $\psi(x)$ and $\pi(x)$ satisfy the usual anticommutation relations

$$\{\psi(t,\mathbf{x}), \pi(t,\mathbf{y})\} = i\delta(\mathbf{x}-\mathbf{y}). \tag{14}$$

The Heisenberg equations for the spinor fields in the external electromagnetic field are

$$i\dot{\psi}(x) = [\psi(x), H(t)], \quad i\dot{\pi}(x) = [\pi(x), H(t)]. \tag{15}$$

Equations (15) may be written in the matrix form

$$i\dot{\psi}(x) = \mathcal{H}(x)\psi(x), \tag{16}$$

where linear (differential in coordinates $\mathbf{x}$ and parametrically depending on t) operator $\mathcal{H}(x)$ acting to a spinor field has the form

$$\mathcal{H}(x) = \left\{ \begin{array}{c} \mathcal{H}^{(A)}(x) \\ \mathcal{H}^{(g)}(x) \end{array} \right\} = \left\{ \begin{array}{c} -i\alpha^k D_k(x) + m\beta \\ -i\frac{1}{h(x)}\alpha^k(x)\nabla_k - iC_0(x) + \frac{1}{h(x)}m\gamma^0 \end{array} \right\}. \tag{17}$$

In Schrödinger representation the secondary quantization procedure consists in expansion of the field $\psi(x)$ with respect to a complete set of solutions $u^{(\pm)}_\alpha(x)$ to the Dirac equation (16):

$$\psi(x) = \sum_\alpha \Big[u^{(-)}_\alpha(x)a_\alpha + u^{(+)}_\alpha(x)b^+_\alpha\Big]. \tag{18}$$

Here, the index α labels the solutions, i.e., it corresponds to the spin state and to the momentum generalized in the presence of external field; the indices $(\pm)$ correspond to the positive- and negative-frequency solutions, i.e., they have the asymptotic behavior $exp(\pm i\omega_\alpha t)$ as $t \to -\infty$ when the external field is switched off or at some initial moment t_0. The standard anticommutation relations hold:

$$\{a_\alpha, a_\beta^+\} = \{b_\alpha, b_\beta^+\} = \delta_{\alpha\beta}.$$

The zeroth component of the charge current $J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ (or $J_\mu(x) = \bar{\psi}(x)\gamma_\mu(x)\psi(x)$ for the case of external gravitational field) determines the scalar product of the spinors u and v:

$$< u, v >= \int_{\Sigma(t)} d^3x \left\{ \begin{array}{c} u^+(x)v(x) \\ \sqrt{-g}\bar{u}(x)\gamma_0(x)v(x) \end{array} \right\}, \tag{19}$$

which does not depend on time when u and v are the solutions of Dirac equation.

The operators $\mathcal{H}^{(A,g)}(x)$ are self-adjoint with respect to the scalar product (19). This means that it posseses a complete orthonormal (in the sense of (19)) system of eigenfunctions $\psi_\alpha^{(\pm)}(x)$ with eigenvalues $\omega_\alpha(t)$:

$$\mathcal{H}^{(A,g)}(x)\psi_\alpha^{(\mp)}(x) = \pm\omega_\alpha(t)\psi_\alpha^{(\mp)}(x), \tag{20}$$

which depend on time as on a parameter. Since the system of functions $\psi_\alpha^{(\pm)}$ is complete at any time (for all space-like hypersurfaces $\Sigma(t)$), the operator of the spinor field $\psi(x)$ may be expanded with respect to it:

$$\psi(x) = \sum_\alpha [\psi_\alpha^{(-)}(x)c_\alpha(t) + \psi_\alpha^{(+)}(x)d_\alpha^+(t)]. \tag{21}$$

The operators $c_\alpha(t)$ and $d_\alpha^+(t)$ introduced in (21) are the Heisenberg annihilation and creation operators for a spinor field in an arbitrary electromagnetic or gravitational background.

Using the expressions (2) or (10) for the canonical momenta and (17) for the operator $\mathcal{H}(x)$, we may, taking into account (20), to write Hamiltonian in the form

$$\begin{aligned} H(t) &= \int_{\Sigma(t)} d^3x \left\{ \begin{array}{c} \psi^+(x)\mathcal{H}^{(A)}(x)\psi(x) \\ \sqrt{-g}\bar{\psi}(x)\gamma^0(x)\mathcal{H}^{(g)}(x)\psi(x) \end{array} \right\} = \\ &= < \psi, \mathcal{H}^{(A,g)}\psi >= \sum_\alpha \omega_\alpha(t)[c_\alpha^+(t)c_\alpha(t) - d_\alpha(t)d_\alpha^+(t)]. \end{aligned} \tag{22}$$

Decomplementation of a spinor field with respect to the system of the eigenfunctions of the operator $\mathcal{H}(x)$ diagonalises the Hamiltonian and the operator coefficients of this decomplementation are the Heisenberg particles operators on space-time hypersurface $\Sigma(t)$ at a moment t. From (22) it is clearly seen that the eigenvalue $\omega_\alpha(t)$ of the operator $\mathcal{H}(x)$ introduced in (17) is just the instantaneous energy in the state α (the energy of the state α on the space-like hypersurface $\Sigma(t)$).

To obtain time evolution of the creation-annihilation operators $c_\alpha(t)$ and $d_\alpha^+(t)$ it is necessary to make use of Heisenberg equations for fields (16).

3 Heisenberg equations for the creation-annihilation operators of particles in nonstationary external fields

To determine time dependence of creation-annihilation operators we insert Heisenberg decomposition of field $\psi(x)$ (21) with respect to eigenfunctions of operator $\mathcal{H}(x)$ (17) into eq. (16):

$$\frac{d}{dt}\sum_\alpha\Big[\psi_\alpha^{(-)}(x)c_\alpha(t)+\psi_\alpha^{(+)}(x)d_\alpha^+(t)\Big]=-i\mathcal{H}(x)\sum_\alpha\Big[\psi_\alpha^{(-)}(x)c_\alpha(t)+\psi_\alpha^{(+)}(x)d_\alpha^+(t)\Big]. \tag{23}$$

Since the operator $\mathcal{H}(x)$ contains derivatives only with respect to coordinates $\mathbf{x}$, it commutes with $c_\alpha(t)$ and $d_\alpha^+(t)$. Multiplying (23) scalary by $\psi_\beta^{(\pm)}(x)$ and taking into account that the functions $\psi_\alpha^{(\pm)}(x)$ form the orthonormal system of eigenfunctions of the operator $\mathcal{H}(x)$ we obtain the Heisenberg equations for the operators $c_\alpha(t)$ and $d_\alpha^+(t)$:

$$\begin{aligned}\dot c_\beta(t) &= -i\omega_\beta(t)c_\beta(t)-\sum_\alpha\Big[<\psi_\beta^{(-)},\dot\psi_\alpha^{(-)}>c_\alpha(t)+<\psi_\beta^{(-)},\dot\psi_\alpha^{(+)}>d_\alpha^+(t)\Big],\\ \dot d_\beta^+(t) &= i\omega_\beta(t)d_\beta^+(t)-\sum_\alpha\Big[<\psi_\beta^{(+)},\dot\psi_\alpha^{(-)}>c_\alpha(t)+<\psi_\beta^{(+)},\dot\psi_\alpha^{(+)}>d_\alpha^+(t)\Big]\end{aligned} \tag{24}$$

If the external field does not depend on time, then $\psi_\alpha^{(\pm)}=const$ (on time), the additional terms under the sums vanish, and (24) becomes the ordinary Heisenberg equations for the creation and annihilation operators.

It is worth-while to establish the connection between the Heisenberg operators $c_\alpha(t)$ and $d_\alpha^+(t)$ and the quasiparticle creation and annihilation operators $\tilde c_\alpha(t)$ and $\tilde d_\alpha^+(t)$ which were constructed in [4, 6] directly from the requirement of diagonality of the instantaneous Hamiltonian. The quasiparticle operators may be expressed in terms of the Schrödinger creation and annihilation operators by the Bogoliubov transformation [7]

$$\begin{aligned}a_\alpha &= \sum_\beta\Big[\Phi_{\alpha\beta}(t)\tilde c_\beta(t)+\Psi_{\alpha\beta}(t)\tilde d_\beta^+(t)\Big],\\ b_\alpha^+ &= \sum_\beta\Big[-\Psi^*_{\alpha\beta}(t)\tilde c_\beta(t)+\Phi^*_{\alpha\beta}(t)\tilde d_\beta^+(t)\Big].\end{aligned} \tag{25}$$

The matrices Φ and Ψ (which, in general, are infinite dimensional) satisfy the conditions which provide the cononicity and invertibility of the transformations (25). They may be expressed in terms of the Schrödinger wave functions $u_\alpha^{(\pm)}(t, \mathbf{x})$ using the condition of diagonality of the Hamiltonian [4, 7]. In terms of $\tilde{c}_\alpha(t)$ and $\tilde{d}_\alpha^+(t)$ one again has the form (22) with the same eigenvalues $\omega_\alpha(t)$ (since the spectrum of a self-adjoint operator is invariant with respect to the choice of diagonal representation).

Substituting decomposition for the spinor field, we obtain the field operator as an expansion with respect to the quasiparticle operators:

$$\psi(x) = \sum_\alpha \Big[\tilde{\psi}_\alpha^{(-)}(x)\tilde{c}_\alpha(t) + \tilde{\psi}_\alpha^{(+)}(x)\tilde{d}_\alpha^+(t)\Big], \tag{26}$$

where the Bogoliubov basis functions $\tilde{\psi}_\alpha^{(\pm)}(x)$ are related to the Schrödinger functions $u_\alpha^{(\pm)}(x)$ eq. (18) by

$$\begin{aligned} \tilde{\psi}_\alpha^{(-)} &= \sum_\beta \Big[\Phi^T_{\alpha\beta}(t)u_\beta^{(-)} - \Psi^+_{\alpha\beta}(t)u_\beta^{(+)}\Big], \\ \tilde{\psi}_\alpha^{(+)} &= \sum_\beta \Big[\Psi^T_{\alpha\beta}(t)u_\beta^{(-)} + \Phi^+_{\alpha\beta}(t)u_\beta^{(+)}\Big]. \end{aligned} \tag{27}$$

It is obvious that a scalar product of the form of (19) is invariant with respect to the canonical transformations of the from of (27).

Since the Hamiltonian is diagonal both in terms of operators $c_\alpha(t)$, $d_\alpha^+(t)$ and $\tilde{c}_\alpha(t)$, $\tilde{d}_\alpha^+(t)$, these operators [and also the basis functions $\psi_\alpha^{(\pm)}(x)$ (eq. (21)) and $\tilde{\psi}_\alpha^{(\pm)}(x)$ (eq. (26))] may differ from each other only by phase factors:

$$\begin{aligned} c_\alpha(t) &= \tilde{c}_\alpha(t)e^{-i\theta_\alpha(t)}, & \psi_\alpha^{(-)}(x) &= e^{\,i\theta_\alpha(t)}\tilde{\psi}_\alpha^{(-)}(x), \\ d_\alpha^+(t) &= \tilde{d}_\alpha^+(t)e^{\,i\theta_\alpha(t)}, & \psi_\alpha^{(+)}(x) &= e^{-i\theta_\alpha(t)}\tilde{\psi}_\alpha^{(+)}(x). \end{aligned} \tag{28}$$

The function $\theta_\alpha(t)$ here is to be determined.

The scalar products that occur in the Heisenberg equation (24) may be expressed in terms of the matrices Φ and Ψ of the Bogoliubov transformations diagonalizing the Hamiltonian. To do this we use the equation

$$< \psi_\beta^{(-)}, \dot{\psi}_\alpha^{(-)} >= e^{-i\theta_\beta(t)} < \tilde{\psi}_\beta^{(-)}, \dot{\tilde{\psi}}_\alpha^{(-)} > e^{\,i\theta_\alpha(t)} + i\dot{\theta}_\alpha(t)\delta_{\alpha\beta}, \tag{29}$$

which follows from (28) and the analogous equations for remaining scalar products in (24).

The scalar product $< \tilde{\psi}_\beta^{(-)}, \dot{\tilde{\psi}}_\alpha^{(-)} >$, included in (29), may be expressed in terms of the matrices Φ and Ψ with using transformations (27), the Dirac equation for the functions $u_\alpha^{(\pm)}(x)$, relations (28) and also eq. (20):

$$\begin{aligned} <\bar{\psi}_\beta^{(-)}, \dot{\bar{\psi}}_\alpha^{(-)}> &= -\left(\dot{\Phi}^+\Phi + \dot{\Psi}^T\Psi^*\right)_{\beta\alpha} - i\omega_\beta(t)\delta_{\alpha\beta}, \\ <\bar{\psi}_\beta^{(-)}, \dot{\bar{\psi}}_\alpha^{(+)}> &= -\left(\dot{\Phi}^+\Psi - \dot{\Psi}^T\Phi^*\right)_{\beta\alpha}. \end{aligned} \tag{30}$$

The similar expressions take place for $<\bar{\psi}_\beta^{(+)}, \dot{\bar{\psi}}_\alpha^{(+)}>$, $<\bar{\psi}_\beta^{(+)}, \dot{\bar{\psi}}_\alpha^{(-)}>$.

As a result substituting (30) in (29) and the scalar products (29) in the Heisenberg equations (24) we obtain equations for the Heisenberg operators $c_\beta(t)$ and $d_\beta^+(t)$ represented through the matrices Φ, Ψ of the Bogoliubov transformations:

$$\begin{aligned} \dot{c}_\beta(t) &= -i\omega_\beta(t)c_\beta(t) - i(\dot{\theta}_\beta(t) - \omega_\beta(t))c_\beta(t) \\ &+ \sum_\alpha e^{-i\theta_\beta(t)}\left(\dot{\Phi}^+\Phi + \dot{\Psi}^T\Psi^*\right)_{\beta\alpha} e^{i\theta_\alpha(t)}c_\alpha(t) \\ &+ \sum_\alpha e^{-i\theta_\beta(t)}\left(\dot{\Phi}^+\Psi - \dot{\Psi}^T\Phi^*\right)_{\beta\alpha} e^{-i\theta_\alpha(t)}d_\alpha^+(t), \end{aligned} \tag{31}$$

and the analogous equation for $\dot{d}_\beta^+(t)$.

The first term in the right-hand side of (31) is equal to the commutator of $c_\beta(t)$ with the Hamiltonian $H(t)$ (22). To obtain the correct result in the limit of stationary field we must require the vanishing of the second term in the right-hand side of (31) fixing by this means the choice of the phase function

$$\theta_\beta(t) = \int_{-\infty}^{t} \omega_\beta(\tau)d\tau. \tag{32}$$

The additional terms in (31) are due to the nonstationarity of the external field. In particular, they vanish in the limit $t \to \pm\infty$ when the external field is switched off and

$$\begin{aligned} \Phi(t) &\underset{t\to-\infty}{\longrightarrow} I, & \Phi(t) &\underset{t\to+\infty}{\longrightarrow} \Phi_+ = const, \\ \Psi(t) &\underset{t\to-\infty}{\longrightarrow} 0, & \Psi(t) &\underset{t\to+\infty}{\longrightarrow} \Psi_+ = const. \end{aligned} \tag{33}$$

As a result, the Heisenberg equations for the particle creation and annihilation operators in an arbitrary nonstationary background take the form

$$\begin{aligned} \dot{c}_\beta(t) &= -i[c_\beta(t), H(t)] \\ &+ \sum_\alpha e^{-i\theta_\beta(t)}\left(\dot{\Phi}^+\Phi + \dot{\Psi}^T\Psi^*\right)_{\beta\alpha} e^{i\theta_\alpha(t)}c_\alpha(t) \end{aligned} \tag{34}$$

$$
\begin{aligned}
&+ \sum_\alpha e^{-i\theta_\beta(t)}\Big(\dot{\Phi}^+\Psi - \dot{\Psi}^T\Phi^*\Big)_{\beta\alpha} e^{-i\theta_\alpha(t)} d^+_\alpha(t), \\
\dot{d}^+_\beta(t) &= [d^+_\beta(t), H(t)] \\
&+ \sum_\alpha e^{i\theta_\beta(t)}\Big(\dot{\Psi}^+\Psi + \dot{\Phi}^T\Phi^*\Big)_{\beta\alpha} e^{-i\theta_\alpha(t)} d^+_\alpha(t) \\
&+ \sum_\alpha e^{i\theta_\beta(t)}\Big(\dot{\Psi}^+\Phi - \dot{\Phi}^T\Psi^*\Big)_{\beta\alpha} e^{i\theta_\alpha(t)} c_\alpha(t).
\end{aligned}
\tag{35}
$$

The presence of the additional terms with the operator $d^+_\alpha(t)$ in (34) and with $c_\alpha(t)$ in (35) is due to the mixing of solutions with the opposite frequencies and corresponds to particle-antipaticle pair production by the external field. The terms with the operators $c_\alpha(t)$ in eq. (34) and $d^+_\alpha(t)$ in eq. (35) describe the effects of scattering of particles by the external field. The sum over the different modes α arises because the variables $\mathbf{x}$ and t can not be separated in an arbitrary external field.

Let us consider the special case of a spatially homogeneous nonstationary electric external field $E(t) = \dot{A}_3(t)$. For the spinor quantized field eqs. (34), (35) were obtained in [3, 4] and have the form

$$
\dot{c}_{\mathbf{p}r}(t) = -i[c_{\mathbf{p}r}(t), H(t)] + \frac{e\sqrt{m^2+p_\perp^2}\,E(t)}{2\omega^2} d^+_{-\mathbf{p}r}(t), \tag{36}
$$

where $\omega^2 = m^2 + p_\perp^2 + (p_3 - eA_3)^2$, $p_\perp = (p_1^2+p_2^2)^{1/2}$ is the momentum transverse with respect to the external field and $r = 1, 2$ corresponds to two possible spin projections onto z axis. The contributions which correspond to the same frequency are absent here, since $\dot{\Psi}^+\Psi + \dot{\Phi}^T\Phi^* = 0$. Note also that in [3] equations for the coefficients matrices of the Bogoliubov transformations equivalent to (34), (35) and (36) were found for the case of a homogeneous electric field. For the case of nonstationary space homogeneous gravitational field the analog of eqs. (34), (35), (36) were obtained in [4].

It should be noted that the developed formalism of Heisenberg quantization may be applied to quantization procedure in external fields of an arbitrary nature. For example, in [8] this procedure was carried out for a neutral scalar field in an external field described by the scalar potential $U(x)$ and for the electromagnetic field in a nonstationary dispersive media as a background field. But for the dispersive media this approach is not so straightforward because of nonlocality on time due to the dispersion.

In conclusion we would like to emphasize that the procedure of Heisenberg quantization may be carried out in universal manner for the spinor field influenced by nonstationary electromagnetic or gravitational background. The only difference arises due to the manifest form of the operator $\mathcal{H}(x)$ which contains all information about an external field. Evidently all the results of this paper may be reformulated for the case of scalar field both in electromagnetic and gravitational background. This approach makes clearer the foundations of the ordinary Bogoliubov diagonalization procedure. Moreover, for the external gravitational background the decomposition of the field into the set of $\mathcal{H}(x)$ eigenfunctions gives rise to one of the concepts of particles in Riemannian space-time.

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A Deterministic Subset of Maxwell–Dirac Electrodynamics

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Abstract

A subset of Majorana solutions of Maxwell–Dirac electrodynamics is defined by the Majorana condition imposed on the spinor field. This subset is not trivial and suggests a natural deterministic interpretation both for spinors with commuting and anticommuting components. A given solution can be obtained from a Majorana solution by a gauge transform if it satisfies a condition that seems to be rather weak for spinors with anticommuting components.

Numerous recent publications reflect growing interest for the interpretation of quantum mechanics. For the development of the discussion there is a constant need of arguments of purely physical character. In this context an example of a non-trivial subset of quantum mechanics that naturally supposes a deterministic interpretation may be instructive.

We start with the equations of Maxwell–Dirac electrodynamics:

$$(i\hat{\partial} + e\hat{A} - m)\Psi = 0 \tag{1}$$

$$\Box A_\mu - A^\nu_{,\nu\mu} = j_\mu \tag{2}$$

$$j_\mu = -e\bar{\Psi}\gamma_\mu\Psi \tag{3}$$

The chiral representation for γ-matrices [1] and a symbol $\hat{A} = A_\mu\gamma^\mu$ are used. This theory is classical in the sense that it is not second-quantized, but it describes, certainly, a wide class of quantum phenomena. We discuss a theory with:

1. c-type spinors (components of the spinor Ψ are c-numbers)
2. a-type spinors (components of the spinor Ψ are anticommuting elements of a Grassman algebra

The former case is simpler and more graphic. Along the lines of the present work it is discussed in [2, 3, cf. also references therein]. However, it is generally accepted nowadays that in a classical theory fermions should be described by anticommuting variables [4]. That is why the present paper deals mainly with the latter case. Nevertheless, main results of works [2, 3] will be presented in parallel as they are basic for a-type spinors too.

Let us define a subset of such solutions of Maxwell–Dirac electrodynamics that the spinor Ψ satisfies the Majorana condition $\Psi = C\bar{\Psi}^T$, where C is the matrix of charge-conjugation [1]. The Majorana condition is an analogue of the reality condition and coincides with it in the Majorana representation [1], and it may be said that we regard the Dirac equation as an equation for real, rather than complex, spinors. See [5] on the possibility of description of charged particles by real fields (not complex ones and not pairs of real fields). Applying charge conjugation to the Dirac equation and using the Majorana condition, we obtain $(i\hat{\partial} - m)\Psi = 0$ and $\hat{A}\Psi = 0$. If $\Psi \neq 0$, the latter equation implies $A_\mu A^\mu = 0$ [6]. If, in addition, the vector A^μ is not zero, then for c-type spinors the equation also implies that there is such λ that $\lambda A^\mu = j^\mu$ [6]. We obtain also that

$$0 = (\hat{\partial}\hat{A} + \hat{A}\hat{\partial})\Psi = 2A^\mu\partial_\mu\Psi + (\partial_\mu A_\nu)\gamma^\mu\gamma^\nu\Psi \tag{4}$$

This equation may be regarded as a system of ordinary (not partial!) differential equations on a curve that in all its points x is tangential to the vector $A^\mu(x)$ (cf. [3]). We may conclude that equations of Maxwell–Dirac electrodynamics for Majorana c-type spinors are equivalent (if $\Psi \not\equiv 0$ and $A \not\equiv 0$) to a system

$$\begin{aligned} (i\hat{\partial} - m)\Psi &= 0 && (5)\\ A_\mu A^\mu &= 0 && (6)\\ \lambda A^\mu &= j^\mu && (7)\\ \Box A_\mu - A^\nu_{,\nu\mu} &= \lambda A_\mu && (8) \end{aligned}$$

For a-type Majorana spinors $j^\mu = 0$, and the last two equations should be replaced by the following ones :

$$\begin{aligned} \hat{A}\Psi &= 0 && (9)\\ \Box A_\mu - A^\nu_{,\nu\mu} &= 0 && (10) \end{aligned}$$

Eqs. 8,6 coincide with the equations of Dirac's "new electrodynamics" [7] up to a constant in the right-hand side of Eq. 6. A solution of Eq. 8 realizes a conditional minimum of the action of the free electromagnetic field with the constraint 6 (cf. [7]). Hence, Eqs. 8,6 describe independent evolution of the electromagnetic field (λ is a Lagrange multiplier). The same statement for Eqs. 10,6 is quite evident. Eq. 6 may be regarded as a nonlinear gauge condition. For an arbitrary 4-potential B^μ there is a gauge-equivalent 4-potential $A^\mu = B^\mu + \partial^\mu \phi$ satisfying Eq. 6, and the function ϕ may be chosen arbitrarily on the hyperplane $x^0 = 0$.

One can see that both for c-spinors and a-spinors a deterministic interpretation seems natural that is similar to Bohm's one [8, 9]. The difference is that the role of quantum potential(s) is played by the ordinary potential of electromagnetic field. An electron may be regarded as a point-like particle with properties that are determined solely by the value of the spinor Ψ in the point of space-time. Possible trajectories are the curves that are tangential to the vector $A^\mu(x)$ in every their point x. In view of Eq. 6 the instantaneous velocity is always equal to the velocity of light. This is consistent with the notion of zitterbewegung and allows smaller mean velocities. It should be stressed that the possibility of this natural deterministic interpretation for the Majorana subset of Maxwell–Dirac electrodynamics depends on two circumstances: the first one is that Eqs. 8,6 or Eqs. 10,6 fix independent evolution of the electromagnetic field, and the second one is that due to Eq. 4 evolution of the spinor on the trajectory is determined by a system of ordinary differential equations.

Systems of Eqs. 5,7,8,6 for c-spinors and of Eqs. 5,9,10,6 for a-spinors are, generally speaking, overdetermined [10], and it is not evident that they have non-trivial solutions (we obtain trivial solutions by setting $\Psi = 0$; the resulting systems of equations for the electromagnetic field are not overdetermined. The system of Eqs. 5,9,10,6 has also trivial solutions that describe free Majorana spinors in zero electromagnetic field). Existence of non-trivial Majorana solutions of the system of Eqs. 5,7,8,6 follows from the results of the work [6]. It may be shown that the system for a-spinors has non-trivial solutions as well. Thus, the subset under consideration can serve at least as a toy model for the discussion of the interpretation of quantum mechanics.

Majorana condition is not invariant with respect to gauge transforms. Gauge invariant conditions may be found that select such solutions of Maxwell–Dirac electrodynamics that may be converted into Majorana solutions by a gauge transform. For c-type spinors the necessary and sufficient condition is that the axial current $\bar{\Psi}\gamma^5\gamma^\mu\Psi$ is zero. For a-type spinors the situation is more complex. If we take seriously Eq. 2 for a-type spinors, we have to admit that both sides of this equation must be zero as they have different degrees (0 and 2 correspondingly) with respect to the generators of the Grassman algebra. Let the components of a-type spinor Ψ be presented in the form

$$\Psi_\alpha(x) = \sum_n \xi_\alpha^{(n)}(x)\theta_n \tag{11}$$

where $\xi^{(n)}$ are Majorana c-type spinors with components $\xi_\alpha^{(n)}$, and θ_n are independent anticommuting generators of the Grassman algebra that are invariant under complex conjugation (involution) [11]. Then for every n a pair $(A, \xi^{(n)})$ should be a solution of the system of Eqs. 5,9,10,6 where the a-spinor Ψ is replaced by the c-spinor $\xi^{(n)}$. Conversely, if for every n $(A, \xi^{(n)})$ satisfies the system of Eqs. 5,9,10,6, then (A, Ψ) also satisfies Eqs. 5,9,10,6. Then in a complex space with coordinates y_n and the dimension (possibly infinite) equal to the number of the generators of the Grassman algebra one may compose 1-forms $\Omega_\alpha = \xi_\alpha^{(n)}\, dy_n$. Then the vector current $j_\mu = -e\bar{\Psi}\gamma_\mu\Psi$ is zero if and only if $\Omega_1^* \wedge \Omega_1 = \Omega_4 \wedge \Omega_4^*$, $\Omega_2^* \wedge \Omega_2 = \Omega_3 \wedge \Omega_3^*$, $\Omega_2^* \wedge \Omega_1 = \Omega_4^* \wedge \Omega_3$, and $\Omega_1^* \wedge \Omega_2 = \Omega_3^* \wedge \Omega_4$. If, for example, $\Omega_1 \wedge \Omega_1^* \wedge \Omega_2^* \not\equiv 0$ and $\Omega_1^* \wedge \Omega_2 \wedge \Omega_2^* \not\equiv 0$, then $\Omega_1 \wedge \Omega_4^* = \Omega_2 \wedge \Omega_3^* = 0$, and the spinor Ψ may be converted into a Majorana spinor by a gauge transform. Thus, for a-spinors solutions of Maxwell–Dirac electrodynamics are Majorana solutions up to a gauge transform if, for example, sufficient transversality conditions are satisfied. These conditions seem to be rather weak.

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Photon polarization operator and the photon elastic scattering amplitude in (2+1) QED in a constant magnetic field

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In recent years investigation of (2+1) quantum field theories has been of great interest. In particular, they are applied widely to model various physical effects in thin layers [1].

In the present paper the photon elastic scattering amplitude in (2+1) QED in an external magnetic field is calculated.

The photon polarization operator in (2+1) QED in the one–loop approximation [2] is writtten as

$$P^{\mu\nu}(x',x) = -ie^2 Sp\Big[\gamma^\mu S(x,x')\gamma^\nu S(x',x)\Big], \tag{1}$$

where γ^μ matrices satisfy the following relations

$$\begin{aligned} &\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\epsilon^{\mu\nu\lambda}\gamma_\lambda,\ g^{\mu\nu} = diag(1,-1,-1), \\ &\gamma^0 = \sigma^3,\ \gamma^1 = i\sigma^1,\ \gamma^2 = i\sigma^2 \end{aligned} \tag{2}$$

and $\sigma^i (i = 1,2,3)$ are Pauli matrices.

We consider the case of a constant magnetic field, determined by the potential

$$A^\mu = (0, 0, x^1 H). \tag{3}$$

In this case the electron propagator has the following form [3]

$$\begin{aligned} &S^c(x,x') = -e^{-i\pi/4} 2\sqrt{\pi}\ * \\ &*\frac{h}{(4\pi)^2}\int_0^\infty ds \frac{e^{-ism^2}}{\sqrt{s}\sin(hs)} \exp\Big[-i\frac{X_0^2}{4s} + i\frac{hX_\perp^2 ctg(hs)}{4} - iuY\frac{h}{2}\Big] * \\ &*\Big\{\frac{1}{2s}\Big[\gamma^0 T - \frac{hs}{\sin(hs)}(\gamma X)_\perp e^{ihs\gamma^0}\Big] + m\Big\} e^{ihs\gamma^0}. \end{aligned} \tag{4}$$

Our notations are

$$\begin{aligned} &X^\mu = x^\mu - x'^\mu, \quad X_\perp^2 = (x^1 - x'^1)^2 + (x^2 - x'^2)^2, \\ &h = eH, \qquad u = x^1 + x'^1, \end{aligned} \tag{5}$$

m is the electron mass.

Calculations of (1) are analogous to those of the photon polarization operator in (3+1) QED in a constant magnetic field [4,5]. The photon polarization operator in (2+1) dimensional QED depends on the operators $\hat{P}^\mu$, $F^{\mu\nu}\hat{P}_\nu$, $F^{\mu\nu}F_{\nu\lambda}\hat{P}^\lambda$, commuting with $\hat{P}^\mu$ operator in the case of a constant magnetic field. Therefore the photon polarization operator is diagonal in the momentum representation and it assumes the form

$$\begin{aligned}\Pi_{\mu\nu}(k,k') &= \int e^{-i(kx-k'x')}\Pi_{\mu\nu}(x,x')d^3xd^3x' = \\ &= (2\pi)^3\delta(k-k')P_{\mu\nu}(k).\end{aligned} \tag{6}$$

In contrast to (3+1) QED, where the photon polarization operator is symmetrical, it is represented in (2+1) dimensional QED as a sum of symmetrical and antisymmetrical terms

$$P^{\mu\nu}(k,H) = P_s^{\mu\nu}(k,H) + P_a^{\mu\nu}(k,H),\ P_s^{\mu\nu} = P_s^{\nu\mu},\ P_a^{\mu\nu} = -P_a^{\nu\mu}. \tag{7}$$

The polarization operator calculated on the mass surface ($m^2 = 0$) determines the photon elastic scattering amplitude

$$T = \frac{1}{2\omega}e_\mu P_{reg}^{\mu\nu}e_\nu, \tag{8}$$

where $\omega = k + 0 =\mid \vec{k} \mid$ is the photon energy, e_μ is a polarization 3-vector of the photon. In this work we renormalized the photon polarization operator in a standard way

$$P_{\mu\nu}^{reg}(k,H) = P_{\mu\nu}(k,H) - P_{\mu\nu}(k,H=0) + P_{\mu\nu}(k), \tag{9}$$

where $P_{\mu\nu}(k)$ is the renormalized photon polarization operator in the case of zero field intensity [6].

According to the optical theorem from the amplitude (8) we find the rate of an electron-positron pair production by a photon

$$w = -2\mathrm{Im}T, \tag{10}$$

and the squared photon mass in a constant magnetic field

$$\delta(m^2) = 2\omega\mathrm{Re}T. \tag{11}$$

Photon polarization operator is determined in (2+1) QED by the one and only linear polarization vector. Presenting it in the form

$$e_\mu = \frac{l_\mu}{(-l^2)^{1/2}} = \frac{1}{\mid \vec{k} \mid}(0,k_2,-k_1),\ l_\mu = \frac{F_{\mu\nu}k^\nu}{(F_{\lambda\rho}^2)^{1/2}}, \tag{12}$$

and taking into account formulas (8) – (9), we finally obtain the amplitude of a photon elastic scattering as follows

$$T = \frac{e^{i\pi/4}e^2m}{(4\pi)^{3/2}\omega}\sqrt{2\frac{H_0}{H}}\int\limits_{-1}^{1}dv\int\limits_0^\infty\sqrt{\rho}d\rho\exp\Bigl(-2i\rho\frac{H_0}{H}\Bigr)\times$$
$$\times\Bigl(\frac{\exp(i\varphi)}{\sin(2\rho)}A(\rho,v)-\frac{1}{2\rho}\Bigl(1-\frac{i}{4\rho}\frac{H}{H_0}\Bigr)\Bigr]. \tag{13}$$

Here $H_0 = \frac{m^2}{e}$ stands for the analogue of the Schwinger critical value of the external field intensity and the functions φ and $A(\rho, v)$ are given by the followings expressions

$$\varphi = \frac{\omega^2}{h}\Bigl[\frac{\rho(1-v^2)}{2}-\frac{\sin(\rho(1-v))\sin(\rho(1+v))}{\sin 2\rho}\Bigr],$$
$$A(\rho,v) = \cos(2\rho v)-i\frac{\omega}{m}\sin(2\rho v)+\frac{H}{4H_0\rho}\cos(2\rho v)\Bigl[-i+\frac{\omega^2\rho(1-v^2)}{eH}\Bigr]-$$
$$-\Bigl(\frac{\omega}{m}\Bigr)^2\frac{\sin(\rho(1+v))\sin(\rho(1-v))}{\sin^2(2\rho)}. \tag{14}$$

Let us now consider the case of relatively weak magnetic fields and high energies of photons, which is expressed by the following inequalities

$$H \ll H_0, \qquad m \ll \omega. \tag{15}$$

In this approximation the region $\rho \ll 1$ gives the main contribution to the amplitude (13), hence expanding the trigonometric functions in (13), (14) we can write (13) as

$$T = -i\frac{e^{i\pi/4}e^2 2m}{(4\pi)^{3/2}\omega}\int\limits_1^\infty\frac{du}{u^{3/2}\sqrt{u-1}}\Bigl[1+\frac{8u-5}{3}\Bigr]\Bigl(\frac{\chi}{4u}\Bigr)^{1/3}G'(z), \tag{16}$$

where $z = (\frac{4u}{\chi})^{2/3}$ and $G(z)$ has the following form

$$G(z) = \int\limits_0^\infty\sqrt{y}dy\exp\Bigl(-iyz-i\frac{y^3}{3}\Bigr). \tag{17}$$

Thus the photon scattering amplitude (16) depends on the external field intensity and the photon energy via the parameter

$$\chi = \frac{H}{H_0}\frac{\omega}{m} = \sqrt{-\frac{e^2(F_{\mu\nu}k^\nu)^2}{m^6}}. \tag{18}$$

The consideration of the amplitude (16) in $\chi \gg 1$ and $\chi \ll 1$ limiting cases leads to the asymptotic expressions

$$
\begin{aligned}
T &= -i\frac{e^2 m 24}{(4\pi)^{3/2}5}\exp(-i\pi/6)\Gamma(1/2)\Big(\frac{\chi}{4\sqrt{3}}\Big)^{1/3}\Gamma(1/3), \quad \chi \gg 1, \\
T &= -\frac{e^2 m\chi^2}{4\pi\ 90\omega} - i\frac{e^2 m}{8\pi\omega}\sqrt{\frac{3\chi\pi}{8}}\exp\Big(-\frac{8}{3\chi}\Big), \quad \chi \ll 1.
\end{aligned} \tag{19}
$$

Comparing our results (19) with the corresponding results in (3+1) QED [7], one can see, that the growth of the photon elastic scattering amplitude in $\chi \gg 1$ limit in (2+1) QED is determined by the factor $\chi^{1/3}$, whereas in (3+1) QED it increases as $\chi^{2/3}$. Thus, considering the case when $\chi \gg 1$ we come to the conclusion that the reduction of the number of dimensions in QED diminishes the dependence of the one-loop contribution to the amplitude of photon elastic scattering by the factor $\chi^{1/3}$.

However we did not find any general regularity, connecting the changes in the number of dimensions in QED with the dependence of the physical values, examined in this work, on the dynamic parameter χ. Indeed, the imaginary part of the scattering amplitude, which in accordance with the optical theorem determines the rate of the electron- positron pair photoproduction contains in (2+1) QED in the limiting case, when $\chi \ll 1$ the preexponentional factor $\sqrt{\chi}$, whereas the preexponentional factor in the similar expression in (3+1) QED is χ. As to the real part of the amplitude in the limiting case, when $\chi \ll 1$ the results in (2+1) QED and (3+1) QED coincide and include the factor χ^2.

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ON MANIFOLD OF EXACT SOLUTIONS OF THE PROBLEM OF BOSONIZATION OF A PAIR OF 2D-ELECTRONS IN A QUANTINIZING MAGNETIC FIELD

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Abstract. It is shown that for a certain relation between the magnetic field strength and the electron charge the non relativistic quantum problem of the correlated motion of a pair of 2D-electrons in a constant and uniform magnetic field admits exact solutions in the form of elementary functions.

In recent times the problem of a pair of 2D-electrons in a magnetic field has attracted attention in connection with experimental and theoretical investigations of the fractional quantum Hall effect [1-3]. It was pointed out in ref.[2] that the half-integer quantum Hall effect can be understood as a result of the coupling of electrons with opposite spins, producing a boson with a charge $2e$,which is in the symmetric Laughlin's state [4].

In is known that for the non relativistic problem of the motion of two 2D-electrons in a constant and uniform magnetic field, in the Pauli equation the spin is separated from the spatial motion, which in turn allows the center-of-mass motion to be separated from the relative motion.

The coordinate part of the relative motion wave function (symmetrical gauging of the vector potential is employed) has the form

$$\psi = \exp(-im\varphi) R(r),$$

the even values of the quantum number m corresponding to the singlet state the odd values to the triplet state. The function $R(r)$ obeys the Schroedinger radial equation

$$\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{m^2}{r^2} - \frac{r^2}{4} - \frac{a}{r} + B_r\right) R(r) = 0. \qquad (1)$$

Here $a = M\left(2ce^3/\left(\hbar^3 H\right)\right)^{1/2}$ and the magnetic length $l = (2\hbar c/(eH))^{1/2}$ is used as a length unit; e, M and H are charge and mass of the electron and the magnetic field strength, respectively. The constant B_r in eq. (1) and the quantized energy of relative motion of electrons E_r are connected by the relation $B_r = 2E_r/(\hbar\omega) + n$, where ω is the cyclotron frequency.

If the Coulomb repulsion of electrons is neglected than $a = 0$ and the solutions of eq. (1) belonging to the metric $L^2(0,\infty; rdr)$ are expressed in terms of elementary functions, and the corresponding eigenvalues are the Landau levels $E_r = \hbar\omega\left(n_r - (m - |m|)/2 + 1/2\right)$, $\quad n_r = 0, 1, \ldots$ The radial quantum number n_r is the number of nodes of the radial wave function $R(r)$.

The presence of Coulomb repulsion results in the splitting of the Landau level; for arbitrary values of the parameter a the eigenfunctions of eq. (1) cannot be expressed even through functions of the hypergeometrical type [5].

In this connection it seems worth nothing that for a certain special mapping of values of the parameter a there are exact solutions of the problem under investigation in terms of elementary functions and there is a simple formula for the energy levels. The above-mentioned mapping of values of the parameter a can easily be established using the Witten technique of super-symmetrical quantum mechanics [6].

Let $U : L^2\left(0,\infty; \exp\left(-r^2/2\right) r^{2|m|+1} dr\right)$ be the unitary representation of the type $U : f(r) \longrightarrow r^{|m|} \exp\left(-r^2/2\right) f(r)$ (see e.g. ref. [7]). For $L^2\left(0,\infty; \exp\left(-r^2/2\right) r^{2|m|+1} dr\right)$ the initial problem of solving eq. (1) is equivalent to that of eigenvalues of the operator

$$\hat{B} = -\partial_r^2 - ((2|m|+1)/r - r)\,\partial_r + a/r. \tag{2}$$

This operator is easily seen to be limited from below and can be represented as

$$\hat{B} = \hat{A}^{+}\hat{A} + k, \tag{3}$$

the eigenvalues of k are connected with the eigenvalues of eq. (1) by the relation $k = 2E_r/(\hbar\omega) + m - |m| - 1$, and the eigenfunctions will satisfy the first order equation

$$\hat{A}\,\phi = 0. \tag{4}$$

Suppose that solutions of the polynomial type exist for eq. (4). Then operator $\hat{A}$ should have the form $\hat{A} = \partial_r - \sum_{s=1}^{n} 1/(r + r_s)$, where r_s are the roots of the polynomial sought for, and n is its power. Here for the operator $\hat{A}^{+}$ in the metric $L^2\left(0,\infty;\exp\left(-r^2/2\right) r^{2|m|+1}dr\right)$ we obtain

$$\hat{A}^{+} = -\partial_r - (2|m|+1)/r + r - \sum_{s=1}^{n} (r+r_s)^{-1}$$

now, comparing eqs. (3) and (2), it is easy to obtain the systems of equations to determine the constant r_s :

$$(2|m|+1)/r_s - r_s - 2\sum_{j\neq s}^{n} (r_j - r_s)^{-1} \quad (n=1). \tag{5}$$

Besides, the following relation must be satisfied:

$$a = (2|m|+1)\sum_{s=1}^{n} r_s^{-1}, \quad k = n. \tag{6}$$

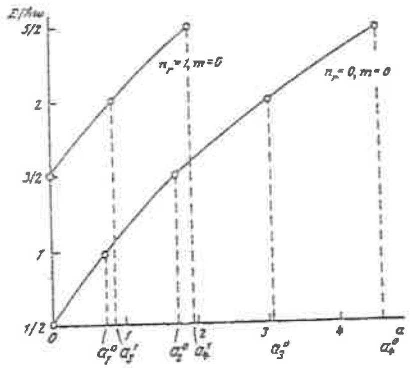

Fig. 1. These curves qualitatively show the behavior of the dependence of $E/\hbar\omega$ on the parameter a. The circles indicate the values of a at which eq. (1) has exact solutions expressable in terms of elementary functions. In the symbol a_j^i the upper index refers to the value of the radial number n_r and the lower upper index refers to n, the power of the polynomial. $a_1^0 = 1$; $a_2^0 = \sqrt{6}$; $a_3^0 = \left(10+\sqrt{73}\right)^{1/2}$; $a_4^0 = \left(50+\sqrt{297}\right)^{1/2}$; $a_3^1 = \left(10-\sqrt{73}\right)^{1/2}$; $a_4^1 = \left(50-\sqrt{297}\right)^{1/2}$.

But if $n = 0$, then $a = 0$ and $k = 0$. Note that by summing all equations of the system (5), one can obtain a somewhat different representation for the parameter $a = \sum_{s=1}^{n} r_s$.

For the case of $m = 0$ corresponding to the upper limit of the multiplet structure of each particular Landau level, the values of the parameter a obtained with the help of eqs. (5) and (6) are shown in fig. 1.

For the application of the obtained results to the calculations in quasi-2D heterostructures, the values of M and e in determination of the value of the parameter a should be understood as effective values of the mass and the charge of a 2D-electron, which can differ from the values for a bulk sample and may also vary from sample to sample.

Recent experimental investigations of quasi-two-dimensional structures [8] reveal anomalies of the cyclotron resonance that manifest themselves at low values of the electron density. The cyclotron resonance is fundamental the study of the dynamical properties of electron systems, and authors of ref. [8] point to the possible role of many-particle effects as an explanation of the anomalies.

It is important to note that the values of the experimental parameters are close to those which are necessary for the test of our theory.

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A PAIR PRODUCTION OF PIONS BY A CONSTANT ELECTROMAGNETIC FIELD

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Abstract

The differential probability for pairs of pions production by an external constant electromagnetic field is found on the base of exact solutions of the equation for pions. Obtained expressions generalise the Schwinger formulas for electron-positron production taking into account the complex structure of pions. At the critical value of the electric field the number of produced pions is increased by about 20% comparing to the pointlike pions. We found also the imaginary part of the effective lagrangian for the electromagnetic field in the presence of pions.

It is known that a constant electric field produces pairs of particles [1].So the probability for the electron-positron production and pairs of scalar pointlike particles production were calculated. But the intensity of the production for real pions will be changed due to its electromagnetic polarizabilities. One of the known ways to obtain it is to use the exact solutions of the equation of motion for particles in the external electromagnetic fields [2]. All physical values can be obtained through the asymptotics of the solutions of the wave equations [3]. For the constant electric field there exist four different simple solutions possess clear physical meaning. These solutions correspond to positive and negative frequencies at $t \to \pm\infty$. Using the Bogolubov transformations the Hamiltonian can be diagonalized. As a result, matrix elements are given by the coefficients of the Bogolubov transformations.

We have recently found some solutions of the equation for real pions [4] in the electromagnetic fields of different configurations [5,6]. In this work we apply the Bogolubov method and obtained solutions [6] for finding the differential probability of pions production in the uniform static magnetic field parallel to the constant electric field. The problem we treat is a generalization of one discussed by Nikishov [2,8,9]. The equation of motion for pions established from the chiral theory [4] and from the phenomenological approach [7] is given by

$$D_\mu^2\phi - D_\mu[(D_\nu\phi)K_{\mu\nu}] - m_{eff}^2\phi = 0, \tag{1}$$

where $m_{eff}^2 = m^2(1 - \beta F_{\mu\nu}^2/2m)$, $D_\mu = \partial_\mu - ieA_\mu$, $\partial_\mu = \partial/\partial x_\mu$, A_μ is the vector potential of the electromagnetic field, $K_{\mu\nu} = (\alpha + \beta)F_{\mu\alpha}F_{\nu\alpha}/m$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength tensor, α, β are the electric and magnetic polarizabilities of a particle respectively, m is the rest mass of a pion. Units are chosen such that $\hbar = c = 1$. Without loss of generality we use the potential as follows

$$A_\mu = (0, x_1 H, -tE, 0). \tag{2}$$

In this case $\mathbf{H}\|\mathbf{E}$ or $\mathbf{E} = \mathbf{n}E$, $\mathbf{H} = \mathbf{n}H$, where $\mathbf{n} = (0,0,1)$ is a unit vector. The solutions to Eq.(1) for the potential (2) exist in the form [6]

$${}^{\pm}_{\pm}\phi_{\mathbf{p},n}(x) = Ne^{i(p_2x_2+p_3x_3)}e^{-\frac{\eta^2}{2}}H_n(\eta)\,{}^{\pm}_{\pm}\Psi(\tau), \tag{3}$$

where $H_n(\eta)$ being Hermit polynomials, $\eta = (eHx_1 + p_2)/\sqrt{eH}$ and ${}^{\pm}_{\pm}\Psi(\tau)$ give four functions having different asymptotics:

$$\begin{gathered} {}_{+}\Psi(\tau) = D_\nu[-(1-i)\tau], \quad {}^{-}\Psi(\tau) = D_\nu[(1-i)\tau], \\ {}^{+}\Psi(\tau) = D_{\nu*}[(1+i)\tau], \quad {}_{-}\Psi(\tau) = D_{\nu*}[-(1+i)\tau], \end{gathered} \tag{4}$$

with $\nu = ik^2/2eEB - 1/2$, $\tau = \sqrt{eE}(x_0 + p_3/eE)$, $B = 1 + WE^2$, $W = (\alpha+\beta)/m$; $D_\nu(x)$ being the Weber-Hermit functions. Here the parameter k^2 which is connected with the energy of the pion in the constant electromagnetic field has the quantized value [5,6]:

$$k^2 = m_{eff}^2 + eHA(2n+1), \tag{5}$$

where $A = 1 - WH^2$ and n being the principal quantum number: $n = 1, 2, \ldots.$ When electromagnetic polarizabilities tend to zero the formula (5) gives us Landau levels of the energy for the particle moving in the constant magnetic field. Our discussion follows very closely the work of Nikishov [2,8]. Functions ${}^{+}_{+}\Psi(\tau)$ correspond to the solutions to Eq.(1) with the positive frequency at $t \to \pm\infty$ and ${}^{-}_{-}\Psi(\tau)$ - with the negative frequency at $t \to \pm\infty$. Solutions (3), (4) are labeled by three conserved numbers p_2, p_3, k^2, where

p_2, p_3 being the momentum projections. The constant N is determined from the condition of the normalization

$$\int {}^{\pm}\phi^*_{\mathbf{p},n}(x) i \overleftrightarrow{\frac{\partial}{\partial t}} {}^{\pm}\phi_{\mathbf{p}',n'}(x) d^3x = \pm\delta(\mathbf{p}-\mathbf{p}')\delta_{n,n'}, \tag{6}$$

where $\mathbf{p} = (p_2, p_3)$ is two-dimensional vector, $\psi \overleftrightarrow{\frac{\partial}{\partial t}} \phi = \psi \frac{\partial}{\partial t}\phi - \phi\frac{\partial}{\partial t}\psi$. The sets of solutions ${}^{\pm}\phi_{\mathbf{p},n}(x)$ and ${}_{\pm}\phi_{\mathbf{p},n}(x)$ are equivalent and therefore they can be connected by the relations [8]

$$\begin{gathered} {}_+\phi_{\mathbf{p},n}(x) = c_{1n}{}^+\phi_{\mathbf{p},n}(x) + c_{2n}{}^-\phi_{\mathbf{p},n}(x), \quad {}^+\phi_{\mathbf{p},n}(x) = c^*_{1n}{}_+\phi_{\mathbf{p},n}(x) - c_{2n}{}_-\phi_{\mathbf{p},n}(x), \\ {}_-\phi_{\mathbf{p},n}(x) = c^*_{2n}{}^+\phi_{\mathbf{p},n}(x) + c^*_{1n}{}^-\phi_{\mathbf{p},n}(x), \quad {}^-\phi_{\mathbf{p},n}(x) = -c^*_{2n}{}_+\phi_{\mathbf{p},n}(x) + c_{1n}{}_-\phi_{\mathbf{p},n}(x), \end{gathered} \tag{7}$$

where $| c_{1n} |^2 - | c_{2n} |^2 = 1$. Coefficients c_{1n}, c_{2n} found from (3), (4), (7) are given by (see [3,8])

$$\begin{gathered} c_{1n} = (2\pi)^{\frac{1}{2}} \Gamma^{-1}\left(\frac{1-i\lambda}{2}\right) \exp[-\frac{\pi}{4}(\lambda - i)], \\ c_{2n} = \exp\left[-\frac{\pi}{2}(\lambda + i)\right], \quad \lambda = \frac{m^2_{eff} + eHA(2n+1)}{eEB} \end{gathered} \tag{8}$$

and Γ being the Γ -function. Values c_{1n}, c_{2n} contain the information about producing pairs of pions in the state n.

The quantized solution to Eq.(1) can be written as

$$\begin{aligned} \phi(x) &= \sum_{\mathbf{p},n} [a_{\mathbf{p},n}(in){}_+\phi_{\mathbf{p},n}(x) + b^+_{\mathbf{p},n}(in){}_-\phi_{\mathbf{p},n}(x)] = \\ &= \sum_{\mathbf{p},n} [a_{\mathbf{p},n}(out){}^+\phi_{\mathbf{p},n}(x) + b^+_{\mathbf{p},n}(out){}^-\phi_{\mathbf{p},n}(x)], \end{aligned} \tag{9}$$

where $a_{\mathbf{p},n}(in)$, $b^+_{\mathbf{p},n}(in)$, $(a_{\mathbf{p},n}(out)$, $b^+_{\mathbf{p},n}(out))$ are the operators of the annihilation for the particle and the operator of the creation for the antiparticle respectively at $t \to -\infty$ $(t \to +\infty)$.

From (7),(9) we arrive at the Bogolubov transformations

$$\begin{gathered} a_{\mathbf{p},n}(out) = c_{1n}a_{\mathbf{p},n}(in) + c^*_{2n}b^+_{\mathbf{p},n}(in), \quad b^+_{\mathbf{p},n}(out) = c_{2n}a_{\mathbf{p},n}(in) + c^*_{1n}b^+_{\mathbf{p},n}(in), \\ a_{\mathbf{p},n}(in) = c^*_{1n}a_{\mathbf{p},n}(out) - c^*_{2n}b^+_{\mathbf{p},n}(out), \quad b^+_{\mathbf{p},n}(in) = -c_{2n}a_{\mathbf{p},n}(out) + c_{1n}b^+_{\mathbf{p},n}(out). \end{gathered} \tag{10}$$

The canonical Bogolubov transformations like (10) are widely used in the superfluid and the superconductors theories. These transformations conserve the commutation relations.

The unitarity operator of the transformation S is defined by [10]

$$S^{-1}a_{\mathbf{p},n}(in)S = a_{\mathbf{p},n}(out), \quad S^{-1}b^{+}_{\mathbf{p},n}(in)S = b^{+}_{\mathbf{p},n}(out),$$

$$S = \prod_n S_n, \quad < 0 \mid = \prod_n < 0_n \mid, \quad < 0_n(out) \mid = < 0_n(in) \mid S_n, \tag{11}$$

$$a_{\mathbf{p},n}(out) \mid 0_n(out) > = b_{\mathbf{p},n}(out) \mid 0_n(out) > = 0,$$

so that S_n found from (10) is given by

$$S_n = c^{*-1}_{1n} \exp\left(- \frac{a_{\mathbf{p},n}(in) b_{\mathbf{p},n}(in) c_{2n}}{c^{*}_{1n}} \right). \tag{12}$$

The amplitude of the probability in the state n is equal

$$< 0_n(out) \mid 0_n(in) > = c^{*-1}_{1n} \tag{13}$$

and the probability for a vacuum remains a vacuum to be

$$C_V = \mid \prod_n c^{*-1}_{1n} \mid^2 = \exp(-\sum_n \ln \mid c_{1n} \mid^2) = \exp[-\sum_n \ln(1 + \mid c_{2n} \mid^2)]. \tag{14}$$

It is easy to check using the commutation relations that the expression holds

$$< 0_n(out) \mid a^{+}_{\mathbf{p},n}(out) a_{\mathbf{p},n}(out) \mid 0_n(in) > = \mid c_{2n} \mid^2 . \tag{15}$$

Then the average number of created pairs of particles from a vacuum is [8]

$$\bar{N} = \int \sum_n \mid c_{2n} \mid^2 dp_2 dp_3 \frac{L^2}{(2\pi)^2}, \tag{16}$$

where L is cutoff along the coordinates.

Now we calculate the value $\bar{N}$ (16). The variables $\eta = (eHx_1 + p_2)/\sqrt{eH}$ and $\tau = \sqrt{eE}(x_0 + p_3/eE)$ which enter into the solutions (3) define the region of forming the process. The coordinates of the center of this region are $t_0 = -p_3/eE$, $x_1 = -p_2/eH$. Therefore we may use the substitution [2]

$$\int dp_2 \to eHL, \qquad \int dp_3 \to eET, \tag{17}$$

where T being the time of the observation. Inserting the value c_{2n} (8) into the variable (16) with the help (17) we obtain the intensity of creating pions

$$I(E,H) = \frac{1}{VT}\bar{N} = \frac{1}{VT}\int \frac{dp_2 dp_3 L^2}{(2\pi)^2}\sum_n \exp\{-\frac{\pi[m_{eff}^2 + eHA(2n+1)]}{eEB}\} = \\ = \frac{e^2EH}{8\pi^2}\,\frac{\exp(-\pi m_{eff}^2/eEB)}{\mathrm{sh}(\pi HA/EB)}. \tag{18}$$

If electromagnetic polarizabilities $\alpha,\beta \to 0$ ($A, B \to 1$, $m_{eff} \to m$) Eq.(18) leads to the well-known expression for the probability of production of scalar pointlike bosons [8]. Taking into account the definitions $B = (1 + (\alpha + \beta)E^2/m)$, $m_{eff}^2 = m^2(1+\beta(E^2-H^2)/m)$ and the approximations $\alpha E^2/m \ll 1$, $\beta E^2/m \ll 1$ which is held for real pions [11] one proceeds by noting the relations

$$\frac{A}{B} \simeq 1 - \frac{\alpha+\beta}{m}(E^2+H^2), \quad \frac{m_{eff}^2}{B} \simeq m^2[1 - \frac{\alpha E^2 + \beta H^2}{m}]. \tag{19}$$

Then Eq.(18) turns into

$$I(E,H) = I^{(0)}[1 + \frac{\alpha+\beta}{m}\pi\frac{H}{E}(E^2+H^2)\coth\frac{\pi H}{E}]\exp[\frac{\pi m}{eE}(\alpha E^2 + \beta H^2)], \tag{20}$$

where $I^{(0)} = (e^2E^2/8\pi^3)\exp(-\pi m^2/eE)$ corresponds to the pointlike scalar boson. It is easily seen that polarizabilities of pions will increase the pairs production. When the magnetic field is switched off then Eq.(18) transforms into

$$I(E) = \lim_{H\to 0} I(E,H) == \frac{e^2E^2B}{8\pi^3}\exp(-\frac{\pi m_{eff}^2}{eEB}). \tag{21}$$

By using the approximations (19) the value (21) is rewritten in the form

$$I(E) = I^{(0)}(1 + \frac{\alpha+\beta}{m}E^2)\exp(\frac{\pi\alpha E m}{e}). \tag{22}$$

In the presence of the magnetic field the average number of created pions is decreased by the factor

$$\frac{I(E,H)}{I(E)} = \frac{\pi H}{EB\mathrm{sh}(\pi HA/EB)}. \tag{23}$$

At $H \gg E$ we have $I(E,H) \to 0$. But in this case the intensity of creating electron-positron pairs will be increased [2].

We can also obtain the expressions (21), (22) using the exact solutions to Eq.(1) in the pure electric field. Then the solutions become

$$^{\pm}_{\pm}\phi_{\mathbf{p}}(x) = N\exp[i(p_1x_1 + p_2x_2 + p_3x_3)]^{\pm}_{\pm}\psi(\tau), \tag{24}$$

where the functions $^{\pm}_{\pm}\psi(\tau)$ are given by (4) with the index ν for cylindrical functions (4):

$$\nu = \frac{i[p_1^2 + p_2^2 + m_{eff}^2]}{2eEB} - \frac{1}{2}. \tag{25}$$

From solutions (24) at $\alpha = \beta = 0$, we shall obtain the solutions to the Klein-Gordon equations which were found by Narozhnyi and Nikishov [3].

The density of created pairs of pions is

$$\mid c_{2p} \mid = \mid \exp(\pi i\nu) \mid^2 . \tag{26}$$

We can obtain the number of created pairs by integrating the expression (3.26) over the momentum $\mathbf{p}$. We may replace

$$\int dp_3 \to eET, \tag{27}$$

and after integrating we have the rate of creating pions (see [9])

$$I(E) = \frac{1}{T}\int \frac{d^3p}{(2\pi)^3} \mid c_{2p} \mid^2 . \tag{28}$$

It is easy to check that the expression (28) (after integrating over p_1,p_2) coincides with (21).

The magnitude of

$$I^{(0)} = \frac{e^2E^2}{8\pi^3}\exp(-\frac{\pi m^2}{eE}) \tag{29}$$

which enters into (22) is small at $E \ll m^2/e$. Since the mass of the pion is more than that of the electron ($m > m_e$) it is easier to produce electron-positron pairs. But if $E \simeq m^2/e$ for pions ($m = 140MeV$, $\alpha = 11 \cdot 10^{-4} fm^3$) $I \simeq 1,19 \cdot I^{(0)}$, $I^{(0)} \simeq 1,1 \cdot 10^{58}(s^{-1} \cdot sm^{-3})$. So the number of produced charged pions, taking into account of polarizabilities is increased by about 20% comparing with the pointlike pions.The $I^{(0)}$ is non-analytic function of $\mathbf{E}$ and it is impossible to obtain (21),(22) using the perturbative theory.

For neutral pions we must set $e = 0$ in Eq.(1). The solutions to Eq.(1) in this case are like solutions for free particles $\phi = C\exp(\pm \imath p_\mu x_\mu)$ but the squared energy of pions is

$$p_0^2 = (p_1^2 + p_2^2)\Big[1 - \frac{\alpha+\beta}{m}(E^2 + H^2)\Big] + p_3^2 + m^2\Big[1 - \frac{\alpha E^2 + \beta H^2}{m}\Big]$$

at $\alpha E^2/m \ll 1$, $\beta H^2/m \ll 1$. In this approximation there is no creation of neutral pions. The same conclusion follows from (22) at $e = 0$. But if $\alpha E^2/m > 1$ the situation is changed due to the complex value of p_0. It occures for huge fields $E > \sqrt{m/\alpha}$. Even for the critical fields $E_c = m^2/e \simeq 3.46 \cdot 10^{18} G$ we have the small value $\alpha E^2/m \simeq 0.05$.

Now we find the imaginary part of the density of the Lagrangian. The squared of the amplitude for a vacuum-vacuum C_V is connected with the imaginary part of the Lagrangian [1]

$$C_V = \exp(-2\mathrm{Im}\ L). \tag{30}$$

From comparing of (14) and (30) we can write

$$VT\mathrm{Im}\ \mathcal{L} = \frac{1}{2}\int\sum_n \ln\mid c_{1n}\mid^2 \frac{dp_2 dp_3 L^2}{(2\pi)^2}, \tag{31}$$

where $\mathcal{L}$ being the density of the Lagrangian L. Substituting values (8) into Eq.(31) and using (17) we have

$$\mathrm{Im}\ \mathcal{L} = \frac{e^2 EH}{8\pi^2}\sum_n \ln(1 + \exp(-\pi\lambda)) =$$

$$\begin{aligned} &\frac{e^2 EH}{8\pi^2}\sum_{n=0}^{\infty}\ln\{1 + \exp\{-\frac{\pi[m_{eff}^2 + eHA(2n+1)]}{eEB}\}\} = \\ &= \frac{e^2 EH}{16\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\frac{\exp(-\pi m_{eff}^2 n/eEB)}{\mathrm{sh}(\pi HAn/EB)}. \end{aligned} \tag{32}$$

The first term of the sum (32) gives us the value $I(E,H)/2$ (18) as it must be. At $\alpha,\beta \to 0$ we arrive to the expression which was obtained by Nikishov [8]. The limit of the expression (32) at $H \to 0$ can be easily obtained. This limit is agreed with Eq.(21),(22). The formula (32) generalizes the Schwinger result [1] in the case the pions polarizabilities are taken into account . The investigation of quantum effects in the strong field is actually important. It is stipulated by the progress in the techniques of the lasers and accelerators of heavy-ions. If the charge of a nucleus exceeds the critical value $Z_c \simeq 170$ then the normal ground state of QED becomes unstable. The electric field will give the emission of positrons and charged vacuum will be formed [12]. Then pairs of particles can be created.

In this paper we considered the pair production of pions by constant electromagnetic fields which are some approximation to the actual field configurations. This approximation can be applied when the size of heterogeneities of fields exceeds the size of the considered system [13]. Particles are created on the typical distance $l = m/eE$ from each other, because the work of the electric field eEl must be equal to the rest energy m of a particle. For the critical field $E_c = m^2/e$ the typical value l is a Compton length $l = 1/m$ which is small. We see that the constant electromagnetic field approximation is valid for the wide class of fields when the typical heterogeneity size, is more than l. When the source of the electromagnetic field is the laser beam with the length wave λ this approximation corresponds to the condition $\lambda > l$ which is easily reached for real laser beams. This is the case of crossed electric and magnetic fields. Here we investigated only the influence of parallel electric and magnetic fields on the pions creation. The method [2,8] allow us to apply obtained solutions [6] for considering other cases of external electromagnetic fields.

In this work we estimated the influence of polarizabilities on creating pions. Therefore it was considered only effective lagrangian for pions up to quadratic terms in $F_{\mu\nu}$. The higher terms in $F_{\mu\nu}$ describe other characteristics of a particle which were not discovered eyt experimentally.

Although the effect due to the complex structure of pions is numerically small it can have a significance at the large strength of the electric field.

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Radiation of relativistic dipoles

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Abstract

A systematic exposition will be given of the classical theory of radiation of relativistic point dipoles. The properties are studied of electromagnetic fields created by a point magnetic moment — magneton. The electromagnetic field tensor of a magneton satisfying Maxwell's equation is obtained. It is shown that, depending on the distance to the magneton $\tilde{r}$ the field tensor splits in a covariant manner into three parts proportional to $\tilde{r}^{-1}$, $\tilde{r}^{-2}$, $\tilde{r}^{-3}$. In a absolute system of rest, where $\beta = \dot{\beta} = \ddot{\beta} = 0$($\beta = \mathbf{u}/c, \mathbf{u}$ is the velocity of magneton), known results are obtained as a particular case. The relationship between the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ radiated by a point magnetic dipole moment μ and a point electric dipole moment ν is derived through the use of dual transformations of the electromagnetic field tensor. It is assumed that each moment is in relativistic and otherwise arbitrary motion. In the relativistic case, as in the nonrelativistic case, the switch $\mu \to \nu$ is accompanied by the replacements $\mathbf{H} \to \mathbf{E}, \mathbf{E} \to \mathbf{H}$. A covariant formalism is developed for describing the electromagnetic fields in the wave zone. The electromagnetic field tensor associated with the radiation is analyzed.

The electrodynamic of relativistic point dipoles has been considered in papers of Frenkel' (see [1]), Bialas [2], Kolsrud and Leer [3], Cohn and Wiebe [4], and others (see also [5]). Here and in what follows we use the theory and formalism of [6].

1 The equations of electromagnetic field

Let's introduce the polarized tensor potential of a relativistic magneton $\mathbf{F}^{\mu\nu} = (\mathbf{F}, \mathbf{G})$, connected with the usual vector potential $A^{\mu} = (\varphi, \mathbf{A})$ by the relation

$$A^{\mu} = \partial_{\nu} F^{\mu\nu}, \tag{1}$$

that is

$$A^{\mu} = \left(-div\mathbf{G}, \frac{1}{c}\frac{\partial \mathbf{G}}{\partial t} + rot\mathbf{F}\right).$$

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In agreement with this definition, the vector **G** is the Hertz vector or the electric polarization potential. The vector **F** in such case can be called the magnetic polarization potential.

The tensors of the magnetic polarization density $M^{\mu\nu} = (\mathrm{N}, \mathrm{M})$ and the density of the "magnetic current" j^{μ} satisfy the relation (see, for example, [7])

$$j^{\mu} = \partial_{\nu} M^{\mu\nu}.$$

It is easy to see that tha Lorentz condition and the condition of contituity of the current density are satisfied automatically

$$\partial_{\nu} A^{\nu} = 0, \quad \partial_{\nu} j^{\nu} = 0.$$

The tensor-potential $F^{\mu\nu}$ satisfies the tensor wave equation [8]

$$\partial^{\rho}\partial_{\rho} F^{\mu\nu} = -\frac{4\pi}{c} M^{\mu\nu}.$$

Differentiating this equation with respect to the coordinates, one can obtain the usual wave equation for a vector potential.

Let us represent the tensor-potential $F^{\mu\nu}$ in a form analogous to the four-dimentional Lienard-Wiechert potential

$$F^{\rho\nu} = -\frac{\mu c}{\tilde{r}_{\sigma} v^{\sigma}} \Pi^{\rho\nu}, \tag{2}$$

where μ is the magnitude of the magnetic moment of the magneton; $v^{\rho} = dr^{\rho}/d\tau$ is its four-dimentional velocity; $\tilde{r}^{\rho} = R^{\rho} - r^{\rho}$ is a four-dimentional vector drawn from the world point of the magneton to the world point of the observer. The mutual orientation of the vectors $\tilde{\mathbf{r}}, \mathbf{R}, \mathbf{r}$ in three-dimentional space is shown in Fig. 1.

The dimentionless spacelike tensor $\Pi^{\mu\nu} = (\mathbf{\Phi}, \mathbf{\Pi})$ is connected with the magnetic polarization tensor by the relation

$$M^{\rho\nu}(t) = \mu \pi^{\rho\nu} \delta\left[\xi - \mathbf{r}(t)\right],$$

where $\pi^{\rho\nu} = \Pi^{\rho\nu}/\gamma$, $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz-factor; ξ is the vector of the "smeared" magneton at the moment of time t, connected with the moment of observation of the radiation $\tilde{t}$ by the relation $\tilde{t} = t - \tilde{r}/c$.

From the condition of spacelikeness of the tensor $\Pi^{\mu\nu}$

$$v_{\mu} \Pi^{\mu\nu} = 0$$

it follows that

$$\mathbf{\Phi} = [\beta \mathbf{\Pi}],$$

and, therefore, in the rest system of the magneton $\Pi_0^{\mu\nu} = (\mathbf{0}, \zeta)$, where ζ is a unit vector, which, according to the hypothesis of Ulenbeck and Goudsmit, we will call the spin vector. The tensor $\Pi^{\mu\nu}$ then represents the laboratory spin tensor (in dementionless form).

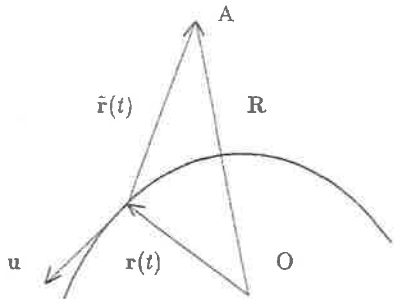

Fig. 1

2 Potentials and fields of a relativistic magneton

According to (1) and (2), the vector potential of the magneton A^σ at the observation point has the form

$$A^\sigma = -\mu c \partial_\nu \frac{\Pi^{\sigma\nu}}{\tilde{r}_\rho v^\rho}, \tag{3}$$

where the derivative ∂_ν is determined by the rules of differentiation of functions with a retarded argument

$$\partial_\nu = \tilde{\partial}_\nu + \frac{\tilde{r}_\nu}{\tilde{r}_\rho v^\rho} \frac{d}{d\tau}, \tag{4}$$

where $\tilde{\partial}_\nu = \partial / \partial \tilde{r}^\nu$.

Differentiating in (3) according to the rules (4), we obtain

$$A^\sigma = -\mu c \left[\frac{\overset{\circ}{\Pi}{}^{\sigma\nu}\, \tilde{r}_\nu}{(\tilde{r}_\rho v^\rho)^2} - \frac{\Pi^{\sigma\nu} \tilde{r}_\nu}{(\tilde{r}_\rho v^\rho)^3} \left(c^2 + \tilde{r}_\rho w^\rho \right) \right], \tag{5}$$

where $w^\rho = dv^\rho / d\tau$ is the four-dimentional acceleration of the magneton, the derivative with respect to the proper time is written with the circle. The components $A^\sigma = (\varphi, \mathbf{A})$ are represented in the form

$$\mathbf{A} = -\frac{\mu}{\gamma^2 [1 - (\mathbf{n}\beta)]^2} \left\{ \frac{1}{\tilde{r} c} \left[(\mathbf{n} - \beta)\, \overset{\circ}{\Pi} \right] + \frac{1}{\tilde{r}^2 \gamma} \left[(\mathbf{n} - \beta) \Pi \right] J \right\}, \tag{6}$$

$$\varphi = \frac{\mu}{\gamma^2 [1 - (\mathbf{n}\beta)]^2} \left\{ \frac{1}{\tilde{r} c} \left(\mathbf{n}\, \overset{\circ}{\Phi} \right) + \frac{1}{\tilde{r}^2 \gamma} (\mathbf{n}\Phi) J \right\},$$

where $\mathbf{n} = \bar{\mathbf{r}}/\bar{r}^0 = \bar{\mathbf{r}}/c(\bar{t} - t)$ is a unit vector directed from the magneton to the observation point,

$$J = 1 + \frac{1}{c^2}\bar{r}_\rho w^\rho = 1 + \frac{1}{c}\bar{r}\gamma^2\{(\mathbf{n}\alpha) - \gamma^2(\alpha\beta)[1 - (\mathbf{n}\beta)]\}.$$

Here we have introduced the notation $\alpha = \dot{\beta} = \mathbf{a}/c$, where $\mathbf{a}$ is the acceleration of the magneton.

Using the standard definition of the electromagnetic field tensor, on the basis of the potentials (5) and differentiation rules (4), we obtain

$$H^{\alpha\beta} = H_1^{\alpha\beta} + H_2^{\alpha\beta} + H_3^{\alpha\beta} \tag{7}$$

$$H_1^{\alpha\beta} = \frac{\mu c}{\bar{r}_\rho v^\rho}\left\{\frac{\overset{\circ\circ}{\Pi}{}^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]}}{(\bar{r}_\rho v^\rho)^2} - \frac{3\bar{r}_\rho w^\rho\, \overset{\circ}{\Pi}{}^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]} + \bar{r}_\rho\, \overset{\circ}{w}{}^\rho\Pi^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]}}{(\bar{r}_\rho v^\rho)^3} + \right.$$

$$\left. + \frac{(\bar{r}_\rho w^\rho)^2\Pi^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]}}{(\bar{r}_\rho v^\rho)^4}\right\},$$

$$H_2^{\alpha\beta} = \frac{\mu c}{(\bar{r}_\rho v^\rho)^2}\left\{2\, \overset{\circ}{\Pi}{}^{\alpha\beta} - \frac{\bar{r}_\rho w^\rho\Pi^{\alpha\beta}}{\bar{r}_\rho v^\rho} - \right.$$

$$- \frac{\overset{\circ}{\Pi}{}^{[\alpha\sigma}v_\sigma\bar{r}^{\beta]} + 2\, \overset{\circ}{\Pi}{}^{[\alpha\sigma}\bar{r}_\sigma v^{\beta]} + \Pi^{[\alpha\sigma}\bar{r}_\sigma w^{\beta]}}{\bar{r}_\rho v^\rho} -$$

$$\left. -3\,\frac{c^2\, \overset{\circ}{\Pi}{}^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]} - \bar{r}_\rho w^\rho\Pi^{[\alpha\sigma}\bar{r}_\sigma v^{\beta]}}{(\bar{r}_\rho v^\rho)^2} + 6\frac{c^2\bar{r}_\rho w^\rho\Pi^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]}}{(\bar{r}_\rho v^\rho)^3}\right\},$$

$$H_3^{\alpha\beta} = -\frac{\mu c}{(\bar{r}_\rho v^\rho)^3}\left\{2\Pi^{\alpha\beta} - 3\frac{\Pi^{[\alpha\sigma}\bar{r}_\sigma v^{\beta]}}{\bar{r}_\rho v^\rho} - 3c^2\frac{\Pi^{[\alpha\sigma}\bar{r}_\sigma\bar{r}^{\beta]}}{(\bar{r}_\rho v^\rho)^2}\right\}.$$

One can show by a direct calculations, taking into account (4), that the tensor $H^{\alpha\beta}$ satisfies Maxwell's equation $\partial_\beta H^{\alpha\beta} = 0$.

In the absolute rest system ($\beta = \dot{\beta} = \ddot{\beta} = 0$) from formulas (7) follow the expressions known in the electrodynamics for the electric and magnatic fields of an absolutely precessing magnetic moment [8].

3 Fields radiated by a relativistic electric dipole moment

We describe the electric dipole moment by means of the tensor polarization potential which is dual of the tensor magnetic potential

$$G^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}.$$

The tensor electric and magnetic polarization densities are related in a corresponding way

$$N^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}M_{\alpha\beta} = (-\mathbf{M}, \mathbf{N}).$$

The duality relation for the electromagnetic field tensor is known to be

$$E^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}H_{\alpha\beta} = (-\mathbf{H}, -\mathbf{E}).$$

The electromagnetic fields of the electric dipole moment are discribed by (cf. [9])

$$\partial^\rho\partial_\rho G^{\mu\nu} = -4\pi N^{\mu\nu},$$

The solution of this equation for a point electric dipole moment can be written

$$G^{\alpha\beta} = -c\nu\frac{\Phi^{\alpha\beta}}{\tilde{r}_\rho v^\rho},$$

where ν is the electric dipole moment.

The dimentionless space-like tensor $\Phi^{\mu\nu} = (-\mathbf{\Pi}, \mathbf{\Phi})$ satisfies the condition

$$v_\mu\Phi^{\mu\nu} = 0,$$

from which we find

$$\mathbf{\Pi} = -[\beta\mathbf{\Phi}].$$

These relations are formally similar to those in the theory of the electromagnetic field radiated by a relativistic magneton. However, since the vector potential A^μ is again given by (1), the electromagnetic field tensor of the electric dipole moment is more complicated. The reason is that the condition $v_\mu\Pi^{\mu\nu} = 0$, which was used previously to simplify this tensor, does not hold in the present case. As a result, additional terms appear in all the structural elements $H^{\alpha\beta}_{1,2,3}$ (see (7), where $\mu \to \nu$; the complete tensor $H^{\alpha\beta}$ is given in [9])

$$H^{\alpha\beta}_{1,add} = \nu\frac{3\tilde{r}_\rho w^\rho}{\tilde{r}_\rho v^\rho}\Pi^{[\alpha\sigma}v_\sigma\tilde{r}^{\beta]},$$

$$H^{\alpha\beta}_{2,add} = -\frac{\nu c}{(\tilde{r}_\rho v^\rho)^2}\left\{\overset{\circ}{\Pi}{}^{[\alpha\sigma} v_\sigma \tilde{r}^{\beta]} + \Pi^{[\alpha\sigma} w_\sigma \tilde{r}^{\beta]}\right\},$$

$$H^{\alpha\beta}_{3,add} = -\frac{\nu c^3}{(\tilde{r}_\rho v^\rho)^3}\left\{\frac{2}{c^2}\Pi^{[\alpha\sigma} v_\sigma v^{\beta]} + \frac{3}{\tilde{r}_\rho v^\rho}\Pi^{[\alpha\sigma} v_\sigma \tilde{r}^{\beta]}\right\},$$

These terms remain even after dual transformation

$$\Pi^{\mu\nu} = -\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\Phi_{\alpha\beta}. \tag{8}$$

Only through a duality transformation of the entire electromagnatic field tensor, in the course of which we also use transformation (8) on the right side, can we find the tensor $E^{\mu\nu}$. The latter is formally identical to $H^{\mu\nu}$ for a relativistic magneton, except that we have

$$H^{\alpha\beta} \rightarrow E^{\alpha\beta}, \mu\Pi^{\alpha\beta} \rightarrow \nu\Phi^{\alpha\beta}. \tag{9}$$

Noting that we have

$$\Pi^{\alpha\beta} = ([\beta\mathbf{\Pi}], \mathbf{\Pi}) \rightarrow \Phi^{\alpha\beta} = ([\beta\mathbf{\Phi}], \mathbf{\Phi}),$$

we can make the transformation of the fields in the switch from the magnetic to the electric dipole moment by means of replacements

$$\mu \rightarrow \nu, \quad -\mathbf{E} \rightarrow \mathbf{H}, \quad \mathbf{H} \rightarrow \mathbf{E}.$$

The same transformation holds in the nonrelativistic case [8].

4 Wave zone and electromagnetic field tensor

The wave zone is dominated by the field which falls off as $1/\tilde{r}$, where $\tilde{r}$ is the distance from the radiating particle (the charge or magneton) to the observation point.

Since the differentiation $\tilde{\partial}_\nu$ in the derivative (see (4))

$$\partial_\nu = \tilde{\partial}_\nu + \frac{\tilde{r}^\nu}{\tilde{r}_\rho v^\rho}\frac{d}{d\tau},$$

which acts on the vector potential, simply increases the rate at which the field falls off, we can assume the following in the wave zone

$$\widetilde{H}^{\mu\nu} = \frac{\tilde{r}^{[\mu}\widetilde{\overset{\circ}{A}{}^{\nu]}}}{\tilde{r}_\rho v^\rho}. \tag{10}$$

The Lorentz condition $\partial_\nu A^\nu = 0$ in the wave zone then become

$$\bar{r}_\mu \widetilde{\overset{\circ}{A}{}^\mu} = 0,$$

where we have retained in $\widetilde{\overset{\circ}{A}{}^\mu}$ only those terms which are proportional to $1/\bar{r}$. From this result we find the familiar relation between the scalar and vector potentials in the case of the Lorentz gauge for the potentials $\widetilde{\varphi} = (\mathbf{n}\widetilde{\mathbf{A}})$.

Differentiation of A^μ with respect to the proper time in the wave zone yields

$$\widetilde{\overset{\circ}{A}{}^\mu} = -\frac{\mu c}{(\bar{r}_\rho v^\rho)^2}\left\{\overset{\circ\circ}{\Pi}{}^{\mu\lambda}\bar{r}_\lambda - \frac{3\,\overset{\circ}{\Pi}{}^{\mu\lambda}\bar{r}_\lambda\bar{r}_\rho w^\rho + \Pi^{\mu\lambda}\bar{r}_\lambda\bar{r}_\rho\,\overset{\circ}{w}{}^\rho}{\bar{r}_\rho v^\rho} + 3\frac{\Pi^{\mu\lambda}\bar{r}_\lambda(\bar{r}_\rho w^\rho)^2}{(\bar{r}_\rho v^\rho)^2}\right\}. \tag{11}$$

Substituting this expression into (10), we find $\widetilde{H}^{\mu\nu} = H_1^{\mu\nu}$, where $H_1^{\mu\nu}$ is given in (7) and corresponds to the field which falls off as $1/\bar{r}$.

It is convenient to write the tensor $\widetilde{H}^{\mu\nu}$ in the more compact form

$$\widetilde{H}^{\mu\nu} = \frac{\mu c}{(\bar{r}_\rho v^\rho)^3} Q^{[\mu}\bar{r}^{\nu]},$$

where, according to (10), Q^μ is the expression in curly brackets in (11).

Using these results and expressions for the spin precession [10], the classical radiation of the point magnetic moment moving at a constant velocity in a arbitrary direction with respect to the field lines of the uniform magnetic and electric fields is analyzed. All characteristics of the radiation agree with the Ternov – Bagrov – Khapaev relativistic quantum theory of the radiation by neutron [11] (see also [16, 12, 13]). It is thus demonstrated that the classical model of radiation with spin flip is valid.

More delail discussion of these questions can be found in the works [14, 15, 16, 17, 18].

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Electron Impact Double Ionization Like a Method for Atomic Spectroscopy

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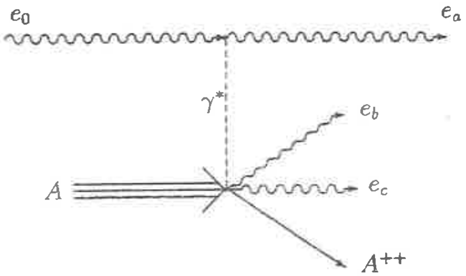
February 28, 1996

Abstract

Two types of (e,3e) collision kinematics are considered from the view point of obtaining direct information about electron-electron correlations in atoms. The first one assumes a symmetric or near-symmetric energy partition between ejected electrons and relatively small momentum transfer from the incident electron. The second assumes a symmetric or near-symmetric energy partition between the scattered electron and one ejected electron. In this case the momentum transfer is large. It is shown that this kinematic regime is better for the investigation of electron-electron correlations in the target.

1. Introduction. The first theoretical papers to consider (e,3e) collisions took an approach that was closely related to (γ,2e) ionisation (Smirnov et al.., 1978; Yudin et al., 1985). With this approach the four-body (e,3e) problem could be reduced to a three-body problem if the incident and the scattered electrons have energies that are large compared to the binding energies of the ejected electrons. In this case the one photon exchange diagram dominates, and the appropriate matrix element T_{bc} takes the form given by the First Born Approximation (FBA):



(1)

Here γ^* is a virtual photon, and the crossed lines mean that the electrons are immersed in the field of A^{++} ion. The conservation of energy and momentum give:

$$E_0 + \varepsilon_0^A = E_a + E_b + E_c + \varepsilon_0^{A^{++}} \tag{2}$$

$$\vec{p}_0 = \vec{p}_a + \vec{p}_b + \vec{p}_c + \vec{q} \tag{3}$$

To simplify futher we restrict ourselves to the He atom, then $\varepsilon_0^{He} = -79eV$, $\varepsilon_0^{He^{++}} = 0$. A projectile with energy transfer $\Delta E = E_0 - E_a$ and momentum transfer Q can be considered to be a virtual photon γ^*. In contrast to the (γ,2e) process, the values ΔE and Q are not connected by a definite dispersion formula. Analytically eq.(1) can be written in the form:

$$T_{bc}^{FBA} = \frac{4\pi}{Q} M(\vec{p}_b, \vec{p}_c; \vec{Q}) \tag{4}$$

with

$$M(\vec{p}_b, \vec{p}_c; \vec{Q}) = \frac{1}{Q} \iint d\vec{r}_1\, d\vec{r}_2 \Phi^{-*}(\vec{p}_b, \vec{p}_c; \vec{r}_1, \vec{r}_2)[e^{i\vec{Q}\vec{r}_1} + e^{i\vec{Q}\vec{r}_2} - 2]\Phi_0(\vec{r}_1, \vec{r}_2) \tag{5}$$

The functions Φ^- and Φ_0 discribe two continuum and two bound electrons respectively in the ion field. Formally, the matrix element for (γ,2e) scattering follows from eq.(5) in the limit $Q \to 0$. Historically the magnitude of

Q in eq.(1) is assumed to be small. The first (e,3e) experiments (Lahmam-Bennani et al, 1989;1992) used these kinematics.

2. Theory: Small Momentum Transfer. The kinematics (1) was proposed for the study of electron-electron correlations in the initial state and assumed some simple models for the final-state wave function Φ^-. This took the form of the product of distorted waves (Dal Capello and Rouso, 1992) or orthogonalised plane waves (Smirnov et al., 1978). The calculations presented by Joulakian et al. (1992) and Dal Cappello and Le Rouso (1992) show a strong dependence on the form chosen for the final state correlations. We can understand this effect qualitatively if we let the function Φ^- take the form:

$$\Phi^-(\vec{p}_b,\vec{p}_c;\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\varphi^-(\vec{p}_b,\vec{r}_1)\varphi^-(\vec{p}_c,\vec{r}_2)+F(\vec{p}_b,\vec{p}_c;\vec{r}_1,\vec{r}_2)+(\vec{p}_b \leftrightarrow \vec{p}_c)] \quad (6)$$

Each hydrogen-like function $\varphi^-(\vec{p},\vec{r})$ with Z = 2 in eq.(6) has the eikonal representation $\varphi^-(\vec{p},\vec{r}) = \exp(\imath\vec{p}\vec{r})\zeta(\vec{p},\vec{r})$, and F includes all post-collision interections of ejected e_b and e_c. If the momenta $p_b, p_c \gg Q$ then the first term in eq.(6) (this term is associated with the so-called shake-off (SO) mechansim of (e,3e) - collisions (McGuire,1982)) gives a very small contribution around the $\vec{Q}$ axis to the matrix element (4) and to the five-fold differential cross section (5DCS) $d^5\sigma/dE_b dE_c d\Omega_a d\Omega_b d\Omega_c$. The estimates give the result:

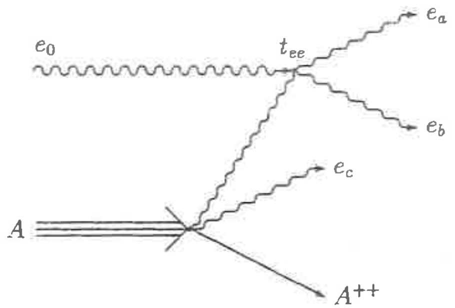
$$d^5\sigma(SO) \sim (\Delta E)^{-8} \quad (7)$$

if $p_a \sim p_c \sim \sqrt{\Delta E}$ and $\Delta E \gg \varepsilon_0^{He}$ (Popov et al., 1994) On the other hand, the second term in eq. (6) describes the so-called two-step mechanism (TS1) (Carlson and Krause, 1965; Tweed, 1973). It can mix the ejected momenta $\vec{p}_b$ and $\vec{p}_c$ during a second collision in a such way, that the value $q = \mid \vec{Q} - \vec{p}_b - \vec{p}_c \mid$ can be small in spite of large values for p_b and p_c. One obtains the estimate (Popov et al., 1994)

$$d^5\sigma(TS1) \sim (\Delta E)^{-3} \quad (8)$$

in the vicinity of $q = 0$. For the case corresponding to $E_0 \sim E_a \gg \Delta E \gg \varepsilon_0^{He}$, $p_b \simeq p_c \simeq \sqrt{\Delta E}$, Q is relativly small, one expects the 5DCS to have a sharp maximum around the $\vec{q}$ axis and little structure around the $\vec{Q}$ axis. This result can be understood in the following way: if $q = 0$ (in (e,2e) theory the domain $q \sim 0$ is called the Bethe-ridge) and we neglect ε_0^{He} compared to ΔE, we have the classical picture of three billiard balls with the residual ion playing no role. In this case the angle between the momenta of the ejected electrons is $\Theta_{bc} \sim \pi - \frac{Q}{\sqrt{\Delta E}} \to \pi$. This can only occur through a two-step mechanism. The first term in the decomposition (6) provides direct information about initial state structure. In this case eq.(5) is a double Fourier-transform of $\Phi_0(\vec{r}_1, \vec{r}_2)$. Its contribution to the 5DCS is concentrated near the $\vec{Q}$ axis, however its magnitude is rather small compared to the other terms in the matrix element (see eqs.(7) and (8)). The second term F in eq.(6) has its largest contribution around the $\vec{q}$ axis and contains information about electron-electron correlations in both the initial and in the final states of the reaction. For this reason, the matrix element T_{bc}^{FBA} has "double model" dependence even for large values of energy transfer ΔE, provided the momenta of the ejected electrons are equal or approximately equal. These kinematics are not suitable for the investigation of electron-electron correlation in atomic wave functions Φ_0.

3. Theory: Big Momentum Transfer. Let us consider now the case of large Q values which can be realized for kinematics where the ejected electron momenta p_a and p_b are nearly equal. These kinematics lead to the Plane Wave Impulse Approximation (PWIA):



(9)

Here t_{ee} is the so-called half-off-shell electron - electron amplitude with a well known analytical expression. The (5DCS) corresponding to the amplitude (9) has the form:

$$\frac{d^5\sigma}{dE_b dE_c d\Omega_a d\Omega_b d\Omega_c} = \frac{2p_a p_b}{(2\pi)^3 p_0} \left| \frac{t_{ee}}{2\pi} \right|^2 \rho(\vec{p}_c, \vec{p}_c + \vec{q}) \qquad (10)$$

with

$$|\frac{t_{ee}}{2\pi}|^2 = \frac{4}{Q^4} f(x) C(y);$$

$$\rho(\vec{p}_c, \vec{p}_c + \vec{q}) = \frac{p_c}{(2\pi)^3} \mid \iint d\vec{r}_1\, d\vec{r}_2 \varphi^{-*}(\vec{p}_c, \vec{r}_1) e^{i(\vec{p}_c+\vec{q})\vec{r}_2} \Phi_0(\vec{r}_1, \vec{r}_2) \mid^2 \tag{11}$$

$$\left\{ \begin{array}{ll} f(x) = \frac{2\pi x}{e^{2\pi x}-1}; & C(y) = [1 + y^4 - y^2 \cos(2x \ln y)] \\ x = \mid \vec{p}_a - \vec{p}_b \mid^{-1}; & y = \frac{|\vec{p}_0 - \vec{p}_b|}{|\vec{p}_0 - \vec{p}_a|} \end{array} \right\}$$

The form (10) is equivalent to (e,2e) binary triple differential cross section (TDCS), but $\rho(\vec{p}_c, \vec{p}_c + \vec{q})$ is the He^+ decay state density or the Fourier transform of the fluctuation function $\chi(\vec{p}, \vec{r})$ (Popov et al., 1994).

$$\chi(\vec{p}, \vec{r}) = \int d\vec{r}' \varphi^{-*}(\vec{p}, \vec{r}') \Phi_0(\vec{r}', \vec{r}) \tag{12}$$

The smaller p_c, the better the approximation (10). For q small, we examine the function ρ in the vicinity of $q = 0$. The 5DCS can be estimated here as (Popov et al., 1994)

$$d^5\sigma(binary) \sim (\Delta E)^{-\frac{3}{2}} \tag{13}$$

The semiclassical ideas of Avaldi et al., (1986) provide a way of treating final state correlations in (8). Because the speed of electron c is small compared to the two other fast electrons, electrons a and b "see" the system ($e + He^{++}$) as an ion He^+, so the distortion of the electrons' paths takes place in the He^+ ion field with Z = 1. The correlation formulas derived by Avaldi et al., (1986) can be used in this case and lead to small angular displacements of the peaks. We expect to "catch" in this way the corrections of a p_0^{-1} order of magnitude to formula (10).

4. Short Conclusion. In order to derive information on two-electron target momentum densities from (e,3e) experiments, binary kinematics are most useful. The proposed binary kinematics are analogous to those used in (e,2e) experiments where one-electron momentum densities are measured. Note that $d^5\sigma(binary)$ is much larger than $d^5\sigma(TS1)$.

5. Experimental Considerations. An experiment has recently been constructed to measure 5DCS for the electron impact double ionization of magnesium. The experiment consists of an electron gun, a scattered electron analyzer and two ejected electron analyzers. Because the magnitude of the 5DCS is small the ejected electron analyzers were designed to accommodate up to eight detectors each at their focal planes. In this way it is possible to sample 64 pairs of angles simultaneously if one considers only events where the ejected electrons pass through different analyzers. Magnesium was chosen as a target because it is helium-like with two s-electrons outside a closed shell. In fact, calculations and experiments have shown that the outer s-electrons in magnesium have characteristics more like doubly excited helium, where electron-electron correlation is large, than like ground state helium. Initial experiments with the instrument have shown that it has sensitvity sufficient to measure 5DCS with sufficient precision to obtain information on initial state correlations.

Quantitative estimates of the reasibility of the corresponding experiments are difficult to do. However, two remarks can be put forward which plead for their feasibility. Firstly, the kinematics of diagram 2 with large E_b (comparable to E_a) and small E_c have a larger cross section, and hence are more favourable than the kinematics with the same energy transfer $(E_b + E_c)$ but an equipartitionning b-c (see, e.g. Duguet *et al* 1991). Second, the feasibility of a triple coincidence experiment is mostly determined by the true to accidental coincidence rates ratio, which is proportional to $d^5\sigma/d^2\sigma_a d^2\sigma_b d^2\sigma_c$ (Lahmam-Bennani *et al* 1991), where $d^2\sigma_i$ is the double differential cross section that determines the single count rate in detector i. In the kinematics (1), small E_b and E_c values together with a large E_a value (i.e. small energy-loss) correspond to a large 5DCS $d^5\sigma$ (hence, to a large "signal"), but also to large DDCS's $d^2\sigma_{a,b,c}$. Since the latter quantities enter as the product of three terms in the denominator, the resulting signal-to-noise ratio might become too small.

6. Acknowlegements. This paper is partially supported by the Russian Foundation for Fundamental Investigations (RFFI).

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PHOTON-LIKE RESONANCE DECAY $\gamma^* \to \nu_i \bar{\nu}_j$ IN A STRONG MAGNETIC FIELD

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Abstract

We considered photoneutrino interactions in a strong magnetic field $B \gg B_o = m_e^2/e_o = 4.41 * 10^{13}G$ in model with mixing. The interactions of real photons proved to be suppressed in such a field. This corresponds to existence of a photon-like resonance with the mass m_γ and at the values $m_\gamma^2 > (m_i + m_j)^2$ the decay mode $\gamma^* \to \nu_i \bar{\nu}_j$ is possible (m_i and m_j stands for neutrino masses). The corresponding rate is calculated and some astrophisical applications are discussed.

1. INTRODUCTION

In a usual QED the renormalization procedure for photon external lines gives a trivial result - the lines turn out to be unchanged:

$$e_\mu \to e_\mu \tag{1}$$

(e_μ is the photon polarization vector)

But the situation is not the same in the presence of an external electromagnetic field, which takes into account in a Furry picture. If the four-momentum conserves (for example, in a constant and homogeneous magnetic field), we obtain

$$e_\mu \to e_\mu + D_{\mu\alpha}(k) \frac{P^{\alpha\beta}(k)}{4\pi} e_\beta, \tag{2}$$

where $D_{\mu\alpha}$ and $P^{\alpha\beta}$ are the photon propagator and polarization tenzor in a Furry picture. At the value of magnetic field strength $B \gg B_o = m_e^2/e_o = 4.41 * 10^{13}G$ this quantities are follows [1, 2]

$$P^{\alpha\beta}(k) = P(k^2, k_\perp^2)(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}), \tag{3a}$$

$$D_{\mu\alpha}(k) = \frac{4\pi}{k^2 - k_\perp^2 - P(k^2, k_\perp^2)} g_{\mu\alpha}, \tag{3b}$$

where $P(k^2, k_\perp^2)$ is the photon polarization operator in such a field, $k^2 = k_0^2 - k_3^2, k_\perp^2 = k_1^2 + k_2^2$ and all indexes run over 0,3 only (third axis is directed along the field). This is one of the consequences of the so called "Two-Dimensional QED Approximation", developed by the authors earlier (TDA) [3]. For the real photon $k^2 - k_\perp^2 = 0$ and using the relation $P(k^2, k^2) \neq 0$ we obtain from (2), (3a,b) the following unexpected result instead of (1):

$$e_\mu \to \frac{k_\mu(ek)}{k^2} \tag{4}$$

(note, that $(ek) \neq 0$ in a general case since a scalar product is defined in a two-dimensional subspase (0,3)).

Thus, we see that a renormalization procedure in TDA leads to a suppression of the mass shell photon interaction because of gauge invariance of vertex functions. In other words, the interacting photon in TDA must be massive (photon-like resonance γ^*). In electroweak theories this leads to a possibility of decay channel $\gamma^* \to \nu_i \bar{\nu_j}$ at the values $m_\gamma^2 > (m_i + m_j)^2$. This channel may be of importance in the forming of massive neutrinos balance at the early stage Universe evolution or as energy loss mechanism of magnetic neutron stars.

At the low energies the photon mass is imaginary $m_\gamma^2 < 0$ and the channel $\nu_i \to \nu_j \gamma^*$ opens [4].

2. THE EFFECTIVE PHOTON MASS

The effective photon mass is given by a relation

$$m_\gamma^2 = -e_\mu P^{\mu\nu} e_\nu. \tag{5a}$$

where a two-dimensional tensor $P_{\mu\nu}$ was defined in Introduction. Taking into account the equation (3a), we obtain

$$m_\gamma^2 = P(k^2, k_\perp^2)[1 - e_\perp^2 + \frac{(ek)_\perp}{k^2}], \tag{5b}$$

where a scalar function $P(k^2, k_\perp^2)$ may be written as follows [1]:

$$P(k^2, k_\perp^2) = \frac{4}{\pi}\alpha\gamma \exp\left(-\frac{k_\perp^2}{2\gamma}\right)\left(\frac{1}{2} + \frac{\xi \ln \xi}{1-\xi^2}\right), \tag{6}$$

$$\gamma = |e_o B| \gg m_e^2, |k^2|;$$

$$\frac{(1-\xi)^2}{\xi} = -\frac{k^2}{m_e^2}, \qquad 0 < \xi \leq 1. \tag{6a}$$

The analitical continuation to the energy range $k^2 > 0$ has a form:

a) $\underline{0 < k^2 < 4m_e^2}$

$$(\frac{1}{2} + \frac{\xi \ln \xi}{1-\xi^2}) \to \frac{1}{2}(1 - \frac{\phi}{\sin\phi}), \tag{7a}$$

$$\frac{k^2}{4m_e^2} = \sin^2\phi/2,$$

b) $\underline{k^2 > 4m_e^2}$. In this case ξ in equation (6) must be replaced by $|\xi|e^{i\pi}$. At the meanings $k^2/m_e^2 \ll 1, k_\perp^2 \ll \gamma$

$$P = -\frac{\alpha k^2 \gamma}{3\pi m_e^2} < 0 \tag{8a}$$

and in according with (5b) the decay $\gamma^* \to \nu_i \bar{\nu}_j$ is forbidden in any case. But at the values $k^2/m_e^2 \gg 1; k^2, k_\perp^2 \ll \gamma$ we have

$$P = \frac{2}{\pi}\alpha\gamma > 0 \tag{8b}$$

and this channel is open. In a frame of reference in which $k_3 = 0$, one can obtain for different polarization states:

$$\vec{e} \perp \vec{B}, \vec{k}: \qquad m_\gamma^2 = 0 \tag{9a}$$

$$\vec{e} \parallel \vec{k}: \qquad m_\gamma^2 = 0; \tag{9b}$$

$$\vec{e} \parallel \vec{B}: \qquad m_\gamma^2 = \frac{2}{\pi}\alpha\gamma. \tag{9c}$$

Thus, the only mode contributing in TDA is (9c), giving a photon-like resonance with the mass depending on the field strength. The resonance lifetime controlled by a QED channel $\gamma^* \to e^+e^-$ and is given by a relation

$$\tau_Q^{-1} = \frac{4\alpha\gamma m_e^2}{k_o(m_\gamma^2 + k_\perp^2)}, \tag{10}$$
$$m_\gamma^2 + k_\perp^2 \gg m_e^2.$$

The QED lifetime "at rest" after subsitution of (9c) is given by an expression

$$\tau_Q^{-1} = 7.16 * 10^{22}(B_o/B)^{1/2}s^{-1} \tag{10a}$$
$$m_\gamma^2 \gg m_e^2.$$

3. A WEAK DECAY $\gamma^* \to \nu_i \bar{\nu}_j$ IN A MODEL WITH MIXING

The matrix element can be easily obtained by crossing from the corresponding results of reference [4]:

$$M = \frac{e_o G \gamma U_{ei} U^*_{ej}}{2(2\pi)^{5/2}} [\bar{u_\nu}(k_j)\gamma^\mu(1+\gamma^5)u_\nu(-k_i)] e^\sigma I_{\sigma\mu}(k), \tag{11}$$

$$I_{\sigma\mu} = A(k^2)(e_{\sigma\alpha}k^\alpha k_\mu + g_{\sigma\mu}k^2 - k_\sigma k_\mu), \tag{11a}$$

$$A|_{k^2 \gg m_e^2} = \frac{8\pi}{k^2}. \tag{11b}$$

Here U is a mixing matrix, $k_{i,j}$ - neutrino four-momentum, $e_{\sigma\mu}$ - absolutly anti-symmetric tensor in (0,3), e^σ stands for polarization vector of "massive photon" with $k^2 = m_\gamma^2 + k_\perp^2 \gg m_e^2$; α, μ, σ run over 0,3. The expression (11) corresponds to a leading contribution of the effective "electron loop" in Feynman diagrams [4].

After some calculations one can obtain the following result for the total decay rate ($k_3 = 0$):

$$\begin{aligned} W = \frac{4k_o \alpha G^2 \gamma^2 R}{3(2\pi)^4 m_\gamma^6} &[m_\gamma^4 - 2m_\gamma^2(m_i^2 + m_j^2) + \\ &+ (m_i^2 - m_j^2)^2]^{1/2} * \\ &*[m_\gamma^4 + m_\gamma^2(m_i^2 + m_j^2) - 2(m_i^2 - m_j^2)^2], \end{aligned} \tag{12}$$

$$R = |U_{ei}U^*_{ej}|^2.$$

Taking into account an evident relation $m_\gamma^2 \gg m_{i,j}^2$ and equation (9c) we arrive at the simple result for photon-like resonance weak decay rate "at rest"

$$W = 3 * 10^{-9}(B/B_0)^{5/2} R\, s^{-1} \tag{13}$$

$$m_\gamma^2 \gg m_e^2.$$

At the field values $B > 10^{17}$ G the QED perturbation theory is failed and one must sum over all loops in QED sector using TDA [3]. In our case this operation reduces to a substitution

$$m_e \to m_e^* = m_e \left(\frac{B}{B_o}\right)^{(\alpha \ln \pi/\alpha)/2\pi} \tag{14}$$

in equations (10-11). But this substitution do not change the order of the rate in the range of field strength $10^{22} G \geq B \gg B_o$ [4].

4.CONCLUSIONS

A relative contribution of the decay channels $\gamma^* \to \nu_i\bar{\nu}_j$ and $\gamma^* \to e^+e^-$ may be written as a ratio of $\tau_W^{-1} = W$ (13) and τ_Q^{-1} (10a):

$$\frac{\tau_W^{-1}}{\tau_Q^{-1}} = 4 * 10^{-32}\left(\frac{B}{B_o}\right)^3 R. \tag{15}$$

This quantily is of the order 10^{-24} when $B \sim 10^{16}G$ (magnetic neutron stars) and of the order 10^{-6} when $B \sim 10^{22}G$ (Big Bang stage). Thus in the second case the decay mode $\gamma^* \to \nu_i\bar{\nu}_j$ could be by one of the main mechanism of massive neutrino production. Note that in any case the weak decay channel gives no information about the neutrino mass because of the relation $m_\gamma \gg m_{i,j}$ and for this aims there are more suitable laboratory experiments on β-decay or neutrino synchrotron radiation [5].

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Semiclassically concentrated states of charged particles in a curved space-time

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Abstract

We define the semiclassical concentration of states of quantum systems described by the Klein–Gordon equations and the Proce equations in external electromagnetic and gravitational fields. We show that the semiclassical concentration can be attained on a classical phase trajectory only.

The present paper develops the method of semiclassically concentrated states [1]–[5] for relativistic wave equations in curved space-time. Approximate solutions satisfying the Klein–Gordon [6] and the Proce [7] equations were previously obtained in a neighborhood of the world line of a charged particle in external gravitational and electromagnetic fields. These solutions were called *semiclassical trajectory-coherent states* (TCS) [8]. The technique for reducing the Klein–Gordon equations to a Schrödinger-type equation for TCS and the Proce equation to a Pauli-type equation was worked out and the corresponding Hamiltonians were calculated. This construction was carried out in geometrical terms. It was established that, with precision up to $O(\hbar^{3/2})$, $\hbar \to 0$, the scalar $\Psi(x)$ and the vector $V^{\nu}(x)$ fields, satisfying the Klein–Gordon and the Proce equations in the class of positive-frequency semiclassically concentrated states, could be interpreted in the standard quantum mechanical way.[1]

We point out two interesting properties of TCS. First, the complete set of these states forms the basis in the space of semiclassically concentrated solutions of the Klein–Gordon and the Proce equations. Second, the semiclassical TCS can be classified as positive- and negative-frequency states. The latter fact deserves detailed consideration. Namely, in an arbitrary Riemann space there is

[1]Numerous papers deal with the problem of the formulation of quantum mechanics on a Riemann manifold (see, e.g., [9]–[11] and references therein).

no natural way for separating the solutions of wave equations into positive- and negative-frequency ones (see, e.g., [12], [13]). On the other hand, this separation is absolutely necessary if we want to interpret the quantum field in terms of particles and antiparticles, and hence, to have well-posed problems and computation of quantum processes. Since TCS can be classified as positive- and negative-frequency ones, they may be widely used in problems of quantum theory in curved space-time.

However, when we construct the semiclassical TCS by using the method of Maslov's complex germ [14], we assume that the corresponding classical equations and their solutions are known. In other words, the space of semiclassically concentrated states is constructed for a given classical trajectory. In the present paper we show that the classical equations of motion appear while the semiclassical concentrated states are constructed in curved (as well as in plane) space-time and that the semiclassical concentration can be attained on a classical trajectory only.

Let us consider the Klein–Gordon equation in curved space-time:

$$\hat{\mathcal{H}}\Psi \left\{\frac{1}{\sqrt{-g}}\hat{\mathcal{P}}_\mu(\sqrt{-g}g^{\mu\nu}\hat{\mathcal{P}}_\nu) - U(x) - m^2c^2\right\}\Psi = 0. \tag{1}$$

Here $g_{\mu\nu}(x)$ is the metric of the Riemann space with signature $(+,-,-,-)$, $g = \det(g_{\mu\nu})$; m is the mass of a particle; c is the velocity of light; $\hat{\mathcal{P}}_\mu = -i\hbar\partial_\mu - \frac{e}{c}\mathcal{A}_\mu(x)$ and $\mathcal{A}_\mu(x)$ are the potentials of the external electromagnetic field. The function $U(x)$ plays the role of an external scalar field.

As is well known, in the linear space of solutions of (1) one can define an invariant scalar product. Let Ψ_1 and Ψ_2 be two (generally speaking) complex solutions of (1), then the scalar product of these solutions can be written as follows

$$\langle\Psi_2|\Psi_1\rangle = \frac{N^2}{2mc}\int\limits_\Sigma d\Sigma^\mu[(\hat{\mathcal{P}}_\mu\Psi_2)^*\Psi_1 + \Psi_2^*(\hat{\mathcal{P}}_\mu\Psi_1)]. \tag{2}$$

Here N is an arbitrary constant, introduced for convenience, Σ is a space-like hypersurface in the Riemann space. Then one can show that if Ψ_1 and Ψ_2 satisfy equation (1), then the scalar product $\langle\Psi_2|\Psi_1\rangle$ is independent of the choice of the hypersurface Σ.

As another example of the wave equation in a curved space, we consider the Proce equation

$$\begin{aligned}(\hat{\mathcal{H}}_p V)^\alpha = -D_\beta W^{\alpha\beta} + \frac{m^2c^2}{\hbar^2}V^\alpha + i\frac{e(g-1)}{c\hbar}F^{\mu\alpha}V_\mu - \\ -2S_{\mu\nu\rho}S^{\mu\nu\alpha}V^\rho + S^{\mu\nu\alpha}W_{\mu\nu} + 2D_\beta(S^{\alpha\beta\sigma}V_\sigma) = 0,\end{aligned} \tag{3}$$

where $W_{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu$, $D_\mu = \nabla_\mu - \frac{ie}{c\hbar}\mathcal{A}_\mu$ is the extended covariant derivative, $\mathcal{A}_\mu$ is the potential of the external electromagnetic field, $F_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$,

$S_{\mu\nu\alpha}$ is the torsion tensor. The scalar product of two arbitrary solution U^μ and V^ν satisfying (3) can be defined as follows

$$\langle U|V\rangle_P = -\int\limits_{\Sigma} d\Sigma_\alpha \{\overset{*}{U}{}^{\mu}(\hat{J}^{\alpha}_{\mu\nu}V^\nu) + (\hat{J}^{\alpha}_{\nu\mu}U^\mu)^* V^\nu\}, \tag{4}$$

where

$$\hat{J}^{\beta}_{\mu\nu} = \frac{i\hbar}{2mc}(D_\mu \delta^\beta_\nu - D^\beta g_{\mu\nu} + 2S^{\beta}_{.\mu\nu}),$$

and Σ is a space-like hypersurface.

By analogy to the plane case, we give the following definition of the semiclassical concentration of a scalar particle in curved space-time:

Definition 1. *The states Ψ of the quantum system (1) will be called semiclassically concentrated on the phase trajectory $z(s) = (p_\mu(s), q^\nu(s))$ belonging to the class $\mathbb{CS}_{\mathrm{KG}}(z(s), \tau(x))$, if*

(i) *the curve $q^\mu(s)$ is time-like, and there exists a family of hypersurfaces $\tau(x) = s$ (where s is a family parameter), defined by the equation*

$$g_{\mu\nu}(\tau)q(\tau)\dot{q}^\mu(\tau)(x^\nu - q^\nu(\tau)) = 0; \tag{5}$$

(ii) *for any operator $\hat{A}$ with symbol $A(p, x, \hbar)$, regularly depending on $\hbar$, we have*

$$\lim_{\hbar\to 0}\langle\Psi|\hat{A}|\Psi\rangle_{\mathrm{KG}} = \lim_{\hbar\to 0}\frac{1}{\|\Psi\|_s}\langle\Psi|\hat{A}|\Psi\rangle_s = A(p(s), q(s), 0). \tag{6}$$

Here we use the notation

$$\langle\Psi_1|\Psi_2\rangle_s = \int\limits_{\tau(x)=s} d\Sigma_\mu\, \dot{q}^\mu \Psi_1^* \Psi_2, \qquad \|\Psi\|_s^2 = \langle\Psi|\Psi\rangle_s. \tag{7}$$

Theorem 1. *If Ψ is a semiclassically concentrated state of class $\mathbb{CS}_{\mathrm{KG}}(z(s), \tau(x))$, then $z(s)$ satisfies the classical Hamilton system with Hamiltonian $\mathcal{H}^{(\mathrm{cl})}(p, x) = c^2\mathcal{P}_\mu\mathcal{P}^\mu - m_0^2c^4 - U(x)$.*

Lemma 1. *If $\Psi \in \mathbb{CS}(z(s), \tau(x))$, then*

$$\frac{D}{ds}\langle\Psi|\hat{A}|\Psi\rangle_{\mathrm{KG}} = \frac{1}{i\hbar}\langle\Psi|[\hat{\mathcal{H}}, \hat{A}]_-|\Psi\rangle_s. \tag{8}$$

Proof. The mean value of the operator $\hat{A}$ can be written as follows

$$\langle\Psi|\hat{A}|\Psi\rangle_{\mathrm{KG}} = \int\limits_{\tau(x)=s} d\Sigma_\mu G^\mu,$$

where

$$G^\mu = \frac{N^2}{2mc} g^{\mu\nu}[(\hat{\mathcal{P}}_\nu \Psi)^* \hat{A}\Psi + \Psi^* \hat{\mathcal{P}}_\nu \hat{A}\Psi].$$

Then

$$\frac{d\langle\hat{A}\rangle}{ds} = \lim_{\Delta s\to 0} \frac{\int\limits_{\tau(x)=s+\Delta s} d\Sigma_\mu G^\mu - \int\limits_{\tau(x)=s} d\Sigma_\mu G^\mu}{\Delta s},$$

because, by condition (6), $G^\mu \to 0$ as we move away from the classical trajectory along the hypersurface $\tau(x) = s$. Therefore, in the numerator we can add any integral along the tubular surface $\sigma(R)$, lying between the hypersurfaces $\tau(x) = s$ and $\tau(x) = s + \Delta s$. Suppose the surface $\sigma(R)$ is infinitely far from $x(s)$, then we have

$$\frac{d\langle\hat{A}\rangle}{ds} = \lim_{\Delta s\to 0} \frac{\Big(\int\limits_{\tau(x)=s+\Delta s} - \int\limits_{\tau(x)=s} + \int\limits_{\sigma(\infty)}\Big) d\Sigma_\mu G^\mu}{\Delta s} = \lim_{\Delta s\to 0}\frac{1}{\Delta s}\int\limits_V d^4x\, \partial_\mu(\sqrt{-g}\, G^\mu).$$

Here we have used the Gauss theorem; V is the volume bounded by the surfaces $\tau(x) = s$, $\tau(x) = s + \Delta s$, and σ. Therefore,

$$\frac{d\langle\hat{A}\rangle}{ds} = \lim_{\Delta s\to 0}\frac{1}{\Delta s}\int\limits_s^{s+\Delta s} ds \int\limits_{\tau(x)=s} \frac{d^4x}{ds}\sqrt{-g}G^\mu{}_{;\nu} = \int\limits_{\tau(x)=s} d\Sigma_\nu\, \dot{q}^\nu G^\mu{}_{;\mu}.$$

Recall that

$$G^\mu{}_{;\mu} = \frac{N^2}{2mc}\frac{1}{i\hbar}(\Psi^*\hat{\mathcal{H}}\hat{A}\Psi - (\hat{\mathcal{H}}\Psi)^*\hat{A}\Psi),$$

since $\hat{\mathcal{H}}\Psi = 0$. Hence, our lemma is proved.

Now let us prove Theorem 1.

Proof. As is well known, the Weyl symbol of the commutator of Weyl-ordered operators with symbols $a(p, x)$ and $b(p, x)$ has the form

$$C(p, x, \hbar) = 2i \sin\Big\{\frac{\hbar}{2}\Big(\frac{\partial}{\partial x^\nu}\frac{\partial}{\partial \pi_\nu} - \frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q^\mu}\Big)\Big\} a(p, x) b(\pi, q)\Big|_{\substack{\pi=p\\ q=x}}. \tag{9}$$

In (8) we pass to the limit as $\hbar \to 0$ and, taking into account (9) and (6), obtain

$$\frac{dA}{ds} = \{\mathcal{H}^{(\mathrm{cl})}, A_0\}_{px}, \tag{10}$$

where the braces denote the Poisson bracket and $A_0 = A(p, x, 0)$. Since the operator $\hat{A}$ is arbitrary, our theorem is proved.

Similarly, for a vector particle in the Riemann–Cartan space we have

Definition 2. *The states V of the quantum system (3) will be called semiclassically concentrated on the phase trajectory $z(s) = (p_\mu(s), q^\nu(s)) \in \mathbb{CS}_P(z(s), \tau(x))$, if*

(i) *the curve $q^\mu(s)$ is time-like, and there exists a family of hypersurfaces $\tau(x) = s$, given by the equation*

$$g_{\mu\nu}(q(\tau))\dot{q}^\mu(\tau)(x^\nu - q^\nu(\tau)) = 0; \tag{11}$$

(ii) *for an arbitrary scalar operator $\hat{A}$ with symbol $A(p, x, \hbar)$, regularly dependent on $\hbar$ on the hypersurface $\tau(x) = s$, we have*

$$\lim_{\hbar\to 0}\langle V|\hat{A}|V\rangle_P = \lim_{\hbar\to 0}\langle V|\hat{A}|V\rangle_s = A(p(s), q(s), 0); \tag{12}$$

(iii) *for an arbitrary 4×4 matrix Γ, for all values of $\hbar \in [0, 1[$, there exist quantum-mechanical means*

$$\langle V|\Gamma A|V\rangle_P \qquad \text{and} \qquad \langle V|\Gamma A|V\rangle_s.$$

Here we use the notation

$$\langle U|V\rangle_s = \int\limits_{\tau(x)=s} d\Sigma_\mu\, \dot{q}^\mu U^+ V. \tag{13}$$

Theorem 2. *If the state $V^\mu(\vec{x}, t, \hbar)$ of the quantum system (3) is semiclassically concentrated and belongs to the class $\mathbb{CS}_P(z(s), \tau(x))$, then $z(s)$ satisfies the classical Hamilton system with Hamiltonian $\mathcal{H}^{(\mathrm{cl})}(p, x) = c^2\mathcal{P}_\mu\mathcal{P}^\mu - m_0^2c^4$.*

To prove this theorem, we shall need:

Lemma 2. *If $V \in \mathbb{CS}_P(z(s), \tau(x))$, then*

$$\frac{D}{ds}\langle V|\hat{A}|V\rangle_P = \frac{1}{i\hbar}\langle V|[\overset{0}{\mathcal{H}}_P, \hat{A}]_-|V\rangle_s. \tag{14}$$

Proof of this lemma literally repeats the proof of Lemma 2.

Proof of Theorem 2. Let us expand the solution of the Proce equation in eigenvectors of the leading symbol of the Hamiltonian calculated at points of the phase trajectory $z(s)$ [7]:

$$V^\mu(\vec{x}, t, \hbar) = e^\mu{}_j(s)\Phi^j(\vec{x}, t, \hbar), \tag{15}$$

$$\overset{0}{\mathcal{H}}{}^\mu{}_\sigma(s)e^\sigma{}_j(s) = \lambda_j(s)e^\mu{}_j(s), \tag{16}$$

where

$$\overset{0}{\mathcal{H}}{}^\mu{}_\sigma = \frac{1}{2mc}[(\mathcal{P}_\alpha\mathcal{P}^\alpha - m^2c^2)\delta^\mu{}_\sigma - \mathcal{P}_\sigma\mathcal{P}^\mu];$$

$$\lambda_0 = -\frac{mc}{2}, \qquad \lambda_a = \frac{1}{2mc}(\mathcal{P}_\alpha \mathcal{P}^\alpha - m^2c^2), \qquad a = 1, 2, 3; \tag{17}$$
$$e_0{}^\mu = N\mathcal{P}^\mu, \qquad \mathcal{P}_\mu e^\mu{}_a = 0.$$

Then, precisely as in the case of Dirac particles [2], we see that the scalar parts of the functions $\Phi^j(\vec{x}, t, \hbar)$ coincide and

$$\lim_{\hbar\to 0} \Phi^0(\vec{x}, t, \hbar) = 0. \tag{18}$$

Then for mean values of the scalar operator $\hat{A}$ we obtain

$$\lim_{\hbar\to 0} \frac{D}{ds}\langle V|\hat{A}|V\rangle_P = \lim_{\hbar\to 0}\langle V|\{[\hat{\mathcal{H}}_{\mathrm{KG}}, \hat{A}]_-\delta_\alpha^\beta - \hat{\mathcal{P}}^\beta[\hat{\mathcal{P}}_\alpha, \hat{A}] - [\hat{\mathcal{P}}^\beta, \hat{A}]_-\hat{\mathcal{P}}_\alpha\}|V\rangle_s.$$

Taking into account (17) and (18), we obtain

$$\dot{A} = \{\mathcal{H}^{(\mathrm{cl})}(p, x), A(p, x, 0)\}_{px}.$$

Since the operator $\hat{A}$ is arbitrary, our theorem is proved.

Acknowledgements

This work was supported by the International Science Foundation under Grant N78300 and by Russian Research Foundation. The third author is a fellow of INTAS Grant 93-2492 and is carried out within the research program of International Center for Fundamental Physics in Moscow.

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SYNCHROTRON AND COMPTON MECHANISMS OF AXION EMISSION BY ELECTRONS IN A MAGNETIZED NEUTRON STAR

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Abstract

The general expressions for the axion luminosity of a magnetized degenerate relativistic electron gas due to synchrotron and Compton mechanisms are derived. Quantitative estimates for the neutron star crust conditions are given. New upper bounds on the axion- electron coupling constant are obtained.

Introduction

Axion is the pseudo-Goldstone boson associated with spontaneously broken the Peccei-Quinn (PQ) symmetry [1]. PQ symmetry gives a natural solution of the strong CP-violation problem. Axion mass and its couplings to stable particles are inversely proportional to the scale of the PQ symmetry breaking v. The origial axion model [2] by S. Weinberg and F. Wilczek assumes $v = v_{BW} = \left(\sqrt{2}G_F\right)^{-1/2} \cong 250$ GeV where v_{BW} is the scale of the electroweak symmetry breaking. The standard axion is ruled out by existing experimental and astrophysical data [3]. Various invisible axion models (with $v \gg v_{BW}$) were constructed [4]. Axions couple to fermions (quarks and leptons) with the axion-fermion couplings $g_{af} = c_f m_f/v$. Here c_f are the model-dependent numbers of order one, m_f are the fermion masses.

The interaction axions a with fermions f is equivalent to the pseudoscalar one in the linear approximation of the original PQ - Lagrangian expansion in powers of (a/v):

$$L_{af} = -ig_{af}\left(\overline{\psi_f}\gamma^5\psi_f\right)a, \tag{1}$$

where $\gamma^5 = -i\gamma^0\,\gamma^1\gamma^2\gamma^3$ - Dirac matrixes, the system of units: $\hbar = c = 1$. The effective Lagrangian (1) is used in our calculations.

1. Invisible axions, if they exist, could carry away large amount of energy from stellar interiors due to enormous mean free path lengths in matter of typical astrophysical densities. The loss energy process in a neutron star, particulary in its crust, may be described as relativistic degenerate electron gas radiations by different mechanisms [5]. The external magnetic field existanse in a stellar matter makes possible the synchrotron axion emission by the electrons $e \longrightarrow e + a$, forbidden in a free case.

The axion synchrotron luminosity, i.e. the energy loss rate of the unit stellar volume due to this process, is given by [6]:

$$Q_a^{ASR} = 2 \int \frac{d^3p}{(2\pi)^3} \int dI^{ASR} n_F(\epsilon) \left[1 - n_F(\epsilon')\right], \tag{2}$$

where dI^{ASR} is the spectral distribution of the individual electron synchrotron radiation intensity. The summation over the initial electron spin states provides the statistical factor two. The integration is over the radiated axion energy spectrum ω and the initial electron momentum $\overrightarrow{p}$. The factor expresses the Pauli's exclusion principle $n_F(\epsilon)\left[1 - n_F\left(\epsilon'\right)\right]$, where $n_F(\epsilon)$ and $n_F\left(\epsilon'\right)$ are the Fermi-Dirac distributions for the electrons in the initial and the final states with energies ϵ and $\epsilon' = \epsilon + \omega$ respectively

$$n_F(\epsilon) = \left[\exp\left(\frac{\epsilon - \mu}{T}\right) + 1\right]^{-1}.$$

The Boltzmann constant is employed $k = 1$. The electron chemical potential μ is $\mu \simeq \epsilon_F = (3\pi^2 n_e)^{1/3} \simeq p_F >> m \quad \left(\epsilon_F = \sqrt{p_F^2 + m^2}\right)$, n_e is the electron concentration, T is the electron temperature.

The formula (2) is valid if the transversal initial and the final electron momentums are relativistic: $p_\perp >> m, \quad p'_\perp >> m$. The magnetic field strength is $H << H_0 = 4,41 \cdot 10^{13} G$ and the electrons motion is semiclassical: $p_F >> T >> \omega_F = eH/\epsilon_F$ (in nonquantizing external magnetic field). The main contribution to the radiation is produced by electrons with momentum values $\left|\overrightarrow{p}\right| \cong p_F$ located near Fermi's sphere. There are two regions of the temperature and densities permiting to estimate the axion loss energy rate by analytical way:

$$\begin{aligned} &1) T >> T_c, \\ &2) T << T_c, \end{aligned} \tag{3}$$

where frequency $\omega = T_c = eHp_F^2/m^3$ corresponds to the spectral distribution maximum (see $dI^{ASR}/d\omega$ from (2)).

The results for the both cases ($i = 1, 2$) may be represented in the unified form:

$$Q_i^{ASR} = c\omega_i^{10/3} g_{ae}^2 T\,(eH/p_F)^{2/3}\,,$$

where c is the numerical coefficient: $c \cong 8.6 \cdot 10^{-3}$, $\omega_1 = T_c$, $\omega_2 = T$.

We also applicate the integral (2) to the case of the hot electron- positron plasma ($T >> p_F >> m$). The electrons and positrons are at phase equilibrium to the photon radiation, so that the chemical potential is $\mu = 0$. If the external magnetic field is nonquantizing $T >> eHT^2/m^3$ the energy loss rate is

$$Q_{pl}^{ASR} = 11 g_{ae}^2 T \left(\frac{eH}{T}\right)^{2/3} \left(\frac{eHT^2}{m^3}\right)^{10/3}.$$

This formula estimates the space-time interstellar matter axion emission.

2. In analogous manner we investigated the axion photoproduction by the magnetized degenerate electron gas due to Compton mechanism $e+\gamma \longrightarrow e+a$. We use the same kinematics as in [7] for the Compton scattering process $e + \gamma \longrightarrow e + \gamma$ in external magnetic field.

In the most interesting case of low temperature values and high densities: $T << T_c$ (see inequality 2) in (3)) the luminosity is:

$$Q^{AC} = 5.6 \cdot 10^{38} g_{ae}^2 \left(\frac{p_F}{m}\right)^{1/3} \left(\frac{H}{10^{13}}\right)^{8/3} \left(\frac{T}{10^9}\right)^{22/3}.$$

3. The main astrophysical method to constrain novel particles properties is the stellar energy loss argument:

$$Q\,(novel \quad particle) < Q\,(standard)\,,$$

where Q is the luminosity due to a certain particle emission. It is possible to find upper bounds on the axion-electron coupling constant g_{ae} by this method exploring the magnetized neutron star model (exactly its crust). We compare axion luminosity due to Compton and synchrotron mechanisms, Q^{AC} and Q^{ASR} respectively, with neutrino energy loss rate $Q^{\nu\tilde{\nu}}$ (see [8]) due to synchrotron mechanism playing a sufficient role in following parameter range: at densities: $\rho = (10^7 \div 10^{14})\ g/cm^3$, at temperature: $T = (10^8 \div 10^{10})K$ and external magnetic field strength $H = (10^{12} \div 10^{14})G$. This conditions are typical for the stellar shell. The best results for the axion-electron coupling are obtained when the parameters ρ, T, H satisfy with low temperature band: $T << T_c$ (inequality 2) from (3)). For the case of the axion synchrotron emission we set $n_e = 10^{33} cm^{-3}$, $T = 10^8 K$, $H = 10^{12} G$ and demand: $Q^{ASR} << Q^{\nu\tilde{\nu}}$. The upper limit is: $g_{ae} < 5.4 \cdot 10^{-14}$ (for the energy scale of the PQ-symmetry breaking: $v \gtrsim 10^{10} GeV$). For the axion Compton photoproduction the calculations are performed for $n_e = 10^{34} cm^{-3}$, $T = 10^{10} K$, $H = 10^{12} G$. The inequality: $Q^{AC} << Q^{\nu\tilde{\nu}}$ leads to $g_{ae} < 1.1 \cdot 10^{-13}$ ($v \gtrsim 1.9 \cdot 10^9 GeV$). But we hope to improve the latter result by taking into account the temperature contribution to electron propagator.

I thank my advisor A.V.Borisov and O.F. Dorofeyev for their aid and fruitful discussion.

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EFFECTS OF ANISOTROPY IN SUPERNOVAE

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Abstract

It is shown that a magnetic field produces the neutrino flux asymmerty at collapsing massive stars. This anisotropy is a possible cause of the non-central supernova explosion. Observable characteristics of this phenomenon are the non-uniform neutrino flux, the high velocity of the pulsar and non-sphericity of the supernova shell.

Supernovae are the most impressive explosions of stars [1, 2], in which a potential energy of $\sim 10^{49} J$ is released. Type II supernovae, which occur at the rate of about 10 per second in the observable Universe and which each produce about 10^{58} neutrinos of about 50 MeV per explosion.

In 1934, Baade and Zwicky [3] showed that only one percent of this energy is sufficient to eject a shell and produce a supernova explosion. At the beginning of the forties, Gamow and Schoenberg [4] proposed a mechanism of energy release in the collapse of a star due to emission of neutrinos.

A huge amount of energy is carried away by neutrinos during a time $\sim 10^{-3}$ sec, which is determined by the rate of the elementary weak process.

High densities, values of temperature and strong magnetic fields are characteristic of the collapse of stars. Modern model scenarios of supernovae (see e.g. [5-8]) make possible to take into account an ever wider set of factors, but discussion of the role of the magnetic field is still only just beginning. In particularly, the magnetic field influence on the neutrinos flux in SN1987A was discussed in [9].

After P - violation discovery in β–decay in external magnetic field by Wu et al. [10] investigations of electromagnetic field influence on above-mentioned reaction was performed [11-14]. Later it was demonstrated that under collapse conditions neutrino fluxes should be anisotropic ones [15, 16] (see also [17-20]). Dominant ν and $\tilde{\nu}$ ejection opposite magnetic field direction leads to acceleration the arising pulsar along magnetic field [15, 16, 21-24].

Among model explanations of high space velocities of pulsars there were suggestions that these velocities are due either to the breakage of correlation in double systems, or to the electromagnetic radiation of magnetic-dipole nature, or to some accidental non-centralness of the explosion in a supernova. For a certain sample of pulsars it is assumed now that the first two points of view definitely cannot describe the observed values of the space velocities of pulsars. In the third one the non-centralness of the explosion is likely to be explained with the anisotropy of neutrino fluxes. Neutrinos will leave the star in an anisotropic way due to the influence of the magnetic field of the collapsing star upon the electroweak neutrino-producing processes. The anisotropy of neutrino fluxes may result in the acceleration of the pulsar in the direction coinciding with the rotation axis of the pulsar.

On the basis of observational data on pulsar radiation in the radio range, using a model of pulsar acceleration due to neutrino ejection, it proved to be possible to establish an observational test [25-27] to determine the modulus of the pulsar spatial velocity and thereby augmenting data on the observed tangential velocities of pulsars.

We should also mention the elongation of supernova shells, which is a significant observed characteristic of supernova remnants [28]. Elongation of a supernova shell along the direction of the pulsar rotation axis can be explained by asymmetry of the fluxes of radiation and particles due to the magnetic field of the pulsar through the region of its magnetic poles. Here, one must also bear in mind that the rotation axis and the magnetic axis of the pulsar do not coincide, that leads to a wider range of angles of escape of the particles and radiation.

The development of the non-sphericity of the thrown away shell after the blast of the supernova may carry traces of the same anisotropy, enhanced by particle fluxes and the radiation from the magnetic poles of the pulsar. The large axis of the shell observed upon the celestial sphere and hiding the pulsar during the initial period of its development will be an alternative to the observation of the pulsar itself the radiation of which does not reach the observer at the Earth. Of course, one may hope that the observation technique will develop to an extent making it possible to detect the signal of the pulsar reflected from other bodies.

H. Bethe [29] argues that convective instabilities are occur in the wake of the shock. These instabilities are caused by neutrinos, which interact in such a way as to heat the matter that lies by far below. The resulting convective overturn transports energy, a portion of which is available to do work on the shock. Herant et al. [30] have extended this ideas. They tested this phenomenon with two-dimensional simulations and found conditions that yield vigorous explosions. The new calculations of Burrows et al. [31] confirmed the overall success of the convective mechanism. They used the best combined treatment of two-dimensional hydrodynamics, convection, the equation of state and neutrino transport.

Some approaches to the interpretation of observed data, obtained with the use of the most recent technology, have not lead unfortunately to the detection of a pulsar in SN1987A at the predicted time moment. It has not been detected up to now. This points to the overestimation of the role of predictions of the steps of the scenarios of the supernova on the basis of the hydrodynamical model calculations. According to our opinion, the models whithout taking into account the role of the magnetic field overlook important factors useful for the interpretation of the observed data of the supernovae evolution.

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ADELIC WAVE FUNCTION OF THE DE SITTER UNIVERSE

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Adelic generalization of the wave function of the Universe, which takes into account usual and p-adic geometries, is considered. It is shown that there exists adelic wave function for the de Sitter minisuperspace model.

Since 1987, p-adic numbers [1] and adeles [2] have been used in theoretical and mathematical physics (for a review, see [3,4]).

Any p-adic number $x \in Q_p$ can be presented by a canonical expansion $x = p^\nu(a_0 + a_1 p + ...)$, $\nu \in Z$, $0 \leq a_i \leq p-1$, where a_i are digits and p is a prime number. p-adic norm is non-archimedean (ultrametric) and for the above representation one has $| x |_p = p^{-\nu}$. It is of special interest the ring of p-adic integers $Z_p = \{x \in Q_p : | x |_p \leq 1\}$.

Real and p-adic numbers can be unified by means of adeles. An adele is an infinite sequence $a = (a_\infty, a_2, ..., a_p, ...)$, where $a_\infty \in R$, and $a_p \in Q_p$ with restriction that $a_p \in Z_p$ for all but a finite number of p. Introducing $A(S) = R \times \prod_{p \in S} Q_p \times \prod_{p \notin S} Z_p$, where S is a finite set of primes, the space of all adeles is a topological ring $\mathcal{A} = \bigcup_S A(S)$.

Ordinary quantum mechanics, which contains complex-valued functions of real variables, can be generalized to p-adic quantum mechanics [5] with complex-valued functions of p-adic variables. In an analogous way one can formulate adelic quantum mechanics [6], which unifies ordinary and p-adic quantum mechanics.

The physical meaning of p-adic and adelic quantum theory seems to be realized in the context of the Planck scale spacetime. According to quantum gravity there is an uncertainty measuring distances, $\Delta x \geq l_0 = \sqrt{\hbar G c^{-3}}$, where $l_0 \sim 10^{-33} cm$ is the Planck length. This can be regarded as a consequence of spacetime quantization with elementary length l_0. If we take $l_0 = 1$ then any p-adic distance $l_p = | n |_p \leq 1$.

Adelic quantum cosmology is the application of adelic quantum theory to description of the Universe as a whole. In other words, it is adelic generalization of ordinary quantum cosmology. If we wish to take into account all possible geometries to study our Universe, then a natural mathematical instrument to do that is just adelic theory.

According to adelic quantum mechanics [6] the ground state of a quantum-mechanical system is of the form

$$\Psi(x) = \Psi_\infty(x_\infty) \prod_{p\in S} \Psi_p(x_p) \prod_{p\notin S} \Omega(|\, x_p \,|_p) \,, \tag{1}$$

where $\Omega(|\, x_p \,|_p) = 1$ if $|\, x_p \,|_p \leq 1$ and $\Omega(|\, x_p \,|_p) = 0$ if $|\, x_p \,|_p > 1$. For a particular cosmological model to be adelic, existence of the p-adic ground-state wave function $\Omega(|\, . \,|_p)$ is a necessary condition.

According to (1) we formulate the adelic ground-state wave function of the Universe as follows:

$$\Psi[h_{ij}] = \Psi_\infty[(h_{ij})_\infty] \prod_{p\in S} \Psi_p[(h_{ij})_p] \prod_{p\notin S} \Omega[|\, (h_{ij})_p \,|_p] \,, \tag{2}$$

where h_{ij}, $(h_{ij})_\infty$ and $(h_{ij})_p$ are adelic, real and p-adic three-metrics on a compact three-surface, respectively. For a reason of simplicity we consider here only gravitational field with the cosmological term Λ.

As a starting point to obtain the wave function of the Universe in the real case one takes a functional integral of the form

$$\Psi_\infty[h_{ij}] = \int \chi_\infty(-S_\infty[g_{\mu\nu}]) \mathcal{D}(g_{\mu\nu})_\infty \,, \tag{3}$$

where χ_∞ is the additive character (exponential function). Integration is taken over some class of four-metrics $g_{\mu\nu}$ which induce the three-metric h_{ij}. S_∞ is the usual Einstein-Hilbert action with the cosmological term. In practice one usually transforms (3) into the corresponding Euclidean version.

To perform p-adic generalization we first make a p-adic counterpart of classical action using its form-invariance under the change of real to the p-adic number field. Then we generalize (3) and introduce p-adic complex-valued wave function

$$\Psi_p[h_{ij}] = \int \chi_p(-S_p[g_{\mu\nu}]) \mathcal{D}(g_{\mu\nu})_p \,, \tag{4}$$

where χ_p is the p-adic additive character, i.e. $\chi_p(x) = exp(2\pi i \{x\}_p)$.

Now, one can write down adelic wave function of the Universe,

$$\Psi[h_{ij}] = \int \chi_\infty(-S_\infty) \mathcal{D}(g_{\mu\nu})_\infty \prod_p \int \chi_p(-S_p) \mathcal{D}(g_{\mu\nu})_p \,, \tag{5}$$

which is the infinite product of (3) and (4). If one can obtain (2) from (5) we will say that such cosmological model is adelic one. More precisely, in the case of a simple minisuperspace model, where the metric is described by a single scale factor, after calculation of the integrals in (5) one has to obtain

$$\Psi(a) = \Psi_\infty(a_\infty) \prod_{p\in S} \Psi_p(a_p) \prod_{p\notin S} \Omega(|a_p|_p)\,, \tag{6}$$

where a, a_∞, a_p are adelic, real and p-adic scale factors, respectively.

Using the Robertson-Walker metric of the form

$$ds^2 = -q^{-1}(t)dt^2 + q(t)d\Omega_3^2\,, \tag{7}$$

which is mathematically convenient in quantum treatment [8] the corresponding adelic action is $S = (S_\infty, S_2, ..., S_p, ...)$, where

$$S_v[q] = \frac{1}{2}\int_{t_1}^{t_2} dt(-\frac{\dot{q}^2}{4} - \lambda q + 1)\,, \quad (v = \infty, 2, ...) \tag{8}$$

is appropriately normalized and λ is a parameter proportional to the cosmological constant Λ.

The minisuperspace propagator is $G_v(q_2, q_1) = \int dT \mathcal{K}_v(q_2, T; q_1, 0)$, where $\mathcal{K}_v(q_2, T; q_1, 0)$ is the usual quantum-mechanical propagator. After integration the classical action (8) becomes

$$S_{cl} = \frac{\lambda^2 T^3}{24} - [\lambda(q_2 + q_1) - 2]\frac{T}{4} - \frac{(q_2 - q_1)^2}{8T}\,, \tag{9}$$

where $q(0) = q_1$ and $q(T) = q_2$. Since (9) is quadratic on q_2 and q_1, we have $\mathcal{K}_v(q_2, T; q_1, 0) = \lambda_v(-8T)\,|\,4T\,|_v^{-\frac{1}{2}} \chi_v(-S_{cl})$, where $\lambda_v(a)$ is a definite complex-valued function [6].

According to the Hartle-Hawking proposal [8] the wave function is $\Psi_v(q) = G_v(q, 0)$. The p-adic wave function of the de Sitter minisuperspace model is

$$\Psi_p(q, \lambda) = \int_{|T|_p \le 1} dT \frac{\lambda_p(-8T)}{|\,4T\,|_p^{\frac{1}{2}}} \chi_p[-\frac{\lambda^2 T^3}{24} + (\lambda q - 2)\frac{T}{4} + \frac{q^2}{8T}]\,, \tag{10}$$

where we specified the range of integration taking $T \in Z_p$.

Now we shall show that the above adelic wave function has the form

$$\Psi(q, \lambda) = \Psi_\infty(q_\infty, \lambda_\infty) \prod_{p\in S} \Psi_p(q_p, \lambda_p) \prod_{p\notin S} \Omega(|q_p|_p)\,, \tag{11}$$

where $S = \{2, 3, p_1, ..., p_n : |\lambda_{p_i}|_{p_i} > 1, (i = 1, 2, ..., n)\}$.

To prove (11) let us consider (10) for $|\lambda|_p \leq 1$ and $p \neq 2,3$. Under these conditions $\Psi_p(q,\lambda) = \int_{|T|_p\leq 1} dT\lambda_p(-2T) \mid T|_p^{-\frac{1}{2}} \chi_p(\frac{\lambda q T}{2} + \frac{q^2}{8T})$, where we used the property $\lambda_p(a^2b) = \lambda_p(b)$. Replacing the variable of integration T by $(8y)^{-1}$ we get $\Psi_p(q,\lambda) = \int_{|y|_p\geq 1} dy\lambda_p(-y) \mid y|_p^{-\frac{3}{2}} \chi_p(\frac{\lambda q}{16y} + q^2 y)$.

For $|q|_p \leq 1$ we obtain $\Psi_p(q,\lambda) = \sum_{\nu=0}^{\infty} p^{-\frac{3\nu}{2}} \int_{|y|_p=p^\nu} dy\lambda_p(-y)\chi_p(q^2 y)$. Taking into account that $\lambda_p(-y) = 1$ if $|y|_p = p^{2k}$, $\sum_{y_0=1}^{p-1} \lambda_p(-y) = 0$ if $|y|_p = p^{2k+1}$, and

$$\sum_{y_0=1}^{p-1} (\frac{y_0}{p}) e^{2\pi i \frac{q_0^2 y_0}{p}} = \begin{cases} \sqrt{p}, & \text{if } p \equiv 1(mod4) \\ i\sqrt{p}, & \text{if } p \equiv 3(mod4) \end{cases},$$

where $(\frac{y_0}{p})$ is the Legendre symbol, one obtains $\Psi_p(q,\lambda) = 1$.

To calculate the wave function for $|q|_p > 1$ it is convenient to introduce a new variable of integration x by $y = x + q^{-1}$. Then one can show that $\Psi_p(q,\lambda) = \chi_p(-q)\Psi_p(q,\lambda)$, and since $\chi_p(-q) \neq 1$ for $|q|_p \geq p$ it follows $\Psi_p(q,\lambda) = 0$. According to the above results we can write

$$\Psi_p(q,\lambda) = \Omega(|q|_p) \tag{12}$$

if $|\lambda|_p \leq 1$ and $p \neq 2,3$.

Since adele $\lambda = (\lambda_\infty, \lambda_2, ..., \lambda_p, ...)$ contains $|\lambda_p|_p > 1$ only for a finite number of p, from (12) it follows (11). Note that obtained adelic wave function has also a place if λ and q are principal adeles, i.e. if $\lambda_\infty = \lambda_p \in Q$ and $q_\infty = q_p \in Q$ for all p.

Summarizing, it is obtained a remarkable result that in the de Sitter minisuperspace model exists ground state $\Omega(|q|_p)$ for all but a finite number of p. By this way it is shown that there exists adelic wave function for the de Sitter model of the Universe.

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ON THE HISTORY OF THE SPECIAL RELATIVITY CONCEPT

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ABSTRACT

This report contains to short excurse in the origin of relativity concept not only for the purpose of supplement a widely spread one- sided image of this important stage of the history of natural science, but also to mind those forgotten approaches to the relativity theory construction that cast light on its close relation to the concepts of classical physics. We emphasize some statements of A. Poincare and H.A. Lorentz which help us to penetrate deeper into the essence of relativity theory.

1. INTRODUCTION

"... Relativity burst upon the world, with a tremendous impact. ...The impact that relativity produced I think has never been equalled either before or since by any scientific idea catching the public mind."

Paul A. M. Dirac (1977)

The special relativity concept created in the the first years of our century, initiated radical transformation of the earlier physical images and became one of the grounds in modern physics. But in spite of its significant place which this theory occupies in the system of modern scientific knowledge, in the historical description of its origin a one-sided approach with substantial gaps became, unfortunately, traditional. In this historiography the period preceding the creation of the relativity theory turned out to be especially underestimated, i.e., it happened when the principle grounds of the new physical theory were put forward to solve contradictions existing in those times.

Such unattentive attitude to the appearance of the principle grounds of the new theory is impossible to explain by to loss of scientific interest to the historical details of the origin of the new scientific concepts. At the same time the principle grounds of a more radical physical theory - quantum mechanics, were developed in physics. But historiographers of quantum mechanics have always regarded this period as a most important element of the deviation from the old ideas of classical physics. The actual era of quantum mechanics is considered to have originated in 1900, the year when M.Planck put forward the hypothesis of discrete energy states of a oscillator and using it derived his formula for the equilibrium black-body radiation spectrum. The subsequent A.Einstein's idea (1905) of photons and L.de Broglie's idea (1923) of a hypothetical wave with a phase velocity related to the velocity of a microparticle were also judged accordingly. In any case, precisely these ideas were always stressed to underlie the wave mechanics created by E. Schrodinger (1926). For a revelatory illustration of the flaws of the historiography of special relativity it is useful to compare the respective presentations of equivalent periods in the development of the two theories, both of which form the foundation of modern physics.

No other physical doctrine excited such widespread interest, as the theory of relativity. The unusual conclusions of the theory on issues seeming most simple always aroused great interest outside the scientific community. Most likely, it was actually because of this widespread popularity of the theory of relativity, organized in the main by men of letters far from science, that its historiographers deviated from an exact and objective description of the history of this most outstanding discovery.

This story, how the historical gaps in the origin of the relativity concept were eliminated in the second half of our century, is the subject of the present report. We also consider those important statements A. Poincare and H. A. Lorentz, which promoted the development of the deeper understanding of the essence of this theory.

2. THE ORIGIN THE INITIAL IDEAS OF SPECIAL RELATIVITY

> "Experiment has provided numerous facts admitting the following generalization: it is impossible to observe absolute motion of matter, or, to be precise, the relative motion of ponderable matter and ether".
>
> *Henri Poincare (1985)*

The descriptions of the history of special relativity, at least those published before 1954, contained no mention whatever of now the initial ideas were formulated during the period preceding its creation. Only the formal utilization was noted, in the works by W. Voigt (1887) and H.A. Lorentz (1892 and 1895), of "local" time in a moving system with the origin of time depending linearly upon the space coordinate.

A truly novel contribution to the historiography of special relativity appeared in 1954 in second volume of the historical work [1] by well-known British mathematician E. Whittaker (the first volume was published in 1910). Whittaker was the first to point out that in 1899 the outstanding French mathematician and theoretical physicist Henri Poincare expressed firm belief in it being essentially impossible to observe absolute motion in optical experiments owing to the relativity principle being obeyed strictly in optical phenomena, also. The scientist confirmed his idea in a talk at the Paris International Physical Congress held in 1900. E.Whittaker also presented next excerpt about prediction new relativistic mechanics from the talk delivered in St Louis Congress of Arts Science by Poincare in 1904 stating: "From all these results there must arise an entirely new kind of dynamics, which will be characterised about all by the rule, that no velocity can exceed the velocity of light." [1, p. 31].

The chapter of the book by Whittaker on special relativity gave rise to lively discussions and, doubtlessly, aroused the big interest of many scientists in independent historical investigations of the period preceding the creation of this theory. As a result, not only was a more detailed investigation of the works by Poincare indicated by Whittaker carried out, but several of his publications [4, 5] were also saved from oblivion. It turned out that the principle of relativity for electromagnetic phenomena was proposed by Poincare even earlier. Thus, the words of Poincare, used as a epigraph to this chapter, were taken by us from his article of 1895 [4]. Further, quoting the Michelson experiment, Poincare stressed that theory must satisfy the above law without any restrictions related to precision.

In paper [6] I personally drew attention to the fact that in the article "Measurement of time" [5] published in 1898 Poincare, in discussing the issue of determining the quantitative characteristics of physical time, arrives at important conclusions, on the conventional essence of the concept of simultaneity, not only representing historical interest, but also permitting to clarify the limited nature of the existing interpretation of the space-time aspect of special relativity. Poincare notes that the postulate of the constant velocity of light "provided us with a new rule for searching for simultaneity", but concerning the assumption made use of here on the independence of the speed of light for the direction of its propagation the author makes the following categorical assertion: "This is postulate without which it would be impossible undertake any measurement of this velocity. The said postulate can never be verified experimentally."[5]. These profound arguments justified Poincare his article the following no less categorical statement: "The simultaneity of two events, or the sequence in which they follow each other, the equality of two time intervals should be determined so as to render the formulation of natural laws as simple as possible. In other words, all these rules, all these definitions are only the fruit of implicit convention." [5].

These precious ideas of great thinker were not applied in any explicit form in the creation of the special theory of relativity, unlike his assertion concerning the principle of relativity being rigorously obeyed by electromagnetic phenomena. Later, also, they were not realized; thus, for instance, the conclusion was not comprehended that the concept of simultaneity, for events occupying different sites, was based on measurement of the speed of light in one direction being essentially impossible without the adoption of a convention on the equality of velocities of light for processes propagating in opposite directions. Convincing evidence that the essence of the above issue was not fully realized by specialists is presented, as it is shown in my article [7], by the publication in several central physical journals of proposals, based on false grounds, to measure the speed of light in a sole direction. Such proposals always implicitly contradict the fundamental principle of causality, and their publication in journals is just as inglorious for the publishers of respectable scientific journals, as discussion in the scientific press of proposals aimed at constructing devices experiencing perpetual motion.

A further development of the idea of determining time on the basis of the postulated constancy of the velocity of light was presented by Poincare in 1900 in an article on the Lorentz theory [8]. In this work the first physical interpretation was given of "local" time introduced by Lorentz as the time corresponding to readings of two clocks synchronized by a light signal under the assumption of a constancy of the velocity of light. This work was ignored by traditional historiography, even though the explanation given by Poincare of the essence of the proper time was repeated literally in 1905 in a work by A.Einstein.

The works, in which the new transformations of space-time coordinates that subsequently occupied the central place in the theory of relativity, should also be attributed to the period preceding the creation of this theory. In the literature the opinion is widespread that these transformations were obtained in their final form by Lorentz in 1904. The fact is less known that they appeared in the book "Ether and matter" by the British theoretical physicist J. Larmor in 1900 [9].[1] And what is totally unknown to historians is that Lorentz first derived the transformations, that subsequently became known, upon the proposal of Poincare, as Lorentz group, in a work of 1899 [10]. In this article were supplemented by factor $\gamma = (1 - v^2/c^2)^{-1/2}$ to the transformations of coordinate $x' = x - vt$ and time $t' = t - vx/c^2$ introduced earlier in the work of 1895 [13]. Only after this supplement new transformations were brought in strict accordance with the invariance of the Maxwell equations and made to satisfy the requirements of a group.

Thus, by the end of the past century the problem of explaining absence

[1]To the presented historical information one must add that the relativistic relation for adding velocities was first obtained by Larmor (see Chapter XI, item 113 of the book [11]) and that the author discussed also the relativistic effect of deceleration of time for electromagnetic processes in a material system travelling through ether (see item 114 in [11]).

of "ether wind" was quite ready for its ultimate solution by the above works by Poincare, Lorentz and Larmor.

3. THE CREATION OF SPECIAL RELATIVITY

> "... Relativity burst upon the world, "The special theory of relativity is not the creation of a single individual, it is due to the joint efforts of a group of great investigators — Lorentz, Poincare, Einstein, Minkowsky."
>
> *Max Born (1959)*

The history of the concluding stage in the creation of the special theory of relativity was only complicated by discrepancies in the estimation of the significance of well-known parallel works and, hence, by the insufficient attention subsequently paid to alternative approaches. These discrepancies reflected, first of all, the objective difficulties in comprehending the theoretical constructions, in same cases, and of apprehending the logic of reasoning, in others. But, regretfully, the tendentious attitude in singling out the recognized as the first one hindered objectiveness in estimating the significance of various publications.

In 1921 an extensive article (about 230 pages volume) [11], written by the future eminent theoretical physicist, at the time a twenty-years-old student of the Munich university, Wolfgang Pauli, was published in the German edition of the Encyclopedia of Mathematical Sciences. This article, later published as book in various languages, still remains one the best expositions of the fundamentals of the special and the general relativity. The article began with a short historical stady, before the publication in 1953 of the book by Whittaker was the most complete and objectuve review of history of special relativity.

In concluding a incomplete list of works were published during the period preceding the creation of the theory Pauli singled out for further discussion "three contributions, by Lorentz [12], Poincare [13] and Einstein [14], which contain the reasoning and the developments that form the basis of the special theory of relativity". Indeed, the grounds do exist for considering the three authors of these fundamental works the creators of the special theory of relativity, even though the contribution of each scientist differs from that of the others. But, in spite of the great success of the article and book by Pauli, many scientists subsequently ignored his historic estimation and adhered in their scientific publications to the widespread version, presented in popular literature, that the sole creator of the theory was Einstein.

The publication in 1935, in the Russian language, of a collection of the classics of relativity, edited by V.K. Frederiks and D.D. Ivanenko [15] turned out to be a digression from the obvious hushing up of the work of H. Poincare [13]. Unlike the collection of the first works on relativity theory, published in Germany in 1913, the Russian edition contained the principal work written by H. Poincare in 1905 [13,b]. The editors pointed out, in the comments to the articles included in the collections, that the main article by Poincare "not only contains Einstein's parallel work, but in certain parts also the more recent - by nearly three years - article by Minkowskii, and partly even exceeds the latter" [15, p. 367], while the fact that this fundamental work had been forgotten was classified as not having analogs in modern phisics. But this high estimate of the work by Poincare only had some influence among theoretical physicists, and did not become known to the historians of science even in Russia. It is no chance that the high estimate of the work by Poincare, given by the editors of the collection in the concluding remarks, was supported and acquired further development in Russia in the work of the next generation of physicists. Thus, in 1973 I compiled and submitted for publication by "Atomizdat" the most complete collection of pioneer works in special relativity theory, which included translations into Russian of articles written by Poincare in 1895-1906 [16]. Subsequently, in 1984, A.A. Logunov published a book under the title "On the works of Henri Poincare ON THE DYNAMICS OF THE ELECTRON" [17].

My proposal to publish a more complete collection of works of the classics of relativity is based on the example of the 1935 collection "The principle of relativity", which reveals that the publication of translations of the original texts of forgotten early works by A. Poincare and G. Larmor would serve as the most objective and effective way to convince the readers of the decisive role of these scientists in creating the concept of relativity and in prepearing a scientific atmosphere for final solution of the problem.

In his book, dedicated to two 1905(06) publications by H. Poincare [13], A.A. Logunov chooses a non-traditional form of exposition for analyzing these works. Instead of usual quotations of fragments from the originals under descussion, the book includes the complete texts of these two articles, published by H. Poincare under the common title of "On the dynamics of the electron", which are time to time interrupted by detailed comments written by A.A. Logunov. These comments, in the main, serve a sole purpose: to show the profound physical meaning and the essential novenlty of particular points and relations established by H. Poincare. Here, A.A. Logunov often inserts into the text of his explanations quotations from earlier articles by Poincare. From these additions it becomes quite clear that the main points of the new theory were put forward by the French scientist long before 1905, while certain new concepts such as "local" time were given a clear explanation of their physical meaning in his earlier articles. At the same time, it becomes clear, how much better, from the point of view of physicists, could the main article of Poincare, intended for mathematical journal "Rendiconti del Circolo Matematico di Palermo" have become, had the earlier explanations or, at least, references to his articles on such explanations of the physical meaning, been utilized.

It is important to note that all the formulae in the articles by Poincare, that are presented in A.A. Logunov's book, are given in accordance with modern notation, which essentially simplifies understanding the theoretical relations.

To conclude this section we note that the history of the creation and development of novel scientific concept is best studied making use of the originals of scientific articles, access to which is significantly simplified owing to the publication of topical collection of old original articles. I have no doubt that, upon acquaintance with the original works of the classics of relativism, any benevolent reader will arrive at the conclusion that special relativity was created by a whole of eminent scientists, — Poincare, Lorentz, Einstein and Minkowski. I discussed detail principal significanceof a contribution of every founder of this theory in concluding article in the

Collection [16].

We now terminate the above fragmentary historical sketch the aim of which was to draw attention to the ideas of Einstein's predecessors, the falling of which into oblivion doubtlessly impoverished the understanding of special relativity for many years. The same idea concerning the limitation of the understanding of this theory was expressed by A.A. Logunov in the preface to his book [18]. by following words: "However, dogmatism and faith, alien to science, but always accompanying it, have done their business. Nearly up to our time have they limited the level of understanding and, consequently, reduced the range of applications of the theory of relativity."

Now we consider question about of the more profound conception of the special relativity, following my book [19] which was published in Italy into the encyclopedic series.

4. THE ESSENCE OF SPECIAL RELATIVITY

"The true relation between real objects are the only reality we are capable of apprehending."

Henri Poincare (1902)

Further we must to realize that the relations are preserved plays a decisive role here, totally in accordance with the simple, but extremely profound assertion made by Poincare [20], adopted as an epigraph for this section of the present article. The term "relativity" occuurring in the title of the theory has a second unexpected justification. Besides the conventional meaning used for establishing in the theory new quantities depending on the relative velocity of motion of reference frames, the term "relativity" may be justified, also, in that the new absolute quantities and invariant relations established by this theory signify conservation of the relations between quantities depending on the respective velocities.

Indeed, the main content of special relativity resides in the general properties of physical phenomena corresponding to the pseudoEuclidean geometry of the four-dimensional world, in which space and time join in a certain entity, independent of the relative motion of inertial reference frames. However, this extremely concise formulation, naturally, requires some decoding, separation of the physical essence from the adopted form of its mathematical expression. It is even useful to digress some time from a form adequate to the content and deal with another plausible expression, so as to reveal in a clear manner the physical essence of the new theory.

The idea of the main content of special relativity was expressed by Minkowski in his famous talk "Space and time" by the following statement termed by the author the postulate of the absolute world: "... the postulate comes to mean that only the four-dimensional world in space and time is given by phenomena, but that the projection in space and in time may still be undertaken with a certain degree of freedom..." [21]. Minkowski's talk began with even sharper words concerning the arbitrariness that arose in the new theory, when space and time quantities were considered separately: "Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality". I do not think Minkowski termed these quantities shadows, because in the new theory they became relative, dependent upon the velocity of relative motion. Most likely, Minkowski implied arbitrariness to signify the apparent contradiction of the obtained results: lengths in each considered reference frame exhibit contraction with respect to any other frame, clocks in each frame slow down relative to other frames. But, anyhow, enrolling quantities in the category of fictions does not free one from the necessity of clarifying the essence of the corresponding effects.

These the "miraculous" reversal of quantities, resulting from comparing lenghts and time intervals, are due to transition from the simultaneity of one frame to the simultaneity of anower frame. We see that the point is that proper simultaneities adopted in different inertial frames differ from each other. It remains for us to clarify the meaning of the central provision of all the theory, the relativity of simultanety, in any words, to understand which common properties of physical processes are reflected in the artificially chosen shift of origins of time at differing points of a moving inertial reference frames. For ultimate clarification of this issue without renouncing arguments based on common sense it is best to turn to the description of velocities of physical processes in a moving frame within the Galilean approach utilizing a unique simultaneity for the two frames being considered.

But before presenting the results of such an analysis we shall recall the main advantage achieved by introducing a shift in the simultaneity along the direction of relative motion of the frames. The shift in simultaneity was introduced under the condition of constancy of the speed of light, and as a result the independence upon direction is obtained of the velocities of all physical processes in each inertial frame from sources at rest in these frames. The calculus of space-time coordinates in each inertial frame was also chosen under the condition that the principle of relativity be satisfied, and therefore the laws of physics turn out to be invariant with respect to relativistic transformations of coordinates. Precisely this represents the content of the correspondence, noted above, of the chosen relativistic metric to the poperties common to physical processes.

Now let us ponder over the main question: what significance has Nature being consistent precisely with the special principle of relativity, and not with the Galileo-Newton-Hertz principle of relativity? Clearly, it means conservation of the form of mathematical equations expressing physicfl laws only under the condition that a relative shift in simultaneity be introduced, when time coordinates of events are calculated in two inertial reference frames moving relative to each other. This means that relative to the simultaneity in the initial frame $K(x, t)$ the reading of a clock in the frame $K'(x', t')$ is ahead by a quantity, that increases linearly along the x'-axis. This shift oa simultaneities does not violate the equivalence of the reference frames, since the reading of the clock in frame $K(x, t)$ will be ahead relative to the simultaneity in frame $K'(x', t')$ by a quantity increasing linearly along the direction opposite to the x-axis.

Hence it should be clear how unjustified it would be to interpret the spetial principle of relativity as the assertion of identity of hcw physical processes proceed in different inertial reference frames moving relative to each other, if the identity of mathematical expressions for the respective physical processes is achieved by taking advantage in these reference frames of noncoinciding times, t and t'. The point is that their main difference consisting in the relative shift of simultaneities means taking into account the general delay of processes along the direction of relative motion of the frames. The principle of relativity being satisfied signifies conservation of kinematical similarity while all processes experience a common delay along the x'-axis. This can be ultimately verified by considering the velocities of processes in amoving inertial reference frame $\hat{K}'(\hat{x}', t)$, the coordinates of which are related to coordinates in the initial frame $K(x, t)$ by the Galileo transformations (1).

Indeed, for the absolute velocity of an arbitrary physical process reproduced at an angle θ in a moving reference frame, utilizing the coordinates $\hat{x}' = x - vt$, $\hat{y}' = y$, $\hat{z}' = z$, $\hat{t}' = t$ we obtain, in accordance with refs. [6,19], the following relation:

$$\hat{u}'(\hat{\theta}') = \frac{u_0(1 - v^2/c^2)}{\left(1 - (v^2/c^2)\sin^2\hat{\theta}'\right)^{1/2} + (u_0 v/c^2)\cos\hat{\theta}'}, \tag{1}$$

where $u_0 = \text{const}(\theta')$ stands for the absolute velocity of the same process, if the coordinates x', y', z', and t' are used.

For the direction along the x'-axis ($\hat{\theta}' = 0$) and the opposite direction $\hat{\theta}' = \pi$ we obtain from (1) the respective velocities

$$\hat{u}'(0) = u_0\frac{1 - v^2/c^2}{1 + u_0 v/c^2}, \quad \hat{u}'(\pi) = u_0\frac{1 - v^2/c^2}{1 - u_0 v/c^2}.$$

Hence for light ($u_0 = c$) we obtain the velocities

$$\hat{u}'(0) = c - v \quad \text{and} \quad \hat{u}'(\pi) = c + v$$

which correspond to the expressions of classical physics and to the problem of "ether wind" that arose in this connection.

Consequently, relativistic theory introduced no changes directly into the motion of a light front in a moving reference frame, while substitution of the constant "c" for the velocities (1) for all direction is due to transition in the moving reference frame from space-time coordinates, $\hat{K}'(\hat{x}', t)$, to the new calculus of coordinates in the same inertial frame, $K'(x', t')$. This cooresponded to the primary provision of the Lorentz theoretical construction concerning the conservation, in an intact form, of classical electrodynamics and optics. The same result transition from the velocities of light $c - v$ and $c + v$ for opposite direction to the constant speed of light "c" was interpreted by Einstein as the result of clarification of the true course of time in a moving frame. We introduce into this assertion only a small, but exstremely significant correction: chanding the notion of the true course of time in some frame signifies a corresponding change of the general course of physical processes in this frame, which can be clearly ilustrated within the preceding approach involving a unique time $\hat{t}' = t$, or $\hat{t} = t'$, for two inertial reference frame.

In the first case ($\hat{t}' = t$) we have isotopic velocities of physical processes in the frame $K(x,t)$ and we fix anisotropic velocities of similar physical processes reproduced in indentical conditions in another inertial frame $\hat{K}'(\hat{x}', t)$. This dependence of the velocity upon the angle, represented by relation (1), exhibits a remarkable peculiarity: in no real experiment can it be distiguished from the case $u = \text{const}(\theta)$, if in Nature there exist no processes with velocities exceeding, within this version of the description, the speed of light in vacuum, i.e. $u_0 < c$. Relation (1) is, naturally, implied to apply to all processes, without exception. The noted remarkable feature of relation (1) follows formally from the fact that the simple transformation of the coordinates of events from the fact that the simple transformation of the coordinates of events from $\hat{K}'(\hat{x}, t)$ to $K'(x', t')$[2] realizes transition to the isotropic velocities $u' = \text{const}(\theta)$. Doubtless, it is of interest, however, to consider in detail the physical reasons underlying the indistinguishability of the obtained angular dedependence (1) and the isotropy of velocities.

It lies in the general property of conservation of kinematical similarity for all physical processes. The velocity angular dependence (1) exhibits the same peculiarity consisting in that the relation between different processes are essentially indistinguishable from the relations between the processes, when the velocities of the processes are independent of the angle. Thus, included in the general, and therefore nonobservable, effects is the difference between the velocities of processes in opposite directions along the $\hat{x}'$-axis. The time required for a certain length to be translated in one direction dif-

[2]These coordinates of event are related according [18, p. 28] as follows:

$$x' = \gamma \hat{x}'; \quad y' = \hat{y}; \quad z' = \hat{z}'; \quad t' = \gamma \left[\hat{t}'\gamma^{-2} - \hat{x}'\frac{v}{c^2}\right].$$

Here $\gamma = (1 - v^2/c^2)^{-1/2}$.

fers from the time required for going in the opposite direction by the same quantity for all physical processes. In other words, the difference between the velocities of light in the positive and in the opposite directions for a moving frame, (1), encountered by classical physics is essentially nonobservable in experiments performed in this frame, only because any other physical process exhibits the same propagation delay in the positive direction with respect to propagation in the opposite direction. Now, does such a nonobservable delay exist for all processes? It exists objectively with respect to processes reproduced in similar conditions in another frame, conventionally regarded as the primary frame. The physical meaning of this delay is totally equivalent to introduction in the moving frame of a proper simultaneity differing from the simultaneity of the primary frame. The limitation of the orthodox interpretation of special relativity consists precisely in that it actually does not reveal the true meaning of the relativity of simultaneity.

The orthodox interpretation of special relativity concentrated on substantiation of a proper basis for calculating space-time coordinates in each individual inertial reference frame. Set aside was the approach initiated by Lorentz, that was based on parallel consideration of two basises for the calculus of coordinates in each of the two inertial reference frames being considered: $K(x, t)$ and $\hat{K}(\hat{x}, t')$ for one, and $K'(x', t')$ and $\hat{K}'(\hat{x}', t)$ for the other. As a result of this economic approach the problem of substantiation acquired a formal solution involving an essential rupture of the commonsense logic. Preliminary consideration of the velocities of physical processes expressed in unified Galilean scales in two inertial frames permits to verify in the simplest manner the relative difference between the courses of processes in the direction of relative of the reference frames. All processes proceed slower in frame K', that in frame K, along the x-axis, but this does not violate equivalence of the frames, since the opposite direction: all the processes in frame K are delayed with respect to the processes in frame K'. The assertion concerning the relative delay of velocities of processes only reveals the physical meaning of the relativity of simultaneity. The role played by the utilized Galilean scales on rulers and faces of clocks is the same as that of reduction to common measurment units of the quantities being compared.

The constancy of the velocity of light exhibite two different aspects.

Thus, the initial provision on the independence of the velocity of light of the motion of the source is something that can be checked experimentally. The assertion of independence of the velocity of the motion of the reference frame has another foundation. Here, instead of the velocities of other physical processes in the given inertial frame is preserved. Precisely because of the relation between the velocities of processes remaining unchanged the proper time introduced in the given frame acquires the status of real time singled out among all possible calculated times by a sole indisputable advantage – it provides for the absolute values of velocities of physical processes originating from sources at rest in the given refence frame being independent of the direction of propagation.[3] But this advantage of choosing for each inertial reference frame its proper basis for calcilating space-time coordinates must not, however, over- shadow the objective relative difference between the velocities along the direction of relative motion of two reference frames. It is merely this fact that is expressed by the difference between the proper simultaneities in these frames leading to the delay of all processes by one and the same quantity depending only on the distance along the x-axis.

In the spirit of the ideas of Lorentz with respect to ether the "ether wind" being nonobservable in the case of light could be explained by the corresponding motion through ether influencing all physical processes. However, imposition by such an explanation of the motion of ether secretly at rest in the initial frame $K(x, t)$ has no sufficient foundation, since, in considering the propagation velocities of the corresponding processes, we obtain, utilizing a unique time $\hat{t} = t'$, asymetric velocities for the nonobservable "ether wind" in the opposite direction in the initial frame $\hat{K}(\hat{x}, t')$. Therefore we are justified in relating the discussed kinematical effects only to the fact itself of relative motion, while their appearance should be explained by the universal dependence of the dynamics of any whatever interaction upon the velocity of relative motion.

[3]Precisely for this reason, to determine the proper time in some inertial refence frame any physical process from a source at rest in the given frame can be utilized, under the assumption that the velocity of the process be independent of the direction in which it propagates. Besides this, clocks previously synchronized at the same point of the frame and then slowly taken apart to different points exhibit readings corresponding to the proper simultaneity of the given inertial frame.

5. CONCLUSION

"A problem arises only when we ... assume or postulate that the same physical situation admite of several ways of description ..."

Albert Einstein (1949)

Revolutionary transformations of basic physical conceptions never proceed smoothly. Giving up conventional views is always painful. Smoothing out the uneven development of knowledge proceeds gradually as the essence of novel concepts is penetrated. Bridges across abysses and crevices separating levels of knowledge are most often built by new generations of scientists, much later than when the new physical theory originates. The process of extending the understanding of a fundamental theory lasts many decades and develops along several main directions. One of these involves revelation of the relation to preceding physical opinions and clarification of the actual degree of novelty inherent in the primary provisions of the discussed theory. Another approach is to clarify the limits justifying application of the theory, based on further development of the understanding of the physical theory.

The latter type of development of the interpretation of a fundamental theory lasts the longest, since it is completed only by the creation of a more general theory ultimately establishing the limits of the given physical theory. Thus, comprehension of classical mechanics, in this respect, was completed only upon creation of the special and general theory of relativity and of quantum mechanics, that imposed limits on its application and explained the reasons of this limitation. The example of classical mechanics also clarified the significance of the criticism, initiated by E.Mach, of the formulation of its laws, originated with Newton, for the subsequent devitation from the conceptions of classical mechanics.

It is no chance that these general issues, related to the knowledge of the essence of physical laws, have been touched upon in the concluding part of my paper on special relativity. I hope to convince the readers that further development of the interpretation of the existing theoretical foundation of the physical science represents a most interesting sphere of scientific activity. The scope of such activities enhancing the profundity of scientific truths, actually already established in physics, can be termed "Foundation of physics", after the title of the international journal that organizes successful discussions of the investigations in this fascinating, and important for the further development of physics, field of scientific activity.

The author sincerely hopes the analisis perfomed in this article and the critical discussion of the simplest of modern physical theories will convince the readers of the existence of more significant possibilities of fruitful activity aimed at the developing the interpretation of other modern theories. Thus, for example, in physics great efforts are still required for clarifying such most important issues, as the reasons underling the appearance of energy nonconservation in the formalism of the geometrized relativistic theory of gravity, and for explaing the astonishing interference phenomenon in experiments involving individual quantum objects for which the theory till now provides a formal description.

Truly, for fruitful activity in the indicated field it is important to free oneself from the prejudice that a physical theory is completed, when a set of mathematical relation is established that describes experimental facts in the respective range of physical phenomena. It must become quite clear that penetration of the essence of profound truths of truly scientific knowledge of Nature merely originates with the establishment of rigorous quantitative laws.

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MAXWELL FORM OF THE EINSTEIN EQUATIONS AND QUASI-FRIEDMANNIAN COSMOLOGY

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The space-time V is a parallelizable differential manifold with tetrad field $e_a = e_{a\mu}(x)dx^\mu$. This tetrad and the constant metric $g_{ab} = diag(-1,1,1,1)$ determine Riemannian metric $g_{\mu\nu} = g^{ab}e_{a\mu}e_{b\nu}$, Riemannian connection ∇_μ, d'Alembertian $\Box = -\nabla^2$, Hodge operator $*$, codifferential $\delta = *d*$, the Laplacian $\Delta = d\delta + \delta d$, Ricci-tensor $R_a = (\Delta - \Box)e_a$. We have (λ is any p-form)

$$*1 =| g |^{1/2}\, d^4x, \quad g = detg_{\mu\nu}, \quad **\lambda = (-1)^{p(4-p)}(sgng)\lambda$$

$$*e_a = *1\ |_{Ker\ e_a} =| *e_a \mid d^3x,$$

$$| *e_a |^2 =| det(g_{\mu\nu} - \epsilon_a e_{a\mu}e_{a\nu})\ |_{Ker\ e_a}|$$

$$\delta e_a = -\nabla^\mu e_{a\mu} =: K_a = -e_a^\mu \partial_\mu \ln | *e_a | = K_{a\mu\nu}g^{\mu\nu}$$

$$\nabla_\mu e_{a\nu} = \epsilon_a e_{a\mu}a_{a\nu} - K_{a\mu\nu} - A_{a\mu\nu}, \quad \epsilon_a = \epsilon^a = g_{aa}$$

$$a_{a\mu} = e^{\nu}_{a}\nabla_{\nu}e_{a\mu} = L_{e_a}e_{a\mu}, \;\; 2K_{a\mu\nu} = -L_{e_a}(g_{\mu\nu} - \epsilon_a e_{a\mu}e_{a\nu})$$

Here $| *e_a |$ is an elementary 3-volume orthogonal e_a-lines, L_{e_a} is a Lie derivative along $e^{\mu}_{a}\partial_{\mu}$, $a_{a\mu}$ is an e_a-lines curvature covector (for quasi-inertial tetrad $K_a = 0$ we have $\epsilon^a a_{a\mu} = 0$), $K_{a\mu\nu}$ is the external curvature 3-tensor of the hyperplane field $Ker \;\; e_a(e_{a\mu}dx^{\mu} = 0)$, $A_{a\mu\nu}$ is an e_a-lines rotation 3-tensor.

The orthogonal expansion for tetrad potentials are as follows:

$$e_a = d\alpha_a + \delta\beta_a + \gamma_a = (\partial_{\mu}\alpha_a - \nabla^{\nu}\beta_{a\nu\mu} + \gamma_{a\mu})dx^{\nu},$$

$$d\gamma_a = \delta\gamma_a = 0$$

$$de_a = d\delta\beta_a, \;\; \delta e_a = \delta d\alpha_a = \Box\alpha_a = K_a$$

Due to Einstein tetrad equation with a matter tensor T_a we have $R_a = 8\pi(T_a - Te_a/2)$. A Maxwell equation (with the electromagnetic potential $A = A_{\mu}dx^{\mu}$ and with 4-current J) and the Einstein equation (with tetrad current S_a, the Hamiltonian density S_{oo}) are following

$$\Delta A = 4\pi J, \;\; \Delta e_a = 8\pi S_a$$

$$S_a = T_a - Te_a/2 + \Box e_a/8\pi$$

$$S_{oo} = T_{oo} + T//2 - a_{o\mu}a^{\mu}_{o} + K_{o\mu\nu}K^{\mu\nu}_{o} + A_{o\mu\nu}A^{\mu\nu}_{o}$$

For the Lorentzian gauge $\nabla^\mu A_\mu = \nabla^\mu e_{a\mu} = 0$ we have $\nabla^\mu J_\mu = \nabla^\mu S_{a\mu} = 0$. In general case (with cosmological Λ-parameter)

$$\delta de_a = 8\pi S_a - dK_a + \Lambda e_a$$

The total 4-momentum for a gravitating physical system on any spacelike hypersurface Σ with 2-boundary $\partial\Sigma$ on V:

$$P_a := -\int_{\partial\Sigma} *de_a = \int_\Sigma *(S_a - dK_a/8\pi + \Lambda e_a/8\pi) = const$$

For the trivial 4-momentum we have $de^a = 0,\ e^a = dx^a$ (the Minkowski vacuum). If $e_o = -dt$ then $P_o = 0$, but $P_i \neq 0$ in general case. In a quasi-Newtonian gravitational field with a negative potential energy we obtain $S_{oo} = -a^2/8\pi$, where a is a free fall acceleration. For weak flat gravitation waves in Minkowski space $h_{22}(t-x) = -h_{33}, h_{23}(t-x)$ we obtain

$$16\pi S_{oo} = (\partial_o h_{22})^2 + (\partial_o h_{22})^2$$

The energy-momentum pseudotensor is not required here.

The tangential deformation of elastic Minkowski vacuum giving the Rindler vacuum

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \to ds^2 = e^{2az}(-dt^2 + dz^2) + dx^2 + dy^2$$

changes his energy-momentum (now $K_3 = -ae^{-az} \neq 0$).

That us take the gravitational Lagrangian L_g in the following form:

$$16\pi L_g = -\nabla_\mu e_{a\nu}\nabla^\nu e^{a\mu} = R - 2\nabla^\mu K_\mu - K^a K_a, \;\; K_\mu = e^a_\mu K_a$$

The gravitational field equations are as following:

$$G_{\mu\nu} + \Lambda(x) g_{\mu\nu} = 8\pi T_{\mu\nu}, \;\; \Lambda(x) := -\frac{1}{2}K^a K_a = -\frac{1}{2}K^2$$

For the fixing the Λ - parameter we suppose in general case

$$T_{a\mu} = -p_a e_{a\mu}$$

This is the Ricci-canonic tetrad gauge condition. If $T_{\mu\nu} = 0$ then $\Lambda = const.$ Here the matter tensor changes the vacuum energy-momentum tensor

$$T^{vac}_{\mu\nu} = -\Lambda g_{\mu\nu}/8\pi = -K^2 g_{\mu\nu}/16\pi$$

For the quasi-Friedmann model we have

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \Sigma^2(d\theta^2 + \sin^2\theta d\varphi^2))$$

$$\Sigma = (sh\chi, \chi, \sin\chi), \;\; k = (-1, 0, 1)$$

$$T_{\mu\nu} = (\rho + p)e_{o\mu}e_{o\nu} + p g_{\mu\nu}, \;\; p = \alpha\rho$$

$$a\ddot{a} - \frac{1}{4}(7 + 3\alpha)\dot{a}^2 + \frac{1}{2}(1 + 3\alpha)k = 0$$

$$\ddot{a} = 2k\frac{1+3\alpha}{7+3\alpha}(\frac{a}{a_o})^{(5+3\alpha)/2}$$

$$a = a_o \to \dot{a} = 0, \quad a = 0 \to \ddot{a} = 0$$

$$\dot{a}^2 = 2k\frac{1+3\alpha}{7+3\alpha}[1 - (\frac{a}{a_o})^{(7+3\alpha)/2}], \quad a = 0 \to \dot{a}^2 = 2k\frac{1+3\alpha}{7+3\alpha}$$

$$\Lambda(t) = \frac{9}{2}(\frac{\dot{a}}{a})^2 = 9\frac{1+3\alpha}{7+3\alpha}[k - k(\frac{a}{a_o})^{(7+3\alpha)/2}]a^{-2}$$

$$(2\frac{1+3\alpha}{7+3\alpha})^{1/2}t = \int [k - k(\frac{a}{a_o})^{(7+3\alpha)/2}]^{-1/2} da$$

$$\rho a^{3(1+\alpha)} = M_o + \frac{3k/8\pi}{7+3\alpha}[6 + (1+3\alpha)(\frac{a}{a_o})^{(7+3\alpha)/2}]a^{1+3\alpha}, \quad M_o = const$$

At the modern age $a \approx 10^{28} cm, \ \Lambda(t) \approx 10^{-56} \ cm^{-2}$.

The conservation of total (matter and vacuum) energy-momentum means the creation of the matter (if $k = 1$). The change of the matter density at the early age may be observed.

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On Weyl equations.

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Abstract

The conservation of the $(\gamma_4 \times \hat{k})$-chiral charge is shown to result in the states $\Psi 1$ and $\Psi 2$, being eigenfunctions of the $(\gamma_4 \times \hat{k})$-chirality operator: $(\gamma_4 \times \hat{k})$. For these states significance the $(\gamma_4 \times \hat{k})$-chirality has certain values. These states break the invariance with respect to rotations in three-dimensional configuration space, however they permit one to introduce mass members without breaking the conservation of $\gamma_4 \times \hat{k}$-chiral symmetry and the corresponding charge (as distinct from γ_5-chiral symmetry, which is broken by introducting a mass).

Introduction.

The Pauli matrices algebra is known to be isomorphic to the quaternion algebra [1]. Thus all results obtained by useing the Pauli matrices can be written in terms of the quaternion calculation. At the same time it is known [2] that the quaternions can be derived from the complex numbers by a so - called doubling procedure , as well as the complex numbers are obtained from the real ones using the same procedure.

The next stage, of using the doubling procedure is a construction of Cayley's algebra , which requires giving up not only commutativity, but also associativity.

1 Operations in Cayley's algebra.

Represent an element of the quaternion algebra in the form: $q = a + b \times \hat{\imath} + c \times \hat{\jmath} + d \times \hat{r}$, where a, b, c, d belong to a field, in particular,the field of real numbers. The quaternions are added (substructed) and multiplied term by term, as polynomials, however it be always should borne in mind that: $\hat{r} = \hat{\imath} \times \hat{\jmath}$; $\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath}$; $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{r} \times \hat{r} = -1$.

An isomorphism of the quaternions and matrices σ_i follows directly from the relation: $1 \to I, \hat{\imath} \to i \times \sigma_2, \hat{\jmath} \to i \times \sigma_1, \hat{r} \to i \times \sigma_3$ (where i is the usual commutative complex unity). The element of Cayley's algebra is an octanion (Q): $Q = q_1 + q_2 \times \hat{k}$, where $q_{1,2}$ - are quaternions; as well as in the case of quaternions $\hat{k} \times \hat{k} = -1, \hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\jmath} \times \hat{k}$, $\hat{k} \times \hat{r} = -\hat{r} \times \hat{k}$.

The octanions are added (substracted) termwise, as well as the quaternions do. Multiplication in Cayley's algebra is usually set by a special table (as is sometimes done for quaternions). However,the results of this table can be obtained by having used a number of simple rules, which considerably simplifies a consideration from the technical point of view. The objects $(\hat{\imath} \times \hat{k}), (\hat{\jmath} \times \hat{k}), (\hat{r} \times \hat{k})$ should be considered anticommutative with $\hat{\imath}, \hat{\jmath}, \hat{r}, \hat{k}$ and between themselves (similarly, in the quaternions algebra , $(\hat{\imath} \times \hat{\jmath})$ anticommutates with $\hat{\imath}$ and $\hat{\jmath}$, but here such a product is unique it is designated by $\hat{r}$) not to be confused. Thus in Cayley's algebra we shall have, e.g. $(\hat{\imath} \times \hat{k}) \times \hat{\imath} = -\hat{\imath} \times (\hat{\imath} \times \hat{k})$; $(\hat{\imath} \times \hat{k}) \times \hat{\jmath} = -\hat{\jmath} \times (\hat{\imath} \times \hat{k})$. Besides, as well as in the quaternion algebra, $(\hat{\imath} \times \hat{k}) = -(\hat{k} \times \hat{\imath})$. It should noted, that the last operation (anticommutation) is feasible, also for the objects being simultaneously: one - inside, other - outside of the brackets, therewith a preservation of brackets is necessary, e.g. $(\hat{\imath} \times \hat{k}) \times \hat{\jmath} = -(\hat{\imath} \times \hat{\jmath}) \times \hat{k}$. Being based on these rules, it is possible to obtain any result from the table of multiplication [3] of Cayley's algebra.

Cayley's algebra is not associative, the quantity being called an associator $[\hat{\imath} \times \hat{\jmath} \times \hat{k}] = \hat{\imath} \times (\hat{\jmath} \times \hat{k}) - (\hat{\imath} \times \hat{\jmath}) \times \hat{k}$ in this algebra, is not equal to zero as easily calculable on the basis of the above-stated rules, and $2 \times \hat{\imath} \times (\hat{\jmath} \times \hat{k})$, at the same time the antiassociator $\{\hat{\imath} \times \hat{\jmath} \times \hat{k}\} = \hat{\imath} \times (\hat{\jmath} \times \hat{k}) + (\hat{\imath} \times \hat{\jmath}) \times \hat{k}$ appears to be equal to zero.

2 Cayley's algebra in the formalism of physical theories.

As far as the the Pauli matrices formalism is a conventional one to construct physical models, and the quaternion formalism is out of practical use , we shall operate not with the quaternion algebra but with the σ-matrices algebra . An extension of the quaternion (σ-matrices) algebra up to Cayley's algebra is realized due tu the object $\hat{k}$: $\hat{k} \times \hat{k} = -1$ unrepresentable in the form of a matrix. Applying the above rules of products, but already not for $\hat{\imath}, \hat{\jmath}, \hat{r}$, and according to isomorphism, we shall have for $i\times\sigma_2, i\times\sigma_1, i\times\sigma_3$, e.g. $(i\times\sigma_1\times\hat{k})\times(i\times\sigma_1) = -(i\times\sigma_1)\times(i\times\sigma_1\times\hat{k})$. Note that the written relation shows, among other things, the opportunity to omit commutative imaginary unity (i), i.e. the opportunity to operate directly with the matrices $I, \sigma_2, \sigma_3, \sigma_1$ and the object $\hat{k}$ following the same rules of multiplication.

3 Gamma — matrices.

As Dirac's gamma-matrices are representable in the form of a direct product of the Pauli matrices , e.g. $\gamma_4 = \sigma_3 \otimes I; \gamma_\alpha = \sigma_2 \otimes \sigma_\alpha$ $(\alpha = 1, 2, 3)$; $\gamma_5 = \sigma_1 \otimes I$, thus having extended the algebra $\{I, \sigma_\alpha\}$ of two-row matrices being in the right-hand side of the direct product up to the algebra $\{I, \sigma_\alpha, (\sigma_\alpha \times \hat{k}), \hat{k}\}$, we obtain, in addition, four γ"matries": $(\gamma_\alpha \times \hat{k}) = \sigma_2 \otimes (\sigma_\alpha \times \hat{k})$ and $(i \times \sigma_2 \otimes I \times \hat{k})$, where is the latter can be written in the form $\sigma_3 \times \sigma_2 \otimes I \times \hat{k} = \gamma_4 \times \gamma_5 \times \hat{k}$. From the aforesaid it is clear that $\gamma_5; \gamma_4; \gamma_\alpha; (\gamma_\alpha \times \hat{k}); (\gamma_4 \times \gamma_5 \times \hat{k})$ will form nine anticommutating matrices, each with the square equal to unity. It is clear as well that γ_4 and γ_5 commutate with $\hat{k}$, and γ_α anticommutate with it.

The brackets here should be treated as well as in Cayley's algebra (it follows from the construction of γ-matrices in the form of a direct product, where Cayley's algebra is realized in its right-hand side).

4 Operator part of Dirac's equation.

It is of interest to consider possible modifications of the operator part of Dirac's equation based on the aforesaid. However simple modifications of the type $\gamma_\mu \times D_\mu \to \Gamma_\mu \times D_\mu$, where $\Gamma_4 = \gamma_4 \times \exp(\gamma_5 \times \hat{k} \times \theta)$, $\Gamma_\alpha = \gamma_\alpha \times \exp(\hat{k} \times \theta_\alpha)$, without summing over α, appear to be unitarily reducible to the original form: $\gamma_\mu \times D_\mu$,where $D_\mu = (\partial_\mu - i \times g \times A^\beta_\mu \times t^\beta)$. Where t^β — are gauge group generators.

5 Chiral symmetry.

Since the operator part of Dirac's equation does not admit simple modifications, it is of interest to consider invariant transformations of wave functions arising as a consequence of the extension of the algebra of γ-matrices due to the element $\hat{k}$.

It is possible to show that mass Lagrangian of the spinor field is invariant with respect to the wave function transformations of :

$\Psi \to \hat{Q} \times \Psi$, where $\hat{Q} = \rho_4 \times (i \times \gamma_4 \times \hat{k}) + \rho_\alpha \times (i \times \gamma_5 \times \gamma_\alpha \times \hat{k})$, and $\rho_\mu \times \rho_\mu = 1, (\mu = 1,2,3,4)$. The operator $\hat{Q}$ is invariant with respect to the parity transformations, charge conjugation and relativistic of rotations (boosts), but is not invariant with respect to rotations in three-dimensional configuration space. At the same time, due to rotations of three-dimensional space the operator $\hat{Q}$ can be reduced to the form $\hat{Q} = (i \times \gamma_4 \times \hat{k})$.

From the aforesaid it is clear that the Lagrangian can be represented in the form of a sum of two parts with the wave functions $\Psi 1 = (1 - i \times \hat{k} \times \gamma_4)\Psi 1$ and $\Psi 2 = (1 + i \times \hat{k} \times \gamma_4)\Psi 2$ respectively (similarly to decomposition of the Lagrangian in a right — and left–hand side in the massless case). Similarly it is possible to speak of conservation of the chiral charge corresponding to the transformation $\Psi \to exp(\hat{k} \times \gamma_4 \times \theta) \times \Psi$, where θ is an independent variable, but it already not a γ_5-chiral charge, and $(\gamma_4 \times \hat{k})$ is a chiral charge, without breaking the conservation of the latter by an introduction of the mass member.

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HYDROGEN-LIKE ENERGY SPECTRUM OF THE EARLY UNIVERSE

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Abstract

The quantum birth of the Universe at Planckian densities is considered allowing for some kinds of matter other than vacuum. The pre-de-Sitter universe looks like a hydrogen atom with the energy equal to that of a universe filled with relativistic gas.

From the viewpoint of the modern cosmologial concepts the de Sitter vacuum, with the equation of state $p = -\varepsilon$, is believed to be an initial stage of evolution of the Universe [1]. It decays into an expanding matter called the Friedmann world. There arises a question: what was before the de Sitter stage? The birth of the Universe is nowadays treated as a quantum tunnelling from "nothing" to the de Sitter vacuum.The idea of a quantum birth of the Universe was first proposed by Tryon [2] and Fomin [3] and developed by many authors.

In the present paper the equations of state other than $p = -\varepsilon$ are taken into account to obtain the wave function and energy spectrum of the quantum Universe as well as the penetrating factor giving the probability of its birth. The problem is formuated as follows. From the Einstein equations for a homogeneous isotropic universe [4,5]

$$\frac{\dot{a}^2}{2} + \frac{4\pi G\varepsilon a^2}{3c^2} = -\frac{kc^2}{2}, \tag{1}$$

$$\ddot{a} = -\frac{4\pi G(\varepsilon + 3p)a}{3c^2}, \tag{2}$$

where a is a scale factor; the parameter $k = 0, +1, -1$ for flat, closed and open models respectively, for the equation of state

$$p = \alpha\varepsilon \tag{3}$$

we obtain the relation

$$\dot{\varepsilon}a + 3(1+\alpha)\dot{a}\varepsilon = 0 \tag{4}$$

at any k, which results in the formula

$$\varepsilon = \varepsilon_{pl}\left(\frac{a}{l_{lp}}\right)^{-3(I+\alpha)}, \tag{5}$$

where $\varepsilon = \varepsilon_{pl}$ at $\alpha = -1$ corresponds to the de Sitter vacuum.
The de Sitter vacuum has Planckian parameters. The quantization procedure reduces to introducing a conformal time in (1) followed by replacing the generalized momentum

$$P = \frac{\partial L}{\partial\left(\frac{da}{d\eta}\right)} = \frac{da}{d\eta}. \tag{6}$$

by the corresponding quantum operator

$$\hat{P} = \frac{1}{i}{l_{pl}}^2\frac{d}{da}. \tag{7}$$

As a result we obtain the Wheeler-DeWitt equation in minisuperspace depending only on a scale factor

$$\frac{d^2\psi}{da^2} - V(a)\psi = 0, \tag{8}$$

DeWitt solved it for the case of a close universe filled with dust using the zero boundary condition for the wave function at the origin [6]. The energy spectrum of the Universe proved to be oscillatory. Later Vilenkin considered a closed zero-energy universe being born of a pure vacuum [7].

In the present paper the energy density is considered to be a superposition of various kinds of matter satisfying the weak energy dominance condition for closed models, namely: vacuum ($\alpha = -1$), domain walls ($\alpha = -\frac{2}{3}$), strings ($\alpha = -\frac{1}{3}$), dust ($\alpha = 0$), relativistic gas ($\alpha = \frac{1}{3}$), bosons and fermions ($\alpha = \frac{2}{3}$), ultrastiff matter ($\alpha = 1$).

Separating a term independent of the scale factor in the potential, we reduce the Wheeler-DeWitt equation to the Schrödinger one with nonzero energy of the Universe in an effective flat space behaving as a relativistic gas moving in the field of other types of matter. The total energy of the system is zero due to a zero Hamiltonian in the Wheeler-DeWitt equation. This may be compared with Rubakov's results on the birth of relativistic particles, while tunnelling the Universe [8]. The Schrödinger equation in the present paper was solved for three cases. First, for nonzero contributions of curvature, strings and vacuum in the potential. Near the minimum the solution is oscillatory as well as DeWitt's. The energy spectrum is described by the formula [9]

$$E_N = \hbar\omega\left(N + \frac{1}{2}\right), \tag{9}$$

where B_2 is a contribution of strings to the total energy density and

$$\omega = \frac{c\sqrt{k - B_2}}{2l_{pl}}, \qquad N = 1, 3, 5, ... \tag{10}$$

The wave function satisfying the boundary condition $\psi(0) = 0$ is

$$\psi = \frac{(k - B_2)^{\frac{1}{8}}}{(2\pi)^{\frac{1}{4}} \sqrt{2^N N! \, l_{pl}}} \exp\left(-\frac{1}{2}\gamma^2 \sqrt{k - B_2}\right) H_N(\gamma), \tag{11}$$

where H_N is a Hermite polynomial, B_2 is a contribution of strings to the total energy density. The condition of level existence

$$E_N < U_{max} \tag{12}$$

takes the form

$$(k - B_2)^{\frac{3}{2}} > 4\left(N + \frac{1}{2}\right). \tag{13}$$

Second, in the pre-de-Sitter domain at small γ only terms with negative powers of γ may be retained in the potential. Then the Schrödinger equation reads

$$\frac{d^2\psi}{d\gamma^2} + \left(\frac{B_5}{\gamma} + \frac{B_6}{\gamma^2} + \frac{2E}{m_{pl}c^2}\right)\psi = 0 \tag{14}$$

Its solution is the wave function [9]

$$\psi = C\rho^{s+1} e^{-\frac{1}{2}\rho} F(-p, 2s + 2, \rho), \tag{15}$$

where F is a degenerate hypergeometric function satisfying the boundary condition $\psi(0) = 0, B_5$ and B_6 are contributions of bosons and fermions and ultrastiff matter respectively,
$\rho = 2\gamma\sqrt{\frac{-2E}{m_{pl}c^2}}, \quad n = \frac{1}{2}B_5\sqrt{\frac{m_{pl}c^2}{-2E}}, \quad n - s - 1 = p = 0, 1, 2, ...; \quad B_5 > 0,$
$s = -\frac{1}{2} + \sqrt{\frac{1}{4} - B_6}$. For $B_6 \leq \frac{1}{4}$ there exist discrete levels (there occurs repulsion for $B_6 < 0$ and attraction for $0 \leq B_6 \leq \frac{1}{4}$ at small γ). The energy spectrum of the Universe is of the form

$$E_p = -\frac{{B_5}^2 m_{pl}c^2}{8\left(p + \frac{1}{2} + \sqrt{\frac{1}{4} - B_6}\right)^2} \tag{16}$$

For $|B_6| \gg p$ $(B_6 < 0)$ we have

$$E_p \approx {B_5}^2 m_{pl}c^2 \left(\frac{1}{8B_6} - \frac{p + \frac{1}{2}}{4B_6\sqrt{-B_6}}\right) \tag{17}$$

Near the potential minimum at $\gamma = -\frac{2B_6}{B_5}$ the spectrum is of the oscillator form. For $\frac{{B_5}^2}{8|B_6|} \ll 1$ the potential well is not too deep to create planckeons. For $|B_6| \ll \frac{1}{4}$

$(B_6 > 0, \quad B_6 < 0)$, $|B_6| \ll p$ $(B_6 < 0)$ the spectrum is hydrogen-like. Third, for very small the solution reduces to

$$\gamma^2 \frac{d^2\psi}{d\gamma^2} + B_6\psi = 0 \tag{18}$$

Its solution is given by the formula [10]

$$\psi = \sqrt{\gamma} \begin{cases} C_1 \cos(b \ln \gamma) + C_2 \sin(b \ln \gamma), & b^2 = B_6 - \frac{1}{4} > 0; \\ C_1 \gamma^b + C_2 \gamma^{-b}, & b^2 = \frac{1}{4} - B_6 > 0; \\ C_1 + C_2 \ln \gamma, & B_6 = \frac{1}{4} \end{cases} \tag{19}$$

satisfying the boundary condition $\psi(0) = 0$.
For $B_6 > \frac{1}{4}$ there occurs a fall to the field centre, which corresponds to $E_0 = -\infty$. Thus we see that the cosmological singularity does not prevent stable existence of the quantum Universe for $B_6 \leq \frac{1}{4}$ as well as the Coulomb singularity allows existence of stable atoms. The analogy between these cases was first proposed by Wheeler in connection with the problem of relativistic collapse [11]. It is of interest to note that the Coulomb law for the Universe potential resembles the asymptotic freedom in quark models of hadrons [4].

The WKB penetration factor reads

$$D = e^{-\frac{2}{3}(k-B_2)^{\frac{3}{2}}}, \qquad (k - B_2)^{\frac{3}{2}} \gg 1. \tag{20}$$

It should be mentioned that there exists an analogy between the tunnelling of the Universe with strings and zero energy and the tunnelling of a particle through a wedge potential [9], since the penetration factor of the Universe has the form similar to those obtained for a wedge potential. The wedge maximum plays the role of the model parameter of the Universe. The energy of a particle corresponds to the contribution of the total energy density. The wedge slope corresponds to the vacuum energy density.

The penetration factor has been first calculated by G.A. Gamow for the case of radioactive nuclei alpha decay [12]. Gamow's procedure was extended in [7] to the case of the Universe birth from pure vacuum. In the absence of strings $(B_2 = 0)$ only closed universes can be born from vacuum (for open ones a_2 as well as V is imaginary), which is well-known . At the same time particles are known to be born in an open universe with a spontaneously broken symmetry when their energy density is negative [13]. In the presence of strings the birth of the Universe becomes possible in open (flat) models if $B_2 < -1$ $(B_2 < 0)$, since $k - B_2 > 0$ and hence V is real. The partial string energy density $\varepsilon_2 = B_2 \varepsilon_{pl} \gamma^{-2}$ is negative in this case. Thus both processes (particle creation, giving rise to the origin of matter in the Universe, and birth of the Universe itself) go along similar lines for open models. At the same time there exist examples of compact flat and hyperbolic spaces given by Zel'dovich, Starobinsky [14] and Fagundes [15] which are allowed to be born of vacuum.

I am indebted to A.I. Studenikin for providing me the possibility of participating in the work of the 7th Lomonosov Conference "Problems of Fundamental Physics".

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ON PARITY CONSERVATION IN WEAK INTERACTIONS AND REASON FOR ORIGINATING SPONTANEOUS β-DECAY
(the hypothesis and project of the test-experiment)

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Abstract

We suggest a new model of weak interactions which does not violate the law of parity conservation. According to the model suggested interaction of non-stable isotope or particle with a relic neutrino-antineutrino pair originates spontaneous weak decay and its spontaneous parity violation. In this framework we suggest a new interpretation of experimental results like those of Wu-type. To verify the hypothesis we suggest a test experiment based on the predicted effect of relic neutrino flow density - β-decay rate dependence.