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Experiments driving theory: Gravitational wave detection and the two body problem in general relativity

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Abstract. Over the last two decades, the search for gravitational waves from coalescing compact binaries by detectors like LIGO and Virgo has crucially required and consequently spurred tremendous progress in the two body problem in general relativity. A broad brush overview of these major developments and the current status of these significant results is presented.

1. Introduction

The problem of motion in general relativity (GR) goes back to 1916 to the post Newtonian (PN) works of Einstein, Droste, and De Sitter. Due to difficulties related to the correct treatment of internal structures of bodies only with the 1938 work of Einstein, Infeld and Hoffmann (EIH) did the GR N-body problem reach its first stage of maturity as discussed in the books of Fock, Infeld & Plebanski, Landau & Lifshitz [1]. In the 1916 paper exploring physical implications of GR, Einstein proposed the existence of gravitational waves (GW) as one of its *important* consequences [2]. Soon after, Einstein (1918) calculated the flux of energy far from source; the famous quadrupole formula, and discussed the related radiation reaction or radiation damping [3]. In 1922 Eddington pointed the inapplicability of this derivation for self-gravitating systems [4]. The complication for the self-gravitating case is fundamental: For self-gravitating systems orders in velocity are related to orders in non-linearity. By the virial theorem, $\Phi = GM/R$ is of the same order as v^2. So reaction terms of order $(v/c)^5$ in a linear theory will be accompanied by terms $(v/c)^3\,\Phi/c^2$, $(v/c)^1\,(\Phi/c^2)^2$ for self-gravitating systems. Thus, higher order v/c or PN calculations (v^2/c^2 is designated as 1PN) requires dealing with higher order non-linearities of Einstein's equations (EE). Landau Lifshiftz (1941) and Fock (1955) extended the quadrupole formula to weakly self-gravitating systems and these constitute two different approaches to GW generation today. Chandrasekhar [5] was first to show conceptually that energy and angular momentum radiated as GW was correctly balanced by the loss of mechanical energy and angular momentum. His work gave astrophysicists confidence that GR was physically reasonable and well behaved. But there was a problem. In the gauge he used, some terms at 2PN were divergent. These divergences cast doubt on the validity of Chandrasekhar's treatment for more mathematically demanding relativists.

2. Discovery of the binary pulsar 1913+16 and its implications

In 1974, R.Hulse and J.Taylor discovered the first binary pulsar 1913+16 [6]. Radio pulsar timing observations allow one to reconstruct the orbit and the related inspiral of this system due to emission of GW. This provides high quality data that is proof that GW exist leading to a Nobel Prize in 1993 [7]. The prospects of testing PN theory against Hulse-Taylor system once again revived more critical questions regarding existing treatment of GW. Does it apply to orbital motion of the two neutron stars (NS) even though it does not apply to their internal structure with strong gravitational fields? Even for weak fields is it a valid approximation of GR due to the divergent terms in the PN equations? One needs methods to treat weak orbital fields without assumption on internal fields and Damour showed that orbits and interactions of stars are independent of compactness modulo tidal distortions (*effacement* of internal structure in GR). The high quality binary pulsar data forced a revisit to approximation methods in GR to remedy the mathematical shortcomings in the existing approaches [8]. Discovery of binary systems with strong self gravity mandated improved approaches to the N-body problem: modern versions of EIH. 1913+16 obliged theorists to go beyond 1PN relativistic effects in the equations of motion (EOM); to 2.5PN ie inclusion of terms of $\mathcal{O}(v^5/c^5)$ beyond Newtonian EOM. Insights of a newer generation more comfortable with techniques in field theory to deal with divergences helped. Damour critically looked at the problem and proposed Riesz regularisation to deal with ultra-violet (UV) divergences and iterated EE to sufficient order of non-linearity to obtain EOM of compact binaries including v^5/c^5 terms (1983). Thorne's review [9] summarized state of the art in techniques involved and results then available. Binary pulsars like 1913+16 provide *direct* observational proof that gravity propagates at speed of light and has a quadrupolar structure. They provide accurate tests in the strong field regime of relativistic gravity and prove that GR is valid beyond the quasi-stationary, weak-field regime. It probed for the first time regimes involving radiative effects and strong fields. GR passed all binary pulsar tests with flying colors. The limits on existence of dipole radiation from binary neutron star systems are not strong. However, stringent limits on dipole radiation (test of quadrupolar structure of gravitation radiation damping) exist currently from di-symmetric pulsar- white dwarf systems, the best being from PSR J1738+0333. Binary pulsar population crucially underlie the estimated number of GW events for GW detectors like LIGO and Virgo and the existence of binary neutron star sources emitting GW for hundreds of million years before coalescing spectacularly in the sensitivity bandwidth of these detectors.

3. Prototype Sources: Coalescing Compact Binaries

The late inspiral and merger epochs of compact binaries of neutron stars or black holes provide us possible strong sources of GW for terrestrial Laser Interferometer GW Detectors like LIGO and Virgo in the 'high' frequency range 10 Hz - 10 kHz. We have guaranteed sources for the GW detectors if there are *enough* of them. The waveform is a chirp (amplitude and frequency increasing with time). GW are weak signals buried in the stronger noise of the detector. One requires matched filtering (MF) both for their detection or extraction and parameter estimation or characterisation. Success of MF requires an accurate model of the signal using GR and favours sources like coalescing compact binaries (CCB) made of neutron stars and black holes over sources like supernovae. This led to spectacular theoretical progress in 2-body problem in GR complementing spectacular progress in the GW detection endeavours [10, 11]. When LIGO was funded in early nineties, and efforts to construct accurate CCB waveforms for NS-NS, BH-BH started, it was soon realised that since CCB were highly relativistic in the final stages of inspiral and merger, far higher order PN accurate waveforms would be needed to accurately describe GW at these late epochs than the 2.5PN adequate for binary pulsar work. Physical insights were essential to simplify the goals and achieve the required accuracy for waveforms to construct template banks.. They include: (i) garden variety inspiralling compact binaries (ICB) would have radiated away their eccentricity and be moving in quasi-circular orbits during the late inspiral, (ii) since matched filtering is sensitive to the phase it is more important to first control higher order phasing than higher order amplitudes i.e. use Newtonian amplitude & best available phasing the so called restricted waveform, (iii) the inspiral can be treated in the adiabatic approximation as sequence of circular orbits which allows one to treat separately the *radiation reaction* effects and the *conservative* effects, (iv) one can go to higher PN orders in the inspiral without getting technically bogged down in controlling the much more difficult higher order conservative PN terms, (v) for compact objects the effects of finite size and quadrupole distortion induced by tidal interactions are of order 5PN. Hence, neutron stars and black holes can be modelled as point particles represented by Dirac δ-functions. Thus modelling ICB waveforms in the adiabatic approximation involves three problems: (a) Motion - given a binary system, iterate EE to discuss conservative motion of the system. Compute conserved energy E (and angular momentum (AM) J for the eccentric case); (b) Generation - given the motion of the binary system on a fixed orbit, iterate EE to compute multipoles of the gravitational field and hence the far zone (FZ) flux of energy $\mathcal{L}$ (and FZ flux of AM $\mathcal{J}$) carried by GW; (c) Radiation Reaction (RR) - given the conserved energy (and AM) radiated flux of energy (and AM), *assume* the balance equations to compute the effect of radiation on the orbit. Compute evolution of frequency $F(t)$ and phase $\phi(t)$ (and radial separation $r(t)$). Non-linear evolution of the orbital phase due to higher order gravitational radiation reaction is the crucial ingredient in constructing templates. Higher order phasing is equivalent to inclusion of higher order gravitational radiation reaction (GRR). 3PN (v^6/c^6) flux beyond leading quadrupole determines 3PN (v^6/c^6) RR relative to leading RR at 2.5PN (v^5/c^5). This became a primary driver for research in the two body problem in GR related to higher PN order EOM (effect of conservative acceleration terms), higher PN order of energy and AM fluxes (effect of radiation reaction acceleration terms), very high order fluxes in the test particle limit, very high order computation of the gravitational self force, numerical relativity for the whole inspiral, merger, ringdown (IMR) phases and finally effective one body for its associated analytical description [10, 12].

4. Approximation Schemes, regimes of validity

There are three main methods to deal with two body problem in GR: Post Newtonian (PN) approach, Perturbation theory (including Gravitational Self force (GSF)) and Numerical Relativity (NR). PN approach expands dynamics and waveforms in powers of v/c. It is valid for any mass ratio, in the near-zone (NZ) and in principle for slow motion, which for self-gravitating bodies implies large separations. Implementation of PN approximation is facilitated if its viewed as downstream of the Post - Minkowskian (PM) approximation, Non-linearity expansion or expansion in G. Unlike the PN approximation, the PM approximation is valid all over the weak-field region including infinity. The implementation of PM approximation is facilitated by use of the Multipole (M) expansion or expansion in irreducible representation of the rotation group using symmetric trace free (STF) tensors or equivalently tensor spherical harmonics. Perturbation theory is suitable to describe motion and radiation of a small body moving around a large body. Here one expands EE around a black hole metric rather than Minkowski metric in the ratio of small mass to big mass. This approach can analytically deal with *extreme mass ratio* inspiral (EMRI), & *quasi-normal mode (QNM) ringing.* Going beyond, the back reaction of the gravitational self-field of the small body modifes its geodetic motion leading to corrections to perturbation results and computed by the GSF formalism. Finally, NR solves EE on computer for coalescence, merger and ringdown beyond inspiral. It can be used for any mass ratio, separation or velocity. The range of validity is constrained by computational resources and accuracy requirements on the numerical solutions. Successful wave-generation formalisms are a subtle cocktail of the above different approaches. The most successful Multipolar Post Minkowskian- Post Newtonian (MPM-PN) is employed to deal with *arbitrary mass ratio* inspiral in the slow-motion weak field regime. In this approach, there are two independent aspects addressing two different problems: (i) the general method (MPM expansion) applicable to extended or fluid sources with compact support, based on the mixed PM and multipole expansion (asymptotically) matched to some PN (slowly moving, weakly gravitating, small-retardation) source. Infra-red (IR) divergences arising from the retardation expansion are dealt with by analytic continuation, (ii) the particular application to describe inspiralling compact binaries (ICB) by use of point particle models. Self-field regularisation to deal with ultra-violet (UV) divergences arising from use of Delta functions to model point particles include Riesz, Hadamard partie finie and dimensional regularisation [10].

5. Implication for Ground Based GW detectors

The availibility of the 2PN EOM from binary pulsar work supplemented by a new insight into treatment of cubic non-linearities facilitated the computation of 2PN phasing of ICB by two independent methods: MPM-PN (Blanchet, Damour, Iyer); Direct Integration of Relaxed Einstein equations (DIRE) (Will, Wiseman). The control of 3PN order was more formidable since ambiguities arise in regularisation methods for the self-field (Hadamard regularisation) that worked till 2PN. Only after almost a decade of struggle and by the use of the gauge invariant *dimensional regularisation* was the problem finally resolved and completed. Computation of 3.5PN GW flux requires and includes control of all the non-linear couplings between multipole moments up to order 3.5PN for general matter sources; those couplings involve in addition to many 'instantaneous' terms, the important contributions of tails, tails-of-tails (weakly dependent on past history) and the non-linear memory (strongly dependent on past history)[10, 12]. A new complication arises in the 4PN EOM that has recently been addressed in the ADM approach by Damour, Jaranowski, Schäfer (2014) [13]. At 4PN, there appear irreducible IR divergences (in any gauge) in the calculation of PN expanded 4PN Hamiltonian of binary system related to fundamental breakdown in formal NZ expn of flat spacetime gravitational propagator found by Blanchet and Damour (1988). At 4PN, impossible in any gauge to express NZ metric and 2-body EOM as functional of instantaneous state of the material source [1].

For NS-NS binaries which are inspiral dominated in the sensitive bandwidth of the present detectors, 3PN conservative acceleration and 3PN RR beyond the leading 2.5PN RR description of inspiral is adequate. For BH-BH binaries whose last stable orbit (LSO) lie in the sensitive detector bandwidth the inspiral phase is not adequate. The merger and ringdown need to be crucially modelled as accurately as possible. This requires NR or an extension of the PN description by effective one body or phenomenological approximants. Extensions of the non-spinning results are available for spinning BH binaries (since BH are spinning), quasi-eccentric binaries (there exist astrophysical mechanisms like the Kozai mechanism leading to binaries that could have eccentricity), tidal effects (NS-NS binaries would tidally distort in their final stages), BH horizon flux (though small could be important at higher PN orders). Other approaches include: Direct Integration of Relaxed EE (DIRE) [15] Strong field point particle limit [16] Effective Field Theory (EFT) [17], Gravitational Self Force (GSF) approaches for EMRI's [18] NR-AR and Self force-PN comparisons.

In the test particle limit, analytical results are available to 22PN for energy flux in the Schwarzschild case [19], 11PN in the Kerr case [20] using the Mano, Suzuki, Takasugi functional series formalism. In the Kerr case 20PN energy flux is also know numerically [21]. These are important for parameter estimation in eLISA. Analytical results for black hole horizon absorbed flux are known to 22.5PN in the Schwarzschild case [20, 21]. and 20PN for Kerr [21]. Experimental mathematics, (high accuracy numerics+ PSLQ integer relation algorithm)

[1] Added in Proof: More recently, the 4PN EOM has been investigated by L. Bernard et al [14] using a Fokker action.

can obtain analytic forms of high PN coefficients (involving Euler-Mascheroni gamma, logs of primes, Riemann zeta function at integers) of linear in mass ratio of binding energy [22].

6. Effective one Body approach

Resummation methods like Padé approximants can be used to extend numerical validity of PN expansions (at least) up to the LSO. Effective-one-body (EOB) approach of Buonanno and Damour introduced in 1998 is a particular non-perturbative resummation of PN-expanded EOM to extend validity of PN results beyond the LSO, and up to the merger [1]. Essential inputs are high order PN and perturbation theory results guided by the insight that non-perturbative effects can be captured analytically if key ingredients that enter the two body dynamics and GW emission are properly resummed about the exact test particle results. Each ingredient is crafted through resummation of the PN expansion to incorporate non-perturbative and strong field effects that are lost when dynamics and waveform (WF) are Taylor expanded in PN orders.

Four essential elements of the EOB approach are: (i) *Hamiltonian* $H_{\rm real}$ describing *conservative* part of relative dynamics of 2 BH (ii) *Radiation-reaction force* $\mathcal{F}_\varphi$ describing loss of (mechanical) angular momentum, and energy, of binary system; (iii) Definition of various multipolar components of *"inspiral-plus-plunge" (metric) waveform* $h_{\ell m}^{\rm insplunge}$; (iv) Attachment of subsequent *"Ringdown waveform"* $h_{\ell m}^{\rm ringdown}$ around certain (EOB-determined) "merger time" t_m. EOB provides analytical description of both motion and radiation of CCB from early inspiral, right thro plunge, merger and final ringdown. The complete waveform contrary to concerns is rather simple!! A smooth continuation of inspiral & sharp transition around merger to ringdown. A new improvement involved replacing the standard PN approach of additive WF by by a *multiplicative* WF; factorized resummed waveforms consisting of a physically motivated *product* of Newtonian waveform and various independent relativistic contributions providing parameter a free resummation for multipolar waveform and energy flux [23].

EOB made several bold quantitative and qualitative predictions concerning dynamics of coalescence and the corresponding radiation of GW, (i) a blurred transition from inspiral to a 'plunge' that is just a smooth continuation of the inspiral, (ii) a sharp transition, around the merger of the black holes, between a continued inspiral and a ring-down signal, and (iii) estimates of the radiated energy and of the spin of the final black hole. (iv) larger energy release for spins parallel to the orbital angular momentum, and to a rotation parameter J/E^2 always smaller than unity at the end of the inspiral (so that a Kerr black hole can form right after the inspiral phase). All these predictions have been broadly confirmed by the results of the later numerical simulations performed by several independent groups. The *waveform simplicity* is due to presence of a single characterstic scale close to merger when radiation reaction, orbital and spin precession time scales become of the same order of magnitude, formation of potential barrier around new born BH filtering direct radiation from the merger burst and highly dissipative nature of disturbances in the BH spacetime because of QNM's. EOB with spin and tidal effects are available. EOB allows to combine in a synergetic manner information from other approaches [1].

7. Pretorius and the Numerical Relativity Breakthrough

In 2005 Pretorius [24] produced the first simulation with large number of orbits through merger using modified harmonic coords, compactification of numerical domain at spatial infinity, singularity excision and damping of constraints. With this amazing breakthrough in NR, one has reliable waveforms for the late inspiral and merger parts of the binary evolution which can be used for constructing templates. In eight months other groups using other methods like Baumgarte Shapiro Shibata Nakamura (BSSN) equations and puncture methods followed [25]. These NR waveforms are calibrated and interpreted by the analytical PN inspiral results. There is exciting progress in matching the PN waveforms to the Numerical Relativity ones.

However, BBH simulations are time consuming and NR simulations currently too expensive to solely build required bank of GW templates and densely fill multidimensional space of BBH physical parameters (masses and spins). The state of the art in NR simulation has progressed tremendously over the decade. From short simulations of about 20 orbits; the new one based on Spectral Einstein Code (SpEC) is 25 times longer [26].. First NR simulation of compact binary whose gravitational waveform long enough to cover entire frequency band of Advanced LIGO (125 orbits; Mass ratio 7; Total mass $45.5M_{\odot}$). EOB models uncalibrated or calibrated against smaller NR WF reproduce NR waveform remarkably well while PN inspiral and calibrated IMR phenomenological display greater disagreement.

8. Concluding remarks

At its centenary, GR has not only has passed many stringent tests in flying colors but has consolidated to become an essential tool for describing the universe on manifold scales from satellites and planets to black holes and the big bang. The GW experiment is driving the GR theory. The experimental challenge to detect GW has led to a remarkable progress in the theoretical two body problem in GR. The classic two body problem in GR pushed by the requirements of an unforgiving technology driven fundamental physics experiment has rejuvenated itself to meet the challenges. If *bliss was it in that dawn to be alive* in the last twenty-five years,then, *to be young in the coming decade would be the very heaven!*

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