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C6 Review of Supersymmetry and Supergravity

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The general status and recent developments in the subjects of supersymmetry and supergravity are reviewed for the benefit of non-specialists. The report is divided into three parts as follows: I. Basics and General Status; II. Selected Recent Work in Global Supersymmetry; and III. Selected Recent Work in Supergravity.

§1. Basics and General Status

Motivation: If supersymmetry or supergravity is the answer, then what is the question? There is a serious reply to this existential query, namely: is there a symmetry principle powerful enough to raise hope for complete unification of the elementary particles and their interactions?

At the global level, supersymmetry is the only invariance compatible with quantum field theory which unifies particles of different spin and internal quantum numbers. It does this by relating bosons and fermions, the two broad classes of particles found in nature.

Local supersymmetry is a gauge principle of a unique type. It is fermionic—the gauge field is a spin 3/2 Rarita-Schwinger field—and it is a significant extension of the general covariance group of relativity. Local supersymmetry automatically requires gravity. The corresponding supergravity field theories therefore link the concept of space-time geometry with the quantum mechanical notions of spin and statistics.

After two years of active research in supergravity, the following key results have emerged:

1) The gravi ton can be unified with very restrictive combinations of lower spin particles in irreducible representations of supersymmetry algebras. The highly gauge invariant field theories of these systems raise hope for the unification of gravitation with the other interactions of elementary particles.

2) An early sign of encouragement was the cancellation in these unified theories of some of the divergences that have always plagued previous matter-gravity systems. Specifically, the divergences oi b-matrix elements in one and two-loop order cancel.

3) Previous consistency problems of the spin 3/2 field such as a causal propagation and negative probabilities are all overcome in supergravity because the interactions respect a fermionic gauge principle.

Despite these achievements there are difficulties. Non-renormalizability looms at the three-loop level and beyond. Phenomenological applications of both supersymmetry and supergravity have interesting features but presently seem unnatural. This report will describe some of these difficulties and some of the dazzling theoretical features of the formalism which have been the subject of recent work.

Basics: The essential idea of supersymmetry is to construct quantum field theories with conserved (Majorana) spinor charges Q_α. These are elements of graded Lie algebras which also involve the Poincaré group generators P^μ and $M^{\mu\nu}$ and other operators. The various supersymmetry algebras which have had applications in theoretical physics are listed in Table I.

The fundamental Poincaré supersymmetry algebra contains TV-spinor charges Q^i $i=1,\cdots,N$ which satisfy

$$\{Q^i_\alpha, \bar{Q}^j_\beta\} = \delta^{ij}(\gamma^\mu)^{\alpha\beta} P_\mu \tag{1}$$

$$[M^{\mu\nu}, Q^i_\alpha] = -i(\sigma^{\mu\nu})_{\alpha\beta} Q^i_\beta \tag{2}$$

$$[P^\mu, Q^i_\alpha] = 0 \tag{3}$$

The presence of the translation generator in the basic anticommutator (1) already suggests the connection between supersymmetry and the structure of space-time which is fully realized in supergravity.

To see that (1-3) imply a symmetry between

Table I. Supersymmetry algebras.

I. Poincaré Supersymmetry:	Golfand and Likhtman[1] Yolkov and Akulov[2] Wess and Zumino[3]
P^{μ}, $M^{\mu\nu}$, $Q_{\alpha}{}^{i}$ $i=1, 2, \ldots N$	
$\{Q, \bar{Q}\}=\gamma\cdot P$	
Simple: $N=l$	
Extended: 7V=2, 3, . . . natural SO(7V) internal symmetry	
Central Charges: U^{ij}, F°	Haag, Lopuszanski and Sohnius[4]
$\{Q, \bar{Q}\}=\gamma\cdot P+iU+\gamma_5 V$	
II. Conformai Supersymmetry	Wess and Zumino[3]
P^{μ}, $M^{\mu\nu}$, $Q_{\alpha}{}^{i}$ $i=1\ldots N$	
D, K^{μ}, A, $S_{\alpha}{}^{i}$ (D—scale, K—conformal, A—axial)	$\{S, \bar{S}\}=\gamma\cdot K$
required SU(N) internal symmetry:	Haag *et al.*[4]
III. DeSitter Supersymmetry	Keck[5] MacDowell and Mansouri[6]
relevant to cosmological terms in supergravity	

particles of different spin is extremely simple. One simply applies the spinor charge Q_{α} to a particle state \p, of definite momentum and helicity. Equation (3) and simple addition of angular momenta then imply

$$Q_{\alpha}|p, s\rangle=a|p, s+\tfrac{1}{2}\rangle+b|p, s-\tfrac{1}{2}\rangle \qquad (4)$$

The right side is a superposition of particles of the same momentum and energy—*ergo* the same mass—and helicities si 1/2. Supersymmetry therefore relates bosons and fermions.

The algebra with one conserved spinor charge is often referred to as "simple supersymmetry" and algebras with more than one spinor charge as "extended supersymmetries." In the latter case there is a unification with internal symmetry. Extended supersymmetry field theories generally have a natural global *SO(N)* internal symmetry, under which the *QÎ* transform as an **jV**-dimensional vector. A larger U(7V) internal symmetry involving j_b rotations of the Q^{i}_{a} can usually be defined.

There are now three possible viewpoints, each of which leads deeper into the subject.

1) The algebraic view which emphasizes the particle representations of supersymmetry algebras and properties of the corresponding field theories;

2) The field multiplet view which emphasizes the structure of sets of fields which transform irreducibly under supersymmetry;

3) The geometric view whose major branches are superspace and a group-theoretic approach *via* curvatures.

We shall adopt 1) in this part of the report because it leads most rapidly to an overall picture of the subject. Recent work in 2) will be described in Part III, while 3) will be discussed by other speakers at this session.

The irreducible representations of supersymmetry always consist of "towers of spins" containing equal numbers of boson and fermion states. They can be derived quite easily using Wigners method of induced representations. One finds that the little algebras of spinor charges are mathematically equivalent to complex Clifford algebras of *N* elements in the massless case and *2N* elements in the massive case. The massless representations (of dimension 27V) are therefore simpler than the massive representations (of dimension 2^{2V}). The number of distinct spins in a representation increases with *N*. The lowest spin representation for a given TV is a tower of spins s=0, 1/2, 1, s_{max}. In the massless case $£_{max}$=(1/4)(Ar+1) or (1/4)JV, for odd and even *N* respectively. For massive particles s_{max}*=(l/2)N.*

Higher spins are a notoriously difficult problem in quantum field theory, and at present consistent local field theories with interactions can be constructed only for spins *s<2.* Therefore a practical limitation for supersymmetric field theories is the restriction to representations with $\wedge_{max}<2$. The spin content of all massless representations with $\wedge_{max}<2$ is given in Table II. Particles always appear in totally antisymmetric tensor representations of the SO(A^) groups, so that the multiplicities are binomial coefficients. For *N=l* supersymmetry, the representations are doublets *(s, s—1/2)* of adjacent spin bosons and fermions.

Almost all theoretical work in the subject concerns properties of the field theories corresponding to the representations in Table II.

a) $\wedge_{max}$ = 1/2. For *N*= 1 we have the Wess-Zumino model with its striking decrease in the number of independent renormalization constants. For *N=2*, the representation of

Table II. Massless representations of Poincaré supersymmetry.

Internal symmetry	Spin	$s=2$	$s=3/2$	$s=1$	$s=1/2$	$s^P=0^\pm$
$N=1$	$s_{max}=2$	1	1			
	$s_{max}=3/2$		1	1		
	$s_{max}=1$			1	1	
	$s_{max}=1/2$				1	1+1
SO(2)	$s_{max}=2$	1	2	1		
	$s_{max}=3/2$		1	2	1	
	$s_{max}=1$			1	2	1+1
	$s_{max}=1/2$				2	2+2
SO(3)	$s_{max}=2$	1	3	3	1	
	$s_{max}=3/2$		1	3	3	1+1
	$s_{max}=1$			1	3+1	3+3
SO(4)	$s_{max}=2$	1	4	6	4	1+1
	$s_{max}=3/2$		1	4	6+1	4+4
	$s_{max}=1$			1	4	3+3
SO(5)	$s_{max}=2$	1	5	10	10+1	5+5
	$s_{max}=3/2$		1	5+1	10+5	10+10
SO(6)	$s_{max}=2$	1	6	15+1	20+6	15+15
	$s_{max}=3/2$		1	6	15	20
SO(7)	$s_{max}=2$	1	7+1	21+7	35+21	35+35
SO(8)	$s_{max}=2$	1	8	28	56	35+35

the algebra must be doubled to be described by a Lagrangian field theory. In both cases a mass term is allowed. (For $N=2$ this gives a supersymmetry algebra with central charges for which the restriction $s_{max}=(1/2)N$ need not apply. See Part II).

b) $s_{max}=1$. These theories are called the globally supersymmetric Yang-Mills theories. The arbitary Yang-Mills internal symmetry group appears as a direct product with the supersymmetry algebra. It is completely independent of the unified $SO(N)$ internal symmetry which is realized globally. The Lagrangians include minimal coupling kinetic terms plus Yukawa and quartic interactions and are therefore conventional renormalizable theoreies. They possess both Poincare and conformai supersymmetry at the classical level. Some of the many interesting properties of these theories will be discussed below.

c) $s_{max}=3/2$. Only free field theories exist for these representations because the rudimentary form of spin 3/2 gauge invariance which remains in the absence of gravity is insufficient for consistent interactions.

d) $s_{max}=2$. The representations include a massless spin 2 particle whose interactions respect the gauge principle of general covariance. The corresponding field theories must possess local rather than only global supersymmetry because the notion of a constant spinor parameter is not covariant. The field theories indeed are the locally supersymmetric unified supergravity theories.

There are more general supersymmetric theories which involve the coupling of different representations. For example, one can couple $s_{max}=1/2$ and $s_{max}=1$ representations using a supersymmetric generalization of Yang-Mills minimal coupling. Futher globally supersymmetric theories with $s_{max}<1$ can be extended to local invariance by coupling to the fields of the supergravity gauge multiplets with $s_{max}=2$.

A Global Supersymmetric Theory. The $N=1$ super-Yang-Mills theory[7] corresponds to the (1, 1/2) doublet representation of simple supersymmetry. The Lagrangian is just the minimal coupling of Yang-Mills potentials $A^a_\mu(x)$ and Majorana spinor fields $\chi^a(x)$ in the adjoint representation of the arbitrary gauge group G:

$$\mathcal{L}=-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}+\tfrac{1}{2}i\bar{\chi}^a\gamma^\mu(D_\mu\chi)^a$$
$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu$$
$$(D_\mu\chi)^a=\partial_\mu\chi^a+gf^{abc}A^b_\mu\chi^c \qquad (5)$$

Before the invention of supersymmetry it would have been very surprising to discover that this rather conventional theory had additional fermionic symmetries. Indeed, for Majorana spinors of the form[3] a=e+if'X7], where s and rj are constant, the Lagrangian changes by a total derivative under the variations

$$\delta A^a_\mu = \frac{i}{\sqrt{2}} \bar{\alpha}\gamma_\mu \chi^a$$

$$\delta \chi^a = \frac{i}{\sqrt{2}} \sigma^{\lambda\rho} F^a_{\lambda\rho}\alpha \tag{6}$$

which embody the bose-fermi character of supersymmetry transformations.

Variations with parameter e correspond to Poincaré supersymmetry transformations with conserved Noether current

$$\mathscr{J}^\mu(x) = -\sigma^{\lambda\rho} F^a_{\lambda\rho}\gamma^\mu \chi^a \tag{7}$$

Variations with parameter TJ are superconformai transformations with Noether current

$$I^\mu(x) = \gamma\cdot x \mathscr{J}^\mu(x) \tag{8}$$

There are no local *invariants* in global supersymmetry or supergravity because of the close connection with space-time translations. At best there are local *densities* such as the Lagrangian above. Care must be taken to include the total derivative term in the calculation of Noether currents.

Basic Supergravity Theory. The *N=l* supergravity theory is the gauge theory of the algebra spanned by P", $M^{\mu\nu}$ and Q_α. The spin content is given by the (2, 3/2) representation. The spin 2 graviton is the gauge quantum of the Poincare group, while its partner, the spin 3/2 graviton, is the gauge particle of supersymmetry. The gauge fields are the vierbein and Rarita-Schwinger fields $V_{a\mu}$ and $<p_\mu$.

The locally supersymmetric field theory of this system[8] can again be expressed as a minimal coupling Lagrangian[9] using the Cartan-Palatini formalism of relativity. Specifically

$$\mathscr{L} = -(1/4\kappa^2) V V^{a\mu} V^{b\nu} R_{\mu\nu ab} - \tfrac{1}{2}\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu \mathscr{D}_\nu \psi_\rho \tag{9}$$

with curvature tensor

$$R_{\mu\nu ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu a}{}^c \omega_{\nu cb} - \omega_{\nu a}{}^c \omega_{\mu cb} \tag{10}$$

and local Lorentz covariant derivative

$$\mathscr{D}_\nu \psi_\rho = (\partial_\nu + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab})\psi_\rho \tag{11}$$

where oj is the spin connection including torsion terms

$$\omega_{\mu ab} = \tfrac{1}{2}[V^\nu_a(\partial_\mu V_{b\nu} - \partial_\nu V_{b\mu}) + V^\rho_a V^\sigma_b(\partial_\sigma V_{c\rho})V^c_\mu + i\kappa^2(\bar{\psi}_\mu\gamma_a\psi_b - \tfrac{1}{2}\bar{\psi}_a\gamma_\mu\psi_b)] - [a\leftrightarrow b] \tag{12}$$

while F^det V. The gravitational coupling constant is K^=ATZG.

The Lagrangian changes by a total derivative under the variations

$$\delta V_{a\mu}(x) = -i\kappa\bar{\varepsilon}(x)\gamma_a\psi_\mu$$

$$\delta\psi_\mu(x) = \kappa^{-1}\mathscr{D}_\mu\varepsilon(x) \tag{13}$$

where e(x) is an arbitrary Majorana spinor function. The variation of $<p_\mu$ involves the covariant derivative of the gauge parameter as is the case for the gauge potentials in Yang-Mills theory and ordinary gravitation. This shows that $(p_\mu$ is indeed the gauge field of supersymmetry transformations.

The transformation rules (13) are the simplest realization of the gauge principle of local supersymmetry which has several unusual theoretical features. For example, the commutator[10] of two variations with parameters ci and e_2 is (when acting on either $V_{a\mu}$ or $é_\mu$)

$$[\delta_s(\varepsilon_1), \delta_s(\varepsilon_2)] = \delta_G(\xi_\mu) + \delta_L(\xi^\mu\omega_{\mu ab}) + \delta_s(-\kappa\xi^\mu\psi_\mu) + \text{eq. of motion terms.} \tag{14}$$

It involves a general coordinate transformation with displacement parameter f^ze^-^ together with field dependent local Lorentz and supersymmetry transformations and equations of motion terms which vanish when z*y*l[=§. At first glance this seems considerably more complicated than the supersymmetry anti-commutator (1). However, global algebraic relations such as (1) are to be understood as acting on the particle states of the theory in an asymptotically flat background geometry. In this limit (14) does imply (1) because the field dependent transformation and equation of motion terms do not contribute. This can be shown using functional Ward identities.

The proof of invariance of global and local supersymmetric field theories (such as (5) and (9)) can be obtained by direct calculation of the variation hSf using the appropriate field transformation laws (such as (6) and (13)). More systematic methods based on field multiplets or super space are available for *N=l* theories, and some of them will be discussed in Part III. At the nitty-gritty algebraic level all of these methods involve

a) detailed Dirac algebra, such as the relations

$$[\sigma^{\mu\nu}, \gamma^{\rho}] = \gamma^{\mu} g^{\nu\rho} - \gamma^{\nu} g^{\mu\rho}$$

and

b) recognition of basic identities such as the gauge field Bianchi identity $\varepsilon^{\mu\nu\rho\sigma}\mathscr{D}_{\nu}F_{\rho\sigma}=0$ or the gravitational Ricci identity

$$[\mathscr{D}_{\mu}, \mathscr{D}_{\nu}]\varepsilon(x) = \tfrac{1}{2}R_{\mu\nu ab}\sigma^{ab}\varepsilon(x),$$

c) use of Fierz rearrangement to show that expressions such as *f*^*hc*^*(^T'l')(X'ïï^l')* vanish because of over-antisymmetrization of spinors.

It is essential, particularly for c) that all spinorial quantitities such as *e*, *%* or be treated as anti-commuting Grassmann variables. It is here that the connection between spin and statistics enters intimately in supersymmetry and supergravity.

Types of Supergravity Theories: The kinds of theories which have been developed so far include

i) Unified supergravity theories which are field theories of particles in representations with ^max=2 of Table II. These representations can be called the gauge multiplets of supergravity.

ii) Matter coupling theories in which the gauge multiplets are coupled to matter multiplets with .y max<1. One can hope to obtain such theories for *N<4* where the relevant representations exist.

iii) Conformai supergravity theories which are gauge theories of the superconformai algebras and which can also be viewed as supersymmetric extensions of Weyl's conformai invariant theory of gravitation.

We will now discuss the general properties and status of these theories without detailed reference to Lagrangians, field variations, etc.

Unified Extended Supergravity Theories : These theories first exhibited the promise of the gauge principle of local supersymmetry for the unification of particle interactions and for a renormalizable theory of gravity. There are 8 distinct theories which may be classified according to the number TV of gauged supersymmetry charges. Each theory has manifest SO(A0 internal symmetry and describes a set of particles where the spin 2 graviton is unified with *N* graviton and lower spin particles in antisymmetric tensor representations of SO(JV) (see Table II). Complete theories are known for A ^ 2 ," A ^ 3 ,[12] *N=4,'*>'** and there are partial results for *N=%.*

In each theory all particle states are connected by supersymmetry and *SO(N)* transformations. It is noteworthy that this unification can include spins 1/2 and 0 which are not associated with gauge fields but are necessary to construct field theories which gauge the extended supersymmetry algebras for TV>3. An approach to supergravity based on algebraic geometry which clarifies the appearance of lower spin particles has recently been proposed by MacDowell.[16]

Unfortunately there is a very complicated non-polynomial structure due to the spinless fields in the extended supergravity theories for *N>4.* In recent work we now have two equivalent forms of the 7V=4 theory. The first form[12] has a non-polynomial structure in which functions such as *[\—/c²(A²\-B²)]~¹* appear. In the second form[13] there is a manifest SU(4) global internal symmetry and a simpler non-polynomial structure with exponential functions $e^{K\varphi}$ of a single scalar field. The equivalence of the two forms involves transformations of the fields which include dual transformation of vector field strengths.

The SO(8) extended supergravity theory is thought to be the largest of the class because the lowest spin representations of extended supersymmetry for *N>9* include spins>5/2 and we are presently unable to describe such high spins in quantum field theory. Theories with 7V=5, 6 are expected to be subcases of the SO(8) theory while the *N=l* representation has the same particle content as iV=8 and is expected to be equivalent. The *N=S* theory has therefore been approached as the next barrier beyond TV—4. Due to the complicated non-polynomial structure, there are at present only partial results[15] which give all terms in the Lagrangian through order /c² in the gravitational coupling. However a supergravity theory of the simple supersymmetry algebra in 11 space-time dimensions has recently been constructed by Cremmer, Julia and Scherk.[17] After compactification of 7 spatial dimensions one expects a 4-dimensional supergravity theory with SO(7) internal symmetry which should be related to the 80(8) theory.

It may seem that the SO(8) supergravity representation is large enough to accomodate all the elementary particles which are known or suggested by present experiments. However, the opposite is true; the SO(8) theory is too small. In a classification of the states with respect to an assumed exactly conserved SU(3) color subgroup of SO(8), Gell-Mann[18] has found that the spin 1 multiplet breaks up into SU(3) (electric charge) quantum numbers as

28=8(0)+1(0)-1(0)+3(-1/3)+3(-1/3)
+ 3(2/3)+3(1/3)+3(1/3)+3(-2/3).

These particles can be identified with colored gluons, the photon, and weak neutral Z boson plus fractionally charged superheavies of the same type as occur in other grand unification attempts. The spin 1/2 states have the following decomposition as Dirac spinors:

3(2/3)+3(—1/3)+3(—1/3)+3(2/3)+6(1/3)
+8(0)+l(—1)+1/X0)+1L(0) and can accomodate the u, d, s, c quarks, a fifth quark flavor which is a color sextet, a neutral octet, one charged lepton (the electron?) and two neutrinos. The particles missing from the scheme are charged vector bosons $W_{\pm}$ and leptons such as ***fi*** and r. The essential reason for this deficiency is that SO(8) is not big enough to contain the product subgroup SU(3)x SU(2)xU(l) of color with weak and electromagnetic flavor. From a purely phenomenological standpoint the A^=9 and 7V=10 supersymmetry algebras have identical .y_{max}=5/2 representations whose lower spin sector is favorable for a grand unification. However, there is so far no indication that a consistent field theory with a "pentatino" and 10 ''gravitons" can be formulated.

It is hard to see how to overcome the problem of constructing a unified realistic supergravity theory. Possible lines of approach, each with its own difficulties, are a) field theories based on several coupled .y_{max}=2 representations; b) superconformai theories which have a natural gauged U(AQ internal symmetry but which are plagued with ghosts (see below), and c) interpretation of the present set of elementary particles as effective low energy excitations which do not correspond exactly to the fields of supergravity theories. The latter might be evident only near the Planck energy of 10^{19} GeV.

We now turn back to the theoretical features of the unified theories and note that the theories were generally constructed in two stages. In the first stage the only coupling constant is the gravitational constant ***t c =*** $(4;rG)^{12}$^10-^{19}GeV-^{1} and the Lagrangians and transformation rules were found by an iterative procedure in ***fc.*** At this stage the SO(7V) internal symmetry is global and there are *(l/2)N(N—l)* vector fields in the adjoint representation of SO(A^) (as can be inferred from the particle content given in Table II) which interact non-minimally via field strengths *F)i=*d_{r}*AV—*d_{d}*Ai'*. At the next stage of construction SO(7V) is gauged using Yang-Mills minimal coupling with charge *e* and then determining additional terms necessary to maintain the fermionic gauge invariance.[19] In this way a marriage of the gauge principles of local internal symmetry and local supersymmetry is achieved, but it is a troubled marriage as we will discuss shortly.

The renormalizability properties of the extended supergravity theories are studied (for *e=0)* by expanding the vierbein V_{aft} about a flat background geometry and considering the properties of radiative corrections in loop graphs involving virtual gravitons, gravitinos, etc. interacting with strength ***tc.*** Since ***tc*** carries negative dimension, high powers of virtual momenta occur in Feynman integrals and one cannot hope for traditional renormalizability where divergences are absorbed in redefinition of a finite number of parameters. Instead one hopes for a finite theory in which divergences may be present offshell but cancel in S-matrix elements. Prior to the development of supergravity it was known that such cancellation occurs in one-loop order in pure general relativity (self-coupled gravitons only) but fails when conventional lower spin matter couplings are added.

One-loop finiteness of the unified theories was first demonstrated[20] by arguments which used global supersymmetry to relate 4-point amplitudes with external lower spin to amplitudes with external gravitons for which a generalization of the known proof of finiteness could be applied. These theoretical arguments were confirmed by explicit calculation of the divergent part of an amplitude in the $N=2$ theory and, subsequently, by other one-loop calculations.[21] Further arguments showed that the local[22] and non-local[23] divergences at the two-loop level cancel on-shell in simple $N=1$ supergravity. It is still unknown whether this result also holds in pure general relativity.

An approach to renormalizability based on locally supersymmetric counter terms was also initiated at an early stage.[24] A locally supersymmetric counter term is an integral expression of schematic form

$$\int d^4x V\{(R..R..\cdots R)+(\bar{\psi}..R..\cdots D\psi)+\cdots\} \tag{15}$$

involving products of variously contracted curvature tensors $R_{\mu\nu ab}$ and terms involving ψ_μ. The integral is invariant under local supersymmetry transformations. When there are L + l factors of R such an integral corresponds by power counting to the possible operator form of a divergence of the sum of all Feynman graphs with L loops in $N=1$ supergravity. It was argued that for L = l and 2 the only possible counter terms vanish when equations of motion are used so that all S-matrix elements are finite. However indication was found for an L = 3 counter term which does not vanish on-shell and is a candidate for a genuine divergence of scattering amplitudes in three-loop order.

There are two more recent developments in the counter term approach. First the $L=3$ counter term mentioned above has been generalized to $N=2$ supergravity,[25] so that unification with internal symmetry does not appear to cure the threatened non-renormalizability. Second, general techniques have been developed for the construction of complete locally supersymmetric counter terms for $N=1$ supergravity[26] (see Part III) which show that there are invariants which correspond to possible divergent scattering amplitudes for any order in perturbation theory beyond L = 3. At this point it cannot be excluded that the coefficients of the candidate divergent counter terms vanish, but no theoretical reason for such a miracle is apparent.

The difficulties of unified extended supergravity theories with gauged internal symmetry arise from cosmological terms, such as the term $(3/2)Pe^2/c^{-4}$ which appears in the Lagrangian density of the SO(2) and SO(3) invariant theories with value dictated by the requirement of local supersymmetry. Quantitatively this term cannot be related to a macroscopic cosmological constant because it is more than 100 orders of magnitude larger than the astronomical upper limit. A field theory with cosmological term cannot be quantized in a flat background geometry because Minkowski space is not a solution of the background field equations. In the present case the maximally symmetric background geometry is a de Sitter space of 0(3, 2) signature. Since this geometry does not have global Cauchy surfaces, it has been thought that a well-posed initial problem and field quantization are not possible. Recent work indicates that these difficulties may be overcome.[27]

There are even more puzzling features associated with the gauged SO(N) internal symmetry for $N=4$ which are also likely to occur for $N>4$. Because there are spinless fields the cosmological term is no longer constant, but is replaced by a scalar field potential $Vf(A, B)$. The two forms of the $N=4$ theory discussed above which are equivalent for zero gauge charge become inequivalent when the internal symmetry is gauged, and the second form leads to a theory with SU(2)xSU(2) gauge group.[28] The problem is that the potentials $f(A, B)$ in all of these theories are unbounded from below, and at the naive level this suggests the absence of a stable vacuum state.

There is now a glimmer of hope that the formidable problems associated with the cosmological term can be overcome. This hope arises in the approach to quantum gravity taken by Hawking and several collaborators in which the topological aspects of geometries summed in the path integral play a key role. Even for pure general relativity, the Euclidean action is unbounded from below, and it is necessary to make a contour rotation in function space to define the path integral.[29] Further Hawking[30] has been led to introduce a cosmological term in the microscopic Lagrangian as a Lagrange multiplier. A picture of the gravitational vacuum as a "space-time foam" of virtual black holes emerges in which the geometry is highly curved on the microscopic scale but appears nearly flat on length scales larger than the Planck length. A microscopic cosmological constant of the same sign and order of magnitude as in supergravity is predicted. These ideas must be developed considerably further before the questions associated with the extended supergravity theories can be settled, but it is important that there is a new avenue of approach involving such challenging and elegant concepts.

Matter Coupling Theories involving the (1, 1/2) and (1/2, 0) multiplets of *N=* supersymmetry were constructed quite early. The successful extension of field theories from global to local supersymmetry by coupling to the (2, 3/2) gauge fields was important in demonstrating that local supersymmetry is a true gauge principle and in determining the basic terms of supergravity Lagrangians. However, matter and gravity are not unified in such theories and it is known by explicit calculation in several cases[21] that one-loop scattering amplitudes are infinite.

Recent progress in the matter coupling includes the discovery[31] of models with a super-Higgs effect in which the gravitino acquires a mass as a consequence of spontaneous breakdown of local supersymmetry. Cosmological terms cancel at the minimum of the scalar field effective potential, so that there is no difficulty in the interpretation of the theory at least at the classical level. Readers should refer to the contribution of J. Scherk to these Proceedings which describes a very general formulation[32] of the super-Higgs effect. There have also been recent constructions of matter coupling theories for *N=2* supergravity. One theory involves an arbitrary number of massless multiplets[33] with *s_m^=l*. Another describes a massive multiplet with *s_m^=\j2* which has a central charge in its algebra.[34]

Superconformai Theories: The general conviction that symmetry is powerful is sufficient to motivate the study of field theories invariant under super conformai algebras which are the largest permitted in the framework studied by Haag *et ai* (and which for *N*= 1 is an algebra with 24 generators). Global Poincaré supersymmetric field theories of massless particles are automatically superconformai invariant as is indicated by the example of super Yang-Mills theory given earlier. Superconformai invariance fails for the supergravity theories discussed until now because the general relativity Lagrangian is not invariant under local conformai or Weyl transformations.

The construction of local superconformai theories has been considered by Kaku, Townsend, and van Nieuwenhuizen.[35] The iV= 1 theory is known in closed form and there are partial results for *N>1*. The construction involved the consideration of potentials and curvatures for the full superconformai group. Constraints on the curvatures were imposed in order to construct the locally invariant action which in final form involves the spin 2 vierbein and self-conjugate spin 3/2 and spin 1 fields *(p„* and *A^*. The full Lagrangian is complicated, but the bilinear kinetic terms are

$$\mathcal{L}^{kin}=(R_{\mu\nu}^2-\tfrac{1}{3}R^2)-\tfrac{3}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2 -\tfrac{2}{3}\bar{\psi}_\mu[\partial\!\!\!/(\Box g_{\mu\nu}-\partial_\mu\partial_\nu)-\tfrac{1}{2}\gamma_5\gamma_\rho\partial_\sigma\Box\varepsilon^{\mu\nu\rho\sigma}]\psi_\nu$$

;i6)

Note that the gravitational terms are those of Weyl's conformai invariant theory, and the superconformai theory can be regarded as the supersymmetric extension thereof.

The full Lagrangian involves one coupling *e* which is the U(l) gauge coupling constant, and one may hope that all forces can be derived in terms of this single parameter. Two problems make it difficult to fulfill this hope. First, there is no known way to recover the macroscopic predictions of general relativity from the Weyl theory and, second, there are negative metric ghosts due to the higher derivative terms, as has been clarified in a recent study of the linearized theory.[36] Progress on these points would open a major new avenue for unification in physics.

§11. Selected Recent Work in Global Supersymmetry

Superconformai Anomalies: The generators S_a of superconformai transformations satisfy *{S, S}=yK* where *Kp* is the conformai generator. *Kp* is conserved classically in a massless field theory of spin 0, 1/2, and 1 but anomalies occur at the quantum level because a length scale is inevitably introduced to define Feynman amplitudes with loops. One should expect analogous anomalies for S_a, and such anomalies were independently discovered by several groups,[37] although with somewhat confused initial interpretations.

In the supersymmetric Yang-Mills theory discussed earlier, there is a complete parallel between the dilatation and superconformai anomalies as indicated in the following table :

	Noether current	Divergence	Dimensional regularization
Dilatations			
	$D^\mu = x^\nu \theta^\mu_\nu$	$\partial_\mu D^\mu = \theta^\mu_\mu$	$\theta^\mu_\mu \sim (n-4)F^2$
Superconformai			
	$I^\mu = \gamma\cdot x\,\mathscr{J}^\mu$	$\partial_\mu I^\mu = \gamma_\mu \mathscr{J}^\mu$	$\gamma_\mu \sigma^{\alpha\beta}\gamma^\mu = (n-4)\sigma^{\alpha\beta}$

The terms which do not vanish for *n^4* indicate schematically how an anomaly can arise in a dimensionally regulated calculation of one-loop amplitudes. This is merely suggestive because dimensional regularization is invalid in supersymmetric theories. However, one may calculate the one-loop amplitude for / ' ' using a regulator-independent method which gives

$$\gamma\cdot\mathscr{J} = -(2/g)\beta(g)\sigma\cdot F\chi \tag{17}$$

to one-loop order, where the renormalization group function /3(g)=$3g^3$CV/167r^2 appears.

The superconformai anomaly may be compared with the axial current and dilatation anomalies

$$\partial\cdot J^5 = \frac{g^2}{16\pi^2} C\nu F\tilde{F}$$
$$\theta^\mu_\mu = \frac{1}{2g}\beta(g)F^2 \tag{18}$$

which are known to all orders in gauge theories of vectors and spinors independent of supersymmetry.

In a class of supersymmetric theories with classical breakdown of conformai invariance *via* mass terms, the operators above transform together under supersymmetry transformations.[38] For example

$$\delta(\partial\cdot J^5) = (1/3)\bar{\varepsilon}\gamma_5\partial\!\!\!/\,\gamma\cdot\mathscr{J}$$
$$\delta\theta^\mu_\mu = -(i/2)\bar{\varepsilon}\partial\!\!\!/\,\gamma\cdot\mathscr{J} \tag{19}$$

At the one-loop level, the quantum anomalies written above also obey these transformation rules.[39] The *yJ?* anomaly is not known beyond one-loop order but one can already see a conflict between (17), (18) and (19) because the axial anamoly contains no higher order radiative corrections (the Adler-Bardeen theorem) but /3(g) and therefore 0£ do. The resolution of this conflict is an interesting open problem which may involve only technical aspects of the renormalization procedure in supersymmetric gauge theories, but may possibly lead to new physics.

Vanishing Two-Loop /3(g): The largest global supersymmetric theory is based on the ^ = 4 representation with £$_{max}$ = 1 and describes Yang-Mills potentials *A%* 4 Majorana spinors X^{Xa}, and 3 scalars and pseudoscalars A^{Xa} and B^{Xa} all in the adjoint representation of the gauge group. The Lagrangian was found by dimensional reduction[40] of the dual fermion model in 10 dimensions and consists of the minimal coupling kinetic terms plus Yukawa and quartic coupling terms all involving the single coupling constant *g*. Earlier results[41] in related theories indicated that /3(g) vanished in one-loop order for this set of fields, but did not vanish in two-loop order, and the question was reopened in connection with the *N=4* theory with its high degree of symmetry. A vanishing two-loop contribution[42] to /3(g) was found! The JV=4 supersymmetric Yang-Mills theory is the only known field theory with no coupling constant renormalization through two-loop order. If this remarkable property remains true to all orders the theory would be exactly conformally covariant.

Central Charges: The concept of central charges in supersymmetry was first discussed by Haag, Lopuszanski and Sohnius[4] who found a consistent modification of the Poincare supersymmetry algebra in which (1) is replaced by

$$\{Q^i_\alpha, \bar{Q}^j_\beta\} = \delta^{ij}\gamma^\mu_{\alpha\beta}P_\mu + i\delta_{\alpha\beta}U^{ij} + \gamma_{5\alpha\beta}V^{ij} \tag{20}$$

Here U^{ij} and V^{ij} are Hermitean scalar and pseudoscalar central charges which are antisymmetric in indices //. By definition central charges commute with all elements of the algebra. Since the charges have dimension 1, they can only occur in field theories with a dimensional parameter such as a mass. The *N=2* massive multiplet with ^$_{max}$=1/2 was the first example of a field theory with central charge.[43]

Our understanding of the physics of central charges has improved greatly recently due largely to work of Witten and Olive.[44] They considered the A'=2 supersymmetry algebra where both central charges are proportional to the alternating symbol e^{ij} and (20) takes the form

$$\{Q^i_\alpha, \bar{Q}^j_\beta\} = \delta^{ij}\gamma\cdot P + i\varepsilon^{ij}(U - i\gamma_5 V) \qquad (21)$$

One result of Witten and Olive is a lower bound on the mass M of particles in a representation of this algebra in terms of central charge eigenvalues. The bound

$$M \geq \sqrt{U^2 + V^2} \qquad (22)$$

is easily derived in the rest frame using the positivity properties of the anti-commutator.

Further analysis of (21) shows that when the bound is not saturated, the irreducible representations are 16 dimensional as in the case of $m \neq 0$ representations of $N=2$ supersymmetry without central charges. When the bound is saturated, the 8x8 matrix on the right side of (19) has 4 vanishing eigenvalues, so that the Clifford algebra is smaller. One then finds massive irreducible representations of dimension 4, as in the case of $m=0$ representations without central charges. The phenomenon of reduced size massive representations had also been noted by Sohnius[45] for $N=2$ and by Fayet[46] for both $N=2$ and $N=4$.

Witten and Olive study the supersymmetry algebra in the $N=2$ super Yang-Mills theory with gauge group SU(2). The fields are the gauge potentials $A^a_\mu(x)$ two triplets of Majorana spinors $\psi^a_i(i=1,2)$, and scalar and pseudoscalar triplets $A^a(x)$ and $B^a(x)$. The Lagrangian

$$\mathcal{L} = \mathcal{L}_{kin} + \tfrac{1}{2}ig\varepsilon^{ij}\varepsilon^{abc}\bar{\psi}^a_i(A^b + i\gamma_5 B^b)\psi^c_j - \tfrac{1}{2}g^2(\varepsilon^{abc}A^b B^c)^2 \qquad (23)$$

consists of covariant kinetic terms and Yukawa and quartic couplings. They calculate the anticommutator $\{Q, \bar{Q}\}$ and find that the central charges appear as surface terms of the form

$$U = \int d^3x\,\partial_i(A^a F^a_{0i} + \cdots)$$

$$V = \int d^3x\,\partial_i(A^a \tfrac{1}{2}\varepsilon^{ijk} F^a_{jk} + \cdots) \qquad (24)$$

Spontaneous breakdown of the SU(2) gauge symmetry can occur in this model preserving a U(1) subgroup, which defines an electromagnetism, and preserving supersymmetry. It is also known[47] that the model contains classical soliton solutions which are magnetic monopoles and dyons and zero energy fermion solutions which are their supersymmetric partners and can be called solitinos. In this situation the central charges become

$$U = \langle A \rangle E$$

$$V = \langle B \rangle G$$

where $\langle A \rangle$ is the vacuum expectation value and E and G are electric and magnetic charges. Central charges are therefore related to topological charges!

The final point of Witten and Olive derives from the observation that for all known Higgs mechanism and soliton states the mass formula

$$M = \langle A \rangle \sqrt{E^2 + G^2} \qquad (26)$$

holds through first order in quantum corrections. This corresponds to saturation of the lower bound discussed earlier, and the massive states therefore span reduced size supersymmetry representations. Higher order quantum corrections are unlikely to introduce new particle states. The reduced size multiplets persist and the mass formula must be exact to all orders, which is a remarkable result in any quantum field theory!

There are several other subjects of recent research in global supersymmetry which cannot be discussed in detail because of the page limitations on this report. Among them are interesting work on two dimensional supersymmetric theories[48] especially the construction of some exact S-matrices[49] the derivation of a supersymmetric non-linear σ-model[50] in four dimensions, and an extensive research program[51] on realistic models for hadron and lepton phenomenology. Many problems have been overcome in constructing phenomenological models and, although complicated, they make characteristic predictions for a new class of particles which carry an additive quantum number R.

§III. Selected Recent Work in Supergravity

Many new results have already been mentioned in Part I, and we discuss a few additional topics here.

Auxiliary Fields and Multiplet Calculus for N=1 Supergravity: The main results here are systematic procedures to construct locally supersymmetric invariants for simple supergravity. There have been applications to matter coupling theories and to locally supersymmetric counter terms.

The recent development was motivated by the calculation of the commutator of two supersymmetry variations, see eq. (14), in which equation of motion terms appeared. Although supergravity can be perfectly well formulated in terms of the physical fields $V_{a\beta}(x)$ and $(p_\mu(x)$, the equation of motion terms mean that $V_{a\mu}(x)$ and $<p_\mu\{x)$ are an incomplete multiplet of fields, which do not realize a closed algebra of local supersymmetry, Lorentz and coordinate transformations independent of dynamics. Similar situations occur in global supersymmetry. For example, the commutator algebra of the super-Yang-Mills theory does not close on the physical fields $Ap(x)$ and %{x\ but an auxiliary pseudoscalar $D(x)$ can be introduced to form a complete multiplet.

In supergravity three groups[52] have recently found that a complete multiplet of fields can be formed from $V_{a\mu}$, $<p_\mu$ and auxiliary axial vector, scalar and pseudoscalar fields called A_μ, S and P. This set of 6 auxiliaries supercedes a larger set found earlier.[53] The local supersymmetry variations of the new multiplet are

$$\delta V_{a\mu}=\kappa\bar{\varepsilon}\gamma_a\psi_\mu$$
$$\delta\psi_\mu=2\kappa^{-1}\mathscr{D}_\mu\varepsilon+\tfrac{1}{3}\gamma_\mu(S-i\gamma_5P)\varepsilon$$
$$+i(A_\mu-\tfrac{1}{3}\gamma_\mu A\!\!\!/)\gamma_5\varepsilon$$
$$\delta P=-\tfrac{1}{2}\bar{\varepsilon}\gamma_5\gamma\cdot\mathscr{R}-\tfrac{1}{2}\kappa\bar{\varepsilon}\gamma\cdot\psi P$$
$$+(i/2)\kappa\bar{\varepsilon}\gamma_5\gamma\cdot\psi S+\tfrac{1}{2}\kappa\bar{\varepsilon}A\cdot\psi$$
$$\delta S=\tfrac{1}{2}\bar{\varepsilon}\gamma\cdot R-\tfrac{1}{2}\kappa\bar{\varepsilon}\gamma\cdot\psi S$$
$$-(i/2)\kappa\bar{\varepsilon}\gamma_5\gamma\cdot\psi P+(i/2)\kappa\bar{\varepsilon}\gamma_5A\cdot\psi$$
$$\delta A_\mu=\tfrac{3}{2}i\bar{\varepsilon}\gamma_5(\mathscr{R}_\mu-\tfrac{1}{3}\gamma_\mu\gamma\cdot\mathscr{R})+\tfrac{1}{2}\kappa\bar{\varepsilon}\psi_\mu P$$
$$+(i/2)\kappa\bar{\varepsilon}\gamma_5\psi_\mu S-\tfrac{1}{2}\kappa\bar{\varepsilon}\gamma\cdot\psi A_\mu+\kappa\bar{\varepsilon}A\!\!\!/\psi_\mu$$
$$-\tfrac{1}{4}\kappa V\varepsilon_{\mu\nu\rho\sigma}A^\nu\bar{\varepsilon}\gamma_5\gamma^\rho\psi^\sigma \qquad (27)$$

where $\mathscr{R}^\mu=V^{-1}\varepsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\mathscr{D}_\rho\psi_\sigma$ is the spin 3/2 Euler variation in supergravity. The commutator of two of these transformations again takes the form of (14) with no equation of motion terms and a more complicated local Lorentz parameter involving the auxiliary fields. Thus a non-linear but closed local algebra has been obtained. It is the universal algebra of all JV=1 supegravity theories encompassing pure supergravity and matter coupling models.

The Lagrangian

$$\mathscr{L}_{SG}=-(1/2\kappa^2)VR-\tfrac{1}{2}V\bar{\psi}_\mu\mathscr{R}^\mu$$
$$+\tfrac{1}{3}V(A_\nu^2-S^2-P^2) \qquad (28)$$

is invariant under (27). From the equations of motion one finds that all auxiliaries vanish, and everything reduces to the form of supergravity discussed in Part I (with new conventions). Matter coupling theories can be reformulated using auxiliary fields with some simplification of structure.[54] The auxiliary field equations relate A_μ, S, and P to the matter fields.

Further progress has emerged from the derivation of the "tensor calculus" based on scalar multiplets by Ferrara and van Nieuwenhuizen[26] and based on vector multiplets by Stelle and West.[55] Both are direct generalizations of well-known aspects of global supersymmetry. We can describe only the basic notions of the scalar multiplet calculus. The vector multiplet calculus is entirely parallel, and has advantages in some applications.

A local scalar multiplet is any set of component fields A, B, $x>$ $F>$ G with the following transformation rules

$$\delta A=\bar{\varepsilon}\chi \qquad \delta B=-i\bar{\varepsilon}\gamma_5\chi$$
$$\delta\chi=\overline{D\!\!\!/}[A-i\gamma_5B]\varepsilon+(F+i\gamma_5G)\varepsilon$$
$$\delta F=\bar{\varepsilon}[\overline{D\!\!\!/}-\tfrac{1}{3}\kappa(S-i\gamma_5P)-(i/2)A\!\!\!/\gamma_5]\chi$$
$$\delta G=i\bar{\varepsilon}\gamma_5[\overline{D\!\!\!/}+\tfrac{1}{3}\kappa(S-i\gamma_5P)+(i/2)A\!\!\!/\gamma_5]\chi \qquad (29)$$

where D_μ is a supercovariant derivative[56] defined to transform without derivatives of $e(x)$. For example $D_\mu A=d_\mu A-l/2ic<p_\mu x$. Scalar multiplets can be formed from matter fields, or from any combinations of $V_{a\mu}$, A_μ, S, P, for which the transformation rules (29) can be established.

A calculus of scalar multiplets can be set up in which the product of two multiplets, the derivative of a multiplet, and a local scalar density can be defined. The density formula tells us that, given the components of any scalar multiplet, the integral

$$I=\int d^4xV\{F+\tfrac{1}{2}\kappa\bar{\psi}\cdot\gamma\chi+\kappa(AS+BP)$$
$$+\tfrac{1}{2}\kappa^2\bar{\psi}_\mu\sigma^{\mu\nu}(A+i\gamma_5B)\psi_\nu\} \qquad (30)$$

is invariant under local supersymmetry transformations. Therefore the problem of constructing supergravity Lagrangians is transformed into the problem of finding local scalar multiplets and applying (30). This is not intrinsically easier, but it does permit input from other approaches. For example, scalar multiplets involving matter fields have been found using global supersymmetry results, while other multiplets involving various combinations of the supergravity gauge multiplet $V_{a\mu}$ <p,t, A_μ S, P have been constructed using results of the superspace approach.[36]

One such multiplet, called the scalar curvature multiplet, has components

$$\tilde{A}=S \qquad \tilde{B}=P$$
$$\tilde{\chi}=\tfrac{1}{2}(\gamma\cdot\mathscr{R}-\gamma\cdot\psi S-i\gamma_5\gamma\cdot\psi P+i\gamma_5 A\cdot\psi)$$
$$\tilde{F}=\tfrac{1}{2}R+\tfrac{1}{2}\bar{\psi}_\mu\gamma^\nu\gamma^\mu\mathscr{R}_\nu+\tfrac{1}{4}\bar{\psi}(S-i\gamma_5 P)\psi^\mu +(i/4)\bar{\psi}\cdot A\gamma_5\gamma\cdot\psi-\tfrac{2}{3}(S^2+P^2+\tfrac{1}{2}A_\mu^2)$$
$$\tilde{G}=-V^{-1}\partial_\mu(VA^\mu)+\tfrac{1}{2}i\bar{\psi}_\mu\gamma_5\mathscr{R}^\mu -\tfrac{1}{2}i\bar{\psi}_\mu\sigma^{\mu\nu}\gamma_5\mathscr{R}_\nu+\tfrac{1}{4}i\bar{\psi}_\mu\gamma_5(S-i\gamma_5 P)\psi^\mu +\tfrac{1}{4}\bar{\psi}\cdot A\gamma\cdot\psi \qquad (31)$$

The scalar density formed from this multiplet according to (30) is .?$_{SG}$ of (28). From similar multiplets containing the Weyl tensor $C_{\mu\nu\rho\sigma}$, one can construct the locally supersymmetric counter terms discussed earlier in connection with the renormalizability question.

The several applications of the multiplet calculus formalism, both in the scalar and vector case, illustrate the power of the formalism. On the other hand, it is clear from (31) that it is not entirely simple. It seems unlikely to me that any simpler procedure will be found because of the close relation of the multiplet calculus to global supersymmetry and superspace.[57] The formalism is presently limited to simple supergravity and we must hope that extensions to N>2 can be found.

New Ghost Couplings : Another curious consequence of the non-closure of the commutator algebra (14) is that the naive generalization of the Faddeev-Popov procedure to supergravity formulated with physical fields $V_{a\mu}$ and <p_μ is incorrect. The usual gauge fixing term[58] is *F((ft)=y"(f>*$_\mu$ and both fixed flat space and covariant *y* matrices may be used. Naively, one would expect a ghost Lagrangian of the form *c^j"£2)*, c, where *c* and c* are commuting Majorana spinor fields, and the additional term *(c*y"(p,) (cp^c),* from the vierbein variation of *F(x),* for covariant *fs. It* is now known that one must add the quartic coupling of the ghost fields *{c*r"c*)(cY,c).* This term has now been derived from the viewpoints of the canonical quantization formalism,[59] BRS identities and unitarity,[60] and from auxiliary fields.[61] In fact, the necessity for quartic coupling terms is quite easily seen from the auxiliary field viewpoint. Naive covariant gauge quantization is correct for the Lagrangian (28), if the auxiliary field terms in the transformation *ô(p*$_\mu$ in (27) are included in the computation of the Fadeev-Popov determinant. However, it is then clear that quartic ghost couplings will arise when auxiliary fields are eliminated, using the equations of motion of the effective Lagrangian.

The Lagrangian of a Rarita-Schwinger field in a fixed external background metric which satisfies the Einstein equations i?$_{\mu\nu}$=0 gives a consistent spin 3/2 field theory. The covariant gauge condition *F(cp)^j"c})*$_\mu$ and weight term *à-y^jj-cp* are then very natural. Nielsen[62] has shown that a third spin 1/2 ghost is then necessary for the unitarity of one-loop amplitudes.

Anomalies'. The conformai[63-65] and axial[64-67] anomalies have now been calculated for particles of spin 3/2, and even for arbitrary spin.[64] Tabulations of the conformai anomaly for various supergravity theories have been made. Curiously the anomaly vanishes in the unified SO(3) theory indicating that this theory will have finite one-loop "S-matrix" even in a background geometry with non-trivial Euler characteristic.

The form of the gravitational axial anomaly is

$$\partial_\mu J^{\mu 5}=\frac{\lambda}{768\pi^2}\varepsilon^{\alpha\beta\gamma\delta}R_{\mu\nu\alpha\beta}R^{\mu\nu}{}_{\gamma\delta}$$

and the value / = 1 obtains for a Majorana spin 1/2 particle. Several groups using different methods have found the value *À=—2* for a Majorana gravitino in an external field with covariant gauge fixing. It has also been found that this result is correct for supergravity,[68] and that the value *Z=—4* found earlier is in error.[69] Finally, there has been a recent study[70] of eigenmodes of wave operators up to spin 2 in a self-dual gravitational instanton background, which have an interesting relation to supersymmetry.

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C6 Super-Higgs Effect in Supergravity

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Supersymmetry assigns equal masses to bosons and fermions in the same multiplet. Since such a degeneracy is not observed in Nature, it is important to break supersymmetry either spontaneously or explicitly. We opt for spontaneous symmetry breaking since the introduction of non symmetrical terms would make the theory lose all predictive power. The recent advances in supergravity, namely the discovery of a minimal set of auxiliary fields, and the establishment of a tensor calculus, allow us to construct the most general coupling of supergravity (2, 3/2) to the scalar multiplet ($1/2, 0^+, 0^-$).[1] We recover as special cases all the previously derived couplings and show that the model depends upon an arbitrary function $G(A, B)$ of the scalar (A) and pseudoscalar (B) fields.

Futher we show that for a very large class of such functions, spontaneous symmetry breaking of supersymmetry takes place. The spinor χ field of the scalar multiplet plays the role of a Goldstone fermion of supersymmetry (Goldstino). It is then absorbed by the spin 3/2 gauge field of supergravity (gravitino), just like in the Higgs model, after which banquet the gravitino becomes massive. In addition, this can occur without developing a cosmological constant, due to a cancellation between terms of opposite signs.

When supergravity[2] was first discovered, it was remarked[3] that the algebra of local supersymmetry transformations did not close unless one used the equations of motion of the spin 3/2 field, a phenomenon which occurs also in flat space supersymmetry when auxiliary (non-propagating) fields are eliminated by use of their equations of motion. This situation was cured by the discovery[4] of a very simple set of 6 auxiliary fields, consisting of an axial vector A, a scalar S and a pseudoscalar P.

This led in turn to the development of a tensor calculus[5] which generalizes to curved space the results originally obtained in flat space by Wess and Zumino. Tensor calculus applies both to scalar and vector multiplets. Since here we are interested in the coupling of supergravity to a scalar multiplet, we give a very short summary of the tensor calculus for scalar multiplets.

A scalar multiplet is a set of 5 objects $\Sigma=(A, B, \chi, F, G)$ which have well defined properties under local supersymmetry transformation. For instance $\delta A=\bar{\epsilon}(x)\chi$, $\delta B=-\bar{\epsilon}(x)\gamma_5\chi$, etc. The fields A, F (B, G) are scalars (pseudoscalars) and χ is a Majorana spinor.

The tensor calculus[5] consists of two basic operations. The first is the multiplication, which to two scalar multiplets Σ_1, Σ_2 associate their product $\Sigma=\Sigma_1\otimes\Sigma_2$. In component form, one has $A=A_1A_2-B_1B_2$ and so on for the