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The white dwarf binary background and LISA

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Abstract. The assumptions used in developing the canonical Galacitic white dwarf binary background level for LISA are investigated. The differences between several models of the white dwarf binary population are described and a technique for comparing the onset of the confusion limit between different population models is introduced.

1. Introduction

The standard galactic white dwarf binary background for LISA that is often used to estimate the parameters of binary black hole inspirals is usually presented as a smooth curve. Included in this curve are many assumptions about the nature of the population synthesis, the structure of the galaxy, and the ability of future (as yet undemonstrated) data analysis techniques. The typical Galactic binary background curve as generated by the LISA sensitivity curve generator [1] is shown in Figure 1. This curve is based upon the work of Hils, Bender and Webbink [2, 3, 4]. The low frequency regime is a confusion-limited signal produced by hundreds to thousands of binaries in each resolvable frequency bin from a one-year observation. The level of this signal depends upon the number of binaries and average chirp mass per bin as well as the total number and spatial distribution of binaries in the Galaxy. At higher frequencies (around 2 mHz in Figure 1) the curve begins to drop dramatically as the signal transitions from confusion-limited to individually resolvable sources. The value of the transition frequency depends on the astrophysics of the Galactic population as well as assumptions about the ability of future data analysis to resolve signals that may overlap in the frequency domain. The smooth curve does not convey the anticipated fluctuations and complexity of the actual spectrum due to a realization of the population of close white dwarf binaries, as seen in Figure 2. Here, we introduce a new way of estimating the transition frequency. We use this measure to evaluate the transition frequency for several different Galactic models in order to highlight the astrophysical contribution to the shape of the white dwarf binary background.

2. Overview of Other Models

Other work in the recent literature has sought to produce different representations of the Galactic binary contribution to the LISA data stream. To varying degrees, these models incorporate different population syntheses, different fidelities to the LISA data stream, and different assumptions about the capabilities of data analysis to resolve individual binaries. Consequently, the range of transition frequencies and confusion levels present in the literature is the result of different ways of determining the transition frequency as well as real astrophysics.

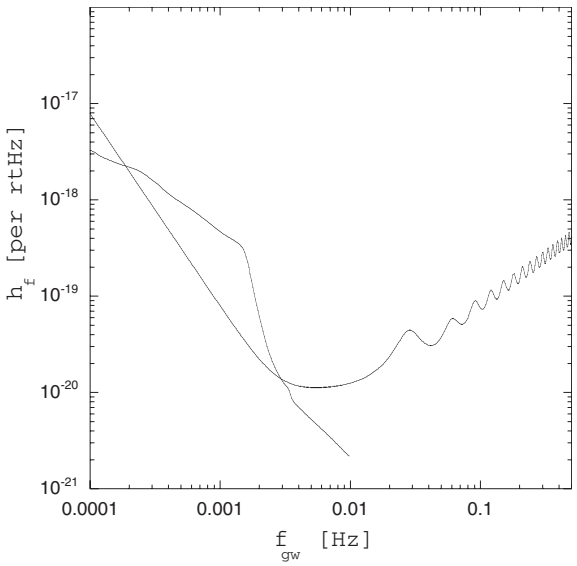

Figure 1. The standard Hils-Bender estimate of the confusion signal due to Galactic white dwarf binaries, shown with the standard LISA sensitivity curve.

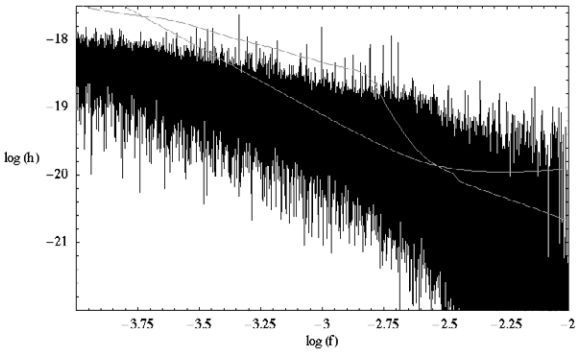

Figure 2. A spectrum for a binary realization of 4×10^6 binaries. Shown with the standard Hils-Bender curve and LISA sensitivity curve.

In this section we describe the variations between several recent models of the Galactic binary contribution.

2.1. Population Synthesis Effects

Although the standard curve includes numerous types of binaries, the signal is dominated by white dwarf binaries above 0.1 mHz. The standard white dwarf curve is based upon an initial mass function to determine the mass of the primary in the progenitor binary and an assumption of 0.14 binaries per decade in orbital period to determine the initial period. Using a prescription from Webbink [5], the final masses and orbital period are found. An assumed stellar birthrate is then used to give a total population of approximately 3×10^7 binaries in the Galaxy. Comparing this number with the known local space density of white dwarf binaries at the time led Hils, Bender and Webbink to reduce the population by a factor of 10 [2]. Using a Monte Carlo sampling from the Hils, Bender and Webbink populations of Galactic binaries [2], Timpano, Rubbo and Cornish [6] have generated a realization of the Galactic white dwarf binaries (among other populations) and have produced a simulated LISA data stream using the reduced total number of binaries (3×10^6). The standard curve is also generated from the reduced population.

A more recent and more sophisticated treatment by Nelemans *et al.* [7] uses binary evolution models as generated by `SeBa` (part of the StarLab suite of codes) [8]. White dwarf cooling models were applied to the resulting population in order to compare the population synthesis with observation. Various stellar birthrates, initial mass functions, and spatial distributions were then used to determine the model that most accurately reproduced current observations. In these models, the most likely total number of binaries is $\sim 27 \times 10^6$. However, when a white dwarf binary background curve generated from Nelemans' population of 27×10^6 binaries is compared with the standard curve generated from a population of 3×10^6 binaries, it is lower [9]. A detailed comparison of the realization of Timpano *et al.* [6] with Nelemans *et al.* [7] shows that differences in the binary evolution have produced an average chirp mass in the standard model that is nearly twice that of Nelemans' model (Nelemans, private communication). By

changing the prescription for evolution in the common envelope phase from energy loss to angular momentum loss, Nelemans has shown that the overall level of the confusion limit can be varied by an order of magnitude [10].

2.2. The Transition Regime

At higher frequencies, the gravitational wave signal from Galactic binaries transitions from a confusion of many sources per resolvable frequency bin to individually resolvable sources. In the canonical white dwarf binary curve, the expected signal strength in this transition zone is modified to include some assumptions about the ability to resolve foreground sources and the effect this has on the ability to determine the properties of other interesting extragalactic sources. The method described in Hils and Bender [3] assumes that the information in three frequency bins are required to completely parameterize (and therefore subtract) the signal from an individual binary. These three bins are then lost at the signal strength of the foreground binary. The effect of this assumption is to produce the dramatic drop in the white dwarf binary curve at around 3 mHz where the average number of binaries per frequency bin drops below 1.

Timpano, Rubbo and Cornish [6] have used a different method of estimating the ability to remove signals in the transition zone. Starting with the full signal from the entire population of binaries, they calculate a running median to determine the background level of the full signal. Next, they determine the bright binaries that stand above this level with a signal to noise of at least 5. These binaries are then competely and exactly removed from the data stream. The process is then repeated on the remaining signal until the number of new bright binaries is less than 1% of the previously found bright binaries. This process ended after 5 iterations and the remaining signal was Gaussian. This process is highly optimistic and therefore represents a lower bound on the cleaned signal. Despite this, the signal remains above the canonical value in the transition region. On the other hand, it is close to an order of magnitude below the canonical curve in the low-frequency confusion-limited region.

Nelemans [7, 10] simply cuts the signal off once the average number of binaries per bin drops below 1.

3. Determining the Transition Frequency

As noted in the previous section, a variety of models of the Galactic binary contribution to the LISA data stream exist in the literature. There are real and significant differences in the transition frequencies of these models, but these differences may be obscured by the fact that different researchers use different prescriptions for determining the value of the transition frequency. In this section, we describe a new way of measuring the transition frequency that does not rely on pseudo data analysis techniques (as found in Hils and Bender [3] and Timpano, Rubbo and Cornish [6]), yet achieves higher realism than simply counting the number of binaries per frequency bin (as is done by Hils, Bender and Webbink [2] and Nelemans *et al.* [7, 10]).

3.1. The ζ Criterion

While ignoring potential data analysis techniques, we have developed a technique for determining the transition frequency at which binaries become individually resolvable that incorporates the spreading of the signal over several frequency bins due to the motion of the LISA constellation [12]. We assume that binaries become individually resolvable when their signals are separated by a few frequency bins that are devoid of signal. We consider the signal strength in a given frequency bin to be drawn from a probability distribution. If the bin contains noise (or is located in the confusion-limited regime), then its signal, $h(f)$, is considered to be drawn

from the following probability distribution function:

$$P(h(f)) = \frac{h(f)}{\sigma_f^2} \exp\left(\frac{-h^2(f)}{2\sigma_f^2}\right) \tag{1}$$

where σ_f is a measure of the averaged spread in signal strength over a suitably small frequency range so that it can be considered constant. From the mean:

$$\langle h \rangle = \int_0^\infty hP(h)dh \tag{2}$$

and the variance:

$$\sigma_h^2 = \int_0^\infty h^2 P(h)dh - \langle h \rangle^2 \tag{3}$$

we can construct the dimensionless quantity:

$$\zeta = \frac{\sqrt{\sigma_h^2}}{\langle h \rangle} \tag{4}$$

that is independent of σ_f. In regions where there are isolated signals separated by empty bins, we expect the signal in a frequency bin to be drawn from a linear combination of two distribution functions: one for the noise and one for the signals. If we assume that the average signal to noise ratio for a signal is b and the probability that a given bin will be empty is a, then the probability distribution takes the form:

$$P'(h(f)) = a\frac{h(f)}{\sigma_f^2} \exp\left(\frac{-h^2(f)}{2\sigma_f^2}\right) + (1-a)\,\frac{h(f)}{(b\sigma_f)^2} \exp\left(\frac{-h^2(f)}{2\,(b\sigma_f)^2}\right). \tag{5}$$

In this case, the value of ζ will be:

$$\zeta = \frac{\left[4\,(a + b^2(1-a)) - \pi\,(a + b(1-a))^2\right]^{1/2}}{\sqrt{\pi}\,(a + b(1-a))}. \tag{6}$$

3.2. Dependence of ζ on N

We have applied this criterion to a number of realizations of the galactic population of white dwarf binaries using the synthesis code of Benacquista, DeGoes and Lunder [11] by calculating:

$$\langle h \rangle_i = \frac{1}{n} \sum_{j=i}^{i+n-1} h(f_j), \tag{7}$$

where n is the frequency range over which we average, and

$$\sigma_{hi}^2 = \langle h^2 \rangle_i - \langle h \rangle_i^2, \tag{8}$$

using a synthesized LISA signal [12]. In the transition region the mean signal strength, read from the amplitude spectral density, is $\simeq 10^{-20}$ Hz$^{-1/2}$. We did not directly introduce any simulated instrumental noise into these models, however numerical round-off errors introduce an effective noise contribution that we estimate from the amplitude spectral density to be $\simeq 10^{-22}$ Hz$^{-1/2}$ in the transition region. Correspondingly, we take $b = 100$ as an approximation for the signal-to-noise ratio in the region of interest. We then took $a = 0.5$ to determine when roughly half

Table 1. Critical frequencies at which ζ passes through 1.2 for different total numbers of Galactic binaries.

N ($\times 10^6$)	Number of Realizations	f_c (mHz)
4	3	2.90 ± 0.07
8	6	4.01 ± 0.11
12	1	4.64
20	2	5.56 ± 0.18

the bins had strong signal and half were empty (or had weak signal). We note that ζ is only weakly dependent upon b in this region and that $b \propto f^{2/3}$, so we need not consider the frequency dependence of ζ. The critical frequency (f_c) at which ζ passes through a predefined value should scale with the number density of binaries per frequency bin as $f_c \propto N^{3/11}$ in the absence of the Doppler spreading of the signal. Including the spreading of the signal (which is proportional to f), would have $f_c \propto N^{3/8}$. Choosing $a = 0.5$ and $b = 100$ gives $\zeta = 1.2$. We have evaluated ζ for a number of realizations with $N = 4,\ 8,\ 20 \times 10^6$ binaries [12] and one realization with $N = 12 \times 10^6$ binaries [13]. The values of f_c at which ζ passes through 1.2 using an average of $n = 20,000$ bins are given in Table 1. The choice of n was made to ensure a reasonably smooth curve for ζ. If smaller values of n are chosen, the effect is to increase the variation in ζ. This, in turn, increases the spread in frequencies through which ζ passes through the threshold value of ζ without significantly altering the value of f_c. A least-squares fit to the data for a function of the form $f_c = \alpha N^\beta$ gives $\beta = 0.39 \pm 0.04$, indicating that ζ does a reasonable job of tracking the transition frequency.

4. Different Galactic Models and the Transition Frequency

Now that we have a tool to explore the behavior of the transition frequency for a range of Galactic models, we apply ζ to study the effect of the disk scale height on the transition frequency. This is motivated by the different scale heights used in the literature. The standard Hils-Bender estimate of the Galactic white dwarf binary population assumes a number density distribution throughout the galaxy given by [2]:

$$\rho(\mathbf{r}) = \rho_0 \exp\left(-R/R_0\right) \exp\left(-\left|z\right|/z_0\right) \tag{9}$$

where R and z are galactocentric cylindrical coordinates. The radial scale is $R_0 = 3500$ pc and the scale height is $z_0 = 90$ pc. The value of the scale height is probably too low and should be between 240 and 500 pc [14]. Nelemans uses a radial scale of $R_0 = 2500$ pc, a scale height of $z_0 = 200$ pc and a density distribution of [7]:

$$\rho(\mathbf{r}) = \rho_0 \exp\left(-R/R_0\right) \operatorname{sech}^2(-z/z_0). \tag{10}$$

Given the variation in scale height in the above models and the range of scale heights described in Nelson *et al.* [14], we have explored the effect that different scale heights have on the transition frequency using ζ. We have also chosen to use a density distribution that incorporates a small amount of cusp to the center of the galaxy [12]:

$$\rho(\mathbf{r}) = \rho_0 \exp\left(-R/R_0\right) \operatorname{sech}^2(-z/z_0), \tag{11}$$

where

$$\rho_0 = \frac{N}{4\pi R R_0 z_0}. \tag{12}$$

The cusp incorporated in this fashion does a reasonable job of reproducing a bulge, and is the actual density distribution used in earlier work by Nelemans [7, 9] (see the footnote in Nelemans [15]). In this context, we note that N, the total number of binaries in the Galaxy, can either be determined by comparison with the local space density or through some global property such as stellar birthrates or supernova Ia rates. If N is determined by the local space density, it will increase in direct proportion to z_0. Consequently, choosing a lower scale height while basing the total number of binaries on the local space density will result in a lower transition frequency. This is shown in Figure 3.

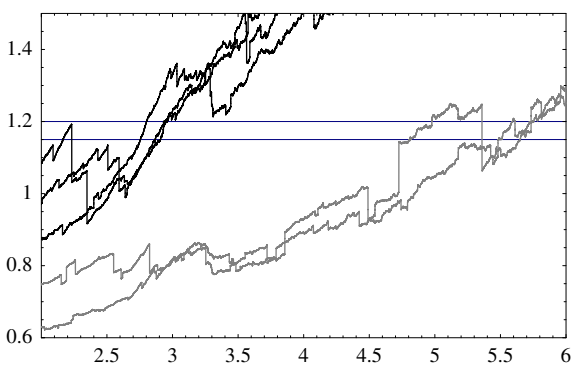

Figure 3. The value of ζ as calculated for 3 realizations of Galactic populations with $z_0 = 100$ pc and $N = 4 \times 10^6$ (black curves) and 2 realizations of Galactic populations with $z_0 = 500$ pc and $N = 20 \times 10^6$ (gray curves). Both populations have the same local space density. The vertical axis is ζ and the horizontal axis is the frequency in mHz. The horizontal lines give the value of ζ for $a = 0.5$ and $b = 100$ for the upper line and $b = 20$ for the lower line.

5. Conclusion

Reasonable variations in the models that are used to generate the Galactic white dwarf binary foreground curve can significantly alter several features of the curve. Differences in binary and stellar evolution models can substantially lower or raise the level of the confusion-limited low frequency regime by changing the average chirp mass of binaries in this frequency range. This effect can be larger than changing the total number of binaries in the Galactic population. Altering the total number of binaries has a more profound effect upon the transition frequency at which binaries may become individually resolvable. Most curves that describe the effective noise level of the Galactic population with respect to detecting extragalactic sources incorporate some model of the efficiency by which these individually resolvable signals can be removed from the LISA data stream. These models are generally overoptimistic in their results. We have introduced a technique for comparing this transition frequency that seems to accurately represent the expected behavior of the transition frequency as a function of the total number of Galactic binaries. We have used ζ to explore the effect of scale height on the transition frequency when local space density is used to determine the total number of Galactic binaries. Finally, we note that a more detailed study of the reasonable parameter space for Galactic populations may enable anaylsis of the overall shape and structure of the LISA curve to place constraints on possible stellar evolution and binary population synthesis models.

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