Blejske delavnice iz fizike — Letnik 12, št. 2
Bled Workshops in Physics — Vol. 12, No. 2

ISSN 1580-4992

Proceedings to the 14th Workshop

What Comes Beyond the Standard Models

Bled, July 11–21, 2011

[Web Edition — 19.12.2011]

Edited by

Norma Susana Mankoč Borštnik
Holger Bech Nielsen
Dragan Lukman

DMFA – založništvo
Ljubljana, december 2011

The 14th Workshop *What Comes Beyond the Standard Models*,
11.– 21. July 2011, Bled

was organized by

Society of Mathematicians, Physicists and Astronomers of Slovenia

and sponsored by

Slovenian Research Agency

Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana

Society of Mathematicians, Physicists and Astronomers of Slovenia

Scientific Committee

John Ellis, CERN
David Gross, KITP
Roman Jackiw, MIT

Organizing Committee

Norma Susana Mankoč Borštnik
Holger Bech Nielsen
Maxim Yu. Khlopov

The Members of the Organizing Committee of the International Workshop "What Comes Beyond the Standard Models", Bled, Slovenia, state that the articles published in the Proceedings to the 14th Workshop "What Comes Beyond the Standard Models", Bled, Slovenia are refereed at the Workshop in intense in-depth discussions.

Contents

Preface in English and Slovenian Language V

Talk Section 1

1 The Cosmic $e^{\pm}$ Anomaly
K. Auchettl and C. Balázs 1

2 Accelerator Probes for New Stable Quarks
K. M. Belostky, M. Yu. Khlopov and K. I. Shibaev 9

3 Explaining Phenomenologically Observed Spacetime Flatness Requires New Fundamental Scale Physics
D. L. Bennett and H. B. Nielsen 16

4 Creating the Universe Without a Singularity and the Cosmological Constant Problem
E. I. Guendelman 27

5 Using Two Measures Theory to Approach Bags and Confinement
E. I. Guendelman 33

6 Predictions for Fermion Masses and Mixing From a Low Energy $SU(3)$ Flavor Symmetry Model With a Light Sterile Neutrino
A. Hernández-Galeana 41

7 Masses and Mixing Matrices of Families of Quarks and Leptons Within the Spin-Charge-Family-Theory, Predictions Beyond the Tree Level
A. Hernández-Galeana and N. S. Mankoč Borštnik 55

8 Towards Nuclear Physics of OHe Dark Matter
M. Yu. Khlopov, A. G. Mayorov and E. Yu. Soldatov 94

9 Fundamental Nonlocality
A. Kleppe 103

10 The *spin-charge-family* Theory is Explaining the Origin of Families, of the Higgs and the Yukawa Couplings
N. S. Mankoč Borštnik 112

11 Seeking a Game in Which the Standard Model Group Shall Win
H. B. Nielsen and D. L. Bennett 149

12 An Idea of New String Field Theory – Liberating Right and Left Movers
H. B. Nielsen and M. Ninomiya 181

13 Degenerate Vacua From Unification of Second Law of Thermodynamics With Other Laws
H.B. Nielsen and M. Ninomiya 199

Discussion Section 213

14 What at all is the Higgs of the Standard Model and What is the Origin of Families?
N. S. Mankoč Borštnik 215

15 On the Problem of Quark-Lepton Families
M. Yu. Khlopov 223

16 Importance of Misunderstanding in Physics Illustrated by Nonexistent Higgs
R. Mirman 227

17 Discussion on the Cosmological Vacuum Energy
H. B. Nielsen and C. Balázs 229

Virtual Institute of Astroparticle Physics Presentation 237

18 VIA Discussions at XIV Bled Workshop
M. Yu. Khlopov 239

Preface

The series of workshops on "What Comes Beyond the Standard Models?" started in 1998 with the idea of Norma and Holger for organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. The picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks and mountaineering, was chosen to stimulate the discussions. The workshops take place in the house gifted to the Society of Mathematicians, Physicists and Astronomers of Slovenia by the Slovenian mathematician Josip Plemelj, well known to the participants by his work in complex algebra.

The idea was successful and has developed into an annual workshop, which is taking place every year since 1998. This year the fourteenth workshop took place. Very open-minded and fruitful discussions have become the trade-mark of our workshop, producing several new ideas and clarifying the proposed ones. The first versions of published works appeared in the proceedings to the workshop.

In this fourteenth workshop, which took place from 11^{th} to 21^{st} of July 2011, we were discussing several topics, not all of them were prepared in time for this proceedings. New experimental results from the accelerators about the properties of families of quarks and leptons, about not yet observed new families or new other kind of particles, about the scalar fields – the Higgs– which has also not been observed, new results from the cosmological and direct measurements of the dark matter, influenced strongly this Bled workshop. As all the new observations influenced all the past Bled workshops.

Since the Higgs has not been observed, was the "approach unifying spin and charges and predicting families" (the *spin-charge-family* theory shortly) again one of the important topics of the workshop. The *spin-charge-family* theory is namely offering the mechanism for generating families, explaining correspondingly not only the origin of families, but also of the family members, having a chance to offer answers also to several other open questions in elementary particle physics and cosmology. The theory sees the Higgs field and the Yukawa couplings as its effective low energy manifestation. The discussions manifested this topic a lot.

There were open discussions about the dark energy in connection with the vacuum energy of the quantum field theory and the possible resolution of this problem with the use of complex action, about triviality of the string theories, about more general approaches to gravity, like it is the one with two measures, about the complex action and its application to the presently observed properties of fermions and bosons.

We discuss (again) a lot about the dark matter direct measurements and possible explanations of the experimental data. While some of us do not see that the so far made direct measurements would be already in contradiction with the prediction of the *spin-charge-family* theory that the fifth family members with the mass of a hundred TeV constitute the dark matter, the others propose that the fifth family ought to be lighter, less than 10 TeV, and of particular properties to fit all the direct observations and propose the OHe solution. There were also different proposals.
Talks and discussions in our workshop are not at all talks in the usual way. Each talk or a discussion on some topics lasted several hours, divided in two hours blocks, with a lot of questions, explanations, trials to agree or disagree from the audience or a speaker side. Most of talks are "unusual" in the sense that they are trying to find out new ways of understanding and describing the observed phenomena.
Although we always hope that the progress made in discussions will reflect in the same year proceedings, it happened many a time that the topics appear in the next or after the next year proceedings. This is happening also in this year. Therefore neither the discussion section nor the talks published in this proceedings, manifest all the discussions and the work done in this workshop. Many a topic, the discussions on which brought new insight into understanding of it, will wait for the next year proceedings.
Several videoconferences were taking place during the Workshop on various topics. It was organized by M.Yu.Khlopov with the use of the facility of the Virtual Institute of Astroparticle physics (wwww.cosmovia.org) of APC. We managed to have ample discussions and we thank all the participants, those presenting a talk and those contributing in these discussions. We all found these teleconferences very appropriate when trying to understand the opinion of those participating virtually.
The reader can find the talks delivered by John Ellis, N.S. Mankoč Borštnik, H.B. Nielsen and A. Romaniouk on www.cosmovia.org,

http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xiv.html

Let us thanks cordially all the participants, those present really and those present virtually, for their presentations and in particular for really fruitful discussions in which we all learn a lot. Thanks also for the good working atmosphere.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(the Organizing comittee)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(the Editors)

Ljubljana, December 2011

1 Predgovor (Preface in Slovenian Language)

Serija delavnic ”Kako preseči oba standardna modela, kozmološkega in elektrošibkega” (”What Comes Beyond the Standard Models?”) se je začela leta 1998 z idejo, da bi organizirali delavnice, v katerih bi udeleženci posvetili veliko časa diskusijam, ki bi kritično soočile različne ideje in teorije. Mesto Bled ob slikovitem jezeru je za take delavnice zelo primerno, ker prijetni sprehodi in pohodi na čudovite gore, ki kipijo nad mestom, ponujajo priložnosti in vzpodbudo za diskusije. Delavnica poteka v hiši, ki jo je Društvu matematikov, fizikov in astronomov Slovenije zapustil v last slovenski matematik Josip Plemelj, udeležencem delavnic, ki prihajajo iz različnih koncev sveta, dobro poznan po svojem delu v kompleksni algebri.

Ideja je zaživela, rodila se je serija letnih delavnic, ki potekajo vsako leto od 1998 naprej. To leto je potekala štirinajstič. Zelo odprte in plodne diskusije so postale značilnost naših delavnic, porodile so marsikatero novo idejo in pomagale razjasniti in narediti naslednji korak predlaganim idejam in teorijam. Povzetki prvih novih korakov in dognanj so izšli v zbornikih delavnic.

Na letošnji, štirinajsti, delavnici, ki je potekala od 11. do 21. malega srpana (julija) 2011, smo razpravljali o več temah, večina je predstavljena v tem zborniku, marsičesa pa nismo uspeli pravočasno premisliti in zapisati. Nove meritve na pospeševalnikih o lastnostih družin kvarkov in leptonov, o tem, da nove družine ali drugačnih novih delcev doslej še niso opazili, o tem, da tudi skalarnega polja, Higgsovega bozona, niso opazili, nove kozmološke in direktne meritve temne snovi so močno odmevale v prispevkih in diskusijah. Tudi dosedanje delavnice so skrbno spremljale delo eksperimentalnih skupin.

Ker skalarnega Higgsa na LHC doslej niso izmerili, je bila *enotna teorija spina, nabojev in družin* (the *spin-charge-family* theory) spet pomembna tema delavnice. Ta predlog namreč ponudi razlago za pojav družin kvarkov in leptonov in pojasni lastnosti družinskih članov. Ponudi odgovore na vprašanja fizike osnovnih delcev in kozmologije. Higgsovo polje in Yukawine sklopitve se pojavijo kot efektivna limita te teorije pri nizkih energijah. Diskusije so se velikokrat dotaknile te teme.

Živahna diskusija je tekla o tem, kako pojasniti temno energijo, kako energija vakuuma prispeva k temni energiji ter kakšno rešitev ponudi kompleksa akcija. Diskutirali smo o drugačnem pogledu na teorijo strun, o tem, kako lahko teorija gravitacije z dvema merama pomaga razumeti pojav inflacije v kozmologiji. Tudi o tem, kako lahko kompleksna akcija pomaga odgovoriti na odprta vprašanja v kozmologiji.

Veliko razprave je bilo (ponovno) na temo direktnih meritev temne snovi in možnih razlagah eksperimentalnih podatkov. Medtem, ko nekateri menimo, da dosedanje direktne meritve niso v nasprotju z napovedmi predloga *enotne teorije spina, nabojev in družin*, da so mase pete družine kvarkov in leptonov, ki gradijo temno snov, okoli 100 TeV ali več, pa drugi predlagajo, da mora biti peta družina lažja kot 10 TeV in s posebnimi lastnostmi, da ustreže vsem direktnim meritvam ter predlagajo kot rešitev OHe. Bili so tudi drugačni predlogi.

Predavanja in razprave na naši delavnici niso predavanja v običajnem smislu. Vsako predavanje ali razprava je trajala več ur, razdelejnih na bloke po dve uri, z veliko vprašanji, pojasnili, poskusi, da bi predavatelj in občinstvo razumeli trditve, kritike in se na koncu strinjali ali pa tudi ne. Večina predavanj je 'neobičajnih' v tem smislu, da poskušajo najti nove matematične načine opisa, pa tudi razumevanja doslej opaženih pojavov.

Čeprav vedno upamo, da bomo vsako leto uspeli zapisati vsa nova dognanja, nastala v ali ob diskusijah, se vseeno mnogokrat zgodi, da se prvi zapisi o napredku pojavijo šele v kasnejših zbornikh. Tako tudi letošnji zbornik ne vsebuje povzetkov vseh uspešnih razprav ter napredka pri temah, predstavljenih v predavanjih. Upamo, da bodo dozoreli do naslednje delavnice in bodo objavljeni v zborniku petnajste delavnice.

Med delavnico smo imeli več spletnih konferenc na različne teme. Organiziral jih je Virtualni institut za astrofiziko iz Pariza (www.cosmovia.org, vodi ga M. Yu. Khlopov). Uspelo nam je odprto diskutirati kar z nekaj laboratoriji po svetu.

Toplo se zahvaljujemo vsem udeležencem, tako tistim, ki so imeli predavanje, kot tistim, ki so sodelovali v razpravi, na Bledu ali preko spleta. Bralec lahko najde posnetke predavanj, ki so jih imeli John Ellis, N.S. Mankoč Borštnik, H.B. Nielsen in A. Romaniouk na spletni povezavi

http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xiv.html

Prisrčno se zahvaljujemo vsem udeležencem, ki so bili prisotni, tako fizično kot virtualno, za njihova predavanja, za zelo plodne razprave in za delovno vzdušje.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(Organizacijski odbor)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(uredniki)

Ljubljana, grudna (decembra) 2011

Talk Section

All talk contributions are arranged alphabetically with respect to the authors' names.

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 1)*
Bled, Slovenia, July 11-21, 2011

1 The Cosmic $e^{\pm}$ Anomaly

K. Auchettl and C. Balázs

Monash Centre for Astrophysics and
ARC Centre of Excellence for Particle Physics at the Tera-scale
School of Physics, Monash University, Victoria 3800 Australia

Abstract. Via a Bayesian likelihood analysis using 219 recent cosmic ray spectral data points we extract the anomalous part of the cosmic $e^{\pm}$ flux. First we show that a significant tension exists between the $e^{\pm}$ related and the rest of the fluxes. Interpreting this tension as the presence of an anomalous component in the $e^{\pm}$ related data, we then infer the values of selected cosmic ray propagation parameters excluding the anomalous data sample from the analysis. Based on these values we calculate background predictions with theoretical uncertainties for PAMELA and Fermi-LAT. We find a statistically significant deviation between the Fermi-LAT $e^- + e^+$ data and the predicted background even when (systematic) uncertainties are taken into account. Identifying this deviation as an anomalous $e^{\pm}$ contribution, we make an attempt to distinguish between various sources that may be responsible for the anomalous $e^{\pm}$ flux.

1.1 Introduction

Cosmic ray observations provided significant and puzzling deviations from theoretical predictions over the last decades. Experiments such as TS [1], AMS [2], CAPRICE [3], MASS [4], and HEAT [5,6] established a hint of an excess of high energy electrons and/or positrons. Measurements of the PAMELA satellite confirmed these results by finding an excess over the theoretical predictions in the $e^-/(e^- + e^+)$ flux for $E > 10$ GeV [7]. The PAMELA data seem to deviate from the theoretical predictions even when experimental and theoretical uncertainties are taken into account [8]. An excess in the $e^- + e^+$ flux was also found by AMS [9], PPB-BETS [10], and HESS [11,12]. Recently the Fermi-LAT satellite confirmed the $e^- + e^+$ excess above 100 GeV [13]. The deviation between the Fermi-LAT data and the theoretical $e^- + e^+$ prediction is significant. This deviation was recently confirmed by the PAMELA collaboration which found the e^- flux to be consistent with the Fermi-LAT data.

Many attempts were to explain the deviation between the data and theory. New physics was invoked ranging from modification of the cosmic ray propagation, through supernova remnants, to dark matter annihilation. Ref. [14] summarizes these speculations. Whether the $e^{\pm}$ anomaly exists depends on the theoretical prediction of the cosmic ray background. The theoretical prediction is challenging because of the lack of precise knowledge of the cosmic ray sources, and because the cosmic ray propagation model has numerous free parameters, such as convection velocities, spatial diffusion coefficients and momentum loss rates.

Motivated by traces of possible new physics in the Fermi-LAT data, we attempt to determine the size of the anomalous contribution in the cosmic $e^{\pm}$ flux. Our method involves the following steps. First we find the cosmic ray propagation parameters that influence the $e^{\pm}$ flux measured by Fermi-LAT and PAMELA the most. Then we subject the cosmic ray data, other than the Fermi-LAT and PAMELA $e^{\pm}$ measurements to a Bayesian likelihood analysis, to determine the preferred values and the 68 % (1-σ) credibility regions of the relevant propagation parameters. Based on the central values and 1-σ credibility regions of these propagation parameters we then predict the background flux, with uncertainties, for Fermi-LAT and PAMELA. Finally, we extract the anomalous part of the spectrum by subtracting the background prediction from the Fermi-LAT and PAMELA measurement.

1.2 Theory of cosmic ray propagation

Cosmic ray propagation is described by the diffusion model [15]. This model assumes homogeneous propagation of charged particles within the Galactic disk and it also takes into account cooling effects. The phase-space density $\psi_a(\vec{r}, p, t)$ of a particular cosmic ray species at a Galactic radius of $\vec{r}$ can be calculated solving the transport equation which has the general form [16]

$$\frac{\partial \psi_a(\vec{r}, p, t)}{\partial t} = Q_a(\vec{r}, p, t) + \nabla \cdot (D_{xx} \nabla \psi_a - \vec{V} \psi_a) - (\frac{1}{\tau_f} + \frac{1}{\tau_r}) \psi_a + \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_a \right) - \frac{\partial}{\partial p} \left(\dot{p} \psi_a - \frac{p}{3} (\nabla \cdot \vec{V}) \psi_a \right). \quad (1.1)$$

Here $q(\vec{r}, p, t)$ is the source term of primary and secondary cosmic ray contributions. The spatial diffusion coefficient D_{xx} has the form

$$D_{xx} = D_{0xx} \beta \left(\frac{R}{\text{GeV}} \right)^{\delta}, \quad (1.2)$$

where $\beta = v/c$, and $R = pc/eZ$ is the magnetic rigidity of the particles which describes a particle's resistance to deflection by a magnetic field. Here Z is the effective nuclear charge of the particle, v is its velocity, p is its momentum, e is its charge, and c is the speed of light. The exponent δ indicates the power law dependence of the spatial diffusion coefficient D_{xx}.

Diffusion in momentum space (diffusive re-acceleration) is described by the coefficient D_{pp}

$$D_{pp} D_{xx} = \frac{4 p^2 {v_A}^2}{3\delta(4 - \delta^2)(4 - \delta) w}. \quad (1.3)$$

Here v_A is the Alfven speed, the parameter w characterises the level of hydromagnetic turbulence experienced by the cosmic rays in the interstellar medium [17]. In Eq.(1.1), $\vec{V}$ is the convection velocity and the parameter τ_f is the time-scale of the fragmentation loss, and τ_r is the radioactive decay time-scale.

The GalProp numerical package solves the propagation equation numerically for $Z \geq 1$ nuclei, as well as for electrons and positrons on a two dimensional spatial grid with cylindrical symmetry in the Galaxy [16]. The input parameter file for GalProp has a number of free parameters which are available for the author to define. These can be classified into a number of subsets: the diffusion of cosmic ray, the primary cosmic ray sources and radiative energy losses of these primary cosmic rays. The diffusion subset is described by the parameters defined above:

$$D_{0xx}, \delta, L, \nu_A, \partial\vec{V}/\partial z. \tag{1.4}$$

The most relevant parameters in the primary cosmic ray source subset are:

$$R_{ref}^{e^-}, \gamma_1^{e^-}, \gamma_2^{e^-}, R_{ref}^{nucleus}, \gamma_1^{nucleus}, \gamma_2^{nucleus}. \tag{1.5}$$

Here $\gamma_1^{e^-}$ and $\gamma_2^{e^-}$ are primary source electron injection indices. They specifying the steepness of the electron injection spectrum, $dq(p)/dp \propto p^{\gamma_i^{e^-}}$, below and above of a reference rigidity $R_{ref}^{e^-}$. There are separate injection indices for nuclei defined by $\gamma_1^{nucleus}$ and $\gamma_2^{nucleus}$ below and above $R_{ref}^{nucleus}$. For further details we refer the reader to [16].

1.3 Parameter space and uncertainties

To reduce the dimension of the parameter space we tested the robustness of the $e^{\pm}$ flux against the variation of nearly all individual parameters and found that it is mostly sensitive to the following propagation parameters:

$$P = \{\gamma_1^{e^-}, \gamma_2^{nucleus}, \delta_1, \delta_2, D_{0xx}\}. \tag{1.6}$$

Here $\gamma_1^{e^-}$ and $\gamma_2^{nucleus}$ are the primary electron and nucleus injection indices, δ_1 and δ_2 are spatial diffusion coefficients below and above a reference rigidity ρ_0, and D_{0xx} determines the normalization of the spatial diffusion coefficient.

Our calculations confirmed the findings of a recent study by [18] that the $e^{\pm}$ flux is sensitive to the change of the Galactic plane height L. Indeed [17] have shown that there is a connection between L and D_{0xx}:

$$D_{0xx} = \frac{2c(1-\delta)L^{1-\delta}}{3\pi w\delta(\delta+2)}. \tag{1.7}$$

Thus, varying the cylinder height amounts to the redefinition of D_{0xx} as also noticed by Ref. [19]. In the light of this, we fix L to 4 kpc and use D_{0xx} as free parameter.

We treat the normalizations of the e^-, e^+, $\bar{p}/p$, B/C, (SC+Ti+V)/Fe and Be-10/Be-9 fluxes as theoretical nuisances parameters.

$$P_{nuisance} = \{\Phi^0_{e^-}, \Phi^0_{e^+}, \Phi^0_{\bar{p}/p}, \Phi^0_{B/C}, \Phi^0_{(SC+Ti+V)/Fe}, \Phi^0_{Be-10/Be-9}\}. \tag{1.8}$$

When evaluating the uncertainties, following [20], we ignore theoretical uncertainties and combine statistical and systematic experimental uncertainties in quadrature

$$\sigma_i^2 = \sigma_{i,statistical}^2 + \sigma_{i,systematic}^2. \tag{1.9}$$

This can be done for Fermi-LAT and the latest PAMELA e^- flux. Unfortunately, systematic uncertainties are not available for the rest of the cosmic ray measurements. When this is the case, as an estimate of the systematics, we define σ_i as the rescaled statistical uncertainty

$$\sigma_i^2 = \sigma_{i,statistical}^2/\tau_i. \tag{1.10}$$

For simplicity, in this study, we use the same scale factor τ_i for all data points where systematic uncertainty is not available. To remain mostly consistent with the work of [20], we set this common scale factor to a conservative value that they use: $\tau_i = 0.2$. We checked that our conclusions only mildly depend on this choice. Further details about our Bayesian parameter inference can be found in [21].

1.4 Experimental data

We included 219 of the most recent experimental data points in our statistical analysis. These contained 114 $e^\pm$ related, and 105 $\bar{p}/p$, B/C, (Sc+Ti+V)/Fe and Be-10/Be-9 cosmic ray flux measurements. These data are summarized in Table 1.1.

1.5 Is there a cosmic ray anomaly?

In this section we investigate whether the present cosmic ray data justify the existence of an anomaly in the $e^\pm$ spectrum. To this end we divide the cosmic ray data into two groups: 114 measurements containing observations of $e^\pm$ fluxes (AMS, Fermi, HESS, and PAMELA) and the rest of 105 data points ($\bar{p}/p$, B/C, (Sc+Ti+V)/Fe, Be-10/Be-9). We perform a Bayesian analysis independently on these two sets of data extracting the preferred values of the propagation parameters.

Fig. 1.1 clearly shows that the two subsets of cosmic ray data are inconsistent with the hypothesis that the cosmic ray propagation model and/or sources implemented in GalProp provides a good theoretical description. Our interpretation of the tension between the $e^\pm$ fluxes and the rest of the cosmic ray data is that the measurements of PAMELA and Fermi-LAT are affected by new physics which is unaccounted for by the propagation model and/or cosmic ray sources included in our calculation.

1.6 The size of the $e^\pm$ anomaly

We use the central values and credibility regions of the parameters determined using the non-$e^\pm$ related data to calculate a background prediction for the $e^\pm$ fluxes. Fig. 1.2 shows the the measured $e^\pm$ fluxes and the calculated background. Statistical and systematic uncertainties combined in quadrature are shown for Fermi-LAT, while ($\tau = 0.2$) scaled statistical uncertainties are shown for PAMELA $e^+/(e^+ + e^-)$ as gray bands. Our background prediction is overlaid as magenta

Measured flux	Experiment	Energy (GeV)	Data points
	AMS [9]	0.60 - 0.91	3
$e^+ + e^-$	Fermi-LAT [13]	7.05 - 886	47
	HESS [11,12]	918 - 3480	9
$e^+/(e^+ + e^-)$	PAMELA [22]	1.65 - 82.40	16
e^-	PAMELA [23]	1.11 - 491.4	39
$\bar{p}/p$	PAMELA [24]	0.28 - 129	23
	IMP8 [25]	0.03 - 0.11	7
	ISEE3 [26]	0.12 - 0.18	6
B/C	Lezniak et al. [27]	0.30 - 0.50	2
	HEAO3 [28]	0.62 - 0.99	3
	PAMELA [29]	1.24 - 72.36	8
	CREAM [30]	91 - 1433	3
(Sc+Ti+V)/Fe	ACE [31]	0.14 - 35	20
	SANRIKU [32]	46 - 460	6
	Wiedenbeck et al. [33]	0.003 - 0.029	3
	Garcia-Munoz et al. [34]	0.034 - 0.034	1
	Wiedenbeck et al. [33]	0.06 - 0.06	1
Be-10/Be-9	ISOMAX98 [35]	0.08 - 0.08	1
	ACE-CRIS [36]	0.11 - 0.11	1
	ACE [37]	0.13 - 0.13	1
	AMS-02 [38]	0.15 - 9.03	15

Table 1.1. Cosmic ray experiments and their energy ranges over which we have chosen the data points for our analysis. We split the data into two groups: e$^{\pm}$ flux related (first five lines in the table), and the rest. We perform two independent Bayesian analyses to show the significant tension between the two data sets.

bands. The central value and the 1-σ uncertainty of the calculated anomaly is displayed as green dashed lines and bands. As the first frame shows the Fermi-LAT measurements deviate from the predicted background both below 10 GeV and above 100 GeV.

In our interpretation the deviation is a statistically significant signal of the presence of new physics in the $e^+ + e^-$ flux. Based on the difference between the central values of the data and the background a similar conclusion can be drawn from PAMELA. Unfortunately, the sizable uncertainties for the PAMELA measurements prevent us to claim a statistically significant deviation. After having determined the background for the e$^{\pm}$ fluxes, we subtract it from the measured flux to obtain the size of the new physics signal. Results for the e$^{\pm}$ anomaly are also shown in Fig. 1.2. As expected based on the background predictions a non-vanishing anomaly can be established for the Fermi-LAT $e^+ + e^-$ flux, while no anomaly with statistical significance can be claimed for PAMELA due to the large uncertainties.

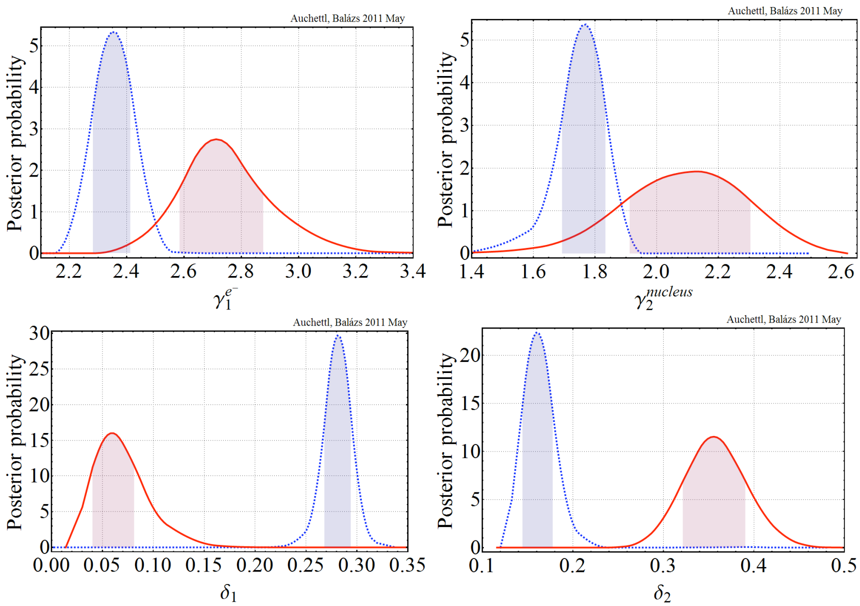

Fig. 1.1. Marginalized posterior probability distributions of propagation parameters listed in Eq.(1.6). The dashed blue curves show results with likelihood functions containing $e^{\pm}$ flux data while the likelihood functions for the solid red curves contain only the rest of the comic ray data. Shaded areas show the 68 % credibility regions. A statistically significant tension between the $e^{\pm}$ and the rest of the data is evident in the lower frames.

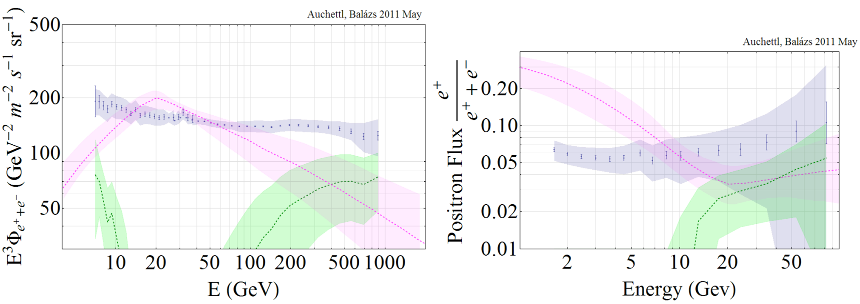

Fig. 1.2. Electron-positron fluxes measured by Fermi-LAT and PAMELA (gray bands) with the extracted size of the $e^{\pm}$ anomaly (green bands). Combined statistical and systematic uncertainties are shown for Fermi-LAT and PAMELA e^-, while ($\tau = 0.2$) scaled statistical uncertainties are shown for PAMELA $e^+/(e^+ + e^-)$. Our background predictions (magenta bands) are also overlaid.

1.7 The source of the anomaly

Based on the available evidence we can only speculate about the origin of the discrepancy between the data and predictions of the cosmic electron-positron spectra. The first obvious assumption is that some aspect of the propagation model used in the present calculation is insufficient for the proper description of the electron-positron fluxes arriving at Earth [39]. In this case there exists no anomaly in the data. Assuming that the propagation model satisfactorily describes physics over the Galaxy the next reasonable thing is to suspect local effects modifying the electron-positron distribution [40]. Further suspicion falls on the lack of sources included in the calculation [41].

Possible new sources of cosmic rays to account for the anomaly have been proposed in two major categories. The first category is standard astrophysical objects such as supernova remnants, pulsars, various objects in the Galactic centre, etc. Finally, more exotic explanations call for new astronomical and/or particle physics phenomena, such as dark matter. In Ref. [21] we compared our extracted signal to recent predictions of anomalous sources. We considered predictions from supernova remnants, nearby pulsars and dark matter annihilation. We concluded that presently uncertainties are too large and prevent us from judging the validity of these as explanations of the anomaly. With more data and more precise calculations the various suggestions of the cosmic $e^- + e^+$ anomaly can be ruled out or confirmed.

Acknowledgements

We are indebted to R. Cotta, A. Donea, J. Lazendic-Galloway, Y. Levin and T. Porter for invaluable discussions on various aspects of cosmic ray physics and likelihood analysis. CB wishes to thank the organizers and participants of the Bled Workshop for stimulating discussions. KA is thankful to P. Chan for help with issues of parallel computing, to M. Jasperse for assistance in Mathematica programming, to J. Lazendic-Galloway, T. Porter and to A. Vladimirov for help with GalProp. This research was funded in part by the Australian Research Council under Project ID DP0877916. The use of Monash University Sun Grid, a high-performance computing facility, is also gratefully acknowledged.

References

1. R. L. Golden, et al., Astrophys. J. **436** (1994) 769–775.
2. J. Alcaraz, et al., Phys. Lett. **B484** (2000) 10–22.
3. M. Boezio, et al., Astrophys. J. **561** (2001) 787–799.
4. C. Grimani, et al., Astron. Astrophys. **392** (2002) 287–294.
5. S. W. Barwick, et al., Astrophys. J. **482** (1997) L191–L194.
6. J. J. Beatty, et al., Phys. Rev. Lett. **93** (2004) 241102.
7. O. Adriani, et al., Nature **458** (2009) 607–609.
8. T. Delahaye, F. Armand, M. Pohl, and P. Salati (2011), arXiv:1102.0744.
9. M. Aguilar, et al., Phys. Rept. **366** (2002) 331–405.

10. S. Torii, et al. (2008), arXiv:0809.0760.
11. F. Aharonian, et al., Phys. Rev. Lett. **101** (2008) 261104.
12. F. Aharonian, et al., Astron. Astrophys. **508** (2009) 561.
13. M. Ackermann, et al., Phys. Rev. **D82** (2010) 092004.
14. P. D. Serpico, Astrophysical models for the origin of the positron 'excess' (2011).
15. V. Ginzburg, et al.,*Astrophysics of cosmic rays*, North Holland, 1990.
16. A. W. Strong, I. V. Moskalenko, and V. S. Ptuskin, Ann. Rev. Nucl. Part. Sci. **57** (2007) 285–327.
17. E. S. Seo, and V. S. Ptuskin, Astrophys. J. **431** (1994) 705–714.
18. R. C. Cotta, et al., JHEP **01** (2011) 064.
19. G. Di Bernardo, et al., Astropart.Phys. **34** (2011) 528–538.
20. R. Trotta, et al., The Astrophysical Journal **729** (2010) 106.
21. K. Auchettl, and C. Balazs (2011), arXiv:1106.4138.
22. O. Adriani, et al., Astropart.Phys. **34** (2010) 1–11.
23. O. Adriani, et al., Phys. Rev. Lett. **106** (2011) 201101.
24. O. Adriani, et al., Phys. Rev. Lett. **105** (2010) 121101.
25. I. V. Moskalenko, et al., Astrophys.J. **565** (2002) 280–296.
26. K. E. Krombel, and M. E. Wiedenbeck, Astrophys. J. **328** (1988) 940–953.
27. J. A. Lezniak, and W. R. Webber, Astrophys. J. **223** (1978) 676–696.
28. J. J. Engelmann, et al., Astronomy and Astrophysics **233** (1990) 96–111.
29. E. M. et al., "The PAMELA Experiment: Preliminary Results after Two Years of Data Taking," in *21st European Cosmic Ray Symposium (ECRS 2008)*, 2008, Proceeding of 21st European Cosmic Ray Symposium, pp. 396–401.
30. H. S. Ahn, et al., Astropart. Phys. **30** (2008) 133–141.
31. A. J. Davis, et al., "On the low energy decrease in galactic cosmic ray secondary/primary ratios," in *Acceleration and Transport of Energetic Particles Observed in the Heliosphere*, edited by R. A. Mewaldt, J. R. Jokipii, M. A. Lee, E. Möbius, & T. H. Zurbuchen , 2000, vol. 528 of *American Institute of Physics Conference Series*, pp. 421–424.
32. M. Hareyama, "SUB-Fe/Fe ratio obtained by Sanriku balloon experiment," in *International Cosmic Ray Conference*, 1999, vol. 3 of *International Cosmic Ray Conference*, pp. 105–+.
33. M. E. Wiedenbeck, and D. E. Greiner, The Astrophysical Journal Letters **239** (1980) L139–L142.
34. M. Garcia-Munoz, et al., "The Energy Dependence of Cosmic-Ray Propagation at Low Energy," in *International Cosmic Ray Conference*, 1981, vol. 9 of *International Cosmic Ray Conference*, pp. 195–+.
35. T. Hams, et al., "^{10}Be/^{9}Be ratio up to 1.0 GeV/nucleon measured in the ISOMAX 98 balloon flight," in *International Cosmic Ray Conference*, 2001, vol. 5 of *International Cosmic Ray Conference*, pp. 1655–+.
36. A. J. Davis, et al., *AIP Conference Proceedings* **528**, 421–424 (2000), URL `http://link.aip.org/link/?APC/528/421/1`.
37. N. E. Yanasak, et al., Advances in Space Research **27** (2001) 727–736.
38. J. Burger, European Physical Journal C **33** (2004) 941–943.
39. A. Tawfik, and A. Saleh (2010), arXiv:1010.5390.
40. M. Pesce-Rollins, and f. t. F. L. collaboration (2009), arXiv:0907.0387.
41. M. T. Frandsen, I. Masina, and F. Sannino (2010), arXiv:1011.0013.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 9)*
Bled, Slovenia, July 11-21, 2011

2 Accelerator Probes for New Stable Quarks

K. M. Belostky[1], M. Yu. Khlopov[1,2,3] and K. I. Shibaev[1]

[1]National Research Nuclear University "Moscow Engineering Physics Institute", 115409 Moscow, Russia
[2] Centre for Cosmoparticle Physics "Cosmion" 115409 Moscow, Russia
[3] APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. The nonbaryonic dark matter of the Universe can consist of new stable double charged particles O^{--}, bound with primordial helium in heavy neutral O-helium (OHe)"atoms" by ordinary Coulomb interaction. O-helium dark atoms can play the role of specific nuclear interacting dark matter and provide solution for the puzzles of dark matter searches. The successful development of composite dark matter scenarios appeals to experimental search for the charged constituents of dark atoms. If O^{--} is a "heavy quark cluster" $\bar{U}\bar{U}\bar{U}$, its production at accelerators is virtually impossible and the strategy of heavy quark search is reduced to search for heavy stable hadrons, containing only single heavy quark (or antiquark). Estimates of production cross section of such particles at LHC are presented and the experimental signatures for new stable quarks are outlined.

2.1 Introduction

The cosmological dark matter can consist of dark atoms, in which new stable charged particles are bound by ordinary Coulomb interaction (See [1–3] for review and references). In order to avoid anomalous isotopes overproduction, stable particles with charge -1 (and corresponding antiparticles), as tera-particles [4], should be absent [5], so that stable negatively charged particles should have charge -2 only.

Such stable double charged particles can hardly find place in SUSY models, but there exist several alternative elementary particle frames, in which heavy stable -2 charged species, O^{--}, are predicted:

(a) AC-leptons, predicted in the extension of standard model, based on the approach of almost-commutative geometry [6–9].
(b) Technileptons and anti-technibaryons in the framework of walking technicolor models (WTC) [10,11].
(c) and, finally, stable "heavy quark clusters" $\bar{U}\bar{U}\bar{U}$ formed by anti-U quark of 4th [6,12–14] or 5th [15] generation.

All these models also predict corresponding +2 charge particles. If these positively charged particles remain free in the early Universe, they can recombine with ordinary electrons in anomalous helium, which is strongly constrained in the terrestrial matter. Therefore cosmological scenario should provide a mechanism, which suppresses anomalous helium. There are two possibilities, requiring two different mechanisms of such suppression:

(i) The abundance of anomalous helium in the Galaxy may be significant, but in the terrestrial matter there exists a recombination mechanism suppressing this abundance below experimental upper limits [6,8].
(ii) Free positively charged particles are already suppressed in the early Universe and the abundance of anomalous helium in the Galaxy is negligible [3,13].

These two possibilities correspond to two different cosmological scenarios of dark atoms. The first one is realized in the scenario with AC leptons, forming neutral AC atoms [8]. The second assumes charge asymmetric case with the excess of O^{--}, which form atom-like states with primordial helium [3,13].

If new stable species belong to non-trivial representations of electroweak SU(2) group, sphaleron transitions at high temperatures can provide the relationship between baryon asymmetry and excess of -2 charge stable species, as it was demonstrated in the case of WTC [10,16–18].

After it is formed in the Standard Big Bang Nucleosynthesis (SBBN), ^4He screens the O^{--} charged particles in composite $(^4He^{++}O^{--})$ *O-helium* "atoms" [13].

In all the proposed forms of O-helium, O^{--} behaves either as lepton or as specific "heavy quark cluster" with strongly suppressed hadronic interaction. Therefore O-helium interaction with matter is determined by nuclear interaction of He. These neutral primordial nuclear interacting objects contribute to the modern dark matter density and play the role of a nontrivial form of strongly interacting dark matter [19,20].

The cosmological scenario of O-helium Universe allows to explain many results of experimental searches for dark matter [3]. Such scenario is insensitive to the properties of O^{--}, since the main features of OHe dark atoms are determined by their nuclear interacting helium shell. It challenges direct experimental search for the stable charged particles at accelerators and such search strongly depends on the nature of O^{--}.

Stable -2 charge states (O^{--}) can be elementary like AC-leptons or technileptons, or look like elementary as technibaryons. The latter, composed of techniquarks, reveal their structure at much higher energy scale and should be produced at LHC as elementary species. They can also be composite like "heavy quark clusters" $\bar{U}\bar{U}\bar{U}$ formed by anti-U quark in one of the models of fourth generation [12,13] or $\bar{u}_5\bar{u}_5\bar{u}_5$ of (anti)quarks $\bar{u}_5$ of stable 5th family in the approach [15].

In the context of composite dark matter scenario accelerator search for stable particles acquires the meaning of critical test for existence of charged constituents of cosmological dark matter.

The signature for AC leptons and techniparticles is unique and distinctive what allows to separate them from other hypothetical exotic particles. In particular, the ATLAS detector has an unique potential to identify these particles and measure their masses.

Test for composite O^{--} can be only indirect: through the search for heavy hadrons, composed of single U or $\bar{U}$ and light quarks (similar to R-hadrons). Here we study a possibility for experimental probe of this hypothesis.

2.2 New stable generations

Modern precision data on the parameters of the Standard model do not exclude [21] the existence of the 4th generation of quarks and leptons.

In one of the approaches the 4th generation follows from heterotic string phenomenology and its difference from the three known light generations can be explained by a new conserved charge, possessed only by its quarks and leptons [12–14,22]. Strict conservation of this charge makes the lightest particle of 4th family (neutrino) absolutely stable, but it was shown in [22] that this neutrino cannot be the dominant form of the dark matter. The same conservation law requires the lightest quark to be long living [12,13]. In principle the lifetime of U can exceed the age of the Universe, if $m_U < m_D$ [12,13].

In the current implementation of the *"spin-charge-family-theory"* [15] there are predicted two sets with four generations each, so that the 4th generation is unstable, while the lightest (5th generation) of the heavy set has no mixing with light families and thus is stable. If $m_{u_5} < m_{d_5}$ and their mass difference is significant, OHe dark matter cosmological scenario can be realized in this theory. For the lower possible mass scale (~ 1TeV) for the 5th generation particles, their search at LHC is possible along the same line as for stable particles of 4th generation in the approach [12–14,22]. In the successive discussion we'll consider stable u-type quark without discrimination of the cases of 4th and 5th generation, denoting the stable quark by U.

Due to their Coulomb-like QCD attraction ($\propto \alpha_c^2 \cdot m_U$, where α_c is the QCD constant) stable double and triple U bound states (UUq), (UUU) can exist [12,4,5,13–15]. The corresponding antiparticles can be formed by heavy antiquark $\bar{U}$. Formation of these double and triple states at accelerators and in cosmic rays is strongly suppressed by phase space constraints, but they can be formed in early Universe and strongly influence cosmological evolution of 4th generation hadrons. As shown in [13], <u>an</u>ti- <u>U</u>-<u>t</u>riple state called <u>anuti</u>um or $\Delta^{--}_{3\bar{U}}$ is of a special interest. This stable anti-Δ-isobar, composed of $\bar{U}$ antiquarks can be bound with ^4He in atom-like state of O-helium [6].

Since simultaneous production of three $U\bar{U}$ pairs and their conversion in two doubly charged quark clusters UUU is suppressed, the only possibility to test the models of composite dark matter from 4th (or 5th) generation in the collider experiments is a search for production of stable hadrons containing single U or $\bar{U}$. U-quark can form lightest (Uud) baryon and $(U\bar{u})$ meson with light quarks and antiquarks. $\bar{U}$ can form the corresponding stable antiparticles, like $\bar{U}\bar{u}\bar{d}$ and $\bar{U}u$. Search for these stable hadrons is similar to the R-hadrons search. The main task will be to distinguish R-hadrons from hadrons, containing quarks of 4th or 5th generation. R-hadrons will be accompanied by supersymmetric particles, what is not the case for 4th or 5th generation hadrons.

2.3 Signatures for U-hadrons in accelerator experiments

In spite of that the mass of U-quarks can be quite close to that of t-quark, strategy of their search should be completely different. U-quark in framework of the considered models is stable and will form stable hadrons at accelerator contrary to t-quark.

Detailed analysis of possibility of U-quark search requires quite deep understanding of physics of interaction between (meta-)stable U-hadrons and nucleons of matter. However, methodic for U-quark search can be described in general, if we know mass spectrum of U-hadrons and (differential) cross sections of their production. Cross section of U-quark production in pp-collisions is presented on the Fig. 2.1. For comparison, cross sections of 4th generation leptons are shown too. Cross sections have been calculated with program CompHEP [23]. Cross sections of U- and D- quarks virtually do not differ. For quarks (U and D) the obtained

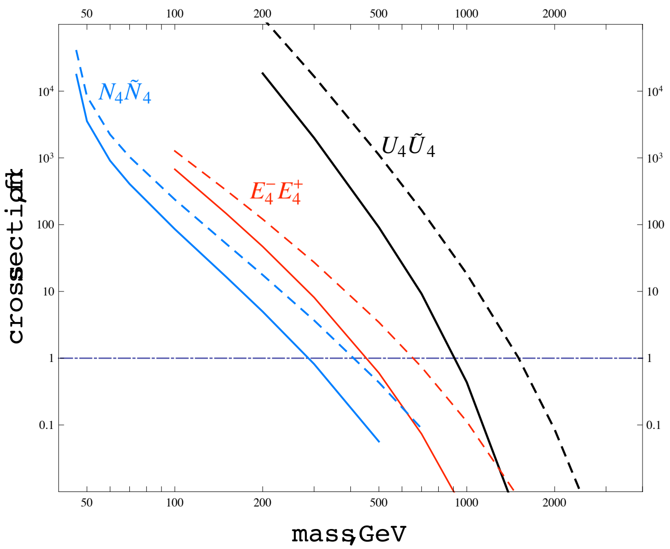

Fig. 2.1. Cross sections of production of 4th generation particles (N, E, U (D)) at LHC. Solid and dashed curves correspond to c.m. energies 7 and 14 TeV respectively. Horizontal dashed line shows approximate level of sensitivity to be reached in 2012 (at the energy 7 TeV).

values were re-scaled in correspondence of estimations done with program Hathor [24]. Heavy stable quarks will be produced with high transverse momentum p_T and velocity, which is less than speed of light. In general, simultaneous measurement of velocity and momentum provides us information about mass of particle. Information on ionization losses are, as a rule, not so good. All these features are typical for any heavy particle, while there can be subtle differences in the shapes of their angle- and p_T-distribution, defined by concrete model, which it predicts. It is the peculiarity of long-lived hadronic nature what can be of special importance for clean selection of events of U-quarks production.

U-quark can form a whole class of U-hadron states which can be considered as stable in the conditions of an accelerator experiment contrary to their relics in Universe. But in any case, as we pointed out, double and triple U-hadronic states cannot be virtually created at collider. Many other hadronic states, whose lifetime exceeds $\sim 10^{-7}$ s, should also look as stable in accelerator experiment. In the Table 1 expected mass spectrum of U-hadrons, obtained with the help of code PYTHIA [25], is presented.

Table 2.1. Mass spectrum and relative yields in LHC for U-hadrons

	Difference of masses of U-hadron and U-quark, GeV	Expected yields in % (in the right columns the yields of long-lived stated are given)	
$\{U\tilde{u}\}^0$, $\{U\tilde{d}\}^+$	0,330	39,5(3)%, 39,7(3)%	
$\{U\tilde{s}\}^+$	0,500	11,6(2)%	
$\{Uud\}^+$	0,579	5,3(1)%	7,7(1)%
$\{Uuu\}_1^{++}$, $\{Uud\}_1^+$, $\{Udd\}_1^0$	0,771	0,76(4)%, 0,86(5)%, 0,79(4)%	
$\{Usu\}^+$, $\{Usd\}^0$	0,805	0,65(4)%, 0,65(4)%	1,51(6)%
$\{Usu\}_1^+$, $\{Usd\}_1^0$	0,930	0,09(2)%, 0,12(2)%	
$\{Uss\}_1^0$	1,098	0,005(4)%	

The lower indexes in notation of U-hadrons in the Table 1 denote the nonzero spin $s = 1$ of the pair of light quarks. ¿From comparison of masses of different U-hadrons it follows that all $s = 1$ U-hadrons decay quickly emitting π-meson or γ-quantum, except for (Uss)-state. In the right column the expected relative yields are presented. Unstable $s = 1$ U-hadrons decay onto respective $s = 0$ states, increasing their yields.

There are two mesonic states being quasi-degenerated in mass: $U\bar{u}$ and $U\bar{d}$ (we skip here discussion of strange U-hadrons). Interaction with the medium composed of u and d quarks transforms U-hadrons into those ones containing u and d (as it is the case in the early Universe [12–14]). The created pair of $U\bar{U}$ quarks will fly out of the vertex of pp-collision as U-hadrons with positive charge in 60% of all U-quark events and as neutrals in 40% (correspondingly, 60% with negative charge and 40% neutrals for $\bar{U}$ hadrons). After traveling through the matter of detectors, at a distance of a few nuclear lengths from vertex, U-hadrons will transform in (roughly) 100% of positively charged hadrons (Uud), while $\bar{U}$ -hadrons will convert in 50% into negatively charged $\bar{U}$ -hadron $(\bar{U}d)$ and in 50% to neutral $\bar{U}$ -hadron $(\bar{U}u)$.

This feature will enable to discriminate the considered case of U-quarks from variety of alternative models, predicting new heavy stable particles.

Note that if the mass of Higgs boson exceeds $2m$, its decay channel into the pair of stable $Q\bar{Q}$ will dominate over the $t\bar{t}$, $2W$, $2Z$ and invisible channel to neutrino pair of 4th generation [26]. It may be important for the strategy of heavy Higgs searches.

2.4 Conclusions

The cosmological dark matter can be formed by stable heavy double charged particles bound in neutral OHe dark atoms with primordial He nuclei by ordinary

Coulomb interaction. This scenario sheds new light on the nature of dark matter and offers nontrivial solution for the puzzles of direct dark matter searches. It can be realized in the model of stable 4th generation or in the approach unifying spin and charges and challenges for experimental probe at accelerators. In the context of this scenario search for new heavy stable quarks acquires the meaning of direct experimental probe for charged constituents of dark atoms of dark matter.

The O^{--} constituents of OHe in the model of stable 4th generation and in the "*spin-charge-family-theory*" are "heavy quark clusters" $\bar{U}\bar{U}\bar{U}$. Production of such clusters (and their antiparticles) at accelerators is virtually impossible. Therefore experimental test of the hypothesis of stable U quark is reduced to the search for stable or metastable U hadrons, containing only single heavy quark or antiquark. The first year of operation at the future 14 TeV energy of the LHC has good discovery potential for U(D)-quarks with mass up to 1.5 TeV, while the level of sensitivity expected in the 2012 at the LHC energy 7 TeV can approach to the mass of 1 TeV. U-hadrons born at accelerator will distinguish oneself by high p_T, low velocity, by effect of a charge flipping during their propagation through the detectors. All these features enable to strongly increase the efficiency of event selection from not only background but also from alternative hypothesis. In particular, we show that the detection of positively charged U-baryon in coincidence with $\bar{U}$-mesons (50% neutrals and 50% negatively charged) provides a distinct signature for the stable U quark. Analysis of other channels of new particles production provides distinctions from the case of R-hadrons. In the latter case all the set of supersymmetric particles should be produced.

It should be noted that the "*spin-charge-family-theory*" predicts together with stable 5th generation also 4th generation of quarks and leptons, which are mixed with the three known families and thus unstable. Experimental probe for new unstable heavy particles implies definite prediction for their mass spectrum and branching ratios for their modes of decay.

Acknowledgments

We would like to thank Norma Mankoč Borštnik, all the participants of Bled Workshop and A.S.Romaniouk for stimulating discussions.

References

1. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, J. Phys.: Conf. Ser. **309** (2011) 012013.
2. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, Bled Workshops in Physics **11** (2010) 73.
3. M. Y. Khlopov, arXiv:1111.2838, to be published in Mod. Phys. Lett. A (2011); arXiv:1111.2887, to be published in *Proc. ICATPP2011*.
4. S. L. Glashow, arXiv:hep-ph/0504287.
5. D. Fargion and M. Khlopov, arXiv:hep-ph/0507087.
6. M. Y. Khlopov, arXiv:astro-ph/0607048.
7. C. A. Stephan, arXiv:hep-th/0509213.
8. D. Fargion *et al*, Class. Quantum Grav. **23** (2006) 7305; M. Y. Khlopov and C. A. Stephan, arXiv:astro-ph/0603187.

9. A. Connes *Noncommutative Geometry* (Academic Press, London and San Diego, 1994).
10. M. Y. Khlopov and C. Kouvaris, Phys. Rev. D **77** (2008) 065002.
11. F. Sannino and K. Tuominen, Phys. Rev. D **71** (2005) 051901; D. K. Hong *et al*, Phys. Lett. B **597** (2004) 89; D. D. Dietrich *et al*, Phys. Rev. D **72** (2005) 055001; D. D. Dietrich *et al*, Phys. Rev. D **73** (2006) 037701; S. B. Gudnason *et al*, Phys. Rev. D **73** (2006) 115003; S. B. Gudnason *et al*, Phys. Rev. D **74** (2006) 095008.
12. K.M.Belotsky *et al*, Gravitation and Cosmology **11** (2005) 3
13. M.Yu. Khlopov, JETP Lett. **83** (2006) 1.
14. K. Belotsky *et al*, arXiv:astro-ph/0602261. K. Belotsky *et al*, Gravitation and Cosmology **12** (2006) 1; K. Belotsky, M.Yu.Khlopov,K.I.Shibaev, Stable quarks of the 4th family? in Eds. N. L. Watson and T. M. Grant: "The Physics of Quarks: New Research." (Horizons in World Physics, V.265), NOVA Publishers, Hauppauge NY, 2009, PP.19-47; arXiv:0806.1067 [astro-ph].
15. N.S. Mankoč Borštnik, Bled Workshops in Physics **11** (2010) 105; A. Borštnik Bračič, N.S. Mankoč Borštnik, Phys. Rev. D **74** (2006) 073013; N.S. Mankoč Borštnik, Mod. Phys. Lett. A **10** (1995) 587; N.S. Mankoč Borštnik, Int. J. Theor. Phys. **40** (2001) 315; G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, New J. of Phys. **10** (2008) 093002.
16. M. Y. Khlopov and C. Kouvaris, Phys. Rev. D **78** (2008) 065040
17. M. Y. Khlopov, *AIP Conf. Proc.* **1241**, 388 (2010).
18. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, Int. J. Mod. Phys. D **19** (2010) 1385.
19. B. D. Wandelt et al., arXiv:astro-ph/0006344; P. C. McGuire and P. J. Steinhardt, arXiv:astro-ph/0105567; G. Zaharijas and G. R. Farrar, Phys. Rev. D **72** (2005) 083502
20. C. B. Dover *et al*, Phys. Rev. Lett. **42** (1979) 1117; S. Wolfram, Phys. Lett. B **82** (1979) 65; G. D. Starkman *et al*, Phys. Rev. D **41** (1990) 3594; D. Javorsek *et al*, Phys. Rev. Lett. **87** (2001) 231804; S. Mitra, Phys. Rev. D **70** (2004) 103517; G. D. Mack *et al*, Phys. Rev. D **76** (2007) 043523;
21. M. Maltoni *et al.*, Phys. Lett. B **476** (2000) 107; V.A. Ilyin *et al.*, Phys. Lett. B **503** (2001) 126; V.A. Novikov *et al.*, Phys. Lett. B **529** (2002) 111; JETP Lett. **76** (2002) 119.
22. K.M.Belotsky, M.Yu.Khlopov and K.I.Shibaev, Gravitation and Cosmology Supplement **6** (2000) 140; K.M.Belotsky *et al.*, Gravitation and Cosmology **11** (2005) 16; K.M.Belotsky *et al.*, Phys.Atom.Nucl. **71** (2008) 147.
23. E.Boos et al, [CompHEP Collaboration], Nucl. Instrum. Methods A **534** (2004) 250.
A.Pukhov et al, CompHEP - a package for evaluation of Feynman diagrams and integration over multi-particle phase space. User's manual for version 3.3, INP MSU report 98-41/542 (arXiv:hep-ph/9908288)
Home page: http://comphep.sinp.msu.ru
24. M. Aliev et al., Computer Phys. Commun. **182** (2011) 1034.
25. T. Sjöstrand et al., Computer Phys. Commun. **135** (2001) 238.
26. K.M. Belotsky et al.,Phys. Rev. D **68** (2003) 054027.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 16)*
Bled, Slovenia, July 11-21, 2011

3 Explaining Phenomenologically Observed Spacetime Flatness Requires New Fundamental Scale Physics

D. L. Bennett[1]* and H. B. Nielsen[2]**

[1]Brookes Institute for Advanced Studies, Bøgevej 6, 2900 Hellerup, Denmark
[2]The Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø

Abstract. The phenomenologically observed flatness - or near flatness - of spacetime cannot be understood as emerging from continuum Planck (or sub-Planck) scales using known physics. Using dimensional arguments it is demonstrated that any immaginable action will lead to Christoffel symbols that are chaotic. We put forward new physics in the form of fundamental fields that spontaneously break translational invariance. Using these new fields as coordinates we define the metric in such a way that the Riemann tensor vanishes identically as a Bianchi identity. Hence the new fundamental fields define a flat space. General relativity with curvature is recovered as an effective theory at larger scales at which crystal defects in the form of disclinations come into play as the sources of curvature.

3.1 Introduction

We address the fundamental mystery of why the spacetime that we experience in our everyday lives is so nearly flat. More provocatively one could ask why the macroscopic spacetime in which we are immersed doesn't consist of spacetime foam[1,2].

This question is approached by putting up a NO-GO for having the spacetime flatness that we observe phenomenologically. This NO-GO builds upon an argumentation that starts with the assumption that spacetime is a continuum down to arbitrarily small scales a with $a << l_{Pl}$ where l_{Pl} is the Planck length.

Earlier one of us (H.B.N.) has attempted to derive reparametrization invariance as a consequence of quantum fluctuations [3]. If reparametrization invariance were for such a reason exact, it would be difficult to see how accepting arbitrarily small length scales could be avoided. So that would nessesitate our assumtion of a total continuum."

This assumption of a continuum at all scales $a << l_{Pl}$ forbids having any form of regulator - e.g., a lattice. With no regulator in place we must expect enormous quantum fluctuations unless we can come to think of some physics that can tame them.

* e-mail: dlbennett99@gmail.com
** e-mail: hbech@nbi.dk

We shall argue quite generally that no form of known physics can accomplish this. For example, the Einstein-Hilbert action

$$\frac{1}{2\kappa}\int d^4x\sqrt{-g}R \tag{3.1}$$

hasn't a chance since at scales a for which $\frac{1}{a^2} >> \frac{1}{\kappa}$ this action is negligible. We shall argue that there does not exist a functional form for an action that can prevent spacetime foam for arbitrarily small scales $a << l_{Pl}$.

As a solution to this problem we propose new fundamental fields at scales $a << l_{Pl}$ that spontaneously break translational invariance. This approach was inspired by the work[4] of Eduardo Guendelman.

3.2 Phenomenological Flatness Impossible if Spacetime Foam Shows Up at Any Scale Including Scales $a \ll l_{Pl}$

Over long distances the spacetime that we experience is - barring the presence of nearby gravitational singularities - very nearly flat. This means that the parallel transport of a vector from a spacetime point A to a distant spacetime point B along say many different pathes should result in a well-defined (small) average rotation angle for the parallel transported vector.

If the connection used for parallel transport takes values in a compact group, a path along which there is strong curvature can have an orbit on the group manifold that is wrapped around the group manifold several or many times depending on the amount of curvature.

Take an S^1 as a prototype compact group manifold. The rotation under parallel transport can be written

$$\Theta = \theta + 2\pi k \tag{3.2}$$

$-\pi < \theta < \pi$ and $k \in \mathbf{Z}$.

For nearly flat space the rotation angle Θ under parallel transport along a path will vary very slowly along the path. The average values of Θ along different pathes are expected to be closely clustered around $\Theta = 0$ and with certainty to lie in the interval $[-\pi, \pi]$.

However, if there were an underlying spacetime foam, then two pathes Γ_1 and Γ_2 connecting the same two widely separated points would in general have vastly different values of Θ say Θ_1 and Θ_2 reflecting the fact that the enormous curvatures encountered in traversing the spacetime foam along the two pathes are completely uncorrelated. If

$$\Theta_1 = \theta_1 + 2\pi k_1 \tag{3.3}$$

and

$$\Theta_2 = \theta_2 + 2\pi k_2 \tag{3.4}$$

we expect k_1 and k_2 to be large and uncorrelated which also means that θ_1 and θ_2 are completely uncorrelated as to their position in the interval $[-\pi, \pi]$.

For example, the pathes Γ_1 and Γ_2 could have Θ_1 and Θ_2 values such that $k_2 >> k_1 >> 1$ while $\Theta_1 \bmod 2\pi = \theta_1$ and $\Theta_2 \bmod 2\pi = \theta_2$ could be such that $\theta_1 > \theta_2$. So it does not necessarily follow from $\Theta_2 > \Theta_1$ that $\theta_2 > \theta_1$.

The fact that Θ and θ can differ by a term that is the product of a uncontrollably large number $|k_2 - k_1|$ multiplied by 2π means that the idea of an average rotation angle when parallel transporting a vector along different pathes between two spacetime points is meaningless. The underlying reason is that in traversing spacetime foam the connection is an uncontrollably rapidly varying function of any path going through spacetime foam. In particular this argument would also apply to pathes connecting spacetime points separated by distances for which spacetime is known phenomenologically to be flat or at least nearly flat.

Without a well defined connection the concept of spacetime flatness is meaningless. The conclusion is that if spacetime foam comes into existence at *any* scale under the scale at which we phenomenologically observe flatness, the possibility for having flat spacetime is forever lost.

It is in particular at scales a with $a << l_{Pl}$ that there is the danger of spacetime foam coming into existence. At these scales the Einstein-Hilbert action would be completely ineffective in preventing spacetime foam. This is the reason for our proposal of new physics at sub-Planckian scales in the form of fundamental fields ϕ^a (that we also call Guendelmann fields) that spontaneously break translational invariace in the vacuum in such a way that the metric can be defined by

$$g_{\mu\nu} = \frac{\partial\phi^a\partial\phi^b}{\partial x^\mu \partial x^\nu}\eta_{ab}. \tag{3.5}$$

With the fundamental fields ϕ^a defined by this form for the metric $g_{\mu\nu}$ it can be shown that the Riemann curvature vanishes identically. The converse can also be shown: the condition $R_{\mu\nu\rho}{}^\sigma = 0$ implies that $g_{\mu\nu}$ must have the form of Eqn.(3.5). It should be stressed that $g_{\mu\nu}$ with the form of Eqn.(3.5) leads to $R_{\mu\nu\rho}{}^\sigma = 0$ as an identity quite independently of any choice of Lagrangian (or lack thereof) and the equations of motion that follow from such a choice.

3.3 There Exists No Action Depending Only on Translationally Invariant Coordinates that can Keep Spacetime Flat at All Scales

We consider the variation of the rotation angle of a vector field (or in general a tensor field) parallel transported around a loop of radius a as a goes to values much Less than the Planck scale compared say to the angle 2π. For this purpose we consider the connection $\Gamma_{\mu\nu}{}^\rho$ integrated around the edge of a disc of radius a:

$$\oint_{\text{disc edge } 2\pi a} \Gamma_{\mu\nu}{}^\rho dx^\nu \overset{Stokes}{\approx} \int_{\text{disc area } \pi a^2} R_{\mu\nu\lambda}^{\rho} dx^\nu dx^\lambda \tag{3.6}$$

3.4 Flatness Requires New Fundamental Fields that Break Translational Invariance Spontaneously at Sub-Planck Scales

We introduce new fundamental fields $\phi^a(x^\mu)$ at scales a with $a << l_{Pl}$ that spontaneously break translational invariance in such a way that the metric is defined by

$$g_{\mu\nu} = \frac{\partial\phi^a \partial\phi^b}{\partial x^\mu \partial x^\nu} \eta_{ab}. \tag{3.7}$$

The new fundamental fields can also be thought of as fundamental absolute coordinates insofar as they break translational invariance. By indexing the new fields (coordinates) ϕ^a with indices $a, b, c, ...$ we are anticipating a later development in which these indices will be seen to be flat indices.

At this point we shall show explicitly the important property that the Riemann tensor $R_{\mu\nu\rho}{}^\sigma$ vanishes identically when the new fundamental coordinates $\phi^a, \phi^b, \phi^c, ...$ are chosen as in Eqn. (3.5). To this end we need Christoffel symbols

$$\Gamma^\rho{}_{\mu\nu} = \frac{1}{2} g^{\rho\tau} \left(\frac{\partial g_{\nu\tau}}{\partial x^\mu} + \frac{\partial g_{\mu\tau}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\tau} \right)$$

in the form

$$\Gamma_{\gamma\mu\nu} = g_{\gamma\rho}\Gamma^\rho{}_{\mu\nu} = \frac{1}{2} \left(\frac{\partial g_{\nu\gamma}}{\partial x^\mu} + \frac{\partial g_{\mu\gamma}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\gamma} \right)$$

into which we substitute Eqn. (3.5)

$$= \frac{1}{2}\eta_{ab} \left(\frac{\partial}{\partial x^\mu} (\frac{\partial\phi^a}{\partial x^\nu}\frac{\partial\phi^b}{\partial x^\gamma}) + \frac{\partial}{\partial x^\nu} (\frac{\partial\phi^a}{\partial x^\mu}\frac{\partial\phi^b}{\partial x^\gamma}) - \frac{\partial}{\partial x^\gamma} (\frac{\partial\phi^a}{\partial x^\mu}\frac{\partial\phi^b}{\partial x^\nu}) \right)$$

which reduces to

$$\eta_{ab} \frac{\partial^2\phi^a}{\partial x^\mu \partial x^\nu} \frac{\partial\phi^b}{\partial x^\sigma}.$$

Going from $\Gamma_{\gamma\mu\nu}$ back to $\Gamma^\rho{}_{\mu\nu} = g^{\gamma\rho}\Gamma_{\gamma\mu\nu}$ yields

$$\Gamma^\rho{}_{\mu\nu} = \eta_{ab} g^{\rho\sigma} \frac{\partial\phi^b}{\partial x^\sigma} \frac{\partial^2\phi^a}{\partial x^\mu \partial x^\nu}. \tag{3.8}$$

We want to show that the the Riemann tensor

$$R_{\mu\nu\lambda}{}^\sigma \doteq \partial_\mu \Gamma_{\nu\lambda}{}^\sigma - \partial_\nu \Gamma_{\mu\lambda}{}^\sigma - \Gamma_{\mu\lambda}{}^\delta \Gamma_{\nu\delta}{}^\sigma + \Gamma_{\nu\lambda}{}^\delta \Gamma_{\mu\delta}{}^\sigma$$

vanishes identically with the choice Eqn. (3.5) for $g_{\mu\nu}$.

We make a small digression in order to establish an intermediate result. Consider the matrix element

$$[g]^{\mu\nu} = \left[(g_{\bullet\bullet})^{-1} \right]^{\mu\nu} = \left[\left(\frac{\partial\phi^a}{\partial\, x^\bullet} \eta_{ab} \frac{\partial\phi^b}{\partial x^\bullet} \right)^{-1} \right]^{\mu\nu} = \left[\left(\frac{\partial\phi^\circ}{\partial x^\bullet} \right)^{-1} \eta_{\circ\circ}^{-1} \left(\frac{\partial\phi^\circ}{\partial x^\bullet} \right)^{-1} \right]^{\mu\nu}$$

$$= \left[\left(\frac{\partial\phi^\circ}{\partial x^\bullet} \right)^{-1} \right]^\mu{}_a \left[(\eta_{\bullet\bullet})^{-1} \right]^{ab} \left[\left(\frac{\partial\phi^\circ}{\partial x^\bullet} \right)^{-1} \right]^\nu{}_b \tag{3.9}$$

where square brackets denote matrix elements with row indices to the left and column indices to the right. The symbols • and ∘ stand for respectively general coordinate and flat coordinate indices and are used to indicate the number and position of otherwise unspecified indices.

Converting from matrix element notation to operator notation according to

$$\left[\left(\frac{\partial\phi^\circ}{\partial x^\bullet}\right)^{-1}\right]^\mu_a = \frac{\partial x^\mu}{\partial \phi^a} \tag{3.10}$$

we have

$$g^{\mu\nu} = \frac{\partial x^\mu}{\partial\phi^a}\eta^{-1\ ab}\frac{\partial x^\nu}{\partial\phi^b}. \tag{3.11}$$

multiplying $g^{\mu\nu}$ into $\frac{\partial\phi^c}{\partial x^\nu}\eta_{bc}$ gives

$$g^{\mu\nu}\frac{\partial\phi^c}{\partial x^\nu}\eta_{bc} = \frac{\partial x^\mu}{\partial\phi^a}\eta^{-1\ ad}\frac{\partial x^\nu}{\partial\phi^d}\frac{\partial\phi^c}{\partial x^\nu}\eta_{bc} = \frac{\partial x^\mu}{\partial\phi^a}\frac{\partial x^\nu}{\partial\phi^a}\frac{\partial\phi^b}{\partial x^\nu} = \frac{\partial x^\mu}{\partial\phi^a}\delta^a_b = \frac{\partial x^\mu}{\partial\phi^b} \tag{3.12}$$

multiply now both sides of (3.8) by $\frac{\partial\phi^c}{\partial x^\rho}$

$$\frac{\partial\phi^c}{\partial x^\rho}\Gamma^\rho{}_{\mu\nu} = \frac{\partial\phi^c}{\partial x^\rho}\underbrace{\eta_{ab}g^{\rho\sigma}\frac{\partial\phi^b}{\partial x^\sigma}}_{\partial x^\rho/\partial\phi^a \text{ from } (3.8)}\frac{\partial^2\phi^a}{\partial x^\mu\partial x^\nu} = \frac{\partial^2\phi^c}{\partial x^\mu\partial x^\nu}$$

which is the intermediate result needed below.

To show that the Riemann $R_{\mu\nu\rho}{}^\sigma$ tensor vanishes identically when $g^{\mu\nu}$ is chosen to have the form of Eqn. (3.5) we shall show that

$$\frac{\partial\phi^b}{\partial x^\sigma}R_{\mu\nu\lambda}^{\ \ \ \sigma} \equiv 0$$

for arbitrary $\partial\phi^b/\partial x^\sigma$. Explicitely

$$\frac{\partial\phi^b}{\partial x^\sigma}R_{\mu\nu\lambda}{}^\sigma = \frac{\partial\phi^b}{\partial x^\sigma}\partial_\mu\Gamma_{\nu\lambda}{}^\sigma - \frac{\partial\phi^b}{\partial x^\sigma}\partial_\nu\Gamma_{\mu\lambda}{}^\sigma - \frac{\partial\phi^b}{\partial x^\sigma}\Gamma_{\mu\lambda}{}^\delta\Gamma_{\nu\delta}{}^\sigma + \frac{\partial\phi^b}{\partial x^\sigma}\Gamma_{\nu\lambda}{}^\delta\Gamma_{\mu\delta}{}^\sigma.$$

The first two terms on the right hand side, i.e.,

$$\frac{\partial\phi^b}{\partial x^\sigma}\partial_\mu\Gamma_{\nu\lambda}{}^\sigma - \frac{\partial\phi^b}{\partial x^\sigma}\partial_\nu\Gamma_{\mu\lambda}{}^\sigma$$

can be written as

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial\phi^b}{\partial x^\sigma}\Gamma_{\nu\lambda}{}^\sigma\right) - \frac{\partial^2\phi^b}{\partial x^\mu\partial x^\sigma}\Gamma_{\nu\lambda}{}^\sigma - \left[\frac{\partial}{\partial x^\nu}\left(\frac{\partial\phi^b}{\partial x^\sigma}\Gamma_{\mu\lambda}{}^\sigma\right) - \frac{\partial^2\phi^b}{\partial x^\nu\partial x^\sigma}\Gamma_{\mu\lambda}{}^\sigma\right].$$

Using (3.4) to rewrite the 1st and 3rd terms of this expression gives

$$\frac{\partial}{\partial x^\mu}\frac{\partial^2\phi^b}{\partial x^\nu\partial x^\lambda} - \frac{\partial^2\phi^b}{\partial x^\mu\partial x^\sigma}\Gamma_{\nu\lambda}{}^\sigma - \frac{\partial}{\partial x^\nu}\frac{\partial^2\phi^b}{\partial x^\mu\partial x^\lambda} + \frac{\partial^2\phi^b}{\partial x^\nu\partial x^\sigma}\Gamma_{\mu\lambda}{}^\sigma.$$

The 1st and 3rd terms cancel since they are totally symmetric under permutations of the indices $\mu\nu\lambda$. Consequently what remains of the first two terms of $(\partial\phi^b/\partial x^\sigma)R_{\mu\nu\lambda}^{\ \ \ \sigma}$ is

$$\frac{\partial^2\phi^b}{\partial x^\nu\partial x^\sigma}\Gamma_{\mu\lambda}{}^\sigma - \frac{\partial^2\phi^b}{\partial x^\mu\partial x^\sigma}\Gamma_{\nu\lambda}{}^\sigma.$$

Using the intermediate result (3.4) in reverse these first two terms of $(\partial\phi^b/\partial x^\sigma)R_{\mu\nu\lambda}^{\ \ \ \ \sigma}$ become

$$\frac{\partial\phi^b}{\partial x^\rho}\Gamma_{\mu\lambda}^{\ \ \sigma}\Gamma_{\nu\sigma}^{\ \ \sigma} - \frac{\partial\phi^b}{\partial x^\rho}\Gamma_{\nu\lambda}^{\ \ \sigma}\Gamma_{\mu\sigma}^{\ \ \rho}$$

which are seen to cancel the last two terms of $(\partial\phi^b/\partial x^\sigma)R_{\mu\nu\lambda}^{\ \ \ \ \sigma}$ (see (3.4) above). As $(\partial\phi^b/\partial x^\sigma)R_{\mu\nu\lambda}^{\ \ \ \ \sigma}$ vanishes identically for arbitrary $\frac{\partial\phi^b}{\partial x^\sigma}$ we can conclude that the Riemann curvature $R_{\mu\nu\lambda}^{\ \ \ \ \sigma}$ vanishes identically when the metric has the form of Eqn. (3.5). This result is not surprising in light of our not having introduced any gravitational sources at the scales a ($a << l_{Pl}$) at which we postulate that the new fundamental fields are instrumental in preventing spacetime foam.

3.5 Using the Postulated Fundamental Fields ϕ^a to Build Flat Spacetime: a Model

Here we put up a model with a term in the Lagrange density $\mathcal{L}$ that depends on the gradient of the new fundamental fields $\phi^a, \phi^b, ...$

$$\mathcal{L} = \mathcal{L}(\cdots, \phi^a, \phi^b, \cdots \partial_\mu\phi^a, \partial_\nu\phi^b, \cdots) = \cdots + \left(t_{ab}(g_{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - p\eta^{ab})\right)^2 + \cdots . \tag{3.13}$$

Such a contribution has terms quartic and quadratic in the gradients that are respectively positive and negative and therefore result in a "Mexican hat potential" as a function of $\partial_\mu\phi^a$. The vacuum solution for such a potential is a constant non-vanishing value $|\partial_\mu\phi^a|_{min}$ of the gradient of the fundamental fields (see Fig. 3.1). Such a vacuum spontaneously breaks translational invariance of course.

Maintaining the constant vacuum value $|\partial_\mu\phi^a|_{min}$ for the gradient of ϕ^a in all of spacetime would lead to divergent values of the fields ϕ^a at large distances. Therefore we take the new fundamental fields to be the complex field Φ^a ($a = 0, 1, 2, 3$)

$$\Phi^a(x^0, x^1, x^2, x^3) \equiv \Phi^a(x) = \chi^a(x)e^{i(\partial_\mu\phi^a)x}. \tag{3.14}$$

For the moment we assume that the modulus $\chi^a(x)$ has the constant value χ_0. In the vacuum it is the gradient of the fundamental field $\phi^a(x)$ which has the value $|\partial_\mu\phi^a|_{min}$ in the vacuum, i.e.,

$$\Phi^a_{vac}(x) = \chi^a_0 e^{i(|\partial_\mu\phi^a|_{min})x} \tag{3.15}$$

We can also say that the condition for having the vacuum value $|\partial_\mu\phi^a|_{min}$ for the gradient is that planes corresponding to adjacent equal values of the complex field $\Phi^a(x)$ are separated in spacetime by a (constant) distance $2\pi/|\partial_\mu\phi^a|_{min}$. Fig. 3.2 shows the variation of the field component Φ^1 as a function x^1

So the requirement of being at the minimum of the potential in Fig. 3.1 (i.e., $\partial_\mu\phi^a = |\partial_\mu\phi^a|_{min}$) defines a (constant) density of planes each of which corresponds to the same value of Φ^1. Fig. 3.3 shows a section of such planes perpendicular to the x^1 axis.

There are similar planes for the other three spacetime axes. Together this system of planes define a lattice with a lattice constant equal to $2\pi/|\partial_\mu\phi^a|_{min}$

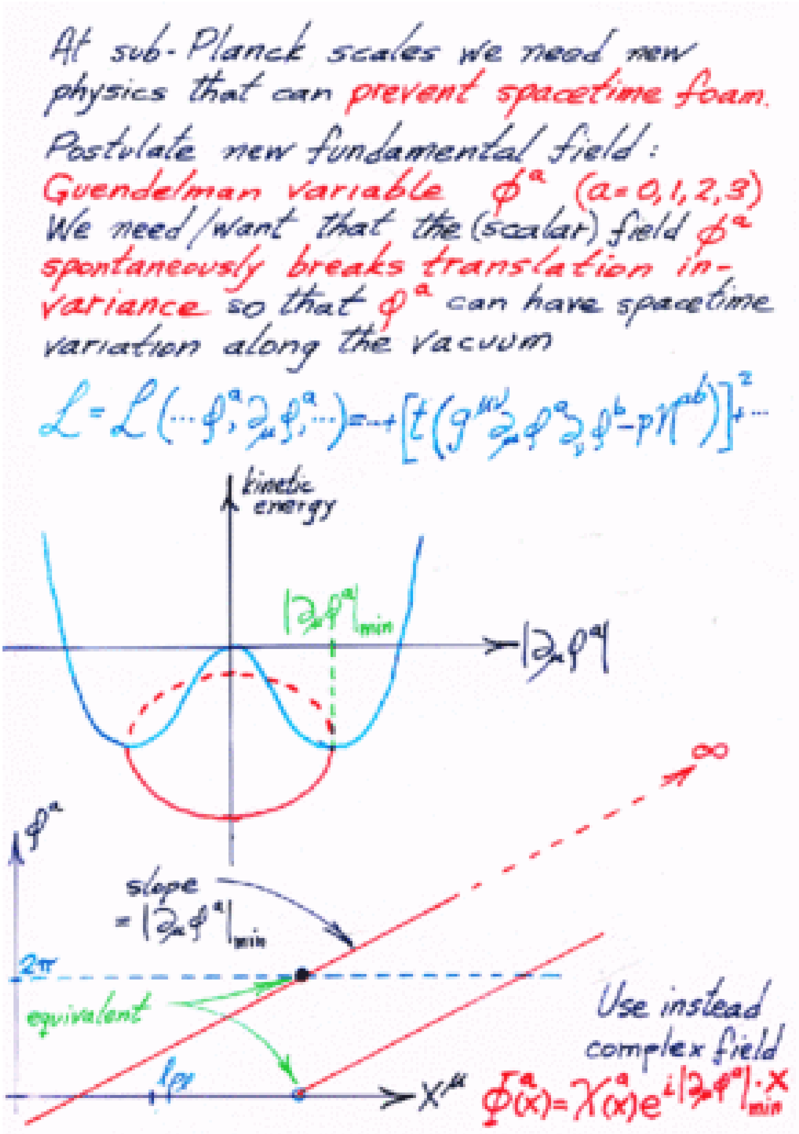

Fig. 3.1. At scales $a << l_{Pl}$ we postulate a new fundamental field Φ that explicitly breaks translational symmetry in the vacuum.

corresponding to the vacuum value for the gradients of the new fundamental fields.

So we have seen that an action containing positive quartic and negative quadratic terms in the gradient of the new proposed fundamental fields ϕ^a (see 3.13) favours the maintenance of a constant density of lattice points with lattice constant $2\pi/|\partial_\mu \phi^a|_{min}$. Any departure from this vacumm density of lattice points (or planes) costs energy because it corresponds to moving away from the minimum at $|\partial_\mu \phi^a| = |\partial_\mu \phi^a|_{min}$.

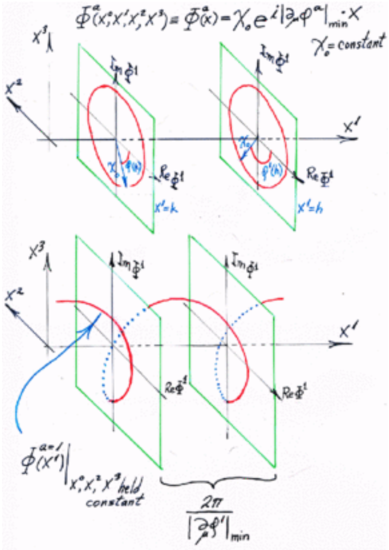

Fig. 3.2. The top part of the figure shows the value of the complex field $\Phi^1(x^1)$ at the arbitrary values $x^1 = k$ and $x^1 = h$. Really $\Phi^1(x^1)$ stands for $\Phi^1(x^0, x^1, x^2, x^2)|_{x^0, x^2, x^3}$held constant In the bottom part of the figure, adjacent identical values of $\Phi^1(x^1)$ define planes of constant x^1 separated by a distance $2\pi/|\partial_\mu \phi^a|_{min}$.

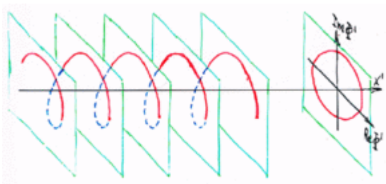

Fig. 3.3. Being in the ground state of the kinetic energy potential of Fig. 3.1 corresponds to the gradients of the fields having the value $|\partial_\mu \phi^a|_{min}$ $a = 0, 1, 2, 3$ which in turn corresponds to equidistant planes with spacing proportional to $2\pi/|\partial_\mu \phi^a|_{min}$. Here are shown several such planes for $a = 1$.

This is the property that we need: an action that fixes the density of lattice points in the sense explained above. In Fig. 3.4 we show a (locally 2-dimensional) appendix that opens off of an otherwise 2-dimensional (flat space) lattice with an almost everywhere fixed density of lattice points (or planes) separated by the distance $2\pi/|\partial_\mu \phi^a|_{min}$. By continuity the lattice planes that go into the appendix must emerge again and rejoin the flat spacetime lattice planes from which they originated.

The crucial point is that the appendix increases the "volume" of spacetime from that corresponding to area of mouth of the appendix to the larger area of the interior of the appendix. But by continuity the number of lattice planes entering and leaving the appendix is the same as the number of planes that enter and leave the area of the appendix mouth without the appendix. Hence the density of lattice

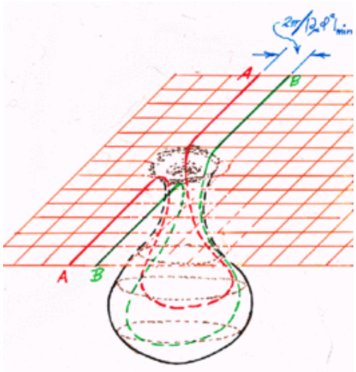

Fig. 3.4. Here we have an almost everywhere regular (i.e., flat spacetime) lattice (here represented by a 2-dimensional grid drawn in perspective). The density of lattice "planes" corresponds to the vacuum value $|\partial_\mu \phi^a|_{min}$ of the gradient of ϕ^a which equivalently means that the distance between planes of some chosen constant value of $\Phi^a(x)$ is $2\pi/|\partial_\mu \phi^a|_{min}$. However in the figure there is also an appendix (bubble) that represents a departure from flat spacetime. If we follow two lattice "planes" **A** and **B** in and out of the appendix we see that underway in the appendix the separation between these "planes" increases because by continuity the same number of lattice "planes" fill a larger volume of spacetime than would be the case without the appendix (i.e., which would be just the "volume" corresponding to the appendix mouth). Hence the density of lattice planes decreases in the appendix to a value $|\partial_\mu \phi^a| < |\partial_\mu \phi^a|_{min}$ which corresponds to an excitation relative to the vacuum state (see Fig. 3.1). Departures from flat spacetime costs energy.

planes (or points) decreases within the appendix relative to the density within the area of the mouth without the appendix.

So the presence of the appendix relative to not having it lowers the density of lattice points in the neighborhood of the appendix. Within the appendix the lattice constant becomes larger than $2\pi/|\partial_\mu \phi^a|_{min}$. This forces the system away from the minimum at $|\partial_\mu \phi^a|_{min}$ in the potential shown in Fig. 3.1. Having the appendix costs energy. Energetically flat spacetime is favoured.

Notice that with an action of the form 3.13 used in our the pivotal relation Eqn. (3.5) is recovered as an equation of motion upon taking a variation w.r.t. $g^{\mu\nu}$

3.6 The Emergence of Genearl Relativity as an Effective Theory at Planck Scale

When the new fundamental fields are introduced as the metric in Eqn. (3.5) we have flat spacetime down to arbitrarily small scales $a << l_{Pl}$. And a consequence we have seen that the Riemann curvature vanishes identically irrespective of what action is used.

Now the question is how do we regain general relativity when we go up to the Planck scale? Here we rely heavily on the work[5] of Hagen Kleinert. In the special case of the model considered above we have seen how the action defines a spacetime a lattice of constant density $|\partial_\mu \phi^a|_{min}$ consisting of planes corresponding to equal values of the the new fundamental complex field $\Phi^a(x)$.

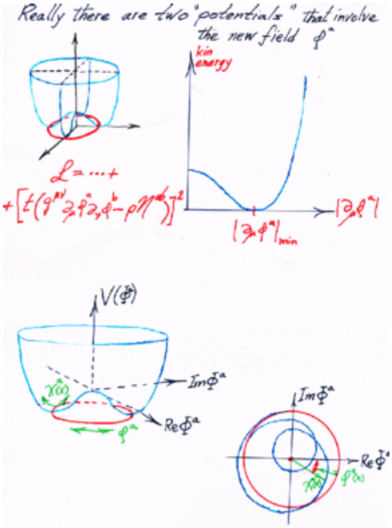

Fig. 3.5. The complex field has two degrees of freedom. In addition to the ϕ-fields already discussed there is also the $\chi(x)$. In the vacuum this degree of freedom is not excited and can be thought of as a soliton with constant topology.

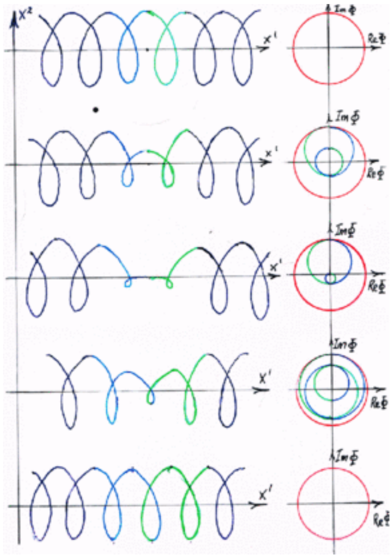

Fig. 3.6. If the $\chi(x)$ degree of freedom is sufficiently excited, the soliton can lose (or gain) a winding. Thinking of the lattice discussed above, changes in the winding number for a soliton can be thought of as the introduction of a crystal defect (dislocation line). It is known (see references to Hagen Klienert) that Einsteinian general relativity can be formulated as a "world crystal" that has dislocation and disclination line defects that give rise to respectively torsion and curvature. This presents a way that the usual general relativity can emerge as an effective theory at say the Planck scale. Recall that at scales $a << l_{Pl}$ where our new fundamental fields are important spacetime is identically flat. So phenomenologically we need a mechanism by which general relativity appears at roughly Planck scale.

Now we think of this lattice as the "world crystal" of Kleinert. Curvature (and torsion if desired) can be introduced respectively as line dislocation and line disclination defects. Fig. 3.6 suggests in a soliton model how a dislocation defect can come about by the loss of a soliton winding. Kleinert demonstrates that the introduction of disclination defects in a regular world crystal by the use multivalued coordinate transformations reproduces general relativity in full. We envision this happening at roughly the Planck scale.

References

1. C. Misner, K. S. Thorne, and J. A. Wheeler,"Gravitation", San Francisco: W. H. Freeman (1973), ISBN 0716703440. See chapter 43 for superspace and chapter 44 for spacetime foam; C. Misner and J. A. Wheeler, "Classical physics as geometry", Ann. Phys. 2 (6): 525 (1957), Bibcode 1957 AnPhy...2..525M. doi:10.1016/0003-4916(57)90049-0, online version (subscription required)
2. Wheeler, John Archibald (1963), Geometrodynamics. New York: Academic Press, LCCN 62-013645; J. Wheeler (1960) "Curved empty space as the building material of the physical world: an assessment", in Ernest Nagel (1962) Logic, Methodology, and Philosophy of Science, Stanford University Press; J. Wheeler (1961) "Geometrodynamics and the Problem of Motion", Rev. Mod. Physics 44 (1): 63, Bibcode 1961RvMP...33...63W. doi:10.1103/RevModPhys.33.63. online version (subscription required); J. Wheeler (1957), "On the nature of quantum geometrodynamics", Ann. Phys. 2 (6): 604.
3. M. Lehto, H. B. Nielsen and M. Ninomiya, "Time Translational Symmetry," Phys. Lett. B **219** (1989) 87; "Semilocality Of One-dimensional Simplicial Quantum Gravity," Nucl. Phys. B **289** (1987) 684; "Diffeomorphism Symmetry In Simplicial Quantum Gravity," Nucl. Phys. B **272** (1986) 228; "Pregeometric Quantum Lattice: A General Discussion," Nucl. Phys. B **272** (1986) 213; "A Correlation Decay Theorem At High Temperature," Commun. Math. Phys. **93** (1984) 483.
4. E. I. Guendelman, Non Singular Origin of the Universe and its Present Vacuum Energy Density, arXiv: 1103.1427v2 [gr-qc] 15 May 2011 and references therein.
5. Hagen Kleinert, *Multivalued Fields in Condensed Matter, Electromagnetism and Gravitation*, World Science Publishing Co. Pte. Ltd. (2008) and references therin.

Bled Workshops in Physics
Vol. 12, No. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 27)*
Bled, Slovenia, July 11-21, 2011

4 Creating the Universe Without a Singularity and the Cosmological Constant Problem

E. I. Guendelman*

Physics Department, Ben Gurion University of the Negev
Beer Sheva 84105, Israel

Abstract. We consider a non singular origin for the Universe starting from an Einstein static Universe in the framework of a theory which uses two volume elements $\sqrt{-g}d^4x$ and Φd^4x, where Φ is a metric independent density, also curvature, curvature square terms, first order formalism and for scale invariance a dilaton field ϕ are considered in the action. In the Einstein frame we also add a cosmological term that parametrizes the zero point fluctuations. The resulting effective potential for the dilaton contains two flat regions, for $\phi \to \infty$ relevant for the non singular origin of the Universe and $\phi \to -\infty$, describing our present Universe. Surprisingly, avoidance of singularities and stability as $\phi \to \infty$ imply a positive but small vacuum energy as $\phi \to -\infty$. Zero vacuum energy density for the present universe is the "threshold" for universe creation. This requires a modified emergent universe scenario, where the universe although very old, it does have a beginning.

The "Cosmological Constant Problem" [1], [2],[3] (CCP), is a consequence of the uncontrolled UV behavior of the zero point fluctuations in Quantum Field Theory (QFT), which leads to an equally uncontrolled vacuum energy density or cosmological constant term (CCT). This CCT is undetermined in QFT, but it is naturally very large, unless a delicate balance of huge quantities, for some unknown reason, conspires to give a very small final result. Also an apparently unrelated question is that of the initial condition for the inflationary universe is very important, it has been addressed for example by assuming a quantum boson condensate in the early universe[4].

Here, we will explore a connection between the question of initial conditions for the Universe with the CCP, we will explore a candidate mechanism where the CCT is controlled, in a the context of a very specific framework, by the requirement of a non singular origin for the universe.

We will adopt the very attractive "Emergent Universe" scenario, where conclusions concerning singularity theorems can be avoided [5], [6], [7], [8], [9], [10], [11], [12] by violating the geometrical assumptions of these theorems. In this scenario [5],[6] we start at very early times ($t \to -\infty$) with a closed static Universe (Einstein Universe).

In [5] even models based on standard General Relativity, ordinary matter and minimally coupled scalar fields were considered and can provide indeed a non

* e-mail: guendel@bgu.ac.il

singular (geodesically complete) inflationary universe, with a past eternal Einstein static Universe that eventually evolves into an inflationary Universe.

Those most simple models suffer however from instabilities, associated with the instability of the Einstein static universe. The instability is possible to cure by going away from GR, considering non perturbative corrections to the Einstein's field equations in the context of the loop quantum gravity[7], a brane world cosmology [8], considering the Starobinski model for radiative corrections (which cannot be derived from an effective action)[9] or exotic matter[10]. In addition to this, the consideration of a Jordan Brans Dicke model also can provide a stable initial state for the emerging universe scenario [11], [12].

In this essay we discuss a different theoretical framework, presented in details in ref.[13] , where such emerging universe scenario is realized in a natural way, where instabilities are avoided and a successful inflationary phase with a graceful exit can be achieved . The model we will use was studied first in [14] (in ref.[13] a few typos in [14] have been corrected and also the discussion of some notions discussed there as well has been improved), however, we differ with [14] in our choice of the state with (here and in ref.[13] with a lower vacuum energy density) that best represents the present state of the universe. This is crucial, since as it should be obvious, the discussion of the CCP depends crucially on what vacuum we take. We will express the stability and existence conditions for the non singular initial universe in terms of the energy of the vacuum of our candidate for the present Universe.

We work in the context of a theory built along the lines of the two measures theory (TMT) [15], [16], [17], [18] which deals with actions of the form,

$$S = \int L_1 \sqrt{-g} d^4x + \int L_2 \Phi d^4x \tag{4.1}$$

where Φ is an alternative "measure of integration", a density independent of the metric, for example in terms of four scalars φ_a ($a = 1, 2, 3, 4$),it can be obtained as follows:

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \tag{4.2}$$

and more specifically work in the context of the globally scale invariant realization of such theories [16], [17], which require the introduction of a dilaton field ϕ. In the variational principle $\Gamma^\lambda_{\mu\nu}$, $g_{\mu\nu}$, the measure fields scalars φ_a and the "matter" - scalar field ϕ are all to be treated as independent variables although the variational principle may result in equations that allow us to solve some of these variables in terms of others, that is, the first order formalism is employed, where any relation between the connection coefficients and the metric is obtained from the variational principle, not postulated a priori. We look at the generalization of these models [17] where an "R^2 term" is present,

$$L_1 = U(\phi) + \epsilon R(\Gamma, g)^2 \tag{4.3}$$

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \tag{4.4}$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), R_{\mu\nu}(\Gamma) = R^{\lambda}_{\mu\nu\lambda} \quad (4.5)$$

$$R^{\lambda}_{\mu\nu\sigma}(\Gamma) = \Gamma^{\lambda}_{\mu\nu,\sigma} - \Gamma^{\lambda}_{\mu\sigma,\nu} + \Gamma^{\lambda}_{\alpha\sigma}\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\lambda}_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma}. \quad (4.6)$$

For the case the potential terms $U = V = 0$ we have local conformal invariance

$$g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu} \quad (4.7)$$

and φ_a is transformed according to

$$\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b) \quad (4.8)$$

$$\Phi \rightarrow \Phi' = J(x)\Phi \quad (4.9)$$

where $J(x)$ is the Jacobian of the transformation of the φ_a fields.

This will be a symmetry in the case $U = V = 0$ if

$$\Omega = J \quad (4.10)$$

Notice that J can be a local function of space time, this can be arranged by performing for the φ_a fields one of the (infinite) possible diffeomorphims in the internal φ_a space.

In the case we have potentials non zero U and V, we give up local conformal invariance, but can retain global scale invariance which is satisfied if [17], [16](f_1, f_2, α being constants),

$$V(\phi) = f_1 e^{\alpha\phi}, U(\phi) = f_2 e^{2\alpha\phi} \quad (4.11)$$

Notice that in this way we have chosen all the conformal breaking to be through the potential, the kinetic terms do not break conformal invariance. In this sense the breaking of conformal invariance is what is usually called a soft breaking. Consideration of cosmological models (in particular emergent models) with "non soft" breaking of conformal invariance has been considered also [19]. A particularly interesting equation is the one that arises from the φ_a fields, this yields $L_2 = M$, where M is a constant that spontaneously breaks scale invariance. The Einstein frame, which is a redefinition of the metric by a conformal factor, is defined as

$$\overline{g}_{\mu\nu} = (\chi - 2\kappa\epsilon R) g_{\mu\nu} \quad (4.12)$$

where χ is the ratio between the two measures, $\chi = \frac{\Phi}{\sqrt{-g}}$, determined from the consistency of the equations to be $\chi = \frac{2U(\phi)}{M+V(\phi)}$. The relevant fact is that the connection coefficient equals the Christoffel symbol of this new metric (for the original metric this "Riemannian" relation does not hold). There is a "k-essence" type effective action, where one can use this Einstein frame metric. As it is standard in treatments of theories with non linear kinetic terms or k-essence models[20]-[23], it is determined by a pressure functional, ($X = \frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$).

$$S_{eff} = \int \sqrt{-\overline{g}} d^4x \left[-\frac{1}{\kappa}\overline{R}(\overline{g}) + p\,(\phi, R) \right] \quad (4.13)$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R} X - V_{eff} \tag{4.14}$$

where V_{eff} is an effective potential for the dilaton field given by

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \tag{4.15}$$

$\overline{R}$ is the Riemannian curvature scalar built out of the bar metric, R on the other hand is the non Riemannian curvature scalar defined in terms of the connection and the original metric,which turns out to be given by $R = \frac{-\kappa(V+M)+\frac{\kappa}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1+\kappa^2\epsilon\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$. This R can be inserted in the action (4.13) or alternatively, R in the action (4.13) can be treated as an independent degree of freedom, then its variation gives the required value as one can check (which can then be reinserted in (4.13)). Introducing this R into the expression (4.15) and considering a constant field ϕ we find that V_{eff} has two flat regions. The existence of two flat regions for the potential is shown to be consequence of the s.s.b. of the scale symmetry (that is of considering $M \neq 0$). The quantization of the model can proceed from (4.13) (see discussion in [13]) and additional terms could be generated by radiative corrections. We will focus only on a possible cosmological term in the Einstein frame added (due to zero point fluctuations) to (4.13), which leads then to the new action

$$S_{eff,\Lambda} = \int \sqrt{-\overline{g}} d^4x \left[-\frac{1}{\kappa}\overline{R}(\overline{g}) + p\,(\phi, R) - \Lambda\right] \tag{4.16}$$

This addition to the effective action leaves the equations of motion of the scalar field unaffected, but the gravitational equations acquire a cosmological constant. Adding the Λ term can be regarded as a redefinition of $V_{eff}\,(\phi, R)$

$$V_{eff}\,(\phi, R) \to V_{eff}\,(\phi, R) + \Lambda \tag{4.17}$$

In this resulting model, there are two possible types of emerging universe solutions, for one of those, the initial Einstein Universe (realized in the region $\phi \to \infty$) can be stabilized due to the nonlinearities of the model, if $\epsilon < 0$, $f_2 > 0$ and $f_2 + \kappa^2\epsilon f_1^2 > 0$ provided the vacuum energy density of the ground state, realized in the region $\phi \to -\infty$, being given by $V_{eff} \to \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda$ is positive, but not very large, since it should be bounded from above by the inequality $\Delta\lambda < \frac{1}{12(-\epsilon)\kappa^2}\left[\frac{f_2}{f_2+\kappa^2\epsilon f_1^2}\right]$. These are very satisfactory results, since it means that the existence and stability of the emerging universe prevents the vacuum energy in the present universe from being very large, but requires it to be positive. The transition from the emergent universe to the ground state goes through an intermediate inflationary phase, therefore reproducing the basic standard cosmological model as well. So, it turns out that the creation of the universe can be considered as a "threshold event" for zero present vacuum energy density, which naturally gives a positive but small vacuum energy density for the present universe.

One may ask the question: how is it possible to discuss the "creation of the universe" in the context of the "emergent universe"?. After all, the Emergent

Universe basic philosophy is that the universe had a past of infinite duration. However, that most simple notion of an emergent universe with a past of infinite duration has been recently challenged by Mithani and Vilenkin [24], at least in the context of a special model. They have shown that a completely stable emergent universe, although completely stable classically, could be unstable under a tunneling process to collapse. On the other hand, an emergent universe can indeed be created by a tunneling process as well.

After creation from "nothing" an emerging universe could last for a long time, provided it is classically stable, that is where the constraints on the cosmological constant for the late universe discussed here come in. If it is not stable, the emergent universe will not provide us with an appropriate "intermediate state" connecting the creation of the universe with the present universe. The existence of this stable intermediate state provides in our picture the reason for the universe to prefer a very small vacuum energy density at late times, since universes that are created, but do not make use of the intermediate classically stable emergent universe will almost immediately recollapse, so they will not be "selected".

The situation is somewhat similar to the reason Carbon is formed at a reasonable rate in stars[25]. There, it is also an appropriate resonance that makes the creation of carbon possible. The analogous role of that resonance, when refering to the creation of the whole universe are the stable emergent universe solution in the picture we are considering and instead of carbon, the "product" we are trying to explain is a small cosmological constant.

Acknowledgements

It is a pleasure for me to thank Norma Mankoč, Holger Nielsen, Maxim Yu. Khlopov and all the organizers and participants of the Bled workshop "On What Comes Beyond The Standard Models?" for a very productive and interesting conference. I would also like to thank Alexander Vilenkin for correspondence concerning his work on instability and possibility of quantum creation of an emergent universe.

References

1. S. Weinberg, Rev. Mod. Phys.**61** (1989) 1.
2. Y. Jack Ng, Int. J. Mod. Phys.**1** (1992) 145.
3. S. Weinberg, astro-ph/0005265.
4. I. Dymnikova, M. Khlopov, Grav.Cosmol.Suppl. **4** (1998).
5. G.F.R. Ellis and R. Maartens, Class. Quantum Grav.**21** (2004) 223.
6. G.F.R. Ellis, J. Murugan and C.G. Tsagas, Class. Quantum Grav. **21** (2004) 233.
7. D.J. Mulryne, R. Tavakol, J.E. Lidsey and G.F.R. Ellis, Phys. Rev.**D71** (2005) 123512.
8. A. Banerjee, T. Bandyopadhyay and S. Chakraborty, Grav.Cosmol.**13** (2007) 290.
9. S. Mukherjee, B.C.Paul, S.D. Maharaj and A. Beesham, arXiv:qr-qc/0505103.
10. S. Mukherjee, B.C.Paul, N.K. Dadhich, S.D. Maharaj and A. Beesham, Class. Quantum Grav.**23** (2006) 6927.
11. S. del Campo, R. Herrera and P. Labrana, JCAP **0907** (2009) 006.

12. S. del Campo, R. Herrera and P. Labrana, JCAP **0711** (2007) 030.
13. E.I. Guendelman, "Non Singular Origin of the Universe and its Present Vacuum Energy Density", e-Print: arXiv:1103.1427 [gr-qc], Int. J. Mod. Phys.**26** (2011) 2951.
14. S. del Campo, E.I. Guendelman, R. Herrera and P. Labrana, JCAP **1006** (2010) 026.
15. Basic idea is developed in E.I. Guendelman and A.B. Kaganovich, Phys. Rev.**D60** (1999) 065004.
16. For a recent review and further references see E.I. Guendelman, A.B. Kaganovich, Plenary talk given at the Workshop on Geometry, Topology, QFT and Cosmology, Paris, France, 28–30 May 2008. e-Print: arXiv:0811.0793 [gr-qc].
17. E.I. Guendelman, Mod. Phys. Lett.**14** (1999) 1043, e-Print: gr-qc/9901017.
18. E.I. Guendelman and O. Katz, Class. Quantum Grav.**20** (2003) 1715, e-Print: gr-qc/0211095.
19. S. del Campo, E. I. Guendelman, A.B. Kaganovich, R. Herrera, P. Labrana, Phys. Lett.**699B** (2011) 211, e-Print: arXiv:1105.0651 [astro-ph.CO].
20. T. Chiba, T.Okabe and M Yamaguchi, Phys.Rev.**D62** (2000) 023511.
21. C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, Phys.Rev.Lett. **85** (2000) 4438.
22. C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt,Phys.Rev.D**63** (2001) 103510.
23. T. Chiba, Phys.Rev.**D66** (2002) 063514.
24. A. T. Mithani, A. Vilenkin, e-Print: arXiv:1110.4096 [hep-th].
25. F. Hoyle, et al., Phys. Rev. Lett.**92** (1953) 1096.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 33)*
Bled, Slovenia, July 11-21, 2011

5 Using Two Measures Theory to Approach Bags and Confinement

E. I. Guendelman*

Physics Department, Ben Gurion University of the Negev
Beer Sheva 84105, Israel

Abstract. We consider the question of bags and confinement in the framework of a theory which uses two volume elements $\sqrt{-g}d^4x$ and Φd^4x, where Φ is a metric independent density. For scale invariance a dilaton field ϕ is considered. Using the first order formalism, curvature (ΦR and $\sqrt{-g}R^2$) terms , gauge field term($\Phi\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}}$ and $\sqrt{-g}F^a_{\mu\nu}F^a_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}$) and dilaton kinetic terms are introduced in a conformally invariant way. Exponential potentials for the dilaton break down (softly) the conformal invariance down to global scale invariance, which also suffers s.s.b. after integrating the equations of motion. The model has a well defined flat space limit. As a result of the s.s.b. of scale invariance phases with different vacuum energy density appear. Inside the bags the gauge dynamics is normal, that is non confining, while for the outside, the gauge field dynamics is confining.

5.1 Introduction and Conclusions

In the bag model of confinement [1] two phases for gauge fields are identified, first the free non confining dynamics for the gauge fields that holds inside the bags, there gauge fields are prevented to flow to the outside (confinement) region by the M.I.T. bag model boundary conditions. On the other hand, in modern cosmology working with different phases is a central theme and also in the context of modern cosmology, as well as for the bag model, we need two phases. In cosmology the two phases they should be connected through cosmological evolution, while in the bag model through the boundary of the bag.

As it is well known, in the context of cosmology, it is very difficult to understand the smallness of the observed present vacuum energy density. This "cosmological constant problem", has been reformulated in the context of the two measures theory (TMT) [2] - [6] and more specifically in the context of the scale invariant realization of such theories [3] - [6]. These theories can provide a new approach to the cosmological constant problem and can be generalized to obtain also a theory with a dynamical space-time [7] . The TMT models consider two measures of integration in the action, the standard $\sqrt{-g}$ where g is the determinant of the metric and another measure Φ independent of the metric. To implement scale invariance (S.I.), a dilaton field is introduced [3] - [6].

* e-mail: guendel@bgu.ac.il

In the TMT theories we obtain drastic modifications of the dynamics of vacuum energy density, which produces naturally a zero cosmological constant and together with this regions of very small vacuum energy density. These ideas work particularly well in the context of scale invariance which can be spontaneously broken by the integration of the equations of motion. What is most important for the present research is that it is the nature of the two measures theories to change not only the dynamics of the vacuum energy density, but also that of the matter itself. For example, in the context of the spontaneously broken scale invariant theories, the dilaton field decouples from the fermionic matter at high densities, avoiding the fifth force problem, see [8], [9]. On the opposite limit, fermionic matter was shown to contribute to the dark energy density for very small densities [10].

In this paper our focus will be on the interplay between gauge field dynamics, in particular confinement properties and the different phases as defined with the help of TMT and whether the possibility of obtaining a confinement phase and a deconfined phase (like in the MIT bag model) can addressed in this context. Using the first order formalism, curvature (ΦR and $\sqrt{-g}R^2$) terms , gauge field terms and dilaton kinetic terms can be introduced in a conformally invariant way. Exponential potentials for the dilaton break down (softly) the conformal invariance down to global scale invariance, which also suffers s.s.b. after integrating the equations of motion. As a result of the s.s.b. of scale invariance phases with different vacuum energy density appear. In this contribution we will review the principles of the TMT and in particular the model studied in [3], which has global scale invariance. Then, we look at the generalization of this model [6] by adding a curvature square or simply "R^2 term" and show that the resulting model contains now two flat regions. The existence of two flat regions for the potential is shown to be consequence of the s.s.b. of the scale symmetry. The model is then further extended to include gauge fields. A gauge field strength squared term coupled to $\sqrt{-g}$, a square root of a gauge field strength squared term coupled to Φ and a mass term for the gauge fields coupled to Φ are the unique candidates which respect local conformal invariance and they can provide a consistent framework to answer the questions posed. For the issue of electric confinement we disregard the mass term and consider only the gauge field strength squared term coupled to $\sqrt{-g}$ and the square root of a gauge field strength squared term coupled to Φ. This square root term has been studied before in order to reproduce confinement behavior [11],[12]-[17]. In the context of the softly broken conformally invariant TMT model it appears however in a particularly natural way. After s.s.b. of scale invariance, the amazing feature that the square root gauge field term is totally screened in the high vacuum energy regions (inside the bags) and acts only outside the bags, reproducing basic qualitative behavior postulated in the M.I.T bag model, also some difficulties present in the original formulation of the square root gauge fields approach to confinement are resolved when the square root term is embedded in the TMT model presented here. . . .

5.2 The Two Measures Theory Fundamentals

We work in the context of a theory built along the lines of the two measures theory (TMT) [2], [3], [4], which deals with actions of the form,

$$S = \int L_1 \sqrt{-g} d^4x + \int L_2 \Phi d^4x \tag{5.1}$$

where Φ is an alternative "measure of integration", a density independent of the metric, for example in terms of four scalars φ_a $(a = 1, 2, 3, 4)$,it can be obtained as follows:

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \tag{5.2}$$

and more specifically work in the context of the globally scale invariant realization of such theories [3], [4], which require the introduction of a dilaton field ϕ. We look at the generalization of these models [4] where an "R^2 term" is present,

$$L_1 = U(\phi) + \epsilon R(\Gamma, g)^2 \tag{5.3}$$

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \tag{5.4}$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda} \tag{5.5}$$

$$R^\lambda_{\mu\nu\sigma}(\Gamma) = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma}\Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\alpha_{\mu\sigma}. \tag{5.6}$$

For the case the potential terms $U = V = 0$ we have local conformal invariance

$$g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu} \tag{5.7}$$

and φ_a is transformed according to

$$\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b) \tag{5.8}$$

$$\Phi \rightarrow \Phi' = J(x)\Phi \tag{5.9}$$

where $J(x)$ is the Jacobian of the transformation of the φ_a fields. This will be a symmetry in the case $U = V = 0$ if

$$\Omega = J \tag{5.10}$$

global scale invariance is satisfied if [4], [3](f_1, f_2, α being constants),

$$V(\phi) = f_1 e^{\alpha\phi}, U(\phi) = f_2 e^{2\alpha\phi} \tag{5.11}$$

In the variational principle $\Gamma^\lambda_{\mu\nu}$, $g_{\mu\nu}$, the measure fields scalars φ_a and the "matter" - scalar field ϕ are all to be treated as independent variables although the variational principle may result in equations that allow us to solve some of these variables in terms of others, that is, the first order formalism is employed, where any relation between the connection coefficients and the metric is obtained from the variational principle, not postulated a priori. A particularly interesting equation is the one that arises from the φ_a fields, this yields $L_2 = M$, where M is a constant that spontaneously breaks scale invariance. The Einstein frame, which is a redefinition of the metric by a conformal factor, is defined as

$$\overline{g}_{\mu\nu} = (\chi - 2\kappa\epsilon R) g_{\mu\nu} \tag{5.12}$$

where χ is the ratio between the two measures, $\chi = \frac{\Phi}{\sqrt{-g}}$, is determined from the consistency of the equations to be $\chi = \frac{2U(\phi)}{M+V(\phi)}$. The relevant fact is that the connection coefficient equals the Christoffel symbol of this new metric (for the original metric this "Riemannian" relation does not hold). There is an effective action, where one can use this Einstein frame metric, it is determined by the pressure functional, ($X = \frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$).

$$S_{eff} = \int \sqrt{-\overline{g}} d^4x \left[-\frac{1}{\kappa}\overline{R}(\overline{g}) + p\,(\phi, R) \right] \tag{5.13}$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R} X - V_{eff} \tag{5.14}$$

where V_{eff} is an effective potential for the dilaton field given by

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \tag{5.15}$$

$\overline{R}$ is the Riemannian curvature scalar built out of the bar metric, R on the other hand is the non Riemannian curvature scalar defined in terms of the connection and the original metric,which turns out to be given by $R = \frac{-\kappa(V+M) + \frac{\kappa}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1+\kappa^2\epsilon\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$. This R can be inserted in the action (5.13) or alternatively, R in the action (5.13) can be treated as an independent degree of freedom, then its variation gives the required value as one can check (which can then be reinserted in (5.13)). Introducing this R into the expression (5.15) and considering a constant field ϕ we find that V_{eff} has two flat regions. The existence of two flat regions for the potential is shown to be consequence of the s.s.b. of the scale symmetry (that is of considering $M \neq 0$).

5.3 Gauge Field Kinetic Terms, Mass Terms and "Confinement Terms" in the Softly Broken Conformally Invariant TMT Model

Now we will see that the incorporation of a term of the form $\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}}$, which in flat space is known to introduce confinement behavior, is in the TMT case quite natural, in fact, there is a good reason to include it, since it respects conformal symmetry if coupled to the new measure Φ . This kind of coupling of a square root gauge field strength to a new measure has been considered in the context of conformally invariant braneworld scenarios[18]-[21], which allow compactification, branes and zero four dimensional cosmological constant. Another place where square root of gauge field square coupled to a modified measure find a natural place is in the formulation of Weyl invariant brane theories[22]-[25]. Black hole solutions in the presence of both a regular Maxwell term and a square root of gauge field square have been also considered [26]. An early model which enriches the ”square root” gauge theory with a dilaton field so that it could describe confined and unconfined regions (bags) was done ”by hand” in [27]. This will be obtained most elegantly however by embedding the square root terms into the TMT formalism.

The reason for the conformal invariance of the $\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}}$ is very simple: conformally invariant terms (with respect to (5.7) , (5.8), (5.9) and (5.10))in TMT are of two kinds, if they multiply the measure Φ they they must have homogeneity 1 with respect to $g^{\mu\nu}$, or if they multiply the measure $\sqrt{-g}$ they they must have homogeneity 2 with respect to $g^{\mu\nu}$, since $\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}} = \sqrt{-F^a_{\mu\nu} F^a_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta}}$, then according to (5.7) $\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}} \to \Omega^{-1} \sqrt{-F^a_{\mu\nu} F^{a\mu\nu}}$ if $\Omega > 0$ and $\Phi \to J\Phi = \Omega\Phi$, so that $\Phi\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}}$ is invariant, provided $J = \Omega > 0$.

A similar story happens with a mass term for the gluon, $A^a_\mu A^a_\alpha g^{\mu\alpha}$ in TMT, this can be a conformally invariant if it goes multiplied with the measure Φ.

Likewise, the conformally invariance implies that a term proportional to $F^a_{\mu\nu} F^a_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta}$ has come multiplied by the Riemannian measure $\sqrt{-g}$, since $\sqrt{-g} F^a_{\mu\nu} F^a_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta}$ is invariant under conformal transformations of the metric. We take therefore for our softly broken conformal invariant model, where we exclude mass terms for the gluons,

$$S = S_L + S_{R^2} - \frac{1}{4} \int d^4x \sqrt{-g} F^a_{\mu\nu} F^a_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta} + \frac{N}{2} \int d^4x \Phi \sqrt{-F^a_{\mu\nu} F^a_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta}} \quad (5.16)$$

here S_L contains the terms linear in the curvature scalar, plus scalar field kinetic terms and potentials and S_{R^2} refers to the R^2 contribution defined before. The consequences of having both a mass term and a confinement term have been explored in [14] where it was shown that in such a case confinement is lost in favor of a Coulomb like behavior, but, as mentioned before, for the purposes of this paper this will not be considered here.

5.4 Description of the Bag Dynamics in the Softly Broken Conformally Invariant TMT Model

Let us proceed now to describe the consequences of the action (5.16). The steps to follow are the same as in the case where we did not have gauge fields.

One interesting fact is that the terms that enter the constraint that determines χ are only those that break the conformal invariance and the constant of integration M. Since the new terms involving the gauge fields do not break the conformal invariance (5.7), (5.8), (5.9), (5.10), the relevant terms that violate this symmetry are only the U and V terms and the constraint remains the same. We can then continue and construct all the equations of motion as before.

The easiest way to summarize the result of such analysis is to consider the effective action in the Einstein frame, as we did in the previous case where we did not have gauge fields. Now, for the case where gauge fields are included in the way described by (5.16), all the equations of motion in the Einstein frame will be correctly described by

$$S_{eff} = \int \sqrt{-\bar{g}} d^4x \left[-\frac{1}{\kappa} \bar{R}(\bar{g}) + p\left(\phi, R, X, F^a_{\mu\nu}, \bar{g}^{\alpha\beta}\right) \right] \tag{5.17}$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R} \left[X + \frac{N}{2} \sqrt{-F^a_{\mu\nu} F^a_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta}} \right] - \frac{1}{4} F^a_{\mu\nu} F^a_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} - V_{eff} \tag{5.18}$$

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \tag{5.19}$$

where

$$\chi = \frac{2U(\phi)}{M + V(\phi)} \tag{5.20}$$

We have again two possible formulations concerning R: Notice first that $\bar{R}$ and R are different objects, the $\bar{R}$ is the Riemannian curvature scalar in the Einstein frame, while R is a different object. This R will be treated in two different ways:

1. First order formalism for R. Here R is a lagrangian variable, determined as follows, R that appear in the expression above for p can be obtained from the variation of the pressure functional action above with respect to R, this gives exactly the expression for R that can be solved for by using the equations of motion in the original frame (and then reexpresing the result in terms of the bar metric), in terms of X, ϕ, etc.

2. Second order formalism for R. R that appear in the action above is exactly the expression for R which can be solved from the equations of motion in terms of X, ϕ, etc. Once again, the second order formalism can be obtained from the first order formalism by solving algebraically R from the eq. obtained by variation of R , and inserting back into the action. Now R is given by

$$R = \frac{-\kappa(V + M) + \kappa\chi\left(X + \frac{N}{2}\sqrt{-F^a_{\mu\nu} F^a_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta}}\right)}{1 + 2\kappa^2\epsilon\left(X + \frac{N}{2}\sqrt{-F^a_{\mu\nu} F^a_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta}}\right)} \tag{5.21}$$

5.5 Regular gauge field dynamics inside the bags

From (5.17), (5.18) and (5.20), we see that the N term, responsible for the confining gauge dynamics, gets dressed in the Einstein frame effective action by the factor $\frac{\chi}{\chi - 2\kappa\epsilon R}$, we will have to check also whether V_{eff} contributes to the gauge field equations of motion.

As we consider regions inside the bags, where $\phi \to -\infty$, we see that χ as given by (5.20), approaches zero for $M \neq 0$, for the case therefore $\epsilon \neq 0$ the N term inside the bags disappears. Notice that if we had not introduced the curvature squared term (i.e. if $\epsilon = 0$) this effect would be absent.

In this same limit and with the same conditions, using only that as $\phi \to -\infty$, $U \to 0$ and $\chi \to 0$, we see that still, in the more complicated theory with gauge fields the same bag constant $V_{eff} \rightarrow \frac{1}{4\epsilon\kappa^2}$ is obtained, so V_{eff} does not contribute to the gauge field equations of motion, but does provide the Bag constant.

In the limit $\phi \rightarrow -\infty$, the only term providing gauge field dynamics is the standard term $-\frac{1}{4}F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}$.

5.6 Confining gauge field effective action outside the bags

We are going to assume $M > 0$, so to keep χ positive and finite everywhere and take now the opposite limit, $\phi \rightarrow +\infty$. Furthermore, the choice $M > 0$ pushes the scalar field outside the bag to large values of ϕ, since the absolute minimum of the effective potential is found for such values, then confining dynamics appears,

$$V_{eff} \rightarrow C + 4B\left[X + \frac{N}{2}\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}}\right]^2 \tag{5.22}$$

and

$$\frac{\chi}{\chi - 2\kappa\epsilon R}\left[X + \frac{N}{2}\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}}\right] \rightarrow$$

$$A\left[1 + 2\kappa^2\epsilon\left[X + \frac{N}{2}\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}}\right]\right]\left[X + \frac{N}{2}\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}}\right] \tag{5.23}$$

where the constants A, B and C are given by, $A = \frac{f_2}{f_2 + \kappa^2\epsilon f_1^2}$, $B = \frac{\epsilon\kappa^2}{4}A$ and $C = \frac{f_1^2}{4f_2}A$. Therefore, the resulting dynamics outside the bag, for $\phi \rightarrow +\infty$ will be described by the effective action (expressing B in terms of A),

$$S_{eff,out} = \int\sqrt{-\bar{g}}d^4x\left[-\frac{1}{\kappa}\bar{R}(\bar{g}) + p_{out}(\phi, X, F)\right] \tag{5.24}$$

$$p_{out}(\phi, X, F) = AX + A\frac{N}{2}\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}} - (1 + N^2\epsilon\kappa^2 A)\frac{1}{4}F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}$$
$$+ AN\epsilon\kappa^2 X\sqrt{-F^a_{\mu\nu}F^a_{\alpha\beta}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}} + A\epsilon\kappa^2X^2 - C \tag{5.25}$$

Full details concerning these developments have been presented elsewhere [28]. Working in the case where gravitation plays an important role, one could also think of using the approach developed here to generalize the "hiding" [29] and "hiding and confining effects"[30], where the confining region is an uncompactified space-time and where charges send the gauge field flux they generate completely into a "flux tube-like" compactified region.

Acknowledgements

It is a pleasure for me to thank Norma Mankoč, Holger Nielsen and all the organizers of the Bled workshop "On What Comes Beyond The Standard Models?" for a very productive and interesting conference.

References

1. A. Chinos, R.L. Jaffe, K. Johnson, C.B. Thorn and V.F. Weisskopf, Phys. Rev. **D9** (1974) 3471.
2. Basic idea is reviewed in E.I. Guendelman and A.B. Kaganovich, Phys. Rev. **D60** (1999) 065004.
3. E.I. Guendelman, Mod. Phys. Lett.**A14** (1999) 1043, e-Print: gr-qc/9901017.
4. E.I. Guendelman, Mod. Phys. Lett.**A14** (1999) 1397.
5. E.I. Guendelman, A.B. Kaganovich, Phys. Rev. **D75** (2007) 083505, e-Print: gr-qc/0607111.
6. E.I. Guendelman and O. Katz, Class.Quant. Grav.**20** (2003) 1715, e-Print: gr-qc/0211095.
7. E.I. Guendelman,Int.J.Mod.Phys. **A25** (2010) 4081, e-Print: arXiv:0911.0178 [gr-qc].
8. E.I. Guendelman, A.B. Kaganovich, Int.J.Mod.Phys. **A17** (2002) 417,e-Print: hep-th/0110040.
9. E.I. Guendelman, A.B. Kaganovich, Ann. of Phys. **323** (2008) 866, e-Print: arXiv:0704.1998 [gr-qc] .
10. E.I. Guendelman, A.B. Kaganovich, Int.J.Mod.Phys. **A21** (2006) 4373.
11. E.I. Guendelman, Int. J. Mod. Phys.**A19** (2004) 3255.
12. P. Gaete and E. I. Guendelman, Phys. Lett. **B640** (2006) 201.
13. P. Gaete, E.I. Guendelman and E. Spallucci, Phys.Lett.**B649** (2007) 218, hep-th/0702067.
14. P. Gaete and E. I. Guendelman, Phys. Lett.**B593** (2004) 151.
15. I. Korover and E.I. Guendelman, Int. J. Mod. Phys.**A24** (2009) 1443.
16. E.I. Guendelman and I. Korover, Mod. Phys. Lett **A25** (2010) 1499.
17. I. Korover, M.Sc. Thesis, Ben Gurion University, 2009.
18. E. I. Guendelman, Phys. Lett. **B412** (1997) 42.
19. E.I. Guendelman, Phys. Lett. **B580** (2004) 87, e-Print: gr-qc/0303048.
20. E.I. Guendelman, E. Spallucci, Phys. Rev. **D70** (2004) 026003, e-Print: hep-th/0311102
21. E.I. Guendelman, e-Print: hep-th/0507092.
22. E.I. Guendelman, A. Kaganovich , E. Nissimov, S. Pacheva, Phys. Rev. **D72** (2005) 086011, e-Print: hep-th/0507193 and references.
23. E. I. Guendelman, A. Kaganovich, E. Nissimov, S, Pacheva, **D66** (2002) 046003, e-Print: hep-th/0203024.
24. E.I. Guendelman, Class. Quant. Grav. **17** (2000) 3673, e-Print: hep-th/0005041.
25. E.I. Guendelman, Phys. Rev.**D63** (2001) 046006, e-Print: hep-th/0006079.
26. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, e-Print: arXiv:1108.0160 [hep-th].
27. E.I. Guendelman , Mod. Phys. Lett. **A22** (2007) 1209.
28. E.I. Guendelman,Int.J.Mod.Phys.**A25** (2010) 4195. e-Print: arXiv:1005.1421 [hep-th].
29. E. Guendelman, A. Kaganovich, E. Nissimov, S. Pacheva, e-Print: arXiv:1108.3735 [hep-th].
30. E. Guendelman, A. Kaganovich, E. Nissimov, S. Pacheva. e-Print: arXiv:1109.0453. [hep-th]

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 41)*
Bled, Slovenia, July 11-21, 2011

6 Predictions for Fermion Masses and Mixing From a Low Energy $SU(3)$ Flavor Symmetry Model With a Light Sterile Neutrino

A. Hernández-Galeana

Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N.,
U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México.

Abstract. I report low energy results on the study of fermion masses and mixing for quarks and leptons, including neutrinos within a $SU(3)$ flavor symmetry model, where ordinary heavy fermions, top and bottom quarks and tau lepton become massive at tree level from **Dirac See-saw** mechanisms implemented by the introduction a new set of $SU(2)_L$ weak singlet vector-like fermions U, D, E, N, with N a sterile neutrino. Light fermions obtain masses from one loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. Recent results shows the existence of a low energy space parameter where this model is able to accommodate the known spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM} as well as the charged lepton masses. Motivated by the recent LSND and MiniBooNe short-baseline neutrino oscillation experiments we fit for the 3+1 scenario the neutrino squared mass differences $m_2^2 - m_1^2 \approx 7.6 \times 10^{-5}$ eV^2, $m_3^2 - m_2^2 \approx 2.43 \times 10^{-3}$ eV^2 and $m_4^2 - m_1^2 \approx 0.29$ eV^2. The model predicts the D vector like quark mass in the range $M_D = (\,350\ -\ 900\,)\,GeV$ and horizontal gauge boson masses of few TeV. These low energy predictions are within LHC possibilities. Furthermore, the above scenario enable us to suppress simultaneously the tree level $\Delta F = 2$ processes for $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ meson mixing mediated by these extra horizontal gauge bosons within current experimental bounds.

6.1 Introduction

The strong hierarchy of quark and charged lepton masses and quark mixing have suggested to many model building theorists that light fermion masses could be generated from radiative corrections[1], while those of the top and bottom quarks as well as that of the tau lepton are generated at tree level. This may be understood as a consequence of the breaking of a symmetry among families (a horizontal symmetry). This symmetry may be discrete [2], or continuous, [3]. The radiative generation of the light fermions may be mediated by scalar particles as it is proposed, for instance, in references [4,5] and this author in [15], or also through vectorial bosons as it happens for instance in "Dynamical Symmetry Breaking" (DSB) and theories like " Extended Technicolor " [6].

In this article we deal with the problem of fermion masses and quark mixing within an extension of the SM introduced by the author[7] which includes a $SU(3)$ gauged flavor symmetry commuting with the SM group. In previous reports[8] we showed that this model has the ingredients to accommodate a realistic spectrum of charged fermion masses and quark mixing. We introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level by the implementation of "Dirac See-saw" mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions. Recently, some authors have pointed out interesting features regarding the possibility of the existence of a sequential fourth generation[9]. Theories and models with extra matter may also provide interesting scenarios for present cosmological problems, such as candidates for the nature of the Dark Matter ([10],[11]). This is the case of an extra generation of vector-like matter, both from theory and current experiments[12]. Due to the fact that the vector-like quarks do not couple to the W boson, the mixing of one U and D vector-like quarks with the SM quarks yield an extended 4×4 non-unitary CKM quark mixing matrix. It has pointed out for some authors that these type of vector-like fermions are weakly constrained from Electroweak Precison Data (EWPD) because they do not break directly the custodial symmetry, then current experimental constraints on vector-like matter come from the direct production bounds and their implications on flavor physics. See ref. [12] for further details on constraints for $SU(2)_L$ singlet vector-like fermions.

Motivated by recent results from the LSND and MiniBooNe short-baseline neutrino oscillation experiments many authors are paying special attention to the study of light sterile neutrinos in the eV-scale to explain the tension in the interpretation of these data[13].

Here we report updated low energy results which accounts for the known quark and charged lepton masses and the quark mixing in a non-unitary $(V_{CKM})_{4\times4}$. We also include a fit for neutrino masses within a "Dirac See-saw" mechanism with a light sterile neutrino of $m_4 \approx 0.54$ eV.

6.2 Model with $SU(3)$ flavor symmetry

6.2.1 Fermion content

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where $SU(3)$ is a flavor symmetry among families and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group of elementary particles. The content of fermions assumes the ordinary quarks and leptons assigned under G as: $\psi_q^o = (3, 3, 2, \frac{1}{3})_L$, $\psi_l^o = (3, 1, 2, -1)_L$, $\psi_u^o = (3, 3, 1, \frac{4}{3})_R$, $\psi_d^o = (3, 3, 1, -\frac{2}{3})_R$, $\psi_e^o = (3, 1, 1, -2)_R$, where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions: Right handed neutrinos $\Psi_\nu^o = (3, 1, 1, 0)_R$, and the $SU(2)_L$ singlet vector-like fermions

$$U_{L,R}^o = (1, 3, 1, \frac{4}{3}) \quad , \quad D_{L,R}^o = (1, 3, 1, -\frac{2}{3}) \tag{6.1}$$

$$N_{L,R}^o = (1, 1, 1, 0) \quad , \quad E_{L,R}^o = (1, 1, 1, -2) \tag{6.2}$$

The above fermion content and its assignment under the group G make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the canonical form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[14]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses together with the radiative corrections.

6.3 Spontaneous Symmetry breaking

The "Spontaneous Symmetry Breaking" (SSB) is proposed to be achieved in the form:

$$G \xrightarrow{\Lambda_1} SU(2) \otimes G_{SM} \xrightarrow{\Lambda_2} G_{SM} \xrightarrow{\Lambda_3} SU(3)_C \otimes U(1)_Q \tag{6.3}$$

Here Λ_1, Λ_2 and Λ_3 are the scales of SSB in order the model to have the possibility to be consistent with the known low energy physics.

6.3.1 Electroweak symmetry breaking

To achieve the spontaneous breaking of the electroweak symmetry to $U(1)_Q$, we introduce the scalars: $\Phi = (3, 1, 2, -1)$ and $\Phi' = (3, 1, 2, +1)$, with the VEVs: $\langle\Phi\rangle^T = (\langle\Phi_1\rangle, \langle\Phi_2\rangle, \langle\Phi_3\rangle)$, $\langle\Phi'\rangle^T = (\langle\Phi'_1\rangle, \langle\Phi'_2\rangle, \langle\Phi'_3\rangle)$, where T means transpose, and

$$\langle\Phi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_i \\ 0 \end{pmatrix} \quad , \quad \langle\Phi'_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ V_i \end{pmatrix} . \tag{6.4}$$

Assuming $(v_1, v_2, v_3) \neq (V_1, V_2, V_3)$ with $v_1^2 + v_2^2 + v_3^2 = V_1^2 + V_2^2 + V_3^2$, the contributions from $\langle\Phi\rangle$ and $\langle\Phi'\rangle$ yield the W gauge boson mass $\frac{1}{2}g^2(v_1^2 + v_2^2 + v_3^2)W^+W^-$. Hence, if we define as usual $M_W = \frac{1}{2}gv$, we may write $v = \sqrt{2}\sqrt{v_1^2 + v_2^2 + v_3^2} \approx$ 246 GeV.

Let me emphasize here that solutions for fermion masses and mixing reported in section 5 suggest that the dominant contribution to Electroweak Symmetry Breaking comes from the weak doublets which couple to the third family.

6.3.2 $SU(3)$ flavor symmetry breaking

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)$, we introduce the scalar fields: η_i, $i = 1, 2, 3$, transforming under the gauge group as $(3, 1, 1, 0)$ and taking the "Vacuum Expectation Values" (VEV's):

$$\langle\eta_3\rangle^T = (0, 0, \mathcal{V}_3) \quad , \quad \langle\eta_2\rangle^T = (0, \mathcal{V}_2, 0) \quad , \quad \langle\eta_1\rangle^T = (\mathcal{V}_1, 0, 0) . \tag{6.5}$$

The above scalar fields and VEV's break completely the $SU(3)$ flavor symmetry. The corresponding $SU(3)$ gauge bosons are defined in Eq.(6.12) through their couplings to fermions. To simplify computations, we impose a $SU(2)$ global symmetry in the gauge boson masses. So, we assume $\mathcal{V}_1 = \mathcal{V}_2 \equiv \mathcal{V}$ in order to cancel mixing between Z_1 and Z_2 horizontal gauge bosons. Thus, a natural hierarchy among the VEVs consistent with the proposed sequence of SSB in Eq.(6.3) is $\mathcal{V}_3 >> \mathcal{V} \gg \sqrt{v_1^2 + v_2^2 + v_3^2} = \frac{v}{\sqrt{2}} \simeq \frac{246\ \text{GeV}}{\sqrt{2}} \backsimeq 173.9\ \text{GeV} \approx m_t$. Hence, neglecting tiny contributions from electroweak symmetry breaking, we obtain for the gauge bosons masses

$$g_H^2 \left\{ \frac{1}{2}(\mathcal{V})^2 [\, Z_1^2 + (Y_1^1)^2 + (Y_1^2)^2 \,] + \frac{1}{6} [\, 2(\mathcal{V}_3)^2 + (\mathcal{V})^2 \,] \, Z_2^2 \right.$$
$$\left. + \frac{1}{4} (\, (\mathcal{V}_3)^2 + (\mathcal{V})^2 \,) [\, (Y_2^1)^2 + (Y_2^2)^2 + (Y_3^1)^2 + (Y_3^2)^2 \,] \right\} \quad (6.6)$$

Them, we may define the horizontal boson masses

$$\begin{aligned} (M_{Z_1})^2 = (M_{Y_1^1})^2 = (M_{Y_1^2})^2 &= M_1^2 \equiv g_H^2 \mathcal{V}^2 \,, \\ (M_{Y_2^1})^2 = (M_{Y_2^2})^2 = (M_{Y_3^1})^2 = (M_{Y_3^2})^2 &= M_2^2 \equiv \frac{g_H^2}{2} ({\mathcal{V}_3}^2 + \mathcal{V}^2) \,, \\ (M_{Z_2})^2 &= 4/3 M_2^2 - 1/3 M_1^2 \end{aligned} \quad (6.7)$$

with the hierarchy $M_{Z_2} \gtrsim M_2 > M_1 \gg M_W$. It is worth to emphasize that this $SU(2)$ global symmetry together with the hierarchy of scales in the SSB yield a spectrum of $SU(3)$ gauge boson masses without mixing in quite good approximation. Actually this global $SU(2)$ symmetry plays the role of a custodial symmetry to suppress properly the tree level $\Delta F = 2$ processes mediated by the M_1 lower scale Z_1, Y_1^1, Y_1^2 horizontal gauge bosons.

6.4 Fermion masses

6.4.1 Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks and Dirac neutrinos. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings, as

$$h\, \bar{\psi}_l^o \, \Phi' \, E_R^o \; + \; h_1 \, \bar{\psi}_e^o \eta_1 \, E_L^o \; + \; h_2 \, \bar{\psi}_e^o \eta_2 \, E_L^o \; + \; h_3 \, \bar{\psi}_e^o \eta_3 \, E_L^o \; + \; M \, \bar{E}_L^o \, E_R^o \; + h.c \quad (6.8)$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_1, h_2 and h_3 are Yukawa coupling constants. When the involved scalar fields acquire VEV's we get, in the gauge basis ${\psi_{L,R}^o}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\, v_1 \\ 0 & 0 & 0 & h\, v_2 \\ 0 & 0 & 0 & h\, v_3 \\ h_1 \mathcal{V} & h_2 \mathcal{V} & h_3 \mathcal{V}_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ b_1 & b_2 & b_3 & c \end{pmatrix} . \quad (6.9)$$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\, \chi_{L,R}$. The orthogonal matrices V^o_L and V^o_R are obtained explicitly in the Appendix A. From V^o_L and V^o_R, and using the relationships defined in this Appendix, one computes

$$V_L^{o\,T} \mathcal{M}^o\, V_R^o = Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) \tag{6.10}$$

$$V_L^{o\,T} \mathcal{M}^o \mathcal{M}^{o\,T}\, V_L^o = V_R^{o\,T} \mathcal{M}^{o\,T} \mathcal{M}^o\, V_R^o = Diag(0, 0, \lambda_-, \lambda_+)\,. \tag{6.11}$$

where λ_- and λ_+ are the nonzero eigenvalues defined in Eqs.(6.51-6.52), $\sqrt{\lambda_+}$ being the fourth heavy fermion mass, and $\sqrt{\lambda_-}$ of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(6.10,6.11) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

6.4.2 One loop contribution to fermion masses

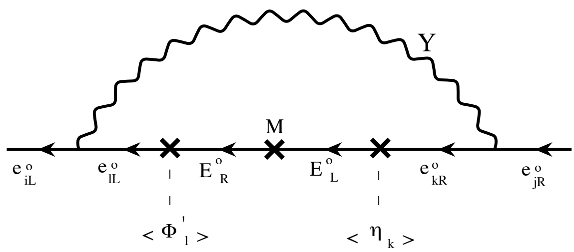

Fig. 6.1. Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}^o_{iL} e^o_{jR}$

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 6.1. The vertices in this diagram read from the $SU(3)$ flavor symmetry interaction Lagrangian

$$\begin{aligned} i\mathcal{L}_{int} = \frac{g_H}{2} \Big\{ &(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o) Z_1^\mu + \frac{1}{\sqrt{3}}(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o) Z_2^\mu \\ &+ (\bar{e^o}\gamma_\mu \mu^o + \bar{\mu^o}\gamma_\mu e^o) Y_1^{1\mu} + (-i\bar{e^o}\gamma_\mu \mu^o + i\bar{\mu^o}\gamma_\mu e^o) Y_1^{2\mu} \\ &+ (\bar{e^o}\gamma_\mu \tau^o + \bar{\tau^o}\gamma_\mu e^o) Y_2^{1\mu} + (-i\bar{e^o}\gamma_\mu \tau^o + i\bar{\tau^o}\gamma_\mu e^o) Y_2^{2\mu} \\ &+ (\bar{\mu^o}\gamma_\mu \tau^o + \bar{\tau^o}\gamma_\mu \mu^o) Y_3^{1\mu} + (-i\bar{\mu^o}\gamma_\mu \tau^o + i\bar{\tau^o}\gamma_\mu \mu^o) Y_3^{2\mu} \Big\}\,, \end{aligned} \tag{6.12}$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(6.8) after the scalar fields get VEV's. The one loop diagram of Fig. 6.1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi} \tag{6.13}$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(6.12), $m^o_3 = -\sqrt{\lambda_-}$ and $m^o_4 = \sqrt{\lambda_+}$ are the See-saw mass eigenvalues, Eq.(6.10), and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix A, we compute

$$\sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) = \frac{a_i\ b_j\ M}{\lambda_+ - \lambda_-}\ F(M_Y, \sqrt{\lambda_-}, \sqrt{\lambda_+})\,, \tag{6.14}$$

$i, j = 1, 2, 3$ and $F(M_Y, \sqrt{\lambda_-}, \sqrt{\lambda_+}) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_+} \ln \frac{M_Y^2}{\lambda_+} - \frac{M_Y^2}{M_Y^2 - \lambda_-} \ln \frac{M_Y^2}{\lambda_-}$. Adding up all the one loop $SU(3)$ gauge boson contributions, we get the mass terms $\bar{\psi}^o_L \mathcal{M}^o_1\ \psi^o_R +$ h.c.,

$$\mathcal{M}^o_1 = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}\,, \tag{6.15}$$

$$\begin{aligned}
R_{11} &= -\frac{1}{4} F_1 (m_{11} + 2 m_{22}) - \frac{1}{12} F_{Z_2} m_{11} + \frac{1}{2} F_2 m_{33}\,, \\
R_{22} &= -\frac{1}{4} F_1 (2 m_{11} + m_{22}) - \frac{1}{12} F_{Z_2} m_{22} + \frac{1}{2} F_2 m_{33}\,, \\
R_{12} &= (\frac{1}{4} F_1 - \frac{1}{12} F_{Z_2}) m_{12} \quad , \quad R_{21} = (\frac{1}{4} F_1 - \frac{1}{12} F_{Z_2}) m_{21}\,, \\
R_{33} &= \frac{1}{3} F_{Z_2} m_{33} - \frac{1}{2} F_2 (m_{11} + m_{22}) \quad , \quad R_{13} = -\frac{1}{6} F_{Z_2} m_{13}\,, \\
R_{31} &= \frac{1}{6} F_{Z_2} m_{31} \quad , \quad R_{23} = -\frac{1}{6} F_{Z_2} m_{23} \quad , \quad R_{32} = \frac{1}{6} F_{Z_2} m_{32}\,,
\end{aligned} \tag{6.16}$$

Here, $F_{1,2} \equiv F(M_{1,2}, \sqrt{\lambda_-}, \sqrt{\lambda_+})$ and $F_{Z2} \equiv F(M_{Z_2}, \sqrt{\lambda_-}, \sqrt{\lambda_+})$, with M_1 , M_2 and M_{Z_2} the horizontal boson masses, Eq.(6.7),

$$m_{ij} = \frac{a_i\ b_j\ M}{\lambda_+ - \lambda_-} = \frac{a_i\ b_j}{a\ b}\ \sqrt{\lambda_-}\ c_\alpha\ c_\beta\,, \tag{6.17}$$

and $c_\alpha \equiv \cos\alpha,\ c_\beta \equiv \cos\beta,\ s_\alpha \equiv \sin\alpha,\ s_\beta \equiv \sin\beta$, as defined in the Appendix A, Eq.(6.53). Therefore, up to one loop corrections we obtain the fermion masses

$$\bar{\psi}^o_L \mathcal{M}^o\ \psi^o_R + \bar{\psi}^o_L \mathcal{M}^o_1\ \psi^o_R = \bar{\chi}_L\ \mathcal{M}\, \chi_R\,, \tag{6.18}$$

with $\mathcal{M} \equiv \left[Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) + {V^o_L}^T \mathcal{M}^o_1\ V^o_R \right]$. Using V^o_L, V^o_R in Eqs.(6.47-6.48) we get the mass matrix in Version I:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\, m_{13} & s_\beta\, m_{13} \\ m_{21} & m_{22} & c_\beta\, m_{23} & s_\beta\, m_{23} \\ c_\alpha\, m_{31} & c_\alpha\, m_{32} & (-\sqrt{\lambda_-} + c_\alpha c_\beta\, m_{33}) & c_\alpha s_\beta\, m_{33} \\ s_\alpha\, m_{31} & s_\alpha\, m_{32} & s_\alpha c_\beta\, m_{33} & (\sqrt{\lambda_+} + s_\alpha s_\beta\, m_{33}) \end{pmatrix}, \qquad (6.19)$$

where the mass entries m_{ij} ; $i, j = 1, 2, 3$ are written as:

$$\begin{aligned} m_{11} &= \tfrac{\eta_+}{a'b'} c_1 H\,, \qquad m_{12} = -\tfrac{\eta_-}{a'b'}\tfrac{b_3}{b} c_1 H\,, \\ m_{21} &= \tfrac{\eta_-}{a'b'}\tfrac{a_3}{a} c_1 H\,,\ \ m_{22} = c_2\left[\tfrac{\eta_+}{a'b'}H + \tfrac{a'b'}{a_3 b_3}(J + \tfrac{\Delta}{2})\right]\,, \\ m_{31} &= \tfrac{\eta_-}{a'b'}\tfrac{a'}{a} c_1 H, \quad m_{32} = c_2\left[\tfrac{a'}{a_3}(\tfrac{\eta_+}{a'b'}H + \tfrac{1}{2}\tfrac{a'b'}{a_3 b_3}\Delta) - \tfrac{b'}{b_3}J\right]\,, \\ m_{13} &= -\tfrac{\eta_-}{a'b'}\tfrac{b'}{b} c_1 H,\ m_{23} = \left[\tfrac{b'}{b_3}(\tfrac{\eta_+}{a'b'}H + \tfrac{1}{2}\tfrac{a'b'}{a_3 b_3}\Delta) - \tfrac{a'}{a_3}J\right]\,, \end{aligned} \qquad (6.20)$$

$$m_{33} = c_2\left(\frac{\eta_+}{a_3 b_3}H + J + \frac{1}{6}\frac{a'^2 b'^2}{a_3^2 b_3^2}\Delta - \frac{1}{3}\left[\frac{a'^2 b'^2}{a_3^2 b_3^2}F_1 + (1 + \frac{a'^2}{a_3^2} + \frac{b'^2}{b_3^2})F_{Z_2}\right]\right)\,,$$

For V_L^o, V_R^o of Eqs.(6.49-6.50) we get the Version II:

$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & c_\beta\, M_{13} & s_\beta\, M_{13} \\ M_{21} & M_{22} & c_\beta\, M_{23} & s_\beta\, M_{23} \\ c_\alpha\, M_{31} & c_\alpha\, M_{32} & (-\sqrt{\lambda_-} + c_\alpha c_\beta\, M_{33}) & c_\alpha s_\beta\, M_{33} \\ s_\alpha\, M_{31} & s_\alpha\, M_{32} & s_\alpha c_\beta\, M_{33} & (\sqrt{\lambda_+} + s_\alpha s_\beta\, M_{33}) \end{pmatrix}, \qquad (6.21)$$

where the mass terms M_{ij} ; $i, j = 1, 2, 3$ may be obtained from those of m_{ij} as follows

$$\begin{aligned} M_{11} &= m_{22},\ \ M_{12} = -m_{21},\ \ M_{13} = m_{23} \\ M_{21} &= -m_{12},\ \ M_{22} = m_{11},\ \ M_{23} = -m_{13} \\ M_{31} &= m_{32},\ \ M_{32} = -m_{31},\ \ M_{33} = m_{33} \end{aligned} \qquad (6.22)$$

$$\eta_- = a_1\, b_2 - a_2\, b_1 \quad , \quad \eta_+ = a_1\, b_1 + a_2\, b_2 \quad , \quad \eta_-^2 + \eta_+^2 = a'^2 b'^2 \qquad (6.23)$$

$$a' = \sqrt{a_1^2 + a_2^2}\ ,\ \ b' = \sqrt{b_1^2 + b_2^2}\ ,\ \ a = \sqrt{a'^2 + a_3^2}\ ,\ \ b = \sqrt{b'^2 + b_3^2}\,, \qquad (6.24)$$

$$c_1 = \frac{1}{2} c_\alpha c_\beta \frac{a_3\, b_3}{a\, b} \frac{\alpha_H}{\pi} \quad , \quad c_2 = \frac{a_3\, b_3}{a\, b}\, c_1 \,, \tag{6.25}$$

$$H = F_2 + \frac{\eta_+}{a_3 b_3}\, F_1 \quad , \quad J = F_{Z_2} + \frac{\eta_+}{a_3 b_3}\, F_2 \quad , \quad \Delta = F_{Z_2} - F_1 \,. \tag{6.26}$$

The diagonalization of $\mathcal{M}$, Eq.(6.19) or Eq.(6.21), gives the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R}$; $\bar{\chi}_L\, \mathcal{M}\, \chi_R = \bar{\Psi}_L\, {V^{(1)}_L}^T \mathcal{M}\, V^{(1)}_R\, \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$${V^{(1)}_L}^T \mathcal{M}\, \mathcal{M}^T\, V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\, \mathcal{M}\, V^{(1)}_R = \mathrm{Diag}(m_1^2, m_2^2, m_3^2, M_F^2) \,, \tag{6.27}$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$\psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R \tag{6.28}$$

6.4.3 Quark Mixing and non-unitary $(V_{CKM})_{4\times 4}$

The interaction of quarks ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$ to the W charged gauge boson is[1]

$$\bar{f^o}_{uL} \gamma_\mu f^o_{dL} W^{+\mu} = \bar{\Psi}_{uL}\, {V^{(1)}_{uL}}^T\, [(V^o_{uL})_{3\times4}]^T\, (V^o_{dL})_{3\times4}\, V^{(1)}_{dL}\, \gamma_\mu \Psi_{dL}\, W^{+\mu} \,, \tag{6.29}$$

hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} \equiv {V^{(1)}_{uL}}^T\, [(V^o_{uL})_{3\times4}]^T\, (V^o_{dL})_{3\times4}\, V^{(1)}_{dL} \,. \tag{6.30}$$

For u-quarks in version I and d-quarks in version II,

$$V^o \equiv [(V^o_{uL})_{3\times4}]^T\, (V^o_{dL})_{3\times4} = \begin{pmatrix} \frac{s_o}{\sqrt{1+r_d^2}} & -c_o & \frac{c^d_\alpha s_o r_d}{\sqrt{1+r_d^2}} & \frac{s^d_\alpha s_o r_d}{\sqrt{1+r_d^2}} \\ \Omega_{11} & \frac{s_o}{\sqrt{1+r_u^2}} & c^d_\alpha\, \Omega_{13} & s^d_\alpha\, \Omega_{13} \\ c^u_\alpha\, \Omega_{31} & \frac{c^u_\alpha s_o r_u}{\sqrt{1+r_u^2}} & c^u_\alpha\, c^d_\alpha\, \Omega_{33} & c^u_\alpha\, s^d_\alpha\, \Omega_{33} \\ s^u_\alpha\, \Omega_{31} & \frac{s^u_\alpha s_o r_u}{\sqrt{1+r_u^2}} & s^u_\alpha\, c^d_\alpha\, \Omega_{33} & s^u_\alpha\, s^d_\alpha\, \Omega_{33} \end{pmatrix} , \tag{6.31}$$

$$\Omega_{11} = \frac{r_u r_d + c_o}{\sqrt{(1+r_u^2)(1+r_d^2)}} \quad , \quad \Omega_{13} = \frac{r_d c_o - r_u}{\sqrt{(1+r_u^2)(1+r_d^2)}} \tag{6.32}$$

[1] Recall that vector like quarks, Eqs.(6.1), are $SU(2)_L$ weak singlets, and so, they do not couple to W boson in the interaction basis.

$$\Omega_{31} = \frac{r_u c_o - r_d}{\sqrt{(1+r_u^2)(1+r_d^2)}} \quad , \quad \Omega_{33} = \frac{r_u r_d c_o + 1}{\sqrt{(1+r_u^2)(1+r_d^2)}} \tag{6.33}$$

$$s_o = \frac{v_2}{v'}\frac{V_1}{V'} - \frac{v_1}{v'}\frac{V_2}{V'} \quad , \quad c_o = \frac{v_1}{v'}\frac{V_1}{V'} + \frac{v_2}{v'}\frac{V_2}{V'} \tag{6.34}$$

$$c_o^2 + s_o^2 = 1 \quad , \quad r_u = (\frac{a'}{a_3})_u \quad , \quad r_d = (\frac{a'}{a_3})_d \tag{6.35}$$

$V_i,\ v_i\ , i = 1, 2$ are related to (e,d) and (u,ν) fermions respectively.

6.5 Numerical results

Using the strong hierarchy for quarks and charged leptons masses and the results in[15], we report here the magnitudes of quark masses and mixing coming from the analysis of a low energy parameter space in this model. For this numerical analysis we used the input global parameters $\frac{\alpha_H}{\pi} = 0.2$, $M_1 = 4$ TeV and $M_2 = 1700$ TeV.

6.5.1 Sector d:

Parameter space: $(\sqrt{\lambda_-})_d = 4.98$ GeV, $(\sqrt{\lambda_+})_d = 500$ GeV, $r_d = 0.052$, $(\eta_+/a_3 b_3)_d = -0.49$, $(\eta_-/\eta_+)_d = 1.3$, $s_\alpha^d = 0.01$, and $s_\beta^d = 0.7056$, lead to the down quark masses: $m_d = 5.4663$ MeV, $m_s = 107.699$ MeV, $m_b = 4.216$ GeV, $M_D = 500.008$ **GeV**, and the mixing matrix

$$V_{dL}^{(1)} = \begin{pmatrix} 0.61120 & -0.79139 & -0.01093 & 9.2\times10^{-5} \\ 0.79127 & 0.61129 & -0.01429 & 1.2\times10^{-4} \\ 0.01799 & 8.04\times10^{-5} & 0.99983 & 0.00152 \\ -1.78\times10^{-4} & -7.96\times10^{-7} & -0.00152 & 0.99999 \end{pmatrix} . \tag{6.36}$$

6.5.2 Sector u:

Parameter space: $(\sqrt{\lambda_-})_u = 358.2$ GeV, $(\sqrt{\lambda_+})_u = 1241.44$ TeV, $r_u = .04$, $(\eta_+/a_3 b_3)_u = -3.20432$, $(\eta_-/\eta_+)_u = 0$, $s_\alpha^u = .01$ and $s_\beta^u = 0.02884$ yield the up quark masses $m_u = 2.4$ MeV, $m_c = 1.2$ GeV, $m_t = 172$ GeV, $M_U = 1241.44$ TeV, and the mixing

$$V_{uL}^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.99900 & 0.04458 & -1.80\times10^{-7} \\ 0 & -0.04458 & 0.99900 & 4.34\times10^{-6} \\ 0 & 3.7366.\times10^{-7} & -4.33\times10^{-6} & 1 \end{pmatrix} . \tag{6.37}$$

The See-saw V^o contribution, Eq.(6.31) with $s_o = -0.417698$, Eq.(6.34) reads

$$V^o = \begin{pmatrix} 0.41713 & 0.90858 & -0.02168 & -2.16\times10^{-4} \\ -0.90871 & 0.41736 & 0.00723 & 7.23\times10^{-5} \\ 0.01562 & 0.01669 & 0.99963 & 0.01 \\ 1.562\times10^{-4} & 1.66\times10^{-4} & 0.0100 & 0.0001 \end{pmatrix} \tag{6.38}$$

6.5.3 $(V_{CKM})_{4\times 4}$

The above up and down quark mixing matrices $V_{uL}^{(1)}$, $V_{dL}^{(1)}$ and V^o yield the quark mixing matrix

$$(V_{CKM})_{4\times 4} = \begin{pmatrix} 0.97428 & 0.22530 & 0.00413 & -3.97\times 10^{-4} \\ -0.22527 & 0.97341 & 0.04133 & -3.96\times 10^{-4} \\ 0.00528 & -0.04120 & 0.99902 & -0.01151 \\ 4.77\times 10^{-5} & -2.25\times 10^{-5} & -0.0100 & 1.15\times 10^{-4} \end{pmatrix} \tag{6.39}$$

Notice that the $(V_{CKM})_{3\times 3}$ sub-matrix is nearly a unitary mixing matrix, which is consistent with the measured values for quark mixing as reported in the PDG [16].

6.5.4 Charged Leptons:

For this sector, the parameter space: $(\sqrt{\lambda_-})_e = 9.14301$ GeV, $(\sqrt{\lambda_+})_e = 23816.4$ TeV, $r_e = 0.05$, $(\eta_+/a_3 b_3)_e = -1.99484$, $(\eta_-/\eta_+)_e = 0$, $s_\alpha^e = 0.001$ and $s_\beta^e = 0.00038$, reproduce the known charged lepton masses: $m_e = 0.511$ MeV , $m_\mu = 105.658$ MeV, $m_\tau = 1776.82$ MeV and $M_E \approx 23816.4$ TeV

6.5.5 Neutrinos 3+1:

For this sector, the parameter space: $(\sqrt{\lambda_-})_\nu = 0.048$ eV, $(\sqrt{\lambda_+})_e = 0.54$ eV, $r_\nu = 0.04$, $(\eta_+/a_3 b_3)_e = 0.01$, $(\eta_-/\eta_+)_e = 4.7$, $s_\alpha^\nu = 0.2$ and $s_\beta^\nu = 0.3992$, fit the neutrinos masses

$$(m_1, m_2, m_3, m_4) = (\ 0.0102\ ,\ 0.0134\ ,\ 0.0511\ ,\ 0.5398\ )\ eV\ , \tag{6.40}$$

the squared mass differences

$$\begin{array}{c} m_2^2 - m_1^2 \approx 7.6\times 10^{-5}\ \text{eV}^2,\ m_3^2 - m_2^2 \approx 2.43\times 10^{-3}\ \text{eV}^2 \\ m_4^2 - m_1^2 \approx 0.29\ \text{eV}^2 \qquad\qquad , \end{array} \tag{6.41}$$

and for charged leptons and neutrinos in version I, the first row of lepton mixing angles

$$\begin{array}{c} (U_{PMNS})_{11} = 0.8145\ , \quad (U_{PMNS})_{12} = 0.5773 \\ (U_{PMNS})_{13} = 0.0422\ , \ (U_{PMNS})_{14} = 1.27\times 10^{-4} \end{array} \tag{6.42}$$

6.5.6 FCNC's in $K^o - \bar{K^o}$ meson mixing

The $SU(3)$ horizontal gauge bosons contribute to new FCNC's, in particular they mediate $\Delta F = 2$ processes at tree level. Here we compute their leading contribution to $K^o - \bar{K^o}$ meson mixing. In the previous scenario the up quark sector does not contribute to $(V_{CKM})_{12}$, and hence the effective hamiltonian from the tree level

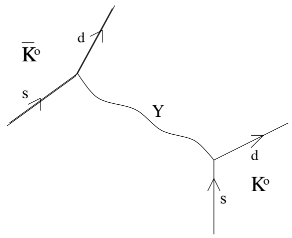

Fig. 6.2. Tree level contribution to $K^o - \bar{K^o}$ from the light $SU(2)$ horizontal gauge bosons.

diagrams, Fig. 6.2, mediated by the $SU(2)$ horizontal gauge bosons of mass M_1 to the $\mathcal{O}_{LL}(\Delta S = 2) = (\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu s_L)$ operator, is given by

$$\mathcal{H}_{eff} = C_{\bar{d}s}\, \mathcal{O}_{LL} \quad , \quad C_{\bar{d}s} \approx \frac{g_H^2}{4} \frac{1}{M_1^2} \frac{r_d^4}{(1+r_d^2)^2} (s_{12}^d)^2 \,, \tag{6.43}$$

and then contribute to the $K^o - \bar{K^o}$ mass difference as

$$\Delta M_K \approx \frac{2\pi^2}{3} \frac{\alpha_H}{\pi} \frac{r_d^4}{(1+r_d^2)^2} (s_{12}^d)^2 \frac{F_K^2}{M_1^2} B_K(\mu) M_K \,. \tag{6.44}$$

It is worth to point out the double mixing suppression in ΔM_K, Eq.(6.44); one from the see-saw mechanism due to the $r_d = (\frac{a'}{a_3})_d$ parameter, and the one from d-quark mixing s_{12}^d. Using the input values: $r_d = 0.052$, $\frac{\alpha_H}{\pi} = 0.2$, $s_{12}^d = 0.79139$, $F_K = 160$ MeV, $M_K = 497.614$ MeV and $B_K = 0.8$, one gets

$$\Delta m_K \approx 2.77 \times 10^{-12}\ \text{MeV} \,, \tag{6.45}$$

which is lower than the current experimental bound[16], $(\Delta m_K)_{Exp} = M_{K_L} - M_{K_S} \approx 3.48 \times 10^{-12}$ MeV. The quark mixing alignment in Eqs.(6.36 - 6.39) avoids tree level contributions to $D^0 - \bar{D^o}$ mixing mediated by the $SU(2)$ horizontal gauge bosons.

6.6 Conclusions

We have reported a low energy parameter space within a $SU(3)$ flavor symmetry model extension, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical spectrum for charged fermion masses and quark mixing. In section 5 we have reported the predicted values for quark and charged lepton masses and quark mixing matrix $(V_{CKM})_{4\times4}$ within allowed experimental values reported in PDG 2010, coming from an input space parameter region with the lower horizontal scale of $M_1 = 4$ TeV and a D vector-like quark mass of the order of 500 GeV. Furthermore, motivated by the recent LSND and MiniBooNe short-baseline neutrino oscillation experiments we are able to fit in the 3+1 scenario the neutrino squared mass differences $m_2^2 - m_1^2 \approx 7.6 \times 10^{-5}\ \text{eV}^2$, $m_3^2 - m_2^2 \approx 2.43 \times 10^{-3}\ \text{eV}^2$ and $m_4^2 - m_1^2 \approx 0.29\ \text{eV}^2$. Hence some of the new particles introduced in this model are within reach at the current LHC experiments, while simultaneously being consistent with present bounds on FCNC in $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ meson mixing.

6.7 Appendix A: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ b_1 & b_2 & b_3 & c \end{pmatrix} \tag{6.46}$$

Using a biunitary transformation $\Psi_L^o = V_L^o \, \chi_L^o$ and $\Psi_R^o = V_R^o \, \chi_R^o$ to diagonalize $\mathcal{M}^o$, the orthogonal matrices V_L^o and V_R^o may be written explicitly as the following two versions

Version I:

$$V_L^o = \begin{pmatrix} \frac{a_2}{a'} & \frac{a_1 a_3}{a' a} & \frac{a_1}{a} \cos\alpha & \frac{a_1}{a} \sin\alpha \\ -\frac{a_1}{a'} & \frac{a_2 a_3}{a' a} & \frac{a_2}{a} \cos\alpha & \frac{a_2}{a} \sin\alpha \\ 0 & -\frac{a'}{a} & \frac{a_3}{a} \cos\alpha & \frac{a_3}{a} \sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \tag{6.47}$$

$$V_R^o = \begin{pmatrix} \frac{b_2}{b'} & \frac{b_1 b_3}{b' b} & \frac{b_1}{b} \cos\beta & \frac{b_1}{b} \sin\beta \\ -\frac{b_1}{b'} & \frac{b_2 b_3}{b' b} & \frac{b_2}{b} \cos\beta & \frac{b_2}{b} \sin\beta \\ 0 & -\frac{b'}{b} & \frac{b_3}{b} \cos\beta & \frac{b_3}{b} \sin\beta \\ 0 & 0 & -\sin\beta & \cos\beta \end{pmatrix} \tag{6.48}$$

Version II:

$$V_L^o = \begin{pmatrix} \frac{a_1 a_3}{a' a} & -\frac{a_2}{a'} & \frac{a_1}{a} \cos\alpha & \frac{a_1}{a} \sin\alpha \\ \frac{a_2 a_3}{a' a} & \frac{a_1}{a'} & \frac{a_2}{a} \cos\alpha & \frac{a_2}{a} \sin\alpha \\ -\frac{a'}{a} & 0 & \frac{a_3}{a} \cos\alpha & \frac{a_3}{a} \sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{pmatrix} , \tag{6.49}$$

$$V_R^o = \begin{pmatrix} \frac{b_1 b_3}{b' b} & -\frac{b_2}{b'} & \frac{b_1}{b} \cos\beta & \frac{b_1}{b} \sin\beta \\ \frac{b_2 b_3}{b' b} & \frac{b_1}{b'} & \frac{b_2}{b} \cos\beta & \frac{b_2}{b} \sin\beta \\ -\frac{b'}{b} & 0 & \frac{b_3}{b} \cos\beta & \frac{b_3}{b} \sin\beta \\ 0 & 0 & -\sin\beta & \cos\beta \end{pmatrix} , \tag{6.50}$$

$$\lambda_\pm = \frac{1}{2} \left(B \pm \sqrt{B^2 - 4D} \right) \tag{6.51}$$

are the nonzero eigenvalues of $\mathcal{M}^{o}\mathcal{M}^{o\,T}$ ($\mathcal{M}^{o\,T}\mathcal{M}^{o}$), with

$$B = a^2 + b^2 + c^2 = \lambda_- + \lambda_+ \quad , \quad D = a^2 b^2 = \lambda_- \lambda_+ \ , \tag{6.52}$$

$$\begin{aligned} \cos\alpha &= \sqrt{\frac{\lambda_+ - a^2}{\lambda_+ - \lambda_-}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_-}{\lambda_+ - \lambda_-}} \ , \\ \cos\beta &= \sqrt{\frac{\lambda_+ - b^2}{\lambda_+ - \lambda_-}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_-}{\lambda_+ - \lambda_-}} \ . \end{aligned} \tag{6.53}$$

$$\begin{aligned} \cos\alpha\ \cos\beta &= \frac{c\,\sqrt{\lambda_+}}{\lambda_+ - \lambda_-} \quad , \quad \cos\alpha\ \sin\beta = \frac{b\,c^2\,\sqrt{\lambda_+}}{(\lambda_+ - b^2)(\lambda_+ - \lambda_-)} \\ \sin\alpha\ \sin\beta &= \frac{c\,\sqrt{\lambda_-}}{\lambda_+ - \lambda_-} \quad , \quad \sin\alpha\ \cos\beta = \frac{a\,c^2\,\sqrt{\lambda_+}}{(\lambda_+ - a^2)(\lambda_+ - \lambda_-)} \end{aligned} \tag{6.54}$$

Note that in the space parameter $a^2 \ll c^2$, b^2 , $\frac{\lambda_-}{\lambda_+} \ll 1$, so that we may approach the eigenvalues as

$$\lambda_- \approx \frac{D}{B} \approx \frac{a^2\,b^2}{c^2 + b^2} \quad , \quad \lambda_+ \approx c^2 + b^2 + a^2 - \frac{a^2\,b^2}{c^2 + b^2} \tag{6.55}$$

Acknowledgments

It is my pleasure to thank the organizers for invitation, useful discussions and for the stimulating Workshop at Bled, Slovenia. The author acknowledgment partial support from the "Instituto Politécnico Nacional", (Grants from EDI and COFAA) and "Sistema Nacional de Investigadores" (SNI) in Mexico.

References

1. S. Weinberg, Phys. Rev. Lett.**29** (1972) 388; H. Georgi and S.L. Glashow, Phys. Rev. **D 7** (1973) 2457; R.N. Mohapatra, Phys. Rev. **D 9** (1974) 3461; S.M. Barr and A. Zee, Phys. Rev. **D 15** (1977) 2652; H. Georgi, "Fermion Masses in Unified models", in *First Workshop in Grand Unification*, ed. P.H. Frampton, S.L. Glashow, and A. Yildiz(1980, Math Sci Press, Brookline, MA); S.M. Barr ,Phys. Rev. **D 21** (1980) 1424; R. Barbieri and D.V. Nanopoulos, Phys. Lett. **B 95** (1980) 43; S.M. Barr ,Phys. Rev. **D 24** (1981) 1895; L.E. Ibanez, Phys. Lett. **B 177** (1982) 403; B.S. Balakrishna, A.L. Kagan and R.N. Mohapatra, Phys. Lett. **B 205** (1988) 345; B.S. Balakrishna, Phys. Rev. Lett. **60** (1988) 1602; K.S. Babu and E. Ma, Mod. Phys. Lett. **A 4** (1989) 1975; H.P. Nilles, M. Olechowski and S. Pokorski, Phys. Lett. **B 248** (1990) 378; R. Rattazzi, Z. Phys.**C 52** (1991) 575; K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **64** (1990) 2747; X.G. He, R.R. Volkas, and D.D. Wu, Phys. Rev. **D 41** (1990) 1630; Ernest Ma, Phys. Rev. Lett. 64 (1990) 2866. B.A. Dobrescu and P.J. Fox, JHEP **0808** (2008) 100; S.M. Barr, Phys. Rev. **D 76** (2007) 105024; S.M. Barr and A. Khan, Phys. Rev. **D 79** (2009) 115005;

2. Sandip Pakvasa and Hirotaka Sugawara, Phys. Lett. **B 73** (1978) 61; Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. **D 25** (1982) 1895; K. S. Babu and X.G. He, ibid. 36 (1987) 3484; Ernest Ma, Phys. Rev. Lett. **B 62** (1989) 61.
3. A. Davidson, M. Koca, and K. C. Wali, Phys. Rev. Lett. **B 43** (1979) 92, Phys. Rev. **D 20** (1979) 1195; C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B 147** (1979) 277; A. Sirlin, Phys. Rev. **D 22** (1980) 971; A. Davidson and K. C. Wali, ibid. 21 (1980) 787.
4. X.G. He, R. R: Volkas, and D. D. Wu, Phys. Rev. **D 41** (1990) 1630; Ernest Ma, Phys. Rev. Lett. **64** (1990) 2866.
5. E. Garcia, A. Hernandez-Galeana, D. Jaramillo, W. A. Ponce and A. Zepeda, Revista Mexicana de Fisica Vol. **48(1)** (2002) 32; E. Garcia, A. Hernandez-Galeana, A. Vargas and A. Zepeda, hep-ph/0203249.
6. N. Chen, T. A. Ryttov, and R. Shrock, arXiv:1010.3736 [hep-ph]; C. T. Hill and E. H. Simmons, Phys. Rept. **381** (2003) 235; *Workshop on Dynamical Electroweak Symmetry Breaking*, Southern Denmark Univ. 2008 (http://hep.sdu.dk/dewsb); R.S. Chivukula, M. Narain, and J. Womersley, in Particle Data Group, J. Phys. G **37** (2010) 1340; F. Sannino, Acta Phys. Polon. B **40** (2009) 3533 (arXiv:0911.0931).
7. A. Hernandez-Galeana, Rev. Mex. Fis. **Vol. 50(5)**, (2004) 522. hep-ph/0406315.
8. A. Hernandez-Galeana, Bled Workshops in Physics, **Vol. 11, No. 2**, (2010) Pag. 60; arXiv:1012.0224; Bled Workshops in Physics,**Vol. 10, No. 2**, (2009) Pag. 67; arXiv:0912.4532;
9. For review and some recent works see for example:
B. Holdom, W.S. Hou, T. Hurth, M. Mangano, S. Sultansoy and G. Unel, "Beyond the 3-generation SM in the LHC era" Workshop, CERN, September 4-5, 2008; arXiv:0904.4698 and references therein.
10. N.S. Mankoč Borštnik, Bled Workshops in Physics,**Vol. 11, No. 2**, (2010) Pag. 89 , Pag. 105; G. Bregar, R.F. Lang and N.S. Mankoč Borštnik, Pag.161; M. Y. Khlopov and N.S. Mankoč Borštnik, Pag.177; arXiv:1012.0224; N.S. Mankoč Borštnik, Bled Workshops in Physics, **Vol. 10, No. 2**, (2009) Pag. 119; G. Bregar and N.S. Mankoč Borštnik, Pag. 149; arXiv:0912.4532;
11. M. Y. Khlopov, A. G. Mayorov, and E. Y. Soldatov; arXiv:1111.3577; M. Y. Khlopov, A. G. Mayorov, and E. Y. Soldatov, Bled Workshops in Physics, **Vol. 11, No. 2**, (2010) Pag. 73; Pag. 185; arXiv:0912.4532; M. Y. Khlopov, A. G. Mayorov, and E. Y. Soldatov, Bled Workshops in Physics, **Vol. 10, No. 2**, (2009) Pag. 79; M.Y. Khlopov, Pag. 155; arXiv:0912.4532;
12. Jonathan M. Arnold, Bartosz Fornal and Michael Trott; arXiv:1005.2185 and references therein.
13. J. Kopp, M. Maltoni and T. Schwetz, Phys. Rev. Lett. **107** (2011) 091801; C. Giunti, arXiv:1111.1069; 1107.1452; F. Halzen, arXiv:1111.0918; Wei-Shu Hou and Fei-Fan Lee, arXiv:1004.2359; O. Yasuda, arXiv:1110.2579; Y.F. Li and Si-shuo Liu, arXiv:1110.5795; B. Bhattacharya, A. M. Thalapillil, and C. E. M. Wagner, arXiv:1111.4225.
14. T. Yanagida, Phys. Rev. D **20** (1979) 2986.
15. A.Hernandez-Galeana, Phys.Rev. **D 76** (2007) 093006.
16. K. Nakamura *et al.*, Particle Data Group, J. Phys. G: Nucl. Part. Phys. **37** (2010) 075021.

BLED WORKSHOPS IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond ... *(p. 55)*
Bled, Slovenia, July 11-21, 2011

7 Masses and Mixing Matrices of Families of Quarks and Leptons Within the Spin-Charge-Family-Theory, Predictions Beyond the Tree Level

A. Hernández-Galeana[1] and N. S. Mankoč Borštnik[2]

[1]Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N., U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México.
[2]Department of Physics, FMF, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

Abstract. The *theory unifying spin and charges and predicting families*, proposed by N.S.M.B., predicts at the low energy regime two (in the mixing matrix elements decoupled) groups of four families. There are two kinds of contributions to mass matrices in this theory. One kind distinguishes on the tree level only among the members of one family, that is among the u-quark, d-quark, neutrino and electron, the left and right handed, while the other kind distinguishes only among the families. Mass matrices for d-quarks and electrons are on the tree level correspondingly strongly correlated and so are mass matrices for u-quarks and neutrinos, up to the term, the Majorana term, which is nonzero only for right handed neutrinos. Beyond the tree level both kinds of contributions start to contribute coherently and it is expected that a detailed study of properties of mass matrices beyond the tree level explains drastic differences in masses and mixing matrices between quarks and leptons. We report in this paper on analysis of one loop corrections to the tree level fermion masses and mixing matrices. Loop diagrams are mediated by the gauge bosons and the two kinds of scalar fields. A detailed numerical analysis of fermion masses and mixing, including neutrinos, within this scenario is in progress and preparation.

7.1 Introduction

The *theory unifying spin and charges and predicting families* (hereafter named the *spin-charge-family-theory* [1–3]), proposed by N.S. Mankoč Borštnik, seems promising to show the right way beyond the *standard model* of fermions and bosons. The reader is kindly asked to learn more about this theory in the refs. [3,1,2] and in the references therein. Following analyses of the ref. [3], we here repeat the parts which are necessary for understanding the starting assumptions and the conclusions to which one loop corrections beyond the tree level lead. We look at the two loop corrections and present for the case that each group of four families would decouple into two times two families numerical results with two loop corrections as well. Some of this results can be found in [4].

The *spin-charge-family-theory* predicts eight massless families of quarks and leptons before the two successive breaks – first from $SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SU(2)_I \times U(1)_I \times SU(3)$ and then from $SU(2)_I \times U(1)_I \times SU(3)$ to $U(1) \times$

$SU(3)$. Mass matrices originate in a simple starting action: They are determined on the tree level by nonzero vacuum expectation values of scalar (with respect to $SO(1,3)$) fields, to which vielbeins and two kinds of spin connection fields contribute. One kind of the spin connection fields includes fields gauging S^{ab}, which are determined by the Dirac gammas (γ^a's), another kind gauges $\tilde{S}^{ab}$, determined by the second kind of gammas $\tilde{\gamma}^a$'s, used in the *spin-charge-family-theory* [1–3] to generate families. Each of the two breaks is triggered by different (orthogonal) superposition of scalar fields. To the first break, besides vielbeins, only the spin connections of one kind contribute. To the second break all the scalar fields contribute.

The mass matrices for eight families appear to be four times four by diagonal matrices, with no mixing matrix elements among the upper four and the lower four families (not in comparison with the life of the universe) also after the two breaks: The upper four families are namely doublets with respect to two $SU(2)$ invariant subgroups (with respect to $SU(2)_{II}$, with generators of the infinitesimal transformations $\vec{\tilde{\tau}}^2$, and the one of the two $SU(2)$ subgroups of $SO(1,3)$, the subgroup $SU(2)_R$ with the generators of the infinitesimal transformations $\vec{\tilde{N}}_R$) of the group defined by $\tilde{S}^{ab}$, and singlets with respect two the other two invariant subgroups ($SU(2)_I$, with the generators $\vec{\tilde{\tau}}^1$, and the $SU(2)_L$, with the generators $\vec{\tilde{N}}_L$). The lower four families are doublets with respect to the two subgroups, the singlets of which are the upper four families.

There are, correspondingly, two stable families: the fifth and the observed first family. The fifth family members are candidates to form the dark matter, the fourth family waits to be observed.

After the first of the two successive breaks (the break from $SO(1,3)\times SU(2)_{II}\times SU(2)_I\times U(1)_{II}\times SU(3)$ in both sectors, S^{ab} and $\tilde{S}^{ab}$, to $SO(1,3)\times SU(2)_I\times U(1)_I\times SU(3)$), which occurs, below $\approx 10^{13}$ GeV, the upper four families become massive. In the second break, which is the *standard model*-like electroweak break, also the lower four families became massive. The second break influences also the mass matrices of the upper four families, although the influence is expected to be small.

Rough estimations made so far [2,5,6] on the tree level, which took into account besides the elementary particle data also the cosmological data, show that the stable of the upper four families might have masses [6] of the order of 100 TeV/c^2. (The ref. [10] discusses also a possibility that the masses are much smaller, of around a few TeV/c^2.) For the lower four families [2,5] we were not really able to predict the masses of the fourth family members, we only estimated for chosen masses of the fourth family members their mixing matrices.

In this paper we are studying, following suggestions from the ref. [3], properties of the mass matrices of twice four families, evaluating loops corrections to the tree level. We namely hope to see already within the one and may be two loops corrections the explanation for the differences in masses and mixing matrices between quarks and leptons, as well as within quarks and within leptons. To the loop corrections the gauge boson fields and both kinds of the scalar field contribute, as explained in the ref. [3].

7.2 Short review of the spin-charge-family-theory

Let us here make a short review of the *spin-charge-family-theory*. The simple starting action for spinors (and gauge fields in $d = (1 + 13)$, ref. [3], Eqs. (3,4)) manifests at the low energy regime after several breaks of symmetries as the effective action (see the ref. [3], Eq. (5)) for eight families of quarks and leptons (ψ), left and right handed

$$\begin{aligned}\mathcal{L}_f = &\bar{\psi}\gamma^n(p_n - \sum_{A,i} g^A\tau^{Ai}A^{Ai}_n)\psi + \\ &\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\ &\text{the rest},\end{aligned} \tag{7.1}$$

where $n = 0, 1, 2, 3$ and

$$\begin{aligned}\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}, \\ \{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB} f^{Aijk}\tau^{Ak}.\end{aligned} \tag{7.2}$$

All the charge (τ^{Ai}, Eq. (7.2)) and the spin ($S^{nn'}; n, n' \in \{0, 1, 2, 3\}$) operators are expressible with S^{ab}. S^{ab} are generators of spin degrees of freedom in $d = (1+13)$, determining all the internal degree of freedom of one family members. Index A enumerates all possible spinor charges and g^A is the coupling constant to a particular gauge vector field A^{Ai}_n, as well as to a scalar field A^{Ai}_s, $s > 3$.

Before the break from $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ $\vec{\tau}^3$ describes the colour charge ($SU(3)$) [1], $\vec{\tau}^1$ the weak charge ($SU(2)_I$) [2], $\vec{\tau}^2$ the second $SU(2)_{II}$ charge [3], and τ^4 determines the $U(1)_{II}$ charge [4]. After the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ $A = 2$ denotes the $U(1)_I$ hyper charge Y $(= \tau^4 + \tau^{23})$ and after the second break of $SU(2)_I \times U(1)_I$ to $U(1)$ $A = 2$ denotes the electromagnetic charge Q $(= S^{56} + \tau^4)$, while instead of the weak charge Q' $(= \tau^{13} - \tau^4 \tan^2\theta_1)$ and $\tau^{1\pm}$ of the *standard model* manifest.

The term in the second row of Eq. (7.1) determines mass matrices of twice four families

$$\begin{aligned}\mathcal{L}_{m_f} &= \psi^\dagger\gamma^0 M\psi \\ &= \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi \;=\; \psi^\dagger\gamma^0\,(\overset{78}{(+)}\; p_{0+} + \overset{78}{(-)}\; p_{0-}\,\psi\,, \\ p_{0s} &= f^\sigma{}_s p_{0\sigma} + \frac{1}{2E}\{p_\alpha, Ef^\alpha{}_a\}_-\,,\; p_{0\sigma} = p_\sigma - \frac{1}{2}S^{ab}\omega_{ab\sigma} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\sigma}\,, \\ \overset{78}{(\pm)} &= \frac{1}{2}\,(\gamma^7 \pm i\gamma^8),\;\; p_{0\pm} = p_{07} \mp i\,p_{08}\,.\end{aligned} \tag{7.3}$$

[1] $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$,

[2] $\vec{\tau}^1 = \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})$

[3] $\vec{\tau}^2 = \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$

[4] $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$

The main argument to take $s = 7, 8$, is (so far) the required agreement with the experimental data. The Dirac spin, described by γ^a's, defines the spinor representations in $d = (1 + 13)$. The second kind of the spin [7,8,3], described by $\tilde{\gamma}^a$'s ($\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$) and anticommuting with the Dirac γ^a ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$), defines the families of spinors. One finds [3]

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0,$$
$$S^{ab} := (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a), \quad \tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a), \quad \{S^{ab}, \tilde{S}^{cd}\}_- = 0. \tag{7.4}$$

The eight massless families ($2^{(1+7)/2-1}$) manifest after the break of $SO(1,7)$ to $SO(1,3) \times SO(4)$ (the break occurs in both sectors, S^{ab} and $\tilde{S}^{ab}$) as twice four families: Four of the families are doublets with respect to two of the four $SU(2)$ invariant subgroups of the groups $SO(4) \times SO(1,3)$ in the $\tilde{S}^{ab}$ sector (namely, with respect to the subgroups with the infinitesimal generators $\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$) and singlets with respect to the remaining two $SU(2)$ invariant subgroups (with the infinitesimal generators $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_{RL}$), while the remaining four families are singlets with respect to the first two and doublets with respect to the remaining two invariant subgroups. At the symmetry level of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ twice four families are massless, the mass matrices M of any family member is equal to zero.

The break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ in both, S^{ab} and $\tilde{S}^{ab}$, sectors is caused by the scalar fields $\vec{\tilde{A}}^2_s$ and $\vec{\tilde{A}}^{\tilde{N}_R}_s$ [5], which gain nonzero vacuum expectation values and determine the mass matrices $\mathcal{M}_{(o)}$ (Eq. 7.12) on the tree level. Only families which couple to these scalar fields become massive. These are four families, which are doublets with respect to the subgroups with generators of the infinitesimal transformations $\vec{\tilde{\tau}}^{(2)}$ $(= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}))$ and $\vec{\tilde{N}}_R$ $(= \frac{1}{2}(\tilde{S}^{23} - i\tilde{S}^{01}, \tilde{S}^{31} - i\tilde{S}^{02}, \tilde{S}^{12} - i\tilde{S}^{03}))$.

The rest four families, which are singlets with respect to these two subgroups, remain massless until the second break of $SU(2)_I \times U(1)_I$ to $U(1)$, in which the scalar fields $\vec{\tilde{A}}^1_s$ and $\vec{\tilde{A}}^{\tilde{N}_L}_s$ gain nonzero vacuum expectation values. These scalar fields couple to the rest four families through $\vec{\tilde{\tau}}^{(1)}$ $(= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}))$ and $\vec{\tilde{N}}_L$ $(= \frac{1}{2}(\tilde{S}^{23} + i\tilde{S}^{01}, \tilde{S}^{31} + i\tilde{S}^{02}, \tilde{S}^{12} + i\tilde{S}^{03}))$. To this, the electroweak break, also scalar fields in the S^{ab} sector contribute. These fields - $A^Q_s, A^{Q'}_s, A^{Y'}_s$ - couple to the family members through the quantum numbers Q, Q' and Y', respectively. While $\vec{\tilde{\tau}}^{(2)}$, $\vec{\tilde{\tau}}^{(1)}$, $\vec{\tilde{N}}_R$ and $\vec{\tilde{N}}_L$ distinguish among the families, but not among the family members, distinguish Q, Q' and Y' among the family members independent of the family index.

[5] The vielbeins and the spin connections are both involved in breaks.

After the break of $SU(2)_{II} \times U(1)_{II} \times SU(2)_I \times SU(3)$ into $SU(2)_I \times U(1)_I \times SU(3)$ the effective Lagrange density for spinors is as follows

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,(\gamma^m\, p_{0m} - M)\psi\,,\\
p_{0m} &= p_m - \{g^1 \vec{\tau}^1\, \vec{A}^1_m + g^Y Y A^Y_m + g^3\, \vec{\tau}^3\, \vec{A}^3_m\\
&\quad + g^2 \cos\theta_2\, Y' A^{Y'}_m + \frac{g^2}{\sqrt{2}}\,(\tau^{2+} A^{2+}_m + \tau^{2-} A^{2-}_m)\}\\
\bar{\psi}\, M\, \psi &= \bar{\psi}\gamma^s\, p_{0s}\,\psi\,,\\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_R} \vec{\tilde{N}}_R \vec{\tilde{A}}^{\tilde{N}_R}_s + \tilde{g}^{\tilde{Y}'}\, \tilde{Y}' \tilde{A}^{\tilde{Y}'}_s + \frac{\tilde{g}^2}{\sqrt{2}}\,(\tilde{\tau}^{2+}\, \tilde{A}^{2+}_s + \tilde{\tau}^{2-}\, \tilde{A}^{2-}_s)\}\,. \quad (7.5)
\end{aligned}$$

In the second row the vector gauge fields which remain massless ($\vec{A}^1_m$, A^Y_m and $\vec{A}^3_m$) and in the third row the massive gauge fields ($A^{Y'}_m$ and $A^{2\pm}_m$) are presented. To the mass matrices of the upper four families $\bar{\psi}\, M\psi$ the vacuum expectation values of the scalar fields $\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\tilde{A}^{\tilde{Y}'}_s$ and $\tilde{A}^{2\pm}_s$, together with the corresponding vielbeins with the scalar index, contribute. The new and the old gauge fields are related as follows: $A^{23}_m = A^Y_m \sin\theta_2 + A^{Y'}_m \cos\theta_2$, $A^4_m = A^Y_m \cos\theta_2 - A^{Y'}_m \sin\theta_2$, $A^{2\pm}_m = \frac{1}{\sqrt{2}}(A^{21}_m \mp i A^{22}_m)$, with the new quantum numbers $Y = \tau^4 + \tau^{23}$, $Y' = \tau^{23} - \tau^4 \tan^2\theta_2$, $\quad \tau^{2\pm} = \tau^{21} \pm i\tau^{22}$ and the new coupling constants of fermions to the massive gauge fields and the massless one become $g^Y = g^4 \cos\theta_2$, $g^{Y'} = g^2 \cos\theta_2$, $\tan\theta_2 = \frac{g^4}{g^2}$, while $A^{2\pm}_m$ have a coupling constant $\frac{g^2}{\sqrt{2}}$.

The new and the old scalar fields are related as: $\tilde{A}^{23}_s = \tilde{A}^{\tilde{Y}}_s \sin\tilde{\theta}_2 + \tilde{A}^{\tilde{Y}'}_s \cos\tilde{\theta}_2$, $\tilde{A}^4_s = \tilde{A}^{\tilde{Y}}_s \cos\tilde{\theta}_2 - \tilde{A}^{\tilde{Y}'}_s \sin\tilde{\theta}_2$, $\tilde{A}^{2\pm}_s = \frac{1}{\sqrt{2}}(\tilde{A}^{21}_s \mp i\tilde{A}^{22}_s)$, while it follows $\vec{\tilde{A}}^2_s = 2(\tilde{\omega}_{58s}, \tilde{\omega}_{57s}, \tilde{\omega}_{56s})$. We shall make a choice in this paper of $\tilde{\theta}_2 = 0$. We also have $\vec{\tilde{A}}^{\tilde{N}_R}_s = 2(\tilde{\omega}_{23s}, \tilde{\omega}_{31s}, \tilde{\omega}_{12s})$, and $\vec{\tilde{N}}_R = \frac{1}{2}\,(\tilde{S}^{23} - i\tilde{S}^{01}, \tilde{S}^{31} - i\tilde{S}^{02}, \tilde{S}^{12} - i\tilde{S}^{03})$, for $s = 7, 8$. The new family quantum numbers are $\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}$, $\tilde{Y}' = \tilde{\tau}^{23} - \tilde{\tau}^4 \tan^2\tilde{\theta}_2$, $\tilde{\tau}^{2\pm} = \tilde{\tau}^{21} \pm i\tilde{\tau}^{22}$.

The reader is kindly asked to look at the ref. [3] for more explanations.

We present in Table 7.1 (from Table VIII. of the ref. [3]) a general shape of mass matrices of all the eight families on the tree level after the break of $SO(2)_{II} \times U(1)_{II}$ into $U(1)_I$. The lower four families stay massless. The u-quark mass matrices (they are determined by $\vec{\tilde{A}}^{\tilde{A}}_{-} = \vec{\tilde{A}}^{\tilde{A}}_7 + i\vec{\tilde{A}}^{\tilde{A}}_8$, for $\tilde{A} = 2, 1, \tilde{N}_R, \tilde{N}_L$) are different than the d-quark ones (they are determined by $\vec{\tilde{A}}^{\tilde{A}}_{+} = \vec{\tilde{A}}^{\tilde{A}}_7 - i\vec{\tilde{A}}^{\tilde{A}}_8$) and e mass matrices differ from the ν ones, while mass matrices for quarks and leptons are identical (ref.[3], they are the same for u-quarks and neutrinos, and for d quarks and electrons. The contribution of the scalar fields causing the Majorana right handed neutrinos (see appendix 7.6) is not added in this table. The contributions below the tree level change the matrix elements and remove the degeneracy between the u-quarks and neutrinos as well as between the d-quarks and electrons. It is expected that they will not appreciably change the symmetry of the matrix elements on the tree level. We shall discuss this in the next section.

To the electroweak break, when $SU(2)_I \times U(1)_I$ breaks into $U(1)$, besides the scalar fields originating in vielbeins and in superposition of spin connection fields of $\tilde{S}^{ab}$ (the ones, which are orthogonal to the ones causing the first break), also the

Σ_i	I_1	I_2	I_3	I_4	II_1	II_2	II_3	II_4
I_1	0	0	0	0	0	0	0	0
I_2	0	0	0	0	0	0	0	0
I_3	0	0	0	0	0	0	0	0
I_4	0	0	0	0	0	0	0	0
II_1	0	0	0	0	$-\frac{1}{2}\,(\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{\tilde{N}^-_R}_{\pm}$	0	$-\tilde{a}^{2-}_{\pm}$
II_2	0	0	0	0	$-\tilde{a}^{\tilde{N}^+_R}_{\pm}$	$\frac{1}{2}(-\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{2-}_{\pm}$	0
II_3	0	0	0	0	0	$-\tilde{a}^{2+}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{23}_{\pm} - \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{\tilde{N}^-_R}_{\pm}$
II_4	0	0	0	0	$-\tilde{a}^{2+}_{\pm}$	0	$-\tilde{a}^{\tilde{N}^+_R}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$

Table 7.1. The mass matrices on the tree level ($\mathcal{M}_{(o)}$) for two groups ($\Sigma = II$ for the upper four, while $\Sigma = I$ for the lower four) families of quarks and leptons after the break of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$. The contribution comes from a particular superposition of spin connection fields, the gauge fields of $\tilde{S}^{ab}$. $(\mp)$ distinguishes u_i from d_i and ν_i from e_i.

scalar fields originating in spin connections of S^{ab} contribute: A^Q_s, $A^{Q'}_s$ and $A^{Y'}_s$. The three additional gauge fields and the lower four families become massive.

The new superposition of gauge fields $\vec{A}^1_m$ and $A^{Y'}_m$ manifest (ref. [3]) leading to one massless A_m ($\equiv A^Q_m$) and three massive gauge fields $A^{Q'}_m$ ($\equiv Z_m$), $A^{\pm}_m$ ($\equiv W^{\pm}_m$).

The effective Lagrange density for spinors is after the electroweak break as follows

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,(\gamma^m\, p_{0m} - M)\psi\,,\\
p_{0m} &= p_m - \{e\,Q\,A_m + g^1\cos\theta_1\,Q'\,Z^{Q'}_m + \frac{g^1}{\sqrt{2}}\,(\tau^{1+}\,W^{1+}_m + \tau^{1-}\,W^{1-}_m) + \\
&\quad + g^2\cos\theta_2\,Y'\,A^{Y'}_m + \frac{g^2}{\sqrt{2}}\,(\tau^{2+}\,A^{2+}_m + \tau^{2-}\,A^{2-}_m)\,,\\
\bar{\psi}\,M\,\psi &= \bar{\psi}\,\gamma^s\,p_{0s}\psi\\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_R}\,\vec{\tilde{N}}_R\,\vec{\tilde{A}}^{\tilde{N}_R}_s + \tilde{g}^{\tilde{Y}'}\,\tilde{Y}'\,\tilde{A}^{\tilde{Y}'}_s + \frac{\tilde{g}^2}{\sqrt{2}}\,(\tilde{\tau}^{2+}\,\tilde{A}^{2+}_s + \tilde{\tau}^{2-}\,\tilde{A}^{2-}_s)\\
&\quad + \tilde{g}^{\tilde{N}_L}\,\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}_s + \tilde{g}^{\tilde{Q}'}\,\tilde{Q}'\,\tilde{A}^{\tilde{Q}'}_s + \frac{\tilde{g}^1}{\sqrt{2}}\,(\tilde{\tau}^{1+}\,\tilde{A}^{1+}_s + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_s)\\
&\quad + e\,Q\,A_s + g^1\,\cos\theta_1\,Q'\,Z^{Q'}_s + g^2\cos\theta_2\,Y'\,A^{Y'}_s\,\}\,,\;\; s \in \{7,8\}\,. \qquad (7.6)
\end{aligned}
$$

The reader is kindly asked to learn how does the operators $\stackrel{78}{(-)}$ transform the weak and the hyper charge of the right handed u_R-quark and ν_R-lepton into those of the left handed ones, while $\stackrel{78}{(+)}$ does the same for the right handed d_R-quark and e_R-lepton, in the ref. [3]. One can learn there also how do the operators $\tilde{N}^{\pm}_R$, $\tilde{N}^{\pm}_L$, $\tilde{\tau}^{2\pm}$ and $\tilde{\tau}^{1\pm}$ transform any member of one family into the same member of another family and what transformations cause any superposition of the operators S^{ab} or of the operators $\tilde{S}^{ab}$ on any family member of any family. A short presentation of these properties is added in the appendix 7.7.

The new fields manifest ($A^{13}_{m} = A_{m} \sin\theta_1 + Z_{m} \cos\theta_1$, $A^{Y}_{a} = A_{m} \cos\theta_1 - Z_{m} \sin\theta_1$, $W^{\pm}_{m} = \frac{1}{\sqrt{2}}(A^{11}_{m} \mp i A^{12}_{m})$), with the new quantum numbers ($Q = \tau^{13} + Y = S^{56} + \tau^4$, $Q' = -Y \tan^2\theta_1 + \tau^{13}$, $\tau^{1\pm} = \tau^{11} \pm i\,\tau^{12}$ and the new coupling constants ($e = g^{Y} \cos\theta_1$ $(\equiv g^{Q})$, $g' = g^{1} \cos\theta_1$ $(\equiv g^{Q'})$ and $\tan\theta_1 = \frac{g^{Y}}{g^{1}}$), are in agreement with the *standard model*. Correspondingly there are new scalar field, new quantum numbers and new coupling constants.

But there are also clearly noticeable differences between the *spin-charge-family-theory* and the *standard model*, presented in Eq. (7.6), which should be sooner or later measurable. Like: i.) The scalar fields explaining the appearance i.a.) of the Higgs in the *standard model*, in the *spin-charge-family-theory* with weak and hyper charges in the adjoint representation since γ^s do what in the *standard model* the weak and hyper charge of the Higgs do, and i.b.) of the Yukawa couplings, which manifest here as new interactions (but this is the case also in the *standard model*). The scalar fields should be measured as several fields, although they effectively manifest as Higgs and Yukawa couplings. ii.) New gauge vector fields $A^{Y'}_{m}$, $A^{2\pm}_{m}$. ii.) New families predicted. iii.) New gauge scalar fields for the upper four families. iv.) New insight into the discrete symmetries, like charge conjugation, parity, charge parity (non conserved) symmetry, matter/anti-matter asymmetry and others.

The scalar fields $\vec{\tilde{A}}^{\tilde{A}}_{s}$, $s \in \{7,8\}$, which gain in this phase transition a nonzero vacuum expectation values, are: $\vec{\tilde{A}}^{\tilde{N}_L}_{s}$ $(= \frac{1}{2}\,(\tilde{\omega}_{23s} - i\,\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\,\tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i\,\tilde{\omega}_{03s})$, $\vec{\tilde{A}}^{1}_{s}$ (again expressible with $\tilde{\omega}_{abs}$, $s \in \{7,8,9,10\}$, transforming as follows $\tilde{A}^{13}_{s} = \tilde{A}_{s} \sin\tilde{\theta}_1 + \tilde{Z}^{\tilde{Q}'}_{s} \cos\tilde{\theta}_1$ $(\tilde{Z}^{\tilde{Q}'}_{s} \equiv \tilde{A}^{\tilde{Q}'}_{s}$, $\tilde{A}_{s} \equiv \tilde{A}^{\tilde{Q}}_{s})$, $\tilde{A}^{\tilde{Y}}_{s} = \tilde{A}_{s} \cos\tilde{\theta}_1 - \tilde{Z}^{\tilde{Q}'}_{s} \sin\tilde{\theta}_1$, $\tilde{W}^{\pm}_{s} = \frac{1}{\sqrt{2}}(\tilde{A}^{11}_{s} \mp i\tilde{A}^{12}_{s})$ $(\tilde{W}^{\pm}_{s} \equiv \tilde{A}^{1\pm}_{s})$, and A^{Q}_{s} $(\equiv A_{s})$, $A^{Q'}_{s}$ $(\equiv Z^{Q'}_{s})$, $A^{Y'}_{s}$, with Q, Q' and Y' defined above and with

$$\vec{\tilde{N}}_{(L,R)} = (\frac{1}{2}(\tilde{S}^{23}\,(+,-)i\tilde{S}^{01}, \tilde{S}^{31}\,(+,-)i\tilde{S}^{02}, \tilde{S}^{12}\,(+,-)i\tilde{S}^{03}).$$

Let us point out again that the upper four families are singlets with respect to $\vec{\tilde{N}}_{L}$ and $\vec{\tilde{\tau}}^{1}$, while the lower four families are singlets with respect to $\vec{\tilde{N}}_{R}$ and $\vec{\tilde{\tau}}^{2}$. At each break the mass matrices on the tree level $\mathcal{M}_{(0)}$ change.

Table 7.2 represents the mass matrices for the lower four families on the tree level. Only the contribution of the scalar fields which originate in the gauge fields of $\tilde{S}^{ab}$ are included into the table. The contribution from terms like $Q\,e\,A^{Q}_{s}$, $g^{Q'}\,Q'\,A^{Q'}_{s}$, $g^{Y'}\,Y'\,A^{Y'}_{s}$, which are diagonal and equal for all the families but distinguish among the members of one family, are not present. The contribution of the scalar fields causing the Majorana right handed neutrinos (see appendix 7.6) is also not added in this table.

The notation $\tilde{a}^{\tilde{A}i}_{\pm} = -\tilde{g}^{\tilde{A}}\,\tilde{A}^{\tilde{A}i}_{\pm}$ is used.

There is a mass term within the *spin-charge-family-theory*, which transform the right handed neutrino to his charged conjugated one, contributing to the (right handed) neutrino Majorana masses. The Majorana terms are expected to be large and might influence strongly the neutrino masses and their mixing matrices. The reader can find more explanation about this term in ref. [3] and in appendix 7.6.

Let us add here, that it is nonzero only for the lower four families. It needs to be studied in more details to say more. These terms are not yet included into Table 7.2.

I_i	1	2	3	4
1	$-\frac{1}{2}\,(\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{\tilde{N}^-_L}_{\pm}$	0	$\tilde{a}^{1-}_{\pm}$
2	$\tilde{a}^{\tilde{N}^+_L}_{\pm}$	$\frac{1}{2}(-\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{1-}_{\pm}$	0
3	0	$\tilde{a}^{1+}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{13}_{\pm} - \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{\tilde{N}^-_L}_{\pm}$
4	$\tilde{a}^{1+}_{\pm}$	0	$\tilde{a}^{\tilde{N}^+_L}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$

Table 7.2. The mass matrices on the tree level ($\mathcal{M}_{(o)}$) for the lower four families ($\Sigma = I$) of quarks and leptons after the electroweak break. Only the contributions coming from the spin connection fields, originating in $\tilde{S}^{ab}$ are presented. $(\mp)$ distinguishes between the values of the u-quarks and d-quarks and between the values of ν and e. The notation $\tilde{a}^{\tilde{A}i}_{\pm} = -\tilde{g}^{\tilde{A}}\,\tilde{A}^{\tilde{A}i}_{\pm}$ is used. The terms coming from spin connection fields originating in $S^{ss'}$ are not presented here. They are the same for all the families, but different for different family members. Also possible Majorana terms are not included.

We present in Table 7.3 the quantum numbers $\tilde{\tau}^{23}$, $\tilde{N}^3_R$, $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$ for all eight families [3]. The first four families are singlets with respect to $\tilde{\tau}^{23}$ and $\tilde{N}^3_R$, while they are doublets with respect to $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$. The upper four families are doublets with respect to $\tilde{\tau}^{23}$ and $\tilde{N}^3_R$ and are singlets with respect to $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$. The representations of families in the technique of the ref. [9] are presented in appendix 7.7, in Table 7.6.

$\Sigma = I/i$	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$	$\tilde{\tau}^{13}$	$\tilde{N}^3_L$	$\Sigma = II/i$	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$	$\tilde{\tau}^{13}$	$\tilde{N}^3_L$
1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0
2	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	2	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
3	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	3	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
4	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	4	$-\frac{1}{2}$	$\frac{1}{2}$	0	0

Table 7.3. The quantum numbers $\tilde{\tau}^{23}$, $\tilde{N}^3_R$, $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$ for the two groups ($\Sigma = II$ for the upper four families and $\Sigma = I$ for the lower four families) of four families are presented [3].

In Table 7.4 we present quantum numbers of all members of a family, any one, after the electroweak break.

When going below the tree level all the massive gauge fields and those scalar fields of both origins, (S^{ab} and $\tilde{S}^{ab}$), to which the family members couple, start to contribute. To the lower four families mass matrices the scalar fields, which are superposition of the $\omega_{sts'}$ field, that is of A^Q_s, $A^{Q'}_s$ and $A^{Y'}_s$, contribute already on the tree level. This was not the case for the upper four families. Contributions of QA^Q_s, $Q'A^{Q'}_s$ and $Y'A^{Y'}_s$ distinguish among all the members of one family, but are the same for a family member belonging to different families. They influence

	Y	Y'	Q	Q'		Y	Y'	Q	Q'
u_R	$\frac{2}{3}$	$\frac{1}{2}\,(1-\frac{1}{3}\tan^2\theta_2)$	$\frac{2}{3}$	$-\frac{2}{3}\tan^2\theta_1$	u_L	$\frac{1}{6}$	$-\frac{1}{6}\tan^2\theta_2$	$\frac{2}{3}$	$\frac{1}{2}(1-\frac{1}{3}\tan^2\theta_1)$
d_R	$-\frac{1}{3}$	$-\frac{1}{2}\,(1+\frac{1}{3}\tan^2\theta_2)$	$-\frac{1}{3}$	$\frac{1}{3}\tan^2\theta_1$	d_L	$\frac{1}{6}$	$-\frac{1}{6}\tan^2\theta_2$	$-\frac{1}{3}$	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\theta_1)$
ν_R	0	$\frac{1}{2}\,(1+\tan^2\theta_2)$	0	0	ν_L	$-\frac{1}{2}$	$\frac{1}{2}\tan^2\theta_2$	0	0
e_R	-1	$\frac{1}{2}\,(-1+\tan^2\theta_2)$	-1	$\tan^2\theta_1$	e_L	$-\frac{1}{2}$	$\frac{1}{2}\tan^2\theta_2$	-1	$-\frac{1}{2}(1-\tan^2\theta_1)$

Table 7.4. The quantum numbers Y, Y', Q, Q' of the members of one family (anyone) [3].

after the second break also the mass matrices of the upper four families. Below the tree level all the gauge fields and dynamical scalar fields start to contribute coherently, as dictated by Eq. (7.6). These contributions are expected to be large for the lower four families, while they influence, since the scale of these two breaks are supposed to be very different, only slightly the upper four family mass matrices. According to the estimations presented in refs. ([3,6]) the changes are within a percent or much less if the masses are large enough (of the order of hundred TeV/c^2 or larger).

We study in this paper properties of both groups of four families, taking the vacuum expectation values of the scalar fields as an input. As we already explained, in the *spin-charge-family-theory* the mass matrices of the family members are within each of the two groups very much correlated. It is the prediction of this theory [3] that there are terms beyond the tree level, which are responsible for the great differences in properties of the family members for the observed three families. It is a hope [3] that the mass matrices can be expressed as follows

$$M = \sum_{k=0,k'=0,k"=0}^{\infty} Q^k\, Q'^{k'}\, Y'^{k"}\, M_{Q\,Q'\,Y'\,kk'k"}\,, \tag{7.7}$$

where Q, Q' and Y' are the operators while the matrices $M_{Q'Y'\,kk'}$ do not, hopefully, depend on the family member, that is that they might be the same for all the members of one family. To neutrino an additional mass matrix might be added, which is zero for all the other family members if the Majorana contribution is taken into account.

While for the lower four families the contributions which depend on Q, Q' (and Y') quantum numbers of each of a family member are expected to be large, this should not be the case for the upper four families (in comparison with the contributions on the tree level).

In the next section we present the loop contributions to the three level mass matrices. The contributions originate in two kinds of scalar fields, namely in $\tilde{\omega}_{abs}$ and in $\omega_{stt'}$, and in the massive gauge fields and affect both groups of four families. First we analyse the effect of one and two loops corrections for the case, that each of four families would decouple into twice 2×2 mass matrices, under the assumption that the lower two families of each group of four families weakly couple to the upper two families of the same group. This assumption seems meaningful from the point of view of mass matrices on the tree level, presented on tables 7.1, 7.2, as well as from the experimental data for the lower three families. The measured values of the mixing matrices for the observed families supports such an assumption for quarks, but not for leptons. We neglect accordingly for this first step the nonzero mass matrix elements between the lower and the upper two families for each group. We then proceed to take into account one loop corrections for all four families of each of the two groups.

7.3 Mass matrices beyond the tree level

It is the purpose of this section (and also of this paper) to manifest that, although in the *spin-charge-family-theory* the matrix elements of different family members are within each of the two groups of four families on the tree level very much correlated, the loop corrections lead to mass matrices, which manifest great differences in properties of the lowest three families.

We show that the one loop corrections originating in the massive gauge fields change masses of families, while they leave mixing matrices unchanged. One loop corrections originating in dynamical scalar fields change both, masses and mixing matrices.

Let us repeat the assumptions [3]: **i.** In the break from $SU(2)_{I}\times SU(2)_{II}\times U(1)_{II}$ to $SU(2)_{I}\times U(1)_{I}$ the superposition of the $\tilde{\omega}_{abs}$ scalar fields which are the gauge fields of $\vec{\tilde{\tau}}^{2}$ and $\vec{\tilde{N}}_{R}$ gain non zero vacuum expectation values, causing nonzero mass matrices for fermions. The lower four families, which do not couple to these scalar fields, remain massless. **ii.** In the electroweak break the superposition of the $\tilde{\omega}_{abs}$ scalar fields which are the gauge fields of $\vec{\tilde{\tau}}^{1}$ and $\vec{\tilde{N}}_{L}$, and the superposition of scalar fields ω_{abs} which are the gauge fields of Q, Q' and Y' gain nonzero vacuum expectation values. (While the scalar gauge fields of Q, Q' and Y' influence masses of all the eight families, the scalar gauge fields of $\vec{\tilde{\tau}}^{1}$ and $\vec{\tilde{N}}_{L}$ influence only the lower four families.) **iii.** There is also a term in loop corrections of a very special products of superposition of $\omega_{abs}, s=5,6,9,\cdots,14$ and $\tilde{\omega}_{abs}$, $s=5,6,7,8$ scalar fields, which couple only to the right handed neutrinos and their charge conjugated states of the lower four families, which might change drastically the properties of neutrinos of the lower four families.

Let us clarify the notation. We have before the two breaks two times ($\Sigma\in\{II, I\}$, II denoting the upper four and I the lower four families) four massless vectors $\psi^{\alpha}_{\Sigma(L,R)}$ for each member of a family $\alpha=\in\{u,d,\nu,e\}$. Let i, $i\in\{1,2,3,4\}$, denotes one of the four family members of each of the two groups of massless families

$$\left(\psi^{\alpha}_{\Sigma(L,R)}\right)^{T}=(\psi^{\alpha}_{\Sigma\,1},\,\psi^{\alpha}_{\Sigma\,2},\,\psi^{\alpha}_{\Sigma\,3},\,\psi^{\alpha}_{\Sigma\,4})_{(L,R)}\,. \tag{7.8}$$

Hence, we have for the lowest four families ($\Sigma=I$) and the u family member ($\alpha=u$)

$$\left(\psi^{u}_{I(L,R)}\right)^{T}=(u,\,c,\,t,\,u_{4})_{(L,R)}\;,$$

u_4 to be recognized as the new, that is the fourth family member. We then have

$$\overline{\psi}^{u}_{I\,L}\,\mathcal{M}^{u\,I}_{(o)}\,\psi^{u}_{I\,R}=\overline{\psi}^{u}_{I\,L\,i}\,\mathcal{M}^{u\,I}_{(o)\,ij}\,\psi^{u}_{I\,R\,j}\,. \tag{7.9}$$

Let $\Psi^{\alpha}_{\Sigma(L,R)}$ be the final massive four vectors for each of the two groups of families, with all loop corrections included

$$\psi^{\alpha}_{\Sigma\,(L,R)} = V^{\alpha}_{\Sigma}\, \Psi^{\alpha}_{\Sigma\,(L,R)}\,,\qquad V^{\alpha}_{\Sigma} = V^{\alpha}_{\Sigma\,(o)}\, V^{\alpha}_{\Sigma\,(1)} \cdots V^{\alpha}_{\Sigma\,(k)} \cdots . \tag{7.10}$$

Then $\Psi^{\alpha\,(1)}_{\Sigma\,(L,R)}$ includes one loop corrections and $\Psi^{\alpha\,(k)}_{\Sigma(L,R)}$ up to (k) loops corrections

$$V^{\alpha}_{\Sigma\,(o)}\, \Psi^{\alpha\,(o)}_{\Sigma\,(L,R)} = \psi^{\alpha}_{\Sigma\,(L,R)}\,,\qquad V^{\alpha}_{\Sigma\,(o)}\, V^{\alpha}_{\Sigma\,(1)} \cdots V^{\alpha}_{\Sigma\,(k)}\, \Psi^{\alpha\,(k)}_{\Sigma(L,R)} = \psi^{\alpha}_{\Sigma(L,R)}\,. \tag{7.11}$$

From the starting action the mass matrices on the tree level follow as presented in Tables (7.1, 7.2). Not being able (yet) to calculate these matrix elements, we take them as parameters. Not (yet) paying attention to the CP non conservation, we assume in this paper that mass matrices are real and symmetric.

We calculate in this paper one and for a simplified version of two decoupled 2×2 families of a four family group two loops corrections to the tree level mass matrices.

The mass matrices, originating in the vacuum expectation values of the scalar fields which are superposition of $\tilde{\omega}_{abd}$ fields (appearing as $\tilde{g}^{\tilde{N}_R}\, \vec{\hat{\tilde{N}}}_R \vec{\tilde{A}}^{\tilde{N}_R}_s$, $\tilde{g}^{\tilde{Y}'}\, \hat{\tilde{Y}}'\, \tilde{A}^{\tilde{Y}'}_s$, $\frac{\tilde{g}^2}{\sqrt{2}}\, \hat{\tilde{\tau}}^{2\pm}\, \tilde{A}^{2\pm}_s$, $\tilde{g}^{\tilde{N}_L}\, \vec{\hat{\tilde{N}}}_L \vec{\tilde{A}}^{\tilde{N}_L}_s$, $\tilde{g}^{\tilde{Q}'}\, \hat{\tilde{Q}}'\, \tilde{A}^{\tilde{Q}'}_s$, $\frac{\tilde{g}^1}{\sqrt{2}}\, \hat{\tilde{\tau}}^{1\pm}\, \tilde{A}^{1\pm}_s$, the reader can find the application of these operators on family members in appendix 7.9), are in this paper assumed to be real and symmetric. On the tree level they manifest as the two by diagonal 4×4 matrices with the symmetry on the tree level presented bellow

$$\mathcal{M}_{(o)} = \begin{pmatrix} -a_1 & e & 0 & b \\ e & -a_2 & b & 0 \\ 0 & b & a_1 & e \\ b & 0 & e & a_2 \end{pmatrix}, \tag{7.12}$$

with the matrix elements $a_1 \equiv a^{\Sigma}_{\pm\,1}$, $a_2 \equiv a^{\Sigma}_{\pm 2}$, $b \equiv b^{\Sigma}_{\pm}$ and $e \equiv e^{\Sigma}_{\pm}$, which are different for the upper ($\Sigma = II$) than for the lower ($\Sigma = I$) four families

$$a_1 = \frac{1}{2}(\tilde{a}^3_{\pm} - \tilde{a}^{\tilde{N}3}_{\pm}) \quad , \quad a_2 = \frac{1}{2}(\tilde{a}^3_{\pm} + \tilde{a}^{\tilde{N}3}_{\pm}) \quad , \quad b = \tilde{a}^{+}_{\pm} = \tilde{a}^{-}_{\pm} \quad , \quad e = \tilde{a}^{\tilde{N}+}_{\pm} = \tilde{a}^{\tilde{N}-}_{\pm}\,. \tag{7.13}$$

The matrix elements for the upper four families ($\Sigma = II$) are: $\tilde{a}^3_{\pm} = \tilde{a}^{23}_{\pm}$, $\tilde{a}^{\tilde{N}3}_{\pm} = \tilde{a}^{\tilde{N}_R 3}_{\pm}$, $\tilde{a}^{\pm}_{\pm} = \tilde{a}^{21}_{\pm} \pm i\, \tilde{a}^{22}_{\pm}$, $\tilde{a}^{\tilde{N}\pm}_{\pm} = \tilde{a}^{\tilde{N}_R 1}_{\pm} \pm i\, \tilde{a}^{\tilde{N}_R 2}_{\pm}$. For the lower four families ($\Sigma = I$) we must take $\tilde{a}^3_{\pm} = \tilde{a}^{13}_{\pm}$, $\tilde{a}^{\tilde{N}3}_{\pm} = \tilde{a}^{\tilde{N}_L 3}_{\pm}$, $\tilde{a}^{\pm}_{\pm} = \tilde{a}^{11}_{\pm} \pm i\, \tilde{a}^{12}_{\pm}$, $\tilde{a}^{\tilde{N}\pm}_{\pm} = \tilde{a}^{\tilde{N}_L 1}_{\pm} \pm i\, \tilde{a}^{\tilde{N}_L 2}_{\pm}$. $(\pm)$ in the denominator distinguishes between the matrix elements for the pair (d and e) $(+)$ and the pair (u and ν) $(-)$. $\bar{\psi} M \psi$ in Eq. (7.1) can, namely, be expressed as

$$\begin{aligned} \bar{\psi} M \psi &= \sum_{s=7,8} \bar{\psi} \gamma^s\, p_{0s}\, \psi = \psi^{\dagger}\, \gamma^0\, (\overset{78}{(-)}\; p_{0-} + \overset{78}{(+)}\; p_{0+})\psi\,, \\ \overset{78}{(\pm)} &= \frac{1}{2}\, (\gamma^7\; \pm i \gamma^8)\,, \\ p_{0\pm} &= (p_{07} \mp i\, p_{08})\,, \quad s \in \{7, 8\}. \end{aligned} \tag{7.14}$$

The reader is kindly asked to learn how do the operators ($\overset{78}{(\mp)}$, any superposition of the operators S^{ab}, or any superposition of the operators $\tilde{S}^{ab}$) apply on any family member of any family in the ref. [3] and in the appendix 7.7, where a short presentation of these properties is made.

To the tree level contributions of the scalar $\tilde{\omega}_{ab\pm}$ fields, diagonal matrices have to be added, the same for all the eight families and different for each of the family member (u, d, ν, e), $a_{\mp} \equiv a^{\alpha}_{\mp}$, which are the tree level contributions of the scalar $\omega_{sts'}$ fields

$$a_{\mp} = e\, Q\, A_{\mp} + g^1\, \cos\theta_1\, Q'\, Z^{Q'}_{\mp} + g^2\, \cos\theta_2\, Y'\, A^{Y'}_{\mp}\,. \tag{7.15}$$

Q, Q' and Y' stay for the eigenvalues of the operators $\hat{Q}, \hat{Q}'$ and $\hat{Y}'$ of the right handed α member of any of the families [6]. Therefore, the tree level mass matrices $\mathcal{M}^{\alpha\,\Sigma}_{(o)}$ are different for the upper ($\Sigma = $ II) than for the lower ($\Sigma = $ I) four families and they are also different for the pairs of (d, e) and (u, ν), but are the same for u and ν and for d and e before adding $a^{\alpha}_{\mp}\, I_{8\times 8}$, which is different for each family member. The matrices M^{α} are indeed 8×8 matrices with two by diagonal 4×4 matrices also after the loops corrections included. The parameters Eq. (7.13), which enter into the tree level mass matrices after the assumptions explained at the beginning of this section, are presented in Table 7.5.

Σ	α							
II	(u, ν)	$\tilde{a}^{\tilde{N}_R 3}_{-}$	$\tilde{a}^{\tilde{N}_R \pm}_{-}$	$\tilde{a}^{23}_{-}$	$\tilde{a}^{2\pm}_{-}$	a^{Q}_{-}	$a^{Q'}_{-}$	$a^{Y'}_{-}$
II	(d, e)	$\tilde{a}^{\tilde{N}_R 3}_{+}$	$\tilde{a}^{\tilde{N}_R \pm}_{+}$	$\tilde{a}^{23}_{+}$	$\tilde{a}^{2\pm}_{+}$	a^{Q}_{+}	$a^{Q'}_{+}$	$a^{Y'}_{+}$
I	(u, ν)	$\tilde{a}^{\tilde{N}_L 3}_{-}$	$\tilde{a}^{\tilde{N}_L \pm}_{-}$	$\tilde{a}^{13}_{-}$	$\tilde{a}^{1\pm}_{-}$	a^{Q}_{-}	$a^{Q'}_{-}$	$a^{Y'}_{-}$
I	(d, e)	$\tilde{a}^{\tilde{N}_L 3}_{+}$	$\tilde{a}^{\tilde{N}_L \pm}_{+}$	$\tilde{a}^{13}_{+}$	$\tilde{a}^{1\pm}_{+}$	a^{Q}_{+}	$a^{Q'}_{+}$	$a^{Y'}_{+}$

Table 7.5. The parameters entering into the tree level mass matrices are presented. The notation $\tilde{a}^{\tilde{A}i}_{\pm} = -\tilde{g}^{\tilde{A}}\, \tilde{A}^{\tilde{A}i}_{\pm}$ (staying for $-\tilde{g}^{\tilde{N}_R}\, \tilde{A}^{\tilde{N}_R i}_{\pm}$, $-\tilde{g}^2\, \tilde{A}^{2i}_{\pm}$, $-\tilde{g}^{\tilde{N}_L}\, \tilde{A}^{\tilde{N}_L i}_{\pm}$, $-\tilde{g}^1\, \tilde{A}^{1i}_{\pm}$), $a^{Q}_{\mp} = g^{Q}\, A^{Q}_{\mp}$ $a^{Q}_{\mp} = g^{Q'}\, A^{Q'}_{\mp}$, $a^{Y'}_{\mp} = g^{Y'} A^{Y'}_{\mp}$ is used.

On the tree level we have

$$\mathcal{M}^{\alpha}_{(o)} = \begin{pmatrix} \mathcal{M}^{\alpha\,\mathrm{II}}_{(o)} & 0 \\ 0 & \mathcal{M}^{\alpha\,\mathrm{I}}_{(o)} \end{pmatrix}\,. \tag{7.16}$$

Since the upper and the lower four family mass matrices appear at two completely different scales, determined by two orthogonal sets of scalar fields, have the two tree level mass matrices $\mathcal{M}^{\alpha\,\Sigma}_{(o)}$ very little in common, only the symmetries and the contributions from Eq. (7.15).

On the tree level we have $\psi^{\alpha}_{\Sigma\,(L,R)} = V^{\alpha}_{\Sigma\,(o)}\, \Psi^{\alpha\,(o)}_{\Sigma\,(L,R)}$ and

$$< \psi^{\alpha}_{\Sigma\,L}|\gamma^0\, \mathcal{M}^{\alpha\,\Sigma}_{(o)}\, |\psi^{\alpha}_{\Sigma\,R} >=< \Psi^{\alpha\,(o)}_{\Sigma\,L}|\gamma^0\, V^{\alpha\,\dagger}_{\Sigma\,(o)}\, \mathcal{M}^{\alpha\,\Sigma}_{(o)}\, V^{\alpha}_{\Sigma(o)}\, |\Psi^{\alpha\,(o)}_{\Sigma\,R\,(o)} >, \tag{7.17}$$

[6] We shall put the operator sign $\hat{O}$ on the operator O only when it is needed so that we can distinguish between the operators and their eigenvalues.

from where the tree level mass eigenvalues follow

$$\mathcal{M}^{\alpha\,\Sigma}_{(o)D} = V^{\alpha\,\dagger}_{\Sigma\,(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o)}\,V^{\alpha}_{\Sigma\,(o)} = \mathrm{diag}(m^{\alpha\,\Sigma}_{(o)\,1}, m^{\alpha\,\Sigma}_{(o)\,2}, m^{\alpha\,\Sigma}_{(o)\,3}, m^{\alpha\,\Sigma}_{(o)\,4}). \tag{7.18}$$

The one loop corrections leads to $\psi^{\alpha}_{\Sigma\,(L,R)} = V^{\alpha}_{\Sigma\,(o)}\,\Psi^{\alpha\,(o)}_{\Sigma\,(L,R)} = V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma\,(1)}\,\Psi^{\alpha\,(1)}_{\Sigma\,(L,R)}$ and $\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)}$ include all the one loop corrections evaluated among the massless states, so that

$$< \psi^{\alpha}_{\Sigma L}|\gamma^0\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)}\,|\psi^{\alpha}_{\Sigma\,R} >=< \Psi^{\alpha\,(o)}_{\Sigma\,L}|\gamma^0\,V^{\alpha\,\dagger}_{\Sigma\,(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)}\,V^{\alpha}_{\Sigma\,(o)}\,|\Psi^{\alpha\,(o)}_{\Sigma R} > . \tag{7.19}$$

The mass matrix including up to one loop corrections is

$$\mathcal{M}^{\alpha\,\Sigma}_{(1)} = V^{\alpha\,\dagger}_{\Sigma\,(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)}\,V^{\alpha}_{\Sigma\,(o)} + M^{\alpha\,\Sigma}_{(o)\,D} = V^{\alpha\,\dagger}_{\Sigma\,(o)}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V^{\alpha}_{\Sigma\,(o)}\,. \tag{7.20}$$

Thus the contribution up to one loop is

$$< \Psi^{\alpha\,(o)}_{\Sigma\,L}|\gamma^0\,V^{\alpha\,\dagger}_{\Sigma\,(o)}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)}))\,V^{\alpha}_{\Sigma\,(o)}\,|\Psi^{\alpha\,(o)}_{\Sigma\,R} >,$$

which can be written as

$$\begin{aligned} &< \psi^{\alpha}_{\Sigma\,L}|\gamma^0\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,|\psi^{\alpha}_{\Sigma\,R} >= \\ &< \Psi^{\alpha\,(1)}_{\Sigma\,L}|\gamma^0\,(V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma\,(1)})^{\dagger}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma\,(1)}|\Psi^{\alpha\,(1)}_{\Sigma\,R} >, \end{aligned} \tag{7.21}$$

with $V^{\alpha}_{\Sigma\,(1)}$ which is obtained from

$$\begin{aligned} \mathcal{M}^{\alpha\,\Sigma}_{(1)\,D} = V^{\alpha\,\dagger}_{\Sigma\,(1)}\,\left[V^{\alpha\,\dagger}_{\Sigma\,(o)}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V_{\alpha\,\Sigma\,(o)}\right]\,V^{\alpha}_{\Sigma\,(1)} = \\ \mathrm{diag}(m^{\alpha\,\Sigma}_{(1)\,1}, m^{\alpha\,\Sigma}_{(1)\,2}, m^{\alpha\,\Sigma}_{(1)\,3}, m^{\alpha\,\Sigma}_{(1)\,4})\,, \end{aligned} \tag{7.22}$$

with $m^{\alpha\,\Sigma}_{(1)\,i}$, $i \in \{1,2,3,4\}$, the mass eigenvalues, which include one loop corrections.

Taking into account corrections up to (k) loops, we have

$$\begin{aligned} &< \psi^{\alpha}_{\Sigma\,L}|\gamma^0\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,k)} + \cdots + \mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,|\psi^{\alpha}_{\Sigma\,R} >= \\ &< \Psi^{\alpha\,(k)}_{\Sigma\,L}|\gamma^0\,(V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma\,(1)}\,\cdots V^{\alpha}_{\Sigma\,(k)})^{\dagger}\,(\mathcal{M}^{\alpha\,\Sigma}{}_{(o\,k)} + \cdots \\ &+\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\;V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma(1)}\,\cdots V^{\alpha}_{\Sigma\,(k)}\,|\Psi^{\alpha\,(k)}_{\Sigma\,R} > . \end{aligned} \tag{7.23}$$

$V^{\alpha}_{\Sigma\,(k)}$ follows from

$$\begin{aligned} &\mathcal{M}^{\alpha\,\Sigma}_{(k)D} = V^{\alpha\,\dagger}_{\Sigma\,(k)}\,[V^{\alpha\,\dagger}_{\Sigma\,(k-1)}\,\cdots \\ &V^{\alpha\,\dagger}_{\Sigma\,(1)}\,V^{\alpha\,\dagger}_{\Sigma\,(o)}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,k)} + \cdots + \mathcal{M}^{\alpha\,\Sigma}_{(o\,1)} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V^{\alpha}_{\Sigma\,(o)}\,V^{\alpha}_{\Sigma\,(1)}\cdots V^{\alpha}_{\Sigma\,(k-1)}\,]\,V^{\alpha}_{\Sigma\,(k)} \\ &= \mathrm{diag}(m^{\alpha\,\Sigma}_{(k)\,1}, \cdots m^{\alpha\,\Sigma}_{(k)\,4}), \end{aligned} \tag{7.24}$$

with $m^{\alpha\,\Sigma}_{(k)\,i}$, $i \in \{1,2,3,4\}$, the mass eigenvalues of the states, which take into account up to (k) loops corrections.

In what follows we shall use the indices Σ and α only when we explicitly calculate mass matrices for a particular group of families and for a particular member, otherwise we shall assume that both indices are all the time present and we shall skip both. Eq.(7.18) will, for example, accordingly read

$$\mathcal{M}_{(o)D} = V^{\dagger}_{(o)}\,\mathcal{M}_{(o)}\,V_{(o)} = diag(m_{(o)\,1}, m_{(o)\,2}, m_{(o)\,3}, m_{(o)\,4})\,, \tag{7.25}$$

with the indices Σ and α assumed, but not written. Similarly Eq. (7.10) reads

$$\begin{aligned} \psi_{(L,R)} &= V\,\Psi_{(L,R)}\,, \\ V &= V_{(o)}\,V_{(1)}\,\cdots\,V_{(k)}\,\cdots\,, \end{aligned} \tag{7.26}$$

connecting the massless ψ and the massive Ψ with all the loop corrections included. In our case, since $\mathcal{M}_{(o)}$ are in this paper assumed to be real and symmetric, $V^{\dagger}_{(o)} = V^{T}_{(o)}$.

Loop corrections (with the gauge and dynamical scalar fields contributing coherently) are expected to cause differences in mass matrices among the family members of the lower four families, and will hopefully explain the experimental data for the so far observed three families of quarks and leptons. The differences among the family members of the upper four families are expected to be small even after taking into account loop corrections, since the contributions to the loop corrections, which distinguish among family members, originate in the ω_{sta} dynamical massive fields, the scalar and vector ones, whose masses can only be of the order of the electroweak scale and this is expected to be for orders of magnitude smaller than the scale of the break of symmetry which brings masses to the upper four families. The only exception is $\tau^{2i} A^{2i}_{m}$.

The contribution which transforms the right handed neutrino into his charged conjugate one, influences only the lower four families, because, by the assumption, the superposition of the $\tilde{\omega}_{abs}$ fields couple only to the lower four families [3]. A short explanation is presented in appendix 7.6.

In appendix 7.7 the matrix $\mathcal{M}_{(o)}$ (Eq.(7.12)) is diagonalized for a general choice of matrix elements, assuming that the matrix is real and symmetric, with the symmetry on the tree level as presented in Eq.(7.12). A possible non hermiticity of the mass matrices on the tree level is neglected. The diagonalizing matrix is presented.

In the ref. [5] the authors, assuming that loop corrections (drastically) change mass matrix elements as they follow on the tree level from the *spin-charge-family-theory*, keep the symmetries of mass matrices as dictated by the *spin-charge-family-theory* on the tree level and fit the mass matrix elements for the lower four families to existing experimental data for a particular choice of masses of the fourth family members.

In this paper we make one loop corrections to the tree level mass matrices, demonstrating that loop corrections may contribute to the tree level mass matrices to the experimentally acceptable direction.

We calculate loop corrections originating in two kinds of the scalar dynamical fields, those originating in $\tilde{\omega}_{abs}$ ($\tilde{g}^{\tilde{Y}'}\ \tilde{Y}'\,\tilde{A}^{\tilde{Y}'}_{s}$, $\frac{\tilde{g}^2}{\sqrt{2}}\,\tilde{\tau}^{2\pm}\,\tilde{A}^{2\pm}_{s}$, $\tilde{g}^{\tilde{N}_{L,R}}\,\vec{\tilde{N}}_{L,R}\,\vec{\tilde{A}}^{\tilde{N}_{L,R}}_{s}$, $\tilde{g}^{\tilde{Q}'}\ \tilde{Q}'\,\tilde{A}^{\tilde{Q}'}_{s}$, $\frac{\tilde{g}^1}{\sqrt{2}}\,\tilde{\tau}^{1\pm}\,\tilde{A}^{1\pm}_{s}$) and those originating in ω_{abs} ($e\,Q\,A_s$, $g^1\cos\theta_1 Q' Z^{Q'}_{s}$, $g^{Y'}\,Y'\,A^{Y'}_{s}$) and in the massive gauge fields ($g^2\cos\theta_2\,Y'\,A^{Y'}_{m}$, $g^1\ \cos\theta_1\,Q'\,Z^{Q'}_{m}$) as it follow from Eq.(7.6).

In appendix 7.8 the corresponding loop corrections are calculated in a general form, which enables to distinguish among members of the scalar fields of both kinds and of the massive gauge fields. The corresponding loop diagrams are presented in Figures 7.1,7.2,7.3.

Fig. 7.1 shows the one loop diagram for the contribution of the terms, either $(-\gamma^0 \stackrel{78}{(\mp)} \frac{\tilde{g}^2}{\sqrt{2}} \hat{\tilde{\tau}}^{2\pm} \tilde{A}^{2\pm}_{\mp})$ or $(-\gamma^0 \stackrel{78}{(\mp)} \frac{\tilde{g}^{\tilde{N}_R}}{\sqrt{2}} \hat{\tilde{N}}^{\pm}_R \tilde{A}^{\tilde{N}_R\pm}_{\mp})$, both presented in Eq.(7.6), when the upper four families are treated. For the lower four families the same diagram shows the one loop corrections induced by either $(-\gamma^0 \stackrel{78}{(\mp)} \frac{\tilde{g}^1}{\sqrt{2}} \hat{\tilde{\tau}}^{1\pm} \tilde{A}^{1\pm}_{\mp})$ or $(-\gamma^0 \stackrel{78}{(\mp)} \frac{\tilde{g}^{\tilde{N}_L}}{\sqrt{2}} \hat{\tilde{N}}^{\pm}_L \tilde{A}^{\tilde{N}_L\pm}_{\mp})$, Eq.(7.6). These fields couple the family members as it is presented in Tables (7.1, 7.2) and demonstrated in the diagram

$$\stackrel{\stackrel{\vec{\tilde{N}}_L}{\leftrightarrow}}{\begin{pmatrix} I_4 & I_3 \\ I_1 & I_2 \end{pmatrix}} \updownarrow \vec{\tilde{\tau}}^1 \quad , \quad \stackrel{\stackrel{\vec{\tilde{N}}_R}{\leftrightarrow}}{\begin{pmatrix} II_4 & II_3 \\ II_1 & II_2 \end{pmatrix}} \updownarrow \vec{\tilde{\tau}}^2 \quad . \tag{7.27}$$

The term $(-\gamma^0 \stackrel{78}{(-)} \frac{\tilde{g}^2}{\sqrt{2}} \hat{\tilde{\tau}}^{2-} \tilde{A}^{2-}_{-})$ applies to u-quarks [ν-leptons], transforming the eighth family right handed u-quark [ν-lepton] (II_4 in the right diagram of Eq.(7.27)) into the fifth left handed one (II_1 in Eq.(7.27)) and the seventh family right handed u-quark [ν-lepton] (II_3) into the sixth family left handed one (II_2), for example. While the term $(-\gamma^0 \stackrel{78}{(-)} \frac{\tilde{g}^{\tilde{N}_R}}{\sqrt{2}} \hat{\tilde{N}}^{-}_R \tilde{A}^{\tilde{N}_R-}_{-})$ transforms the eighth family right handed u-quark [ν-lepton] (II_4 in the right diagram of Eq.(7.27)) into the seventh left handed one (II_3 in Eq.(7.27)) and the sixth family right handed u-quark [ν-lepton] (II_2 in Eq.(7.27)) into the fifth family left handed on (II_1) and equivalently for the lower four families. That is, the term $(-\gamma^0 \stackrel{78}{(-)} \frac{\tilde{g}^1}{\sqrt{2}} \hat{\tilde{\tau}}^{1-} \tilde{A}^{1-}_{-})$ transforms the fourth family right handed u-quark [ν-lepton] (I_4 in the left diagram of Eq.(7.27))) into the first left handed one (I_1) and the third family right handed u-quark [ν-lepton] (I_3) into the second family left handed one (I_2) . The term $(-\gamma^0 \stackrel{78}{(-)} \frac{\tilde{g}^{\tilde{N}_L}}{\sqrt{2}} \hat{\tilde{N}}^{-}_L \tilde{A}^{\tilde{N}_L-}_{-})$ transforms correspondingly the fourth family right handed u-quark [ν-lepton] (I_4) into the third family left handed one (I_3) .

Correspondingly Fig. 7.1 represents the one loop diagrams for the d-quark and e-leptons for either the upper of the lower four families if $(-\gamma^0 \stackrel{78}{(-)} \frac{\tilde{g}^A}{\sqrt{2}} \hat{\tilde{\tau}}^{A\pm} \tilde{A}^{A\pm}_{-})$ (where index A denotes 2 or $\tilde{N}_R$ for the upper four families, and $\hat{\tilde{\tau}}^{Ai}$ correspondingly $\hat{\tilde{\tau}}^{2i}$ and $\hat{\tilde{N}}^{i}_R$, and 1 or $\tilde{N}_L$ for the lower four families, and $\hat{\tilde{\tau}}^{Ai}$ correspondingly $\hat{\tilde{\tau}}^{1i}$ and $\hat{\tilde{N}}^{i}_L$) is replaced by $(-\gamma^0 \stackrel{78}{(+)} \frac{\tilde{g}^A}{\sqrt{2}} \hat{\tilde{\tau}}^{A\pm} \tilde{A}^{A\pm}_{+})$, Eq. (7.6).

In Fig. 7.2 the terms which in the loop corrections contribute to diagonal matrix elements of the u-quarks [ν-leptons] and d-quarks [e-leptons] of each of the four members of the upper four and the lower four families are presented. Similarly as in Fig. 7.1, the terms $(-\gamma^0 \stackrel{78}{(\mp)} \tilde{g}^2 \hat{\tilde{\tau}}^{23} \tilde{A}^{23}_{\mp})$ and $(-\gamma^0 \stackrel{78}{(\mp)} \tilde{g}^{\tilde{N}_R} \tilde{N}^3_R \tilde{A}^{\tilde{N}^3_R}_{\mp})$, (Eq. (7.6)) contribute to the upper four families distinguishing among families and among the family members pairs (u , ν), (−), and (d , e), (+), while the terms

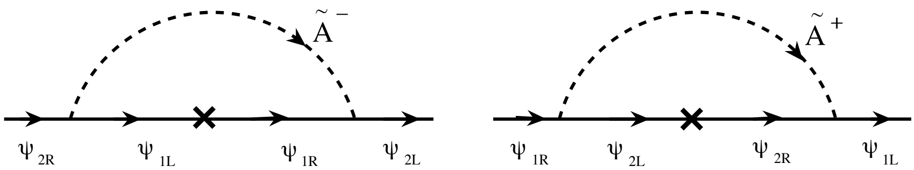

Fig. 7.1. One loop contributions originating in $\tilde{A}^{\pm}$ scalar fields, where $\tilde{A}^{\pm}$ stays for $\tilde{A}^{2\pm}_{\pm}$ or $\tilde{A}^{\tilde{N}_R\pm}_{\pm}$ when the upper four families are treated, while it stays for $\tilde{A}^{1\pm}_{\pm}$ or $\tilde{A}^{\tilde{N}_L\pm}_{\pm}$ when the lower four families are treated. Each of the massless states $\psi_{i(R,L)}$ in the figure, staying instead of $\psi^{\alpha}_{\Sigma(R,L)i}$ where $\Sigma = II$ determines the upper four families group membership and $\Sigma = I$ the lower four families group membership, should correspondingly carry also the family member index $\alpha = (u, d, \nu, e)$ the group family index $\Sigma = II, I$, where II denotes the upper four and I for the lower four families, besides the family index $i = (1, 2, 3, 4)$, which distinguishes among the familes within each of the two groups.

$(-\gamma^0 \stackrel{78}{(\mp)} \tilde{g}^1 \hat{\tilde{\tau}}^{13} \tilde{A}^{13}_{\mp})$ and $(-\gamma^0 \stackrel{78}{(\mp)} \tilde{g}^{\tilde{N}_L} \hat{\tilde{N}}^3_L \tilde{A}^{\tilde{N}^3_L}_{\mp})$ contribute to the mass terms of the lower four families.

The eigenvalues of the operators $\hat{\tilde{\tau}}^{2\,3}$, $\hat{\tilde{N}}^3_R$, $\hat{\tilde{\tau}}^{1\,3}$, and $\hat{\tilde{N}}^3_L$ are presented in Table 7.3.

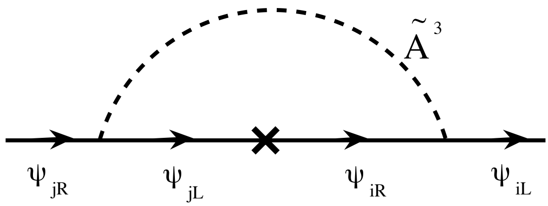

Fig. 7.2. One loop contributions originating in $\tilde{A}^3$ scalar fields, where $\tilde{A}^3$ stays for $\tilde{A}^{23}_{\pm}$ or $\tilde{A}^{\tilde{N}_R 3}_{\pm}$ when the upper four families are treated, while it stays for $\tilde{A}^{13}_{\pm}$ or $\tilde{A}^{\tilde{N}_L 3}_{\pm}$ when the lower four families are treated. The rest of comments are the same as in Fig. 7.1. Each of states carries also the family member index α, the Σ index determining one of the two four families groups and the index i which distinguishes among the families within each of the two groups of four families.

The same Fig. 7.2 represents also the one loop contribution of the dynamical scalar fields originating in S^{ab}, namely of $e\,\hat{Q}\,A_{\mp}$, $g^1\,\cos\theta_1\,\hat{Q}'\,Z^{Q'}_{\mp}$ and $g^2\cos\theta_2\,\hat{Y}'\,A^{Y'}_{\mp}$, $((-)$ for u-quarks and ν-leptons, $(+)$ for d-quarks and e-leptons), if these fields replace $\tilde{A}^3_{\mp}$. These diagonal terms are the same for all the four families of any of the two groups, but since the operators $\hat{Q}$, $\hat{Q}'$ and $\hat{Y}'$ have different eigenvalues on each of the family members (u, d, ν, e), Table 7.4, these matrix elements are different for different family members.

Fig. 7.3 represents the contribution of the massive gauge field $A^{Y'}_m$, originating in the dynamical part of the Lagrange density in Eq. (7.6) $(g^2\cos\theta_2\,\hat{Y}'\,A^{Y'}_m)$. Replacing $A^{Y'}_m$ by $Z^{Q'}_m$ the same figure represents also the contribution of the term $g^1\,\cos\theta_1\,\hat{Q}'\,Z^{Q'}_m$, Eq. (7.6). Both contributions distinguish among the family members and are the same for all the eight families. The quantum numbers Q' and Y' are presented in Table 7.4.

In all the loop corrections the strength of couplings ($\tilde{g}^{(2,1)}$, $\tilde{g}^{(\tilde{N}_R,\tilde{N}_L)}$), the application of the operators ($\hat{\tilde{\tau}}^{(2,1)i}$, $\hat{\tilde{N}}^{i}_{(R,L)}$), as well as the masses of the dynamical fields playing, as usually, an essential role, must be taken into account.

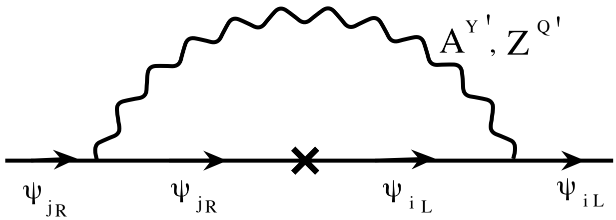

Fig. 7.3. One loop contributions from the massive gauge vector fields $A^{Y'}_m$ and $Z^{Q'}_m$. Each of states carry in addition to $i, j = (1, 2, 3, 4)$, which distinguishes the four members of each of the two group of four families, also the group index $\Sigma = II, I$, for the upper and the lower four families, respectively, and the family member index $\alpha = (u, d, \nu, e)$.

In appendix 7.8 the explicit evaluations of all the above discussed loop contributions are derived in a general form, that is as functions of parameters which determine a particular contribution.

The influence of a particular contribution to the mass matrices, and accordingly also to mixing matrices, depends strongly on whether the upper or the lower four families are concerned, on the family members involved and on the family quantum number of states involved in the corrections.

The final mass matrices, manifesting the Lagrange density $\psi^{\dagger}\gamma^{0}\gamma^{s}p_{0s}\psi$ with one loop corrections to which the scalar dynamical and massive gauge fields contribute, have the shape, presented in Eq. (7.21), (7.22), with $V_{\alpha\,\Sigma(1)}$ which is obtained from Eq. (7.22), and which correspondingly determines mixing matrices with the one loop corrections included, for each of the two groups of four families ($\Sigma = II, I$) and for each of the family member $\alpha \in \{u, d, \nu, e\}$. The graphic representation of these loop corrections can be seen in Figs. (7.1, 7.2, 7.3).

Matrices $\mathcal{M}^{\alpha\,\Sigma}_{(1)} = \mathcal{M}^{\alpha\,\Sigma}_{(1)\,\tilde{S}} + \mathcal{M}^{\alpha\,\Sigma}_{(1)\,S} + \mathcal{M}^{\alpha\,\Sigma}_{(1)\,V}$ are written in terms of the parameters presented in Tables (7.5, 7.7) and used in Eqs. [(7.46),(7.47), (7.51), and (7.65)], respectively, of appendix 7.8.

The tree level masses ($m^{\alpha\,\Sigma}_{(1)\,i}$, $i \in \{1, 2, 3, 4\}$) and diagonalizing matrices ($V_{\alpha\,\Sigma\,(o)}$) are presented in Eqs. (7.32, 7.33) of appendix 7.7 as functions of parameters from Table 7.5.

To the mass matrices up to one loop (Eqs.(7.20,7.43)) contribute7.49))

$$\begin{aligned} \mathcal{M}^{\alpha\,\Sigma}_{(1)} &= \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)\tilde{S}} + \mathcal{M}^{\alpha\,\Sigma}_{(1)S} + \mathcal{M}^{\alpha\,\Sigma}_{(1)V} + \mathcal{M}^{\alpha\,\Sigma}_{(o)D}\,, \\ &= V^{\alpha\,\dagger}_{\Sigma(o)}\,(\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o1)\tilde{S}} + \mathcal{M}^{\alpha\,\Sigma}_{(o1)S} + \mathcal{M}^{\alpha\,\Sigma}_{(o1)V} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V^{\alpha}_{\Sigma(o)}\,, \end{aligned} \tag{7.28}$$

where $\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)\tilde{S}}$ are contributions of the scalar gauge fields originating in $\tilde{\omega}_{abs}$, $\mathcal{M}^{\alpha\,\Sigma}_{(1)S}$ are contributions from $\omega_{sts'}$ and $\mathcal{M}^{\alpha\,\Sigma}_{(1)V}$ determine one loop corrections from the massive boson fields. The detailed calculations are done in appendices (7.7,7.8). From Eqs. (7.56, 7.55) one reads details for $\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)\tilde{S}} = V^{\alpha\,\dagger}_{\Sigma(o)}\ \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)\tilde{S}}\ V^{\alpha}_{\Sigma(o)}$. Details about $\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)S} = V^{\alpha\,\dagger}_{\Sigma(o)}\ \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)S}\ V^{\alpha}_{\Sigma(o)}$ are written in Eq. (7.63) and details about $\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)V} = V^{\alpha\,\dagger}_{\Sigma(o)}\ \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)V}\ V^{\alpha}_{\Sigma(o)}$ are written in Eq. (7.68).

To obtain masses and diagonalizing matrices for each family member α of both groups of families Σ the diagonalization (Eq. (7.22)) must be performed.

7.4 General properties of the mass matrices

All the expressions, needed for the evaluation of masses and diagonalizing matrices of each family member $\alpha = (u\,, d\,, \nu\,, e)$ for either the upper ($\Sigma = \mathrm{II}$) or the lower ($\Sigma = \mathrm{I}$) four families, on the tree level or with the loop corrections included, are presented in appendices 7.7 and 7.8. The final mass matrix including one loop corrections is the sum of the three matrices presented in Eqs. (7.56), 7.63, 7.68 of appendix 7.8. We take the mass matrix elements on the tree level as well as the masses of the scalar and gauge fields as free parameters, fitting them to the existing experimental data.

All the free parameters which determine the mass matrices on the tree level can be read from Eqs. (7.13, 7.15) and in Table 7.5. Since the contributions from the scalar fields $\tilde{\omega}_{abs}$ to the tree level mass matrix are the same for u-quark and ν-lepton and the same for d-quark and e-lepton, while they are different for each of these two pairs (matrix elements of $(u\,,\nu)$ differ from those of $(d\,,e)$), there are four free parameters due to these contributions and the additional three parameters which originate in the scalar ω_{abs} fields, all together therefore seven free parameters for each of the two pairs on the tree level.

The loop corrections originate in massive fields, that is in dynamical scalar and vector boson fields. The scalar fields $\tilde{g}^{\tilde{N}_R}\ \hat{\vec{\tilde{N}}}_R \vec{\tilde{A}}^{\tilde{N}_R}_{\mp}$ and $\tilde{g}^2\ \hat{\vec{\tilde{\tau}}}^2\ \vec{\tilde{A}}^2_{\mp}$ contribute to masses of the upper four families, while $\tilde{g}^{\tilde{N}_L}\ \hat{\vec{\tilde{N}}}_L \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$ and $\tilde{g}^1\ \hat{\vec{\tilde{\tau}}}^1\ \vec{\tilde{A}}^1_{\mp}$ contribute to masses of the lower four families. On the other side, the scalar fields contributions $e\,\hat{Q}\,A_{\mp}$, $g^{Q'}\ \hat{Q'}\,Z^{Q'}_{\mp}$ and $g^{Y'}\ \hat{Y'}\,A^{Y'}_{\mp}$, and the gauge fields contributions $g^{Y'}\ \hat{Y'}\,A^{Y'}_{m}$ and $g^{'}\ \hat{Q'}\,A^{Q'}_{m}$ "see" only the family member index α and not the family index i. Their masses and coupling constants are presented in Table 7.7 of appendix 7.8. We use their masses as free parameters as well.

Since there is no experimental data for the upper four families, we can try to learn from the proposed procedure by taking into account evaluations of properties for the fifth family quarks [6] more about the mass differences of the family members of the upper four families.

7.4.1 Properties of the lower two families for each of the two groups of four families below the tree level

We study the influence of one loop corrections on the mass matrices and mixing matrices of quarks and leptons for the 2×2 case, for $\tilde{a}^{i+}_{\pm} = \tilde{a}^{i-}_{\pm} = 0$, for $i = (2,1)$, $i = 2$ for $\Sigma = \mathrm{II}$ and $i = 1$ for $\Sigma = \mathrm{I}$. This assumption seems acceptable as a first step for the lower group of four families, while, since we have almost no knowledge about the upper four families (except rough estimations evaluated when using the *spin-charge-family-theory* to explain dark matter content of our universe and the direct measurements of the dark matter [6]), it is questionable for the upper group of four families.

Taking the results from appendix 7.7, Eq. (7.39), which is applicable for either the upper or the lower group of four families and for any family member, one recognizes that $m^{u\,\Sigma}_{(o)2} - m^{u\,\Sigma}_{(o)1} = m^{\nu\,\Sigma}_{(o)2} - m^{\nu\Sigma}_{(o)1} = \sqrt{(\tilde{a}^{i3}_{-})^2 + (2\,\tilde{a}^{i+}_{-})^2}$, $i = (2,1)$, for $\Sigma = (II, I)$, respectively, and $m^{d\,\Sigma}_{(o)2} - m^{d\,\Sigma}_{(o)1} = m^{e\,\Sigma}_{(o)2} - m^{e\,\Sigma}_{(o)1} = \sqrt{(\tilde{a}^{i3}_{+})^2 + (2\,\tilde{a}^{i+}_{+})^2}$, $i = (2,1)$, for $\Sigma = (II, I)$, respectively. This is in complete disagreement with the experimental data for u-quarks and neutrinos of the lowest two of the lower group of four families, and not so bad for d-quarks and electrons of the first two families, where it almost works, as it is well known. We namely have [11] $(m^s - m^d)$ $(= (m^{d\,I}_{(o)2} - m^{d\,I}_{(o)1})) = [\,(101.0 \pm 25) - (4.1 - 5.8)\,]$ MeV and $(m^{\mu} - m^{e})$ $(= m^{e\,I}_{(o)2} - m^{e\,I}_{(o)1}) = [(105.65837) - (0,5109989)\,]$ MeV. It is therefore on the loop corrections to correct the disagreements.

For the lowest two families there are three matrix elements on the tree level (Eqs. (7.13,7.15)), a_1 $(= -\frac{1}{2}(\tilde{a}^{3}_{\pm} - \tilde{a}^{\tilde{N}3}_{\pm}) + a_{\mp})$, a_2 $(= -\frac{1}{2}(\tilde{a}^{3}_{\pm} + \tilde{a}^{\tilde{N}3}_{\pm}) + a_{\mp})$, (Eq. (7.15)), with , $a_{\mp} = e\,Q\,A_{\mp} + g^1\, \cos\theta_1\, Q'\, Z^{Q'}_{\mp} + g^2\, \cos\theta_2\, Y'\, A^{Y'}_{\mp}$, and e $(= \tilde{a}^{\tilde{N}^{+}}_{\pm} = \tilde{a}^{\tilde{N}^{-}}_{\pm})$, which we shall take as free parameters. (The definition of $a_i, i = 1, 2$, is now slightly changed by taking into account contributions of Eqs. (7.13) and (7.15).

7.4.2 Properties of the two groups of four families below the tree level

We study the influence of one and two loop corrections on the mass matrices and correspondingly on masses and mixing matrices of quarks and leptons for the lower and the upper group of four families (7.6), taking as an input, that is as free parameters, the parameters from Tables 7.5, 7.7. The loop corrections due to two kinds of scalar fields and to massive gauge fields are presented in Figs. (7.1, 7.2, 7.3).

The tree level mass matrices of each group after the electroweak break is presented in Eq. (7.32). The one loop contributions originating in the scalar fields ($\tilde{g}^{\tilde{N}_R}\, \hat{\vec{\tilde{N}}}_R \vec{\tilde{A}}^{\tilde{N}_R}_{\mp}$, $\tilde{g}^2\, \hat{\vec{\tilde{\tau}}}^2\, \vec{\tilde{A}}^2_{\mp}$) must be added to the tree level mass matrices of the upper group of four families only, while those originating in the scalar fields ($\tilde{g}^{\tilde{N}_L}\, \hat{\vec{\tilde{N}}}_L \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$, $\tilde{g}^1\, \hat{\vec{\tilde{\tau}}}^1\, \vec{\tilde{A}}^1_{\mp}$) contribute to mass matrices of the lower four families only. Both are presented in appendix 7.8 in Eq. (7.56).

The contributions of the scalar fields ($e\,\hat{Q}\,A_{\mp}$, $g^1\, \cos\theta_1\, \hat{Q}'\, Z^{Q'}_{\mp}$, $g^2 \cos\theta_2\, \hat{Y}'\, A^{Y'}_{\mp}$) are presented in Eq.(7.63), these ones contribute to both, the upper four and the lower four families. The contributions to the upper four families is much weaker than to the lower four, due to much larger tree level masses of the upper four families. The contributions depend on the family members quantum numbers Q' and Y' and due to $\overset{78}{(\mp)}$ $p_{0\mp}$ distinguish also among (u, ν) and (d, e) pairs.

The massive gauge vector fields $g^1 \cos\theta_1 \,\hat{Q}' \,Z^{Q'}_m$, $g^2 \cos\theta_2 \,\hat{Y}' \,A^{Y'}_m$ contributions differ for different members of a family as well. Their influence on the upper four and the lower four family members depends again on the three level mass matrices. These contributions are presented in Eq.(7.68).

7.5 Discussion and Conclusions

We analysed in this paper the properties of twice four families as they follow from the *spin-charge-family-theory* when loop corrections, discussed in the ref. [3], are taken into account. Having experimental results only for the lowest three of the lower four families, most discussions in this paper concern the lower group of four families.

In the *spin-charge-family-theory* [3] fermions carry two kinds of spin and correspondingly interact with the two kinds of spin connection fields. One kind of spin determines at low energies, after several breaks of the starting symmetry, the spin and the charges of fermions, the second kind determines families.

After several breaks from $SO(1,13)$ to $SO(1,7) \times U(1) \times SU(3)$ and further to $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ there are eight massless families [7], which after the break to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ manifest as a massive and a massless group of four families. Correspondingly, after this break ($SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$) also vector bosons involved in this break, become massive. This break is (assumed to be) triggered by the superposition of the scalar fields $\tilde{S}^{ab}\,\tilde{\omega}_{abs}$, which are triplets with respect to the two $SU(2)$ (with the generators of the infinitesimal transformations $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$).

At the electroweak break (from $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1,3) \times U(1) \times SU(3)$) the lowest four families become massive too, while staying decoupled from the upper four families. The vector bosons involved in this break, become massive. This break is (assumed to be) triggered by the superposition of the scalar fields $\tilde{\omega}_{abs}$, which are triplets with respect to the two $SU(2)$ (with the generators of the infinitesimal transformations $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and the superposition of the scalar fields $\omega_{s'ts}$, which are singlets with respect to the tree $U(1)$ ($A^{Y'}_s$, $A^{Q'}_s$, A^{Q}_s).

In this contribution we report the obtained analytical forms of the upper and lower 4x4 mass matrices taking into account all contributions from dynamical scalars and gauge bosons up to one loop corrections. At present we are carrying out a detailed numerical analysis trying to fit within this scenario the known quark and lepton masses and mixing matrices, including the neutrino properties.

7.6 APPENDIX: Majorana mass terms

There are mass terms within the *spin-charge-family-theory*, which transform the right handed neutrino to his charged conjugated one, contributing to the right

[7] The massless ness of the eight families is in this paper, following the paper [3], is just assumed. In the ref. [12], and in the references presented there, it is proven for a toy model that after the break there can exist massless families of fermions.

handed neutrino Majorana masses [3]

$$\begin{aligned}
&\psi^\dagger \gamma^0 \overset{78}{(-)}\, p_{0-}\, \psi\,,\\
&p_{0-} = -(\tilde{\tau}^{1+}\, \tilde{A}^{1+}_{-} + \tilde{\tau}^{1-}\, \tilde{A}^{1-}_{-})\, \mathcal{O}^{[+]}\, \mathcal{A}^{\mathcal{O}}_{[+]},\\
&\mathcal{O}^{[+]} = \overset{78}{[+]}\overset{56}{(-)}\overset{9\,10}{(-)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}\,.
\end{aligned} \tag{7.29}$$

One easily checks, using the technique with the Clifford objects (see ref. [3]) that $\gamma^0 \overset{78}{(-)}\, p_{0-}$ transforms a *right handed neutrino* of one of the *lower four families* into the charged conjugated one, belonging to the same group of families. It does not contribute to masses of other leptons and quarks, right or left handed. Although the operator $\mathcal{O}^{[+]}$ appears in a quite complicated way, that is in the higher order corrections, yet it might be helpful in explaining the properties of neutrinos. The operator $-(\tilde{\tau}^{1+}\, \tilde{A}^{1+}_{-} + \tilde{\tau}^{1-}\, \tilde{A}^{1-}_{-})\, \mathcal{O}^{[+]}\, \mathcal{A}^{\mathcal{O}}_{[+]}$ gives zero, when being applied on the upper four families, since they are singlets with respect to $\tilde{\tau}^{1\pm}$.

7.7 APPENDIX: Diagonalization of 4×4 tree level mass matrix

We take mass matrices on the tree level as they follow from the *spin-charge-family-theory*, Eq. (7.6). The part determined by the $\tilde{\omega}_{abs}$ fields is presented in tables (7.1, 7.2), for the upper four and the lower four families, respectively.

After assuming that real and symmetric matrices are good approximation for both groups of families (this is a good enough approximation for the lower four families, if we neglect the CP nonconserving terms, while for the upper four families we have no information yet about the discrete CP nonconserving symmetry either from studying the *spin-charge-family-theory* or from the experimental point of view) the mass matrices presented in tables (7.1, 7.2) and in Eq. (7.12) are 4×4 matrices

$$\mathcal{M}_{(o)} = \begin{pmatrix} -a_1 & e & 0 & b \\ e & -a_2 & b & 0 \\ 0 & b & a_1 & e \\ b & 0 & e & a_2 \end{pmatrix}, \tag{7.30}$$

with $a_1\,, a_2\,, b$ and e explained in Eq (7.14) of sect. 7.3. These matrix elements are different for the upper four families ($a_1 = \frac{1}{2}(\tilde{a}^{23}_{\pm} - \tilde{a}^{\tilde{N}^3_R}_{\pm} + a^{\alpha}_{\pm})\,, a_2 = \frac{1}{2}(\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm} + a^{\alpha}_{\pm})\,, b = \tilde{a}^{2+}_{\pm} = \tilde{a}^{2-}_{\pm}\,, e = \tilde{a}^{\tilde{N}^+_R}_{\pm} = \tilde{a}^{\tilde{N}^-_R}_{\pm}$) and different for the lower four families ($a_1 = \frac{1}{2}(\tilde{a}^{13}_{\pm} - \tilde{a}^{\tilde{N}^3_L}_{\pm} + a^{\alpha}_{\pm})\,, a_2 = \frac{1}{2}(\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm} + a^{\alpha}_{\pm})\,, b = \tilde{a}^{1+}_{\pm} = \tilde{a}^{1-}_{\pm}\,, e = \tilde{a}^{\tilde{N}^+_L}_{\pm} = \tilde{a}^{\tilde{N}^-_L}_{\pm}$) and also different for each of the family member ($\alpha \in \{u, d, \nu, e\}$), distinguishing in between the two pairs (d, e) (+) and (u, ν) (−) and in the term $a^{\alpha}_{\pm}$), with (7.15) $a_{\mp} = e\,Q\,A_{\mp} + g^1 \cos\theta_1\, Q'\, Z^{Q'}_{\mp} + g^2 \cos\theta_2\, Y'\, A^{Y'}_{\mp}$, where Q, Q' and Y' stay for the quantum numbers for the right handed members of one (anyone) family ($\alpha \in \{u, d, \nu, e\}$).

We present in Table 7.6 the representation of the right handed u_R-quark of a particular colour and the right handed colourless ν-lepton for all the eight families with the basic massless states expressed with the Clifford algebra objects [8]. Table is taken from ref. [3]. The quantum numbers, which each of these eight families carries, are presented in Table 7.3. The same quantum family numbers carry any member of a family ($\alpha \in \{u, d, \nu, e\}$), the left or right handed, colourless or of any colour.

$I_{R\,1}$	u_R^{c1}	$\stackrel{03}{[+i]}\,\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\,\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$I_{R\,2}$	u_R^{c1}	$\stackrel{03}{[+i]}\,\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$I_{R\,3}$	u_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$I_{R\,4}$	u_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\,\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$II_{R\,1}$	u_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\,\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$II_{R\,2}$	u_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\,\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$II_{R\,3}$	u_R^{c1}	$\stackrel{03}{[+i]}\,\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\,\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\,\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$
$II_{R\,4}$	u_R^{c1}	$\stackrel{03}{[+i]}\,\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[-]}\;\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\,\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\,\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}$

Table 7.6. Eight families of the right handed u_R quark with the spin $\frac{1}{2}$, the colour charge $\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$ and of the colourless right handed neutrino ν_R of the spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. $S^{ab}, a, b \in \{0, 1, 2, 3, 5, 6, 7, 8\}$ transform u_R^{c1} of the spin $\frac{1}{2}$ and the chosen colour $c1$ to all the members of the same colour: to the right handed u_R^{c1} of the spin $-\frac{1}{2}$, to the left u_L^{c1} of both spins ($\pm\frac{1}{2}$), to the right handed d_R^{c1} of both spins ($\pm\frac{1}{2}$) and to the left handed d_L^{c1} of both spins ($\pm\frac{1}{2}$). They transform equivalently the right handed neutrino ν_R of the spin $\frac{1}{2}$.

While the diagonal matrix elements, originating in ω_{abs} scalar vacuum expectation values, are expected to cause large (and desired) changes in mass matrices for the lower four families, their contribution to the mass matrices of the upper four families is expected to be very small, because of the difference in the strength of the tree level contributions from $\tilde{\omega}_{abs}$ sectors in both groups of four families.

The matrix of Eq.(7.30) can be diagonalized by the orthogonal matrix $V_{(o)}$ (Eq. (7.18))

$$V^T_{(o)} \mathcal{M}_{(o)}\, V_{(o)} = \mathcal{M}_{(o)D} = \mathrm{diag}(m_{(o)1}, m_{(o)2}, m_{(o)3}, m_{(o)4}) \,. \tag{7.31}$$

The diagonal contributions $a_{\mp}$ to mass matrices (Eq.(7.15)), the same for all the eight families, do not influence the diagonalization.

Four eigenvalues $m_{(o)i}$ $(i = 1, 2, 3, 4)$ of the tree level mass matrices $\mathcal{M}_{(o)}$, different for each of the two groups of four families, and also different for different family members (due to the diagonal contribution of Eq. (7.15) and to ($\mp$), are expressible in terms of Δ_o andΛ_o

$$\begin{aligned}
&\eta^4 - \Delta_o \eta^2 + \Lambda_{(o)} = 0\,, \quad \Delta_{(o)} = a_1^2 + a_2^2 + 2b^2 + 2e^2\,,\\
&\Lambda_{(o)} = (a_1 a_2 + b^2 - e^2)^2\,,\\
&m^{\alpha}_{(o)1} = -\eta_1 + a^{\alpha}\,, \quad m^{\alpha}_{(o)2} = -\eta_2 + a^{\alpha}\,, \quad m^{\alpha}_{(o)3} = \eta_1 + a^{\alpha}\,,\\
&m^{\alpha}_{(o)4} = \eta_2 + a^{\alpha}\,,\\
&\eta_1 = \frac{1}{\sqrt{2}}\sqrt{\Delta_{(o)} - R_1 R_2}\,, \quad \eta_2 = \frac{1}{\sqrt{2}}\sqrt{\Delta_{(o)} + R_1 R_2}\,,\\
&R_1 = \sqrt{(a_1 + a_2)^2 + 4b^2}\,, \quad R_2 = \sqrt{(a_2 - a_1)^2 + 4e^2},\\
&R_1^2 R_2^2 = \Lambda^2_{(o)} - 4\Lambda_{(o)}\,, \quad \eta_1^2 + \eta_2^2 = \Delta_{(o)}\,,\\
&\eta_1^2 \eta_2^2 = \Lambda_{(o)}\,, \quad \eta_2^2 - \eta_1^2 = R_1\, R_2\,.
\end{aligned} \tag{7.32}$$

Computing the eigenvectors, we obtain the orthogonal matrix $V_{(o)}$

$$V_{(o)} = \begin{pmatrix} s_1 & -s_2 & -s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ -s_3 & -s_4 & s_1 & s_2 \\ -s_4 & s_3 & -s_2 & s_1 \end{pmatrix}, \qquad s_1\, s_3 = s_2\, s_4\,. \tag{7.33}$$

s_1, s_2, s_3, s_4 are mixing angles defined in terms of the parameters and eigenvalues as follows

$$\begin{aligned}
s_1 &= \frac{1}{2}\sqrt{\frac{(\eta_2 + a_2)^2 - (a_1 + \eta_1)^2}{\eta_2^2 - \eta_1^2}}\,, \quad s_2 = \frac{1}{2}\sqrt{\frac{(\eta_2 + a_1)^2 - (a_2 + \eta_1)^2}{\eta_2^2 - \eta_1^2}}\\
s_3 &= \frac{1}{2}\sqrt{\frac{(\eta_2 - a_2)^2 - (a_1 - \eta_1)^2}{\eta_2^2 - \eta_1^2}}\,, \quad s_4 = \frac{1}{2}\sqrt{\frac{(\eta_2 - a_1)^2 - (a_2 - \eta_1)^2}{\eta_2^2 - \eta_1^2}}
\end{aligned} \tag{7.34}$$

It is easy to check the orthogonality of $V_{(o)}$, $V^T_{(o)} V_{(o)} = I$, and Eq.(7.31).

The matrix $V_{(o)}$, which is different for the upper ($\Sigma = II$) than for the lower four families ($\Sigma = I$) and different for the pair (u, ν) than the pair (d, e) transforms the massless states ψ (ψ^{α}_{Σ}) into the massive basis (Eq.(7.11)) $\Psi^{(o)}$ ($\Psi^{\alpha\,(o)}_{\Sigma}$)

$$V^{\alpha}_{\Sigma(o)}\, \Psi^{\alpha\,(o)}_{\Sigma} = \psi^{\alpha}_{\Sigma}\,, \;\; \Sigma = II, I, \;\; \alpha \in \{u,\; d,\; \nu,\; e\}. \tag{7.35}$$

7.7.1 Some useful relationships

$$
\begin{aligned}
s_1^2\, s_2^2 + s_3^2\, s_4^2 &= \frac{e^2\,((a_2+a_1)^2+2b^2)}{R_1^2\, R_2^2} \quad ,\\
s_1^2\, s_4^2 + s_2^2\, s_3^2 &= \frac{b^2\,((a_2-a_1)^2+2e^2)}{R_1^2\, R_2^2} \quad ,\\
s_1^2\, s_3^2 &= \frac{b^2\, e^2}{R_1^2\, R_2^2}\,,\\
s_1\, s_2(s_1^2-s_2^2) + s_3\, s_4(s_4^2-s_3^2) &= \frac{e(a_2-a_1)\,((a_2+a_1)^2+2b^2)}{R_1^2\, R_2^2}\,,\\
s_1\, s_4(s_1^2-s_4^2) + s_2\, s_3(s_2^2-s_3^2) &= \frac{b(a_2+a_1)\,((a_2-a_1)^2+2e^2)}{R_1^2\, R_2^2}\,,\\
s_1\, s_2(s_4^2-s_3^2) + s_3\, s_4(s_1^2-s_2^2) &= 2s_1\, s_3(s_1\, s_4 - s_2\, s_3) = \frac{2eb^2(a_2-a_1)}{R_1^2\, R_2^2}\,,\\
s_1\, s_4(s_2^2-s_3^2) + s_2\, s_3(s_1^2-s_4^2) &= 2s_1\, s_3(s_1\, s_2 - s_3\, s_4) = \frac{2be^2(a_2+a_1)}{R_1^2\, R_2^2}\,,\\
s_1\, s_3(s_1^2+s_3^2-s_2^2-s_4^2) &= (s_1\, s_2 - s_3\, s_4)(s_1\, s_4 - s_2\, s_3) = \frac{be(a_2^2-a_1^2)}{R_1^2\, R_2^2}\,. \qquad (7.36)
\end{aligned}
$$

7.7.2 2×2 matrices in the limit $b = 0$ within the 4×4 ones

We study here the limit when the off diagonal matrix elements b in Eq. (7.12,7.30) are small in comparison with the other nonzero matrix elements. We put in what follows $b = 0$. The mass matrices of Eq. (7.30) then simplifies into two by diagonal 2×2 matrices. In this limit it follows, after using the relation

$$
\left(\frac{1}{2}\left[a_1 + a_2 \pm \sqrt{(a_2-a_1)^2+4e^2}\right]\right)^2 = \frac{1}{2}\left[a_1^2 + a_2^2 + 2e^2 \pm \sqrt{(a_1+a_2)^2}\,\sqrt{(a_2-a_1)^2+4e^2}\right]
$$

in Eq. (7.32),

$$
\eta_{1,2} = \frac{1}{2}\left[a_1 + a_2 \mp \sqrt{(a_2-a_1)^2+4e^2}\right]. \qquad (7.37)
$$

Now η_i, $i = 1,2$ obey relations: $\eta_i^2 - (a_1+a_2)\,\eta_i + a_1 a_2 - e^2 = 0$, $\eta_2 + \eta_1 = a_1 + a_2$, $\eta_2 - \eta_1 = \sqrt{(a_2-a_1)^2+4e^2}$, $\eta_1\,\eta_2 = a_1 a_2 - e^2$. Correspondingly one finds: $(\eta_2 - a_2)^2 - (a_1 - \eta_1)^2 = (\eta_1 + \eta_2 - a_1 - a_2)(\eta_2 - \eta_1 - a_2 + a_1)$ and $(\eta_2 - a_1)^2 - (a_2 - \eta_1)^2 = (\eta_1 + \eta_2 - a_1 - a_2)(\eta_2 - \eta_1 + a_2 - a_1)$.

From the above equations it follows that $s_3 = 0 = s_4$ and

$$
s_1 = \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1+(\frac{2e}{a_2-a_1})^2}}\right)}\,, \quad s_2 = \sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1+(\frac{2e}{a_2-a_1})^2}}\right)}\,. \qquad (7.38)
$$

The masses of the first two families in each group of four families are then

$$m^{\alpha}_{(o)1} = -\eta_1 + a^{\alpha}\,, \quad m^{\alpha}_{(o)2} = -\eta_2 + a^{\alpha}\,,$$
$$m^{\alpha}_{(o)3} = \eta_1 + a^{\alpha}\,, \quad m^{\alpha}_{(o)4} = \eta_2 + a^{\alpha}\,, \tag{7.39}$$

in this case of the two by two diagonal matrices

$$\eta_{1,2} = \frac{1}{2}\,(a_1 + a_2 \mp \sqrt{(a_2 - a_1)^2 + 4e^2}).$$

7.8 APPENDIX: Mass matrices with one loop gauge and scalar corrections included

According to Eq. (7.6) to one loop corrections to the tree level mass matrices $\mathcal{M}_{(o)}$ the scalar fields of the two kinds and the massive gauge fields contribute. As discussed in sect. 7.3 to the loop corrections contribute:

i.) The scalar fields expressible with $\tilde{\omega}_{abs}$ contribute after the electroweak break to masses of both groups of four families. The scalar fields ($\tilde{g}^{\tilde{N}_R}\, \vec{\tilde{N}}_R \vec{\tilde{A}}^{\tilde{N}_R}_{\mp}$, $\tilde{g}^2\, \vec{\tilde{\tau}}^2\, \vec{\tilde{A}}^2_{\mp}$) contribute to masses of the upper four families, while ($\tilde{g}^{\tilde{N}_L}\, \vec{\tilde{N}}_L \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$, $\tilde{g}^1\, \vec{\tilde{\tau}}^1\, \vec{\tilde{A}}^1_{\mp}$) contribute to masses of the lower four families. Each group of these scalar fields appear at a different scale. The contributions in both groups of scalar fields distinguish among the pairs $(u\,,\nu)$ and $(d\,,e)$ due to the term $\overset{78}{(\mp)}\; p_{0\mp}$ in Eq.(7.3), which contributes to $(u\,,\nu)$ for $(-)$ and to $(d\,,e)$ for $(+)$.

ii.) The scalar fields expressible with ω_{abs}

$$(e\,Q\,A_{\mp}\,,\; g^1\; \cos\theta_1\; Q'\,Z^{Q'}_{\mp}\,,$$

$g^2 \cos\theta_2\, Y'\, A^{Y'}_{\mp}$). These scalar fields, which gain masses during the electroweak break, contribute to only the diagonal matrix elements, distinguishing among the family members $\alpha \in \{u, d, /nu, e\}$ through the eigenvalues of the operators Q, Q' and Y' and through the term $\overset{78}{(\mp)}\; p_{0\mp}$. The effect of their loops contributions depends strongly on the three level mass matrices.

iii.) The massive gauge vector fields $g^1 \cos\theta_1\, Q'\, Z^{Q'}_m$, $g^2 \cos\theta_2\, Y'\, A^{Y'}_m$ contributions differ for different members of a family according to the eigenvalues of the operators Q' and Y' and due to $\overset{78}{(\mp)}\; p_{0\mp}$. Their influence on the upper four and the lower four family members depends on the three level mass matrices.

The one loop contributions to the tree level mass matrices are illustrated in figures 7.1,7.2,7.3 presented in section 7.3.

We discuss these contributions separately for both kinds of scalar fields and for gauge bosons.

According to Eqs.(7.10,7.23) the contributions taking into account up to (k) loops corrections read

$$\psi^{\dagger}_L\, \gamma^0\, (\mathcal{M}_{(o\,k)} + \cdots + \mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\, \psi_R = \psi^{\dagger}_L\, \gamma^0\, \mathcal{M}_{(k)}\, \psi_R$$
$$\Psi^{(k)\,\dagger}_L\, \gamma^0\, (V_{(o)}\, V_{(1)}\, \cdots V_{(k)})^{\dagger}\, (\mathcal{M}_{(o\,k)} + \cdots + \mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\; V_{(o)}\, V_{(1)}\, \cdots V_{(k)}\, \Psi^{(k)}_R\,, \tag{7.40}$$

where $\psi_{(L,R)}$ are the massless states and $\Psi^{(k)}_{(L,R)}$ the massive ones when (k) loops corrections are taken into account

$$\psi_{(L,R)} = V_{(o)}\, V_{(1)} \cdots V_{\Sigma\,(k)}\, \Psi^{(k)}_{(L,R)} \tag{7.41}$$

and we skipped the indices Σ and α, assuming that they are present and will be determined when numerical calculations will be performed.

Accordingly we have up to one loop corrections

$$\begin{aligned} &\Psi^{(1)\,\dagger}_{L}\, \gamma^0\, (V_{(o)}\, V_{(1)})^{\dagger}\, (\mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\; V_{(o)}\, V_{(1)}\, \Psi^{(1)}_{R} = \\ &\qquad \psi^{\dagger}_{L}\, \gamma^0\, (\mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\, \psi_R\,, \\ &\mathcal{M}_{(o\,1)} = \tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}} + \mathcal{M}_{(o\,1)_{S}} + \mathcal{M}_{(o\,1)_{V}}\,, \end{aligned} \tag{7.42}$$

where $\mathcal{M}_{(o\,1)}$ stays for the sum of the one loop contributions of the scalar fields originating in $\tilde{\omega}_{ab\pm}$ ($\vec{\tilde{A}}^{\tilde{N}_{(L,R)}}_{\mp}$, $\vec{\tilde{A}}^{(2,1)}_{\mp}$), we write them as $\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}$, of those originating in $\omega_{st\pm}$ ($A_{\mp}$, $Z^{Q'}_{\mp}$, $A^{Y'}_{\mp}$), we write them as $\mathcal{M}_{(o\,1)_{S}}$ and of those originating in the massive boson fields ($Z^{Q'}_{m}$, $A^{Y'}_{m}$), we write them as $\mathcal{M}_{(o\,1)_{G}}$. All these contributions will be calculated in the next three subsections for the tree level mass matrices $\mathcal{M}_{(o)}$ from Eq.(7.12).

The mass matrix up to one loop is (Eq.(7.20))

$$\mathcal{M}_{(1)} = V^{\dagger}_{(o)}\, (\mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\, V_{(o)}\,, \tag{7.43}$$

with $\mathcal{M}_{(o\,1)} = (\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}} + \mathcal{M}_{(o\,1)_{S}} + \mathcal{M}_{(o\,1)_{V}})$ to be calculated in the subsections of this appendix and with $V_{(1)}$ which follows from (Eq.(7.24))

$$\mathcal{M}_{(1)D} = V^{\dagger}_{(1)}\, [V^{\dagger}_{(o)}\, (\mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\, V_{(o)}\,]\, V_{(1)} = \mathrm{diag}(m_{(1)\,1}, \cdots m_{(1)\,4})\,. \tag{7.44}$$

Let $m_{(o)\,i}$ $i = 1,2,3,4$ be the diagonal mass eigenvalues from Eqs. (7.25,7.31) (each carrying the quantum number of the family member α and the group index Σ)

$$\mathcal{M}_{(o)D} = V^{\dagger}_{(o)}\, \mathcal{M}_{(o)}\, V_{(o)} = \mathrm{diag}(m_{(o)\,1}, m_{(o)\,2}, m_{(o)\,3}, m_{(o)\,4})\,, \tag{7.45}$$

from appendix 7.7.

Let M_A stays for the masses of all fields A^A_a ($\vec{\tilde{A}}^{2}_{\mp}$, $\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}$, $\vec{\tilde{A}}^{1}_{\mp}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$, $A^{Q}_{\mp}$, $Z^{Q'}_{\mp}$, $A^{Y'}_{\mp}$, $Z^{Q'}_{m}$, $A^{Y'}_{m}$) contributing to loop corrections to the tree level masses as presented in Figs. (7.1, 7.2, 7.3) and let g^A and τ^A stay for the corresponding coupling constants (as presented in Table 7.7) and the eigenvalues of the operators $\hat{\tau}^A$.

Then the one loop contributions of both kinds of the scalar fields can be read from Figs. (7.1, 7.2) leading to

$$\Sigma^A_{kS} = m_{(o)k}\, \frac{(g^A \vec{\tau}^A)^2}{16\pi^2}\, \frac{(M_A)^2}{(M_A)^2 - (m_{(o)k})^2}\quad \ln \frac{(M_A)^2}{(m_{(o)k})^2}\,. \tag{7.46}$$

We must keep in mind that τ^A ($\equiv (Y', Q', Q)$, applied on the right handed massless states, or, if taking the hermitean conjugate value of the mass term $(\psi^{\dagger}\, \gamma^0 \stackrel{78}{(\pm)}\, p_{0\pm}\, \psi)^{\dagger}$ of Eq. (7.3),

A^A_a	$\tilde{A}^{\tilde{N}_R(1,2)}_{\mp}$	$\tilde{A}^{\tilde{N}_R 3}_{\mp}$	$\tilde{A}^{2(1,2)}_{\mp}$	$\tilde{A}^{2 3}_{\mp}$	$\tilde{A}^{\tilde{N}_L(1,2)}_{\mp}$	$\tilde{A}^{\tilde{N}_L 3}_{\mp}$	$\tilde{A}^{1(1,2)}_{\mp}$	$\tilde{A}^{1 3}_{\mp}$	$A^{Y'}_{\mp}$	$A^{Q'}_{\mp}$	$A^{Q}_{\mp}$	$A^{Y'}_m$	$Z^{Q'}_m$
g^A	$\tilde{g}^{\tilde{N}_R}$	$\tilde{g}^{\tilde{N}_R}$	$\tilde{g}^2$	$\tilde{g}^2$	$\tilde{g}^{\tilde{N}_L}$	$\tilde{g}^{\tilde{N}_L}$	$\tilde{g}^1$	$\tilde{g}^1$	$g^{Y'}$	$g^{Q'}$	g^Y	$g^{Y'}$	$g^{Q'}$
M_A	$M_{\tilde{N}_R}$	$M_{\tilde{N}_R 3}$	$M_{\tilde{2}}$	$M_{\tilde{2}3}$	$M_{\tilde{N}_L}$	$M_{\tilde{N}_L 3}$	$M_{\tilde{1}}$	$M_{\tilde{1}3}$	$M_{Y'S}$	$M_{Q'S}$	M_{QS}	$M_{Y'}$	$M_{Q'}$

Table 7.7. Notation for masses of dynamical scalar and vector boson fields. We used in Table 7.7 the notation $M_{\tilde{N}_R} = M_{\tilde{N}_R 1} = M_{\tilde{N}_R 2}$, $M_{\tilde{2}} = M_{\tilde{2}1} = M_{\tilde{2}2}$, $M_{\tilde{N}_L} = M_{\tilde{N}_L 1} = M_{\tilde{N}_L 2}$ and $M_{\tilde{1}} = M_{\tilde{1}1} = M_{\tilde{1}2}$.

on the left handed massless states, which brings the same result, with the eigenvalues presented in Table 7.4) are the same for all the families of both groups and so are M_A ($\equiv (M_{Y'}, M_{Q'}, M_Q)$) and that accordingly contributions $\Sigma^{(Y',Q',Q)}_k$ of the scalar fields $A^{Y'}_{\mp}$, $A^{Q'}_{\mp}$ and $A^{Q}_{\mp}$ bring different contributions for different families only through $m^{\alpha\,\Sigma}_{(o)k}$. The contributions of ($\vec{\tilde{A}}^{2}_{\mp}$, $\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}$, $\vec{\tilde{A}}^{1}_{\mp}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$) are different for different group of families and different members of one group, while they distinguish among the family members only through dependence of the fields on masses ($m^{\alpha\,\Sigma}_{(o)k}$) and on ($\mp$).

To evaluate the contributions from the gauge fields as presented in Fig. 7.3 we must evaluate

$$\Sigma^A_{kV} = m_{(o)k}\,\frac{(g^A)^2\,\tau^{A_L}\,\tau^{A_R}}{4\pi^2}\,\frac{(M_A)^2}{(M_A)^2-(m_{(o)k})^2}\quad \ln\frac{(M_A)^2}{(m_{(o)k})^2}\,, \tag{7.47}$$

where τ^A_L, τ^A_R are the eigenvalues of the operators Y' and Q' applied on the left (τ^A_L) and on the right (τ^A_R) handed member of a family α ($=\in \{u, d, \nu, e\}$) in the massless basis, M_A stays for $M_{Y'}$ and $M_{Q'}$. In Table 7.8 we present the values $(g^A)^2\,\tau^{A_L}\,\tau^{A_R}$ for the members of a family α=(u, d, ν, e). Let us add that

$$\begin{aligned}(g^{Y'})^2\,Y'^{\alpha}_L\,Y'^{\alpha}_R &= -g^4\sin\theta_2\,\tau^{4\,\alpha}\,g^2\cos\theta_2\,Y'^{\alpha}_R \;= -(g^{Y'})^2\,\tau^{4\,\alpha}\,Y'^{\alpha}_R\,,\\(g^{Q'})^2\,Q'^{\alpha}_L\,Q'^{\alpha}_R &= -g^1\cos\theta_1\,Q'^{\alpha}_L\,g^Y\sin\theta_1\,Y^{\alpha}_R \;= -e^2\,Q'^{\alpha}_L\,Q'^{\alpha}_R\,,\end{aligned} \tag{7.48}$$

where it is $\tau^{4\,\alpha} = \tau^{4\,\alpha}_L = \tau^{4\,\alpha}_R$.

α	u	d	ν	e
$\frac{(g^{Y'})^2\,Y'^{\alpha}_L\,Y'^{\alpha}_R}{4\pi^2}$	$-\frac{\alpha}{12\pi}\frac{1}{\cos^2\theta_1}\cdot\left(1-\frac{1}{3}\tan^2\theta_2\right)$	$\frac{\alpha}{12\pi}\frac{1}{\cos^2\theta_1}\cdot\left(1+\frac{1}{3}\tan^2\theta_2\right)$	$\frac{\alpha}{4\pi}\frac{1}{\cos^2\theta_1}\cdot\left(1+\tan^2\theta_2\right)$	$-\frac{\alpha}{4\pi}\frac{1}{\cos^2\theta_1}\cdot\left(1-\tan^2\theta_2\right)$
$\frac{(g^{Q'})^2\,Q'^{\alpha}_L\,Q'^{\alpha}_R}{4\pi^2}$	$-\frac{\alpha}{3\pi}\cdot\left(1-\frac{1}{3}\tan^2\theta_1\right)$	$-\frac{\alpha}{6\pi}\cdot\left(1+\frac{1}{3}\tan^2\theta_1\right)$	0	$-\frac{\alpha}{2\pi}\cdot\left(1-\tan^2\theta_1\right)$

Table 7.8. The couplings $\frac{(g^{Y'})^2\,Y'^{\alpha}_L\,Y'^{\alpha}_R}{4\pi^2}$ are presented, evaluated for the members of a (any) family $\alpha = (u, d, \nu, e)$. $Y'^{\alpha}_{(L,R)}$ and $Q'^{\alpha}_{(L,R)}$ are the eigenvalues of the operators $\hat{Y}'$ and $\hat{Q}'$ applied on the left (Y'^{α}_L, Q'^{α}_L) and on the right (Y'^{α}_R, Q'^{α}_R) handed member of a family α ($=\in \{u, d, \nu, e\}$) in the massless basis.

We evaluate in the next subsections the one loop corrections for all the three kinds of fields.

The corresponding mass matrices including one loop corrections (Eq. (7.43)) $\mathcal{M}^{\alpha\,\Sigma}_{(1)}$ are the sum of contributions of two kinds of massive scalar dynamical fields $\mathcal{M}^{\alpha\,\Sigma}_{(1)\,\tilde{S}}$ (Eq. (7.56) and $\mathcal{M}^{\alpha\,\Sigma}_{(1)\,S}$ (Eq. (7.63)), of massive vector boson fields $\mathcal{M}^{\alpha\,\Sigma}_{(1)\,V}$ (Eq. (7.68)) and of the tree level mass matrices $V^{\alpha\dagger}_{\Sigma\,(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o)}\,V^{\alpha}_{\Sigma(o)}$

$$V^{\alpha\dagger}_{\Sigma\,(o)}\,(\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)\,\tilde{S}} + \mathcal{M}^{\alpha\,\Sigma}_{(o\,1)\,S} + \mathcal{M}^{\alpha\,\Sigma}_{(o\,1)\,V} + \mathcal{M}^{\alpha\,\Sigma}_{(o)})\,V^{\alpha}_{\Sigma\,(o)}\,. \tag{7.49}$$

We obtain the masses and the diagonalizing matrices, within one loop corrections, from Eq. (7.44).

7.8.1 Scalar fields – $\vec{\tilde{A}}^{\tilde{N}(R,L)}_{\mp}$, $\vec{\tilde{A}}^{(2,1)}_{\mp}$ – contributions to one loop corrections to the mass matrices

We shall first study the one loop corrections to the tree level mass matrices from the scalar fields originating in $\tilde{\omega}_{abs}$ ($\vec{\tilde{A}}^{\tilde{N}(R,L)}_{\mp}$, $\vec{\tilde{A}}^{(2,1)}_{\mp}$), which distinguish among all the families. They also distinguish among the family members α through the dependence on $(\mp)$ and through the tree masses $m^{\alpha\,\Sigma}_{(o)k}$. The tree level diagonalizing matrices $V^{\alpha}_{\Sigma(o)}$ also depend on $(\mp)$, that is they are different for the pair $(u\,,\nu)$ than for the pair (d, e). The corresponding diagrams are presented in Figs. (7.1, 7.2).

The operators $\vec{\tilde{N}}_R\,\vec{\tilde{A}}^{\tilde{N}_R}$ and $\vec{\tilde{\tau}}^2\,\vec{\tilde{A}}^2$ transform the members of the upper four families while $\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}$ and $\vec{\tilde{\tau}}^1\,\vec{\tilde{A}}^1$ transform the members of the lower four families, both kinds of transformations are presented in Eq. (7.50)

$$\overset{\tilde{N}^i_L}{\leftrightarrow}\begin{pmatrix} I_4 & I_3\\ I_1 & I_2\end{pmatrix}\updownarrow \tilde{\tau}^{1i} \quad , \quad \overset{\tilde{N}^i_R}{\leftrightarrow}\begin{pmatrix} II_4 & II_3\\ II_1 & II_2\end{pmatrix}\updownarrow \tilde{\tau}^{2i} \quad . \tag{7.50}$$

Let us repeat that the upper four families are doublets with respect to $\vec{\tilde{N}}_R$ and doublets with respect to $\vec{\tilde{\tau}}^2$ and that they are singlets with respect to $\vec{\tilde{N}}_L$ and singlets with respect to $\vec{\tilde{\tau}}^1$, while the lower four families are doublets with respect to $\vec{\tilde{N}}_L$ and doublets with respect to $\vec{\tilde{\tau}}^1$ and that they are singlets with respect to $\vec{\tilde{N}}_R$ and singlets with respect to $\vec{\tilde{\tau}}^2$. Accordingly the mass matrices 8×8 stay to be two by diagonal matrices 4×4 also after the loops corrections.

Let us, to treat both groups of families formally all at once, accept the notation.
i. Let the scalar fields $\vec{\tilde{A}}^{\tilde{N}(R,L)}_{\mp}$ be denoted by $\vec{\tilde{A}}^{\tilde{N}}_{\mp}$, and $\vec{\tilde{A}}^{(2,1)}_{\mp}$ by $\vec{\tilde{A}}_{\mp}$.
ii. Let masses of these dynamical scalar fields be different for different components of $\tilde{A}^{\tilde{N}i}_{\mp}$, so that $\tilde{A}^{\tilde{N}1}_{\mp}$ and $\tilde{A}^{\tilde{N}2}_{\mp}$ have equal masses, and $\tilde{A}^{\tilde{N}3}_{\mp}$ a different one. Equivalent assumption is made for the massless of the components of $\vec{\tilde{A}}_{\mp}$.

Let $(M_{\tilde{\tau}}\,, M_{\tilde{\tau}3})$ represents the masses of the dynamical scalar fields $\vec{\tilde{A}}^2_{\mp}$, ($M_{\tilde{2}1} = M_{\tilde{2}2}$ and $M_{\tilde{2}3}$ from Table 7.7) when treating the upper four families, as well as the masses of the scalar fields $\vec{\tilde{A}}^1_{\mp}$, ($M_{\tilde{1}1} = M_{\tilde{1}2}$ and $M_{\tilde{1}3}$ from Table 7.7) when treating the lower four families. Let $(M_{\tilde{N}}\,, M_{\tilde{N}3})$ represents the masses of the scalar fields $\vec{\tilde{A}}^{\tilde{N}(R,L)}_{\mp}$ ($M_{\tilde{N}_{(R,L)}1} = M_{\tilde{N}_{(R,L)}2}$ and $M_{\tilde{N}_{(R,L)}3}$ from Table 7.7). We shall distinguish between the two groups of families when pointing out the differences and when looking for the numerical evaluations. The masses of the two kinds of the scalar fields differ for many orders of magnitude.
iii. Correspondingly let $m_{(o)i}\,, i\in\{1,2,3,4\}$, be the tree level masses of either the upper ($m^{\alpha\Sigma=II}_{(o)i}$) or the lower four families ($m^{\alpha\Sigma=I}_{(o)i}$) for each of the family member $\alpha=(u\,,d\,,\nu\,,e)$ and $\tilde{g}$ ($\tilde{g}^{(2,1)}$) determines the couplings to the fields $\vec{\tilde{A}}_{\mp}$ and $\tilde{g}^{\tilde{N}}$ ($\tilde{g}^{\tilde{N}(R,L)}$) the couplings to the fields $\vec{\tilde{A}}^{\tilde{N}}_{\mp}$ as presented in Table 7.7.

From the diagrams in Figs. (7.1,7.2) one loop contributions of the fields $\vec{\tilde{A}}_{\pm}$ and $\vec{\tilde{A}}^{\tilde{N}}_{\pm}$ follow

$$
\begin{aligned}
\tilde{\Sigma}^{3}_{k\tilde{S}} &= m_{(o)k}\,\frac{(\tilde{g})^2}{4}\,\frac{1}{16\,\pi^2}\,\frac{(M_{\tilde{\tau}3})^2}{(M_{\tilde{\tau}3})^2-(m_{(o)k})^2}\;\ln\frac{(M_{\tilde{\tau}3})^2}{(m_{(o)k})^2}\,,\\
\tilde{\Sigma}^{\pm}_{k\tilde{S}} &= m_{(o)k}\,\frac{(\tilde{g})^2}{2}\,\frac{1}{16\,\pi^2}\,\frac{(M_{\tilde{\tau}})^2}{(M_{\tilde{\tau}})^2-(m_{(o)k})^2}\;\ln\frac{(M_{\tilde{\tau}})^2}{(m_{(o)k})^2}\,,\\
\tilde{\Sigma}^{\tilde{N}3}_{k\tilde{S}} &= m_{(o)k}\,\frac{(\tilde{g}^{\tilde{N}})^2}{4}\,\frac{1}{16\,\pi^2}\,\frac{(M_{\tilde{N}3})^2}{(M_{\tilde{N}3})^2-(m_{(o)k})^2}\;\ln\frac{(M_{\tilde{N}3})^2}{(m_{(o)k})^2}\,,\\
\tilde{\Sigma}^{\tilde{N}\pm}_{k\tilde{S}} &= m_{(o)k}\,\frac{(\tilde{g}^{\tilde{N}})^2}{2}\,\frac{1}{16\,\pi^2}\,\frac{(\tilde{g}^{\tilde{N}})^2}{(M_{\tilde{N}})^2-(m_{(o)k})^2}\;\ln\frac{(M_{\tilde{N}})^2}{(m_{(o)k})^2}\,,
\end{aligned}
\tag{7.51}
$$

with $k \in \{1,2,3,4\}$ and where also the eigenvalues of the operators $\vec{\tilde{\tau}}^{A}$ are already taken into account. From here the contributions to the $\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}$ mass matrix of Eq.(7.51) follows when by taking into account Eq. (7.50) the transformations

$$
\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,ij} = \sum_{k=1}^{4} V_{(o)\,ik}\,V_{(o)\,jk}\,\tilde{\Sigma}^{(3,\tilde{N}3)}_{k\tilde{S}} \quad , \quad \tilde{\Sigma}^{(\pm,\tilde{N}\pm)}_{\tilde{S}\,ii} = \sum_{k=1}^{4} V_{(o)\,ik}\,V_{(o)\,ik}\,\tilde{\Sigma}^{(\pm,\tilde{N}\pm)}_{k\tilde{S}} \tag{7.52}
$$

are performed and $\tilde{\Sigma}^{(3,\tilde{N}3)}_{ij}$ built into the mass matrix $\tilde{\mathcal{M}}_{(o1)_{\tilde{S}}}$ of Eq. (7.53).

$$
\tilde{\mathcal{M}}_{(o1)_{\tilde{S}}} =
\begin{pmatrix}
\tilde{\Sigma}^{3}_{11}+\tilde{\Sigma}^{\tilde{N}3}_{11}+\tilde{\Sigma}^{\pm}_{44}+\tilde{\Sigma}^{\tilde{N}\pm}_{22} & -\tilde{\Sigma}^{\tilde{N}3}_{12} & 0 & -\tilde{\Sigma}^{3}_{14}\\
-\tilde{\Sigma}^{\tilde{N}3}_{12} & \tilde{\Sigma}^{3}_{22}+\tilde{\Sigma}^{\tilde{N}3}_{22}+\tilde{\Sigma}^{\pm}_{33}+\tilde{\Sigma}^{\tilde{N}\pm}_{11} & -\tilde{\Sigma}^{3}_{23} & 0\\
0 & -\tilde{\Sigma}^{3}_{23} & \tilde{\Sigma}^{3}_{33}+\tilde{\Sigma}^{\tilde{N}3}_{33}+\tilde{\Sigma}^{\pm}_{22}+\tilde{\Sigma}^{\tilde{N}\pm}_{44} & -\tilde{\Sigma}^{\tilde{N}3}_{34}\\
-\tilde{\Sigma}^{3}_{14} & 0 & -\tilde{\Sigma}^{\tilde{N}3}_{34} & \tilde{\Sigma}^{3}_{44}+\tilde{\Sigma}^{\tilde{N}3}_{44}+\tilde{\Sigma}^{\pm}_{11}+\tilde{\Sigma}^{\tilde{N}\pm}_{33}
\end{pmatrix}.
\tag{7.53}
$$

The matrix $\tilde{\mathcal{M}}_{(o1)_{\tilde{S}}}$ (and correspondingly all the matrix elements) should carry the indices α and Σ, since $m_{(o)k}$ and $V_{(o)}$ carry the indices α and Σ while $M_{\tilde{\tau}}$, $M_{\tilde{\tau}3}$, $M_{\tilde{\tau}}$ and $M_{\tilde{\tau}3}$ carry the index Σ. Correspondingly the matrix of Eq. (7.53) applies to any family member of either the upper or the lower group of four families.

To obtain the mass matrix up to one loop $\mathcal{M}_{(1)}$ (Eqs.(7.20, 7.43)) one needs to find $V^{\dagger}_{(o)}\,(\mathcal{M}_{(o\,1)} + \mathcal{M}_{(o)})\,V_{(o)} \;= V^{\dagger}_{(o)}\,\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}\,V_{(o)} \;+ V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_{S}}\,V_{(o)} \;+ V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_{V}}\,V_{(o)}\,.$

Let us therefore calculate here $\tilde{\mathcal{M}}_{(1)\tilde{S}}$ $(\equiv \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(1)\tilde{S}}) = V^{\dagger}_{(o)}\,\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}\,V_{(o)}$. Introducing

$$
\begin{aligned}
\tilde{\Sigma}_1 &= \tilde{\Sigma}^{3}_{1} + \tilde{\Sigma}^{\tilde{N}3}_{1} + \tilde{\Sigma}^{\pm}_{4} + \tilde{\Sigma}^{\tilde{N}\pm}_{2} \quad , \quad \tilde{\Sigma}_2 = \tilde{\Sigma}^{3}_{2} + \tilde{\Sigma}^{\tilde{N}3}_{2} + \tilde{\Sigma}^{\pm}_{3} + \tilde{\Sigma}^{\tilde{N}\pm}_{1} \quad ,\\
\tilde{\Sigma}_3 &= \tilde{\Sigma}^{3}_{3} + \tilde{\Sigma}^{\tilde{N}3}_{3} + \tilde{\Sigma}^{\pm}_{2} + \tilde{\Sigma}^{\tilde{N}\pm}_{4} \quad , \quad \tilde{\Sigma}_4 = \tilde{\Sigma}^{3}_{4} + \tilde{\Sigma}^{\tilde{N}3}_{4} + \tilde{\Sigma}^{\pm}_{1} + \tilde{\Sigma}^{\tilde{N}\pm}_{3} \quad ,\\
\tilde{\Sigma}_{(1-2)} &= \tilde{\Sigma}^{\tilde{N}3}_{1} - \tilde{\Sigma}^{\tilde{N}3}_{2} \quad , \quad \tilde{\Sigma}_{(3-4)} = \tilde{\Sigma}^{\tilde{N}3}_{3} - \tilde{\Sigma}^{\tilde{N}3}_{4}, \quad ,\\
\tilde{\Sigma}_{(1-4)} &= \tilde{\Sigma}^{\tilde{N}3}_{1} - \tilde{\Sigma}^{\tilde{N}3}_{4} \quad , \quad \tilde{\Sigma}_{(2-3)} = \tilde{\Sigma}^{\tilde{N}3}_{2} - \tilde{\Sigma}^{\tilde{N}3}_{3} \quad ,
\end{aligned}
\tag{7.54}
$$

we obtain for the mass matrix elements $(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{ij}$ when taking into account the matrix $V_{(o)}$ from Eq.(7.33) and Eq. (7.54)

$$
\begin{aligned}
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{11} = \tilde{\Sigma}_1 + 2(s_1^2 s_2^2 + s_3^2 s_4^2)(\tilde{\Sigma}_2 - \tilde{\Sigma}_1 - \tilde{\Sigma}_{(1-2)}) + 2(s_1^2 s_4^2 + s_2^2 s_3^2)(\tilde{\Sigma}_4 - \tilde{\Sigma}_1 - \tilde{\Sigma}_{(1-4)}) \\
&+4 s_1^2 s_3^2 (\tilde{\Sigma}_3 - \tilde{\Sigma}_1 + \tilde{\Sigma}_{(3-4)} - \tilde{\Sigma}_{(2-3)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{22} = \tilde{\Sigma}_2 + 2(s_1^2 s_2^2 + s_3^2 s_4^2)(\tilde{\Sigma}_1 - \tilde{\Sigma}_2 + \tilde{\Sigma}_{(1-2)}) + 2(s_1^2 s_4^2 + s_2^2 s_3^2)(\tilde{\Sigma}_3 - \tilde{\Sigma}_2 - \tilde{\Sigma}_{(2-3)}) \\
&+4 s_1^2 s_3^2 (\tilde{\Sigma}_4 - \tilde{\Sigma}_2 - \tilde{\Sigma}_{(3-4)} - \tilde{\Sigma}_{(1-4)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{33} = \tilde{\Sigma}_3 + 2(s_1^2 s_2^2 + s_3^2 s_4^2)(\tilde{\Sigma}_4 - \tilde{\Sigma}_3 - \tilde{\Sigma}_{(3-4)}) + 2(s_1^2 s_4^2 + s_2^2 s_3^2)(\tilde{\Sigma}_2 - \tilde{\Sigma}_3 + \tilde{\Sigma}_{(2-3)}) \\
&+4 s_1^2 s_3^2 (\tilde{\Sigma}_1 - \tilde{\Sigma}_3 + \tilde{\Sigma}_{(1-2)} + \tilde{\Sigma}_{(1-4)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{44} - \tilde{\Sigma}_4 + 2(s_1^2 s_2^2 + s_3^2 s_4^2)(\tilde{\Sigma}_3 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(3-4)}) + 2(s_1^2 s_4^2 + s_2^2 s_3^2)(\tilde{\Sigma}_1 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(1-4)}) \\
&+4 s_1^2 s_3^2 (\tilde{\Sigma}_2 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(2-3)} - \tilde{\Sigma}_{(1-2)})\,,
\end{aligned}
$$

$$
\begin{aligned}
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{12} = [s_1 s_2 (s_1^2 - s_2^2) - s_3 s_4 (s_3^2 - s_4^2)]\,(\tilde{\Sigma}_2 - \tilde{\Sigma}_1 - \tilde{\Sigma}_{(1-2)}) \\
&+[s_1 s_2 (s_3^2 - s_4^2) - s_3 s_4 (s_1^2 - s_2^2)](\tilde{\Sigma}_4 - \tilde{\Sigma}_3 - \tilde{\Sigma}_{(3-4)}) + 2 s_1 s_3 (s_1 s_4 - s_2 s_3)(\tilde{\Sigma}_{(1-4)} - \tilde{\Sigma}_{(2-3)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{24} = s_1 s_3 (s_1^2 + s_3^2 - s_2^2 - s_4^2)(\tilde{\Sigma}_2 + \tilde{\Sigma}_4 - \tilde{\Sigma}_1 - \tilde{\Sigma}_3) \\
&+(s_1 s_2 - s_3 s_4)(s_1 s_4 - s_2 s_3)(\tilde{\Sigma}_{(2-3)} - \tilde{\Sigma}_{(1-4)} - \tilde{\Sigma}_{(1-2)} - \tilde{\Sigma}_{(3-4)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{14} = [s_1 s_4 (s_1^2 - s_4^2) + s_2 s_3 (s_2^2 - s_3^2)](\tilde{\Sigma}_1 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(1-4)}) \\
&+[s_1 s_4 (s_2^2 - s_3^2) + s_2 s_3 (s_1^2 - s_4^2)](\tilde{\Sigma}_2 - \tilde{\Sigma}_3 + \tilde{\Sigma}_{(2-3)}) - 2 s_1 s_3 (s_1 s_2 - s_3 s_4)(\tilde{\Sigma}_{(1-2)} + \tilde{\Sigma}_{(3-4)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{23} = [s_1 s_4 (s_1^2 - s_4^2) + s_2 s_3 (s_2^2 - s_3^2)](\tilde{\Sigma}_2 - \tilde{\Sigma}_3 + \tilde{\Sigma}_{(2-3)}) \\
&+[s_1 s_4 (s_2^2 - s_3^2) + s_2 s_3 (s_1^2 - s_4^2)](\tilde{\Sigma}_1 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(1-4)}) + +2 s_1 s_3 (s_1 s_2 - s_3 s_4)(\tilde{\Sigma}_{(1-2)} + \tilde{\Sigma}_{(3-4)})\,,
\end{aligned}
$$

$$
\begin{aligned}
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{34} = [s_1 s_2 (s_1^2 - s_2^2) + s_3 s_4 (s_4^2 - s_3^2)](\tilde{\Sigma}_3 - \tilde{\Sigma}_4 + \tilde{\Sigma}_{(3-4)}) \\
&+[s_1 s_2 (s_4^2 - s_3^2) + s_3 s_4 (s_1^2 - s_2^2)](\tilde{\Sigma}_2 - \tilde{\Sigma}_1 - \tilde{\Sigma}_{(1-2)}) + -2 s_1 s_3 (s_1 s_4 - s_2 s_3)(\tilde{\Sigma}_{(1-4)} + \tilde{\Sigma}_{(2-3)})\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{21} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{12}\,,\; (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{31} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13}\,,\; (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{41} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{14}\,, \\
&(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{32} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{23}\,,\; (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{43} = (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{34}\,. \qquad (7.55)
\end{aligned}
$$

All the matrix elements $(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{ij}$ carry the indices α, which distinguishes among family members, and Σ, which distinguishes between the two groups of four families. The matrix $\tilde{\mathcal{M}}^{\alpha\Sigma}_{(1)\tilde{S}}$ is accordingly

$$
\begin{aligned}
\tilde{\mathcal{M}}^{\alpha\Sigma}_{(1)\tilde{S}} &= V^{\alpha\dagger}_{\Sigma(o)}\, \tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o1)\tilde{S}}\, V^{\alpha}_{\Sigma(o)} \\
&= \begin{pmatrix}
(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{11} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{12} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{14} \\
(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{12} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{22} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{23} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13} \\
(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{23} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{33} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{34} \\
(\tilde{\mathcal{M}}_{(1)\tilde{S}})_{14} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{13} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{34} & (\tilde{\mathcal{M}}_{(1)\tilde{S}})_{44}
\end{pmatrix}^{\alpha\Sigma}_{\tilde{S}}\,. \qquad (7.56)
\end{aligned}
$$

The matrix $\tilde{\mathcal{M}}_{(1)\tilde{S}}$ carry the indices α (distinguishing among family members) and Σ (distinguishing between the two groups of four families), which are added to the matrix.

Contributions from scalar fields $\vec{\tilde{A}}^{\tilde{A}}_{\pm}$ which couple two families We explain in details the contribution to loop corrections from the scalar fields $\vec{\tilde{A}}^{\tilde{A}}_{\pm}$, representing $\vec{\tilde{A}}^{2}_{\pm}$ and $\vec{\tilde{A}}^{\tilde{N}_R}_{\pm}$ in the case of the upper four families and $\vec{\tilde{A}}^{1}_{\pm}$ and $\vec{\tilde{A}}^{\tilde{N}_L}_{\pm}$ for the lower four families. We work in the massless basis. Let these fields act between the families (i, j) accordingly to

Eq.(7.50). Let these two families are the two states of the fundamental representation of the associated $SU(2)$ flavour symmetry (with the corresponding infinitesimal generators of the group, which are either $\vec{\tilde{\tau}}^1$ or $\vec{\tilde{N}}_L$ for the lower four families or $\vec{\tilde{\tau}}^2$ or $\vec{\tilde{N}}_R$ for the upper four families). The fields $\vec{\tilde{A}}^{\tilde{A}}_{\pm}$ couple to the families (i,j) (that is to the massless states $\psi^{\alpha}_{\Sigma(L,R)i}$, we here omit the indices α and Σ) as follows

$$\frac{\tilde{g}^{\tilde{A}}}{2}\,\Big[\big(\overline{\psi}_{j\,(L,R)}\,\psi_{i\,(R,L)}+\overline{\psi}_{i\,(L,R)}\,\psi_{j\,(R,L)}\big)\ \tilde{A}^{\tilde{A}1}_{\pm} + \big(i\,\overline{\psi}_{j\,(L,R)}\,\psi_{i\,(R,L)}-i\,\overline{\psi}_{i\,(L,R)}\,\psi_{j\,(R,L)}\big)\ \tilde{A}^{\tilde{A}2}_{\pm} + \big(\overline{\psi}_{i\,(L,R)}\,\psi_{i\,(R,L)}-\overline{\psi}_{j\,(L,R)}\,\psi_{j\,(R,L)}\big)\ \tilde{A}^{\tilde{A}3}_{\pm}\Big]\,. \tag{7.57}$$

For particular values of the indices $\alpha \in (u\,,d\,,\nu\,,e)$ and $\Sigma \in (II, I)$, the pair of the families (i,j) is associated to the subset of tree level mass parameters from $\mathcal{M}_{(o)} \equiv \mathcal{M}^{\alpha\,\Sigma}_{(o)}$, Eq.(7.30). In Table 7.9 these tree level matrix elements are presented for the case $(i=4, j=1)$. Using

	ψ_{4R}	ψ_{1R}
$\bar{\psi}_{4L}$	a_2	b
$\bar{\psi}_{1L}$	b	$-a_1$

Table 7.9. 2×2 tree level parameters for $i=4, j=1$ family indices

the scalar couplings of Eq. (7.57) and the involved tree level mass parameters we can draw the one loop diagrams of Figs. 7.1 and 7.2. From these diagrams the one loop contributions of the fields $\vec{\tilde{A}}^{\tilde{A}}_{\pm}$ follow

$$\bar{\psi}^{\alpha}_{\Sigma\,iL}\left(\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,ii}+\tilde{\Sigma}^{(\pm,\tilde{N}\pm)}_{\tilde{S}\,jj}\right)\psi^{\alpha}_{\Sigma\,iR}-\bar{\psi}^{\alpha}_{\Sigma\,iL}\,\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,ij}\,\psi^{\alpha}_{\Sigma\,jR} + \bar{\psi}^{\alpha}_{\Sigma\,jL}\left(\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,jj}+\tilde{\Sigma}^{(\pm,\tilde{N}\pm)}_{\tilde{S}\,ii}\right)\psi^{\alpha}_{\Sigma\,jR}-\bar{\psi}^{\alpha}_{\Sigma\,jL}\,\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,ij}\,\psi^{\alpha}_{\Sigma\,iR}\,, \tag{7.58}$$

with $\tilde{\Sigma}^{(3,\tilde{N}3)}_{\tilde{S}\,ij}$ and $\tilde{\Sigma}^{(\pm,\tilde{N}\pm)}_{\tilde{S}\,ii}$ defined in Eqs. (7.52, 7.51).

7.8.2 Scalar fields – $\vec{A}^{Y'}_{\mp}$, $\vec{A}^{Q'}_{\mp}$, $\vec{A}^{Q}_{\mp}$ – contributions to one loop corrections to the mass matrices

The one loop corrections of the scalar fields originating in $\omega_{sts'}$ — $\vec{A}^{Y'}_{\mp}$, $\vec{A}^{Q'}_{\mp}$, and in $\vec{A}^{Q}_{\mp}$ — $(e\,Q\,A^{Q}_{\mp},\ g^1\,\cos\theta_1\,Q'\,Z^{Q'}_{\mp},\ g^2\cos\theta_2\,Y'\,A^{Y'}_{\mp})$ are presented in Fig. 7.2. Their contributions to the mass matrix $\mathcal{M}_{(o1)_S}$ depend on a family member α through different values for each of the two pairs $(u\,,\nu)$ and $(d\,,e)$ $(\mp)$, through the dependence of the tree level masses $(m_{(o)i})$ on α, and also through the eigenvalues of the operators $(\hat{Y}',\hat{Q}',\hat{Q})$ on different family members $\alpha=(u\,,d\,,\nu\,,e)$ as already explained and also presented in Table 7.4. Their contributions depend also on the group $(\Sigma=(II\,,I))$ and family indices $(i=(1\,,2\,,3\,,4))$ through $(m^{\alpha\,\Sigma}_{(o)i})$.

Let here $Q^{\alpha}, Q^{'\alpha}, Y^{'\alpha}$ stay for the eigenvalues of the corresponding operators on the states ψ^{α} (the states are indeed $\psi^{\alpha}_{\Sigma\,i}$, carrying also the family and the group indices as presented in Eq. (7.8) and skipped here). And let $M_{Q\,S}\,,M_{Q'\,S}$ and $M_{Y'\,S}$ represent the masses of the scalar dynamical fields $A^{Q}_{\mp}\,,Z^{Q'}_{\mp}$ and $A^{Y'}_{\mp}$, respectively.

We have equivalent expressions to those of Eq. (7.51)

$$\begin{aligned}
\Sigma^{Y'\alpha}_{kS} &= m_{(o)k}\,\frac{(g^{Y'}\,Y'^{\alpha})^2}{16\pi^2}\;\frac{(M_{Y'S})^2}{(M_{Y'S})^2-(m_{(o)k})^2}\;\ln\frac{(M_{Y'S})^2}{(m_{(o)k})^2}\,,\\
\Sigma^{Q'\alpha}_{kS} &= m_{(o)k}\,\frac{(g^{Q'}\,Q'^{\alpha})^2}{16\pi^2}\;\frac{(M_{Q'S})^2}{(M_{Q'S})^2-(m_{(o)k})^2}\;\ln\frac{(M_{Q'S})^2}{(m_{(o)k})^2}\,,\\
\Sigma^{Q\alpha}_{kS} &= m_{(o)k}\,\frac{(g^{Q}\,Q^{\alpha})^2}{16\pi^2}\;\frac{(M_{QS})^2}{(M_{QS})^2-(m_{(o)k})^2}\;\ln\frac{(M_{QS})^2}{(m_{(o)k})^2}\,,
\end{aligned} \tag{7.59}$$

where, as already explained, $m_{(o)k}$ ($m^{\alpha\,\Sigma}_{(o)k}$) are the masses, depending on the member of a family α and on the group of four families (Σ), evaluated on the tree level.

Let us evaluate $\Sigma^{\alpha\Sigma(Y'^{\alpha},Q'^{\alpha},Q^{\alpha})}_{ij}$, similarly as in Eq. (7.52), pointing out that they depend on $\Sigma=(II,I)$ through masses ($m^{\alpha\,\Sigma}_{(o)}$) and through $V_{(o)ik}$ ($\equiv V^{\alpha}_{\Sigma(o)\,ik}$)

$$\Sigma^{\alpha\,\Sigma(Y'^{\alpha},Q'^{\alpha},Q^{\alpha})}_{S\,ij}=\sum_{k=1}^{4}V^{\alpha}_{\Sigma(o)\,ik}\,V^{\alpha}_{\Sigma(o)\,jk}\,\Sigma^{\alpha\,\Sigma(Y'^{\alpha},Q'^{\alpha},Q^{\alpha})}_{kS}\,. \tag{7.60}$$

we end up with the matrix $\mathcal{M}_{(o1)_S}$, which carry the indices α and Σ ($\mathcal{M}^{\alpha}_{\Sigma(o1)_S}$). Due to Eq. (7.43) we need to calculate to obtain the mass matrices up to the one loop corrections included $\mathcal{M}_{(1)}$ ($V^{\dagger}_{(o)}\,(\mathcal{M}_{(o\,1)}+\mathcal{M}_{(o)})\,V_{(o)}\;=V^{\dagger}_{(o)}\,\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}\,V_{(o)}\;+V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_S}\,V_{(o)}\;+V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_V}\,V_{(o)}$.

Let us calculate here therefore $V^{\alpha\dagger}_{\Sigma\,(o)}\,\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)_S}\,V^{\alpha}_{\Sigma(o)}$, which distinguish among members of a family (α) and between the two groups of families(Σ), using Eqs. (7.43, 7.59, 7.60)

$$\begin{aligned}
&\left(V^{\alpha\,\dagger}_{\Sigma(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)_S}\,V^{\alpha}_{\Sigma\,(o)}\right)_{ij}=\\
&=\sum_{(l,k,r)=1}^{4}(V^{\alpha}_{\Sigma\,(o)li}\,V^{\alpha}_{\Sigma(o)lk})\,(V^{\alpha}_{\Sigma(o)rk}\,V^{\alpha}_{\Sigma(o)rj})\,(\Sigma^{Y'\alpha}_{kS}+\Sigma^{Q'\alpha}_{kS}+\Sigma^{Q\alpha}_{kS})^{\alpha\,\Sigma}\\
&=\delta_{ik}\,\delta_{jk}\,(\Sigma^{Y'\alpha}_{kS}+\Sigma^{Q'\alpha}_{kS}+\Sigma^{Q\alpha}_{kS})^{\alpha\,\Sigma}\,.
\end{aligned} \tag{7.61}$$

We have for $\mathcal{M}^{\alpha\,\Sigma}_{(1)_S}$

$$\mathcal{M}^{\alpha\,\Sigma}_{(o1)_S}=\left(V^{\alpha\,\dagger}_{\Sigma(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)_S}\,V^{\alpha}_{\Sigma\,(o)}\right) \tag{7.62}$$

$$\begin{pmatrix}
\Sigma^{Y'\alpha}_{1S}+\Sigma^{Q'\alpha}_{1S}+\Sigma^{Q\alpha}_{1S} & 0 & 0 & 0\\
0 & \Sigma^{Y'\alpha}_{2S}+\Sigma^{Q'\alpha}_{2S}+\Sigma^{Q\alpha}_{2S} & 0 & 0\\
0 & 0 & \Sigma^{Y'\alpha}_{3S}+\Sigma^{Q'\alpha}_{3S}+\Sigma^{Q\alpha}_{3S} & 0\\
0 & 0 & 0 & \Sigma^{Y'\alpha}_{4S}+\Sigma^{Q'\alpha}_{4S}+\Sigma^{Q\alpha}_{4S}
\end{pmatrix}^{\alpha\Sigma}_{S}.$$

7.8.3 Gauge bosons – $A^{Y'}_m$, $Z^{Q'}_m$ – contribution to one loop corrections to the mass matrices 4×4

We study the one loop contributions to the tree level mass matrices from the gauge fields $A^{Y'}_m$ and $Z^{Q'}_m$. According to ref. [3] $A^{Y'}_m$ gains a mass after the phase transition from $SU(2)_I\times SU(2)_{II}\times U(1)_{II}$ into $SU(2)_I\times U(1)_I$ (and becomes a superposition of $\vec{A}^{2}_m$ and A^{4}_m fields), while $Z^{Q'}_m$ gains a mass after the electroweak break (from $SU(2)_I\times U(1)_I$ into $U(1)$) (and becomes a superposition of $\vec{A}^{1}_m$ and A^{Y}_m fields). The one loop corrections of both vector fields to the tree level mass matrices are presented in Fig. 7.3. After the electroweak break Eq. (7.6) determines the covariant moments of all the eight families. These two massive vector fields influence mass matrices of the upper and the lower four families.

According to ref. [3] before the phase transitions ψ_R transform under $SU(2)_{II} \times U(1)_{II}$, that is with respect to $\vec{\hat{\tau}}^2$ and $\hat{\tau}^4$, as $(2, \tau^4)$, $\tau^4 = \frac{1}{6}\,(-\frac{1}{2})$ for quarks (leptons), while ψ_L transform under $SU(2)_{II} \times U(1)_{II}$ as $(1, \tau^4)$. From the kinetic term of Eq.(7.6) the gauge couplings to $A^{Y'}_m$ is

$$\left[g^{Y'}\,\hat{Y}'\,(\,\psi_L + \psi_R)^{\alpha}_{\Sigma}\right] A^{Y'}_m\,, \tag{7.63}$$

where massless states $\psi^{\alpha}_{\Sigma(L,R)}$ carry indices Σ (distinguishing the upper, $=$ II, and the lower, $=$ I, four families), α ($= (u\,,d\,,\nu\,,e)$, which distinguishes a family members) and the family index ($i = 1, 2, 3, 4$) of each group. We further have $g^{Y'}\,\hat{Y}'\;\psi^{\alpha}_{L} = -g_4\;\sin\theta_2\;\hat{\tau}^4\;\psi^{\alpha}_{\Sigma L}$, $g^{Y'}\,\hat{Y}'\;\psi^{\alpha}_{\Sigma R} = g_2\;\cos\theta_2\;\hat{Y}'\,\psi^{\alpha}_{\Sigma R}$ (see Table 7.4). $\vec{\hat{\tau}}^2$, $\vec{\hat{\tau}}^1$ and $\hat{\tau}^4$ distinguish only among family members.

According to ref. [3] massless states ψ_R transform as $(1, Y)$ under $SU(2)_I \times U(1)_Y$, that is with respect to $\vec{\hat{\tau}}^1$ and $\hat{Y}$, while ψ_L transforms as $(2, Y)$ under $SU(2)_I \times U(1)_Y$. Accordingly, Eq.(7.6) dictates the following couplings of $Z^{Q'}_m$ to fermions in a massless basis [8]

$$\left[g^{Q'}\,\hat{Q}'\,(\,\psi^{\alpha}_{\Sigma L} + \psi^{\alpha}_{\Sigma R})\right] Z^{Q'}_m\,, \tag{7.64}$$

where massless basis $\psi_{(L,R)}$ carry indices Σ , α and the family index ($i = 1, 2, 3, 4$) of each group, and $g^{Q'}\hat{Q}'\;\psi^{\alpha}_{\Sigma L} = g_1\;\cos\theta_1\;\hat{Q}'\,\psi^{\alpha}_{\Sigma L}$ and $g^{Q'}\hat{Q}'\;\psi^{\alpha}_{\Sigma R} = -g_Y\;\sin\theta_1\;\hat{Q}'\,\psi^{\alpha}_{\Sigma R}$ (see Table 7.4).

The internal fermion lines in the diagram of Fig. 7.3 represent the massive basis $\Psi^{(o)}$ (carrying the index Σ, α and i, $\Psi^{\alpha(o)}_{\Sigma\,i}$) and the masses $m^{\alpha\Sigma}_{(o)i}$ are diagonal values, eigenvalues, of the 4×4 matrix, belonging to the family member α and the four family group Σ, $\mathcal{M}^{\alpha\,\Sigma}_{(o)}$.

Let $Y'^{\alpha}(L, R)$, $Q'^{\alpha}_{(L,R)}$ stay for the eigenvalues of the corresponding operators on the states $\psi^{\alpha}_{\Sigma\,(L,R)\,i}$ (Eq. (7.8)) and let $M_{Y'}$ and $M_{Q'}$ represent the masses of the vector bosons $A^{Y'}_m$ and $Z^{Q'}_m$ ($\equiv A^{Q'}_m$), respectively. From the diagram in Fig. 7.3 then follow expressions, equivalent to those from Eqs. (7.51, 7.59)

$$\begin{aligned}
\Sigma^{Y'\alpha}_{kV} &= m_{(o)k}\,\frac{(g^{Y'})^2\,Y'^{\alpha}_L\,Y'^{\alpha}_R}{4\,\pi^2}\;\frac{(M_{Y'})^2}{(M_{Y'})^2-(m_{(o)k})^2}\;\;\ln\frac{(M_{Y'})^2}{(m_{(o)k})^2}\,,\\
\Sigma^{Q'\alpha}_{kV} &= m_{(o)k}\,\frac{(g^{Q'})^2\,Q'^{\alpha}_L\,Q'^{\alpha}_R}{4\,\pi^2}\;\frac{(M_{Q'})^2}{(M_{Q'})^2-(m_{(o)k})^2}\;\;\ln\frac{(M_{Q'})^2}{(m_{(o)k})^2}\,,
\end{aligned} \tag{7.65}$$

where, as already explained, $m_{(o)k}$ are the masses of the (upper or lower) four families on the tree level, and should carry the indices of the group Σ and of the family member α and $Y'^{\alpha}_{(L,R)}$ and $Q'^{\alpha}_{(L,R)}$ are the eigenvalues of the operators when applied to the right (Y'^{α}_R, Y'^{α}_R) or to the left (Y'^{α}_L, Y'^{α}_L) handed member of a family (α) in the massless basis.

Let us evaluate $\Sigma^{\Sigma(Y'^{\alpha},Q'^{\alpha})}_{ij}$, similarly as in Eqs. (7.52,7.60, pointing out that they depend on $\Sigma = (\mathrm{II}, \mathrm{I})$ through masses ($m^{\alpha\,\Sigma}_{(o)}$) and through $V_{(o)ik}$ ($\equiv V^{\alpha}_{\Sigma(o)\,ik}$) and are

[8] Before the electroweak break the lower four families are massless and the massive gauge field $A^{Y'}_m$ contribute to masses of only the upper (massive) four families. After the electroweak break the lower four families become massive as well. Correspondingly both massive gauge fields, $A^{Y'}_m$ and $A^{Q'}_m \equiv Z^{Q'}_m$, contribute to masses of the upper and the lower four families.

different for each of the family member α

$$\Sigma^{\Sigma(Y'^{\alpha},Q'^{\alpha})}_{V\,ij} = \sum_{k=1}^{4} V^{\alpha}_{\Sigma(o)\,ik}\, V^{\alpha}_{\Sigma(o)\,jk}\, \Sigma^{(Y'^{\alpha},Q'^{\alpha})}_{kV}\,. \tag{7.66}$$

We end up with the matrix $\mathcal{M}_{(o1)_V}$, which carry the indices α and Σ ($\mathcal{M}^{\alpha}_{\Sigma(o1)_V}$). Due to Eq. (7.43) we need to calculate, to obtain the mass matrices up to the one loop corrections included, $\mathcal{M}_{(1)}$ $(V^{\dagger}_{(o)}\,(\mathcal{M}_{(o\,1)}+\mathcal{M}_{(o)})\,V_{(o)} = V^{\dagger}_{(o)}\,\tilde{\mathcal{M}}_{(o\,1)_{\tilde{S}}}\,V_{(o)} + V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_S}\,V_{(o)} + V^{\dagger}_{(o)}\,\mathcal{M}_{(o\,1)_V}\,V_{(o)}$.

Let us calculate here $V^{\alpha\dagger}_{\Sigma\,(o)}\,\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)_V}\,V^{\alpha}_{\Sigma(o)}$, which distinguish among members of a family (α) and between the two groups of families(Σ), using Eqs. (7.43, 7.59, 7.60)

$$\begin{aligned}\left(V^{\alpha\,\dagger}_{\Sigma(o)}\,\mathcal{M}^{\alpha\,\Sigma}_{(o\,1)_V}\,V^{\alpha}_{\Sigma\,(o)}\right)_{ij} &= \sum_{(l,k,r)=1}^{4} (V_{(o)li}\,V_{(o)lk})\,(V_{(o)rk}\,V_{(o)rj})\,(\Sigma^{Y'\alpha}_{kV}+\Sigma^{Q'\alpha}_{kV}+\Sigma^{Q\alpha}_{kV})\\ &= \delta_{ik}\,\delta_{jk}\,(\Sigma^{Y'\alpha}_{kV}+\Sigma^{Q'\alpha}_{kV}+\Sigma^{Q\alpha}_{k})\,.\end{aligned} \tag{7.67}$$

We have for $\mathcal{M}^{\alpha\,\Sigma}_{(1)_V}$ the expression with only diagonal terms

$$\begin{aligned}&\mathcal{M}^{\alpha\,\Sigma}_{(1)_V} = V^{\alpha\dagger}_{\Sigma\,(o)}\,\tilde{\mathcal{M}}^{\alpha\,\Sigma}_{(o\,1)_V}\,V^{\alpha}_{\Sigma(o)}\\ &\begin{pmatrix} \Sigma^{Y'\alpha}_{1V}+\Sigma^{Q'\alpha}_{1V} & 0 & 0 & 0\\ 0 & \Sigma^{Y'\alpha}_{2V}+\Sigma^{Q'\alpha}_{2V} & 0 & 0\\ 0 & 0 & \Sigma^{Y'\alpha}_{3V}+\Sigma^{Q'\alpha}_{3V} & 0\\ 0 & 0 & 0 & \Sigma^{Y'\alpha}_{4V}+\Sigma^{Q'\alpha}_{4V}\end{pmatrix}^{\alpha\,\Sigma}_{V},\end{aligned} \tag{7.68}$$

like in Eq. (7.63). The contribution to the one loop corrections originating in the massive vector boson fields $A^{Y'}_m$ and $Z^{Q'}_m$ leads to the diagonal mass matrices $\mathcal{M}^{\alpha\,\Sigma}_{(1)_V}$.

The mass matrices of Eq.(7.68) demonstrate that the one loop contributions from $A^{Y'}$ and $A^{Y'}$ gauge bosons give corrections to the tree level mass eigenvalues, but not change the off diagonal terms.

7.9 APPENDIX: Short presentation of technique [1,7–9], taken from [3]

In this appendix a short review of the technique [7–9], initiated and developed by one of the authors when proposing the *spin-charge-family-theory* [1,2] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anticommuting (Grassmann) coordinates [7–9], if the dimension of ordinary space is also d and further developed by both authors of the technique. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra which anti commute with one another. The technique was further developed in the present shape together with H.B. Nielsen [7–9] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s. In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s. The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The objects γ^a and $\tilde{\gamma}^a$ have properties 7.4,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{7.69}$$

for any d, even or odd. I is the unit element in the Clifford algebra.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,,\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,,
\end{aligned} \tag{7.70}$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{7.71}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (7.69) and formally unitary, i.e. $\gamma^{a\,\dagger}\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(7.71) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(7.70) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4,\\
&S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned} \tag{7.72}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\begin{aligned}
\Gamma^{(d)} &:= (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,\\
\Gamma^{(d)} &:= (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,.
\end{aligned} \tag{7.73}$$

One can proceed equivalently for $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(7.71) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$\begin{aligned}
&\overset{ab}{(k)}:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \overset{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,\\
&\overset{+}{\circ}:= \frac{1}{2}(1+\Gamma)\,, \qquad \overset{-}{\bullet}:= \frac{1}{2}(1-\Gamma),
\end{aligned} \tag{7.74}$$

where $k^2 = \eta^{aa}\eta^{bb}$. One can easily check by taking into account the Clifford algebra relation (Eq.7.69) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.7.70) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$S^{ab} \stackrel{ab}{(k)} = \frac{1}{2} k \stackrel{ab}{(k)}, \qquad S^{ab} \stackrel{ab}{[k]} = \frac{1}{2} k \stackrel{ab}{[k]},$$
$$\tilde{S}^{ab} \stackrel{ab}{(k)} = \frac{1}{2} k \stackrel{ab}{(k)}, \qquad \tilde{S}^{ab} \stackrel{ab}{[k]} = -\frac{1}{2} k \stackrel{ab}{[k]}, \tag{7.75}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognizethat γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \; \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]}, \; \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \; \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)},$$
$$\tilde{\gamma}^a \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \; \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]}, \; \tilde{\gamma}^a \stackrel{ab}{[k]} = \; i \stackrel{ab}{(k)}, \; \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)} \; . \tag{7.76}$$

From Eq.(7.76) it follows

$$S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[k]}\stackrel{cd}{[k]},$$
$$S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2} \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2} \stackrel{ab}{(k)}\stackrel{cd}{(k)},$$
$$S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[k]}\stackrel{cd}{(k)},$$
$$S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(k)}\stackrel{cd}{[k]} \; . \tag{7.77}$$

From Eqs. (7.77) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0,$$
$$\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}, \quad \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]},$$
$$\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0,$$
$$\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)} \; . \tag{7.78}$$

We recognize in the first equation of the first row and the first equation of the second row the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$\stackrel{ab}{(\tilde{\pm} i)} = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad \stackrel{ab}{(\tilde{\pm} 1)} = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \tag{7.79}$$

one recognizes that

$$\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{(\tilde{-k})}\stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]} = i \stackrel{ab}{(k)}, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]} = 0 \, . \tag{7.80}$$

Recognizing that

$$\stackrel{ab}{(k)}^{\dagger} = \eta^{aa} \stackrel{ab}{(-k)} , \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]} , \tag{7.81}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned} &< \stackrel{ab}{(k)}^{\dagger} \stackrel{ab}{(k)} >= 1 , \\ &< \stackrel{ab}{[k]}^{\dagger} \stackrel{ab}{[k]} >= 1 . \end{aligned} \tag{7.82}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of d/2, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(7.72)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{array}{c} \stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0 \\ \stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0 \\ \stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0 \\ \vdots \\ \stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\;d-2}{[-k_{d-1\;d-2}]}\;\psi_0 \\ \stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0 \\ \vdots \end{array} \tag{7.83}$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$

for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (7.72)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (7.72)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra S^{ab} and $\tilde{S}^{ab}$

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}\,,
\end{aligned} \tag{7.84}
$$

a left handed ($\Gamma^{(1,13)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hypercharge (2/3) and colour $(1/2\,,1/(2\sqrt{3}))$, for example, can be written as

$$
\begin{aligned}
&\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;|\psi\rangle = \\
&\frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)||\\
&(\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi\rangle\,.
\end{aligned} \tag{7.85}
$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (7.84)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (7.84)), generate families from the starting u_R quark, transforming u_R quark from Eq. (7.85) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hypercharge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$ from Eq. (7.85) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hypercharge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$

$$
\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}\;. \tag{7.86}
$$

Below some useful relations [2] are presented

$$
\begin{aligned}
&N^{\pm}_{+} = N^1_{+} \pm i\,N^2_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,,\quad N^{\pm}_{-} = N^1_{-} \pm i\,N^2_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,,\quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\;\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,,\quad \tau^{2\mp} = (\mp)\;\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\;\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,,\quad \tilde{\tau}^{2\mp} = (\mp)\;\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\;.
\end{aligned} \tag{7.87}
$$

References

1. N.S. Mankoč Borštnik, Phys. Lett. **B 292** (1992) 25, J. Math. Phys. **34** (1993) 3731 Int. J. Theor. Phys. **40** 315 (2001), Modern Phys. Lett. **A 10** (1995) 587, Proceedings of the 13^{th} Lomonosov conference on Elementary Particle Physics in the EVE of LHC, World Scientific, (2009) p. 371-378, hep-ph/0711.4681 p.94, arXiv:0912.4532 p.119.
2. A. Borštnik, N.S. Mankoč Borštnik, hep-ph/0401043, hep-ph/0401055, hep-ph/0301029, Phys. Rev. **D 74** (2006) 073013, hep-ph/0512062.

3. N.S. Mankoč Borštnik, http://arxiv.org/abs/1011.5765, http://arXiv:1012.0224, p. 105-130.
4. A. Hernandez-Galeana, N.S. Mankoč Borštnik, http://arXiv:1012.0224, p. 166-176.
5. G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, hep-ph/0711.4681, New J. of Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0612250, p.25-50.
6. G. Bregar, N.S. Mankoč Borštnik, Phys. Rev. **D 80** (2009) 083534.
7. N.S. Mankoč Borštnik, J. of Math. Phys. **34** (1993) 3731.
8. N.S. Mankoč Borštnik, H.B. Nielsen, J. of Math. Phys. **43** (2002) 5782, hep-th/0111257,
9. N.S. Mankoč Borštnik, H. B. Nielsen, J. of Math. Phys. **44** (2003) 4817, hep-th/0303224.
10. M.Y. Khlopov, A.G. Mayorov, E.Yu. Soldatov, http://arXiv:1012.0224, p. 73–88, Proceedings Bled 2010.
11. K. Nakamura *et al.*, (Particle Data Group), J. Phys. G: **37** 075021 (2010).
12. D. Lukman, N.S. Mankoč Borštnik, H.B. Nielsen, New J. of Phys. **13** (2011) 103027, hep-th/1001.4679v5.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 94)*
Bled, Slovenia, July 11-21, 2011

8 Towards Nuclear Physics of OHe Dark Matter

M. Yu. Khlopov[1,2,3], A. G. Mayorov[1] and E. Yu. Soldatov[1]

[1]National Research Nuclear University ”Moscow Engineering Physics Institute”, 115409 Moscow, Russia
[2] Centre for Cosmoparticle Physics ”Cosmion” 115409 Moscow, Russia
[3] APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. The nonbaryonic dark matter of the Universe can consist of new stable charged particles, bound in heavy ”atoms” by ordinary Coulomb interaction. If stable particles O^{--} with charge -2 are in excess over their antiparticles (with charge +2), the primordial helium, formed in Big Bang Nucleosynthesis, captures all O^{--} in neutral ”atoms” of O-helium (OHe). Interaction with nuclei plays crucial role in the cosmological evolution of OHe and in the effects of these dark atoms as nuclear interacting dark matter. Slowed down in terrestrial matter OHe atoms cause negligible effects of nuclear recoil in underground detectors, but can experience radiative capture by nuclei. Local concentration of OHe in the matter of detectors is rapidly adjusted to the incoming flux of cosmic OHe and possess annual modulation due to Earth's orbital motion around the Sun. The potential of OHe-nucleus interaction is determined by polarization of OHe by the Coulomb and nuclear force of the approaching nucleus. Stark-like effect by the Coulomb force of nucleus makes this potential attractive at larger distances, while change of polarization by the effect of nuclear force gives rise to a potential barrier, preventing merging of nucleus with helium shell of OHe atom. The existence of the corresponding shallow well beyond the nucleus can provide the conditions, at which nuclei in the matter of DAMA/NaI and DAMA/LIBRA detectors have a few keV binding energy with OHe, corresponding to a level in this well. Annual modulation of the radiative capture rate to this level can reproduce DAMA results. The OHe hypothesis can qualitatively explain the controversy in the results of direct dark matter searches by specifics of OHe nuclear interaction with the matter of underground detectors.

8.1 Introduction

Ordinary matter around us consists of neutral atoms, in which electrically charged nuclei are bound with electrons. Few years ago we proposed that in the similar way the dark matter consists of dark atoms, in which new stable charged particles are bound by ordinary Coulomb interaction (See [1–3] for review and references). In order to avoid anomalous isotopes overproduction, stable particles with charge -1 (and corresponding antiparticles), as tera-particles [4], should be absent [5], so that stable negatively charged particles should have charge -2 only.

Elementary particle frames for heavy stable -2 charged species are provided by: (a) stable ”antibaryons” $\bar{U}\bar{U}\bar{U}$ formed by anti-U quark of fourth generation [6–9] (b) AC-leptons [9–11], predicted in the extension [10] of standard model, based on the approach of almost-commutative geometry [12]. (c) Technileptons and anti-technibaryons [13] in the framework of walking technicolor models (WTC) [14]. (d) Finally, stable charged clusters $\bar{u}_5\bar{u}_5\bar{u}_5$ of (anti)quarks $\bar{u}_5$ of 5th family can follow from the approach, unifying spins and charges [15]. Since all these models also predict corresponding +2 charge antiparticles, cosmological scenario should provide mechanism of their suppression, what can naturally take place in the asymmetric case, corresponding to excess of -2 charge species, O^{--}. Then their positively charged antiparticles can effectively annihilate in the early Universe.

If new stable species belong to non-trivial representations of electroweak SU(2) group, sphaleron transitions at high temperatures can provide the relationship between baryon asymmetry and excess of -2 charge stable species, as it was demonstrated in the case of WTC [13,16–18].

After it is formed in the Standard Big Bang Nucleosynthesis (SBBN), ^4He screens the O^{--} charged particles in composite $(^4He^{++}O^{--})$ *O-helium* "atoms" [7].

In all the proposed forms of O-helium, O^{--} behaves either as lepton or as specific "heavy quark cluster" with strongly suppressed hadronic interaction. Therefore interaction with matter of O-helium is determined by nuclear interaction of its helium shell. These neutral primordial nuclear interacting objects contribute to the modern dark matter density and play the role of a nontrivial form of strongly interacting dark matter [19,20].

The qualitative picture of OHe cosmological evolution [1,2,7,11,13,17,21] was recently reviewed in [3]. Here we concentrate on some open questions in the properties of O-helium dark atoms and their interaction with matter, which are crucial for our explanation of the puzzles of dark matter searches.

8.2 O-helium interaction with nuclei

8.2.1 Structure of O^{--} atoms with nuclei

The properties of OHe interaction with matter are determined first of all by the structure of OHe atom that follows from the general analysis of the bound states of non-hadronic negatively charged particles X with nuclei in a simple model [22], in which the nucleus is regarded as a sphere with uniform charge density. Spin dependence is not taken into account so that both the particle and nucleus are considered as scalars.

Variational treatment of the problem [22] gives for

$$0 < a = ZZ_x \alpha A m_p R < 1$$

the Coulomb binding energy like in hydrogen atom, while at

$$2 < a < \infty$$

for large nuclei X is inside nuclear radius and the harmonic oscillator approximation is valid. Here α is the fine structure constant, $R = d_o A^{1/3} \sim 1.2A^{1/3}/(200MeV)$ is the nuclear radius, Z is the electric charge of nucleus and Z_x is the electric charge of negatively charged particle X with the mass $m_o = S_3$ TeV. The reduced mass is $1/m = 1/(Am_p) + 1/m_o$ and for $Am_p \ll m_o$ is $m \approx Am_p$.

In the case of OHe ($Z_x = 2$, $Z = 2$,$A = 4$)

$$a = ZZ_x \alpha A m_p R \leq 1,$$

proving its Bohr-atom-like structure, assumed in our earlier papers [7–9,13,17,18,23]. However, the size of He, rotating around O^{--} in this Bohr atom, turns out to be of the order and even a bit larger than the radius r_o of its Bohr orbit, and the corresponding correction to the binding energy due to non-point-like charge distribution in He is significant. The variational approach [22] gives in the limit of small a the expression for binding energy

$$E_b(a) = (\frac{1}{2}a^2 - \frac{2}{5}a^4)/(Am_pR^2). \quad (8.1)$$

Therefore the hydrogen-like Bohr atom binding energy of OHe

$$E_b = \frac{1}{2}Z^2Z_x^2\alpha^2Am_p = 1.6\,\text{MeV}$$

is corrected for helium final size effect as follows:

$$E_b = \frac{1}{2}Z^2Z_x^2\alpha^2Am_p - \frac{2}{5}Z^4Z_x^4\alpha^4A^3m_p^3R^2 \approx 1.3\,\text{MeV}. \quad (8.2)$$

Bohr atom like structure of OHe seems to provide a possibility to use the results of atomic physics for description of OHe interaction with matter. However, the situation is much more complicated. OHe atom is similar to the hydrogen, in which electron is hundreds times heavier, than proton, so that it is proton shell that surrounds "electron nucleus". Nuclei that interact with such "hydrogen" would interact first with strongly interacting "protonic" shell and such interaction can hardly be treated in the framework of perturbation theory. Moreover in the description of OHe interaction the account for the finite size of He, which is even larger than the radius of Bohr orbit, is important. One should consider, therefore, the analysis, presented below, as only a first step approaching true nuclear physics of OHe.

8.2.2 Potential of O-helium interaction with nuclei

The approach of [1–3] assumes the following picture of OHe interaction with nuclei: OHe is a neutral atom in the ground state, perturbed by Coulomb and nuclear forces of the approaching nucleus. The sign of OHe polarization changes with the distance: at larger distances Stark-like effect takes place - the Coulomb force of nucleus polarizes OHe so that He is put behind O^{--} and nucleus is attracted by the induced dipole moment of OHe, while as soon as the perturbation by nuclear force starts to dominate the nucleus polarizes OHe in the opposite way so that He is virtually situated more close to the nucleus, resulting in a dipole Coulomb barrier for helium shell in its merging with the approaching nucleus. Correct mathematical description of this change of OHe polarization, induced by the simultaneous action of Coulomb force and strongly nonhomogeneous nuclear force needs special treatment. For the moment we use the analogy with Stark effect in the ground state of hydrogen atom and approximate the form of dipole Coulomb barrier by the Coulomb barrier in the theory of α decay, corrected for the Coulomb attraction of nucleus by O^{--}. When helium is completely merged with the nucleus the interaction is reduced to the oscillatory potential of O^{--} with homogeneously charged merged nucleus with the charge $Z + 2$.

Therefore OHe-nucleus potential has qualitative feature, presented on Fig. 8.1 by solid line. To simplify the solution of Schrodinger equation the potential was approximated in [1,2] by a rectangular wells and wall, shown by dashed lines on Fig. 8.1. The existence of potential barrier U_2 in region II causes suppression of reactions with transition of OHe-nucleus system to levels in the potential well U_1 of the region I. It results in the dominance of elastic scattering while transitions to levels in the shallow well U_3 (regions III-IV) should dominate in reactions of OHe-nucleus capture.

Schrodinger equation for OHe-nucleus system is reduced to the problem of relative motion for the reduced mass $m = \frac{Am_p m_o}{Am_p + m_o}$ in the spherically symmetric potential, presented on Fig. 8.1. If the mass of OHe $m_o \gg Am_p$, center of mass of OHe-nucleus system

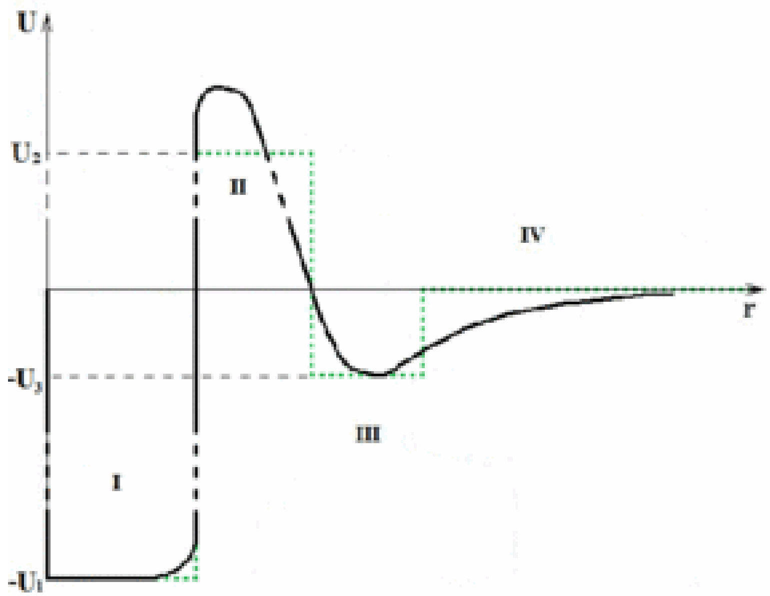

Fig. 8.1. The potential of OHe-nucleus system and its rectangular well and wall approximation.

approximately coincides with the position of O^{--} and the reduced mass is approximately equal to the mass of nucleus Am_p, where A is its atomic weight.

Solutions of Schrodinger equation for each of the four regions, indicated on Fig. 8.1, are given in textbooks (see e.g.[24]) and their sewing determines the condition, under which a low-energy OHe-nucleus bound state appears in the region III.

Strictly speaking, we should deal with a three-body problem for the system of He, nucleus and O^{--} and the correct quantum mechanical description should be based on the cylindrical and not spherical symmetry. In the lack of the exact solution of the problem we present here qualitative arguments for the existence and properties of OHe-nucleus bound states.

8.3 OHe in the direct searches for dark matter

8.3.1 O-helium in the terrestrial matter

The evident consequence of the O-helium dark matter is its inevitable presence in the terrestrial matter, which appears opaque to O-helium and stores all its in-falling flux.

After they fall down terrestrial surface, the in-falling OHe particles are effectively slowed down due to collisions with matter, which are dominantly elastic as follows from our description of OHe-nucleus interaction. Then they drift, sinking down towards the center of the Earth with velocity [7]

$$V = \frac{g}{n\sigma v} \approx 80 S_3 A_{med}^{1/2} \text{ cm/s}. \tag{8.3}$$

Here $A_{med} \sim 30$ is the average atomic weight in terrestrial surface matter, $n = 2.4 \cdot 10^{24}/A$ is the number density of terrestrial atomic nuclei, σv is the rate of nuclear collisions and $g = 980 \text{ cm/s}^2$.

In underground detectors, OHe "atoms" are slowed down to thermal energies and give rise to energy transfer $\sim 2.5 \cdot 10^{-4}$ eVA/S_3, far below the threshold for direct dark matter detection. It makes this form of dark matter insensitive to the severe CDMS [25] and XENON100 [26] constraints. However, OHe induced processes in the matter of underground detectors can result in observable effects. These effects strongly depend on the details of the OHe interaction with nuclei.

It should be noted that the nuclear cross section of the O-helium interaction with matter escapes the severe constraints [19] on strongly interacting dark matter particles (SIMPs) [19,20] imposed by the XQC experiment [27]. Therefore, a special strategy of direct O-helium search is needed, as it was proposed in [28].

Near the Earth's surface, the O-helium abundance is determined by the equilibrium between the in-falling and down-drifting fluxes.

At a depth L below the Earth's surface, the drift timescale is $t_{dr} \sim L/V$, where $V \sim 400 S_3$ cm/ s is the drift velocity (8.3) and $m_o = S_3$ TeV is the mass of O-helium. It means that the change of the incoming flux, caused by the motion of the Earth along its orbit, should lead at the depth $L \sim 10^5$ cm to the corresponding change in the equilibrium underground concentration of OHe on the timescale $t_{dr} \approx 2.5 \cdot 10^2 S_3^{-1}$ s.

The equilibrium concentration, which is established in the matter of underground detectors at this timescale, is given by [29]

$$n_{oE} = n_{oE}^{(1)} + n_{oE}^{(2)} \cdot \sin(\omega(t - t_0)), \tag{8.4}$$

where $\omega = 2\pi/T$, T is the period of Earth's orbital motion around Sun and t_0 is the phase. So, there is a averaged concentration given by

$$n_{oE}^{(1)} = \frac{n_o}{320 S_3 A_{med}^{1/2}} V_h \tag{8.5}$$

and the annual modulation of concentration characterized by the amplitude

$$n_{oE}^{(2)} = \frac{n_o}{640 S_3 A_{med}^{1/2}} V_E. \tag{8.6}$$

Here V_h is velocity (220 km/s) of Solar System in the Galaxy, V_E is velocity (29.5 km/s) of Earth's orbital motion around Sun and $n_0 = 3 \cdot 10^{-4} S_3^{-1}$ cm^{-3} is the local density of O-helium dark matter.

8.3.2 OHe in the underground detectors

The explanation [1,29] of the results of DAMA/NaI [30] and DAMA/LIBRA [31] experiments is based on the idea that OHe, slowed down in the matter of detector, can form a few keV bound state with nucleus, in which OHe is situated **beyond** the nucleus. Therefore the positive result of these experiments is explained by annual modulation in reaction rate of radiative capture of OHe

$$A + (^4He^{++}O^{--}) \rightarrow [A(^4He^{++}O^{--})] + \gamma \tag{8.7}$$

by nuclei in DAMA detector.

Solution of Schrodinger equation determines the condition, under which a low-energy OHe-nucleus bound state appears in the shallow well of the region III and the range of nuclear parameters was found [1–3], at which OHe-sodium binding energy is in the interval 2-4 keV.

The rate of radiative capture of OHe by nuclei can be calculated [1,29] with the use of the analogy with the radiative capture of neutron by proton with the account for: i) absence of M1 transition that follows from conservation of orbital momentum and ii) suppression of E1 transition in the case of OHe. Since OHe is isoscalar, isovector E1 transition can take place in OHe-nucleus system only due to effect of isospin nonconservation, which can be measured by the factor $f = (m_n - m_p)/m_N \approx 1.4 \cdot 10^{-3}$, corresponding to the difference of mass of neutron, m_n, and proton, m_p, relative to the mass of nucleon, m_N. In the result the rate of OHe radiative capture by nucleus with atomic number A and charge Z to the energy level E in the medium with temperature T is given by [1,29]

$$\sigma v = \frac{f\pi\alpha}{m_p^2} \frac{3}{\sqrt{2}} (\frac{Z}{A})^2 \frac{T}{\sqrt{Am_pE}}. \tag{8.8}$$

Formation of OHe-nucleus bound system leads to energy release of its binding energy, detected as ionization signal. In the context of our approach the existence of annual modulations of this signal in the range 2-6 keV and absence of such effect at energies above 6 keV means that binding energy E_{Na} of Na-OHe system in DAMA experiment should not exceed 6 keV, being in the range 2-4 keV. The amplitude of annual modulation of ionization signal can reproduce the result of DAMA/NaI and DAMA/LIBRA experiments for $E_{Na} = 3\,\text{keV}$. The account for energy resolution in DAMA experiments [32] can explain the observed energy distribution of the signal from monochromatic photon (with $E_{Na} = 3\,\text{keV}$) emitted in OHe radiative capture.

At the corresponding nuclear parameters there is no binding of OHe with iodine and thallium [1].

It should be noted that the results of DAMA experiment exhibit also absence of annual modulations at the energy of MeV-tens MeV. Energy release in this range should take place, if OHe-nucleus system comes to the deep level inside the nucleus. This transition implies tunneling through dipole Coulomb barrier and is suppressed below the experimental limits.

For the chosen range of nuclear parameters, reproducing the results of DAMA/ NaI and DAMA/LIBRA, the results [1] indicate that there are no levels in the OHe-nucleus systems for heavy nuclei. In particular, there are no such levels in Xe, what seem to prevent direct comparison with DAMA results in XENON100 experiment [26]. The existence of such level in Ge and the comparison with the results of CDMS [25] and CoGeNT [33] experiments need special study. According to [1] OHe should bind with O and Ca, what is of interest for interpretation of the signal, observed in CRESST-II experiment [34].

In the thermal equilibrium OHe capture rate is proportional to the temperature. Therefore it looks like it is suppressed in cryogenic detectors by a factor of order 10^{-4}. However, for the size of cryogenic devices less, than few tens meters, OHe gas in them has the thermal velocity of the surrounding terrestrial matter and this velocity dominates in the relative velocity of OHe-nucleus system. It gives the suppression relative to room temperature only $\sim m_A/m_o$. Then the rate of OHe radiative capture in cryogenic detectors is given by Eq.(8.8), in which room temperature T is multiplied by factor m_A/m_o. Note that in the case of $T = 70\,\text{K}$ in CoGeNT experiment relative velocity is determined by the thermal velocity of germanium nuclei, what leads to enhancement relative to cryogenic germanium detectors.

8.4 Discussion

The cosmological dark matter can be formed by stable heavy charged particles bound in neutral dark atoms by ordinary Coulomb attraction. Analysis of the cosmological data and atomic composition of the Universe gives the constrains on the particle charge showing that only −2 charged constituents, being trapped by primordial helium in neutral O-helium states, can avoid the problem of overproduction of the anomalous isotopes of chemical elements, which are severely constrained by observations.

This scenario can be realized in different frameworks, in particular in Minimal Walking Technicolor model or in the approach unifying spin and charges and contains distinct features, by which the present explanation can be distinguished from other recent approaches to this problem [35] (see also review and more references in [36]).

It should be noted that O-helium, being an α-particle with screened electric charge, can catalyze nuclear transformations, which can influence primordial light element abundance and cause primordial heavy element formation. It is especially important for quantitative estimation of role of OHe in Big Bang Nucleosynthesis and in stellar evolution. These effects need a special detailed and complicated study and this work is under way. Our first steps in the approach to OHe nuclear physics seem to support the qualitative picture of OHe cosmological evolution described in [1–3,7,11,13,17,21] and based on the dominant role of elastic collisions in OHe interaction with baryonic matter.

Cosmological model of O-helium dark matter can even explain puzzles of direct dark matter searches. The explanation is based on the mechanism of low energy binding of OHe with nuclei. We have found [1,2] that within the uncertainty of nuclear physics parameters there exists their range at which OHe binding energy with sodium is equal to 4 keV and there is no such binding with iodine and thallium. Annual modulation of the energy release in the radiative capture of OHe to this level explains the results of DAMA/NaI and DAMA/LIBRA experiments.

With the account for high sensitivity of our results to the values of uncertain nuclear parameters and for the approximations, made in our calculations, the presented results can be considered only as an illustration of the possibility to explain effects in underground detectors by OHe binding with intermediate nuclei. However, even at the present level of our studies we can make a conclusion that effects of such binding should strongly differ in detectors with the content, different from NaI, and can be absent in detectors with very light (e.g. ^3He) and heavy nuclei (like xenon). Therefore test of results of DAMA/NaI and DAMA/LIBRA experiments by other experimental groups can become a very nontrivial task. Recent indications to positive result in the matter of CRESST detector [34], in which OHe binding is expected together with absence of signal in xenon detector [26], may qualitatively favor the presented approach. For the same chemical content an order of magnitude suppression in cryogenic detectors can explain why indications to positive effect in CoGeNT experiment [33] can be compatible with the constraints of CDMS experiment.

An inevitable consequence of the proposed explanation is appearance in the matter of underground detectors anomalous superheavy isotopes, having the mass roughly by m_o larger, than ordinary isotopes of the corresponding elements.

It is interesting to note that in the framework of the presented approach positive result of experimental search for WIMPs by effect of their nuclear recoil would be a signature for a multicomponent nature of dark matter. Such OHe+WIMPs multicomponent dark matter scenarios naturally follow from AC model [11] and can be realized in models of Walking technicolor [16].

The presented approach sheds new light on the physical nature of dark matter. Specific properties of dark atoms and their constituents are challenging for the experimental search. The development of quantitative description of OHe interaction with matter confronted with the experimental data will provide the complete test of the composite dark matter model. It challenges search for stable double charged particles at accelerators and cosmic rays as direct experimental probe for charged constituents of dark atoms of dark matter.

Acknowledgments

We would like to thank Norma Mankoč Borštnik, all the participants of Bled Workshop and to A.S. Romaniouk for stimulating discussions.

References

1. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, J. Phys.: Conf. Ser **309** (2011) 012013.
2. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, Bled Workshops in Physics **11** (2010) 73.
3. M. Y. Khlopov, arXiv:1111.2838, to be published in Mod. Phys. Lett. A (2011); arXiv:1111.2887, to be published in *Proc. ICATPP2011*.
4. S. L. Glashow, arXiv:hep-ph/0504287.
5. D. Fargion and M. Khlopov, arXiv:hep-ph/0507087.
6. K.M.Belotsky *et al*, Gravitation and Cosmology **11** (2005) 3.
7. M.Yu. Khlopov, JETP Lett. **83** (2006) 1.
8. K. Belotsky *et al*, arXiv:astro-ph/0602261; K. Belotsky *et al*, Gravitation and Cosmology **12** (2006) 1; K. Belotsky, M.Yu.Khlopov,K.I.Shibaev, Stable quarks of the 4th family? in Eds. N. L. Watson and T. M. Grant: "The Physics of Quarks: New Research." (Horizons in World Physics, V.265), NOVA Publishers, Hauppauge NY, 2009, PP.19-47; arXiv:0806.1067 [astro-ph].
9. M. Y. Khlopov, arXiv:astro-ph/0607048.
10. C. A. Stephan, arXiv:hep-th/0509213.
11. D. Fargion *et al*, Class. Quantum Grav. **23** (2006) 7305; M. Y. Khlopov and C. A. Stephan, arXiv:astro-ph/0603187.
12. A. Connes *Noncommutative Geometry* (Academic Press, London and San Diego, 1994).
13. M. Y. Khlopov and C. Kouvaris, Phys. Rev. **D 77** (2008) 065002.
14. F. Sannino and K. Tuominen, Phys. Rev. **D 71** (2005) 051901; D. K. Hong *et al*, Phys. Lett. **B 597** (2004) 89; D. D. Dietrich *et al*, Phys. Rev. **D 72** (2005) 055001; D. D. Dietrich *et al*, Phys. Rev. **D 73** (2006) 037701; S. B. Gudnason *et al*, Phys. Rev. **D 73** (2006) 115003; S. B. Gudnason *et al*, Phys. Rev. **D 74** (2006) 095008.
15. N.S. Mankoč Borštnik, Bled Workshops in Physics **11** (2010) 105; A. Borštnik Bračič, N.S. Mankoč Borštnik, Phys. Rev. **D 74** (2006) 073013; N.S. Mankoč Borštnik, Mod. Phys. Lett. **A 10** (1995) 587; N.S. Mankoč Borštnik, Int. J. Theor. Phys. **40** (2001) 315; G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, New J. of Phys. **10** (2008) 093002.
16. M. Y. Khlopov and C. Kouvaris, Phys. Rev. **D 78** (2008) 065040.
17. M. Y. Khlopov,*AIP Conf. Proc.* **1241**, 388 (2010).
18. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, Int. J. Mod. Phys **D 19** (2010) 1385.
19. B. D. Wandelt et al., arXiv:astro-ph/0006344; P. C. McGuire and P. J. Steinhardt, arXiv:astro-ph/0105567; G. Zaharijas and G. R. Farrar, Phys. Rev. **D 72** (2005) 083502.
20. C. B. Dover *et al*, Phys. Rev. Lett. **42** (1979) 1117; S. Wolfram, Phys. Lett. **B 82** (1979) 65; G. D. Starkman *et al*, Phys. Rev. **D 41** (1990) 3594; D. Javorsek *et al*, Phys. Rev. Lett. **87** (2001) 231804; S. Mitra, Phys. Rev. **D 70** (2004) 103517; G. D. Mack *et al*, Phys. Rev. **D 76** (2007) 043523.
21. M. Y. Khlopov, arXiv:0801.0167 [astro-ph]; M. Y. Khlopov, arXiv:0801.0169 [astro-ph].
22. R. N. Cahn and S. L. Glashow, Science **213** (1981) 607; M. Pospelov, Phys. Rev. Lett. **98** (2007) 231301; K. Kohri and F. Takayama, Phys. Rev. **D 76** (2007) 063507.
23. M. Y. Khlopov, arXiv:0806.3581 [astro-ph].
24. L. D. Landau, E. M. Lifshitz *Quantum Mechanics: Non-Relativistic Theory* (Fizmatlit, Moscow, 2004).

25. D. S. Akerib *et al.* [CDMS Collaboration], Phys. Rev. Lett. **96** (2006) 011302; Z. Ahmed *et al.* [CDMS Collaboration], Phys. Rev. Lett. **102** (2009) 011301; N. Mirabolfathi *et al.* [CDMS Collaboration], Nucl. Instrum. Methods **A 559** (2006) 417.
26. E. Aprile *et al.* [XENON100 Collaboration], Phys. Rev. Lett. **105** (2010) 131302.
27. D. McCammon *et al*, Nucl. Instrum. Methods **A 370** (1996) 266; D. McCammon *et al*, Astrophys. J. **576** (2002) 188
28. K. Belotsky *et al*, arXiv:astro-ph/0606350.
29. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, Bled Workshops in Physics **11** (2010) 185.
30. R. Bernabei *et al.*, Rivista Nuovo Cimento **26** (2003) 1.
31. R. Bernabei *et al.* [DAMA Collaboration], Eur.Phys.J **C 56** (2008) 333.
32. R. Bernabei *et al.* [DAMA Collaboration], Nucl. Instrum. Methods **A 592** (2008) 297.
33. C. E. Aalseth *et al.*, Phys. Rev. Lett. **107** (2011) 141301.
34. G. Angloher *et al.*, arXiv:1109.0702 [astro-ph.CO].
35. F. Petriello and K. M. Zurek, JHEP **0809** (2008) 047; R. Foot, Phys. Rev. **D 78** (2008) 043529; J. L. Feng, J. Kumar and L. E. Strigari, Phys. Lett. **B 670** (2008) 37; J. L. Feng, J. Kumar, J. Learned and L. E. Strigari, JCAP **0901** (2009) 032; E. M. Drobyshevski, Mod. Phys. Lett. **A 24** (2009) 177; B. Feldstein, A. L. Fitzpatrick and E. Katz, JCAP **1001** (2010) 020; Y. Bai and P. J. Fox, JHEP **0911** (2009) 052; B. Feldstein, A. L. Fitzpatrick, E. Katz and B. Tweedie, JCAP **1003** (2010) 029; A. L. Fitzpatrick, D. Hooper and K. M. Zurek, Phys. Rev. **D 81** (2010) 115005; S. Andreas, C. Arina, T. Hambye, F. S. Ling and M. H. G. Tytgat, Phys. Rev. **D 82** (2010) 043522; D. S. M. Alves, S. R. Behbahani, P. Schuster and J. G. Wacker, JHEP **1006** (2010) 113; V. Barger, M. McCaskey and G. Shaughnessy, Phys. Rev. **D 82** (2010) 035019; C. Savage, G. Gelmini, P. Gondolo and K. Freese,Phys. Rev. **D 83** (2011) 055002; D. Hooper, J. I. Collar, J. Hall and D. McKinsey, Phys. Rev. **D 82** (2010) 123509; S. Chang, R. F. Lang and N. Weiner, Phys. Rev. Lett. **106** (2011) 011301; S. Chang, N. Weiner and I. Yavin, Phys. Rev. **D 82** (2010) 125011; V. Barger, W. Y. Keung and D. Marfatia, Phys. Lett. **B 696** (2011) 74; A. L. Fitzpatrick and K. M. Zurek, arXiv:1007.5325 [hep-ph]; T. Banks, J. F. Fortin and S. Thomas, arXiv:1007.5515 [hep-ph]; B. Feldstein, P. W. Graham and S. Rajendran, Phys. Rev. **D 82** (2010) 075019; De-Chang Dai, K. Freese, D. Stojkovic, JCAP **0906** (2009) 023; Jia-Ming Zheng *et al.*, Nucl. Phys. **B 854** (2012) 350.
36. G. B. Gelmini, Int. J. Mod. Phys. **A 23** (2008) 4273; E. Aprile, S. Profumo, New J. of Phys. **11** (2009) 105002; J. L. Feng, Ann. Rev. Astron. Astrophys.**48** (2010) 495.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 103)*
Bled, Slovenia, July 11-21, 2011

9 Fundamental Nonlocality

A. Kleppe⋆

SACT, Stensgt. 40 B, Oslo, Norway

Abstract. According to the Random Dynamics approach the structured physical reality that we observe emerges from a fundamentally chaotic and (almost) random primal layer. Properties that we take for granted, like space and time, causality and locality, are all derived. A mild nonlocality is however believed to remain even after the introduction of locality, but this (classical) nonlocality is very different from the quantum nonlocality discussed by Einstein, Rosen, Podolsky, as well as by John Bell. How to get to grips with the classical nonlocality of Random Dynamics, and how is it related to quantum nonlocality?

9.1 Introduction

Locality is a property that is mostly taken for granted in field theory. Perhaps that is the reason why it actually means so seldom is discussed at great length.

We usually think of locality in terms of information being localized, and propagating from one spacetime point to another by at most the speed of light. Nonlocality, on the other hand, refers to a situation where information can spread out ″instantaneously″ over a large distance.

In the Random Dynamics approach[1], locality is not perceived as fundamental. The reason is that the primal Random Dynamics ″world machinery″ $\mathcal{M}$ is a very general, random mathematical structure which merely contains non-identical elements and some set-theoretical notions. From this $\mathcal{M}$, differentiability and a concept of distance (geometry), as well as space and time, Lorentz invariance, locality, and eventually all other physical concepts, are to be derived.

But even after locality is derived, some smeared out left-over nonlocal effects remain, showing up in coupling constants (which feel an average over spacetime, and also depend on such averages). This remaining (mild) nonlocality is moreover supported by the Multiple Point Principle (MPP)[2].

In a nonlocal theory, the degrees of freedom are functions of more than one spacetime point. This allows for making predictions in a noncausal way, i.e. to get information about parts of the Universe that are at a spacelike interval from ourselves.

At our everyday, classical level, reality is however convincingly ″local″, which is reflected in that

- The Laws of Nature - the equations of physics - are local.
- Special relativity advocates locality.
- The continuity equation tells that there are no jumps!

Interactions are thus generically local, taking place in one spacetime point, implying that one spatio-temporal site x_μ is assigned to each degree of freedom. This is closely related to the idea of causation being $\mathtt{local}$: A can influence B provided B is ″within reach″.

⋆ e-mail: astri.kleppe@gmail.com

In a local theory things thus occur locally, and the action can be factorized; $S = S_1 + S_2 + ...$, where each S_j depends on the fields in limited regions of x_μ-spacetime.

This is intuitively graspable, locality seems to be a quite manageable and transparent concept. It is however not such a simple business to establish the conditions for having locality; one should not forget that Isaac Newton struggled with this notion, and locality was in reality only restored with relativity.

The question of the conditions for locality has recently been addressed by Don Bennett and Holger Bech Nielsen, who in the context of their model for explaining the phenomenologically observed spacetime flatness[3], state that to get locality we need:

- reparametrization invariance.
- spontaneous breakdown of translational invariance by new fields.

9.2 Nonlocality in Random Dynamics and the Multiple Point Principle scheme

One can imagine a scenario where nature is nonlocal at a fundamental level, yet effectively local at larger scales. the Multiple Point Principle scheme even postulates nonlocality at extremely long distances, as long as the long-distance nonlocality is invariant under diffeomorphisms or reparametrization.

In a case of extreme nonlocality, a term in the Lagrangian could depend on many space-time points, whereby interactions would occur between all space-time points at once, and the action would no longer be a sum of terms depending on limited space-time regions.

The nonlocality of Random Dynamics and MPP does however not appear in the action, but rather as signals propagated at once all over space and time: *"a mild form of nonlocality consisting of an interaction that is the same between any pair of points in spacetime independent of the distance between these points"*[4].

This means a nonlocality which cannot transmit (information carrying) wave pulses, while it *can* be used to transmit (unmodulated flat) waves which carry no information.

An interaction which is the same between the fields at any pair of spacetime points would however not be perceived as a nonlocal effect (regardless of the distance between the points), but rather as a background phenomenon which so to speak appears in the constants of nature. This is the point of view which is advocated in the Multiple Point Principle scenario. It is argued that since in a local theory, the dynamical (bare) physical constants can only depend on local spacetime points, there is only an (indirect) dependency on the past, but not on the future. How then can a bare cosmological constant at the stage of the Big Bang be finetuned to obtain the tiny value of the dressed cosmological constant of today - it seems to need some kind of advanced fortunetelling anticipating the coming stages of the universe. This can be solved precisely by assuming that the strict principle of locality is broken, with a nonlocality that admits a certain degree of the needed "clairvoyance", i.e. influence from the future. This nonlocality is precisely the "mild", classical nonlocality of Random Dynamics and the MPP scheme.

One may however still stumble over the concept of "an interaction that is the same between any pair of points in spacetime independent of the distance between these points".

9.2.1 Interpreting nonlocality

Locality is expressed in terms of a Lagrangian or an action that depends on one spacetime point, while a nonlocal action is a function of several spacetime points. In the local theory interactions are well localized and causality is well defined, while in the nonlocal case what is near and what is distant becomes blurred, since there can in principle exist connections over very large distances.

Intuitively, physical *distance* is a notion connected to motion, to the transportation from point A to point B. When it's fast to travel between A and B, the distance is small. So when there are correlations between events in two separated points A and B, one may say that the distance between A and B (from a correlation perspective) is small or even irrelevant. A situation where the interaction is the same between any pair of points in spacetime is more extreme.

What is an interaction that is independent of the distance? One can imagine a space with "distance independent" distances - an example being the metric space M with a discrete metric d such that if $x, y \in M$, the distance

$$d(x, y) = \begin{cases} 1 \text{ if } x \neq y \\ 0 \text{ if } x = y \end{cases} \tag{9.1}$$

Another example of a space with "distance independence" is an N-dimensional space where N is very large. In such a space the *expected distance* between a pair of points that are independently selected at random, is approximately $\sqrt{N/6}$. That is, the expected distance between two random points is about the same irrespectively which random points we choose. This result is valid for any bounded N-space (and if the Universe is unbounded, we can always cut out a bounded subspace that will contain our pairs of points).

But it's hard to imagine that the postulated interaction which is "the same between any pair of points in spacetime independent of the distance between these points" should indicate that we live in a N-dimensional space, and instead of overinterpreting geometry, one may question what in this connection is meant by 'interaction'. In the usual understanding, an interaction transmits (energy and) information between two spacetime points A and B. In order to carry information one needs a wave packet (with a beginning and an end) which will always be a superposition of many wavelengths (each with its own phase velocity). For a packet the frequency is some function of the wave length: $\omega = \omega(\lambda)$, and the velocity of the packet (the group velocity) is $d\omega/d\lambda < c$ (which for a real plane wave is zero, since $\omega(\lambda)$ is a constant).

If one instead imagines an (ideal) plane wave with no beginning and no end, which travels between two separated spacetime points A and B with arbitrarily large (i.e. $\gg c$) (phase) velocity (which poses no phenomenological problems, since such an unmodulated wave cannot carry any information between two different points), it cannot be used to create causal events between A and B. This is how the interaction appearing in the definition of the mild nonlocality of Random Dynamics and MPP should be understood, i.e. not an interaction in the causal sense but rather in the sense that a plane wave can "leave" from A and subsequently "arrive" at B, and since "leaving" and "arriving" occur simultaneously one can establish "interactions" (that do not convey information of course) by arbitrarily assigning "departures" and "arrivals" to any pair of spacetime points.

9.3 Quantum nonlocality

The mild nonlocality postulated by Random Dynamics and the Multiple Point Principle, is obviously not the same as the quantum nonlocality which is at hand when measurements on two or more distant quantum systems turn out to be correlated in a way that defies classical description.

Such quantum mechanical effects, interpreted as nonlocal, or non-separable, are the Aharohnov-Bohm effect[5] or the nonlocality discussed by Einstein, Rosen and Podolsky (EPR)[6]. While the Aharonov-Bohm effect implies nonlocality in the equations of motion, the nonlocality discussed by EPR, concerns nonlocal correlations.

When Einstein, Rosen and Podolsky in 1935 criticized the idea of the wave function as a complete description of the system, the argument was that the wave function doesn't capture some local, real properties that are part of the physical (quantum) reality. They argued that the wave function doesn't tell the whole story, and therefor there must be some more - hidden - variables. They in reality stated that quantum theory needs these additional variables to restore causality and locality.

The EPR demand that quantum mechanics should be a complete theory is not a requirement for a deterministic theory, but for a theory such that there is locality and separability for composite systems. Each separate component should thus be characterized by its own separate properties, and it should be impossible to alter the properties on a distant system by acting on a local system.

The classical example brought up by EPR is a spin zero particle which decays into two entangled spin 1/2 particles A and B. When a measurement performed on A results in a definite outcome a, a subsequent measurement on B will have an outcome which is complementary to a. Since there is supposedly no interaction between the states, the anti-correlation of the two outcomes gives the impression of originating from some pre-existing determined values of the measurement results. The situation appears to involve local, real properties that are nevertheless not captured by the wave equation. There seems to be some information that is unaccounted for by quantum theory, which therefore according to EPR, is incomplete.

So is there some kind of information flux between A and B, or are there some hidden variables that are somehow at hand, like a pool of information from which reality serves itself? By formulating the requirements made by Einstein, Rosen and Podolsky as probability constraints, John Bell[7] in 1964 developed a strategy which offers a testable difference between the predictions of quantum theory and the predictions of local hidden variable theories. He showed that the probability constraints are equivalent to the requirement that statistical correlations between separated systems should be reducible to a common cause, and with the assumption of Einsteinian locality and the assumption of physical realism (in the sense that particle properties, i.e. spin, mass, position, etc, are taken to be 'real'), he derived a joint probability distribution for measurements on two separate particles, expressed as an inequality demonstrating that the particles cannot be as strongly correlated as predicted by quantum mechanics.

In Bell's own words, *"in a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that a theory could not be Lorentz invariant"*.

Experimental evidence[8] based on Bell's inequality implies that the local realism favoured by Einstein yields predictions that disagree with those of quantum mechanical theory, thus ruling out hidden variable theories. That is, no physical theory of local hidden variables can reproduce all of the predictions of quantum mechanics.

Bell concluded that we must either accept nonlocality or abandon local realism. In a many-world interpretation, however, the observed correlations do not demand the introduction of nonlocality, since measurements are then allowed to have non-unique outcomes. Thus Bell's conclusion is maybe not absolutely conclusive, and both Bell's assumptions and his conclusion are indeed subject of ongoing discussions[9].

One problem with Bell's reasoning is that it assumes that the measurements performed at each site can be chosen independently of each other and of the hidden variable that is presumeably being measured. In order for the argument for his inequality to follow, Bell assumed counterfactual definiteness, namely that it is meaningful to speak about what the result of the experiment would have been if different choices had been made. In a deterministic theory the measurements chosen by the experimenters at each site, are predetermined by the laws of physics. From a deterministic perspective it thus doesn't make sense to speak about what would have happened if different measurements had been made. The chosen measurements can moreover be determined in advance, and the outcomes of the measurement at one site can be affected by the measurement performed at the other, without information traveling faster than the speed of light.

One way to evade Bell's theorem is therefore to assume superdeterminism, a term describing a class of completely deterministic theories. Since counterfactual definiteness does not apply to deterministic theories, in a (hypothetical) superdetemined theory his assumption is overthrown, and therefore his entire reasoning.

In 1985, John Bell discussed superdeterminism in a BBC interview[10]:

There is a way to escape the inference of superluminal speeds and spooky action at a distance. But it involves absolute determinism in the universe, the complete absence of free will. Suppose the world is super-deterministic, with not just inanimate nature running on behind-the-scenes clockwork, but with our behavior, including our belief that we are free to choose to do one experiment rather than another, absolutely predetermined, including the "decision" by the experimenter to carry out one set of measurements rather than another, the difficulty disappears. There is no need for a faster than light signal to tell particle A what measurement has been carried out on particle B, because the universe, including particle A, already "knows" what that measurement, and its outcome, will be.

9.3.1 Joint measurability

Another issue[12] that has been addressed in the discussions about Bell's theorem, is Bell's assumption of joint measurability, the idea that two properties can be measured without mutual interference.

In a classical theory with hidden variables, with two sites A and B where measurements are performed, Bell introduced a parameter (or strategy) λ, which locally characterizes the measurement outcomes for each system. The local separability postulated by EPR reads

$$P(a, b|A, B, \lambda) = P(a|A, \lambda)P(b|B, \lambda) \tag{9.2}$$

where $P(a|A, \lambda)$ is the probability that given λ, the outcome of the measurement on A is a. Suppose λ has values running over a set λ_j and each λ_j has a probability $\rho(\lambda_j)$ of being selected. Then

$$P(a, b|A, B) = \Sigma_{j=1}^{k} P(a, b|A, B, \lambda_j)\rho(\lambda_j) \tag{9.3}$$

is the joint probability for the measurement results. The correlator $\Sigma_{a,b}P(a, b|A, B)$ represents the expectation that the measurements on A and B are correlated.

If the measurements on the first site can have two outcomes A and A', and the outcomes on the second site are B and B', one form of Bell's inequality[11] states that in a classical theory (i.e. any theory of hidden variables), a certain combination of correlations $E(A, B) + E(A, B') + E(A', B) - E(A', B')$ is limited by

$$-2 \leq E(A, B) + E(A, B') + E(A', B) - E(A', B') \leq 2 \tag{9.4}$$

If these correlations were independent, the absolute value of the sum could be as much as 4; the mathematical formalism of quantum mechanics however predicts a maximum value of $2\sqrt{2}$. The enigma is why the predicted value isn't maximal. It may be due to relativistic causality[13]. If that is the case, nonlocality and causality might determine all of quantum mechanics. This leads to the notion of axiomatic nonlocality.

9.4 Axiomatic nonlocality

The Aharanov-Bohm effect and the EPR paradox both arise within nonrelativistic quantum theory. Locality on the other hand, is closely tied up with relativity, since a local variable only causes events within its future light cone, just as it can only be caused by events in its past light cone. It is thus conceivable that nonlocality is brought about by quantum mechanics and relativity taken together[13].

Special relativity is as non-nonlocal and causal as anything can be - causality literally resides within the walls of the light cone. Quantum theory, on the other hand, doesn't altogether satisfy the locality principle, but however nonlocal quantum correlations may be, they still preserve relativistic causality in the sense that they cannot be used to transmit signals, i.e. no measurement results are so correlated that they allow signaling between two distant systems. So even if relativity and nonlocality together seem *almost* impossible, it looks like quantum mechanics somehow reconcile them.

One way of examining the relation between quantum mechanics and relativity is to consider the "level underneath", and formulate an axiomatic basis for quantum theory in analogy with the axioms underlying relativity. Special relativity can be deduced from two axioms:

- the equivalence of two inertial reference frames
- the constancy of the speed of light

Imagining an analogous axiomatic basis for quantum physics, we can attempt to deduce quantum theory from:

- relativistic causality
- nonlocality

By formulating nonlocality as an axiom, one no longer has to explain it; on the other hand, quantum indeterminacy and limits on measurements may now appear as a consequence of the presence of a nonlocal action.

If nonlocality is accepted as physical reality, relativity however implies causal ambiguity. In the EPR system, when something has an effect on A, the wavefunction of B should be 'simultaneously' effected. But in relativity, simultaneity is an ambiguous concept, the succession of events will for example in the case of the EPR experiment, depend on the chosen reference frame (in some frame the measurement on A precedes the measurement on B, while in another frame the course of events is reversed).

The attempts to derive a theory from axiomatic causality and nonlocality moreover lead to the conclusion that quantum mechanics is not the only theory which emerges from the demand of simultaneous causality and nonlocality. Quantum theory is only one of a class of theories consistent with axiomatic causality and nonlocality, and it is not even the most nonlocal theory. It has been argued that quantum mechanics not only reconciles relativity and nonlocality, but it might also be the *unique* theory combining them.

9.5 Quantum information

Physical information is limited by all kinds of constraints. The transmission of signals is limited by the speed of light; and erasure of information is a dissipative process involving compression of phase space, and therefore irreversible.

It was Szilard who in 1929 invented the concept of a bit of information, the acquisition of one bit being associated with the entropy $\Delta S = k \ln 2$, since it involves choosing from two possibilities, 0 or 1.

The corresponding unit of quantum information is the qubit, which is a vector in a 2-dimensional complex vector space with inner product. In hommage to the bit concept, the elements of an ON basis in this space are called $|0>$ and $|1>$; and a normalized vector can be represented as $|\psi>= a|0> +b|1>$, $|a|^2 + |b|^2 = 1$, where a, b are complex. One can perform measurements that project $|\psi>$ onto the basis ($|0>, |1>$), with non-deterministic outcomes: the probability of obtaining $|0>$ or $|1>$ is $|a|^2$ and $|b|^2$, respectively.

The quantum state of N qubits can be expressed as a vector in a space of dimension 2^N, and with an ON basis for this space where each qubit has a definite value $|0>$ or $|1>$, the N-qubit quantum state is $|1000110...0110>$.

One difference between classical and quantum information is the no cloning theorem: it is impossible to clone a quantum state, because quantum information cannot be copied with perfect fidelity. *If* it were possible to perfectly clone a quantum state, we could defy Heisenberg by measuring an observable of the copy without disturbing the original. That acquiring quantum information causes a disturbance is thus connected with the no cloning theorem.

9.5.1 Quantum computation and nonlocality

A quantum computer cannot do anything that a classical computer cannot do, but it does everything much much faster.

Quantum computers operate with probabilistic algorithms, meaning that if we run the same program twice we probably obtain different results, because of the randomness of the quantum measurement process. In order to describe the whole N-qubit quantum state, one might try a probabilistic classical algorithm, in which the outcome is not uniquely determined by the input. One may hope for a local simulation in which each qubit has a definite value at each time step, and each quantum gate can act on the qubits in various possible ways.

But Bell's theorem precisely addresses the impossibility of such a project: there is no local probabilistic algorithm that can reproduce the conclusions of quantum mechanics.

The reason is how quantum information is organized. Think of a 3N -qubit quantum system where $N \gg 1$. Choose a random state S of the 3N -system, and then divide the system into three subsystems, each with N qubits.

Send the subsystems to different locations in the world, say Paris, Copenhagen and Bled, and then investigate S by making measurements. We imagine having several copies of S to measure on, the only restriction being that the measurements are limited to be carried out within a subsystem - in Paris, in Copenhagen or in Bled; no collective measurements outside of the subsystems' boundaries are allowed. Then for a typical state of the 3N -qubit system, the measurements will tell almost nothing about S. Almost all the information that distinguishes one state from another resides in the nonlocal correlations between measurement outcomes in subsystems. These are the nonlocal correlations which Bell recognized as an essential part of the physical description.

If we choose a state for the 3N qubits randomly, we almost always find that the entropy of each subsystem is very close to $S = N - 2^{(N+1)}$. N is thus the maximal possible value of the entropy, corresponding to the case in which the subsystems carries no exponentially small amount of information when looking at each subsystem separately.

So measurements reveal very little information if we don't consider how the measurement results in Paris, Copenhagen and Bled were correlated with each other. The correlations are in a way part of a 'collective' measurement, and with knowledge of the correlations we can in principle completely reconstruct the state.

Nonlocal correlations are however very fragile and tend to rapidly decay. One reason is that the quantum system is in contact with a 'heat bath', namely its surrounding. Interactions between a quantum system and its environment establish nonlocal correlations between them, and the quantum information that was initially encoded in a device with time become encoded in the correlations between the device and its surrounding; and then the information is in reality lost.

An example is Schrödinger's cat: the state $|cat\rangle = (|dead\rangle + |alive\rangle)$ is in principle possible, but it is seldom observed, because it is so extremely unstable.

Once the state $|cat\rangle$ is prepared, the quantum information encoded in the superposition of $|dead\rangle$ and $|alive\rangle$ will immediately be transferred to correlations between $|cat\rangle$ and the environment, and becomes completely inaccessible. To measure on $|cat\rangle$ inevitably means to project it onto the state $|alive\rangle$ or the state $|dead\rangle$.

It was actually Schrödinger who coined the term entanglement to describe this peculiar connection between quantum systems[14]: *When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.*

...

"Another way of expressing the peculiar situation is: the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts, even though they may be entirely separate and therefore virtually capable of being best possibly known, i.e., of possessing, each of them, a representative of its own. The lack of knowledge is by no means due to the interaction being insufficiently known – at least not in the way that it could possibly be known more completely – it is due to the interaction itself."

9.6 Nonlocality in Random Dynamics

In Random Dynamics, nonlocality is perceived as the fundamental state of affairs. The notion of axiomatic quantum nonlocality avocates a similar approach. In the course of the Random Dynamics derivation of physics from the fundamental set $\mathcal{M}$, we should therefore take into account the idea of quantum mechanics being the unique theory that encompasses both nonlocality and relativity, indeed reconciling them.

References

1. "Some remarks on random dynamics", H.B. Nielsen and N. Brene Proc. of the 2nd Nishinomiya Yukawa Memorial Symposium on String Theory, Kyoto University, 1987 (Springer, Berlin, 1988); "Random Dynamics and relations between the number of

fermion generations and the fine structure constants," H. B. Nielsen (Bohr Inst.) NBI-HE-89-01, Jan 1989. 50pp. Talk presented at Zakopane Summer School, May 41 - Jun 10, 1988. Published in Acta Phys.Polon.B20:427, 1989; "Origin of Symmetries", C. D. Froggatt and H. B. Nielsen, World Scientific (1991).
2. D. L. Bennett, C. D. Froggatt, H. B. Nielsen, "Nonlocality as an explanation for finetuning and field replication in nature", NBI-95-15, GUTPA/95/04/1.
3. D. L. Bennett and H. B. Nielsen, "Explaining phenomenologically observed spacetime flatness requires new fundamental scale physics", Proceedings to the 14th Workshop, What Comes Beyond the Standard Models, Bled, July 1121, 2011.
4. D.L. Bennett and H.B. Nielsen, Proceedings of Institute of Mathematics of NAS of Ukraine 2004, Vol. 50, Part 2, 629636, The Multiple Point Principle: Realized Vacuum in Nature is Maximally Degenerate.
5. Aharonov, Y; Bohm, D (1959), "Significance of electromagnetic potentials in quantum theory". Physical Review **115** (1959) 485-491.
6. A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?" Phys. Rev. **47** (1935) 777.
7. J.S. Bell, "On the Einstein-Podolsky-Rosen Paradox" Physics **1** (1964) 195-200.
8. A. Aspect, Bell's inequality test: more ideal than ever, Nature **398** (1999) 189; A. Aspect, P. Grangier, and G. Roger (1982): 1804-1807; "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities", Physical Review Letters **49** (2): 9194.
9. Fergus Ray-Murray http://oolong.co.uk/Causality.html
10. J.S. Bell in a BBC Radio interview with Paul Davies, 1985
11. J. F. Clauser, M. A. Horne, A. Shimoney and R. A. Holt, Phys. Rev. Lett. 23 (1969) 880.
12. Thomas Brody (1993), "The Philosophy Behind Physics" (Springer-Verlag, Heidelberg Berlin).
13. S. Popescu and D. Rohrlich, "Nonlocality as an axiom for quantum theory", arXiv:quant-ph/950809v1, 1995; "Causality and nonlocality of axioms for quantum mechanics", TAUP 2452-97.
14. E. Schrödinger (1935), "Discussion of probability relations between separated systems". Mathematical Proceedings of the Cambridge Philosophical Society 31 (04): 555563; Schrödinger E (1936). "Probability relations between separated systems". Mathematical Proceedings of the Cambridge Philosophical Society 32 (03): 446452.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 112)*
Bled, Slovenia, July 11-21, 2011

10 The *spin-charge-family* Theory is Explaining the Origin of Families, of the Higgs and the Yukawa Couplings

N. S. Mankoč Borštnik

Department of Physics, FMF, University of Ljubljana, Jadranska 19, 1000 Ljubljana

Abstract. The *spin-charge-family* theory [10–14] offers a possible explanation for the assumptions of the *standard model* – for the charges of a family members, for the gauge fields, for the appearance of families, for the the scalar fields – interpreting the *standard model* as its low energy effective manifestation. The *spin-charge-family* theory predicts at the low energy regime two decoupled groups of four families of quarks and leptons. The predicted fourth family waits to be observed, while the stable fifth family is the candidate to form the dark matter. The Higss and Yukawa couplings are the low energy effective manifestation of several scalar fields, all with the bosonic (adjoint) representations with respect to all the charge groups, with the family groups included. Properties of the families are analysed and relations among coherent contributions of the loop corrections to fermion properties discussed, including the one which enables the existence of the Majorana neutrinos. The appearance of several scalar fields is presented, their properties discussed, it is explained how these scalar fields can effectively be interpreted as the *standard model* Higgs (with the fermion kind of charges) and the Yukawa couplings, and a possible explanation why the Higgs has not yet been observed offered. The relation to proposals that the Yukawas follow from the $SU(3)$ family (flavour) group, having the family charges in the fundamental representations of these groups, is discussed. The *spin-charge-family* theory predicts that there are no supersymmetric partners of the observed fermions and bosons.

10.1 Introduction

The *standard model* offered more than 35 years ago an elegant next step in understanding the origin of fermions and bosons. It is built on several assumptions leaving many open questions to be answered in the next step of theoretical interpretations. A lot of proofs and calculations have been done which support the *standard model*.

The measurements so far offer no sign which would help to make the next step beyond the *standard model*.

To explain the assumptions, which are the building blocks of the *standard model*, and to make successful new step beyond it any new proposal, model, theory must in my opinion at least i.) explain the origin of families and their mass matrices, predict the number of families, and accordingly explain the Yukawa couplings, mixing matrices and masses of family members, ii.) explain the origin of the *standard model* scalar field, the Higgs, and its connections to fermion masses and Yukawa couplings, and iii.) explain the origin of the dark matter. These three open questions are to my understanding so tightly connected that they call for common explanation.

There are many proposals in the literature [1–9] extending the *standard model*. No one explains, to my knowledge, the origin of families. Without a theory which is offering the answers to at least the above questions the suggestions for what and how should experiments search for new events can hardly be successful.

The *theory unifying spin and charges and predicting families* [10,11,19,13,14,12], to be called the *spin-charge-family* theory, seems promising in answering these, and several other questions, which the *standard model* leaves unanswered.

The *spin-charge-family* theory assumes in $d = (1 + (d - 1))$, $d = 14$ (or larger), a simple starting action for spinors and the gauge fields: Spinors carry only two kinds of the spins (no charges), namely the one postulated by Dirac 80 years ago and the second kind proposed by the author of this paper. There is no third kind of a spin. Spinors interact with only vielbeins and the two kinds of the corresponding spin connection fields.

After the breaks of the starting symmetry, leading to the low energy regime, the simple starting action (Eq.(10.4)) manifests two decoupled groups of four families of quarks and leptons, with only the left handed (with respect to $d = (1 + 3)$) members of each family carrying the weak charge while the right handed ones are weak chargeless. The fourth family is predicted [11,13] to be possibly observed at the LHC or at somewhat higher energies, while the stable fifth family members, forming neutral (with respect to the colour and electromagnetic charge) baryons and the fifth family neutrinos are predicted to explain the origin of the dark matter [14].

The spin connections, associated with the two kinds of spins, together with vielbeins, all behaving as scalar fields with respect to $d = (1 + 3)$, are with their vacuum expectation values at the two $SU(2)$ breaks responsible for the nonzero mass matrices of fermions and also for the masses of the gauge fields. The spin connections with the indices of vector fields with respect to $d = (1 + 3)$, manifest after the break of symmetries as the known gauge fields.

Although the properties of the scalar fields, that is their vacuum expectation values, coupling constants and masses, can not be calculated without the detailed knowledge of the mechanism of breaking the symmetries, and have been so far only roughly estimated, yet one can see, assuming breaks which lead to observable phenomena at the low energy regime, how properties of the scalar fields determine the fermion mass matrices, manifesting effectively as the *standard model* Higgs and its Yukawa couplings.

According to the *spin-charge-family* theory the break of the starting symmetry is caused by nonzero expectation values of vielbeins and both kinds of the spin connection fields which are scalars with respect to $SO(1,3)$ symmetry. At the symmetry of $SO(1,7) \times U(1)_{II} \times SU(3)$ there are $(2^{\frac{1+7}{2}-1})$ massless left handed (with respect to $SO(1,7)$) families [1], which stay massless until the symmetry $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ breaks. It is namely assumed that at the symmetry of $SO(1,7) \times U(1)_{II} \times SU(3)$ we indeed are left with the symmetry $SO(1,7)_\gamma \times U(1)_{II\,\gamma} \times SU(3)_\gamma \times SO(1,7)_{\tilde{\gamma}}$, while all possible other families which we started with become massive during the breaks up to this point.

Four of the eight families are doublets with respect to the generators (Eqs. (10.14, 10.15, 10.16)) $\vec{\tilde{\tau}}^2$ and with respect to $\vec{\tilde{N}}_R$, while they are singlets with respect to $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$. The other four families are singlets with respect to the first two kinds of generators and doublets with respect to the second two kinds ($\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$) of generators.

Each member of these eight massless families carries before the two successive breaks, from $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ (first to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$

[1] We studied in the refs. [17,16] a toy model, in which fermions, gauge and scalar fields live in M^{1+5}, which is broken into $M^{1+3} \times$ an infinite disc. One can find several conditions, under which only left handed family members stay massless.

and then to $SO(1,3) \times U(1) \times SU(3)$) the quantum numbers of the two $SU(2)$, one $U(1)$ and one $SU(3)$ charges and the quantum numbers of the subgroups to which each of the four families belong with respect to $\vec{\tilde{\tau}}^{(2,1)}$ and $\vec{\tilde{N}}_{(R,L)}$.

Analysing properties of each family member with respect to the quantum numbers of the spin and the charges subgroups, $SO(1,3)$, $SU(2)_I$, $SU(2)_{II}$, $U(1)_{II}$ and $SU(3)$, we find that each family includes left handed (with respect to $SO(1,3)$) weak $SU(2)_I$ charged quarks and leptons and right handed (again with respect to $SO(1,3)$) weak $SU(2)_I$ chargeless quarks and leptons. While the right handed members (with respect to $SO(1,3)$) are doublets with respect to $SU(2)_{II}$, the left handed fermions are singlets with respect to $SU(2)_{II}$.

Each member of eight massless families of quarks and leptons carries the $SU(2)_{II}$ charge, in addition to the family quantum number and the quantum numbers of the *standard model*. This quantum number determines, together with the $U(1)_{II}$ charge, after the first of the two breaks the *standard model* hyper charge and the fermion quantum number ($-\frac{1}{2}$ for leptons and $\frac{1}{6}$ for quarks).

Each break is triggered by the vielbeins and by one or both kinds of the spin connection fields. To be in agreement in the low energy regime with the *standard model* assumptions supported by the experimental data, the gauge fields which are scalars with respect to $SO(1,3)$ and have appropriate symmetries are assumed to contribute. In the break of $SU(2)_I \times SU(2)_{II} \times U(1)_{II}$ to $SU(2)_I \times U(1)_I$ the scalar (with respect to $d = (1+3)$) fields originating in vielbeins and in the second kind of spin connection fields belonging to a triplet with respect to the $SU(2)_{II}$ symmetry in the $\tilde{S}^{ab}$ sector (with the generators $\tilde{\tau}^{2i} = c^{2i}{}_{ab}\tilde{S}^{ab}$, $\{\tilde{\tau}^{2i}, \tilde{\tau}^{2j}\}_- = \varepsilon^{ijk}\tilde{\tau}^{2k}$) and with respect to one of the two $SU(2)$ from $SO(1,3)$ again in the $\tilde{S}^{ab}$ sector (with the generators $\tilde{N}^i_R = c^{Ri}{}_{ab}\tilde{S}^{ab}$, $\{\tilde{N}^i_R, \tilde{N}^j_R\}_- = \varepsilon^{ijk}\tilde{N}^k_R$) gain nonzero vacuum expectation values.

The upper four families, which are doublets with respect to these two $SU(2)$ groups, become massive and so does the $SU(2)_{II}$ gauge vector field (in the adjoint representation of $SU(2)_{II}$ in the S^{ab} sector with $\tau^{2i} = c^{2i}{}_{ab}S^{ab}$, $\{\tau^{2i}, \tau^{2j}\}_- = \varepsilon^{ijk}\tau^{2k}$). The lower four families, which are singlets with respect to $\tilde{\tau}^{2i}$ and $\tilde{N}^i_R$, the two $SU(2)$ subgroups in the $\tilde{S}^{ab}$ sector, stay massless.

This is assumed to happen below the energy scale of 10^{13} GeV, that is below the unification scale of all the three charges, and also pretty much above the electroweak break.

The lower four families become massive at the electroweak break, when $SU(2)_I \times U(1)_I$ breaks into $U(1)$. To this break the vielbeins and the scalar part of both kinds of the spin connection fields contribute, those which are triplets with respect to the two remaining invariant $SU(2)$ subgroups in the $\tilde{S}^{ab}$ sector, $\tilde{\tau}^{1i}$ ($\tilde{\tau}^{1i} = c^{1i}{}_{ab}\tilde{S}^{ab}$) and $\tilde{N}^i_L$ ($\tilde{N}^i_L = c^{Li}{}_{ab}\tilde{S}^{ab}$), as well as the scalar gauge fields of Q, Q' and Y' (all expressible with S^{ab}). In this break also the $SU(2)_I$ weak gauge vector field becomes massive [2].

Although the estimations of the properties of families done so far are very approximate [13,14], yet the predictions give a hope that the starting assumptions of the *spin-charge-family* theory are the right ones:

i. Both existing Clifford algebra operators determine properties of fermions. The Dirac γ^a's manifest in the low energy regime the spin and all the charges of fermions (like in the Kaluza-Klein[like] theories [3]). The second kind of the spin, forming the equivalent representations with respect to the Dirac one, manifests families of fermions.

---

[2] The fourth family is not in contradiction with the measurements [20]

[3] The Kaluza-Klein[like] theories [21,22] have difficulties with (almost) masslessness of the spinor fields at the low energy regime. In the refs. [16,17] we are proposing possible solutions to these kind of difficulties.

ii. Fermions carrying only the corresponding two kinds of the spin (no charges) interact with the gravitational fields – the vielbeins and (the two kinds of) the spin connections. The spin connections originating in the Dirac's gammas manifest at the low energy regime the known gauge fields. The spin connections originating in the second kind of gammas are responsible, together with the vielbeins and the spin connections of the first kind, for the masses of gauge fields and fermions.
iii. The assumed starting action for spinors and gauge fields in d-dimensional space is simple: In d-dimensional space all the fermions are massless and interact with the corresponding gauge fields of the Poincaré group and the second kind of the spin connections, the corresponding Lagrange densities for the gauge fields are linear in the two Riemann scalars.

The project to come from the starting action through breaks of symmetries to the effective action at low (measurable) energy regime is very demanding. Although one easily sees that a part of the starting action manifests, after the breaks of symmetries, at the tree level the mass matrices of the families and that a part of the vielbeins together with the two kinds of the spin connection fields manifest as scalar fields, yet several proofs are still needed besides those done so far [16,17] to guarantee that the *spin-charge-family* theory does lead to the measured effective action of the *standard model*. Very demanding calculations in addition to rough estimations [11,13,14] done so far are needed to show that predictions agree also with the measured values of masses and mixing matrices of the so far observed fermion families, explaining where do large differences among masses of quarks and leptons, as well as among their mixing matrices originate.

Let us point out that in the *spin-charge-family* theory the scalar (with respect to $(1 + 3)$) spin connection fields, originating in the Dirac kind of spin, couple only to the charges and spin, contributing on the tree level equally to all the families, distinguishing only among the members of one family (among the u-quark, d-quark, neutrino and electron, the left and right handed), the other scalar spin connection fields, originating in the second kind of spin, couple only to the family quantum numbers. Both kinds start to contribute coherently only beyond the tree level and a detailed study should manifest the drastic differences in properties of quarks and leptons: in their masses and mixing matrices [10,19]. It is a hope that the loop corrections will help to understand the differences in properties of fermions, with neutrinos included and the calculations will show to which extent are the Majorana terms responsible for the great difference in the properties of neutrinos and the rest of the family members.

In this work the mass matrices of the two groups of four families, the two groups of the scalar fields giving masses to the two groups of four families and to the gauge fields to which they couple, and the gauge fields are studied and their properties discussed, as they follow from the *spin-charge-family* theory. Many an assumption, presented above, allowed by the *spin-charge-family* theory, is made in order that the low energy manifestation of the theory agrees with the observed phenomena, but not (yet) proved that it follows dynamically from the theory.

This paper manifests that there is a chance that the properties of the observed three families naturally follow from the *spin-charge-family* theory when going beyond the tree level, although on the tree level the mass matrices of leptons and quarks are (too) strongly related. A possible explanation is made why the observed family members differ so much in their properties. It is also explained why does the *spin-charge-family* theory predict two stable families, and why and how much do the fifth family hadrons differ in their properties from the first family ones, offering the explanation for the existence of the dark matter.

In the refs. [11,13] we studied the properties of the lower four families under the assumption that the loop corrections would not change much the symmetries of the mass matrices of the family members, as they follow from the *spin-charge-family* theory on the tree level, but would take care of differences in properties among members. Relaxing strong connections between the mass matrices of the u-quark and neutrino and the d-quark and electron, we were able to predict some properties of the fourth family members and their mixing matrix elements with the three so far measured families. This paper is bringing a possible justification for these relaxation. The concrete evaluations of the properties of the mass matrices beyond the tree level are in progress. First steps are done in the contribution with A. Hernández-Galeana [15], while more detailed analyses of the mass matrices with numerical results are in preparation.

The *standard model* is presented as a low energy effective theory of the *spin-charge-family* theory. Also some attempts in the literature to understand families as the $SU(3)$ flavour extension of the *standard model* are commented.

The *spin-charge-family* does not at support the existence of the supersymmetric partners of the so far observed fields.

10.2 The spin-charge-family theory from the starting action to the standard model action

Let us in this section add to the introduction into the *spin-charge-family* theory, made in the previous section, the mathematical part. The theory assumes that the spinor carries in $d(= (1+13))$-dimensional space two kinds of the spin, no charges [10]: **i.** The Dirac spin, described by γ^a's, defines the spinor representations in $d = (1+13)$, and correspondingly in the low energy regime after several breaks of symmetries and before the electroweak break, the spin ($SO(1,3)$) and all the charges (the colour $SU(3)$, the weak $SU(2)$, the hyper charge $U(1)$) of quarks and leptons, left handed weak charged and right handed weakless, the left and the right handed distinguishing also in the weak charge, as assumed by the *standard model*. **ii.** The second kind of the spin [18], described by $\tilde{\gamma}^a$'s ($\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$) and anticommuting with the Dirac γ^a ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$), defines the families of spinors, which at the symmetries of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ manifest two groups of four massless families.

There is no third kind of the Clifford algebra objects. The appearance of the two kinds of the Clifford algebra objects can be understood as follows: If the Dirac one corresponds to the multiplication of any spinor object B (any product of the Dirac γ^a's, which represents a spinor state when being applied on a spinor vacuum state $|\psi_0>$) from the left hand side, the second kind of the Clifford objects can be understood (up to a factor, determining the Clifford evenness ($n_B = 2k$) or oddness ($n_B = 2k+1$) of the object B as the multiplication of the object from the right hand side

$$\tilde{\gamma}^a B\,|\psi_0>:= i(-)^{n_B} B\gamma^a\,|\psi_0>, \tag{10.1}$$

with $|\psi_0>$ determining the spinor vacuum state. Accordingly we have

$$\begin{aligned}
&\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0,\\
&S^{ab} := (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a), \quad \tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a), \quad \{S^{ab}, \tilde{S}^{cd}\}_- = 0.
\end{aligned} \tag{10.2}$$

More detailed explanation can be found in appendix 10.7. The *spin-charge-family* theory proposes in $d = (1+13)$ a simple action for a Weyl spinor and for the corresponding gauge fields

$$\begin{aligned}
S = &\int d^dx\, E\, \mathcal{L}_f + \\
&\int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R}),
\end{aligned} \tag{10.3}$$

$$\begin{aligned}
\mathcal{L}_f &= \frac{1}{2}\,(E\bar{\psi}\,\gamma^a p_{0a}\psi) + h.c.,\\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, Ef^{\alpha}{}_a\}_-,\\
p_{0\alpha} &= p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + h.c.\;,\\
\tilde{R} &= \frac{1}{2}\,f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^{c}{}_{b\beta}) + h.c.\;. \qquad (10.4)
\end{aligned}$$

Here [4] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. To see that the action (Eq.(10.4)) manifests after the break of symmetries [11,19,13] all the known gauge fields and the scalar fields and the mass matrices of the observed families, let us rewrite formally the action for a Weyl spinor of (Eq.(10.4)) as follows

$$\begin{aligned}
\mathcal{L}_f = \bar{\psi}\gamma^n(p_n - \sum_{A,i} g^A\tau^{Ai}A^{Ai}_n)\psi +\\
\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\;\psi\} +\\
\text{the rest}, \qquad (10.5)
\end{aligned}$$

where $n = 0, 1, 2, 3$ with

$$\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\;S^{ab},\\
\{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB}f^{Aijk}\tau^{Ak}. \qquad (10.6)
\end{aligned}$$

All the charge (τ^{Ai} (Eqs. (10.6), (10.15), (10.16)) and the spin (Eq. (10.14)) operators are expressible with S^{ab}, which determine all the internal degrees of freedom of one family.

Index A enumerates all possible spinor charges and g^A is the coupling constant to a particular gauge vector field A^{Ai}_n. Before the break from $SO(1,3)\times SU(2)_I\times SU(2)_{II}\times U(1)_{II}\times SU(3)$ to $SO(1,3)\times SU(2)_I\times U(1)_I\times SU(3)$, τ^{3i} describe the colour charge ($SU(3)$), τ^{1i} the weak charge ($SU(2)_I$), τ^{2i} the second $SU(2)_{II}$ charge and τ^4 determines the $U(1)_{II}$ charge. After the break of $SU(2)_{II}\times U(1)_{II}$ to $U(1)_I$ stays $A = 2$ for the $U(1)_I$ hyper charge Y and after the second break of $SU(2)_I \times U(1)_I$ to $U(1)$ stays $A = 2$ for the electromagnetic charge Q, while instead of the weak charge Q' and $\tau^{\pm}$ of the *standard model* manifest.

The breaks of the starting symmetry from $SO(1,13)$ to the symmetry $SO(1,7)\times SU(3)\times U(1)_{II}$ and further to $SO(1,3)\times SU(2)_I\times SU(2)_{II}\times U(1)_{II}\times SU(3)$ are assumed to leave the low lying eight families of spinors massless [5]. After the break of $SO(1,13)$ to $SO(1,7)\times SU(3)\times U(1)$ there are eight such families ($2^{8/2-1}$), all left handed with respect to $SO(1,13)$.

---

[4] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^{\alpha}{}_b = \delta^a{}_b,\; e^a{}_\alpha f^{\beta}{}_a = \delta^{\beta}_{\alpha}$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[5] We proved that it is possible to have massless fermions after a (particular) break if we start with massless fermions and assume particular boundary conditions after the break or the "effective two dimensionality" cases [18,17,16]

Accordingly the first row of the action in Eq. (10.5) manifests the dynamical fermion part of the action, while the second part manifests, when $\omega_{ab\sigma}$ and $\tilde{\omega}_{ab\sigma}$ ($\sigma \in (7,8)$) fields gain nonzero vacuum expectation values, the mass matrices of fermions on the tree level. Scalar fields contribute also to masses of those gauge fields, which at a particular break lose symmetries. It is assumed that the symmetries in the $\tilde{S}^{ab}\tilde{\omega}_{abc}$ and in the $S^{ab}\omega_{abc}$ part break in a correlated way, triggered by particular superposition of scalar (with respect to the rest of symmetry) vielbeins and spin connections of both kinds (ω_{abc} and $\tilde{\omega}_{abc}$). I comment this part in sections 10.2.2, 10.3. The Majorana term, manifesting the Majorana neutrinos, is contained in the second row as well (10.3.1). The third row in Eq. (10.5) stays for all the rest, which is expected to be at low energies negligible or might slightly influence the mass matrices beyond the tree level.

The generators $\tilde{S}^{ab}$ (Eqs. (10.14), (10.15), (10.16)) transform each member of one family into the corresponding member (the same family member) of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ (Eq.(10.2,10.66)).

Correspondingly the action for the vielbeins and the spin connections of S^{ab}, with the Lagrange density $\alpha\, E\, R$, manifests at the low energy regime, after breaks of the starting symmetry, as the known vector gauge fields – the gauge fields of $U(1)$, $SU(2)$, $SU(3)$ and the ordinary gravity, contributing also to the break of symmetries and correspondingly to the masses of the gauge fields and fermions, while $\tilde{\alpha}\, E\, \tilde{R}$ are responsible for off diagonal mass matrices of the fermion members and also to the masses of the gauge fields. Beyond the tree level all the massive fields contribute coherently to the mass matrices.

After the electroweak break the effective Lagrange density for spinors looks like

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,(\gamma^m\, p_{0m} - M)\psi\,,\\
p_{0m} &= p_m - \{e\, Q\, A_m + g^1 \cos\theta_1\, Q'\, Z^{Q'}_m + \frac{g^1}{\sqrt{2}}\,(\tau^{1+}\, W^{1+}_m + \tau^{1-}\, W^{1-}_m) + \\
&\quad + g^2 \cos\theta_2\, Y'\, A^{Y'}_m + \frac{g^2}{\sqrt{2}}\,(\tau^{2+}\, A^{2+}_m + \tau^{2-}\, A^{2-}_m)\,,\\
\bar{\psi}\, M\, \psi &= \bar{\psi}\,\gamma^s\, p_{0s}\psi\\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_R}\, \vec{\tilde{N}}_R\, \vec{\tilde{A}}^{\tilde{N}_R}_s + \tilde{g}^{\tilde{Y}'}\, \tilde{Y}'\, \tilde{A}^{\tilde{Y}'}_s + \frac{\tilde{g}^2}{\sqrt{2}}\,(\tilde{\tau}^{2+}\, \tilde{A}^{2+}_s + \tilde{\tau}^{2-}\, \tilde{A}^{2-}_s)\\
&\quad + \tilde{g}^{\tilde{N}_L}\, \vec{\tilde{N}}_L\, \vec{\tilde{A}}^{\tilde{N}_L}_s + \tilde{g}^{\tilde{Q}'}\, \tilde{Q}'\, \tilde{A}^{\tilde{Q}'}_s + \frac{\tilde{g}^1}{\sqrt{2}}\,(\tilde{\tau}^{1+}\, \tilde{A}^{1+}_s + \tilde{\tau}^{1-}\, \tilde{A}^{1-}_s)\\
&\quad + e\, Q\, A_s + g^1\, \cos\theta_1\, Q'\, Z^{Q'}_s + g^2 \cos\theta_2\, Y'\, A^{Y'}_s\}\,,\;\; s \in \{7,8\}\,.
\end{aligned}
\tag{10.7}
$$

The term $\bar{\psi}\, M\, \psi$ determines the tree level mass matrices of quarks and leptons. The contributions to the mass matrices appear at two very different energy scales due to two separate breaks. Before the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ the vacuum expectation values of the scalar fields appearing in p_{0s} are all zero. The corresponding dynamical scalar fields are massless. All the eight families are massless and the vector gauge fields A^{Ai}_m, $A = 2;$ in Eq. (10.5) are massless as well. To the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ the scalar fields from the first row in the covariant momentum p_{0s}, that is the two triplets $\vec{\tilde{A}}^{\tilde{N}_R}_s$ and $\vec{\tilde{A}}^2_s$ are assumed to contribute, gaining non zero vacuum expectation values. The upper four families, which are doublets with respect to the infinitesimal generators of the corresponding groups, namely $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, become massive. No scalar fields of the kind ω_{abs} is assumed to contribute in this break. Therefore, the lower four families, which are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, stay massless. Due to the break of $SU(2)_{II} \times U(1)_{II}$ symmetries in the space of $\tilde{S}^{ab}$ and S^{ab}, the gauge fields $\vec{A}^2_m$ become massive. The gauge vector fields $\vec{A}^1_m$ and $\vec{A}^Y_m$ stay massless at this break.

To the break of $SU(2)_{I} \times U(1)_{I}$ to $U(1)$ the scalar fields from the second row in the covariant momentum p_{0s}, that is the triplets $\vec{\tilde{A}}_{s}^{\tilde{N}_{L}}$ and $\vec{\tilde{A}}_{s}^{1}$ and the singlet $\tilde{A}_{s}^{4}$, as well as the ones from the third row originating in ω_{abc}, that is (A_s, $Z_s^{Q'}$, $A_s^{Y'}$), are assumed to contribute, by gaining non zero vacuum expectation values.

This electroweak break causes non zero mass matrices of the lower four families. Also the gauge fields $Z_m^{Q'}$, W_m^{1+} and W_m^{1-} gain masses. The electroweak break influences slightly the mass matrices of the upper four families, due to the contribution of A_s, $Z_s^{Q'}$, $A_s^{Y'}$ and $\tilde{A}_s^4$ and in loop corrections also $Z_m^{Q'}$ and $W_m^{\pm}$.

To loops corrections of both groups of families the massive vector gauge fields contribute. The dynamical massive scalar fields contribute only to families of the group to which they couple.

The detailed explanation of the two phase transitions which manifest in Eq. (10.7) is presented in what follows.

10.2.1 Spinor action through breaks

In this subsection properties of quarks, u and d, and leptons, ν and e, of two groups of four families are presented at the stage of

$$\begin{aligned} & SO(1,3)_{\gamma} \times SO(1,3)_{\tilde{\gamma}} \times SU(2)_{I\,\gamma} \times SU(2)_{I\,\tilde{\gamma}} \\ & \times SU(2)_{II\,\gamma} \times SU(2)_{II\,\tilde{\gamma}} \times U(1)_{II\,\gamma} \times U(1)_{II\,\tilde{\gamma}} \\ & \times SU(3)_{\gamma}, \end{aligned} \tag{10.8}$$

when eight families are massless, and then when in the two successive breaks, in which first four and then the last four families gain masses. Half of the eight massless families are doublets with respect to the subgroup $SU(2)_{\tilde{\gamma}\,R}$ of $SO(1,3)_{\tilde{\gamma}}$ and with respect to $SU(2)_{II\,\tilde{\gamma}}$ and singlets with respect to the $SU(2)_{\tilde{\gamma}\,L}$ subgroup of $SO(1,3)_{\tilde{\gamma}}$ and with respect to $SU(2)_{I\,\tilde{\gamma}}$, the rest four families are singlets with respect to the subgroup $SU(2)_{\tilde{\gamma}\,R}$ and with respect to $SU(2)_{II\,\tilde{\gamma}}$ while they are doublets with respect to the $SU(2)_{\tilde{\gamma}L}$ and with respect to $SU(2)_{I\,\tilde{\gamma}}$. The two indices γ and $\tilde{\gamma}$ are to point out that there are two kinds of subgroups of $SO(1,7)$, those in the S^{ab} (taking care of the spin and charges) and those in the $\tilde{S}^{ab}$ (taking care of the families) sector. We shall in what follows omit these two indices, keeping in mind that there are two kinds of groups and subgroups.

At the break of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ the four families coupled to the scalar fields which gain at this break nonzero vacuum expectation values become massive, while the four families which do not couple to these scalar fields stay massless, representing four families of left handed weak charged colour triplets quarks (u_L, d_L), right handed weak chargeless colour triplets quarks (u_R, d_R), left handed weak charged colour singlets leptons (ν_L, e_L) and right handed weak chargeless colour singlets leptons (ν_R, e_R). After the second break the members of the lowest of the upper four families, sharing after the second break the charges and spin of the lowest four families, are the candidates to form the dark matter.

After the second break from $SO(1,3)\times SU(2)_I\times U(1)_I\times SU(3)$ to $SO(1,3)\times U(1)\times SU(3)$ the last four families become massive due to the nonzero vacuum expectation values of the rest of scalar fields. Three of the families represent the observed families of quarks and leptons, so far included into the *standard model*, if we do not count the right handed νs (which carry the additional charge Y', not assumed in the *standard model*, as also all the other members do).

The technique [18], which offers an easy way to keep a track of the symmetry properties of spinors, is used as a tool to clearly demonstrate properties of spinors. This technique is explained in more details in appendix 10.7. In this subsection only a short introduction, needed to follow the explanation, is presented. Mass matrices of each groups of four families, on the tree and below the tree level, originated in the scalar gauge fields, which at each of the two breaks gain a nonzero vacuum expectation values, will be discussed in section 10.3.

Following the refs. [18] we define nilpotents ($\stackrel{ab}{(k)}^2 = 0$) and projectors ($\stackrel{ab}{[k]}^2 = \stackrel{ab}{[k]}$) (Eq. (10.54) in appendix 10.7)

$$\begin{aligned}
&\stackrel{ab}{(\pm i)}: = \frac{1}{2}(\gamma^a \mp \gamma^b), \; \stackrel{ab}{[\pm i]}:= \frac{1}{2}(1 \pm \gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = -1, \\
&\stackrel{ab}{(\pm)}: = \frac{1}{2}(\gamma^a \pm i\gamma^b), \; \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = 1,
\end{aligned} \tag{10.9}$$

as eigenvectors of S^{ab} as well as of $\tilde{S}^{ab}$ (Eq. (10.55) in appendix 10.7)

$$S^{ab} \stackrel{ab}{(k)}= \frac{k}{2} \stackrel{ab}{(k)}, \quad S^{ab} \stackrel{ab}{[k]}= \frac{k}{2} \stackrel{ab}{[k]}, \quad \tilde{S}^{ab} \stackrel{ab}{(k)}= \frac{k}{2} \stackrel{ab}{(k)}, \quad \tilde{S}^{ab} \stackrel{ab}{[k]}= -\frac{k}{2} \stackrel{ab}{[k]}\,. \tag{10.10}$$

One can easily verify that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$ (Eq. (10.56) in appendix 10.7)

$$\gamma^a \stackrel{ab}{(k)}= \eta^{aa} \stackrel{ab}{[-k]}, \; \gamma^b \stackrel{ab}{(k)}= -ik \stackrel{ab}{[-k]}, \; \gamma^a \stackrel{ab}{[k]}= \stackrel{ab}{(-k)}, \; \gamma^b \stackrel{ab}{[k]}= -ik\eta^{aa} \stackrel{ab}{(-k)}, \tag{10.11}$$

$$\tilde{\gamma^a} \stackrel{ab}{(k)}= -i\eta^{aa} \stackrel{ab}{[k]}, \; \tilde{\gamma^b} \stackrel{ab}{(k)}= -k \stackrel{ab}{[k]}, \; \tilde{\gamma^a} \stackrel{ab}{[k]}= \; i \stackrel{ab}{(k)}, \; \tilde{\gamma^b} \stackrel{ab}{[k]}= -k\eta^{aa} \stackrel{ab}{(k)}\,. \tag{10.12}$$

Correspondingly, $\tilde{S}^{ab}$ generate the equivalent representations to representations of S^{ab}, and opposite. Defining the basis vectors in the internal space of spin degrees of freedom in $d = (1 + 13)$ as products of projectors and nilpotents from Eq. (10.9) on the spinor vacuum state $|\psi_0>$, the representation of one Weyl spinor with respect to S^{ab} manifests after the breaks the spin and all the charges of one family members, and the gauge fields of S^{ab} manifest as all the observed gauge fields. $\tilde{S}^{ab}$ determine families and correspondingly the family quantum numbers, while scalar gauge fields of $\tilde{S}^{ab}$ determine, together with particular scalar gauge fields of S^{ab}, mass matrices, manifesting effectively as Yukawa fields and Higgs.

Expressing the operators γ^7 and γ^8 in terms of the nilpotents $\stackrel{78}{(\pm)}$, the mass term in Eqs. (10.5, 10.7) can be rewritten as follows

$$\begin{aligned}
\bar{\psi} M \psi &= \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi = \psi^\dagger \gamma^0 \,(\stackrel{78}{(-)}\; p_{0-} + \stackrel{78}{(+)}\; p_{0+})\psi\,, \\
\stackrel{78}{(\pm)} &= \frac{1}{2}\,(\gamma^7 \; \pm i\,\gamma^8)\,, \\
p_{0\pm} &= (p_{07} \mp i\, p_{08})\,.
\end{aligned} \tag{10.13}$$

After the breaks of the starting symmetry (from $SO(1,13)$ through $SO(1,7)\times U(1)_{II}\times SU(3)$) to $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ there are eight ($2^{\frac{8}{2}-1}$) massless families of spinors. (Some support for this assumption is made when studying toy models [17,16].)

Family members $\alpha \in (u, d, \nu, e)$ carry the $U(1)_{II}$ charge (the generator of the infinitesimal transformations of the group is τ^4, presented in Eq. (10.16)), the $SU(3)$ charge (the generators are $\vec{\tau}^3$, presented in Eq. (10.16)) and the two $SU(2)$ charges, $SU(2)_{II}$ and

$SU(2)_{I}$ (the generators are presented in Eq. (10.15) as $\vec{\tau}^2$ and $\vec{\tau}^1$, respectively). Family members are in the representations, in which the left handed (with respect to $SO(1,3)$) carry the $SU(2)_{I}$ (weak) charge (with the corresponding generators $\vec{\tau}^1$), while the right handed carry the $SU(2)_{II}$ charge (with the corresponding generators $\vec{\tau}^2$).

Each family member carries also the family quantum number, which concern $\tilde{S}^{ab}$ and is determined by the quantum numbers of the two $SU(2)$ from $SO(1,3)$ (with the generators $\vec{\tilde{N}}_{(L,R)}$, Eq. (10.16)) and the two $SU(2)$ from $SO(4)$ (with the generators $\vec{\tilde{\tau}}^{(1,2)}$, Eq. (10.15)).

Properties of families of spinors can transparently be analysed if using our technique. We arrange products of nilpotents and projectors to be eigenvectors of the Cartan subalgebra $S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}$ and, at the same time, they are also the eigenvectors of the corresponding $\tilde{S}^{ab}$, that is of $\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}$.

Below the generators of the infinitesimal transformations of the subgroups of the group $SO(1,13)$ in the S^{ab} and $\tilde{S}^{ab}$ sectors, responsible for the properties of spinors in the low energy regime, are presented.

$$\begin{aligned}
\vec{N}_{\pm}(=\vec{N}_{(L,R)}) :&= \frac{1}{2}(S^{23} \pm i S^{01}, S^{31} \pm i S^{02}, S^{12} \pm i S^{03})\,, \\
\vec{\tilde{N}}_{\pm}(=\vec{\tilde{N}}_{(L,R)}) :&= \frac{1}{2}(\tilde{S}^{23} \pm i \tilde{S}^{01}, \tilde{S}^{31} \pm i \tilde{S}^{02}, \tilde{S}^{12} \pm i \tilde{S}^{03})
\end{aligned} \tag{10.14}$$

determine representations of the two $SU(2)$ subgroups of $SO(1,3)$,

$$\begin{aligned}
\vec{\tau}^1 :&= \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,, \quad \vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78})\,, \\
\vec{\tilde{\tau}}^1 :&= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67},\, \tilde{S}^{57} + \tilde{S}^{68},\, \tilde{S}^{56} - \tilde{S}^{78})\,, \quad \vec{\tilde{\tau}}^2 := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67},\, \tilde{S}^{57} - \tilde{S}^{68},\, \tilde{S}^{56} + \tilde{S}^{78}),
\end{aligned} \tag{10.15}$$

determine representations of $SU(2)_{I} \times SU(2)_{II}$ of $SO(4)$ and

$$\begin{aligned}
\vec{\tau}^3 :=&\ \frac{1}{2}\{S^{9\,12} - S^{10\,11}\,, S^{9\,11} + S^{10\,12},\, S^{9\,10} - S^{11\,12}, \\
&S^{9\,14} - S^{10\,13},\, S^{9\,13} + S^{10\,14}\,, S^{11\,14} - S^{12\,13}, \\
&S^{11\,13} + S^{12\,14},\, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\
\tau^4 :=&\ -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,, \quad \tilde{\tau}^4 := -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})\,,
\end{aligned} \tag{10.16}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$.

It is assumed that at the break of $SO(1,13)$ to $SO(1,7) \times U(1)_{II} \times SU(3)$ all spinors but one become massive, which then manifests eight massless families generated by those generators of the infinitesimal transformations $\tilde{S}^{ab}$ which belong to the subgroup $SO(1,7)$. Some justification for such an assumption can be found in the refs. [17,16].

At the stage of the symmetry $SO(1,3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ each member of a family appears in eight massless families. Each family manifests at this symmetry eightplets of u and d quarks, left handed weak charged and right handed weak chargeless (of spin $(\pm\frac{1}{2})$) in three colours, and the colourless eightplet of ν and e leptons, left handed weak charged and right handed weak chargeless (of spin $(\pm\frac{1}{2})$).

In Table 10.1 the eightplet of quarks of a particular colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$) and the $U(1)_{II}$ charge ($\tau^4 = 1/6$) is presented in our technique [18], as products of nilpotents and projectors.

In Table 10.2 the eightplet of the colourless leptons of the $U(1)_{II}$ charge ($\tau^4 = -1/2$) is presented in the same technique.

i		$\|^a\psi_i>$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(1,7)}=1,\ \Gamma^{(6)}=-1$, of quarks							
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\|\stackrel{56}{(+)}\stackrel{78}{(+)}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\|\stackrel{56}{(+)}\stackrel{78}{(+)}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\|\stackrel{56}{[-]}\stackrel{78}{[-]}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\|\stackrel{56}{[-]}\stackrel{78}{[-]}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\|\stackrel{56}{[-]}\stackrel{78}{(+)}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\|\stackrel{56}{[-]}\stackrel{78}{(+)}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\|\stackrel{56}{(+)}\stackrel{78}{[-]}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\|\stackrel{56}{(+)}\stackrel{78}{[-]}\|\|\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[-]}\ \stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{2}{3}$

Table 10.1. The 8-plet of quarks - the members of $SO(1,7)$ subgroup of the group $SO(1,13)$, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$ is presented in the technique [18]. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour $(1/2, 1/(2\sqrt{3}))$. Here $\Gamma^{(1,3)}$ defines the handedness in $(1+3)$ space, S^{12} defines the ordinary spin (which can also be read directly from the basic vector, both vectors with both spins, $\pm\frac{1}{2}$, are presented), τ^{13} defines the third component of the weak charge, τ^{23} the third component of the $SU(2)_{II}$ charge, τ^4 (the $U(1)$ charge) defines together with τ^{23} the hyper charge ($Y = \tau^4 + \tau^{23}$), $Q = Y + \tau^{13}$ is the electromagnetic charge. The vacuum state $|\psi_0>$, on which the nilpotents and projectors operate, is not shown. The basis is the massless one. The reader can find the whole Weyl representation in the ref. [19].

In both tables the vectors are chosen to be the eigenvectors of the operators of handedness $\Gamma^{(n)}$ and $\tilde{\Gamma}^{(n)}$, the generators τ^{13} (the member of the weak $SU(2)_I$ generators), τ^{23} (the member of $SU(2)_{II}$ generators), τ^{33} and τ^{38} (the members of $SU(3)$), $Y (= \tau^4 + \tau^{23})$ and $Q (= Y + \tau^{13})$. They are also eigenvectors of the corresponding $\tilde{S}^{ab}$, $\tilde{\tau}^{Ai}$, $A = 1, 2, 4$ and $\tilde{Y}$ and $\tilde{Q}$. The tables for the two additional choices of the colour charge of quarks follow from Table 10.1 by changing the colour part of the states [19], that is by applying τ^{3i}, which are not members of the Cartan subalgebra, on the states of Table 10.1.

Looking at Tables (10.1, 10.2) and taking into account the relation $\stackrel{78}{(-)}\stackrel{78}{(+)} = -\stackrel{78}{[-]}$ from Eq. (10.58) in appendix 10.7 and the relation $\gamma^0 \stackrel{03}{(+i)} = \stackrel{03}{[-i]}$ from Eq. (10.11) one notices that the operator $\gamma^0 \stackrel{78}{(-)}$ (Eq.(10.13)) transforms the right handed u_R^{c1} from the first row of Table 10.1 into the left handed u_L^{c1} of the same spin and charge from the seventh row of the same table, and that it transforms the right handed ν_R from the first row of Table 10.2 into the left handed ν_L presented in the seventh row of the same table, just what the Higgs and γ^0 do in the *standard model*. Equivalently one finds that the operator $\gamma^0 \stackrel{78}{(+)}$ transforms the right handed d_R^{c1}-quark from the third row into the left handed one (of the same spin and colour) presented in the fifth row of Table 10.1 and that it transforms the right handed e_R from the third row of Table 10.2 into the left handed one (of the same spin) presented in the fifth row of Table 10.2.

i		$\vert^a\psi_i>$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(1,7)} = 1,\ \Gamma^{(6)} = -1$, of quarks							
1	ν_R	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{[-]}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{[-]}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[-]}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{[-]}\Vert\overset{9\,10}{(+)}\ \overset{11\,12}{(+)}\ \overset{13\,14}{(+)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

Table 10.2. The 8-plet of leptons - the members of $SO(1,7)$ subgroup of the group $SO(1,13)$, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$ is presented in the massless basis. It contains the colour chargeless left handed weak charged leptons and the right handed weak chargeless leptons. The rest of notation is the same as in Table 10.1.

The superposition of generators $\tilde{S}^{ab}$ forming eight generators ($\tilde{N}^{\pm}_{R,L}, \tilde{\tau}^{(2,1)\pm}$) presented in appendix 10.7, Eq. (10.67), generates families, transforming each member of one family into the same member of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ (Eq.(10.2)). The eight families of the first member of the eightplet of quarks from Table 10.1, for example, that is of the right handed u^{c1}_R-quark with spin $\frac{1}{2}$, are presented in the left column of Table 10.3. The generators ($\tilde{N}^{\pm}_{R,L}, \tilde{\tau}^{(2,1)\pm}$) (Eq. (10.67)) transform the first member of the eightplet from Table 10.2, that is the right handed neutrino ν_R with spin $\frac{1}{2}$, into the eightplet of right handed neutrinos with spin up, belonging to eight different families. These families are presented in the right column of the same table. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators ($N^{\pm}_{R,L}, \tau^{(2,1)\pm}$) on this particular member.

Let us point out that the break of $SO(1,7)$ into $SO(1,3) \times SU(2)_{II} \times SU(2)_I$, assumed to leave all the eight families massless, allows to divide eight families into two groups of four families. One group of families contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to this two groups. When gaining non zero vacuum expectation values, these scalar fields determine nonzero mass matrices of the four families, to which they couple. These happens at some scale, assumed that it is much higher than the electroweak scale.

The group of four families, which are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, stay massless unless the gauge scalar fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, together with the gauge scalars of Q , Q' and Y', gain a nonzero vacuum expectation values at the electroweak break. Correspondingly the decoupled twice four families, that means that the matrix elements between these two

I_R	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
II_R	u_R^{c1}	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
III_R	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
IV_R	u_R^{c1}	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
V_R	u_R^{c1}	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
VI_R	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
VII_R	u_R^{c1}	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
$VIII_R$	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$

Table 10.3. Eight families of the right handed u_R^{c1} quark with spin $\frac{1}{2}$, the colour charge ($^{c1} = (\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3}))$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$) from Eq. (10.67). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (10.67)) transform u_R^{c1} of spin $\frac{1}{2}$ and the chosen colour ^{c1} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_R of spin $\frac{1}{2}$ to all the colourless members of the same family.

groups of four families are equal to zero, appear at two different scales, determined by two different breaks.

To have an overview over the properties of the members of one (any one of the eight) family let us present in Table 10.4 quantum numbers of particular members of any of the eight families: The handedness $\Gamma^{(1+3)}(= -4iS^{03}S^{12})$, S^{03}_L, S^{12}_L, S^{03}_R, S^{12}_R, τ^{13} (of the weak $SU(2)_I$), τ^{23} (of $SU(2)_{II}$), the hyper charge $Y = \tau^4 + \tau^{23}$, the electromagnetic charge Q, the $SU(3)$ status, that is, whether the member is a member of a triplet (the quark with the one of the charges $\{(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (-\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})\}$) or the colourless lepton, and Y' after the break of $SU(2)_{II} \times U(1)_{II}$ into $U(1)_I$.

Before the break of $SU(2)_{II} \times U(1)_{II}$ into $U(1)_I$ the members of one family from Tables 10.1 and 10.2 share the family quantum numbers presented in Table 10.5: The "tilde handedness" of the families $\tilde{\Gamma}^{(1+3)}(= -4i\tilde{S}^{03}\tilde{S}^{12})$, $\tilde{S}^{03}_L, \tilde{S}^{12}_L, \tilde{S}^{03}_R, \tilde{S}^{12}_R$ (the diagonal matrices of $SO(1,3)$), $\tilde{\tau}^{13}$ (of one of the two $SU(2)_I$), $\tilde{\tau}^{23}$ (of the second $SU(2)_{II}$).

We see in Table 10.5 that the first four of the eight families are singlets with respect to subgroups determined by $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, and doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, while the rest four families are doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, and singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$.

When the break from $SU(2)_I \times SU(2)_{II} \times U(1)_{II}$ to $\times SU(2)_I \times U(1)_I$ appears, the scalar fields, the superposition of $\tilde{\omega}_{abs}$, which are triplets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ (are assumed to) gain a nonzero vacuum expectation values. As one can read from Eq. (10.5) these scalar fields cause nonzero mass matrices of the families which are doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ and correspondingly couple to these scalar fields with nonzero vacuum expectation values. The four families which do not couple to these scalar fields stay massless. The vacuum expectation value of $\tilde{A}^4_{\pm} = 0$ is assumed to stay zero at the first break. In this break also the vector (with respect to (1+3)) gauge fields of $\vec{\tilde{\tau}}^2$ (the generators of $SU(2)_{II}$) become massive.

	$\Gamma^{(1+3)}$	S^{03}_L	S^{12}_L	S^{03}_R	S^{12}_R	τ^{13}	τ^{23}	Y	Q	$SU(3)$	Y'
u_{Li}	-1	$\mp\frac{i}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{2}{3}$	triplet	$-\frac{1}{6}\tan^2\theta_2$
d_{Li}	-1	$\mp\frac{i}{2}$	$\pm\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{6}$	$-\frac{1}{3}$	triplet	$-\frac{1}{6}\tan^2\theta_2$
ν_{Li}	-1	$\mp\frac{i}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	singlet	$\frac{1}{2}\tan^2\theta_2$
e_{Li}	-1	$\mp\frac{i}{2}$	$\pm\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1	singlet	$\frac{1}{2}\tan^2\theta_2$
u_{Ri}	1	0	0	$\pm\frac{i}{2}$	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	triplet	$\frac{1}{2}(1-\frac{1}{3}\tan^2\theta_2)$
d_{Ri}	1	0	0	$\pm\frac{i}{2}$	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$	triplet	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\theta_2)$
ν_{Ri}	1	0	0	$\pm\frac{i}{2}$	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	0	0	singlet	$\frac{1}{2}(1+\tan^2\theta_2)$
e_{Ri}	1	0	0	$\pm\frac{i}{2}$	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$	-1	-1	singlet	$-\frac{1}{2}(1-\tan^2\theta_2)$

Table 10.4. The quantum numbers of the members – quarks and leptons, left and right handed – of any of the eight families ($i \in \{I, \cdots, VIII\}$) from Table 10.3 are presented: The handedness $\Gamma^{(1+3)} = -4iS^{03}S^{12}$, $S^{03}_L, S^{12}_L, S^{03}_R, S^{12}_R$, τ^{13} of the weak $SU(2)_I$, τ^{23} of the second $SU(2)_{II}$, the hyper charge $Y\,(=\tau^4+\tau^{23})$, the electromagnetic charge $Q\,(=Y+\tau^{23})$, the $SU(3)$ status, that is, whether the member is a triplet – the quark with the one of the charges determined by τ^{33} and τ^{38}, that is one of $\{(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (-\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})\}$ – or a singlet, and the charge $Y'\,(=\tau^{23}-\tau^4\,\tan^2\theta_2)$.

i	$\tilde{\Gamma}^{(1+3)}$	$\tilde{S}^{03}_L$	$\tilde{S}^{12}_L$	$\tilde{S}^{03}_R$	$\tilde{S}^{12}_R$	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}'$	$\tilde{Y}$	$\tilde{Q}$
I	-1	$\frac{i}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
II	-1	$-\frac{i}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
III	-1	$\frac{i}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
IV	-1	$-\frac{i}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
V	1	0	0	$-\frac{i}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	-1
VI	1	0	0	$\frac{i}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
VII	1	0	0	$-\frac{i}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
VIII	1	0	0	$\frac{i}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0

Table 10.5. Quantum numbers of a member of the eight families from Table 10.3, the same for all the members of one family, are presented: The "tilde handedness" of the families $\tilde{\Gamma}^{(1+3)} = -4i\tilde{S}^{03}\tilde{S}^{12}$, the left and right handed $SO(1,3)$ quantum numbers (Eq. (10.14), $\tilde{S}^{03}_L, \tilde{S}^{12}_L$, $\tilde{S}^{03}_R, \tilde{S}^{12}_R$ of $SO(1,3)$ group in the $\tilde{S}^{mn}$ sector), $\tilde{\tau}^{13}$ of $SU(2)_I$, $\tilde{\tau}^{23}$ of the second $SU(2)_{II}$, $\tilde{\tau}^4$ (Eq. (10.16)), $\tilde{Y}'\,(=\tilde{\tau}^{23}-\tilde{\tau}^4\,\tan\tilde{\theta}_2)$, taking $\tilde{\theta}^2=0$, $\tilde{Y}\,(=\tilde{\tau}^4+\tilde{\tau}^{23})$, $\tilde{Q} = (\tilde{\tau}^4+\tilde{S}^{56})$.

In the successive (electroweak) break the scalar gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, coupled to the rest of eight families, gain nonzero vacuum expectation values. Together with them also the scalar gauge fields $A^{Y'}_s$, $A^{Q'}_s$ and A^Q_s (the superposition of $\omega_{sts'}$ spin connection fields) gain nonzero vacuum expectation values. The scalar fields $\vec{\tilde{A}}^1_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$, A^Q_s, $A^{Q'}_s$ and $A^{Y'}_s$ determine mass matrices of the last four massless families. At this break also the vector gauge fields of $\vec{\tau}^1$ become massive.

The second break, which (is assumed to) occurs at much lower energy scale, influences slightly also properties of the upper four families.

There is the contribution which appears in the loop corrections as the term bringing nonzero contribution only to the mass matrix of neutrinos, transforming the right handed neutrinos to the left handed charged conjugated ones. It looks like that (for a particular choice of operators and parameters) such a Majorana mass term appears for the lower four families only. We discuss the Majorana neutrino like contribution in subsect. 10.3.1.

Let us end this subsection by admitting that it is assumed (not yet showed or proved) that there is no contributions to the mass matrices from $\psi^{\dagger}_{L}\,\gamma^0\gamma^s\,p_{0s}\,\psi_R$, with $s = 5, 6$. Such a contribution to the mass term would namely mix states with different electromagnetic charges (ν_R and e_L, u_R and d_L), in disagreement with what is observed.

10.2.2 Scalar and gauge fields in $d = (1+3)$ through breaks

In the *spin-charge-family* theory there are the vielbeins $e^s{}_\sigma$

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ 0 & e^s{}_\sigma \end{pmatrix}$$

in a strong correlation with the spin connection fields of both kinds, with $\tilde{\omega}_{st\sigma}$ and with $\omega_{ab\sigma}$, with indices $s, t, \sigma \in \{5, 6, 7, 8\}$, which manifest in $d = (1+3)$-dimensional space as scalar fields after particular breaks of a starting symmetry. Phase transitions are (assumed to be) triggered by the nonzero vacuum expectation values of the fields $f^\alpha{}_s\,\tilde{\omega}_{ab\alpha}$ and $f^\alpha{}_s\,\omega_{ab\alpha}$.

The gauge fields then correspondingly appear as

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ e^s{}_\mu = e^s{}_\sigma E^\sigma{}_{Ai} A^{Ai}_\mu & e^s{}_\sigma \end{pmatrix},$$

with $E^{\sigma Ai} = \tau^{Ai}\,x^\sigma$, where A^{Ai}_μ are the gauge fields, corresponding to (all possible) Kaluza-Klein charges τ^{Ai}, manifesting in $d = (1+3)$. Since the space symmetries include only S^{ab} ($M^{ab} = L^{ab} + S^{ab}$) and not $\tilde{S}^{ab}$, there are no vector gauge fields of the type $e^s{}_\sigma \tilde{E}^\sigma{}_{Ai}\tilde{A}^{Ai}_\mu$, with $\tilde{E}^\sigma{}_{Ai} = \tilde{\tau}_{Ai}\,x^\sigma$. The gauge fields of $\tilde{S}_{ab}$ manifest in $d = (1+3)$ only as scalar fields.

The vielbeins and spin connection fields from Eq. (10.4) ($\int d^dx\, E\,\,(\alpha\, R + \tilde{\alpha}\,\tilde{R})$) are manifesting in $d = (1+3)$ in the effective action, if no gravity is assumed in $d = (1+3)$ ($e^m{}_\mu = \delta^m{}_\mu$)

$$\begin{aligned} S_b = \int\, d^{(1+3)}x\,\{-\frac{\varepsilon^A}{4}\, F^{Aimn}\, F^{Ai}{}_{mn} + \frac{1}{2}\,(m^{Ai})^2\; A^{Ai}_m A^{Ai\,m} + \\ \text{contributions of scalar fields}\}. \end{aligned} \tag{10.17}$$

Masses of gauge fields of the charges τ^{Ai}, which symmetries are unbroken, are zero, nonzero masses correspond to the broken symmetries.

In the breaking procedures, when $SO(1,7) \times U(1)_{II} \times SU(3)$ breaks into $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$, there are eight massless families of quarks and leptons (as discussed above) and massless gauge fields $SU(2)_I$, $SU(2)_{II}$, $U(1)_{II}$ and $SU(3)$. Gravity in $(1+3)$ is not discussed.

In the break from $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ the scalar fields originating in $f^\alpha{}_s\,\tilde{\omega}_{ab\alpha} = \tilde{\omega}_{abs}$ gain nonzero vacuum expectation values causing the break of symmetries, which manifests on the tree level in masses of the superposition of gauge fields $\vec{A}^2_m$ and A^4_m, as well as in mass matrices of the upper four families.

To the break from $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1,3) \times U(1) \times SU(3)$ both kinds of scalar fields, a superposition of $f^\alpha{}_s\,\tilde{\omega}_{ab\alpha} = \tilde{\omega}_{abs}$ and $f^\alpha{}_s\,\omega_{st\alpha} = \omega_{s't s}$, with $(s', t) = \{(5,6), (7,8)\}$; $s = \in \{7, 8\}$ and A^4_s, contribute which manifests in the masses of $W^\pm_m$, Z_m and in mass matrices of the lower four families.

Detailed studies of the appearance of breaks of symmetries as follow from the starting action, the corresponding manifestation of masses of the gauge fields involved in these breaks, as well as the appearance of the nonzero vacuum expectation values of the scalar (with respect to (1+3)) fields which manifest in mass matrices of the families involved in particular breaks are under consideration. We study in the refs. [16,17] on toy models possibilities that a break (such a break is in the discussed cases the one from $SO(1,13)$ to $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$, via $SO(1,7) \times U(1)_{II} \times SU(3)$) can end up with massless fermions. We found in the ref. [16] for a toy model scalar vielbein and spin connection fields which enable massless fermions after the break. We were not been able yet, even not for this toy model, to solve the problem, how do particular scalar fields causing a break of symmetries appear and what fermion sources are responsible for their appearance.

In this paper it is (just) assumed that there occur nonzero vacuum expectation values of particular scalar fields, which then cause breaks of particular symmetries, and change properties of gauge fields and of fermion fields.

Although the symmetries of the vacuum expectation values of the scalar fields are known when the break of symmetries is assumed, yet their values (numbers) are not known. Masses and potentials determining the dynamics of these scalar fields are also not known, and also the way how do scalar fields contribute to masses of the gauge fields, on the tree and below the tree level, waits to be studied.

Let us repeat that all the gauge fields, scalar or vectors, either originating in ω_{abc} or in $\tilde{\omega}_{abc}$ are after breaks in the adjoint representations with respect to all the groups, to which the starting groups break.

Scalar and gauge fields after the break from $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ Before the break of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1) \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1) \times SU(3)$ the gauge fields $\vec{A}^2_m$ ($A^{21}_m = \omega_{58m} + \omega_{67m}$, $A^{22}_m = \omega_{57m} - \omega_{68m}$, $A^{23}_m = \omega_{56m} + \omega_{78m}$), $\vec{A}^1_m$ ($A^{11}_m = \omega_{58m} - \omega_{67m}$, $A^{12}_m = \omega_{57m} + \omega_{68m}$, $A^{13}_m = \omega_{56m} - \omega_{78m}$) and A^4_m are all massless.

After the break the gauge fields $A^{2\pm}_m$, as well as one superposition of A^{23}_m and A^4_m, become massive, while another superposition (A^Y_m) and the gauge fields $\vec{A}^1_m$ stay massless, due to the (assumed) break of symmetries.

The fields $A^{Y'}_m$ and $A^{2\pm}_m$, manifesting as massive fields, and A^Y_m which stay massless, are defined as the superposition of the old ones as follows

$$\begin{aligned} A^{23}_m &= A^Y_m \sin\theta_2 + A^{Y'}_m \cos\theta_2, \\ A^4_m &= A^Y_m \cos\theta_2 - A^{Y'}_m \sin\theta_2, \\ A^{2\pm}_m &= \frac{1}{\sqrt{2}}(A^{21}_m \mp i A^{22}_m), \end{aligned} \tag{10.18}$$

for $m = 0,1,2,3$ and a particular value of θ_2. The scalar fields $A^{Y'}_s$, $A^{2\pm}_s$, $A^{Y'}_s$, which do not gain in this break any vacuum expectation values, stay masses. This assumption guarantees that they do not contribute to masses of the lower four families on the tree level.

The corresponding operators for the new charges which couple these new gauge fields to fermions are

$$Y = \tau^4 + \tau^{23}, \quad Y' = \tau^{23} - \tau^4 \tan^2\theta_2, \quad \tau^{2\pm} = \tau^{21} \pm i\tau^{22}. \tag{10.19}$$

The new coupling constants become $g^Y = g^4 \cos\theta_2$, $g^{Y'} = g^2 \cos\theta_2$, while $A^{2\pm}_m$ have a coupling constant $\frac{g^2}{\sqrt{2}}$.

In the break also the scalar fields originating in $\tilde{\omega}_{abs}$ contribute, and symmetries in both sectors, $\tilde{S}^{ab}$ and S^{ab}, are broken simultaneously. The scalar fields $\vec{\tilde{A}}^2_s$ (which are the superposition of $\tilde{\omega}_{abs}$, $\tilde{A}^{21}_s = \tilde{\omega}_{58s} + \tilde{\omega}_{67m}$, $\tilde{A}^{22}_s = \tilde{\omega}_{57s} - \tilde{\omega}_{68s}$, $\tilde{A}^{23}_s = \tilde{\omega}_{56s} + \tilde{\omega}_{78s}$) gain a nonzero vacuum expectation values.

We have for the scalar fields correspondingly

$$\begin{aligned} \tilde{A}^{23}_s &= \tilde{A}^{\tilde{Y}}_s \sin\tilde{\theta}_2 + \tilde{A}^{\tilde{Y}'}_s \cos\tilde{\theta}_2, \\ \tilde{A}^{4}_s &= \tilde{A}^{\tilde{Y}}_s \cos\tilde{\theta}_2 - \tilde{A}^{\tilde{Y}'}_s \sin\tilde{\theta}_2, \\ \tilde{A}^{2\pm}_s &= \frac{1}{\sqrt{2}}(\tilde{A}^{21}_s \mp i\tilde{A}^{22}_s), \end{aligned} \tag{10.20}$$

for $s = 7,8$ and a particular value of $\tilde{\theta}_2$. These scalar fields, having a nonzero vacuum expectation values, define according to Eq. (10.7) mass matrices of the upper four families, which are doublets with respect to $\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$.

To this break and correspondingly to the mass matrices of the upper four families also the scalar fields which couple to the upper four families

$$\vec{\tilde{A}}^{\tilde{N}_R}_s = (\,\tilde{\omega}_{23s} - i\,\tilde{\omega}_{01s}\,, \tilde{\omega}_{31s} - i\,\tilde{\omega}_{02s}\,, \tilde{\omega}_{12s} - i\,\tilde{\omega}_{03s}\,) \tag{10.21}$$

contribute. The lower four families, which are singlets with respect to both groups, stay correspondingly massless.

The corresponding new operators are then

$$\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}\,, \quad \tilde{Y}' = \tilde{\tau}^{23} - \tilde{\tau}^4 \tan^2\tilde{\theta}_2\,, \quad \tilde{\tau}^{2\pm} = \tilde{\tau}^{21} \pm i\tilde{\tau}^{22}\,, \quad \vec{\tilde{N}}_R\,. \tag{10.22}$$

New coupling constants are correspondingly $\tilde{g}^{\tilde{Y}} = \tilde{g}^4 \cos\tilde{\theta}_2$, $\tilde{g}^{\tilde{Y}'} = \tilde{g}^2 \cos\tilde{\theta}_2$, $\tilde{A}^{2\pm}_s$ have a coupling constant $\frac{\tilde{g}^2}{\sqrt{2}}$, and $\vec{\tilde{A}}^{\tilde{N}_R}_s$ $\tilde{g}^{\tilde{N}_R}$.

Scalar and gauge fields after the break from $SU(2)_I \times U(1)_I$ to $U(1)$ To the electroweak break, when $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ breaks into $SO(1,3) \times U(1) \times SU(3)$, both kinds of the scalar spin connection fields are assumed to contribute, that is a superposition of $\tilde{\omega}_{abs}$, which is orthogonal to the one trigging the first break

$$\begin{aligned} \tilde{A}^{13}_s &= \tilde{A}_s \sin\tilde{\theta}_1 + \tilde{Z}_s \cos\tilde{\theta}_1, \\ \tilde{A}^{\tilde{Y}}_s &= \tilde{A}_s \cos\tilde{\theta}_1 - \tilde{Z}_s \sin\tilde{\theta}_1, \\ \tilde{W}^{\pm}_s &= \frac{1}{\sqrt{2}}(\tilde{A}^{11}_s \mp i\tilde{A}^{12}_s)\,, \end{aligned} \tag{10.23}$$

and

$$\vec{\tilde{A}}^{\tilde{N}_L}_s \tag{10.24}$$

and a superposition of $\omega_{sts'}$

$$\begin{aligned} A^{Q}_s &= \sin\theta_1\, A^{13}_s + \cos\theta_1\, A^{Y}_s\,, \quad A^{Q'}_s = \cos\theta_1\, A^{13}_s - \sin\theta_1\, A^{Y}_s\,, \\ A^{Y'}_s &= \cos\theta_2\, A^{23}_s - \sin\theta_2\, A^{4}_s\,. \end{aligned} \tag{10.25}$$

$s \in (7,8)$. While the superposition of Eqs.(10.23, 10.24) couple to the lower four families only, since the lower four families are doublets with respect to $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}^{\tilde{N}_L}$, and the upper four families are singlets with respect to $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}^{\tilde{N}_L}$, the scalar fields A^{Q}_s, $A^{Q'}_s$ and $A^{Y'}_s$ (they are a superposition of $\omega_{sts'}$; $s,t \in (5,\cdots,14)$; $s' = 7,8$) couple to all the eight families, distinguishing among the family members.

Correspondingly a superposition of the vector fields $\vec{A}^{1}_{m}$ and A^{4}_{m},

$$\begin{aligned} A^{13}_{m} &= A_{m} \sin\theta_{1} + Z_{m} \cos\theta_{1}, \\ A^{Y}_{m} &= A_{m} \cos\theta_{1} - Z_{m} \sin\theta_{1}, \\ W^{\pm}_{m} &= \frac{1}{\sqrt{2}}(A^{11}_{m} \mp i A^{12}_{m}), \end{aligned} \tag{10.26}$$

that is $W^{\pm}_{m}$ and Z_{m}, become massive, while A_{m} stays with $m = 0$. The new operators for charges are

$$\begin{aligned} Q &= \tau^{13} + Y = S^{56} + \tau^{4}, \\ Q' &= -Y \tan^{2}\theta_{1} + \tau^{13}, \\ \tau^{1\pm} &= \tau^{11} \pm i\tau^{12}, \end{aligned} \tag{10.27}$$

and the new coupling constants are correspondingly $e = g^{Y} \cos\theta_{1}$, $g' = g^{1} \cos\theta_{1}$ and $\tan\theta_{1} = \frac{g^{Y}}{g^{1}}$, in agreement with the *standard model*. We assume for simplicity that in the scalar sector of ω_{stc} – $\omega_{s,t,s'}$ – the same θ_{1} determines properties of the coupling constants as it does in the vector one – $\omega_{s,t,m}$.

In the sector of the $\tilde{\omega}_{abs}$ scalars the corresponding new operators are

$$\begin{aligned} \tilde{Q} &= \tilde{\tau}^{13} + \tilde{Y} = \tilde{S}^{56} + \tilde{\tau}^{4}, \\ \tilde{Q}' &= -\tilde{Y} \tan^{2}\tilde{\theta}_{1} + \tilde{\tau}^{13}, \\ \tilde{\tau}^{1\pm} &= \tilde{\tau}^{11} \pm i\tilde{\tau}^{12}, \end{aligned} \tag{10.28}$$

with the new coupling constants $\tilde{e} = \tilde{g}^{Y} \cos\tilde{\theta}_{1}$, $\tilde{g'} = \tilde{g}^{1} \cos\tilde{\theta}_{1}$ and $\tan\tilde{\theta}_{1} = \frac{\tilde{g}^{Y}}{\tilde{g}^{1}}$.

To this break and correspondingly to the mass matrices of the lower four families also the scalar fields $\vec{\tilde{A}}^{\tilde{N}_{L}}_{s}$ (orthogonal to $\vec{\tilde{A}}^{\tilde{N}_{R}}_{s}$) contribute.

All the scalar fields presented in this and the previous subsection are massive dynamical fields, coupled to fermions and governed by the corresponding scalar potentials, for which we assume that they behave as normalizable ones (at least up to some reasonable accuracy).

10.3 Mass matrices on the tree level and beyond in the spin-charge-family theory

In the two subsections (10.2.2, 10.2.2) of section 10.2.2 properties of scalar and gauge fields after each of the two successive breaks are discussed. The appearance of the vacuum expectation values of some superposition of two kinds of spin connection fields and vielbeins, all scalars with respect to $(1 + 3)$, is assumed. These scalar fields determine in the *spin-charge-family* theory mass matrices of fermions and masses of vector gauge fields on the tree level. It is the purpose of this section to discuss properties of families of fermions after these two breaks, on the tree level and beyond the tree level. Properties of the family members within the pairs (u, ν) and (d, e) are, namely, on the tree level very much related and it is expected that hopefully loop corrections (in all orders) make properties of the lowest three families in agreement with the observations.

The starting fermion action (Eq. 10.5)) manifests after the two successive breaks of symmetries in the effective low energy action presented in Eq. (10.7). The mass term (Eq. (10.13)) manifests correspondingly in the fermion mass matrices.

Let us repeat the assumptions made to come from the starting action to the low energy effective action: **i.** In the break from $SU(2)_{I} \times SU(2)_{II} \times U(1)_{II}$ to $SU(2)_{I} \times U(1)_{I}$

the superposition of the $\tilde{\omega}_{abs}$ scalar fields which are the gauge fields of $\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$, with the index $s \in (7,8)$ gain non zero vacuum expectation values. **ii.** In the electroweak break the superposition of the $\tilde{\omega}_{abs}$ scalar fields which are the gauge fields of $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$, and the superposition of scalar fields $\omega_{s'ts}$ $(s, s', t \in (7,8))$ which are the gauge fields of Q, Q' and Y' gain nonzero vacuum expectation values.

The first break leaves the lower four families, which are singlets with respect to the groups ($\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$) involved in the break, massless. At the electroweak break all the families become massive. While the scalar fields coupled with $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$ to fermions influence only the lower four families, the scalar gauge fields coupled with Q, Q' and Y' to fermions influence mass matrices of all the eight families.

To loop corrections the gauge vector fields, the scalar dynamical fields originating in $\omega_{s'ts}$ and in $\tilde{\omega}_{abs}$ contribute, those to which a particular group of families couple. Let us tell that there is also a contribution to loop corrections, manifesting as a very special products of superposition of $\omega_{abs}, s = 5, 6, 9, \cdots, 14$ and $\tilde{\omega}_{abs}$, $s = 5, 6, 7, 8$ fields, which couple only to the right handed neutrinos and their charge conjugated states of the lower four families. This term might strongly influence properties of neutrinos of the lower four families.

Table 10.6 represents the mass matrix elements on the tree level for the upper four families after the first break, originating in the vacuum expectation values of two superposition of $\tilde{\omega}_{abs}$ scalar fields, the two triplets of $\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$. The notation $\tilde{a}^{\tilde{A}i}_{\pm} = -\tilde{g}^{\tilde{A}i}\, \tilde{A}^{\tilde{A}i}_{\pm}$ is used. The sign $(\mp)$ distinguishes between the values of the two pairs (u-quarks, ν-lepton) and (d-quark, e-lepton), respectively. The lower four families, which are singlets with respect to the two groups ($\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$), as can be seen in Table 10.4, stay massless after the first break.

	I	II	III	IV	V	VI	VII	VIII
I	0	0	0	0	0	0	0	0
II	0	0	0	0	0	0	0	0
III	0	0	0	0	0	0	0	0
IV	0	0	0	0	0	0	0	0
V	0	0	0	0	$-\frac{1}{2}\,(\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{\tilde{N}^-_R}_{\pm}$	0	$-\tilde{a}^{2-}_{\pm}$
VI	0	0	0	0	$-\tilde{a}^{\tilde{N}^+_R}_{\pm}$	$\frac{1}{2}(-\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{2-}_{\pm}$	0
VII	0	0	0	0	0	$-\tilde{a}^{2+}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{23}_{\pm} - \tilde{a}^{\tilde{N}^3_R}_{\pm})$	$-\tilde{a}^{\tilde{N}^-_R}_{\pm}$
VIII	0	0	0	0	$-\tilde{a}^{2+}_{\pm}$	0	$-\tilde{a}^{\tilde{N}^+_R}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{23}_{\pm} + \tilde{a}^{\tilde{N}^3_R}_{\pm})$

Table 10.6. The mass matrix for the eight families of quarks and leptons after the break of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$. The notation $\tilde{a}^{\tilde{A}i}_{\pm}$ stays for $-\tilde{g}^{\tilde{A}i}\, \tilde{A}^{\tilde{A}i}_{\pm}$, $(\mp)$ distinguishes u_i from d_i and ν_i from e_i, index i determines families.

Masses of the lowest of the higher four family were evaluated in the ref. [14] from the cosmological and direct measurements, when assuming that baryons of this stable family (with no mixing matrix to the lower four families) constitute the dark matter.

The lower four families obtain masses when the second $SU(2)_I \times U(1)_I$ break occurs, at the electroweak scale, manifesting in nonzero vacuum expectation values of the two triplet scalar fields $\tilde{A}^{1i}_s$, $\tilde{A}^{\tilde{N}_L i}_s$, and the $U(1)$ scalar fields $\tilde{A}^4_s$, as well as A^Q_s, $A^{Q'}_s$ and $A^{Y'}_s$, and also in nonzero masses of the gauge fields $W^{\pm}_m$ and Z_m.

Like in the case of the upper four families, also here is the mass matrix contribution from the nonzero vacuum expectation values of $f^{\sigma}{}_s\, \tilde{\omega}_{ab\sigma}$ on the tree level the same for (u-quarks and ν-leptons) and the same for (d-quarks and e-leptons), while $(\mp)$ distinguishes between the values of the u-quarks and d-quarks and correspondingly between the values of ν and e. The contributions from A^Q_s, $A^{Q'}_s$ and $A^{Y'}_s$ to mass matrices are different for different family members and the same for all the families of a particular family member.

Beyond the tree level all mass matrix elements of a family member become dependent on the family member quantum number, through coherent contributions of the vector and all the scalar dynamical fields.

Table 10.7 represents the contribution of $\tilde{g}^{\tilde{1}i}\, \tilde{\tau}^{\tilde{1}i}\, \tilde{A}^{\tilde{1}i}_{\mp}$ and $\tilde{g}^{\tilde{N}_L}\, \tilde{N}^i_L\, \tilde{A}^{\tilde{N}_L i}_{\mp}$, to the mass matrix elements on the tree level for the lower four families after the electroweak break. The contributions from $e\, Q A^Q_s$, $g^{Q'} Q' A^{Q'}_s$ and $g^{Y'}\, Y' A^{Y'}_s$, which are diagonal and equal for all the families, but distinguish among the members of one family, are not present. The notation $\tilde{a}^{\tilde{A}i}_{\pm} = -\tilde{g}^{\tilde{A}i}\, \tilde{A}^{\tilde{A}i}_{\mp}$ is used, $\tilde{\tau}^{Ai}$ stays for $\tilde{\tau}^{1i}$ and $\tilde{N}^i_L$ and correspondingly also the notation for the coupling constants and the triplet scalar fields is used.

	I	II	III	IV
I	$-\frac{1}{2}\,(\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{\tilde{N}^-_L}_{\pm}$	0	$\tilde{a}^{1-}_{\pm}$
II	$\tilde{a}^{\tilde{N}^+_L}_{\pm}$	$\frac{1}{2}(-\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{1-}_{\pm}$	0
III	0	$\tilde{a}^{1+}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{13}_{\pm} - \tilde{a}^{\tilde{N}^3_L}_{\pm})$	$\tilde{a}^{\tilde{N}^-_L}_{\pm}$
IV	$\tilde{a}^{1+}_{\pm}$	0	$\tilde{a}^{\tilde{N}^+_L}_{\pm}$	$\frac{1}{2}\,(\tilde{a}^{13}_{\pm} + \tilde{a}^{\tilde{N}^3_L}_{\pm})$

Table 10.7. The mass matrix on the tree level for the lower four families of quarks and leptons after the electroweak break. Only the contributions coming from the terms $\tilde{S}^{ab}\, \tilde{\omega}_{abs}$ in p_{0s} in Eq.(10.7) are presented. The notation $\tilde{a}^{\tilde{A}i}_{\pm}$ stays for $-\tilde{g}\, \tilde{A}^{\tilde{A}i}_{\pm}$, where $(\mp)$ distinguishes between the values of the (u-quarks and d-quarks) and between the values of (ν and e). The terms coming from $S^{ss'}\, \omega_{ss't}$ are not presented here. They are the same for all the families, but distinguish among the family members.

The absolute values of the vacuum expectation values of the scalar fields contributing to the first break are expected to be much larger than those contributing to the second break ($|\frac{\tilde{A}^{1i}_s}{\tilde{A}^{2i}_s}| \ll 1$).

The mass matrices of the lower four families were studied and evaluated in the ref. [13] under the assumption that if going beyond the tree level the differences in the mass matrices of different family members start to manifest. In this ref. we assumed the symmetry properties of the mass matrices from Table 10.7 and fitted the matrix elements to the experimental data for the three observed families within the accuracy of the experimental data. We were not able to determine masses of the fourth family. Taking the fourth family masses as parameters we were able to calculate matrix elements of mass matrices, predicting mixing matrices for all the members of the four lowest families.

In Table 10.8 we present quantum numbers of all members of a family, any one, after the electroweak break. It is easy to show that the contribution of complex conjugate to $\psi^{\dagger}_L\, \gamma^0\, (\stackrel{78}{(-)}\, p_{\mp}\, \psi_R$ gives the same value.

Table 10.9 presents the quantum numbers $\tilde{\tau}^{23}$, $\tilde{N}^3_R$, $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$ for all eight families. The first four families are singlets with respect to $\tilde{\tau}^{2i}$ and $\tilde{N}^i_R$, while they are doublets with respect to $\tilde{\tau}^{1i}$ and $\tilde{N}^i_L$ (all before the break of symmetries). The upper four families are

	Y	Y′	Q	Q′		Y	Y′	Q	Q′
u_R	$\frac{2}{3}$	$\frac{1}{2}(1-\frac{1}{3}\tan^2\theta_2)$	$\frac{2}{3}$	$-\frac{2}{3}\tan^2\theta_1$	u_L	$\frac{1}{6}$	$-\frac{1}{6}\tan^2\theta_2$	$\frac{2}{3}$	$\frac{1}{2}(1-\frac{1}{3}\tan^2\theta_1)$
d_R	$-\frac{1}{3}$	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\theta_2)$	$-\frac{1}{3}$	$\frac{1}{3}\tan^2\theta_1$	d_L	$\frac{1}{6}$	$-\frac{1}{6}\tan^2\theta_2$	$-\frac{1}{3}$	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\theta_1)$
ν_R	0	$\frac{1}{2}(1+\tan^2\theta_2)$	0	0	ν_L	$-\frac{1}{2}$	$\frac{1}{2}\tan^2\theta_2$	0	0
e_R	-1	$\frac{1}{2}(-1+\tan^2\theta_2)$	-1	$\tan^2\theta_1$	e_L	$-\frac{1}{2}$	$\frac{1}{2}\tan^2\theta_2$	-1	$-\frac{1}{2}(1-\tan^2\theta_1)$

Table 10.8. The quantum numbers Y, Y', Q, Q' of the members of a family.

correspondingly doublets with respect to $\tilde{\tau}^{2i}$ and $\tilde{N}^i_R$ and are singlets with respect to $\tilde{\tau}^{1i}$ and $\tilde{N}^i_L$.

$\Sigma = I/i$	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$	$\tilde{\tau}^{13}$	$\tilde{N}^3_L$	$\Sigma = II/i$	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$	$\tilde{\tau}^{13}$	$\tilde{N}^3_L$
1	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
2	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	2	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
3	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
4	0	0	$\frac{1}{2}$	$\frac{1}{2}$	4	$\frac{1}{2}$	$\frac{1}{2}$	0	0

Table 10.9. The quantum numbers $\tilde{\tau}^{23}$, $\tilde{N}^3_R$, $\tilde{\tau}^{13}$ and $\tilde{N}^3_L$ for the two groups of four families are presented.

10.3.1 Mass matrices beyond the tree level

While the mass matrices of (u and ν) have on the tree level the same off diagonal elements and differ only in diagonal elements due to the contribution of $e\, QA^Q_s$, $g^{Q'}Q'A^{Q'}_s$ and $g^{Y'}\, Y'A^{Y'}_s$ and the same is true for (d and e), loop corrections, to which massive gauge fields and dynamical scalar fields of both origins ($\tilde{\omega}_{abs}$ and $\omega_{s't s}$) contribute coherently, are expected to change mass matrices of the lower four families drastically. For the upper four families, for which the diagonal terms from $e\, QA^Q_s$, $g^{Q'}Q'A^{Q'}_s$ and $g^{Y'}\, Y'A^{Y'}_s$ are almost negligible, since they are the same for all eight families, loop corrections are not expected to bring drastic changes in mass matrices between different family members. On the tree level the mass matrices demonstrate twice four by diagonal matrices (this structure stays unchanged also after taking into account loop corrections in all orders)

$$M^{\alpha}_{(o)} = \begin{pmatrix} M^{\alpha\, II}_{(o)} & 0 \\ 0 & M^{\alpha\, I}_{(o)} \end{pmatrix}, \tag{10.29}$$

where $M^{\alpha\, II}_{(o)}$ and $M^{\alpha\, I}_{(o)}$ have the structure

$$M_{(o)} = \begin{pmatrix} -a_1 & b & 0 & c \\ b & -a_2 & c & 0 \\ 0 & c & a_1 & b \\ c & 0 & b & a_2 \end{pmatrix}, \tag{10.30}$$

with the matrix elements $a_1 \equiv a^{\Sigma}_{\pm\,1}$, $a_2 \equiv a^{\Sigma}_{\pm 2}$, $b \equiv b^{\Sigma}_{\pm}$ and $e \equiv e^{\Sigma}_{\pm}$. The values a_1, a_2, b and c are different for the upper ($\Sigma = II$) and the lower ($\Sigma = I$) four families, due to two different scales of two different breaks. One has

$$a_1 = \frac{1}{2}(\tilde{a}_{\pm}^{(1,2)3} - \tilde{a}_{\pm}^{\tilde{N}(R,L)3}) \quad , \quad a_2 = \frac{1}{2}(\tilde{a}_{\pm}^{(1,2)3} + \tilde{a}_{\pm}^{\tilde{N}(R,L)3}) \quad ,$$
$$b = \tilde{a}_{\pm}^{\tilde{N}(R,L)+} = \tilde{a}_{\pm}^{\tilde{N}(R,L)-} \,, \qquad c = \tilde{a}_{\pm}^{(1,2)+} = \tilde{a}_{\pm}^{(1,2)-} \quad . \tag{10.31}$$

For the upper four families ($\Sigma = II$) we have correspondingly $\tilde{a}_{\pm}^{3} = \tilde{a}_{\pm}^{23}$, $\tilde{a}_{\pm}^{\tilde{N}3} = \tilde{a}_{\pm}^{\tilde{N}_R 3}$, $\tilde{a}_{\pm}^{\pm} = \tilde{a}_{\pm}^{21} \pm i\, \tilde{a}_{\pm}^{22}$, $\tilde{a}_{\pm}^{\tilde{N}\pm} = \tilde{a}_{\pm}^{\tilde{N}_R 1} \pm i\, \tilde{a}_{\pm}^{\tilde{N}_R 2}$ and for the lower four families ($\Sigma = I$) we must take $\tilde{a}_{\pm}^{3} = \tilde{a}_{\pm}^{13}$, $\tilde{a}_{\pm}^{\tilde{N}3} = \tilde{a}_{\pm}^{\tilde{N}_L 3}$, $\tilde{a}_{\pm}^{\pm} = \tilde{a}_{\pm}^{11} \pm i\, \tilde{a}_{\pm}^{12}$, $\tilde{a}_{\pm}^{\tilde{N}\pm} = \tilde{a}_{\pm}^{\tilde{N}_L 1} \pm i\, \tilde{a}_{\pm}^{\tilde{N}_L 2}$.

To the tree level contributions of the scalar $\tilde{\omega}_{ab\pm}$ fields, diagonal matrices $a_{\pm}$ have to be added, the same for all the eight families and different for each of the family member (u, d, ν, e), $(\hat{a}_{\mp} \equiv a_{\mp}^{\alpha})\,\psi$, which are the tree level contributions of the scalar $\omega_{sts'}$ fields

$$\hat{a}_{\pm} = e\,\hat{Q}\,A_{\pm} + g^{1}\,\cos\theta_1\,\hat{Q}'\,Z_{\pm}^{Q'} + g^{2}\,\cos\theta_2\,\hat{Y}'\,A_{\pm}^{Y'} \,. \tag{10.32}$$

Since the upper and the lower four family mass matrices appear at two completely different scales, determined by two orthogonal sets of scalar fields, the two tree level mass matrices $\mathcal{M}_{(o)}^{\alpha\,\Sigma}$ have very little in common, besides the symmetries and the contributions from Eq. (10.32).

Let us introduce the notation, which would help to make clear the loop corrections contributions. We have before the two breaks two times ($\Sigma \in \{II, I\}$, II denoting the upper four and I the lower four families) four massless vectors $\psi_{\Sigma(L,R)}^{\alpha}$ for each member of a family $\alpha \in \{u, d, \nu, e\}$. Let i, $i \in \{1, 2, 3, 4, \}$ denotes one of the four family members of each of the two groups of massless families

$$\psi_{\Sigma(L,R)}^{\alpha} = (\psi_{\Sigma\,1}^{\alpha},\, \psi_{\Sigma\,2}^{\alpha},\, \psi_{\Sigma\,3}^{\alpha},\, \psi_{\Sigma\,4}^{\alpha})_{(L,R)} \,. \tag{10.33}$$

Let $\Psi_{\Sigma(L,R)}^{\alpha}$ be the final massive four vectors for each of the two groups of families, with all loop corrections included

$$\psi_{\Sigma\,(L,R)}^{\alpha} = V_{\Sigma}^{\alpha}\,\Psi_{\Sigma\,(L,R)}^{\alpha} \,,$$
$$V_{\Sigma}^{\alpha} = V_{\Sigma\,(o)}^{\alpha}\,V_{\Sigma\,(1)}^{\alpha} \cdots V_{\Sigma\,(k)}^{\alpha} \cdots \,. \tag{10.34}$$

Then $\Psi_{\Sigma(L,R)}^{\alpha\,(k)}$, which include up to (k) loops corrections, read

$$V_{\Sigma\,(o)}^{\alpha}\,\Psi_{\Sigma\,(L,R)}^{\alpha\,(o)} = \psi_{\Sigma\,(L,R)}^{\alpha} \,,$$
$$V_{\Sigma\,(o)}^{\alpha}\,V_{\Sigma\,(1)}^{\alpha} \cdots V_{\Sigma\,(k)}^{\alpha}\,\Psi_{\Sigma(L,R)}^{\alpha\,(k)} = \psi_{\Sigma(L,R)}^{\alpha} \,. \tag{10.35}$$

Correspondingly we have

$$< \psi_{\Sigma\,L}^{\alpha}|\gamma^{0}\,(M_{(o\,k)}^{\alpha\,\Sigma} + \cdots + M_{(o\,1)}^{\alpha\,\Sigma} + M_{(o)}^{\alpha\,\Sigma})\,|\psi_{\Sigma\,R}^{\alpha} >=$$
$$< \Psi_{\Sigma\,L}^{\alpha\,(k)}|\gamma^{0}\,(V_{\Sigma\,(o)}^{\alpha}\,V_{\Sigma\,(1)}^{\alpha} \cdots V_{\Sigma\,(k)}^{\alpha})^{\dagger}\,(M_{(o\,k)}^{\alpha\,\Sigma} + \cdots$$
$$+ M_{(o\,1)}^{\alpha\,\Sigma} + M_{(o)}^{\alpha\,\Sigma})\;V_{\Sigma\,(o)}^{\alpha}\,V_{\Sigma(1)}^{\alpha} \cdots V_{\Sigma\,(k)}^{\alpha}\,|\Psi_{\Sigma\,R}^{\alpha\,(k)} > . \tag{10.36}$$

Let us repeat that to the loop corrections two kinds of the scalar dynamical fields contribute, those originating in $\tilde{\omega}_{abs}$ ($\tilde{g}^{\tilde{Y}'}\;\hat{\tilde{Y}}'\,\tilde{A}_{s}^{\tilde{Y}'}$, $\frac{\tilde{g}^{2}}{\sqrt{2}}\,\hat{\tilde{\tau}}^{2\pm}\,\tilde{A}_{s}^{2\pm}$, $\tilde{g}^{\tilde{N}_{L,R}}\,\hat{\vec{\tilde{N}}}_{L,R}\,\vec{\tilde{A}}_{s}^{\tilde{N}_{L,R}}$, $\tilde{g}^{\tilde{Q}'}\,\hat{\tilde{Q}}'\;\tilde{A}_{s}^{\tilde{Q}'}$, $\frac{\tilde{g}^{1}}{\sqrt{2}}\,\hat{\tilde{\tau}}^{1\pm}\,\tilde{A}_{s}^{1\pm}$) and those originating in ω_{abs} ($e\,\hat{Q}\,A_{s}$, $g^{1}\,\cos\theta_1\,\hat{Q}'\,Z_{s}^{Q'}$, $g^{Y'}\cos\theta_2\,\hat{Y}'\,A_{s}^{Y'}$) and the massive gauge fields ($g^{2}\cos\theta_2\,\hat{Y}'\,A_{m}^{Y'}$, $g^{1}\,\cos\theta_1\,\hat{Q}'\,Z_{m}^{Q'}$) as it follow from Eq.(10.7).

In the ref. [15] the loop diagrams for these contributions to loop corrections are presented and numerical results discussed for both groups of four families. The masses and coupling constants of dynamical scalar fields and of the massive vector fields are taken as an input and the influence of loop corrections on properties of fermions studied.

Let us arrange mass matrices, after the electroweak break and when all the loop corrections are taken into account, as a sum of matrices as follows

$$M^{\alpha\,\Sigma} = \sum_{k=0,k'=0,k''=0}^{\infty} (Q^{\alpha})^{k}\,(Q'^{\alpha})^{k'}\,(Y'^{\alpha})^{k''}\,M^{\alpha\,\Sigma}_{Q\,Q'\,Y'\,k\,k'\,k''}\,. \tag{10.37}$$

To each family member there corresponds its own matrix $M^{\alpha\,\Sigma}$. It is a hope, however, that the matrices $M^{\alpha\,\Sigma}_{Q\,Q'\,Y'\,k\,k'\,k''}$ might depend only slightly on the family member index α ($M^{\alpha\,\Sigma}_{Q\,Q'\,Y'\,k\,k'\,k''} = M^{\Sigma}_{Q\,Q'\,Y'\,k\,k'\,k''}$) and the eigenvalues of the operators $(\hat{Q}^{\alpha})^{k}\,(\hat{Q}'^{\alpha})^{k'}\,(\hat{Y}'^{\alpha})^{k''}$ on the massless states $\psi^{\alpha}_{\Sigma\,R}$ make the mass matrices $M^{\alpha\,\Sigma}$ dependent on α. To masses of neutrinos only the terms $(Q^{\alpha})^{0}\,(Q'^{\alpha})^{0}\,(Y'^{\alpha})^{k''}\,M^{\alpha\,\Sigma}_{Q\,Q'\,Y'\,k\,k'\,k''}$ contribute.

There is an additional term, however, which does not really speak for the suggestion of Eq. (10.37). Namely the term in loop corrections which transforms the right handed neutrinos into their left handed charged conjugated ones and which manifests accordingly the Majorana neutrinos. These contribution is presented in Eq. (10.38) of the subsection 10.3.1. It concerns only the lower group of four families and might contribute a lot, in addition to the "Dirac masses" of Eq. (10.37), to the extremely small masses of the observed families of neutrinos. This term needs, as also all the loop corrections to the tree level mass matrices for all the family members, additional studies.

More about the mass matrices below the tree level can be found in the ref. [15].

Majorana mass terms in the spin-charge-family theory There are mass terms within the *spin-charge-family* theory, which transform the right handed neutrino to its charged conjugated one, contributing to the right handed neutrino Majorana masses

$$\begin{aligned}
&\psi^{\dagger}\,\gamma^{0}\,\overset{78}{(-)}\,p_{0-}\,\psi\,,\\
&p_{0-} = -(\tilde{\tau}^{1+}\,\tilde{A}^{1+}_{-} + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_{-})\,\mathcal{O}^{[+]}\,\mathcal{A}^{\mathcal{O}}_{[+]},\\
&\mathcal{O}^{[+]} = \overset{78}{[+]}\,\overset{56}{(-)}\,\overset{9\,10}{(-)}\,\overset{11\,12}{(-)}\,\overset{13\,14}{(-)}\,.
\end{aligned} \tag{10.38}$$

One easily checks, using the technique with the Clifford objects, that $\gamma^{0}\,\overset{78}{(-)}\,p_{0-}$ transforms a *right handed neutrino* of one of the *lower four families* into the charged conjugated one, belonging to the same group of families. It does not contribute to masses of other leptons and quarks, right or left handed. Although the operator $\mathcal{O}^{[+]}$ appears in a quite complicated way, that is in the higher order corrections, yet it might be helpful when explaining the properties of neutrinos. The operator $-(\tilde{\tau}^{1+}\,\tilde{A}^{1+}_{-} + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_{-})\,\mathcal{O}^{[+]}\,\mathcal{A}^{\mathcal{O}}_{[+]}$ gives zero, when it applies on the upper four families, since the upper four families are singlets with respect to $\tilde{\tau}^{1\pm}$.

This term needs further studies.

10.4 Scalar fields of the spin-charge-family theory manifesting effectively as the standard model Higgs

Before starting to interpret the *standard model* as an effective approach of the *spin-charge-family* theory let me remind the reader that effective interactions are commonly used in many body systems. The Heisenberg model, for example, uses the "spin-spin" interactions for describing ferromagnetic properties of materials, replacing the complicated electromagnetic interactions among many particles involved.

The right handed neutrino is a regular family member in the *spin-charge-family* theory. In the *standard model* the right handed neutrinos were left out only because of historical reasons: neutrinos were assumed to be massless and the right handed neutrinos have in the *standard model* all the charges equal to zero. Accordingly there was no need to postulate the existence of an additional gauge field to which the right handed neutrinos, together with all the quarks and leptons, would couple.

Let us point out the starting assumptions of the *standard model* from the point of view of symmetries. The infinitesimal generators of the symmetry groups assumed by the *standard model*

$$[SO(1,3) \times SU(3) \times SU(2) \times U(1)]_{(\nu,s)} \tag{10.39}$$

determine singlet (scalar), spinor (fundamental), vector and $\cdots$ (staying for infinite many) representations, all corresponding to the same algebra of the infinitesimal generators of the group $SO(1,3)$; singlet, spinor, vector (adjoint), $\cdots$ representations determined by the algebra of the $SU(3)$, of $SU(2)$ and of $U(1)$ infinitesimal generators. Index (ν, s) is meant to point out that spinors are coupled to the vector fields according to the action of the *standard model*: Massless spinors with the charges in the fundamental representations of the groups couple to massless vectors with the charges in the adjoint representations of the same groups as described by the Lagrange density presented in the first line of Eq. (10.5) – if the assumptions are made that only the left handed spinors carry the weak charge while the right handed ones are weakless and that the appropriate values for the family members with respect to the $U(1)$ charge group are chosen. While in the *spin-charge-family* theory it is straightforward to see that the $U(1)_{II,I}$ charges representations, as well as colour singlets, are of the spinor kind, since they all originate in the starting spinor multiplet, in the *standard model* the spinor representations of all the charges can only be assumed.

After assuming that triplets and singlets of the $SU(3)$ charge, doublets and singlets of the $SU(2)$ charge and singlets of $U(1)$ charge are of the spinor origin, then it seems natural from the point of view of representations that all of them constitute members of one family. The *standard model* makes in addition the assumption about the relation between the spinor representation of $SO(1,3)$ and the representations of the charges. (It turns out that these choices lead to also other useful properties.) That each vector carries only the charge through which it couples to fermions seems an elegant (the simplest one) assumption. There are families of spinors assumed in addition, all massless, all equal with respect to the groups (Eq. (10.39)).

To make spinors and weak bosons massive the *standard model* assumes the Higgs (the scalar representation of $SO(1,3)$) with the spinor charges with respect to the rest of the groups of Eq. (10.39) and with the values of $U(1)$ group which ensure with the Higgs "dressed" right handed members of any family to have the charges of the left handed partners: u and ν are "dressed" with anti-Higgs, d and e with Higgs. The action added to the massless ones (for the massless spinors and vectors) takes care of the interaction of the vector fields with this scalar field and of the scalar potential. Mass matrices for each of the family members are "put by hand" (just assumed) in addition.

To include into Eq. (10.39) the scalar field and to point out all the relations – interactions – assumed by the *standard model* we could correspondingly rewrite Eq. (10.39) as follows

$$\begin{aligned}&\{[SO(1,3) \times SU(3) \times SU(2) \times U(1) \times SU(3)_{fa}]_{(\nu,s)^{*}}\\ &\times[SO(1,3) \times SU(3) \times SU(2) \times U(1)]_{(\nu,sc)^{**}}\}^{***} ,\end{aligned} \tag{10.40}$$

with the indices in Eq. (10.40) which tell us the limitation of the choice: i. The index $(\nu, s)^*$ tells that spinor charges are in relation with the handedness and that no vector field is assumed for the family charge $SU(3)_{fa}$. ii. The index $(\nu, sc)^{**}$ tells us that for the scalar field a very particular representations of the charge groups is allowed (Higgs is a colour singlet, weak doublet and of particular $U(1)$ charge). iii. The index *** tells us that the *standard model* is offering no explanation for the appearance of the families and the Yukawa couplings.

The *spin-charge-family* theory starting symmetry group is

$$[SO(1,13)_\gamma \times SO(1,13)_{\tilde{\gamma}}]_{\nu,s} \tag{10.41}$$

with the action (Eq. (10.4)) which couples vielbeins and spin connection fields with spinors. Since there are two kinds of spins (S^{ab} and $\tilde{S}^{ab}$) there are also two kinds of the spin connection fields. To observe one kind of spin as the spin and all the charges of fermions, the break of the starting symmetry must occur. Accordingly it is meaningful to replace Eq. (10.41) with

$$[SO(1,13)_\gamma \times SO(1,13)_{\tilde{\gamma}}]_{(\nu,s)^\diamond} \tag{10.42}$$

where the index $(\nu, s)^\diamond$ points out that several particular breaks of the starting symmetry (phase transitions) happen. At the stage when the starting symmetry has broken to the symmetry $(SO(1,3)_\gamma \times SO(1,3)_{\tilde{\gamma}} \times SU(2)_\gamma \times SU(2)_{\tilde{\gamma}} \times U(1)_\gamma \times U(1)_{\tilde{\gamma}} \times SU(3)_\gamma)$, that is before the electroweak break, there are four massless families with the members: coloured quarks and colourless leptons, the left handed members all weak charged and the right handed members all weak chargeless, of a very determined $U(1)$ charge (just the ones assumed by the *standard model*), all in the spinor representations, since all the members of one family follow from the starting left handed massless spinor representation. All the families are equivalent. No assumption, except the one that each phase transition is connected with a particular break of a symmetry, need to be made. The way of breaking the starting symmetry determines also that the number of massless families is before the electroweak break equal to four, rather than to the observed three.

The mass term appears after the break by itself. There are several scalar fields, all with the charges in the adjoint representations, which determine after the phase transition triggered by their nonzero vacuum expectation values the properties of families of quarks and leptons.

Mass matrices of fermions of the lower four families are in the *spin-charge-family* theory, according to Eq.(10.7), on the tree level determined by the scalar fields through the operator

$$\hat{\Phi}^{I}_{\mp} = \stackrel{78}{(\mp)} \{\tilde{g}^{\tilde{N}_L}\, \vec{\tilde{N}}_L\, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp} + \tilde{g}^1\, \vec{\tilde{\tau}}^1\, \vec{\tilde{A}}^1_{\mp} + e\, Q\, A^Q_{\mp} + g^{Q'}\, Q'\, Z^{Q'}_{\mp} + g^{Y'}\, Y'\, A^{Y'}_{\mp}\}. \tag{10.43}$$

The operator $\stackrel{78}{(\mp)}$, appearing in Eq. (10.43) on the left hand side, transforms all the quantum numbers of the right handed quarks and leptons to those of the left handed ones, except the handedness, for the transformation of which in the *standard model* as well as in the *spin-charge-family* theory γ^0 takes care. One can formally replace the operator $\stackrel{78}{(\mp)}$ with the operator $\sum_{\alpha,i} |\psi^\alpha_{ILi} >< \psi^\alpha_{IRi}|$,

$$\stackrel{78}{(\mp)} \Longrightarrow \sum_{\alpha,i} |\psi^\alpha_{ILi} >< \psi^\alpha_{IRi}|, \tag{10.44}$$

$\alpha \in \{u, d, \nu, e\}$ and $i \in \{1,2,3,4\}$. Both operators of Eq. (10.44) do the same: Transform the right handed member of any family with the left handed one of the same family, doing what the Higgs does (up to its vacuum expectation value) when "dressing" right handed quarks and leptons. The great difference among these three operators is that the operator $\stackrel{78}{(\mp)}$ follows from the simple starting action, while the Higgs and the operator $\sum_{\alpha,i} |\psi^{\alpha}_{ILi} >< \psi^{\alpha}_{IRi}|$ are put by hand.

The product of the operators γ^0 and $\stackrel{78}{(\mp)}$ transforms the right handed quarks and leptons into the left handed ones, as explained in section 10.2 and can be read in Tables 10.1 and 10.2. The part of the operator $\hat{\Phi}^{I}_{\mp}$, that is $\{\tilde{g}^{\tilde{N}_L}\, \vec{\tilde{N}}_L\, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp} + \tilde{g}^1\, \vec{\tilde{\tau}}^1\, \vec{\tilde{A}}^{1}_{\mp} + e\, Q\, A^{Q}_{\mp} + g^{Q'}\, Q'\, Z^{Q'}_{\mp} + g^{Y'}\, Y'\, A^{Y'}_{\mp}\}$, takes care of the mass matrices of quarks and leptons. The application of $\{\tilde{g}^{\tilde{N}_R}\, \vec{\tilde{N}}_R\, \vec{\tilde{A}}^{\tilde{N}_R}_{\mp} + \tilde{g}^2\, \vec{\tilde{\tau}}^2\, \vec{\tilde{A}}^{2}_{\mp}\}$ on the lower four families is zero, since the lower four families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$. In the loop corrections besides the massive scalar fields - $\vec{\tilde{A}}^{1}_{\mp}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{\mp}$, $A^{Q}_{\mp}$, $Z^{Q'}_{\mp}$ and $A^{Y'}_{\mp}$ - also the massive gauge vector fields - $Z^{Q'}_{m}$, $A^{1\pm}_{m} = W^{\pm}_{m}$, $A^{Y'}_{m}$ and $A^{2\pm}_{m}$ - start to contribute coherently.

We do not (yet) know the properties of the scalar fields, their vacuum expectation values, masses and coupling constants. We may expect that they behave similarly as the Higgs field in the *standard model*, that is that their dynamics is determined by potentials which make contributions of the scalar fields renormalizable. Starting from the spin connections and vielbeins we only can hope that at least effectively at the low energy regime, that is in the weak field regime, the effective theory is behaving as a renormalizable one.

Yet we can estimate masses of gauge bosons under the assumption that the scalar fields, which determine mass matrices of fermion family members, are determining effectively also masses of gauge fields, like this is assumed in the *standard model*. If breaking of symmetries occurs in both sectors in a correlated way (I have assumed so far that this is the case) then symmetries of the vielbeins $f^{\alpha}{}_{a}$ and the two spin connection fields, $\omega_{abc} = f^{\alpha}{}_{c}\, \omega_{ab\alpha}$ and $\tilde{\omega}_{abc} = f^{\alpha}{}_{c}\, \tilde{\omega}_{ab\alpha}$, change simultaneously.

To see the *standard model* as an effective theory of the *spin-charge-family* theory let us assume the existence of the scalar fields, which by "dressing" right handed family members, ensure them the weak and the hyper charge of their left handed partners. Therefore, we shall replace the part $\stackrel{78}{(-)}$ of the operator $\hat{\Phi}^{I}_{\mp}$ in Eq.(10.43), which transforms the weak chargeless right handed u_R quark of a particular hyper charge Y ($\frac{2}{3}$) of any family into the weak charged u_R quark with Y of the left handed u_L ($\frac{1}{6}$), while γ^0 changes its right handedness into the left one, with the scalar field of Table 10.10, which has the appropriate weak and hyper charge. Although all the scalar fields, as all the gauge fields, of the *spin-charge-family* theory are bosons manifesting their properties – the charges of all origins – in the adjoint representations, we replace, in order to mimic the *standard model*, the part $\stackrel{78}{(-)}$ by the scalar field with the charges originating in S^{ab} in the fundamental representation. This scalar field must be a colourless weak doublet with the hyper charges $Y = -\frac{1}{2}$ for (u_R and ν_R) and $Y = \frac{1}{2}$ for ((d_R) and e_R), while only the components with the electromagnetic charge $Q = (\tau^{13} + Y)$ equal to zero are allowed to have nonzero vacuum expectation values. This scalar field must be a dynamical field, with a nonzero vacuum expectation value, massive, and governed by a hopefully renormalizable potential. This means that averaging over all the scalar fields appearing in Eq. (10.43) manifests as the assumed scalar field and the Yukawa couplings of the *standard model*.

We can simulate the part $\stackrel{78}{(-)}$ with the scalar field Φ^{I}_{-}, presented in Table 10.10, which "dresses" u_R and ν_R in the way assumed by the *standard model*, and we simulate the part $\stackrel{78}{(+)}$ with the scalar field Φ^{I}_{+} from Table 10.10, which "dresses" d_R and e_R.

	Φ^I	τ^{13}	τ^{23}	τ^4	Y	Q	colour charge
Φ^I_-	$\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\|\|\,\stackrel{9\,10}{[+]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	colourless
Φ^I_+	$\stackrel{56}{[-]}\,\stackrel{78}{[+]}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	colourless

Table 10.10. One possible choice of the weak and hyper charge components of the scalar fields carrying the quantum numbers of the *standard model* Higgs, presented in the technique [10,18], chosen to play the role of the *standard model* Higgs. Both states on the table are colour singlets, with the weak and hyper charge, which if used in the *standard model* way "dress" the right handed quarks and leptons so that they carry quantum numbers of the left handed partners. The state $(\Phi^I_- \, u_R)$, for example, carries the weak and the hyper charge of u_L. "Dressing" the right handed family members with the $\Phi^I_\mp$ manifests effectively as the application of the operators $\stackrel{78}{(\mp)}$ (Eq.(10.43)) on the right handed family members.

In the *spin-charge-family* theory mass matrices are determined on the tree level by $\{\tilde{g}^{\tilde{N}_L}\, \tilde{N}^i_L\, \tilde{A}^{\tilde{N}_L\, i}_\mp + \tilde{g}^1\, \tilde{\tau}^{1i}\, \tilde{A}^{1i}_\mp + e\, Q\, A^Q_\mp + g^{Q'}\, Q'\, Z^{Q'}_\mp + g^{Y'}\, Y'\, A^{Y'}_\mp\}$ (Eq.(10.43)). Let this operator be called $\hat{\Phi}^{\nu I}_\mp$

$$\hat{\Phi}^{\nu I}_\mp = \{\tilde{g}^{\tilde{N}_L}\, \tilde{N}^i_L\, \tilde{A}^{\tilde{N}_L\, i}_\mp + \tilde{g}^1\, \tilde{\tau}^{1i}\, \tilde{A}^{1i}_\mp + e\, Q\, A^Q_\mp + g^{Q'}\, Q'\, Z^{Q'}_\mp + g^{Y'}\, Y'\, A^{Y'}_\mp\}\,. \tag{10.45}$$

In the attempt to see the *standard model* as an effective theory of the *spin-charge-family* theory the *standard model* Higgs together with the Yukawa couplings can be presented as the product of $\Phi^I_\mp$ and $\hat{\Phi}^{\nu I}_\mp$. The role of $\Phi^I_\mp$ in this product is to "dress" the right handed quarks and leptons with the weak charge and the appropriate hyper charge, while $\hat{\Phi}^{\nu I}_\mp$ effectively manifests on the tree level as the Higgs and the Yukawa couplings together.

Masses of the vector gauge fields as well as the properties of the scalar fields should in the *spin-charge-family* theory be determined by studying the break of symmetries. We discuss in subsect. 10.2.2 the break, but the detailed calculations are very demanding and we have not (yet) been able to perform them.

One can extract some information about properties of the scalar fields in Eq.(10.45) from the masses of the so far observed quarks and leptons and the weak boson masses. From the covariant momentum after the electroweak break

$$p_{0m} = p_m - \frac{g^1}{\sqrt{2}}[\tau^{1+}\, A^{1+}_m + \tau^{1-}\, A^{1-}_m] + g^1 \sin\theta_1\, Q\, A^Q_m + g^1 \cos\theta_1\, Q'\, A^{Q'}_m\,, \tag{10.46}$$

with θ_1 equal to θ_W, with the electromagnetic coupling constant $e = \sin\theta_W$, the charge operators $Q = \tau^{13} + Y$, $Q' = \tau^{13} - \tan^2\theta_W\, Y$, and with the gauge fields $W^\pm_m = A^{1\pm}_m = \frac{1}{\sqrt{2}}\,(A^{11}_m \mp A^{12}_m)$, $A_m = A^Q_m = A^{13}_m \sin\theta_W + A^Y_m \cos\theta_W$ and $Z_m = A^{Q'}_m = A^{13}_m \cos\theta_W - A^Y_m \sin\theta_W$, we estimate

$$(p_{0m}\, \hat{\Phi}^I_\mp)^\dagger\, (p_0{}^m\, \hat{\Phi}^I_\mp) = \{\, \frac{(g^1)^2}{2}\, A^{1+}_m\, A^{1-\, m} + (\frac{g^1}{2\,\cos\theta_1})^2\, A^{Q'}_m\, A^{Q'\, m}\}\, \mathrm{Tr}(\Phi^{\nu I\dagger}_\mp\, \Phi^{\nu I}_\mp)\,. \tag{10.47}$$

$\Phi^{\nu I}_\mp$ are determined in Eq. (10.45), while the states $\Phi^I_\mp$ from Table 10.10 are normalized to unity as explained in the refs. [18] and in the appendix. Assuming, like in the *standard model*, that $\mathrm{Tr}(\Phi^{\nu I\dagger}_\mp\, \Phi^{\nu I}_\mp) = \frac{v^2}{2}$, we extract from the masses of gauge bosons one information about the vacuum expectation values of the scalar fields, their coupling constants and their masses. Mass matrices of quarks and leptons offer additional information about the scalar fields of the *spin-charge-family* theory. Measuring charged and neutral currents, decay rates of hadrons, the scalar fields productions in the fermion scattering events and their decay properties provides us with additional information.

Studying neutral and charged currents and possible scalar field productions and decays are important next step to be done.

Let me conclude this section by the observation that the colourless scalar with the weak charges in the fundamental representation of the $SU(2)$ group is a strange object from the point of view of the fact that all the known fields are either fermions in the fundamental representations with respect to the charge groups or they are (vector) bosons in the adjoint representations with respect to the charge groups. In the *spin-charge-family* theory the scalar fields carry all the charges (with the family quantum numbers included) in the adjoint representations. It is challenging to prove or disprove whether or not the *standard model* can be interpreted as an effective low energy manifestation of the *spin-charge-family* theory. And that several scalar fields of the *spin-charge-family* theory with all the charges in the adjoint representations of the corresponding groups effectively manifest as the Higgs with the charges in the fundamental representations and the Yukawa couplings.

10.5 Models with the $SU(3)$ flavour groups and the spin-charge-family theory

In section 10.4 we look at the *standard model* assumptions from the point of view of the *spin-charge-family* theory. There are many attempts in the literature to connect families of quarks and leptons with the fundamental representations of the $SU(3)$ gauge group. Let me comment a quite simplified version of the assumptions presented in the refs. [23–25] from the point of view of the *spin-charge-family* theory.

Let us therefore assume that the three so far observed families of quarks and leptons, neutrinos will be treated as ordinary family members, if all massless, manifest the "flavour" symmetry

$$\begin{aligned}&\{[SO(1,3) \times SU(3) \times SU(2) \times U(1) \times SU(3)_{fa}]_{(\nu,s)^{*\bullet}}\\&\times[SO(1,3) \times SU(3) \times SU(2) \times U(1)]_{(\nu,sc)^{**}}\}\\&\{\times[SO(1,3) \times SU(3) \times SU(2) \times U(1)] \times SU3_{fasc}\}^{\bullet\bullet}\,,\end{aligned} \tag{10.48}$$

with the indices in Eq. (10.48) which tell us the limitations of the choice: i. The index $(\nu,s)^{*\bullet}$ tells that spinor charges are in relation with the handedness, that no vector gauge field is assumed for the family charge $SU(3)_{fa}$, and that family (flavour) groups are different for different family members; The left handed quarks have different $SU(3)$ family charge than the right handed ones and the right handed u and the right handed d have each their own $SU(3)$ family charge. Similarly the left handed leptons have their own $SU(3)$ family charge which differs from the $SU(3)$ family charges of the right handed ν and the right handed e, and the right handed $SU(3)$ family charge of ν is different than the $SU(3)$ family charge of e. ii. The index $(\nu,sc)^{**}$ tells us, as before, that for the scalar field a very particular choice of representations of the charge groups is allowed (Higgs is a colour singlet, weak doublet and of particular $U(1)$ charge). iii. The index $^{\bullet\bullet}$ tells us that there are scalar fields which are singlets with respect to the groups $[SO(1,3) \times SU(3) \times SU(2) \times U(1)]$ (colourless, weakless, with zero hyper charge scalars) which accordingly do not couple to the gauge vector fields of the charges $SU(3), SU(2), U(1)$. They carry family charges in the fundamental, anti-fundamental or singlet representations of the groups, depending to which family members do they couple. To u quarks the scalar which is a triplet with respect to the family $SU(3)$ charge of u_L and d_L, anti-triplet with respect to the family $SU(3)$ charge of u_R and chargeless with respect to the family $SU(3)$ charge of the right handed d_R quarks. Equivalently there is the scalar which is again a triplet with respect to the family $SU(3)$ charge of u_L and d_R, anti-triplet with respect to the family $SU(3)$ charge of d_R and $SU(3)$ family chargeless with respect to the family $SU(3)$ charge of the right handed u_R quarks. These scalars have no couplings to the leptons. In the lepton sector goes equivalently.

It is assumed in addition that Yukawa scalar fields are, like the Higgs, the dynamical fields, which by gaining nonzero vacuum expectation values, break the family (flavour) symmetry.

In the *spin-charge-family* theory the families appear as representations of the group with the infinitesimal generators $\tilde{S}^{ab}$, forming the equivalent representations with respect to the group with the generators S^{ab}. These latter generators determine spin of fermions and their charges. The fields gauging the group S^{ab} determine after the break of symmetries in the low energy regime all the known gauge fields, which are vectors in $(1+3)$ with the charges in the adjoint representations.

The scalar fields which are gauge fields of the $\tilde{\tau}^{1i}$ $(= \tilde{c}^{\tilde{1}i}{}_{ab}\, \tilde{S}^{ab})$ and $\tilde{N}^{i}_{L}$ $(= \tilde{c}^{\tilde{N}_L i}{}_{ab}\, \tilde{S}^{ab})$ in the adjoint representations of the two $SU(2)$ groups determine together with the singlet scalar fields which are the gauge fields of Q, Q' and Y' (all three are expressible with S^{ab}) after the electroweak break the mass matrices of four families of quarks and leptons, and correspondingly the Yukawa couplings, masses and mixing matrices of the (lowest) four families of quarks and leptons. The scalar fields, with their nonzero vacuum expectation values, determine mass matrices of quarks and leptons on the tree level and contribute to masses of weak bosons. Below the tree level the dynamical scalar fields of both origins and the massive gauge fields bring coherent contributions to the tree level mass matrices.

While on the tree level the off diagonal matrix elements of the u-quark mass matrix are equal to the off diagonal matrix elements of the ν-lepton mass matrix, and the off diagonal matrix elements of the d-quark mass matrix equal to the off diagonal matrix elements of the e-lepton mass matrix, the loop corrections change this picture drastically, hopefully reproducing the experimentally observed properties of fermions.

In the *spin-charge-family* theory there is no scalar field in the fundamental representation of the weak charge, the "duty" of which is in the *standard model* to take care of the weak and the hyper charge of the right handed family members.

All the fermion charges, with the family quantum number included, are described by the fundamental representations of the corresponding groups, and all the bosonic fields, either vectors or scalars, have their charges in the adjoint representations.

The refs. [23–25] try to explain the appearance of mass matrices of the *standard model*, which manifest in the Higgs fields and the Yukawa matrices, by taking the Yukawas as dynamical fields. Yukawa scalar fields with their bi-fundamental representations of the $SU(3)$ flavour group are an attempt to continue with the assumption of the *standard model* that there exist scalar fields with charges in the fundamental representations. To do the job Yukawa scalar field are assumed to be in the bi-fundamental representations. It seems a very nontrivial task to make use of the analyses of the experimental data presented as a general extension of the *standard model* in the refs. [24,25] for the *spin-charge-family* theory as it manifests in the low energy region.

10.6 Conclusions

The *spin-charge-family* theory [10,11,19,12–14] is offering the way beyond the *standard model* by proposing the mechanism for generating families of quarks and leptons and consequently predicting the number of families at low (sooner or later) observable energies and the mass matrices for each of the family member (and correspondingly the masses and the mixing matrices of families of quarks and leptons).

The *spin-charge-family* theory predicts the fourth family to be possibly measured at the LHC or at some higher energies and the fifth family which is, since it is decoupled in the mixing matrices from the lower four families and it is correspondingly stable, the candidate to form the dark matter [14].

The proposed theory also predicts that there are several scalar fields, taking care of mass matrices of the two times four families and of the masses of weak gauge bosons. At low energies these scalar dynamical fields manifest effectively pretty much as the *standard model* Higgs field together with Yukawa couplings, predicting at the same time that observation of these scalar fields is expected to deviate from what for the Higgs the *standard model* predicts.

To the mass matrices of fermions two kinds of scalar fields contribute, the one interacting with fermions through the Dirac spin and the one interacting with fermions through the second kind of the Clifford operators (anticommuting with the Dirac ones, there is no third kind of the Clifford algebra object). The first one distinguishes among the family members, the second one among the families. Beyond the tree level these two kinds of scalar fields and the vector massive fields start to contribute coherently, leading hopefully to the measured properties of the so far observed three families of fermions and to the observed weak gauge fields.

In the ref. [11,13] we made a rough estimation of properties of quarks and leptons of the lower four families as predicted by the *spin-charge-family* theory. The mass matrices of quarks and leptons turns out to be strongly related on the tree level. Assuming that loop corrections change elements of mass matrices considerably, but keep the symmetry of mass matrices, we took mass matrix element of the lower four families as free parameters. We fitted the matrix elements to the existing experimental data for the observed three families within the experimental accuracy and for a chosen mass of each of the fourth family member. We predicted then elements of the mixing matrices for the fourth family members as well as the weakly measured matrix elements of the three observed families.

In the ref. [14] we evaluated the masses of the stable fifth family (belonging to the upper four families) under the assumption that neutrons and neutrinos of this stable fifth family form the dark matter. We study the properties of the fifth family neutrons, their freezing out of the cosmic plasma during the evolution of the universe, as well as their interaction among themselves and with the ordinary matter in the direct experiments.

In this paper we study properties of the gauge vector and scalar fields and their influence on the properties of eight families of quarks and leptons as they follow from the *spin-charge-family* theory on the tree and below the tree level after the two successive breaks, from $SO(1,3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_{I} \times U(1)_{I} \times SU(3)$ and further to $SO(1,3) \times U(1) \times SU(3)$, trying to understand better what happens during these two breaks and after them.

We made assumptions about succesive breaks of the starting symmetry since we are not (yet) able to evaluate how do the breaks occur and what does trigger them.

In the break from $SO(1,3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_{I} \times U(1)_{I} \times SU(3)$ several scalar fields (the superposition of ${f^{\sigma}}_{s}\, \tilde{\omega}_{abs}\,, s = (7,8)$) contribute to the break as gauge triplet fields of $\vec{\tilde{N}}_{R}$ and of $\vec{\tilde{\tau}}^{2}$, gaining nonzero vacuum expectation values. Correspondingly they cause nonzero mass matrices of the upper four families to which they couple and nonzero masses of vector fields, the superposition of the gauge triplet fields of $\vec{\tau}^2$ and of the gauge singlet field of τ^4. Since these scalar fields do not couple to the lower four families (they are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$) the lower four families stay massless at this break.

At the successive break, that is at the electroweak break, several other combinations of $f^{\sigma}{}_s\, \tilde{\omega}_{ab\sigma}$, the gauge triplets of $\vec{\tilde{N}}_L$ and of $\vec{\tilde{\tau}}^1$ (which are orthogonal to previous triplets), together with some combinations of scalar fields $f^{\sigma}{}_s\, \omega_{s't\sigma}$, the gauge fields of Q, Q' and Y', gain nonzero vacuum expectation values, contributing correspondingly to mass matrices of the lower four families and to masses of the gauge fields $W^{\pm}_m$ and Z_m, influencing slightly, together with the massive vector fields, also mass matrices of the upper four families.

Although mass matrices of the family members are in each of the two groups of four families very much related on the tree level (u-quarks are related to ν-leptons and d-quarks to e-leptons), the loop corrections, in which the scalar fields of both kinds contribute, those distinguishing among the families and those distinguishing among the family members $(u\,, d\,, \nu\,, e)$, together with the massive vector gauge fields which distinguish only among family members, start to hopefully (as so far done calculations [15] manifest) explain why are properties of the so far observed quarks and leptons so different. Numerical evaluations of the loop corrections to the tree level are in preparation (the ref. [15]).

It might be, however, that the influence of a very special term in higher loop corrections, which influences only the neutrinos, since it transforms the right handed neutrinos into the left handed charged conjugated ones, is very strong and might be responsible for the properties of neutrinos of the lower three families.

To simulate the *standard model* the effective low energy model of the *spin-charge-family* theory is made in which the operator, which in the *spin-charge-family* theory transforms the weak and hyper charges of right handed quarks and leptons into those of their left handed partners, is replaced by a weak doublet scalar, colour singlet and of an appropriate hyper charge, while the scalar dynamical fields of the *spin-charge-family* theory determine the Yukawa couplings. This weak doublet scalar "dresses" the right handed family members with the appropriate weak charge and hyper charge behaving as the Higgs of the *standard model*. It is further tried to understand to which extent can the scalar fields originating in $\tilde{\omega}_{abs}$ and ω_{abs} spin connection dynamical fields (all in the adjoint representations with respect to all the gauge groups) be replaced by a kind of a "bi-fundamental" (with respect to their several family groups) Yukawa scalar dynamical fields of the models presented in the refs. [25,24], in which fermion families are assumed to be members of several $SU(3)$ family (flavour) $SU(3)$ groups. It seems so far that it is hard to learn something from such, from the point of view of the *spin-charge-family* theory, very complicated models extending further the *standard model* assumption that the scalar (the Higgs) has charges in the fundamental representations. The so far very successful Higgs is in the *spin-charge-family* theory seen as an effective object, which can not very easily be extended to Yukawas.

Let me repeat that the *spin-charge-family* theory does not support the existence of the supersymmetric partners of the so far observed fermions and gauge bosons (assuming that there exist fermions with the charges in the adjoint representations and bosons with the charges in the fundamental representations). The supersymmetry does not show up at least up to the unification scale of all the charges.

Let me add that if the *spin-charge-family* theory offers the right explanation for the families of fermions and their quantum numbers as well as for the gauge and scalar dynamical fields, then the scalar dynamical fields represent new forces, as do already – in a hidden way – the Yukawas of the *standard model*.

Let me point out at the end that the *spin-charge-family* theory, offering explanation for the appearance of spin, charges and families of fermions, and for the appearance of gauge vector and scalar boson fields at low energy regime, still needs careful studies, numerical ones and also proofs, to demonstrate that/whether this is the right next step beyond the *standard model*.

10.7 Appendix: Short presentation of technique [10,18]

I make in this appendix a short review of the technique [18], initiated and developed by me when proposing the *spin-charge-family* theory [10,11,19,12–14] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anticommuting (Grassmann) coordinates [18], if the dimension of ordinary space is also d. There are two kinds of operators in the Grasmann space, fulfilling the Clifford algebra which anticommute with one another. The technique was further developed in the present shape together with H.B. Nielsen [18] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s. In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be orientates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s. The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The objects γ^a and $\tilde{\gamma}^a$ have properties (10.2),

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{10.49}$$

for any d, even or odd. I is the unit element in the Clifford algebra.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,,\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,,
\end{aligned} \tag{10.50}$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{10.51}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (10.49) and formally unitary, i.e. $\gamma^{a\,\dagger}\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(10.51) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(10.50) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4,\\
S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned} \tag{10.52}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,$$
$$\Gamma^{(d)} := (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,. \tag{10.53}$$

One can proceed equivalently for $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(10.51) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma,\ \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows (Eq. (10.9))

$$\stackrel{ab}{(k)}: = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,$$
$$\stackrel{+}{\circ}: = \frac{1}{2}(1+\Gamma)\,, \qquad \stackrel{-}{\bullet}:= \frac{1}{2}(1-\Gamma), \tag{10.54}$$

where $k^2 = \eta^{aa}\eta^{bb}$. One can easily check by taking into account the Clifford algebra relation (Eq.10.49) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.10.50) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$S^{ab}\ \stackrel{ab}{(k)}= \frac{1}{2}\, k\ \stackrel{ab}{(k)}\,, \qquad S^{ab}\ \stackrel{ab}{[k]}= \frac{1}{2}\, k\ \stackrel{ab}{[k]}\,,$$
$$\tilde{S}^{ab}\ \stackrel{ab}{(k)}= \frac{1}{2}\, k\ \stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab}\ \stackrel{ab}{[k]}= -\frac{1}{2}\, k\ \stackrel{ab}{[k]}\,, \tag{10.55}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize (Eq. 10.11,10.12) that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\gamma^a\ \stackrel{ab}{(k)}= \eta^{aa}\ \stackrel{ab}{[-k]},\ \gamma^b\ \stackrel{ab}{(k)}= -ik\ \stackrel{ab}{[-k]},\ \gamma^a\ \stackrel{ab}{[k]}=\stackrel{ab}{(-k)},\ \gamma^b\ \stackrel{ab}{[k]}= -ik\eta^{aa}\ \stackrel{ab}{(-k)}\,,$$
$$\tilde{\gamma^a}\ \stackrel{ab}{(k)}= -i\eta^{aa}\ \stackrel{ab}{[k]},\ \tilde{\gamma^b}\ \stackrel{ab}{(k)}= -k\ \stackrel{ab}{[k]},\ \tilde{\gamma^a}\ \stackrel{ab}{[k]}=\ i\ \stackrel{ab}{(k)},\ \tilde{\gamma^b}\ \stackrel{ab}{[k]}= -k\eta^{aa}\ \stackrel{ab}{(k)}\ . \tag{10.56}$$

From Eq.(10.56) it follows

$$S^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc}\ \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)}= \frac{i}{2}\eta^{aa}\eta^{cc}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,$$
$$S^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2}\ \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]}= -\frac{i}{2}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,$$
$$S^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\ \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]}= -\frac{i}{2}\eta^{aa}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,$$
$$S^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\ \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)}= \frac{i}{2}\eta^{cc}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]}\ . \tag{10.57}$$

From Eqs. (10.57) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$\begin{array}{llll}
\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, & \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\stackrel{ab}{[k]}\,, & \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\stackrel{ab}{[-k]}\,, & \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,, \\
\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, & \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, & \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, & \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,, \\
\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, & \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, & \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, & \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,, \\
\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, & \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, & \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, & \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\,.
\end{array} \tag{10.58}$$

We recognize in the first equation of the first row and the first equation of the second row the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$\stackrel{ab}{\tilde{(\pm i)}} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{\tilde{(\pm 1)}} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{10.59}$$

one recognizes that

$$\stackrel{ab}{\tilde{(k)}}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{\tilde{(-k)}}\stackrel{ab}{(k)} = -i\eta^{aa}\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{\tilde{(k)}}\stackrel{ab}{[k]} = i\stackrel{ab}{(k)}\,, \quad \stackrel{ab}{\tilde{(k)}}\stackrel{ab}{[-k]} = 0\,. \tag{10.60}$$

Recognizing that

$$\stackrel{ab}{(k)}^{\dagger} = \eta^{aa}\stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,, \tag{10.61}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned}
&< \stackrel{ab}{(k)}^{\dagger}\stackrel{ab}{(k)} >= 1\,, \\
&< \stackrel{ab}{[k]}^{\dagger}\stackrel{ab}{[k]} >= 1\,.
\end{aligned} \tag{10.62}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of d/2, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(10.52)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{array}{c}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\vdots \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{[-k_{d-1\;d-2}]}\,\psi_0 \\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\vdots
\end{array} \tag{10.63}$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (10.52)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (10.52)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra S^{ab} and $\tilde{S}^{ab}$

$$
\begin{aligned}
& S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}, \\
& \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}, \qquad (10.64)
\end{aligned}
$$

a left handed ($\Gamma^{(1,13)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$
\begin{aligned}
& \stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\; |\psi\rangle = \\
& \frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)|| \\
& (\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle \,. \qquad (10.65)
\end{aligned}
$$

This state is an eigen state of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (10.64)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (10.64)), generate families from the starting u_R quark, transforming u_R quark from Eq. (10.65) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$ from Eq. (10.65) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$
\tilde{S}^{01} \;\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\; \stackrel{03}{[+i]}\stackrel{12}{[+]} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} \;. \qquad (10.66)
$$

Below some useful relations [11] are presented

$$
\begin{aligned}
&N^{\pm}_{+} = N^{1}_{+} \pm i\, N^{2}_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\, N^{2}_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\ \stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\ \stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\ \stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\ \stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\ .
\end{aligned}
\tag{10.67}
$$

References

1. R. Jackiw, K. Johnson, Phys. Rev. D **8** (1973) 2386.
2. W. Weinberg, Phys. Rev D **19** (1979) 1277, S. Dimopoulos, L. Susskind, Nucl. Phys. **B 155** (1979) 237.
3. T. Appelquist, J. Terning, Phys. Rev. D (1994) 2116.
4. P.Q. Hung, Phys. Rev. Lett. **80** (1998) 3000, P.H. Frampton, P.Q. Hung, M. Sher, Phys. Rept. **330** (2000) 263.
5. S.J. Huber, C. A. Lee, Q. Shafi, Phys. Lett. **B 531**(2002) 112.
6. A.J. Buras, B. Duling, T. Feldmann, arxiv:1002.2126v3.
7. Yu.A. Simonov, arxiv:1004.2672v1.
8. T.A. Ryttov, R. Shrock, arxiv: 1004.2075.
9. Z. Kakushadze, S.H.H. Tye, arxiv: 9605221. Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, N
10. N.S. Mankoč Borštnik, Phys. Lett. **B 292** (1992) 25, J. Math. Phys. **34** (1993) 3731 Int. J. Theor. Phys. **40** 315 (2001), Modern Phys. Lett. **A 10** (1995) 587, *Proceedings of the* 13^{th} *Lomonosov conference on Elementary Physics Prticle physics in the EVE of LHC*, World Scientific, (2009) p. 371-378, hep-ph/0711.4681 p.94, arXiv:0912.4532 p.119. hep-ph/0711.4681, p. 94-113, http://viavca.in2p3.fr, arxiv:0812.0510[hep-ph],1-12, arXiv:0912.4532, p.119-135, http://arxiv.org/abs/1005.2288.
11. A. Borštnik, N.S. Mankoč Borštnik, hep-ph/0401043, hep-ph/0401055, hep-ph/0301029, Phys. Rev. **D 74** (2006) 073013, hep-ph/0512062.
12. N.S. Mankoč Borštnik, arxiv:101.5765v3, arxiv:1112.0889.
13. G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, hep-ph/0711.4681, p. 53-70, New J. Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0612250, p.25-50.
14. G. Bregar, N.S. Mankoč Borštnik, Phys. Rev. **D 80** (2009) 083534.
15. A. Hernández-Galeana, N.S. Mankoč Borštnik, arXiv:1012.0224, p. 166-1766. To appear in the Proceedings to the 14^{th} workshop What Comes Beyond the Standard Models, Bled, 11 - 21 of July, 2011.
16. D. Lukman, N.S. Mankoč Borštnik, H.B. Nielsen, http://arxiv.org/abs/1001.4679, New J. Phys. **13** (2011) 103027.
17. N.S. Mankoč Borštnik, H.B. Nielsen, Phys. Lett. **B 633** (2006) 771-775, hep-th/0311037, hep-th/0509101, Phys. Lett. **B 644** (2007)198-202, hep-th/0608006, Phys. Lett. **B 10** (2008)1016., D. Lukman, N.S. Mankoč Borštnik, H.B. Nielsen, Bled, July 19 - 31, 2004, hep-ph/0048751v4 (2010), New J. Phys. **13** (2011) 103027.
18. N.S. Mankoč Borštnik, J. of Math. Phys. **34** (1993) 3731, N.S. Mankoč Borštnik, H.B. Nielsen, J. of Math. Phys. **43** (2002) 5782, hep-th/0111257, J. of Math. Phys. **44** (2003) 4817, hep-th/0303224.

19. A. Borštnik Bračič, N. S. Mankoč Borštnik, *Proceedings to the Euroconference on Symmetries Beyond the Standard Model*, Portorož, July 12 - 17, 2003, p. 31-57, hep-ph/0401043, hep-ph/0401055.
20. V.A. Novikov, L.B. Okun, A.N. Rozanov, M.I. Vysotsky, arxiv:0111028, M. Maltoni, V.A. Novikov, L.B. Okun, A.N. Rozanov, and M.I. Vysotsky, Phys. Lett. **B 476** (2000) 107.
21. E. Witten, Nucl. Phys. **B 186** (1981) 412; Princeton Technical Rep. PRINT -83-1056, October 1983.
22. The authors of the works presented in *An introduction to Kaluza-Klein theories*, Ed. by H. C. Lee, World Scientific, Singapore 1983.
23. R.S. Chivukula and H. Georgi, Phys. Lett. **B** 188 (1987) 99.
24. G. D'Ambrosio, G. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B 645** (2002) 155–187, arXiv:hep-ph/0207036.
25. R. Alonso, M.B. Gavela, L. Merlo, S. Rigolin, arXiv: 1103.2915v1[hep-ph].
26. C.D. Froggatt, H.B. Nielsen, Nucl. Phys.P**B 147** (1979) 277.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 149)*
Bled, Slovenia, July 11-21, 2011

11 Seeking a Game in Which the Standard Model Group Shall Win

H. B. Nielsen[1]⋆ and D. L. Bennett[2]⋆⋆

[1]The Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø
[2]Brookes Institute for Advanced Studies, Bøgevej 6, 2900 Hellerup, Denmark

Abstract. It is attempted to construct a group-dependent quantity that could be used to single out the Standard Model *group* $S(U(2) \times U(3))$ as being the "winner" by this quantity being the biggest possible for just the Standard Model group. The suggested quantity is first of all based on the inverse quadratic Casimir for the fundamental or better smallest faithful representation in a notation in which the adjoint representation quadratic Casimir is normalized to unity. Then a further correction is added to help the wanted Standard Model group to win and the rule comes even to involve the abelian group $U(1)$ to be multiplied into the group to get this correction be allowed. The scheme is suggestively explained to have some physical interpretation(s). By some appropriate procedure for extending the group dependent quantity to groups that are not simple we find a way to make the Standard Model Group the absolute "winner". Thus we provide an indication for what could be the reason for the Standard Model Group having been chosen to be the realized one by Nature.

11.1 Introduction

It is one of the great questions asked in connection with our Bled Conference: Why Nature has selected just those gauge groups, which we find? Of course so far the only gauge group found is that of the Standard Model. Thus it is a priori this gauge group, which we should attempt to explain; then the theory, we might invent for that purpose, may or may not suggest further gauge groups as for instance the hierarchy of gauge groups suggested in the model of Norma Mankoc et al.[1]. One of us (H.B.N) and Rugh and Surlykke [3] estimated quantitatively the amount of information contained in the knowledge of the gauge group, and with N. Brene[2] we found that defining a quantitative concept of skewness - lack of automorphisms - appropriately we could declare the Standard Model Group to be characterized as essentially the most "skew".

In the present article we should go for inventing a somewhat different *group dependent quantity* than the "skewness" [2], and then imagine that Nature for some reason has selected just that group, which, say,*maximizes* this group dependent quantity. This means that we strictly speaking in a phenomenological way attempt to adjust the rules of a competition between groups and seek to adjust the rules, so as to make an already selected winner, the Standard Model Group, to win the game. It is a bit as a great dictator seeking to make, say, his son become the winner of a sport game by cleverly adjusting the rules of the game, so

⋆ e-mail: hbech@nbi.dk
⋆⋆ e-mail: dlbennett99@gmail.com

that he wins. In an analogously "nepotistic" way we shall seek to arrange the game so as to make the Standard Model group win the game.

It should be said though, that in inventing the game we also look at some physical model behind, much inspired indeed by our long ongoing project of Random Dynamics[5,7,6]. Strictly speaking there may be even a couple of routes suggesting, what groups are "best" based on the ideas of Random Dynamics, that the fundamental laws of Nature are extremely complicated. That is to say, if indeed the laws of nature were fundamentally in some way random and very complicated, what would then be the characteristic property -the strength so to speak - of the group combination that would appear as the gauge group effectively as seen by relative to say Planck scale physicists working at low energy?:

a) The one route is based on, that the gauge symmetry appears at first by accident *approximately*, but that then quantum fluctuations take over and cause the gauge group to appear effectively as an exact gauge group [6]. In such a philosophy the group with the best chance should be the group for which a gauge theory most easily can appear just by accident. Suggestively such a favoured group should be one for which, say a lattice gauge theory, could most easily turn out to appear with approximately gauge invariant action by accident [14,6]. This in turn is at least suggestively argued to occur for a gauge group, for which the range in the configuration space, over which the action has not to vary according to the rule of gauge symmetry is in some way - that may be hard to make precise though - so *small as possible*. If namely the range over which the variation of the action shall be small is "small", then there is the better chance to get it constant approximately there just by accident. This argumentation is then turned into saying, that the range of variation of the link variables caused by a gauge transformation say associated with a site in the lattice should be as "small" as possible in order to make the gauge group most likely to occur by accident. Now we typically imagine the lattice link variables to be or at least be represented as matrix elements of some representation of the gauge group. Well, at least we typically take the action contribution from one plaquette to be a trace of some representation of the gauge group. Normally we have the "intuitive" or conventional expectation that although the most general action contribution $S_{\square}$ should be a linear expansion on traces/characters for all the possible representations of the gauge group, the traces of the smallest representations would somehow dominate this expansion. Such an expected dominance of the small representation trace in the action means that the variation of the action as function of the physical combination of the link variables - i.e. the plaquette variables - vary relatively slowly over the group. But if we can get the action in this sense vary relatively slowly over the gauge group, it may mean that it also suggestively varies relatively slowly, when we vary the gauge. If somehow we have a "setting" - meaning say that everything is written basically in terms of matrices in some low representation - so that the variation of the action along the group is relatively slow, then very likely one would think that also the variation in this "setting" of any possible candidate for a term as a function of some gauge transformation would a priori be relatively slow. In other words we say that a good 'setting" for making the variation a priori along the gauge variation "small" is one in which the plaquette action is dominated by a "low" representation/character - meaning say low quadratic Casimir for it -.

It should be had in mind that the quadratic Casimir is crudely a measure for how much variation the representation matrix in the representation in question varies as function of the group element it represents. If then we imagine that in the lattice model, which supposed to be the fundamental model, the group is represented by a certain representation rather than directly as an abstract group element, the variation of the "fundamental" lattice model variables are in some sense - that may not be so clear though - more slowly varying as function of the group elements the smaller the quadratic Casimir for the representation functioning as the fundamental fields. But the slower this variation is the less extensive region is passed by the "fundamental" lattice variables and the easier it would therefore be that by accident under a gauge transformation the action that were at first not taken to be gauge invariant would be so by accident nevertheless. By this argumentation it is here argued in words that may really be meaningless that a *a small quadratic Casimir for the representation which is used by Nature as the "fundamental" lattice field degrees of freedom makes it more likely for the gauge group in question to occur by accident in an a priori random action theory*. The point should then crudely be that we should look among all groups and seek which ones have the representations with smallest quadratic Casimir for representations that must still be faithful in order to at all represent the group in question. The smaller these faithful representations that could be used the better should be the chance for the group to be the one realized in Nature.

b) The second route - which were the one, we started working on - involves several assumptions which we have worked on before, but it may become too much for this route being trustable unless we somehow can get the number of assumptions somewhat reduced. Most importantly we assume "Multiple Point Principle" [11] on which we have worked much and which states that there are several degenerate vacua. That is to say the coupling constants get - mysteriously ? - adjusted so as to make the theory discussed just sit on a phase transition, where several phases meet. The next assumption then is that after such an adjustment of the lattice action coefficients - which are basically the coupling constants being adjusted by the "Multiple point principle" - we look for that group which gives us the numerically biggest value in the vacuum realized (we argue it is the Coulomb phase one) of the plaquette action $S(U(\Box))$. For this latter assumption we may loosely say, well it means minimizing the energy density may be. Or we may involve the complex action model [9] and argue that a big contribution for the plaquette action may likely lead to a big contribution also numerically for the imaginary part of the action. Since now it is the main point of this complex action model to minimize the imaginary part of the action the best chance for a certain gauge theory to be realized should then be, if it can give the numerically biggest imaginary part. But assuming real and imaginary parts to depend in roughly the similar way on the variables this would then favour groups with that numerically large plaquette action. We shall go into this a bit complicated route to get a suggestive game for the groups in section 11.8.

The game proposed at the end in the present article is a somewhat new one, but actually one of the present authors (H.B.N.) and Niels Brene[2] long ago had a slightly different proposal for the game to be won by the Standard Model *group*, namely that it should be the *most skew* in certain sense, which we even made quantitative. Of course "skew" for a group means that it has relatively few automorphisms. Honestly speaking we did not yet publish what to do with the Abelian invariant subgroups so we strictly seeking took the competition between the groups having just one $U(1)$ invariant subgroup. The precise quantity for the game to minimize were then the number $\#Out(G)$ of outer automorphisms divided by the logarithm of the rank r of the group. In reality what comes to count a lot in this game about skewness turns out to be the division out of subgroups of the center, which is what distinguishes the various groups having the same Lie algebra. We shall see below that in order to finally adjust the game of the present article based in stead at first on the quadratic Casirmir for a faithful and small representation to really get the Standard Model group win is again to allow this "division out of a subgroup of the center". This means the distinction of the *group* (rather than the Lie algebra) is to give a lot of points in the game. So at the end we might be forced to let the game depend much on the property of the group rather than of the Lie algebra, and that may presumably be the main lesson that it is *the group property rather than the Lie algebra properties, that really matters to select the Standard Model*.

We shall therefore in the following section 11.2 review the seemingly so important distinction between group and Lie algebra, and call attention to that we even though we can claim that the phenomenology of Standard Models gives us not only the Lie algebra but also the Lie group, so that this distinction really has a phenomenological significance in fact in terms of the representations of quarks and leptons. In the following section 11.4 we shall then discuss that at least reasonable notation independent quantities have to be chosen for the competition, so that the game will not vary unreasonably by varying notations and normalizations. This suggests essentially to use the Dynkin index which is precisely being an important index, because it is somewhat sensible with respect to independence of notation as a start. Then in the next section 11.5 we shall present the group theoretical values of interest for our proposed game, i.e. the Dynkin indices [4]essentially and the corrections connected with the group rather than the Lie algebra for at first the simple groups. How to combine the simple groups by a kind of averaging may open up for a bit freedom and therefore nepotism to let the Standard Model Group win, but really there is not so much to do to help the Stand Model Group, it must essentially fight for itself. This discussion is put into section 11.6. The conclusive discussion of the game is put into section 11.7. The model behind of the somewhat more complicated nature involving "multiple point principle" is put in section 11.8. A by itself very interesting motivation for our a bit complicated multiple point principle route is, that it goes in connection with very old attempts of ours to fit the fine structure constants.

11.2 Phenomenological significance of Group rather than Lie Algebra

A priori one might say that it is only the gauge Lie algebra of the Yang Mills theory that matters, since the Yang Mills field theories are constructed alone from the knowledge of the Lie algebra of the gauge group. So from this point of view one can say that the Standard Model group (without now stressing the word group it means that we think of the Lie algebra) is $U(1) \times SU(2) \times SU(3)$. However, we can, and we shall in this article, assign a "phenomenological meaning" to the gauge *group* rather than just the Lie algebra by associating the choice of the *group* (among the several groups having the same Lie *algebra*) with the system of representations under which the various matter fields - the Fermions and the Higgs fields - transform. The reader should have in mind that while all the possible representations for quarks and leptons and the Higgs or thinkable additions to the Standard Model are allowed a priori, we may prevent some by requiring representation of a certain *group*. Indeed it is only some of the representations of the *Lie algebra* of the Standard Model, as we might denote the Lie algebra of $U(1) \times SU(2) \times SU(3)$, which are also representations of the various *groups* having the same Lie algebra, such as $U(1) \times SU(2)/Z_2 \times SU(3)/Z_3$, $S(U(2) \times U(3))$, $U(2) \times SU(3)$ etc. For example the group $SO(3) = SU(2)/Z_2$ has the same Lie algebra as $SU(2)$, but as is rather well known while $SU(2)$ has all the representations of the Lie algebra - it is indeed the *covering group* of say $SO(3)$ -both with half integer and integer (weak iso)spin, the group $SO(3) = SU(2)/Z_2$ has *only* as true representations the (weak iso)spin integer ones. Since the left handed quarks and leptons belong to the weak isospin =1/2 representation of $SU(2)$, which is not allowed as true representation of $SO(3)$, we can conclude that a group with the same Lie algebra as the Standard Model using $SO(3)$ instead of $SU(2)$ would be an example of a *group* that could *not* be used in the Standard Model. It would e.g. *not* be allowed to claim that $U(1) \times SO(3) \times SU(3)$ were the Standard

Model *group*, because it could not have the left handed quarks and leptons and the Higgs as representations.

So you see that there are many groups that are forbidden as Standard Model groups, but e.g. the covering group $\mathbf{R} \times SU(2) \times SU(3)$ for which all representations of the Lie algebra are also allowed representations of the (covering) group could at first not be prevented as "the group for the Standard Model".

However, it is our philosophy to impose a **phenomenological extra requirement to select the group**, which deserves to be called the *Standard Model Group* (SMG). The idea is to among the various groups with the Standard Model Lie algebra, which are allowed in the sense of having all the representations present in the Standard Model, we believe in, to select as the Standard Model Group to be that one (or several ?) which is *most informative* w.r.t. selecting, which representations are allowed, so that just knowing this group tells us as much as possible about, which representations occur in nature as presently known. With requirement of the *most informative* group about the representations in the Standard Model we should of course not accept the covering group $\mathbf{R} \times SU(2) \times SU3)$, which would give no information, provided we can at all find a group with the Standard Model Lie algebra which would exclude some representations (which of course should be some representations not found in nature so far). Such a more informative group giving correct information about representations found empirically is the group denoted $S(U(2) \times U(3)) = (U(1) \times SU(2) \times SU(3))/Z_6$. The symbol $U(2)$ in this symbol $S(U(2) \times U(3))$ alludes to it being constructed as a pair of a 2×2 unitary matrix (meaning one in the group $U(2)$) and the $U(3)$ symbol alludes to an 3×3 unitary matrix (i.e. one in $U(3)$) and then the extra condition being imposed by the S in front that the product means that the determinant of the two unitary matrices put into a 5×5 matrix shall be unity. Seen in this way it is rather obvious that the here proposed "Standard Model Group" $S(U(2) \times U(3))$ is a subgroup of $SU(5)$ as a group and not only as far as the Lie algebra is concerned. One can even say that some of victories of $SU(5)$ concerning the weak hypercharges of the particles in the Standard Model can be ascribed to the information gotten out of the from $SU(5)$ surviving sub*group* $S(U(2) \times U(3))$. The second way of denoting the same *group* $S(U(2) \times U(3))$ is $U(1) \times SU(2) \times SU(3)/Z_6$ and it describes it as first considering the *group* $U(1) \times SU(2) \times SU(3)$ and then divide out its center a certain subgroup isomorphic to the group of integers counted modulo 6, called here Z_6. This special subgroup is generated by the group element $(2\pi, -\mathbf{1}, \exp(2\pi/3)\mathbf{1})$ of $U(1) \times SU(2) \times SU(3)$ and the elements generated by it being divided out. This means that one divides out the invariant subgroup generated by this element $(2\pi, -\mathbf{1}, \exp(2\pi/3))$ so as to construct the corresponding factor group. We here counted the length around of the $U(1)$ as being $6 * 2\pi$, so that the sixth power of the generating element $(2\pi, -\mathbf{1}, \exp(2\pi/3)\mathbf{1})$ becomes the unit element in $U(1) \times SU(2) \times SU(3)$. One might also describe this group starting from the covering group $\mathbf{R} \times SU(2) \times SU(3)$ dividing out the subgroup generated by essentially the same element as we just used $(2\pi, -\mathbf{1}, \exp(2\pi/3)\mathbf{1})$.

It should be remarked that by this division out of group isomorphic to the integers modulo 6 we get the three invariant Lie *algebras* for respectively $U(1)$, $SU(2)$, and $SU(3)$ linked together. While the Lie group $U(1) \times SU(2) \times SU(3)$ is the cross product of three factors, the suggested phenomenological *group* for the Standard Model, or for nature we could almost say, $S(U(2) \times U(3)) = (U(1) \times SU(2) \times SU(3))/Z_6$ is *not* a cross product of any corresponding groups. This corresponds to that the rules for hypercharge quantisation which follows from the "phenomenologically"supported group $S(U(2) \times U(3))$ are such that the hypercharge values $y/2$ allowed by this *group* depends on the representations of the non-abelian Lie algebras $SU(2)$ and $SU(3)$.

It should be remarked immediately that this type of bringing an abelian group $U(1)$ together with non-abelian groups by division out of a discrete subgroup is a rather characteristic property of the Standard Model Group $S(U(2) \times U(3))$. That means then that it is "talent for the Standard Model Group", in the sense that among Lie groups with similar rank or similar dimension as this Standard Model Group" there are not many that can claim to divide out in the nontrivial way a bigger discrete group than this Z_6, which is divided out in the Standard Model Group case. So if we want to "help" the "Standard Model Group" $S(U(2) \times U(3))$ to win a game, we should it give many points to have such a "division out" with a relatively large group, so that $S(U(2) \times U(3))$ can win on its Z_6.

For example in the article by one of us and N. Brene and one of us [2] in which we claimed that having few automorphisms was what singled out the Standard Model Group $S(U(2) \times U(3))$ among other groups with the same number of abelian dimensions, it were in reality the division out of the discrete subgroup Z_6 causing a connection between SU(3) and U(1) that removed some separate automorphism acting on U(1) separately and one on SU(3) separately replacing it by only a common automorphism for them both that helped to make the Standard Model Group more skew so as to win the game for being "skewest".

11.3 Introductory guidance for what game to propose

One could imagine several directions for speculations giving ideas about what type of games among groups one should attempt in order to seek a game suitable for the Standard Model Group to win.

Some such inspiration ways of thinking could be:

- One idea would be that the Standard Model Group is the end or close to the end of a perhaps long series of group break downs - you could think of Normas theory in which it comes after several break downs of some $SO(N, 1)$ at higher energies- and thus one could almost in Darwinistic terms think about what would be the typical way for a group to break and under such a breaking, is there some property that gets enhanced by the breaking. By this we mean: Is there some property - expressed by number say - of the group surviving the break down that will typically or always be bigger than for the group that broke down to it. If we have such a quantity we would - if it is true that there are many breakings - expect it to be so big for the Standard Model Group that making a game for such a quantity would likely make the Standard Model win or at least get close to win. There are of course some quantities that do get say smaller each time the group breaks, namely the dimension or the rank. So in such a many breaking philosophy we would expect that the Standard Model Group would have - in some sense - very low dimension and very low rank, say. But it is difficult to say what to compare. At least we must admit that some groups have smaller rank and/or smaller dimension than the Standard Model Group, so these simple ideas were not quite so useful.
  One route though might be to require for instance that the gauge group we look for should have a system of Weyl fermions that are both mass protected and nevertheless leads to no anomalies in the gauge charges. Then one could even add (extra) assumptions about that the representations of the Weyl fermions be in some sense small or simple.
- Alternatively we could think somewhat in the direction of the landscape model (from string theory) [18] that there are many a priori possible vacua having different gauge groups. Then we need some extra speculation or assumption about which of these vacua then have the best chance of be the one in which we come to live, or which gets

realized at all. To selects such vacuum and thus the gauge group to be found, one might first think of the anthropic principle: then it would we should speculate about which gauge group would be the most favourable for humans.

One could also say we need a theory for initial conditions to tell us which vacuum should be selected to be produced in the beginning and then likely survive. Here the complex action model of one of us(H.B.N.) and Ninomiya could come in as a candidate to select a vacuum. In fact the main point of this complex action model ends up being that the initial conditions get settled in such a way as to minimize the imaginary part of the action S_I evaluated for the whole history of the Universe though both past and future. Since so enormously much of the universe is practically empty - i.e. vacuum - it is clear minimizing such an imaginary part of the action S_I will in very first approximation mean that *that* vacuum should be selected to exist through most of time and space, which has the smallest imaginary part of the Lagrangian density $\mathcal{L}_I$. Without knowing what the imaginary part of the Lagrangian density in the correct fundamental theory is it is of course somewhat difficult to guess how to get this imaginary part of the Lagrangian density minimized, but we could attempt the following loose argument: Suppose that the imaginary part of the Lagrangian density has a very similar form as the real part, just with different coefficients. Then we would guess that to find the minimal imaginary Lagrangian density we might instead seek an extremal real part, and then hope that this is where most likely the extreme imaginary part will also be. Such a search for an extremal real part of the Lagrange density might by itself be supported by other arguments without using complex action model. In fact we could speculate that somehow the most stable vacuum were one with smallest energy density. Ignoring or approximating away the kinetic part of the energy density extremizing the energy density would lead to extremizing the Lagrangian density among the possible vacua.

Such a search for a numerically largest plaquette action - if one thinks i a lattice gauge theory model terms - could thus be an idea that could be supported by several speculations; either our selection by complex action model or by some minimization of energy or Lagrangian density.

But the Lagrangian density in a certain vacuum of course depends on the coupling constants or equivalently on the coefficients to the various terms that may occur in say a lattice gauge theory. Therefore such a minimization of the plaquette action among the different vacua requires that we have in addition a method for calculating these coefficients or coupling constants for all the different theories with their different gauge groups, which were what were to compete. Now at this point we propose our determination of the coupling constant by means of our principle of multiple point principle (MPP)[11]. This principle MPP means that the vacuum sits at a phase transition point as function of these coupling constants. But now it sounds, that we have really put too many unreliable assumptions on top of each other so that the chance of the all being true gets very low: existence of an imaginary action, vacuum being selected to have it minimal, the imaginary action of the vacuum being minimal just when real part is extremal too, the multiple point principle of couplings being chosen to just sit on the phase borders(at some multiple point, where several phases meet). And then to make use of this long series of assumptions we have to make the approximations to be used to estimate the size of the plaquette action under the MPP etc, assumptions. It is actually this series of ideas that were the point of the route of section 11.8. But probably it can only be excused by saying, that doing such a series of speculations we have at least an attempt to a connected picture and should have a better chance of stumbling on to a correct proposal for the game, that is characterizing the Standard Model Group, because we should not make any totally stupid and wrong things, if we are in some at least thinkable scheme.

- One very attractive way to proceed would be a genuine Random Dynamics[5,7,6] . In principle we might imagine a quantum field theory, which instead of being assumed translational invariant is assumed to have a quenched random (glassy) Lagrangian density[14] or action for the unit cell if we think of the model as regularized to let us say a lattice type theory. We may even take the number of degrees of freedom to vary in a quenched random way from cell to cell in the lattice. So we take it, that there is connected to each 4-cube in a lattice at random - quenched randomly - chosen a number of degrees of freedom. Next also in the quenched random way an action contribution expression is chosen, and that expression delivers then the action contribution from the cell considered, and it depends of course from assumed locality only on the degrees of freedom of that cell and the neighboring few cells. Such a model on a lattice and with locality and background geometry put in but otherwise with quenched random action and number of degrees of freedom could be considered a Random Dynamics model[5,7,14] .

  According to our old idea [6] there can in say a lattice model occur effectively gauge invariance without it being put in to the extend that a photon without mass can appear in a model with no exact gauge invariance. Let us though mention that this phenomenon of a gauge symmetry appearing by itself, as one might say, comes about that at a stage we write the theory as having a formal gauge symmetry looking at first as if it were Higgsed. Then it is the Higgs degrees of freedom in the formally gauge invariant description, that by quantum fluctuations wash out, so as to become ordinary massive particles or just an unimportant field not accessible at low energy. The idea should now be that the quenched random theory proposed here as a manifestation of the Random Dynamics project would in a way similar to the one described in [6] be rewritable into a theory with some formal gauge invariance, which then due to quantum fluctuations could appear at the end not Higgsed (although it looked at first Higgsed). Thus some gauge symmetries would come out as observable at long distances, giving rise to say massless photons gluons etc..

  Thinking in terms of such a quenched random theory producing effective although at first formal gauge symmetries it becomes in principle a matter of a may be hard - but presumably doable - computer calculation to find out which gauge groups occur and how often in this "by itself way"[6], provided though that we put in the definite rules for the quenched random distribution of the action and the number of degrees of freedom per cell. However, it might very likely turn out that this specific choice of a quenched random distribution of the degrees of freedom numbers per cell and the action per cell will be of little significance as to how the model will show up at long distances, and what gauge groups will appear.

  Such an insensitivity to the details of the quenched random probability set up may though be just the wishful thinking of Random Dynamics, that at the end it is features of the theory determined by looking only at long distances (or in other regimes, where the "poor" physicist can get access), that determine the effective laws of nature which we see. In any case it would be a very important project to by computer or just theoretically find out which gauge groups preferentially would come out by themselves from such quenched random lattice theory with even a quenched random number of degrees of freedom (varying from cell to cell). In the spirit of the present article the idea of course would be a bit speculatively to figure out what properties of a group would make it likeliest that just that group in question would appear by itself.

  How now to get an idea of which groups would most likely come out of such a quenched random theory ? Well, in order that one can get the formally exact gauge symmetry to appear effectively so as to deliver massless gauge bosons effectively it is needed, that at the starting level the gauge symmetry is there approximately, because it is the rudiment of the fundamentally not present gauge symmetry being broken, that leads to the "Higgs" or Higgs like effects breaking at first seemingly spontaneously the global gauge symmetry, that has to be small enough for being destructed by quantum fluctuations. In other words we only can get sufficient quantum fluctuations to bring the formal gauge symmetry which we might invent to become physically effective at long distances provided the original gauge breaking were small enough to be beaten by the quantum fluctuations. So we are in fact asking for which gauge groups are likely to occur by quenched random accident in small regions of the lattice theory as *approximate* gauge symmetries. Now let us think of seeking such a locally accidentally approximate gauge symmetry by starting to look for it say near some starting point in the configuration space of the theory locally and then estimate the chance, that going further and further away from this point the action will by accident not change more than some limit corresponding to the limit for getting it finally appear as a long distance gauge symmetry. Now a gauge transformation in a lattice theory is to be thought about as if we locally have the possibility of transforming the configuration by means of any (gauge) group element. So we now ask for how to get the best chance for that we acting with any element in the group corresponding to approximate realization of the gauge symmetry of the action at a certain site. When we here talk about a site, it is just meant that in many places one can presumably find some way of transforming the even random number locally of degrees of freedom in a neighborhood so as to approximately (but approximately only) not change the action (contribution from that region). To begin with in asking for approximate symmetry of the action at first when the gauge group elements of the transformation are near to the unit element it is mainly the Lie algebra that must be relevant. The chance for having by accident the same action as one goes further and further away by transforming with elements which lie longer and longer away from the unit element gets of course smaller and smaller the longer away we go to ask for this accidental symmetry. So it makes it most likely to find an accidental symmetry for a given group, when the action of the group changes the variables in the quenched random theory as little as possible. In the extreme case, when the variables of the quenched random theory were not transformed at all the invariance of the action would of course be guaranteed, but that would be a trivial case, that would of course at the end not lead to any effective gauge theory at long distances. So we must ask for a slow variation, but there should be some variation. To make it easy - or at least for start - we shall think of the degrees of freedom among the quenched random ones being roughly representation matrix elements. That is to say we may think of that there are among the quenched random number of degrees of freedom locally some we may think of as matrix elements and of the proposed transformation law as a linear representation of the group. In this way we allow ourselves to think of the speed with which the configuration moves when varying the group element in the (local) gauge transformation as motion speed for a representation matrix. This latter speed is proportional to the square root of the quadratic Casimir c_R for the representation in question R. So we see that the chance of getting an approximate symmetry under such a one point local gauge transformation is biggest, if the representation to which we relate it has the smallest quadratic Casimir, because then so to speak the speed of moving of the configuration - approximated by the matrix elements of the representation R - when we move the group element is the smallest. Since we thought of starting at around the unit group element and got the normalization for the speed to consider specified by the Lie algebra, we would naturally count the quadratic Casimir c_R normalized by setting the quadratic Casimir for the adjoint representation, which is the representation on the Lie algebra itself, equal to unity.

In this way we get from the Random Dynamics picture of looking for approximate gauge symmetry by accident the suggestion of selecting the game to be:

Which group has the smallest quadratic Casimir for its smallest faithful representation in a notation normalize to let the adjoint representation quadratic Casimir be normalized to be 1.

Typically of course it will be in the local cases, wherein the representation matrix that we can use as an approximation to the local variables is one with the smallest quadratic casimir that will be most important for finding approximate gauge symmetry by accident, because it is these cases that have the biggest chance. It is therefore we in practice must think of the relevant representation R as being the one with the smallest quadratic casimir. The representation with smallest quadratic casimir is practically the same as the "fundamental" representation. Thus we arrive essentially to that the game to win for being the most likely group to appear approximately by accident is the one which has the smallest fundamental representation quadratic casimir c_F. The ratio of the two quadratic Casimirs, the fundamental and the adjoint, is actually such an interesting quantity group theoretically that it got essentially - i.e. apart from some dimension of representation factors - the name Dynkin-index.

In the spirit of the just above it is clear that if we could somehow "divide out" part of the center this would make the group smaller(in volume) and thus easier to get realized as approximately a good symmetry (i.e. an approximate symmetry of the action) by accident. We should therefore let such a division out of the center count extra, enhancing the success of the group to win the bigger the subgroup divided out. As is explained a bit more in the following section 11.4 it is suggested that we should improve our quantity to be minimized to $c_F/(\#\text{center divided out})^{2/d}$. We can namely crudely consider c_F as proportional to the $2/d$th power of the volume of the group in the sense, that since c_F is a quadratic form in the "distance" in the group the volume of a d-dimensional group gets by varying this c_R from representation to representation its volume changed proportional to the dth power of the square root of c_R. If one therefore change the volume by some other effect effectively, namely by dividing out a subgroup of the center having #centerpart divided out elements - which will of course diminish the volume by a factor $\frac{1}{\#\text{center divided out}}$, this would correspond to replacing c_F by an effective quadratic Casimir $c_F/\#\text{centerpart divided out}$

11.4 Requirements of correct behavior under group volume scalings

It is important to fix the precise quantity to be proposed as the one that the group winning should say maximize so that this quantity shall not be notation dependent but as stable under change of conventions as possible. It is therefore we had to take the ratio of two relatively easy to select representations. If we had namely not taken a ratio this way the quadratic Casimirs would depend on the notation for normalizing quadratic Casimirs.

For giving a possible good physical sense to this ratio it is immediately obvious that a meaning of the type that this ratio denotes the square of the speed of motion of the group element in the two different representations discussed is called for. If now the true physical quantity to be argued for were indeed rather a total volume ratio we can see that a volume correction for say the "fundamental" representation would have to come in just the right power to combine in a physically consistent way with the speed ratio already being present in the proposed $1/c_F$. This considerations leads rather quickly to that our first proposal $1/c_f$ can only be corrected by a division out of a center subgroup of order $\#center$ by the factor $(\#center)^{2/d}$, where d is the dimension of the group.

That is to say that the quantity to be say maximized would in order to combine the volume dependence correctly

$$(\#center(dividedout))^{2/d}/c_F. \tag{11.1}$$

The to be minimized quantity could then be of course the inverse of this

$$c_F/(\#\text{center divided out})^{2/d}.$$

11.5 What Scores do Different (Simple) Groups get?

Before we in the next subsection 11.5.2 shall tell about how one extracts from the literature the values for the quantity $1/c_F$, we may put forward some features of how the competition goes by mentioning a few remarks:

- **Large rank behavior** As is well known the simple Lie algebras are classified into four infinite series and further some "exceptional" Lie algebras. For the infinite series it actually turns out that if we allow the smallest quadratic Casimir representation F to be the one making c_F smallest we get for the algebras for "large N" - meaning the late algebras in the infinite chain - that

$$\frac{1}{c_F} \to 2 \text{ for the rank } r \to \infty. \tag{11.2}$$

  This is a very important property for our project because you could add a formally ∞ to the rank r region and the function of the algebra $1/c_F$ would remain a continuous function and that now on a compactified space of algebras. These means that there should exist one (or perhaps several) largest value for $1/c_F$. So we can really expect to find a presumably single winner among the simple algebras - or we might have got an infinite limit, but that luckily does not happen -.
- **The front field** The winner number one among the simple Lie algebras turns out to be $SU(2) = A_1$, since it gets using the general formula for A_1

$$\frac{1}{c_F} = \frac{2}{1-\frac{1}{N^2}} \text{ for } SU(N) = A_{N-1}, \tag{11.3}$$

  that

$$\frac{1}{c_F(SU(2))} = \frac{2}{1-1/4} = \frac{8}{3}. \tag{11.4}$$

  This $8/3$ is the absolutely record for any simple Lie algebra, and so $SU(2) = A_1$ is the "gold medal winner" among simple Lie groups.
  If we use the correction factor $(\frac{1}{\#\text{center-elements divided out}})^{\frac{2}{d}}$, which we mentioned above it happens that it is also bigger for $SU(2) = A_1$ than for any other simple Lie algebra. In fact it is for $SU(2)$ equal to $2^{2/3}$ = 1.587401052, so that the full score with this factor included becomes for the gold winner $SU(2)$ equal to $\frac{8}{3} * 1.587401052 =$ 4.233069472 So the winner just even more certainly becomes $SU(2)$.
  It is of course comforting for our model that this absolute winner among the simple Lie algebras is at least one of the invariant subliealgebras of the Standard Model Lie algebra $U(1) \times SU(2) \times SU(3)$.

But now comes for our scheme a problem: The silver winner among the simple Lie algebras using only the ratio of the quadratic Casimirs $\frac{1}{c_F}$ is *not* as we might hope for the Standard Model algebras to win the $A_2 = SU(3)$ algebra, but rather $SU(3)$ is beaten by $SO(5) = B_2 \approx Sp(4) = C_2$ which obtain the score

$$\frac{1}{c_F(C_2)} = 12/5 \tag{11.5}$$

obtained from the general formula $\frac{1}{c_F(B_r)} = \frac{4(r+1)}{2r+1} = \frac{2N+4}{N+1}$ where $N = 2r$ by putting $r = 2$ or equivalently $N = 4$. The $SO(5) = B_2$ Lie algebra is isomorphic to the symplectic one C_2; to get the four dimensional representation, which is the vector representation V for the symplectic C_2, we must for $SO(5) = B_2$ use the spinor representation.
Now the for our hoped for explanation of the Standard Model a bit unfortunate fact is that the $SU(3) = A_2$ algebra only reach the score $\frac{1}{c_F(A_2)} = \frac{2}{1-1/3} = \frac{9}{4} < \frac{12}{5}$. So in the pure use of $1/c_F$ the phenomenologically relevant $SU(3) = A_2$ lost and only obtained the bronze medal. Of course it is still promising that it got a medal at all, but we could have said that we got the two genuine simple Lie groups if the winning gold and silver to be the two phenomenologically found ones. But alas, it were not like that completely!
But now we have already mentioned the idea of the extra factor $(\#center)^{\frac{2}{d}}$, where d is the dimension of the algebra.
For $SU(3)$ and $SO(5) \approx Sp(4)$ the extra factor turns out:

$$\text{For } SU(3) = A_2 : (\#center)^{2/d} = 3^{1/4} = 1.316074013 \tag{11.6}$$

$$\text{For } Sp(4) = SO(5) = B_2 = C_2 : (\#center)^{2/d} = 2^{1/5} = 1.148698355. \tag{11.7}$$

Thus we get for the full scores when this factor is included:

$$\text{For } SU(3) = A_2 : \frac{(\#center)^{2/d}}{c_F} = 3^{1/4} * 9/4$$
$$= 1.316074013 * 9/4 = 2.961166529 \tag{11.8}$$

$$\text{For } Sp(4) = C_2 = SO(5) = B_2 : \frac{(\#center)^{2/d}}{c_F} = 2^{1/5} * 12/5$$
$$= 1.148698355 * 12/5 = 2.756876052. \tag{11.9}$$

So we see that the extra factor from dividing out the center just barely brought the $SU(3)$ algebra in front of $SO(5) \approx Sp(4)$ by .20 out of ca 2.9 meaning by 7%.
This looks extremely promising for the Standard Model indeed doing very well in the game provided we include "dividing out the center" factor $(\#center)^{2/d}$. The only two genuine simple Lie algebras in the Standard Model then come out with respectively gold and silver medals, $SU(2)$ with gold, $SU(3)$ with silver.

- **The problem of** $U(1)$ With the $U(1)$ there are several problems, which we must discuss:
  - 1. Since the adjoint representation should be considered either as non existing or as trivial we must consider the quadratic Casimir for the the Abelian $U(1)$ as either $C_A(U(1)) = 0$ or at best for our hopes for favouring the Standard Model ill-defined.
    Actually there is a possibility for making some sense of the ratio C_A/C_F if we could somehow arbitrarily select one representation of $U(1)$ given by some "charge" q_A to be considered formally the "adjoint" A and then another one with another "charge" q_F to be the F-representation. Then one would naturally say that the Casimir is the square of the "charge" so that $C_A = q_A$ and $C_F = q_F$. In this case of course our competition quantity $\frac{1}{c_F} = \frac{C_A}{C_F} = \frac{q_A}{q_F}$. But what shall be considered A and what F ?

– 2. The idea that one could "divide out the center" of one of the genuine simple Lie groups such as $SU(2)$ or $SU(3)$ were meant to mean that after having divided it out we got instead the groups $SU(2)/Z_24$ and $SU(3)/Z_3$ instead. But then we should only be allowed to use as F the representations that are representations of these *groups*. But then the representations F which we used in the construction of our $1/c_F$'s above are *not allowed*. That in turn would mean that we would have instead of the F we used in the cases mentioned and actually typically to use rather the adjoint representation itself, so as to get for the competing quantity $1/c_F$ now replaced by $1/c_A = C_A/C_A = 1$. If we do that we loose a factor bigger than 2 for the algebras in the strong field. That is not compensated by the extra factor $(\#center)^{2/d}$ and if this extra factor is only achievable by paying the price that only the adjoint representation get allowed to be used as F, then it is better for winning the game to give up the extra factor.
– 3. If the total group - the cross product say of several simple factors - has a $U(1)$-factor in it, one can divide out a subgroup of the center that could be e.g. Z_2, if we have $SU(2)$ and Z_3 if we have $SU(3)$ in such a way that this divided out subgroup of the center is not subgroup neither of the genuine simple Lie group nor of the $U(1)$ separately. If one divides say a Z_3 or a Z_2 out in such a way, then it does not prevent that there can be a representation which with respect to $SU(2)$ or $SU(3)$ corresponds to the F we used in our above calculation and which managed to make these simple algebras win the game.
In this way we can claim that we have a way - by means of using a $U(1)$ - to both get the favourable F representation used to let our favourites win, and at the *same time* get "division out of the center" take place.
This situation seems so favorable and really needed to get win for a simple Lie algebra by the help of the extra factor, that unless it requires a very high price in form of some loss in the final score, it seems to be very needed to include a $U(1)$-factor in the total group.
So here we have essentially argued that unless the rules for the Abelian $U(1)$ get adjusted in detail to be very unfavourable for winning then because of the otherwise impossible combination of the extra factor and the representation F, it becomes needed to have a $U(1)$ included in the total group.

11.5.1 Standard Model Group very promising, crude review

Let us here argue how one with very little (extra) assumptions about the averaging, when having a team of Lie algebras, is to be taken, can argue for the Standard Model group being the winner among teams of Lie algebras:

We must of course have some rule for making a score for a group that is not simple fro the score numbers for those simple invariant subgroups of the group. One can imagine several weightings such as e.g. weighting the individual simple group scores by the dimensions of the simple groups. But of course our derivation that the Standard Model wins the game would be most convincing if it could be done with so mild assumptions as possible concerning these rules of combining. Otherwise we could be accused for having adjusted the rule of weighting so as to favor the Standard Model (if we do not succeed in arguing that it does not matter much what rule we use, then of course we shall assume some rule that favours the Standard Model, so as to see if it is at least possible to make the Standard Model win in such a way.)

If we cannot get the "extra factor from dividing out center(subgroup)" $(\#center)^{2/d}$, the largest achievable score for any simple Lie algebra and therefore also for any (sensible average, which of course ca never be bigger than the quantities from which it is averaged) average over a "team" (a non-simple group) becomes the 8/3 which is the biggest achievable value for $1/c_F$, being reached for the simple algebra $SU(2)$. To reach a score higher than these $8/3 = 2.6667$ we have to obtain the extra factor $(\#\mathtt{center})^{2/d}$, but for that we *need* to have a $U(1)$. So we suggestively should have a $U(1)$ combined with an $SU(2)$ and then have a center Z_2 divided out in a way that is not a subgroup of neither $SU(2)$ nor $U(1)$. But that means we have now suggestively reached $U(2)$. It could now seemingly be possible that what would win would be just one or a cross product of several $U(2)$'s. We can namely with a sensible averaging not get a different score for a group and this group crossed with itself a number of times. But now we already argued that we needed the $U(1)$. So we ask, could we not apply this same $U(1)$ several times instead of just to help $SU(2)$ to get a high score? Actually we may use it again, but we cannot use it to help another algebra, which has again a Z_2 center to get divided out. The problem is that if we attempt that, we shall miss the allowance to use the representation F we used for both the $SU(2)$ and the other algebra, that also has the center being Z_2. That would mean that the score for either $SU(2)$ or the other Lie algebra would miss more than a factor 2 in the score. If, however, we can divide out a center which is a Z_n with an odd n so that it has no common factor with the 2 in Z_2, there will be no such problem. So e.g. a Z_3 would be o.k.. Such an extension with a Lie algebra that had a group from which we could divide out e.g. Z_3 could be added without the need for any further $U(1)$. From what we already saw about the individual scores for the simple Lie algebras or rather groups the silver medal winner were already $SU(3)$. So now we must ask, if it so to speak would pay in terms of getting the best score average for the full group if we to our first suggestion $U(2)$ add/extend with the $SU(3)$. Because of the ambiguity coming from that we do not clearly have settled how to count the $U(1)$ we do not know, if the addition of the proposed $SU(3)$ will pay. It is namely so: If the averaging of $SU(2)$ with the $U(1)$ has brought this average from the $8/3 * 2^{2/3} = 4.233069472$ below the score-value of $SU(3)$ being $9/4 * 3^{1/4} = 2.961166529$, then it will pay to include the $SU(3)$. That might happen, but we must admit that it dependence on the exact averaging rule, as well as on what one puts the score for $U(1)$ in itself. So honestly we only got to, that it is possible to imagine an averaging procedure, that would make the Standard Model win!

11.5.2 Extraction of the $1/c_F$

In [17] we find for the quadratic Casimir C_A

$$C_A = \eta g, \tag{11.10}$$

where g is the dual Coxeter number, while η is a notation-dependent normalization constant, which is defined via the formula

$$C_R = \frac{\eta}{2} \sum_{i=1}^{r} \sum_{j=1}^{r} (a_i + 2) G_{ij} a_j \tag{11.11}$$

for the quadratic Casimir in the representation R. Here again the quantities a_i for $i = 1, 2, ..., r$ are the Dynkin labels for this representation R. Finally the r is the rank of the group, and G_{ij} is the inverse of the Cartan matrix.

Using still [17] the "second index" $I_2(V)$ for the "vector" representation V given as

$$I_2(V) = \frac{\eta}{2} \text{ for SU(N) and Sp(N)}, \tag{11.12}$$

$$I_2(V) = \eta \text{ for SO(N)}. \tag{11.13}$$

and the relation

$$I_2(R) = \frac{N_R}{N_A} C_R, \tag{11.14}$$

where N_R is the dimension of the general representation R while N_A is that of the adjoint representation A, we get

$$\frac{1}{c_F} = \frac{C_A}{C_F} = \frac{C_A}{C_V} == \frac{2N_V g}{N_A} \text{ for SU(N) and Sp(N)}, \tag{11.15}$$

$$\frac{1}{c_F} = \frac{C_A}{C_F} = \frac{C_A}{C_V} = \frac{N_V g}{N_A} \text{ for SO(N)}. \tag{11.16}$$

(We here took it that the "smallest" representation F were indeed the "vector" representation V, which is not always the case) Herein we shall then insert the dual Coxeter numbers g, which are

$$g_{A_r} = r+1 = N \text{ for } A_r = SU(N) \text{ where } r = N-1, \tag{11.17}$$

$$g_{B_r} = 2r-1 = N-2 \text{ for } B_r = SO(N) \text{ for N odd and } r = \frac{N-1}{2}, \tag{11.18}$$

$$g_{C_r} = r+1 \text{ for symplectic groups } C_r \tag{11.19}$$

$$g_{D_r} = 2r-2 = N-2 \text{ for N even and } D_r = SO(N) \text{ where } r = N/2, \tag{11.20}$$

$$g_{G_2} = 4 \text{ for} G_2, \tag{11.21}$$

$$g_{F_4} = 9 \text{ for } F_4, \tag{11.22}$$

$$g_{E_6} = 12 \text{ for } E_6, \tag{11.23}$$

$$g_{E_7} = 18 \text{ for } E_7, \tag{11.24}$$

$$g_{E_8} = 30 \text{ for } E_8. \tag{11.25}$$

and then we obtain e.g.

$$\text{For } A_r = SU(r+1) : \frac{C_A}{C_V} = \frac{2g_{A_r} N_V}{N_A} = \frac{2(r+1)(r+1)}{(r+1)-1} = \frac{2}{1-1/(r+1)} = \frac{2}{1-1/N} \tag{11.26}$$

$$\text{For } B_r = SO(2r+1) : \frac{C_A}{C_V} = \frac{g_{B_r} N_V}{N_A} = \frac{(2r-1)(2r+1)}{r(2r+1)} = 2 - 1/r, \tag{11.27}$$

$$\text{For } C_r = Sp(2r) : \frac{C_A}{C_V} = \frac{2g_{C_r} N_V}{N_A} = \frac{2(r+1)2r}{r(2r+1)} = \frac{4(r+1)}{2r+1} = \frac{2N+4}{N+1} \tag{11.28}$$

$$\text{For } D_r = SO(2r) : \frac{C_A}{C_V} = \frac{g_{D_r} N_V}{N_A} = \frac{(2r+1)2r}{r(2r-1)} = \frac{4r+2}{2r-1} = \frac{2N+2}{N-1} \tag{11.29}$$

But now we must admit that those "vector" representations V which we here used are not in all cases the smallest neither as concerns the quadratic Casimir nor w.r.t. dimensions. This is the case for the relatively low rank r $SO(N)$ groups. They have namely spinor representations. In fact we have for an even N that $SO(N) = SO(2r) = D_r$ have a spinor representation - we shall have the chiral irreducible representation - of dimension $2^{r-1} = 2^{N/2-1}$; for odd-N we have $SO(N) = SO(2r+1) = B_r$ a spinor representation of dimension $2^r = 2^{(N-1)/2}$.

We may read this problem off in the list of what [17] propose as "reference representations". Here the odd-N SO(N) algebras B_1, B_2, B_3, and B_4 are proposed represented as reference representations by their spinor representations, while it is for $r \geq 5$ the B_r have as their reference representations the vector representations V. Similarly it is proposed to use as "reference representations" for the even SO(N)-algebras in the case D_3 meaning $SO(6)$, while for $r \geq 4$ we use the "vector" representation.

11.6 How to Combine Scores to Scores for Non-Simple Groups ?

When we combine simple gauge groups into semisimple group we have to postulate some rule for combining and in some way averaging our quantities for the various simple groups. We might think of more complicated rules but in the light of the "theory behind" the favoring of the groups

11.6.1 The $U(1)$ problem, what to take for its C_A/C_F

Since the Lie algebra of $U(1)$ has a trivial adjoint representation it has really no meaning to talk about C_A for $U(1)$, or we might say it is zero, but a zero is not so useful for our normalization.

We could propose instead to replace the adjoint representation by the "unit charge" representation of the $U(1)$ and use that as a normalization representation. Now we should ask as we did for the other groups: can we find a "smaller" representation ? That should now be one with a smaller charge, but such a smaller charge would only be allowed if we used a bigger version of the $U(1)$ circle. So keeping the group unchanged there are no smaller charges allowed. Looked upon this way we can say that the $U(1)$ is analogous to the E_8 algebra for which there is no smaller representation than the adjoint one. Therefore we get for E_8 that $C_A/C_F = 1$. Therefore we should by analogy also take 1 for the abelian group $U(1)$. Then it may not matter so much whether we really have and use an adjoint representation.

11.6.2 Volume product weighting, a proposal

One way to combine into some average the scores of the different simple groups going into not simple Lie group is suggested by having in mind that

- a: We thought of the chance of getting symmetry by accident crudely being a good symmetry for some a priori "random" action suggesting that it is the volume of possible set of field configurations in which the group transformation brings a state around by transformation under the group that counts. (This volume should be minimized to make the chance for having the accidental symmetry by accident maximized.)
- b: We should attempt to count in such a way that just putting some repetition of the group as a cross product should not change the chances; rather it should be the type and structure of the group occurring that we should get information about.
- c: When we have say a cross product of groups the image in the configuration space should also have the character of being a product, so that the volume of the combined group representation would become a product of the volumes of the components.
- d: The quantity, which we used $\frac{C_A}{C_F} * (\#center)^{2/d}$ were - by the accident of our notation - as going inversely as the $2/d$th power of the volume in the configuration space relative to some more crudely chosen group volume. (indeed we selected this group volume by means of the commutation rules so as make it given by the quadratic Casimir for the adjoint representation.)

The way suggested by this thinking is that we should use logarithms of our numbers used for scores and weight them by the *dimensions* of the groups. That is to say we propose the quantity:

$$T = \frac{\sum_S d_S \ln(\text{“}\frac{C_A}{C_F}\text{''}|_S * (\#center)^{2/d}|_S)}{\sum_S d_S} = \frac{1}{\sum_S d_S} \ln \prod_S \text{“}\frac{C_A}{C_F}\text{''}|_S^{d_S} (\#center_S)^2. \quad (11.30)$$

Here S symbolizes the various simple Lie algebras going into the non-simple group, we consider. So e.g. in the case of the Standard Model Group $S(U(2) \times U(3))$ this S runs over the three Lie algebras,$S = U(1), SU(2),$ and $SU(3)$.

Having already found above the scores for $U(1)$ being in our way counted as 1 meaning a 0 when we take the logarithm (this were somewhat not quite clean, but the most reasonable), for $SU(2)$ the seemingly everyone beating $\frac{8}{3} * 2^{2/3} = 4.233069472$, and for the $SU(3)$ score $\frac{9}{4} * 3^{2/8} = 2.961166529$, we may as an example evaluate the by Nature beloved Standard Model Group:

$$T_{SMG=S(U(2)\times U(3))} = \frac{1*0+3*\ln 4.233069472+8*\ln 2.96116652}{1+3+8}$$
$$= 0 + 0.360731843 + 0.723722192 = 1.084454035. \quad (11.31)$$

This means that the averaged score for the Standard Model group should be counted as having this averaged quantity as its logarithm, so that it becomes itself:

$$\exp T_{SMG=S(U(2)\times U(3))} = 2.95782451. \quad (11.32)$$

This score by the Standard Model shall be compared to other obviously competing candidates such as $U(2)$. We should remember that without the company of the $U(1)$ the $SU(2)$ is not allowed to gain its $2^{2/3}$-factor, so without this $U(1)$ it would not even have 8/3 = 2.666666667(because we could not use the representation F being the spin 1/2) and could not compete. With the inclusion of $SU(2)$ having to carry along the $U(1)$ - with only its 1 score - we get this 4.233069472 formally for $SU(2)$ cut down to its 3/4th power, meaning 2.951151786 for $U(2)$. It is really a very tight game but it is the Standard Model that wins over even the $U(2)$! That it must be like that is also signaled by that fact, that the number for $SU(3)$ when the center-factor is counted is 2.96116652 and brings the average for the Standard model group up. This makes us look for if $U(3)$ could now beat the Standard Model Group? Well $U(3)$ would score the 8/9th power of these 2.96116652 giving 2.624690339, which is less than the score of the Standard Model Group 2.95782451.

It should be remembered, that the application of this formula should be done only, when there are sufficient $U(1)$'s to make the simple groups S over which we sum get their F-representations used realized. It is really the importance of the $SU(2)$ and the $SU(3)$ groups sharing their $U(1)$. This is only possible because their center Z_N's have mutually prime numbers n, namely 2 and 3. It is this collaboration between the two by sharing the burden of the $U(1)$ which they need for getting their center-factors $2^{2/3}$ and $3^{2/8}$ respectively, that brings the Standard Model Group $S(U(2) \times U(3))$ to win. All of the three simple groups collaborate to win.

We leave it to the reader to check that no other combination of groups can beat the Standard Model Group! Most of the competers are soon loosing out, because it is only the small rank simple groups that get the high scores.

11.7 Conclusion on the Game Found so far

Let us summarize the most important of the games discussed - the game between "teams" meaning Groups that are not necessarily simple, so that they appear as combinations of simple algebras. Here the proposal for game quantity is the w.r.t. to dimension averaged logarithm of the quantity originally proposed $\frac{C_A}{C_F}$ including - if allowed without spoiling the representation F used - a $center$-factor $(\#center)^{2/d}$. To bring the total averaged logarithm T for a group that is typically not simple to be compared to the previously discussed numbers it may be best to exponentiate it back by taking as the "team-score" (meaning score for groups, that are not necessarily simple) $\exp T$ for the combined group in question.

For this dimension averaged quantity we found that the Standard Model Group $S(U(2) \times U(3))$ (as suggested from the representations of the quarks and leptons found in nature) is the maximal score of

$$\exp T_{SMG=S(U(2)\times U(3))} = 2.95782451. \quad (11.33)$$

This $SMG = S(U(2) \times U(3))$ is extremely tightly followed by $U(2)$ which got

$$\exp T_{U(2)} = 2.951151786. \quad (11.34)$$

But it were the Standard Model, that won!

If the reader would accept that the rules of the game were chosen in a reasonable simple way, one would say, that it is very remarkable, that we have been able to present a game giving just the Standard Model Group the best score! It should be expected there were a reason, that should be found, to explain, why precisely this group with the highest score in our sense should be the realized one. So this finding should possibly bring us to get to an understanding of the question: Why the standard model group?

11.8 Our Early Model with MPP and Numerically Maximizing Plaquette action

We came into the ideas of the present article by in a lattice gauge theory speculating about some reason for that the energy per plaquette normalized in some way should be minimized. Of course such an energy of a plaquette contribution to the energy depends on the couplings constants, the fine structure constants, and so we would have to combine such a looking for minimal energy (or a minimal action),with some assumption about what the coupling constants would be with different possibilities for the gauge group. As such a machinery to provide the gauge couplings we then had in mind to assume the idea of multiple point principle MPP, which means in a lattice gauge theory that the gauge coupling parameters shall be adjusted so as to get the lattice theory go to a "multiple point", i.e. a point in coupling constant space where several phases meet.

We shall not too deeply into the calculations needed in the present article. What we have to do is to use the constraints on the coupling constants imposed by the requirement of the several phases just meet, that is to say the couplings are in this sense "critical". In principle we can include several possibly only as lattice artifact relevant parameters among the here mentioned "coupling constants". Using such constraints which in principle are constraints which we can calculate we should be able to have estimates of coupling constants even for groups, which are not realized, but only thought of as possibilities. In this way we become able to estimate questions such as what would the energy or action (whatever we ask for) per plaquette in the lattice theory be, if the group were say G. Thus we can in such a scheme ask for maximizing e.g. say the action of the plaquettes.

Now the question is, if we can make the details of the here proposed scheme so that we get the groups classified much the same way as we have in the present article proposed partly by phenomenological guessing. Indeed it seems that the scheme with use of MPP to restrict the coupling constants a then maximizing the plaquette action normalized in an appropriate way with the square of the dimension involved.

11.8.1 On our Finestructure Constant Fitting in New Light

One possibly great feature of using the scheme with MPP and maximizing plaquette action is that it together with the selection of the group also provide the coupling constants, so that we in addition to the prediction of the gauge group as we did in the present article get a *related* prediction about the fine structure constants. This can hopefully soon bring us to present a fit to the latter in such a *combined* scheme. That might open up for making interesting phenomenology on the details of the model-type proposed by fitting both the gauge group and the fine structure constants.

11.9 Further Speculations for a Reason for the Selection

11.9.1 What is Good for Prevention of Spontaneous Breaking of Gauge Symmetry

We should imagine a gauge glass or just a glassy structure in the sense that the action is given with terms which vary from point or lattice cell to point in a quenched random way. This is what we mean here in the abstract sense by "glass" that the theory or its action involves a lot of quenched random - meaning fixed randomly before you integrate to make the partition function or the Feynman path integral - variables, so that in a way one could think of it as, the theory itself being random. It is even random in a non-translational invariant way in as far as it varies from point to point or from little lattice neighborhood to the next little lattice neighborhood.

The main point, we now want to point out is that if we let the quenched random theory not a priori obey gauge symmetry and gauge symmetry has to come out the way suggested in [6] the gauge theory that we might formally think about is also in the danger of being broken - spontaneously - by the ground state not having the the plaquette variables driven to a center element - as is required for the invariance under a global gauge transformation of the vacuum - but to some non-central element. Honestly speaking: in the quenched random model it will almost certainly happen that here and there in space(time) will be plaquette variables, which actually will lead to the minimum energy density, say by standing at some non-central element. If it stands at a central element, it is not so serious, since we can essentially just think of all the elements being displaced by a right translation and that after such a transformation the central element at the bottom of the energy were transformed into the unit element so that we can really think of it as if it had the bottom at the unit element. But for a noncentral element being at first at the bottom we cannot transform it to the unit element without changing the system physically. So if truly a non-central value occurs for the vacuum field it means indeed that the global part of the gauge group in question has broken spontaneously.

11.10 Conclusion and Outlook

We have in the present contribution put up an attempt to by combined looking at some physical ideas behind and on the goal of making the Standard Model group win produce some function defined for compact Lie groups with the property that it singles out just the Standard Model group $S(U(2) \times U(3)$ as being *the* Lie group for which this function has its biggest value. Indeed we managed - in an almost satisfactory way - to construct such function in a reasonable simple way. The procedure for evaluating our proposed function is like this:

- A) For each of the non-abelian simple Lie algebras of the Lie algebra we construct the quantity
$$\frac{C_A}{C_F} * (\#center)^{2/d}, \tag{11.35}$$
where C_A is the quadratic Casimir for the adjoint representation A of the simple non-abelian group in question. The quadratic Casimir C_F is for a "smaller" representation if possible and this representation F shall be chosen at the end with the purpose of making the quantity (11.35) as large as possible. Typically the representation F will be the "fundamental" representation.The quantity $\#center$ is the number of elements in the center of the covering group for the Lie algebra in question, and d is its dimension.
- B)Next average the logarithm of this quantity over all the simple non-abelian Lie weighting with the dimensions d of the Lie algebras including the abelian components in the total Lie algebra for the whole group counted to give 0 in logarithm (as if the quantity (11.35) were 1 for $U(1)$). This average is presented in equation (11.30).
- C) There is, however, an important restriction forbidding, that unless there are enough $U(1)$'s included in the group and they have got to a sufficient degree some (discrete) center-subgroup divided out of the cross product of the covering groups and and the $U(1)$'s in a way connecting the groups to be no longer just a cross product of separated groups, we cannot use the formula above. This restriction shall be understood to mean: Firstly: Under the division out of the (non-trivial) subgroup of the center of the cross product of the abelian and non-abelian simple groups we must not identify the center elements of any of the simple groups so that we obtain a factor group, which no longer has the representation F for that simple groups as a representation without allowing phase ambiguities. (This will typically mean that there must be no element in the invariant discrete subgroup divided out which is trivial w.r.t. to the $U(1)$'s, because that would typically lead to that the representation F would not be a representation of the group)
Secondly: In order to obtain the factor $(\#center)^{2/d}$ for one of the simple Lie algebras - averaged between - it is required that the discrete subgroup divided out has indeed a factor group in correspondence to the center of the simple group in question. (This requirement implies that the divided out discrete subgroup of the center of the product of the covering groups and the $U(1)$'s should (at least) has as many elements as the product of the numbers $\#center$ for all the simple groups for which the factor $(\#center)^{2/d}$ in (11.35) is to be used.).
- D) The quantity - the score so to speak - which should be largest possible for the Lie group to be realized in Nature should under the restrictions in C) be the average constructed under B).

The really remarkable fact of the present article is that *The Standard Model Group as phenomenologically defined partly under use of its physically realized representations of quarks and leptons and the Higgs turns out to be precisely that (compact) Lie group which gives the biggest value for the average constructed under B) with the restrictions C) imposed!*

In fact one gets for the exponential of the average over the logarithms as told in B) the number **2.95782451**.

The Standard Model Group is, however, remarkably closely followed by the Lie group $U(2)$ for which the exponentiated average becomes: 2.951151786. They only deviate on the fourth significant digit, and difference is only of the order of 0.007 compared to almost 3.

In our opinion the procedure for constructing the function of the compact Lie groups, the score in the game so to speak, is so simple that one would say it is pretty remarkable that it should give just the Standard Model Group, which is realized in Nature at least for the energetically accessible physics in practice, to have the biggest average. after all there are many groups which nature could have chosen, if one did not impose the phenomenological or other restrictions. Of course the Standard Model Group is the only fitting if we do not include parts of the group, which are not at all seen experimentally at present. But that just means that the Standard Model Group is - we could say - measured to be the true model. In the present paper we search for some theoretical assumption as simple as possible, that could single out and point to just this special group $S(U(2) \times U(3))$, which is the by the representations of quarks and leptons *group* with the Standard Model Lie algebra, and we found the principle of maximizing the quantity $\exp T$ where T is the average described! It singles out the *right* group for nature!

11.10.1 Taking serious that it is not an accident

The point of such an exercise as the present one is of curse to get some hints as to what is the reason Nature has just chosen the Standard Model Group and not some other group among the after all pretty many groups she could have chosen between.

It were above suggested that the quantity in which the Standard Model Group is excellent is that compared to a normalization given by the quadratic Casimir C_A for the adjoint representation A the group has (a) very small representation(s) in terms of some quadratic Casimir C_F for a representation, which we above have thought upon as a representation related to the fields in the e.g. lattice gauge theory model working in Nature. The thing that seems to be important is that compared to some "natural" distance measure on the group (related to the quadratic Casimir for the adjoint representation C_A) the way it is possible to make it move the fields in some appropriate representation F is very slow. That is to say you may move the group element a lot but the fields only tinily for the groups having high scores in our game. Such a property of it being easy to push the fields only little around for the group element moving much without making the transformation completely zero (i.e. still using a true representation F) seems to be what our result points to as the important principle used to select just the model, which nature has chosen.

We suggested that such a selection were likely to be the result, if the gauge group had in reality appeared by first getting an approximate gauge group by accident. Then the gauge symmetry should for practical purposes have become exact due to quantum fluctuations. But the important point to extract is that the choice of the Standard Model group suggests that the group that can be represented, on fields say, being most tinily moved around under a by some adjoint representation related normalization of distances in the group is the group most beloved by nature to be realized.

Acknowledgement

One of us (H.B.N.) thanks for being allowed to have room and stay as emeritus at the Niels Bohr Institute at Copenhagen University, since 1st of September, when the writing of this proceeding took place.

11.11 Appendix: A Pedagogical Calculation Procedure for our Purpose

Having in mind that one of the main ideas for why our proposed quantity C_A/C_F - i.e. the ratio of the quadratic Casimir for the adjoint representation C_A divided by some representation F having the smallest quadratic Casimir C_F among faithful representations

```
In[1]:= ListPlot[{{4, 2.08}, {5, 1.777}, {6, 1.385}, {7, 1.218}, {8, 1.000}},
        PlotStyle → PointSize[0.02]]
```

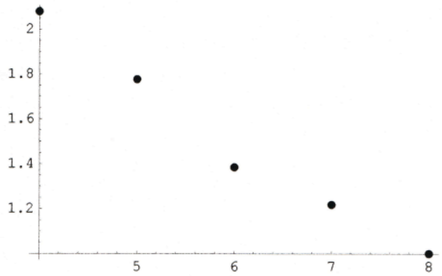

Fig. 11.1. The ratio C_A/C_F for E groups plotted as a function of rank. Here we have used that $E_5 = SO(10)$ and $E_4 = SU(5)$.

- is suggested to be that this quantity C_A/C_F, the bigger it, is favours the chance that the group in question should be the one realized in nature because a big C_A/C_F means that varying the potential gauge an amount measured by the Killing form normalized by the adjoint representation Casimir being put to say 1 makes the variation in the link variables supposedly in the representation F minimal, we shall here present as a couple of examples a practical calculation of our quantity, almost making clear its physical significance for our purpose.

Remember that we in our speculative arguments for which group would most easily become a gauge group by accidentally being so near to being it that a quantum fluctuation effect might set in and make it practically an exact gauge symmetry we used the normalization of the Killing form to make the quadratic Casimir for the adjoint representation say 1 so that we thereby got a physically meaningful distance concept on the group manifold. Under use of this distance concept we then asked how the field theory variables say the link variables in some lattice gauge theory formulation will vary for a given - unit- variation of the gauge group element. If one in the link variable space use the distance concept derived from the trace of the square of difference of the couple of representation matrices corresponding to the link variable, the ratio of the infinitesimal distance in the group manifold relative to the corresponding distance in the link variable will be given by the square root of the ratio C_A/C_F, where F is the representation used to represent the link variable. The main idea were that the chance to find an approximate symmetry under the transformation of the various link variables as transformed under the gauge transformation will be bigger the bigger the ratio C_A/C_F and so the group "to be realized by accident" with best chance is expected to be the group with largest C_A/C_F- value.

In this appendix we shall present a way to calculate this ratio C_A/C_F by using just the Cartan algebra representations in a way that allows us to calculate simply the ratio of the average of the square of the root vector length compared to the corresponding average of the weight vectors for the representation F.

Let us provide a couple of examples:

- $A_1 = SU(2)$:
  In this case the smallest representation - w.r.t. say the quadratic Casimir - is $F = \underline{2}$. As is well known the root system for the $F = \underline{2}$ consists of two weights both of half length of the roots. Thus we find if we say roots have length $\sqrt{2}$ - as is usual -
  - Adjoint : Average of the squared roots $\frac{2+2+0}{3} = \frac{4}{3}$.

– F: Average of the squared weights $\frac{1/2+1/2}{2} = \frac{1}{2}$.

So we obtain by taking the ratio of these averages:

$$\frac{C_A}{C_F}|_{A_1} = \frac{4/3}{1/2} = \frac{8}{3} \tag{11.36}$$

- G_2 : The root system for the exceptional Lie group G_2 consists of two regular hexagons with centers in the zero point, the one rotated by 30^0 w.r.t. the other one and one $1/\sqrt{3}$ times the other one. The Lie algebra of G_2 has an $SU(3) = A_2$ subgroup corresponding to the roots of the bigger one of the two hexagons. The "smallest" representation F for the G_2 is seven dimensional and consists w.r.t. the $SU(3)$ subgroup of a triplet an antitriplet and a singlet. This means that the weight system for this representation F consists of the six roots in the smaller of the two hexagons and in addition a weight at zero. Then we have for the average of the squares of the distances from zero for the weights
  – Adjoint representation: $average = \frac{6*2+6*2/3}{14} = \frac{8}{7}$.
  – F : $average = \frac{6*2/3+0}{7} = \frac{4}{7}$

  We thus find that
$$\frac{C_A}{C_F}|_{G_2} = \frac{8/7}{4/7} = 2 \tag{11.37}$$
- $B_2 = SO(5) = C_2 = Sp(4)$: The root system for these isomorphic Lie algebras consists of the corners and the midpoints of the sides of a square (with side 2 say). There are thus 8 roots. The "smallest" representation F is a four dimensional one with the roots with the weights sitting in the four centers for the four squares with side 1 into which the coordinate axes divide the mentioned square of side 2. Then we get for the averages of the squares of the distances from the zero in the root and weight systems:
  – Adjoint : $average = \frac{4*1+4*2+2*0}{10} = \frac{6}{5}$
  – F : $average = \frac{4*1/2}{4} = \frac{1}{2}$

  So our competition number becomes
$$\frac{C_A}{C_F}|_{B_2} = \frac{6/5}{1/2} = \frac{12}{5} \tag{11.38}$$
- $A_2 = SU(3)$: For $SU(3)$ the root system is a regular hexagon around zero, and we take the length of the roots as usual to be $\sqrt{2}$. The "smallest" representation F is the quark or we can equally well take the antiquark representation $\underline{3}$. The weight system for say the quark representation forms is a triangle centered around zero and having the side length $\sqrt{2}$ like we took the roots to have. Thereby the distances of the weights from zero become $\sqrt{2}3$. So the averages of the squares of the distances from zero becomes:
  – Adjoint: $average = \frac{6*2+2*0}{8} = \frac{3}{2}$
  – F= "quark"representation : $average = \frac{3*2/3}{3} = \frac{2}{3}$

  So we obtain for our ratio
$$\frac{C_A}{C_F}|_{A_2} = \frac{3/2}{2/3} = \frac{9}{4} \tag{11.39}$$
- F_4 : The root system for the exceptional Lie algebra F_4, Φ, is described as contained in $V = \mathbf{R}^4$ and consisting of those vectors α with length 1 or $\sqrt{2}$ for which the coordinates obey that 2α having all coordinates integer and that so that for each 2α these coordinates are either all even or all odd. There are 48 roots in this system.

  These 48 roots are easily seen to fall into one group of 16 of length 1 for which the coordinates are all $\pm 1/2$, one group of 24 of length $\sqrt{2}$ having two coordinates 0 and two ± 1, and 8 roots have just one coordinate equal to ± 1 and the other coordinates being 0.

  The average square distance of these roots together with the 4 Cartan group basis vectors with 0 distance so to speak becomes $\frac{16*1+8*1+24*2+4*0}{48+4} = \frac{72}{52} = \frac{18}{13}$.

We used here the Cartan algebra only, but since these Cartan algebra elements can be transformed around to go into the non-Cartan algebra so at the end the average "charges" must be the same and thus this restriction would not matter.

11.12 Appendix 2: Calculation of C_A/C_F

In order to calculate the ratios of quadratic Casimirs we shall here rewrite a list of the adjoint representations for the Lia algebras:

$$A_n: \quad (1,0,0,...,0,0,1) \quad n(n+2) \tag{11.40}$$

$$B_n: \quad (0,1,0,...0,0,0) \quad n(2n+1) \tag{11.41}$$

$$C_n: \quad (2,0,0,...,0,0,0) \quad n(2n+1) \tag{11.42}$$

$$D_n: \quad (0,1,0,...,0,0,0) \quad n(2n-1) \tag{11.43}$$

$$G_2: \quad (1,0) \quad 14 \tag{11.44}$$

$$F_4: \quad (1,0,0,0) \quad 52 \tag{11.45}$$

$$E_6: \quad (0,0,0,0,0,1) \quad 78 \tag{11.46}$$

$$E_7: \quad (1,0,0,0,0,0,0) \quad 133 \tag{11.47}$$

$$E_8: \quad (0,0,0,0,0,0,1,0) \quad 248 \tag{11.48}$$

Here the orders of the Dynkin labels correspond to enumerating the being successive in the chain except for the E-algebras for which the largest number though is assigned to the node which is both an end node and attached to the node having three neighbours. In the cases of B_n and C_n it is the nth node that is respectively the short and the long simple roots. In cases F_4 and G_2 the short roots are numbered with the largest numbers.

In the same notation we also copy in what we can call Simple irreducible representations of the simple Lie algebras:

$$\text{For } A_n \quad (1,0,...,0,0) \quad \dim = n+1 \tag{11.49}$$

$$\text{or} \quad (0,0,...,0,1) \quad \dim = \overline{n+1} \tag{11.50}$$

$$\text{For } B_n \quad (1,0,...,0) \quad \dim = 2n+1 \tag{11.51}$$

$$\text{and} \quad (0,0,...,0,1) \quad 2^n \tag{11.52}$$

$$\text{For } C_n \quad (1,0,...,0,0) \quad \dim = 2n \tag{11.53}$$

$$\text{For } D_n \quad (1,0,...,0,0) \quad \dim = 2n \tag{11.54}$$

$$\text{and} \quad (0,0,...,0,1) \quad \dim = 2^{n-1} \tag{11.55}$$

$$\text{or} \quad (0,0,...,0,1,0) \quad \dim = 2^{n-1} \tag{11.56}$$

$$\text{For } G_2 \quad (0,1) \quad \dim = 7 \tag{11.57}$$

$$\text{For } F_4 \quad (0,0,0,1) \quad \dim = 26 \tag{11.58}$$

$$\text{For } E_6 \quad (1,0,0,0,0,0) \quad \dim = 27 \tag{11.59}$$

$$\text{or} \quad (0,0,0,0,1,0) \quad \dim = \overline{27} \tag{11.60}$$

$$\text{For } E_7 \quad (0,0,0,0,0,1,0) \quad \dim = 56 \tag{11.61}$$

$$\text{For } E_8 \quad (0,0,0,0,0,0,1,0) \quad \dim = 248 \tag{11.62}$$

In order to use the equation

$$C_R = \frac{\eta}{2} \sum_{i=1}^{r} \sum_{j=1}^{r} (a_i + 2) G_{ij} a_j \tag{11.63}$$

for the quadratic Casimir C_R of a representation R being in our cases of interest, we must know the matrix elements of the "Metric tensors for the weight spaces" G_{ij} (or the inverse Cartan matrix) at the relevant places: For R being the adjoint representation A we have for the A_n Lie algebra both $a_1 = 1$ and $a_n = 1$ while the other Dynkin labels $a_i = 0$. For the other algebras than the A_n-series, we have only one Dynkin label different from zero, and that is

For Adjoint Representations

$$B_n: \ a_2(Adj \quad B_n) = 1; \tag{11.64}$$
$$C_n: \ a_1(Adj \quad C_n) = 2; \tag{11.65}$$
$$D_n: \ a_2(Adj \quad D_n) = 1; \tag{11.66}$$
$$G_2: \ a_1(Adj \quad G_2) = 1; \tag{11.67}$$
$$F_4: \ a_1(Adj \quad F_4) = 1; \tag{11.68}$$
$$E_6: \ a_6(Adj \quad E_6) = 1; \tag{11.69}$$
$$E_7: \ a_1(Adj \quad E_7) = 1; \tag{11.70}$$
$$E_8: \ a_7(Adj \quad E_8) = 1. \tag{11.71}$$

For the simple representations mentioned in the list above we have correspondingly that the only non-zero Dynkin labels are

For simple representations:

$$\text{For } A_n: \quad a_1(A_n, n+1) = 1, \tag{11.72}$$
$$\text{or} \quad a_n(A_n, \overline{n+1}) = 1; \tag{11.73}$$
$$\text{For } B_n: \quad a_1(B_n, 2n+1) = 1, \tag{11.74}$$
$$\text{and for the spinor rep.} \quad a_n(B_n, 2^n) = 1; \tag{11.75}$$
$$\text{For } C_n: \quad a_1(C_n, 2n) = 1; \tag{11.76}$$
$$\text{For } D_n: \quad a_1(D_n, 2n) = 1, \tag{11.77}$$
$$\text{and for spinors} \quad a_n(D_n, 2^{n-1}) = 1, \tag{11.78}$$
$$\text{or} \quad a_{n-1}(D_n, 2^{n-1}, *) = 1; \tag{11.79}$$
$$\text{For } G_2: \quad a_2(G_2, 7) = 1; \tag{11.80}$$
$$\text{For } F_4: \quad a_4(F_4, 26) = 1; \tag{11.81}$$
$$\text{For } E_6: \quad a_1(E_6, 27) = 1, \tag{11.82}$$
$$\text{or} \quad a_5(E_6, \overline{27}) = 1; \tag{11.83}$$
$$\text{For } E_7: \quad a_6(E_7, 56) = 1; \tag{11.84}$$
$$\text{For } E_8: \quad a_7(E_8, 248) = 1 \tag{11.85}$$

So except for the case of A_n, in which we need the $G_{1,n} = \frac{1}{n+1}$ and $G_{n,1} = \frac{1}{n+1}$ matrix elements also, we only need the diagonal elements and the sums of the elements in the columns of the metric tensor matrices for the weight or the inverse Cartan matrices. We therefore here present these diagonal series of elements:

Diagonal Elements of Weight Space Metric

$$A_n : G_{1,1} = \frac{1*n}{n+1}, G_{2,2} = \frac{2(n-1)}{n+1}, ..., G_{(n-1),(n-1)} = \frac{(n-1)*1}{n+1}, G_{n,n} = \frac{n*1}{n+1}; \tag{11.86}$$

$$B_n : G_{1,1} = 1, G_{2,2} = 2, ..., G_{(n-1),(n-1)} = n-1, G_{n,n} = \frac{n}{4}; \tag{11.87}$$

$$C_n : G_{1,1} = \frac{1}{2}, G_{2,2} = 1, ...G_{(n-1),(n-1)} = \frac{n-1}{2}, G_{n,n} = \frac{n}{2}; \tag{11.88}$$

$$D_n : G_{1,1} = 1, G_{2,2} = 2, ..., G_{n-2,n-2} = n-2, G_{n-1,n-1} = \frac{n}{4}, G_{n,n} = \frac{n}{4}; \tag{11.89}$$

$$G_2 : G_{1,1} = 2, G_{2,2} = \frac{2}{3}; \tag{11.90}$$

$$F_4 : G1,1 = 2, G_{2,2} = 6, G_{3,3} = 3, G_{4,4} = 1; \tag{11.91}$$

$$E_6 : G_{1,1} = \frac{4}{3}, G_{2,2} = \frac{10}{3}, G_{3,3} = 6, G_{4,4} = \frac{10}{3}, G_{5,5} = \frac{4}{3}, G_{6,6} = 2; \tag{11.92}$$

$$E_7 : G_{1,1} = 2, G_{2,2} = 6, G_{3,3} = 12, G_{4,4} = \frac{15}{2}, G_{5,5} = 4, G_{6,6} = \frac{3}{2}, G_{7,7} = \frac{7}{2}; \tag{11.93}$$

$$E_8 : G_{1,1} = 4, G_{2,2} = 14, G_{3,3} = 30, G_{4,4} = 20, G_{5,5} = 12, G_{6,6} = 6, G_{7,7} = 2, G_{8,8} = 8; \tag{11.94}$$

In addition we needed the sums over the columns and thus we present these sums e.g.

$$Sum(A_n) = (\sum_{i=1}^{n} G_{i,1}, \sum_{i=1}^{n} G_{i,2}, ..., \sum_{i=1}^{n} G_{i,n}) \tag{11.95}$$

and get the following:

$$Sums(A_n) = \left(\frac{n}{2}, n-1, \frac{3n}{2} - 3, ..., n-1, \frac{n}{2}\right) \tag{11.96}$$

$$Sums(B_n) = \left(1(n-1/2), 2(n-1), 3(n-3/2), \ldots, \frac{(n+1)(n-1)}{2} = (n-1)(n-(n-1)/2), \frac{n}{4}\right) \tag{11.97}$$

$$Sums(C_n) = \left(\frac{n}{2}, \frac{2n-1}{2}, \frac{3n-3}{2}, ..., \frac{(n+2)(n-1)}{4}, \frac{(n+1)n}{4}\right) \tag{11.98}$$

$$Sums(D_n) = \left(n-1, 2n-3, 3n-6, ..., \frac{(n-1)n}{2}, \frac{(n-1)n}{2}\right) \tag{11.99}$$

$$Sums(G_2) = \left(3, \frac{5}{3}\right) \tag{11.100}$$

$$Sums(F_4) = \left(8, 15, \frac{21}{2}, \frac{11}{2}\right) \tag{11.101}$$

$$Sums(E_6) = (8, 15, 21, 15, 8, 11) \tag{11.102}$$

$$Sums(E_7) = \left(17, 33, 48, \frac{75}{2}, 26, \frac{27}{2}, \frac{49}{2}\right) \tag{11.103}$$

$$Sums(E_8) = (46, 91, 135, 110, 84, 57, 29, 68) \tag{11.104}$$

Denoting the jth element in these Sums by an index j like e.g. $Sums(G_2)_j$ we can then write the expression for the typical cases above of a "simple" representation where the a_j alone is different from zero:

$$C_F = \frac{\eta}{2}(G_{j,j} + 2Sums_j) \tag{11.105}$$

We can thus by insertion obtain:

Simple Quadratic Casimirs

$$C_F(A_n) = \frac{\eta}{2}(\frac{n}{n+1} + 2 * \frac{n}{2}) = \frac{\eta}{2} * \frac{n(n+2)}{n+1} \tag{11.106}$$

$$C_{F\ vector}(B_n) = \frac{\eta}{2}(1 + 2 * (n - \frac{1}{2}) = \eta * n \text{for 2n+1} \tag{11.107}$$

$$C_{F\ spinor}(B_n) = \frac{\eta}{2}(\frac{n}{4} + 2 * \frac{n^2}{4}) = \frac{\eta * (n + 2n)}{8} \tag{11.108}$$

$$C_F(C_n) = \frac{\eta}{2}(\frac{1}{2} + 2 * \frac{n}{2}) = \eta * \frac{(2n+1)}{4} \tag{11.109}$$

$$C_{F\ vector}(D_n) = \frac{\eta}{2}(1 + 2 * (n - 1)) = \eta * (n - 1/2) \tag{11.110}$$

$$C_{F\ spinor}(D_n) = \frac{\eta}{2}(\frac{n}{4} + 2 * \frac{n(n-1)}{4}) = \eta * \frac{2n^2 - n}{8} \tag{11.111}$$

$$C_F(G_2) = \frac{\eta}{2}(\frac{2}{3} + 2 * \frac{5}{3}) = \eta * 2 \tag{11.112}$$

$$C_F(F_4) = \frac{\eta}{2}(1 + 2 * \frac{11}{2}) = \eta * 6 \tag{11.113}$$

$$C_F(E_6) = \frac{\eta}{2}(\frac{4}{3} + 2 * 8) = \eta * \frac{26}{3} \tag{11.114}$$

$$C_F(E_7) = \frac{\eta}{2}(\frac{3}{2} + 2 * \frac{27}{2}) = \eta * \frac{57}{4} \tag{11.115}$$

$$C_F(E_8) = \frac{\eta}{2}(2 + 2 * 29) = \eta * 30 \tag{11.116}$$

These quadratic Casimirs can be compared with e.g. the corresponding ones for the adjoint representations, and then the normalization - symbolized by the factor η, which we thus avoid having to choose. Thereby we obtain the ratio which were our first proposal in the present article for quantity about which to hold the game. These adjoint representation quadratic Casimirs become:

Quadratic Casimirs for Adjoint Representations

$$\begin{aligned} C_A(A_n) &= \frac{\eta}{2}(2 * Sums(A_n)_1 + 2 * Sums(A_n)_n \\ &\quad + G_{1,1}(A_N) + G_{n,n}(A_n) + G_{n,1}(A_n) + G_{1,n}(A_n)) \\ &= \frac{\eta}{2}(2 * \frac{n}{2} + 2 * \frac{n}{2} + \frac{n}{n+1} + \frac{n}{n+1} + \frac{1}{n+1} + \frac{1}{n+1}) = \eta * (n+1) \end{aligned} \tag{11.117}$$

$$C_A(B_n) = \frac{\eta}{2}(2 * Sums(B_n)_2 + G_{2,2}) = \frac{\eta}{2}(2 * 2(n-1) + 2) = \eta * (2n - 1) \tag{11.118}$$

$$C_A(C_n) = \frac{\eta}{2}(2 * 2 * Sums(C_n)_1 + 2^2 * G_{1,1}) = \frac{\eta}{2}(4 * \frac{n}{2} + 2 * \frac{1}{2}) = \eta * (n+1) \tag{11.119}$$

$$C_A(D_n) = \frac{\eta}{2}(2 * Sums(D_n)_2 + G_{2,2}) = \frac{\eta}{2}(2 * (2n-3) + 2) = \eta * 2(n-1) \tag{11.120}$$

$$C_A(G_2) = \frac{\eta}{2}(2 * Sums(G_2)_1 + G_{1,1}) = \frac{\eta}{2}(2 * 3 + 2) = \eta * 4 \tag{11.121}$$

$$C_A(F_4) = \frac{\eta}{2}(2 * Sums(F_4)_1 + G_{1,1}) = \frac{\eta}{2}(2 * 8 + 2) = \eta * 9 \tag{11.122}$$

$$C_A(E_6) = \frac{\eta}{2}(2 * Sums(E_6)_1 + G_{1,1}(E_6)) = \frac{\eta}{2}(2 * 11 + 2) = \eta * 12 \tag{11.123}$$

$$C_A(E_7) = \frac{\eta}{2}(2 * Sums(E_7)_7 + G_{7,7}(E_7)) = \frac{\eta}{2}(2 * 17 + 2) = \eta * 18 \tag{11.124}$$

$$C_A(E_8) = \frac{\eta}{2}(2 * Sums(E_8)_7 + G_{7,7}(E_8)) = \frac{\eta}{2}(2 * 29 + 2) = \eta * 30 \tag{11.125}$$

(These numbers are indeed $\eta * g$, where g is the dual Coxeter number, see above) Combining these calculations of quadratic casimirs we then finally obtain by taking the ratios our competition quantity C_A/C_F.

Our Ratio of Adjoint to "Simplest" Quadratic Casimirs C_A/C_F

$$\frac{C_A}{C_F}|_{A_n} = \frac{2(n+1)^2}{n(n+2)} = \frac{2(n+1)^2}{(n+1)^2-1} = \frac{2}{1-\frac{1}{(n+1)^2}} \tag{11.126}$$

$$\frac{C_A}{C_{F\,vector}}|_{B_n} = \frac{2n-1}{n} = 2 - \frac{1}{n} \tag{11.127}$$

$$\frac{C_A}{C_{F\,spinor}}|_{B_n} = \frac{2n-1}{\frac{2n^2+n}{8}} = \frac{16n-8}{n(2n+1)} \tag{11.128}$$

$$\frac{C_A}{C_F}|_{C_n} = \frac{n+1}{n/2+1/4} = \frac{4(n+1)}{2n+1} \tag{11.129}$$

$$\frac{C_A}{C_{F\,vector}}|_{D_n} = \frac{2(n-1)}{n-1/2} = \frac{4(n-1)}{2n-1} \tag{11.130}$$

$$\frac{C_A}{C_{F\,spinor}}|_{D_n} = \frac{2(n-1)}{\frac{2n^2-n}{8}} = \frac{16(n-1)}{n(2n-1)} \tag{11.131}$$

$$\frac{C_A}{C_F}|_{G_2} = \frac{4}{2} = 2 \tag{11.132}$$

$$\frac{C_A}{C_F}|_{F_4} = \frac{9}{6} = \frac{3}{2} \tag{11.133}$$

$$\frac{C_A}{C_F}|_{E_6} = \frac{12}{\frac{26}{3}} = \frac{18}{13} \tag{11.134}$$

$$\frac{C_A}{C_F}|_{E_7} = \frac{18}{\frac{57}{4}} = \frac{72}{57} = \frac{24}{19} \tag{11.135}$$

$$\frac{C_A}{C_F}|_{E_8} = \frac{30}{30} = 1 \tag{11.136}$$

11.13 Appendix 3: Checks and overview of C_A/C_F

It may be comforting that one can put the calculations in section 11.12, i.e. appendix 2, up to a few cross checks, such as checking that isomorphic algebras give the same ratio C_A/C_F as of course they shall for a notation independent quantity:

- $A_1 \approx B_1 \approx C_1$

$$\frac{C_A}{C_F}|_{A_1} = \frac{8}{3}, \tag{11.137}$$

$$\frac{C_A}{C_{F\,spinor}}|_{B_1} = \frac{16*1-8}{1*(2*1+1)} = \frac{8}{3}, \tag{11.138}$$

$$\frac{C_A}{C_F}|_{C_1} = \frac{4(1+1)}{2*1+1} = \frac{8}{3}. \tag{11.139}$$

- $A_1 \times A_1 \approx D_2$

$$\frac{C_A}{C_F}|_{A_1} = \frac{2}{1-\frac{1}{(1+1)^2}} = \frac{8}{3} \tag{11.140}$$

$$\frac{C_A}{C_{F\,spinor}}|_{D_2} = \frac{16(2-1)}{2*(2*2-1)} = \frac{8}{3} \tag{11.141}$$

- $B_2 = SO(5) \approx C_2 = Sp(4)$

$$\frac{C_A}{C_{F\ spinor}}|_{B_2} = \frac{16*2-8}{2*(2*2+1)} = \frac{12}{5} \tag{11.142}$$

$$\frac{C_A}{C_F}|_{C_2} = \frac{4*(2+1)}{2*2+1} = \frac{12}{5} \tag{11.143}$$

- $D_3 = SO(6) \approx A_3 = SU(4)$

$$\frac{C_A}{C_{F\ spinor}}|_{D_3} = \frac{16(3-1)}{3*(2*3-1)} = \frac{32}{15} \tag{11.144}$$

$$\frac{C_A}{C_F}|_{A_3} = \frac{2*(3+1)^2}{(3+1)^2-1} = \frac{32}{15}. \tag{11.145}$$

Further we should note that for $D_4 = SO(8)$ (w.r.t. Lie algebra) there is symmetry between the spinor and vector representations, which are both 8-dimensional. Thus we should have $\frac{C_A}{C_{F\ spinor}}|_{D_4} = \frac{C_A}{C_{F\ vector}}|_{D_4}$. Indeed we find

$$\frac{C_A}{C_{F\ spinor}}|_{D_4} = \frac{16(4-1)}{4*(2*4-1)} = \frac{12}{7} \tag{11.146}$$

$$\frac{C_A}{C_{F\ vector}}|_{D_4} = \frac{4(4-1)}{2*4-1} = \frac{12}{7}. \tag{11.147}$$

We should also expect approximately the same large N behavior behavior for $SO(N)$, whether it be for even N for which we have $D_{N/2}$, or for odd N for which we have $B_{(N-1)/2}$. Let us indeed formally consider these two Lie algebras:

$$\frac{C_A}{C_{F\ vector}}|_{D_{N/2}} = \frac{4(n-1)}{2n-1} = \frac{4(N/2-1)}{2N/2-1} = \frac{2N-4}{N-1} \tag{11.148}$$

$$\frac{C_A}{C_{F\ vector}}|_{B_{(N-1)/2}} = \frac{2n-1}{n} = \frac{(N-1)-1}{(N-1)/2} = \frac{2N-4}{N-1}. \tag{11.149}$$

Remarkable we get even exactly the same formal expressions $\frac{2N-4}{N-1}$.

Similarly we may compare the spinor representation for F using ratios $C_A/C_{F\ spinor}$ for $B_{(N-1)/2}$ and $D_{N/2}$:

$$\frac{C_A}{C_{F\ spinor}}|_{B_{(N-1)/2}} = \frac{16n-8}{n(2n+1)} = \frac{16*(N-1)/2-8}{(N-1)/2*(2(N-1)/2+1)}$$
$$= \frac{8N-16}{(N^2-N)/2} = \frac{16(N-2)}{N(N-1)} \tag{11.150}$$

$$\frac{C_A}{C_{F\ spinor}}|_{D_{N/2}} = \frac{16(n-1)}{n(2n-1)} = \frac{16(N/2-1)}{N/2*(N-1)} = \frac{16(N-2)}{N(N-1)} \tag{11.151}$$

So in spite of the fact that the dimensionality of the spinor representations is not a smooth function of N but rather jumps up and down with the even or oddness of N, we got formally the same formula for our ratio for competition becomes the same written as a function of the N of $SO(N)$.

11.13.1 The speculation of the high rank groups almost giving same C_A/C_F

We have already seen that for large rank r the infinite series of Lie algebras have our C_A/C_F going to 2. This is not so surprising from the thinking that as the rank goes up the root systems and the weight system for F (the "smallest" representation) become more and more rich in number of roots and weights as the rank goes up. But then we might consider the root and weight distributions to be more and more statistical understandable. And if so then we might expect that the small details in the Dynkin diagram deviating from just a long chain of single line connected nodes like in the A_n's would have less and less effect and so the approach to a single number common for all Lie algebras.

References

1. N.S. Mankoč Borštnik, Phys. Lett. **B 292** (1992) 25, J. Math. Phys. **34** (1993) 3731, Int. J. Theor. Phys. **40** 315 (2001), Modern Phys. Lett. **A 10** (1995) 587, http://viavca.in2p3.fr, arxiv:0812.0510[hep-ph],1-12, arXiv:0912.4532, p.119-135, http://arxiv.org/abs/1005.2288, J. of Math. Phys. **34** (1993). A. Borštnik, N.S. Mankoč Borštnik, hep-ph/0401043, hep-ph/0401055, p. 31-57, hep-ph/0301029, Phys. Rev. **D 74** (2006) 073013, hep-ph/0512062, A. Hernández-Galeana, N.S. Mankoč Borštnik, p. 166-176, arXiv:1012.0224. D. Lukman, N.S. Mankoč Borštnik, H.B. Nielsen, http://arxiv.org/abs/1001.4679, New J. Phys. **13** (2011) 103027, hep-ph/0048751v4 (2010). G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, hep-ph/0711.4681, New J. of Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0612250, p.25-50, N.S. Mankoč Borštnik, H.B. Nielsen, Phys. Lett. **B 633** (2006) 771-775, hep-th/0311037, hep-th/0509101, Phys. Lett. **B 644** (2007)198-202, hep-th/0608006, Phys. Lett. **B 10** (2008)1016, J. of Math. Phys. **34** (1993) 3731, *J. of Math. Phys.* **43** (2002) 5782, hep-th/0111257, J. of Math. Phys. **44** (2003) 4817, hep-th/0303224.
2. H. B. Nielsen and N. Brene, "Spontaneous Emergence Of Gauge Symmetry," IN *KRAKOW 1987, PROCEEDINGS, SKYRMIONS AND ANOMALIES*, 493-498 AND COPENHAGEN UNIV. - NBI-HE-87-28 (87,REC.JUN.) 6p H. B. Nielsen and N. Brene, Physicalia Magazine, The Gardener of Eden, 12 (1990) 157; NBI-HE-89-38; H. B. Nielsen and N. Brene, Phys. Lett. B **223** (1989) 399.
3. H.B. Nielsen, S.E. Rugh and C. Surlykke, Seeking Inspiration from the Standard Model in Order to Go Beyond It, Proc. of Conference held on Korfu (1992).
4. Philippe Di Francesco, Pierre Mathieu, David Sénéchal, Conformal Field Theory, 1997 Springer-Verlag New York, ISBN 0-387-94785-X.
5. RANDOM DYNAMICS: H. B. Nielsen, "Dual Strings,", "Fundamentals of quark models", In: Proc. of the Seventeenth Scott. Univ. Summer School in Physics, St. Andrews, august 1976. I.M. Barbour and A.T. Davies(eds.), Univ. of Glasgow , 465-547 (publ. by the Scott.Univ. Summer School in Physics, 1977);
   H.B. Nielsen, Har vi brug for fundamentale naturlove(in Danish) (meaning:"Do we need laws of Nature?") *Gamma 36 page 3-16, 1978*(1. part) and *Gamma 37 page 35-46, 1978* (2. part);
   H.B. Nielsen and C. D. Froggatt, Nucl. Phys. **B164**(1979) 114 - 140.
6. D. Førster, H.B. Nielsen, and M. Ninomiya, "Dynamical stability of local gauge symmetry. Creation of light from chaos." Phys. Lett. **B94**(1980) 135 -140
7. H.B. Nielsen, Lecture notes in Physics **181**, "Gauge Theories of the Eighties" In: Proc. of the Arctic School of Physics 1982, Akaeslompolo, Finland, Aug. 1982. R. Raitio and J. Lindfors(eds.). Springer, Berlin, 1983,p. 288-354.
   H.B. Nielsen, D.L. Bennett and N. Brene: "The random dynamics project from fundamental to human physics". *In:* Recent developments in quantum field theory. J. Ambjoern, B.J. Durhuus and J.L. Petersen(eds.), Elsvier Sci.Publ. B.V., 1985, pp. 263-351
8. H.B. Nielsen and D. L. Bennett, "The Gauge Glass: A short review", Elaborated version of talk at the Conf. on Disordered Systems, Copenhagen, September 1984. Nordita preprint 85/23.

9. see for example some papers in this Bled Proceedings series: H. B. Nielsen and M. Ninomiya, arXiv:1008.0464 [physics.gen-ph].
H. B. Nielsen and M. Ninomiya, Int. J. Mod. Phys. A **24** (2009) 3945 [arXiv:0802.2991 [physics.gen-ph]].
H. B. Nielsen and M. Ninomiya, arXiv:0711.3080 [hep-ph].
10. H. B. Nielsen and M. Ninomiya, JHEP **0603** (2006) 057 [hep-th/0602020].
11. D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A **9** (1994) 5155 [arXiv:hep-ph/9311321].
D. L. Bennett, arXiv:hep-ph/9607341.
D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A **14** (1999) 3313 [arXiv:hep-ph/9607278].
12. D. L. Bennett, C. D. Froggatt and H. B. Nielsen, "Nonlocality as an explanation for fine tuning in nature," (CITATION = C94-08-30);
13. D. L. Bennett, C. D. Froggatt and H. B. Nielsen, "Nonlocality as an explanation for fine tuning and field replication in nature," arXiv:hep-ph/9504294.
14. Physics Letters B Volume 208, Issue 2, 14 July 1988, Pages 275-280, http://arxiv.org/abs/hep-ph/9311321
http://arxiv.org/abs/hep-ph/9607341
Physics Letters B Volume 178, Issues 2-3, 2 October 1986, Pages 179-186
H.B. Nielsen and N. Brene, Gauge Glass, Proc. of the XVIII International Symposium on the Theory of Elementary Particles, Ahrenshoop, 1985 (Institut fur Hochenergiphysik, Akad. der Wissenschaften der DDR, Berlin-Zeuthen, 1985);
15. D. L. Bennett, "Who is Afraid of the Past" (A resume of discussions with H.B. Nielsen during the summer 1995 on Multiple Point Criticallity and the avoidance of Paradoxes in the Presence of Non-Locality in Physical Theories), talk given by D. L. Bennett at the meeting of the Cross-displiary Initiative at Niels Bohr Institute on September 8, 1995. QLRC-95-2. D. L. Bennett, arXiv:hep-ph/9607341. H. B. Nielsen and C. Froggatt, arXiv:hep-ph/9607375.
16. A J Macfarlane and Hendryk Pfeiffer, J. Phys. A: Math. Gen. 36 (2003) 2305 – 200 2232317 PII: S0305-4470(03)56335-1 Representations of the exceptional and other Lie algebras with integral eigenvalues of the Casimir operator.
17. T. van Ritbergen, A. N. Shellekens, J. A. M. Vermaseren, UM-TH-98-01 NIKHEF-98-004 Group theory factors for Feynman ... www.nikhef.nl/ form/maindir/oldversions/. ../packages/.../color.ps
18. See M. Douglas, "The statistics of string / M theory vacua", JHEP 0305, 46 (2003). arXiv:hep-th/0303194; S. Ashok and M. Douglas, "Counting flux vacua", JHEP 0401, 060 (2004). Frederik Denef; Douglas, Michael R. (2006). "Computational complexity of the landscape". Annals of Physics 322 (5): arXiv:hep-th/0602072. Bibcode 2007AnPhy.322.1096D. doi:10.1016/j.aop.2006.07.013. S. Weinberg, "Anthropic bound on the cosmological constant", Phys. Rev. Lett. 59, 2607 (1987). S. M. Carroll, "Is our universe natural?", arXiv:hep-th/0512148. M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, "Dimensionless constants, cosmology and other dark matters", arXiv:astro-ph/0511774. F. Wilczek, "Enlightenment, knowledge, ignorance, temptation," arXiv:hep-ph/0512187. See also the discussion at [1]. See, e.g. Alexander Vilenkin (2006). "A measure of the multiverse". Journal of Physics A: Mathematical and Theoretical 40 (25): arXiv:hep-th/0609193. Bibcode 2007JPhA...40.6777V. doi:10.1088/1751-8113/40/25/S22. Abraham Loeb (2006). "An observational test for the anthropic origin of the cosmological constant" (subscription required). JCAP 0605: 009. http://www.arxiv.org/astro-ph/0604242. Jaume Garriga and Alexander Vilenkin (2006). "Anthropic prediction for Lambda and the Q catastrophe" (subscription required). Prog. Theor.Phys. Suppl. 163: 245@57. arXiv:hep-th/0508005. Bib-

code 2006PThPS.163..245G. doi:10.1143/PTPS.163.245. http://www.arxiv.org/hep-th/0508005. Delia Schwartz-Perlov and Alexander Vilenkin (2006). "Probabilities in the Bousso-Polchinski multiverse" (subscription required). JCAP 0606: 010. http://www.arxiv.org/hep-th/0601162. L. Smolin, "Did the universe evolve?," Classical and Quantum Gravity 9, (1992). L. Smolin, The Life of the Cosmos (Oxford, 1997) L. Susskind, "The anthropic landscape of string theory", arXiv:hep-th/0302219. L. Susskind, The cosmic landscape: string theory and the illusion of intelligent design (Little, Brown, 2005). M. J. Rees, Just six numbers: the deep forces that shape the universe (Basic Books, 2001). R. Bousso and J. Polchinski, "The string theory landscape", Sci. Am. 291, (2004). Lubos Motl's blog criticized the anthropic principle and Peter Woit's blog frequently attacks the anthropic string landscape.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond ... *(p. 181)*
Bled, Slovenia, July 11-21, 2011

12 An Idea of New String Field Theory – Liberating Right and Left Movers*

H. B. Nielsen[1] and M. Ninomiya[1]

[1]Niels Bohr Institute, University of Copenhagen,
17 Blegdamsvej, DK 2100 Denmark
[2]Okayama Institute for Quantum Physics,
Kyo-yama 1-9-1 Kitaku, Okayama 700-0015, Japan

Abstract. We develop the idea for a new string field theory of ours that was proposed earlier in a very rudimentary form in a talk in the Symposium of Tohwa University [1]. The main point is to describe the system of strings in the Universe by means of the images of the τ-derivatives of the right and left mover parts, $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ respectively. The major progress since the Tohwa-talks [1] is to imagine a discretization of the $\tau_R = \tau - \sigma$, $\tau_L = \tau + \sigma$ variables for the right and left movers respectively. We then observe that, by using $\dot{X}^\mu_{R,L}$ values at only the discretized even-numbered sites, we can set the commutation rules for second quantization without any contradiction. In fact we can quantize the objects described by these even numbered images. A light-cone frame description of the string field theory in this way is presented.

12.1 Introduction

There has been various proposals for how to make a second quantized string theory [2–5]. It is meant a formalism in which it is possible to have a physical space analogous to the Fock space in quantum field theory so as to describe an arbitrary number of strings [6].

It is the purpose of the present article to propose the basic ideas for a different type of string field theory than the usual one. It may be denoted by the code words that in our formalism right and left movers are liberated. It is clear that the present formalism described here is different from the earlier attempts by the fact that in our present scheme we do not distinguish as many different (Fock space like) states as the other string field theories do.

In fact when two strings pass through the same point of the 25 dimensional space at the same moment of time, one just have four pieces of strings with common point as the end points.

But then it is a priori not obvious which pairs out of these four pieces are to be considered belonging to the "same string". In the earlier string field theories it is, so to speak, part of the physical degrees of freedom of the multi-string state which parts of the strings make up one string and which part of the next string does. In our formalism two states including the same pieces of strings are considered the same state. That is to say, in our Fock space like formalism we do not distinguish the two states, which only deviate

* OIQP-11-11

from each other with respect to which of the pieces of strings are on one string and which are on the the next string.

In this sense our formalism or model has much fewer Fock space like states than the usual formalism. However it should be noted that both our formalism and earlier string field theories have at the end infinitely many states. Thus in this sense of just counting it does not make much sense to say our formalism has fewer states: But it is clear that some of the states in our formalism must correspond to several states in the earlier formalisms.

One might speculate that a reason for the relatively large degree of complication in the earlier formalisms such as Kaku-Kikkawa and Witten, etc. [2–5], is due to the fact that from the point of view of our formalism, these formalisms must carry around with an appreciable amount of superfluous information,i.e. superfluous degree of freedom.

Precisely these extra degrees of freedom telling which pieces of string hang together making up string number 1, string number 2, and so on are physically what changes just in the very moment of a scattering. Indeed what must happen - in a classical way of thinking - for locality reasons is that pieces of strings are shuffled around from one string to the other one in the infinitesimally short moment of time in which only locally interacting strings interact. At this infinitesimal time the places in 25 dimensional space where some piece of string is present cannot change. There is not sufficient change for the positions or momentum densities of the pieces of string going on in the zero time allotted to it.

So the only thing that can truly happen during the basically zero length of time in which the strings are in touch and thus allowed to interact from the principle of locality, is that the various pieces of strings can be redistributed among the string number 1, string number 2, etc. But as just described then this redistribution that can happen is in our own formulation only a thought property without physical content, while in the older string field theory it is physical.

Here we have already alluded to the major point of our formalism that nothing happens during the scattering. In a way scattering is in our formalism immaterial!

Actually our string field theory makes nothing happen at all. In our formalism we, do not describe the string themselves. Rather we make our formalism to concern instead the right and left moving components of the string position vector in 25 + 1 dimensional space time, $X^M(\tau, \sigma) = X^M_R(\tau, \sigma) + X^M_L(\tau, \sigma)$. It is well known that considering closed strings - so that we can avoid here the end point reflections - the right mover $X^M_R(\tau - \sigma)$ and the left one $X^M_L(\tau + \sigma)$ only depend respectively on $\tau_R = \tau - \sigma$ and $\tau_L = \tau + \sigma$. That is to say that if we describe them then in terms of these variables - τ_R and τ_L respectively thinking of them as replacements for σ in the right mover $X^\mu_R(\tau - \sigma)$ and in the left mover $X^\mu_L(\tau + \sigma)$ not vary in any time anymore. Looking at it in this way a description using right and left movers instead of the usual $X^\mu(\tau, \sigma)$ leads to the possibility of no development at all: As time goes nothing happens provided we use the variables $\tau_R = \tau - \sigma$, $\tau_L = \tau + \sigma$ respectively for right and left moving part. During the very moments of scattering also nothing has time to happen in our point of view. Different way of concerning pieces of string organised to form string number 1, string number 2, etc. should not be counted as physical degrees of freedom (in our scheme).

Let us immediately conclude that, as we shall see below the fundamental formalism in our description does not have genuine time development. We could say that it is a kind of Heisenberg picture. However, it is important to notice that this does not mean that in practice one cannot describe scattering and calculate Veneziano amplitudes. Having prepared a state with typically several strings present one can translate such a state into Fock space like state in our formalism. Next we then ask what is the probability for finding a certain set of "outgoing" strings. To evaluate an answer for that sort of S-matrix question one now has to also translate the final state into a last state of the Heisenberg nature of our picture and further one formally calculates as if the S-matrix were the unit operator. The translations into our formalism of single string descriptions - which involves calculations of left and right movers and their quantum fluctuations - can turn out complicated enough that even the (trivial) overlap $\langle f|i\rangle$ can turn out to become the Vaneziano model.

In the following section 2, we start by describing how we think of the $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ images in a discretized way as consisting of chains of "objects". In section 3, we shall report the theorem about classical string scattering which is absolutely crucial for our formalism. It is this theorem that guarantees the conservation of the images in 25 dimensional space-time of the maps $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ from all the strings present. In subsection 3.2 we tell how our formalism is to be adjusted to having also open strings. In this case we have, contrary to the only closed string case, not two but only one image of $\dot{X}^\mu_{R,L}$ space. In section 4 we review a bit string theory and point to some problems with constructing our type of Fock space (for "objects"). In section 5 we mention the remaining reparametrization freedom of the τ_R and τ_L variables. In section 6 we discuss that because we only represent the even-numbered objects in our formalism the "odd" ones have to be recovered from the conjugate variables of the even ones. The second quantization which is the main point of a string field theory is then presented in section 7. In section 8 the lack of time dependence or the Heisenberg picture nature of our formalism is explained. This constitutes a worry for our model being totally trivial but in section 9 we suggest that in spite of that our model is presumably able to provide the Veneziano model scattering amplitudes, and if so of course it would have proven its right to be considered a string field theory. In section 10 we mention some of the technicalities still waiting before our formalism should be considered. In section 11 we present conclusion and a bit of outlook.

12.2 Our main point, the "objects"

It should be stressed that we in the present paper do not as in other string field theories start from string creating or string annihilating operators. Rather we first relate each string to an in principle infinite number of "objects" which in turn are related to the right and left mover fields $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ on the string.

For a given string state the right and left mover τ-derived fields

$$\dot{X}^\mu_R(\tau_R) = \dot{X}^\mu_R(\tau - \sigma) \tag{12.1}$$

and

$$\dot{X}^\mu_L(\tau_L) = \dot{X}^\mu_L(\tau + \sigma) \tag{12.2}$$

are because of the periodicity in σ periodic functions of respectively $\tau_R = \tau - \sigma$ and $\tau_L = \tau + \sigma$ with the same period as in σ for $X^\mu(\sigma, \tau)$.

Since the constraint conditions as is well known or seen below are $(\dot{X}^\mu_R)^2 = 0 = (\dot{X}^\mu_L)^2$ a period of $\dot{X}^\mu_R(\tau_R)$ or of $\dot{X}^\mu_L(\tau_L)$ is a closed circle on the light cone in $25+1$ dimensional Minkowski space time. (We shall for closed strings think of two different Minkowski spaces, one for R and one for L, while for open strings only one Minkowski space time).

Now we imagine a discretization of the parameters τ_R and τ_L. I.e. we approximate them by some integers in some way, and we can distinguish points in the discretization chains as being "even" or "odd". To each such discretization point we associate, what we call, an "object". Thus we got to each string associated a right and left chain of "objects" on the light cones in one or two $25+1$ dimensional Minkowski spaces. Instead of going for directly making an annihilation operator for a whole string our main idea is to make annihilation operators for the "objects" and then you may construct an annihilation operator for a whole string as the product of those for all the associated "objects".

It should be noted that there are a couple of small technical modifications to the just mentioned construction of "objects":

1)We only make the Fock space to describe our theory from the "even" numbered "objects"

2)We shall rather use integrals over small regions about the discretization points than $\dot{X}_R^\mu$ and $\dot{X}_L^\mu$ themselves as the "positions" for the "objects"

Also have in mind that since there are (∞) many objects per string the number of strings is not related to the number of "objects".

12.3 Our basic theorem for classical string scattering

This basic theorem stating, that the images of $\dot{X}_R^\mu$ and $\dot{X}_L^\mu$ are totally conserved for a system of strings even if they scatter, made up the major part of the talk in Tohwa symposium [1]. The rest of the proceedings of Tohwa were, from our present point of view, a too complicated and not the right attempt to our present string field theory, although in a very basic sense the present article repeats the main ideas in the Tohwa proceedings.

Our theorem sounds: For classical dual closed strings the over all the strings present, (enumerated by $i = 1, 2, \cdots, N$) united images of the maps $X_R^\mu : \{\tau_R\} \to 25+1$ dimensional Minkowski space, i.e. $\{\dot{X}_R^\mu(\tau_R) | \tau_R \in \text{"period interval"}\}$ is constant as a function of time X^0 even when scatterings occur, (but may be only up to a null-set).

Here "period interval" means the interval over which the variable $\tau_R = \tau - \sigma$ runs around the string in question. Often one would use a notation $\tau_R \in$ "period interval" $= [\tau - 2\pi, \tau]$.

When we talk about a moment of time X^0 in this theorem the idea is to consider all these (σ, τ)-combinations for the various strings for which the $X^0(\sigma, \tau) = X^0$ the for the moment characteristic time X^0 value. Typically this will mean that there is for a fixed moment X^0 relations between σ and τ being different for the different classical strings. It is, however, possible that we could have chosen the specific gauge - if we wish so even for all the strings present - that $X^0(\sigma, \tau) = \tau$. In this gauge - which is rather pedagogical - we can simply consider τ being the time, so that considering the moment X^0 means considering $\tau = X^0$.

That we in the theorem assume the strings to be "dual" means that we let their motion be described by the usual string theory dynamics, say by the Nambu action. Since we as our main point talk about the right and left movers, $X_R^\mu(\tau_R)$ and $X_L^\mu(\tau_L)$, we must of course in order to get to them choose some gauge - in the usual way too - so that the equations of motion for $X^\mu(\sigma, \tau)$ simplifies to the massless Klein-Gordon form

$$(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2}) X^\mu(\sigma, \tau) = 0 \tag{12.3}$$

or

$$(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma})(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma}) X^\mu(\sigma, \tau) = 0 \tag{12.4}$$

so that we have the general solution to $X^\mu(\sigma, \tau)$ for each separate string on the form $X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma)$. These functions $X_R^\mu(\tau - \sigma)$ and $X_L^\mu(\tau + \sigma)$ are not completely uniquely determined from the here given definition in as far as, we of course could add a constant $(25+1)$-vector to $X_R^\mu(\tau - \sigma)$ and subtract the same vector from $X_L^\mu(\tau + \sigma)$. This is one of the "technical" reasons that we formulate the theorem using $\dot{X}_R^\mu(\tau - \sigma) = \frac{d}{d\tau} X_R^\mu(\tau - \sigma)$ rather than simply by using $X_R^\mu(\tau - \sigma)$ itself. The constant vector which could be added and subtracted is namely differentiated away (to 0) and thus this ambiguity mentioned does not influence $\dot{X}_R^\mu(\tau - \sigma)$, but only $X_R^\mu(\tau - \sigma)$ itself.

That the strings are assumed "classical" means that we do not in the theorem like in usual string theory invoke quantum mechanics, but consider a theory of ideal infinitely thin strings obeying the equations of motion derived from the Nambu-action in a classical mechanics way.

The concept "image" in the theorem simply is meant in the function theory sense of being the set of points in the target space (here the 25+1 dimensional Minkowski space-time M_{25+1}) into which some τ_R or τ_L is mapped. For string number i say in the gauge with $\tau = time$ mentioned the "image" is the image for the function

$$\dot{X}^{\mu}_{R}(\tau - \sigma) : [0, 2\pi] \to M_{25+1}.$$

This image is of course formally written

$$\text{“image”} = \left\{ \dot{X}^{\mu}(\tau_R) | \tau_R \in \text{“period interval”} \right\} \subseteq M_{25+1}$$

where

$$\text{“period interval”} = [\tau - 2\pi, \tau].$$

As we are in the theorem working classically there is of course nothing special about just the $26 = 25 + 1$ dimensions required by bosonic string theory quantum mechanically. So that we talk about $25 + 1$ dimensions here shall only be considered a pedagogical trick to guide the quantum string theorist to see how our thinking is related to usual quantum string theory.

When we only formulated the theorem for $\dot{X}^{\mu}_{R}(\tau - \sigma)$ but not for $\dot{X}^{\mu}_{L}(\tau + \sigma)$ also, it were just for simplicity of formulation. Of course the completely analogous theorem holds for the left mover $X^{\mu}_{L}(\tau + \sigma)$, i.e. for the united images for these left mover functions, is also valid.

12.3.1 The proof of the Theorem.

The proof may be performed by writing down the right and left mover representations for the various strings number $i = 1, 2, ..., N$ and then use that classically the scattering must take place by reshuffling of the various pieces of strings specified by the point at which two scattering strings have a point in common. We shall, however, not go through such a calculation, which may be found in [1].

It may, however, be easier to argue as we already essentially did in the introduction: classically infinitely thin strings will generically only meet and interact momentarily. So all the time, seen from some frame, except for a null-set the strings do not touch. During the major time intervals in between the strings are completely free. For closed strings - as we assume at first - e.g. $\dot{X}^{\mu}_{R}(\tau_R)$ is a periodic function of τ_R with the period of that for σ on the string. That implies that the image from each string under $\dot{X}^{\mu}_{R}(\tau_R) = \dot{X}^{\mu}_{R}(\tau - \sigma)$ is actually unchanged. Varying τ just means shuffling σ around the circle, i.e. period. This is true for all the strings string 1, string 2, $\cdots$ present at the time considered. Thus also the united image I_{mR} is constant as long as time stays inside one of the intervals in which there are no interactions. But that were up to a null-set all time, and during these intervals the $\dot{X}^{\mu}_{R}(\tau_R)$ and $\dot{X}^{\mu}_{L}(\tau_L)$ cannot change either. So we have proven the theorem that I_{mR} and I_{mL} are totally independent of the cutting moment of time space-like surface. One should note that this theorem already means that there are two time conserved quantities $\dot{X}^{\mu}_{R}(\tau_R)$ and $\dot{X}^{\mu}_{L}(\tau_L)$ per point on the strings. Since this is essentially the dimension of the phase space of the string-system, so many conserved quantities mean that at least in the first crude estimate the many string system is a solvable mechanical system!

12.3.2 Open string cases

Above we considered a string theory with only closed strings. The extension to one with also open strings is, however, quite easy, but interestingly turns out to mean that we have only one common mover image $I_m = I_{mR} \bigcup I_{mL}$ to consider. It is only this united mover-image which is conserved in the case of there also being open strings.

Locally on the interior of the string there is no difference between an open and closed string and we can for both open and closed find the solution of the form

$$X^{\mu}(\sigma,\tau) = X_R^{\mu}(\tau-\sigma) + X_L^{\mu}(\tau+\sigma). \tag{12.5}$$

The part $X_R^{\mu}(\tau-\sigma)$ represents a pattern moving along the string "to the right" if we consider τ the time. But then when this pattern reaches the end of an open string this pattern a priori disappears because there is then no more any string on which to find just that pattern of $X_R^{\mu}(\tau-\sigma)$. This would a priori spoil our theorem for open strings, and indeed it does spoil it. However, we can very luckily make a very similar theorem working also for open strings.

Indeed the point is that at the end points of the open strings we have the well known boundary condition

$$0 = \frac{\partial X^{\mu}(\sigma,\tau)}{\partial\sigma} = \frac{\partial X_R^{\mu}(\tau-\sigma)}{\partial\sigma} + \frac{\partial X_L^{\mu}(\tau+\sigma)}{\partial\sigma} = -\dot{X}_R^{\mu}(\tau-\sigma) + \dot{X}_L^{\mu}(\tau+\sigma) \tag{12.6}$$

valid for the endpoint σ-values, say $\sigma = 0, \pi$.
For example $\sigma = 0$ we simply obtain

$$\dot{X}_R^{\mu}(\tau) = \dot{X}_L^{\mu}(\tau). \tag{12.7}$$

This equation must actually be true for all τ and thus the right mover function $\dot{X}_R^{\mu}(\tau)$ and the left mover function $\dot{X}_L^{\mu}(\tau)$ do actually coincide. It is now easy to see that as some pattern in $\dot{X}_R^{\mu}(\tau-\sigma)$ runs into the end point an individual pattern just runs out of the same endpoint now as represented by the left mover $\dot{X}_L^{\mu}(\tau+\sigma)$ instead of by the right mover. But that means that the total image of *both* $\dot{X}_R^{\mu}(\tau-\sigma)$ *and* $\dot{X}_L^{\mu}(\tau+\sigma)$ *becomes constant in time* just as in our theorem for the closed strings. Now it is just that you have to unite both the $\dot{X}_R^{\mu}(\tau_R)$ images and the $\dot{X}_L^{\mu}(\tau_L)$ images before you the set $I_{mR} \bigcup I_{mL}$ which is conserved.

Of course when in the only closed string theory you have separate conservation in time of I_{mR} and I_{mL} then also the union of these two sets will be conserved. In this sense you have a stronger statement in the only closed string than in a model with also open strings.

Concerning the question of whether the string theory also is essentially solvable due to our theorem in the open string theory one has to contemplate if the curve-structure total image $I_{mR} \cup I_{mL}$ has enough information to match the degrees of freedom of the strings. In fact it turns that the open string theory is also solvable quite similarly to the closed string one. In fact the crucial point is that there is in the union set $I_{mR} \bigcup I_{mL}$ for every little (infinitesimal) piece of string both a little piece corresponding to the right mover pattern and the left mover pattern in that piece. Theory we get that - ignoring the discussion of the time and longitudinal dimensions, and only caring for the transverse dimensions - the development of the strings is totally - except for null-set - predicted by the conserved quantities, namely our $I_{mR} \bigcup I_{mL}$.

So in both cases, with and without open strings, we argue that string theory is essentially solvable.

12.4 How to make a string field theory

With the understanding in mind that the whole state classically of a system of strings is describable by the unification sets I_{mR} and I_{mL} (in the closed string case), we at first get to the idea that a second quantized string theory should be given by a Fock space (system) in which one can put some "objects" into two Minkowski-spaces M_{25+1R} and M_{25+1L} in a way as to form 1-dimensional curves. The idea then is that these curves of objects in M_{25+1R} and M_{25+1L} shall then be identified with the unified images I_{mR} and I_{mL}.(see figure 1, in section 11).

Remembering the usual string theory constraints

$$(\dot{X}^{\mu}_{(\sigma,\tau)})^2 + (X'^{\mu}(\sigma,\tau))^2 = 0 \,,$$
$$\dot{X}^{\mu}(\sigma,\tau)X'_{\mu}(\sigma,\tau) = 0 \,, \tag{12.8}$$

which in terms of the right and left mover fields become

$$(\dot{X}^{\mu}_{R}(\tau_R))^2 = 0 \tag{12.9}$$

and

$$(\dot{X}^{\mu}_{L}(\tau_L))^2 = 0 \tag{12.10}$$

we see that the union of images I_{mR} and I_{mL} are contained in the light cones in the two Minkowski spaces, M_{25+1R} and M_{25+1L}. Thus it will only be necessary to have the possibility to put the "objects" that shall make up the I_{mR} and I_{mL} onto these light cones.

12.4.1 More quantization.

With the crude starting idea of representing the united images I_{mR} and I_{mL} by putting 1-dimensional curves of what we call "objects" onto the light cones in the two Minkowski space-times we of course now want to translate the idea into a truly quantized string theory so as to obtain along these lines a true string field theory.

What we lack in the quantization compared to the Fock space describing "objects" forming chains representing the I_{mR} and I_{mL} is to make use of $X^{\mu}_{R}(\tau_R)$ and $X^{\mu}_{L}(\tau_L)$ being operators, and thus of course having the correct commutation relations

$$\left[\dot{X}^{\mu}_{R}(\tau_R), \dot{X}_{R}(\tau'_R)\right] = \delta'(\tau_R - \tau'_R)2\pi\alpha' \tag{12.11}$$

Thinking now of the analogue of second quantization by means of a Fock space of a theory (quantum field theory) constructed from the wish to form states with an arbitrary number of particles, we remember that the Fock-space construction must be based on a system of single particle basis states. These basis states in turn are (typically) eigenstates of a system of *commuting* operators. For example one can construct the usual quantum field theory by three momentum eigenstates. Then the Fock-space basis vectors can be characterized by a number of particles distributed into a set of such three momentum states

$$|\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_N\rangle \tag{12.12}$$

In analogy to this Fock-space construction we shall now - for the purpose of our string field theory - construct a Fock-space based on states characterized by a distribution of a number of the "objects" - we talked about - into states described by points on the light cones in the two Minkowski spaces M_{25+1R} and M_{25+1L}. This would then mean that states, on the light cones, should be described as eigenstates of the $\dot{X}^{\mu}_{R}(\tau_R)$ or $\dot{X}^{\mu}_{L}(\tau_L)$, now considered as operators.

12.4.2 The commutation problem.

But then we meet the problem, that since the $\dot{X}^\mu_R(\tau_R)$ for different τ_R values do not commute, so that we cannot just consider a single set of 26 operators commuting with themselves and each other. If namely $\dot{X}^\mu_R(\tau_R)$ and $\dot{X}^\mu_R(\tau_R)$ do not commute they cannot be used directly to construct a basis. The trick we shall use to overcome this problem is the following:

We first imagine that we discretize the variables $\tau_R = \tau - \sigma$ and $\tau_L = \tau + \sigma$ so that we imagine τ_R and τ_L to take in some way or another only integer values. For example it could be that τ_R and τ_L were taken to be a constant times integers. Really we shall take some more complicated rule for discretization. Then the important point of our trick is to *throw away the points of discretized τ_R and τ_L values for which the integer is odd.* We throw these points away in the sense that we do not associate an "object" directly to the "odd" points, but rather only to the even τ_R or τ_L points.

The crux of the matter then is that if we discretize the δ'-function commutator $[\dot{X}^\mu_R(\tau_R), \dot{X}^\nu] = 2\pi\alpha'\eta^{\mu\nu}\delta'(\tau_R - \tau'_R)$ in the obvious way leads to that this commutator is only non-zero when the integers corresponding to τ_R and τ'_R deviate from each other by ± 1. Thus if we only keep in our attempts to associate "objects" directly by the τ_R's associated with the even integers we avoid the problem that the different relevant $\dot{X}^\mu_R(\tau_R)$'s do not commute.

12.5 Inclusion of reparametrization in τ_R and τ_L

After having presented the crude idea of discretizing the right mover variable $\tau_R = \tau - \sigma$ and the left mover variable $\tau_L = \tau + \sigma$ we should like to be a bit more specific taking into account that there is a left over part in Nambu action originally present reparametrization of the coordinates (σ, τ), namely that we can even after the gauge choice still transform

$$\begin{aligned} \tau_R &\to \tau'_R = f(\tau_R)\ , \\ \tau_L &\to \tau'_L = g(\tau_R)\ . \end{aligned} \tag{12.13}$$

Here the transformation functions f and g can be any pair of increasing functions consistent with the periodicity conditions for τ_R and τ_L. In literature one often works with complexified τ_R and τ_L under names z and $\bar{z}$ and correspondingly f and g being analytic, but we use the more "physical" τ_R and τ_L being real.

In principle we could fix this left over reparametrization and discretize in any way we like, but it is rather suggestive to define as the variables to be represented as the position of the "objects" on the light cones in M_{25+1R} and M_{25+1L} not simply $\dot{X}^\mu_R(\tau_R)$ and $\dot{X}^\mu_L(\tau_L)$, but rather

$$\int_{\substack{\text{REGION OF}\\ \text{DISCRETIZED}\\ \text{POINT}}} \dot{X}^\mu_R(\tau_R) d\tau_R\ , \tag{12.14}$$

and

$$\int_{\substack{\text{REGION OF}\\ \text{DISCRETIZED}\\ \text{POINT}}} \dot{X}^\mu_L(\tau_L) d\tau_L\ . \tag{12.15}$$

That is to say: By any discretization there is a natural range covered by the n'th point $\tau_R(n)$ in the series of discrete points, say from the middle point between $\tau_R(n-1)$ and $\tau_R(n)$, i.e. $\frac{1}{2}(\tau_R(n-1) + \tau_R(n))$ to the middle point between $\tau_R(n)$ and $\tau_R(n+1)$, i.e. $\frac{1}{2}(\tau_R(n) + \tau_R(n+1))$. To reduce the reparametrization dependence - but we do not at first remove it - we could then define our "object" - position $X^\mu_R(n)$ by the integral over the "covered region" by the $\tau_R(n)$ included discrete point

$$J^\mu_R(n) \hat{=} \int_{\frac{1}{2}(\tau_R(n)+\tau_R(n-1))}^{\frac{1}{2}(\tau_R(n)+\tau_R(n+1))} \dot{X}^\mu_R(\tau_R) d\tau_R\ , \tag{12.16}$$

and analogously for the left mover

$$J^{\mu}_{L}(n) = \int_{\frac{1}{2}(\tau_L(n)+\tau_L(n-1))}^{\frac{1}{2}(\tau_L(n)+\tau_L(n+1))} \dot{X}^{\mu}_{L}(\tau_L)d\tau_L\ . \tag{12.17}$$

By doing so we avoid some of the reparametrization dependence, so that the only reparametrization dependence left comes from the reshuffling of the separation points as $\frac{1}{2}(\tau_R(n)+\tau_R(n-1))$ and $\frac{1}{2}(\tau_L(n)+\tau_L(n-1))$ separating the small regions "objects" by the discrete points.

This remaining reparamentrization dependence could be suggestively gauge fixed by fixing one of the 26-vector components of $X^{\mu}_{R}(n)$ and of $X^{\mu}_{L}(n)$. One could e.g. fix the time components $X^{0}_{L}(n)$ and $X^{0}_{R}(n)$ to some small constants, but it would be better and match better with more usual string theory formalism to use infinite momentum frame. In infinite momentum frame coordinates for the 25 + 1 dimensional target space time, we choose the coordinates $X^{+}, X^{-}, X^{1}, X^{2}, \ldots, X^{24}$. The metric tensor $\eta_{\mu\nu}$ is taken in this infinite momentum frame as

$$\begin{aligned} \eta_{\mu\nu} &= -\delta_{\mu\nu} \qquad \text{for} \quad \mu,\nu = 1,2,\ldots,24 \\ \eta_{+-} &= 2 \end{aligned} \tag{12.18}$$

In this infinite momentum frame we suggestively fix $J^{+}_{R}(n)$ and $J^{+}_{L}(n)$ to some small values as the parametrization.

If we only keep the even n integrals $J^{\mu}_{R}(n)$ and $J^{\mu}_{L}(n)$ and replace as a sufficiently good approximation for the light discretization the delta prime function $\delta'(\tau_R - \tau'_R)$ in the commutator for the derivatives $\dot{X}^{\mu}_{R}(\tau_R)$ by a discretized approximation we can arrange that all the kept even n $J^{\mu}_{R}(n)$'s commute with each other. What does not commute is the $X^{\mu}_{R}(n)$ for say an even n are the two nearest neighbors $X^{\mu}_{R}(n-1)$ and $X^{\mu}_{R}(n+1)$ (which are "odd" and thus not directly included in the Fock space construction).

12.6 On recovering the odd-n $X^{\mu}_{R}(\tau_R(n))$'s

In order to achieve an effective commutation of the different

$$J^{\mu}_{R}(n) = \int_{\frac{1}{2}(\tau_R(n-1)+\tau_R(n))}^{\frac{1}{2}(\tau_R(n)+\tau_R(n+1))} \dot{X}^{\mu}_{R}(\tau_R)d\tau_R,$$

we threw out of consideration at first the $J^{\mu}_{R}(n)$ quantities for odd values of the integer n. Since we argued in the classical discussion in the beginning of this article that the full function set $\dot{X}^{\mu}_{R}$ is needed to describe the strings and that it would thus not be satisfactory to leave out every second point (after discretization), some way of recovering the $J^{\mu}_{R}(n)$-degrees of freedom for n being odd is needed. The point of this recovery of the odd n $J^{\mu}_{R}(n)$'s being some physical variables must of course be that they are essentially the canonically conjugate variables to the even-n $J^{\mu}_{R}(n)$'s. It has turned out that we have some technical details in order to fully realize such a correspondence between the odd n $J^{\mu}_{R}(n)$-variables and the conjugate momenta to the even-n $J^{\mu}_{R}(n)$'s. So let us in the present article be satisfied by establishing this interpretation of the odd $X^{\mu}_{R}(n)$-variables as being given by the conjugate momenta of even $X^{\mu}_{R}(n)$-variables in the crudest approximation of looking only locally along a string, but ignoring the problems of restricting to $\dot{X}^{\mu}_{R}(\tau_R)$ to be for each string a periodic function, and also at first the problems of the constraint equations. Under this simplifying "approximation" we see that we can simply construct an odd-n $X^{\mu}_{R}(n)$ to be proportional to the difference between the conjugate momenta of the $J^{\mu}_{R}(n \pm 1)$ for the nearest neighbors in the discretized series (representing τ_R). That is to say that if we denote by

$$\Pi_{\mu R}(m) \tag{12.19}$$

the canonical conjugate of $X_R^\mu(m)$ then we see that we can use - up to an unimportant constants or quantities commuting with the even-m $X_R^\mu(m)$'s - that

$$J^\mu(n) \propto \Pi_\mu(n+1) - \Pi_\mu(n-1) \, . \tag{12.20}$$

We can note that considering locally the situation along the string there is - in first approximation - a match of degrees of freedom: There are (in the limit of many discretized values per unit length in τ_R) approximately equally many even and odd numbers locally. Of course there are equally many conjugate variables $\Pi_{\mu R}(m)$ (with m even) as there are variables $X_R^\mu(m)$ to which they are conjugate. Thus there in first approximation just the right number of conjugate variables to represent the odd-n $J_R^\mu(n)$ quantities. To get the right commutation property approximating a derivative of a delta that a $X_R^\mu(n)$ shall commute with all the other $X_R^\mu(m)$'s except for the two nearest neighbors $X_R^\mu(m \pm 1)$, we of course need the odd $X_R^\mu(m)$ to be a linear combination of the two neighboring conjugate variables. The sign must be so that it is the difference to simulate the derivative of delta function $\delta'(\tau_R - \tau_R')$.

12.7 Second Quantization

Let us stress the main idea of our string field theory attempt - before going into the technical details and problems of interpretation in reality - by mentioning creation and annihilation operators for the already earlier mentioned “objects”. In fact whenever we have as suggested above a Fock space described theory in which the states are described by how some “objects” are distributed in some set of variables, X_R^μ (we have here deliberately left out the enumeration of the “object” n) we can, provided the objects are either bosons or fermions, construct for every possible value of the set of variables X_R^μ a pair of a creation and an annihilation variable $a^+(X_R^\mu)$ and $a(X_R^\mu)$. Of course the meaning is as usual that by acting with $a^+(X_R^\mu)$ an “object” is put into the state for a single “object” in which the X_R^μ-variable takes the value X_R^μ mentioned in the creation operator symbol $a^+(X_R^\mu)$. Similarly of course the annihilation operator $a(X_R^\mu)$ removes one “object” with variables X_R^μ if there are any such “objects”; if not it just gives zero.

Analogous to how one in quantum field theory can construct second quantized field operators $\phi(\vec{x})$ as Fourier-expanded in the annihilation operators $a(\vec{p})$ we can in our formalism use Fourier transforms of our $a(X_R^\mu)$-operators to construct annihilation operators for “objects”with given values of the conjugate momenta ($\Pi_{\mu R}$). (For simplicity you should here rather think of a non-relativistic quantum field theory than a relativistic one with all the for our analogy at first complicating details of a Dirac sea, etc). Once we can Fourier transform to obtain creation $\phi^+(\Pi_{\mu R})$ and annihilation operators for “objects” with definite $\Pi_{\mu R}-$ eigenvalues, we can easily extend that to make annihilation $a_0(X_{0R}^\mu)$ and creation operators $a_0^+(X_{0R}^\mu)$ for odd-“objects” in the space of “odd-object” X_{0R}^μ into which we let the $X_R^\mu(m)$ for m odd take their values.

At least if we have in mind some chain of the “objects” present, then we can corresponding to such a chain construct a series of odd-numbered “objects” by means of the neighboring object Π_R^μ. This means that by creating by means of the creation operators $a_e^+(X_{eR}^\mu)$ an appropriate superposition of states representing chains of “even” objects one gets corresponding to that also a distribution for the odd-objects. Of course since “odd” and “even” J_R^μ's like the $\dot{X}_R^\mu(\tau_R)$'s, which they represent, do not commute and thus of course one cannot produce by the object creating operators $a_e^+(X_R^\mu)$ a one string state in the Fock space with well defined values for the X_R^μ's for both "odd" and "even". If one, however, is satisfied to make only either the even points or odd points it should be possible. This is just analogous to that you can only create particles into states in agreement with Heisenberg's uncertainty relations.

We shall of course have in the case of only closed string theories both have a Fock space in which one can put in "objects" being interpreted as having "positions" given by the right mover X_R^μ and another Fock space being interpreted as for left mover even "objects". The full Fock space for the total theory of only closed strings shall then be the Cartesian product $H_R \otimes H_L$ of the left mover and right mover Fock spaces, where we denoted the Fock space for "even" rightmover objects H_R while that for leftmovers were denoted H_L.

If one considers a string theory with open strings also, the rightmover waves get converted into left mover waves on the strings at the end points. Thus our theorem only constitutes the conservation of the X_R^μ's - or $\dot{X}_R^\mu$ image and the X_L^μ's or $\dot{X}_L^\mu$ image together. We must therefore in the open string theory put together "objects" for both left and right movers into a single sort of "objects" - and still we only represent the "objects" in the formalism, which correspond to even points on the τ_R and τ_L discretizations. - so as to only construct one single Fock space for "objects" H_μ. Here H_μ is a Fock space with states constructed as states with a set of a mixture of right mover and left mover "objects" (leaving only "even type") are present. We do not even distinguish in our formalism for the open-string case between "objects" being right mover or left mover. If we consider them - as we shall for the $\dot{X}_R^\mu, \dot{X}_L^\mu$ degrees of freedom; contrary to Fermionic, Neveu-Schwarz-Ramond - say bosons we postulate even symmetry of the wave function (= Fock space states) under the permutation of a rightmover "object" with a left mover one, and not only under permutation of right with right and of left with left.

12.8 Lack of Time Development

We have now above sketched how to build up a Fock space for describing a string theory, meaning that we have put forward the idea for a "string field theory". If should be stressed that this formalism idea of ours is (in an abstract sense) a Heisenberg picture formalism. By this we mean that the Fock space state is not like in the Schrødinger picture to be developed in time. Really we have transformed the string theory so much that we do not have any time in it any longer. Actually the reader will remember that it were absolutely crucial for our string field theory that the equation of motion for the field $X^\mu(\sigma,\tau)$ on the string were solved (essentially) by writing this field $X^\mu(\sigma,\tau)$ as a sum of a right mover $X_R^\mu(\tau-\sigma) = X_R^\mu(\tau_R)$ field and a left mover field $X_L^\mu(\tau+\sigma) = X_L^\mu(\tau_L)$. That means, that we solved the equations of motion and went into a description using $X_R^\mu(\tau_R)$ and $X_L^\mu(\tau_L)$ which has not even the time τ for the internal string motion in it any more.

But now the true greatness of the observations leading to our formation were (above) that *even during the scattering processes* - considered classically - *the images of the* $\dot{X}_R^\mu(\tau)_R$ *and* $\dot{X}_L^\mu(\tau_L)$ *remain unchanged* up to a null-set. That is to say even with scattering there should (in the considered approximation) be no change in the description in terms of our Fock spaces). So in our formalism - if taken as an ontological model - the scattering process is not in correspondence with anything physically existing. One may be worried that we in our formalism have thrown away too many degrees of freedom, although it is in principle only a nul-set compared to the rest of the degrees of freedom - the ones on the pieces of the strings not hit in the scattering -. You remember also, that it is precisely this "null-set" of information which in fact concerns how the different pieces of strings are glued together, which distinguishes our string field theory from the earlier competing string field theories of Kaku-Kikkawa and Witten. In the following section we shall discuss the problems of our string field theory.

12.9 Some Problems and Explanations

We can indeed be worried by getting proposed a model string field theory, in which the scattering of strings is only something, we think about, but which has no ontological content.

Actually we shall, however, claim that this "no true scattering" in the ontological sense is indeed in fact o.k. The first argument suggesting that this type of model is indeed acceptable as a string field theory is to point out that one can in principle - and we hope in a later publication show that it is quite easily actually possible to do the calculations - obtain the Veneziano model scattering amplitudes from our string field theory.

In principle we have in our string field theory a Heisenberg picture formulation. If one wants to calculate an S-matrix in a Heisenberg picture model one must implement the initial state $|i\rangle$ for which we want $\langle f|S|i\rangle$ as a $t \to -\infty$ or initial state. This means that if in the state $|i\rangle$ we have a number of strings in various states, we at first think of $|i\rangle$ as being of the form

$$|i\rangle = |\{X_i^\mu(\sigma, \tau_{\text{init}})\}\rangle \tag{12.21}$$

if the string states were given as eigenstates of the $X_i^\mu(\sigma, \tau = \tau_{\text{init}})$ for string number i in the initial state. Really an eigenstate of the $X^\mu(\sigma, \tau_{\text{init}})$ is not realistic because there will in practice rather be eigenstates of mass squares of the strings. That would mean states which could be written as superpositions of eigenstates for $X^\mu(\sigma, \tau)$'s. The point is that we should write explicitly the zero-point fluctuation into the superposition coefficients. At least it requires a little bit of calculation to transcribe the practically interesting string states into eigenstates of the string position $X^\mu(\sigma, \tau)$ or what for our purpose is more important into eigenstates for $X_R^\mu(\tau - \sigma)$ and $X_L^\mu(\tau + \sigma)$; but here it must be kept in mind that $X_R^\mu(\tau - \sigma)$ e.g. do not commute with themselves for different σ-values (or $\tau_R - \sigma$ values). So we can first rewrite a given string state - as e.g. a mass eigenstate for string - into eigenstates of the right and left mover $\dot{X}_R^\mu(\tau_R)$ $X_L^\mu(\tau_L)$ say *after we have discretized* and then use the proposed trick in this article that *only keeps the even-numbered points of* τ_R *and* τ_L *in the discretization*.

The important point to stress it that one can by a moderate amount of calculation translate a given initial state $|i\rangle$ in forms of e.g. mass and momentum eigenstates for (isolated) strings into a linear combination of eigenstates of the $\dot{X}_R^\mu(n)$ and $\dot{X}_L^\mu(n)$ for n even, which represent $\dot{X}_R^\mu(\tau_R)$ and $\dot{X}_L^\mu(\tau_L)$. Imagining that we have written the states of all the strings in the incomming state $|i\rangle$ into eigenstate of the even-numbered $\dot{X}_R^\mu(n)$ and $\dot{X}_L^\mu(n)$ we can then set up a Fock space state, in which there are present the "objects" corresponding to just the strings present in the state $|i\rangle$. The main point is that states given as string states for several strings such as $|i\rangle$ can be translated into superpositions of states with given distributions of "objects" and thereby into a Fock space state in our model.

Analogously one can of course corresponding to a thinkable final state $|f\rangle$ described as some - well possibly different - number of strings, construct a state in our Fock space (or in closed strings only case in the Cartesian product of our two Fock spaces) for "objects", corresponding to $|f\rangle$.

If we denote the Fock space (or Cartesian product of two) states correspondence to $|i\rangle$ and $|f\rangle$ as

$$\begin{aligned} |i\rangle &\to |i\rangle_{\text{Fock}} \\ |f\rangle &\to |f\rangle_{\text{Fock}} \end{aligned} \tag{12.22}$$

then the usual S-matrix element $\langle f|S|i\rangle$ becomes in our Heisenberg picture model

$$\langle f|S|i\rangle = \langle f_{Fock}|i\rangle_{Fock}. \tag{12.23}$$

That is to say, that formally we use the unit operator as the S-matrix. As we have, however, already pointed out the rewriting from the physical string language to our Fock space involves making explicit zero-point fluctuations of the strings and thus is sufficiently complicated, that the mathematical expressions, we shall finally obtain as function of the external momenta (i.e. the momenta of the strings in the initial $|i\rangle$ and final state $|f\rangle$) will not be completely trivial. Rather it is indeed not at all excluded that we should indeed, as physically expected, since we after all started with a string theory, obtain a Veneziano model scattering amplitude for the S-matrix.

Indeed we can see some intuitive arguments, that an integral form of the type expressing the Veneziano models will appear naturally in the evaluation of the overlap $\langle f_{Fock}|i_{Fock}\rangle$.

In spite of the zero point fluctuations, we must include in the descriptions of $|i_{Fock}\rangle$ and $|f_{Fock}\rangle$, there is still a correlation between neighboring "even" "objects" or ($X_R^\mu(n)$ and $X_L^\mu(n)$), so that an overlap contribution in which the series of "objects" lying along a pieces of string in one of the strings in $|i\rangle$, does go into a similar piece in the series in one of the strings in the final state $|f\rangle$ will become a larger contribution (numerically) compared to a contribution in which the piece of string in $|i\rangle$ is matched to several pieces of strings in the $|f\rangle$ strings. This favouring with respect to getting large overlap contribution, when neighboring "objects" in the $|i\rangle$-state are analogously neighboring in the $|f\rangle$-state, is of course what corresponds to a continuous curve being transformed from $|i\rangle$ to $|f\rangle$. That is to say, that if we represent a chain or curve ($\tau_R \in a$ period) of $\dot{X}_R^\mu(\tau_R)$ fluctuating around a classical curve, then the favoured overlap is the one in which the curves in $|i\rangle$ are transformed so in a way with as little splitting as possible into the $|f\rangle$ state. This kind of speculation suggests, that we shall evaluate the overlap $\langle f_{Fock}|i_{Fock}\rangle$ (giving us the S-matrix element $\langle f|S|i\rangle$) as a perturbation series, in which the successive terms are classified according to the number of breaking points in the (classical representative) curves of "objects" (on the light cone in $25 + 1$ space time). That is to say the dominant term should be the one with the lowest number of breaks; the next has more breaks and there are in principle an infinite number of terms having more and more breakings.

By this consideration of using for calculating the main term is some S-matrix element $\langle f|S|i\rangle = \langle f_{Fock}|i_{Fock}\rangle$ the contribution in which there has been made the lowest number of breakings and rejoining the $\dot{X}_R^\mu(\tau_R)$ and $\dot{X}_L^\mu(\tau_L)$ curves from the $|i\rangle$ state to go into matching the curves in the $|f\rangle$-state. But now even for a minimal number of breakings of the $\dot{X}_{R-}^\mu$ and $\dot{X}_{L-}^\mu$ curves in order to go from $|i\rangle$ to $|f\rangle$ the needed breaking places on the chains of "objects" can be placed at different places in the chains. Since even in the speculated approximation of the overlap contributions with fewest breaks dominating the different contributions just deviating by different places on the chains of the breaking will contribute - in principle - in a comparable way and all have to be included in this dominant approximation.

Our suggestion - which really has to be true - is that this summation over *different breaking places on the chains* but keeping the total number of breaking the same (namely the minimal number needed) is of course in the limit of the discretization being in infinitesimal steps truly an integral that shall turn out to be identifiable with the integrations in the Veneziano model for the scattering in question.

This suggestion is almost unavoidably true from the point of view that we have constructed our model from describing physical strings and that is already well understood how these integrations of the Veneziano model are related either to true Minkowski space gluing and splitting of strings, or better to complex analytical continuation of such integrals of the points of splitting and joining.

In this article we have formulated the theory in Minkowski space-time as is the true physical situation, but in formal string theory one often write instead the integrals over some $z = \tau + i\sigma$ and $\bar{z} = \tau - i\sigma$ rather than using the real valued $\tau_R = \tau - \sigma$ and $\tau_L = \tau + \sigma$ used above in the present article, but it is for the moment our belief that the difference is just a technical detail of a contour deformation of the integration.

12.10 Waiting Technicalities

We must admit that we have in the present only delivered the crude but crucial ideas for our string field theory, but we have not really yet put forward the detailed formalism, as is indeed required. Basically we have of course to go through in a string field theory (=a second quantized single string theory) the various gauge choices, constraint, and anomaly problem one has in a single string theory.

We shall really go not through these problems of proper quantization in the present article, but rather postpone it to a later article or leave it for a reader to formulate our model in some cleverly chosen gauge etc.

Let us, however, here give an idea about what problems we have in mind in order to bring our ideas to provide a definite string field theory formalism:

1) To make the formalism definite one has either to choose the reparametrization of (σ, τ) inside the strings completely and even to discretize in a definite way so that our $X_R^\mu(n)$'s and $X_L^\mu(n)$'s obtain a well defined meaning (rather than being gauge-monsters) or one has to formulate a remaining part of the gauge freedom is propagated into our "object" and later Fock-space formulation. By this is meant that if we do not choose the gauge (meaning reparametrization of (σ, τ)) to the end we must a gauge freedom surviving into the Fock space formulation.

One solution to this problem which we are working on consists in using infinite momentum frame and using the requirement that

$$X_R^+(n) = E \tag{12.24}$$

where E is a certain constant to fix together the discretization and the reparametrization of the type

$$\tau_R \to \hat{\tau}_R(\tau_R) \tag{12.25}$$

where $\hat{\tau}_R$ is any increasing function of τ_R. Having assumed (12.24) we can namely easily say that we choose τ_R to be proportional to the number in the chain of the "objects". In this way then the gauge of the rudimentary gauge freedom (12.25) gets chosen.

If we now choose then the constraints equation

$$\begin{aligned} 0 &= X_R^\mu(n) X_R^\nu(n) g_{\mu\nu} \\ &= 2X_R^+(n) X^-(n) - \vec{X}_{RT}(n) \vec{X}_{RT}(n) \end{aligned} \tag{12.26}$$

which restricts $X_R^\mu(n)$ to lay on the light-cone in the $25 + 1$ dimensional (formal) Minkowski space-time, gets further restricted by and the $X_R^\mu(n)$'s turn out to lay on a parabola on this light cone only. That means that only the 24 "transverse" coordinates $\vec{X}_{RT}(n)$ of X_R^μ, meaning the ones perpendicular to both the + and - coordinates, are independent variables since the + coordinate $X_R^+(n) = E$ by and the $X_R^-(n)$ then becomes the one given from the constraints,

$$X_R^-(n) = \frac{\vec{X}_{RT}(n) \cdot \vec{X}_{RT}(n)}{2E}. \tag{12.27}$$

To make these special constructions for $X_R^-(n)$ and $X_R^+(n)$ provides a problem - which would like to postpone to our later publication - because there is no obvious generalization of how we construct the odd $X^{\mu}_{R}(n \text{ odd})$ in terms of the conjugate variables to the even n, $\vec{X}_{RT}(n \text{ even})$ neighbors for special + and - components.

For the transverse coordinates of the odd n, $X^{\mu}_{R}(n)$'s, we have to put for n odd

$$\vec{X}_{R}(n) \propto \vec{\Pi}_{R}(n+1) - \vec{\Pi}(n-1) \tag{12.28}$$

in order to have the commutators between the neighboring "objects" $X^{\mu}_{R}(n)$ corresponding to $\delta'(\tau_R - \tau'_R)$ proportional commutator required for the $\dot{X}^{\mu}_{R}(\tau_R)$'s.

The same commutation rule can of course not be arranged for the $X^{-}_{R}(n)$'s if we let them be just fixed numbers for both odd and even n and for $X^{-}_{R}(n)$'s being quadratic expressions in the transverse degrees of freedom also the commutators will be a priori more complicated. So to organize a consistent picture involving also the + and - components is postponed to a later article.

12.10.1 Status of the Odd "Objects".

Let us towards the end stress that in our formalism where the Fock space(s) describe directly only the even "objects" the odd ones have to be constructed - formally - from the differences of the conjugate momenta $\vec{\Pi}_{RT}(n+1) - \vec{\Pi}_{RT}(n-1)$. But this means that if the order in which we imagine the even "objects"are organized into chains corresponding to strings get changed - by scattering - then it is even so that the from the even objects constructed odd ones are not only put into a different ordering, but there are even some odd ones being replaced by different ones. If a piece of a chain of even "objects" is attached to another piece than before then a new odd object that were not present before is being included. But it is only of the order of a few odd "objects" that get replaced, while the numbers of both odd and even objects in total in a number of strings goes to infinite in the limit of discretization being very fine. In this sense the number of replaced odd objects can be considered a null-set and thus in some sense be negligible.

12.11 Conclusion and Outlook

We have put forward some ideas for constructing a string field theory, in the sense of a second quantized theory of strings. The basic idea is *not to simply construct a Fock space in which one has states with various numbers of strings* such as one would at first think were to be done, and such as the already known theories, Kaku Kikkawa or Witten, have it. Rather we do not use the strings themselves but rather formally split them up into right mover and left mover degrees of freedom, mainly represented by $\dot{X}^{\mu}_{R}(\tau_R)$ and $\dot{X}^{\mu}_{L}(\tau_L)$. These separated degrees of freedom are then described by means of one-dimensional chains of point constituents (after discretization) called "objects" (rather than for the strings themselves). It is for these "objects" we construct a Fock space. See Figure 12.1 for an illustration of how the "objects" sit in one of the (two or one) Minkowski spaces. Well, for the case of only closed strings we actually use *two* Fock spaces, one for the "objects" related to the right movers $\dot{X}^{\mu}_{R}(\tau_R)$ and one to the $\dot{X}^{\mu}_{L}(\tau_L)$. Since each string is described by a large variable number of objects there is no (immediate) relation between the number of "objects" in our Fock space(s) and the number of strings. Rather the strings may split and unite without change in our Fock space(s). So it is in principle o.k. and can still allow for string scatterings even if there is no time development of our Fock space states - for "objects" - at all. Indeed we initiated this article by arguing for a theorem for strings in classical approximation: For a theory with only closed strings the images under the mappings

$$\dot{X}^{\mu}_{R} : \bigcup_{i} \{\tau_R \text{ range for string i}\} \longrightarrow M^{(R)}_{25+1} (\text{a Minkowski space time}) \tag{12.29}$$

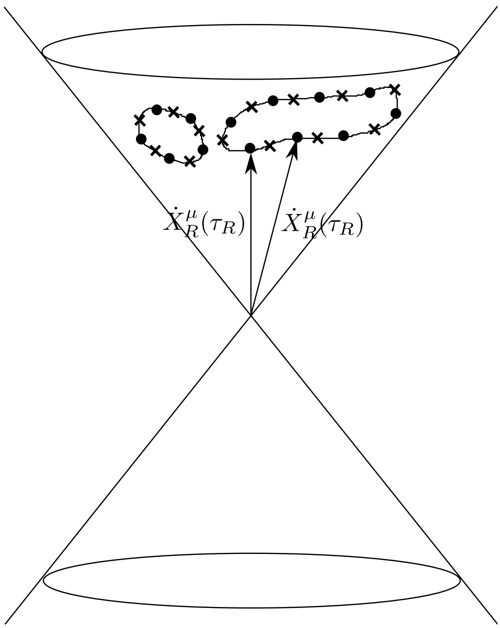

Fig. 12.1. Symbolic drawing of (one of) the 25 + 1 Minkowski spaces with the light cone, where the 25 +1 axes are $\dot{X}^0, \dot{X}^1, \dot{X}^2$, etc., and on the latter a couple of closed curves being the images of a couple of strings. The crosses and dots alternating along these curves symbolize the "positions" of the respectively "even" and "odd" "objects". The shown two curves correspond to that there are two strings represented in the example drawn.

and

$$\dot{X}^{\mu}_{L} : \bigcup_i \{\tau_L \text{ range for string } i\} \longrightarrow M^{(L)}_{25+1} \text{(another Minkowski space time)} \tag{12.30}$$

are conserved in time. This theorem implies a huge number of conservation laws, apart from null-sets the same number of degrees of freedom as the strings themselves. In this sense it means that up to the null-sets mentioned classical string theory, even with scattering (splitting and unification of strings), is a solvable theory. In the language if representing the strings by these images,

$$I_R = \bigcup_{i=1}^{n} \left\{ \dot{X}^{\mu}_{R}(\tau_R) \middle| \, \tau_R \in \text{ range for string } i \right\} \tag{12.31}$$

$$I_L = \bigcup_{i=1}^{n} \left\{ \dot{X}^{\mu}_{L}(\tau_L) \middle| \, \tau_L \in \text{ range for string } i \right\} \tag{12.32}$$

there is no (time) development at all. This means that the string theory - even of several splitting and uniting strings - has been $\underline{\mathit{trivialized}}$. Our string field theory formulated in terms of our "objects" means that we make used of this trivialized formulation of (classical) string theory. It is therefore our "objects" and thus also the Fock spaces describing them do not develop at all.

One might therefore feel tempted to believe that we cannot obtain a scattering amplitudes - meaning the Veneziano model - in our scheme, since if scattering is not revealed by any change in the Fock spaces, how could there be any scattering amplitude? Surprisingly enough we hope, however, to have given suggestive arguments that indeed one can rather easily obtain Veneziano scattering amplitudes from our formalism!

In some sense this surprise is related to the neglected "null-sets". Such null-sets may in fact be thought of as telling about how pieces of strings are glued together.

Whenever an applier of our formalism would ask for a scattering amplitude he would have calculated a not so simple wave function in our Fock space for "objects" for the set of strings in the initial state $|i\rangle$. Similarly he must calculate a wave function for the system of outgoing strings for which he asks the S-matrix element. This S-matrix element is then computed as the overlap of the two constructed Fock space states. I.e. they are basically calculated as if the "fundamental" S-matrix (in our Fock space) were just the unit operator. We gave arguments for series expansion of the in this way constructed S-matrix leading easily to integrals very similar to the way that Veneziano models are expressed.

Some of the major technical points or ideas put forward in the present article were to make our Fock space for the "objects" *only for the in a discretization of* τ_R *(or* τ_L*) even numbered points*.

Another detail is that in the formulation of the $\dot{X}^\mu_L$ and $\dot{X}^\mu_L$ the usual consequence of the Virasoro - algebra restrictions becomes - classically at least - simply that our "objects" must lay on the light cone in the spaces $M^{(R)}_{25+1}$ and $M^{(L)}_{25+1}$ respectively.

Yet a suggested technical detail were to use rather than the $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ themselves to connect with the "objects" these quantities integrated up over a small (discretization scale) interval in τ_R respectively τ_L,

$$J^\mu_R(n) = \int_{\tau_R(n)-\frac{\Delta\tau_R}{2}}^{\tau_R(n)+\frac{\Delta\tau_R}{2}} \dot{X}^\mu_R(\tau_R d\tau_R) \tag{12.33}$$

and

$$J^\mu_L(n) = \int_{\tau_L(n)-\frac{\Delta\tau_L}{2}}^{\tau_L(n)+\frac{\Delta\tau_L}{2}} \dot{X}^\mu_L(\tau_L d\tau_L) \tag{12.34}$$

(Here $\Delta\tau_R$ and $\Delta\tau_L$ are disretization intervals.) Then reparametrization symmetry in τ_R respectively τ_L can be gauge fixed in say an infinite momentum frame by putting the $X^+_{R,L}$ =constant. This would bring the $X^\mu_{R,L}$'s to be only on a certain "parabola" lying on the light cone.

We hope in forthcoming paper to a) Specify how by construction we achieve $(x^\mu_R(n))^2 = 0$ and thereby deduce the mass spectrum for a closed chain of "objects" and thus of a string. b) Complete the evaluation of the Veneziano scattering amplitude c) Investigate if we really obtain unitarity corrections to Veneziano amplitudes by evaluating the overlaps contributions in which there are more breaks in the correspondence between the incomming and the final state chains of "objects" used to evaluate the overlap $\langle f|i\rangle$, that should give the S-matrix element.

Acknowledgement

The authors acknowledge K. Murakami for informing them many relevant references. M.N. would like to thank the Niels Bohr Institute for hospitality extended to him during his stay there. H.B.N. thanks for room and allowance to stay at NBI as an emeritus professor to the Niels Bohr Institute. One of us (M.N.) is supported by the JSPS Grant-in-Aid for Scientific Research Nos. 21540290 and 23540332.

References

1. H.B. Nielsen and M. Ninomiya, “A new type of string field theory”, in 10th Tohwa International Symposium on string theory, AIP conf. Proceedings Vol. 607 issue 1; arXiv hep-th/0111240 v1.
2. M. Kaku and K. Kikkawa, Phys. Rev. **D 10** (1974) 1110; **D 10** (1974) 1823.
3. E. Witten, Nucl. Phys. **B 268** (1986) 253-294; Phys. Rev. **D 46** (1992) 5467-5473.
   N. Seiberg and E. Witten, JHEP **09** (1999) 032.
   M. Saadi and B. Zwiebach, Ann. Phys. **192** (1989) 213.
   B. Zwiebach, Nucl. Phys. **B 390** (1993) 33.
4. H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Phys. Rev. **D 34** (1986) 2360; **D 35** (1987) 1318.
   T. Kugo, in Quantum Mechanics of Fundamental Systems 2, chap. 11, eds. C. Teitelboim and J. Zanelli (Plenum Pub. 1989).
5. W. Siegel and B. Zwiebach, Nucl. Phys. **B 263** (1986) 105; Nucl. Phys. **B 282** (1987) 125; Nucl. Phys. **B 299** (1988) 206.
   W. Taylor and B. Zwiebach, arXiv: hep-th/0311017v3.
   K. Ohmori, arXiv: hep-th/0102085.
   L. Rastelli, arXiv: hep-th/0509129.
   M. Schnabl, arXiv: 1004.4858.
6. H.B.Nielsen (Nordita) 'An almost physical interpretation of the integrand of the n-point Veneziano model', preprint at Niels Bohr Institute.
   Paper presented at the XV Int. Conf. on High Energy Physics. Kiev, USSR, 1970 (See p. 445(in Veneziano's talk)). There was an earlier version without the word 'almost' in the title. Leonard Susskind, Yeshiva University preprint and Phys Rev Letters; L. Susskind, Phys. Rev D Vol 1 p. 1182 (1970); Eduardo Galli and L. Susskind, Phys. rev D. 1 p. 1189 (1970) Y. Nambu, Lectures at the Copenhagen Summer Symposium (1970); Nambu,Y., Dual Model Of Hadrons, EFI-70-07 Feb. 1970, 14pp.
   See also: Green, Schwarz, Witten, SuperString Theory vol. 1 and 2. Cambridge University Press, 1987 (many references therein). H. B. Nielsen, “String from Veneziano model,” arXiv:0904.4221 [hep-ph].

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
***What Comes Beyond* . . .** *(p. 199)*
Bled, Slovenia, July 11-21, 2011

13 Degenerate Vacua From Unification of Second Law of Thermodynamics With Other Laws

H. B. Nielsen[1] and M. Ninomiya[2]

[1]Niels Bohr Institute, Blegdamsvej 17,
DK2100 Copenhagen, Denmark
[2]Okayama Institute for Quantum Physics,
Kyoyama 1-9, Kita-ku, Okayama 700-0015, Japan

Abstract. Using our recent attempt to formulate second law of thermodynamics in a general way into a language with a probability density function, we derive degenerate vacua. Under the assumption that many coupling constants are effectively "dynamical" in the sense that they are or can be counted as initial state conditions, we argue in our model behind the second law that these coupling constants will adjust to make several vacua all having their separate effective cosmological constants or, what is the same, energy densities, being almost the same value, essentially zero. Such degeneracy of vacuum energy densities is what one of us works on a lot under the name "The multiple point principle" (MPP).

13.1 Introduction

The second law of thermodynamics [1–3] concerns, contrary to the other laws, the question of initial state and further seemingly straightly violates the time-reversal symmetry of the other laws. Even if time reversal symmetry is slightly broken in the Standard Model, at last CPT is not broken, and the breaking is anyway so tiny that it does not support the violation of time reversal invariance of the order of that of the second law. This arrow of time problem [4] at first seems to violate any hope of constructing a model or theory behind the second law without violating the usual symmetries of the other (time development) laws, especially CPT or time reversal symmetry. However, we believe to have actually presented such a model, and S. Hawking and J. Hartle's [9] no boundary initial conditions also present a model [5–8] that should indeed both have the second law for practical purposes and obey the usual symmetries. Really our model ends up very close to the Hartle-Hawking's one, but we think that ours is in principle more general. We see the connection so that by using imaginary time by Hawking et al have effectively got an imaginary part of the action come in. Our model [5–8,10] could be formulated as having a general complex action where real and imaginary parts are in principle independent functions to be chosen only respecting the symmetries and dimensionwise requirements etc.

Since we ended up with a reasonable picture for second law without too detailed assumptions about the real and imaginary parts of the action we might claim the generalization somewhat successful.

So far we worked purely classically to avoid at first the unpleasant quantum features of quantum mechanics for such a second law discussion that there does not truly exist a clean history path being true but rather a mysterious functional integral over many paths.

We did not so far go in detail with the question that such a purely classical model could definitively not be good enough at the end.

¿From an esthetic and simplicity point of view it would seem that a priori one should at first seek to construct models like the ones mentioned, since that is what we could consider "unification" of the second law with the rest of the laws and their symmetries. Also one could easily imagine that some law behind the second law could exist and possibly give a bit more information than just the second law itself, so that if we could guess it or find—it is perhaps Hartle-Hawking's no boundary—then we could use it for more. In our previous articles we in principle sought to discuss just a general formulation of such a law behind the second law by simply stating that is must—at least—be of the form of providing for every time track—i.e. equation of motion solution—a probability density $P(\text{path})$ in phase space. We think of the paths as associated with points in a phase space by simply choosing a standard moment of time say $t = t_{st}$ and letting the phase space point associated with the path be the ordered set of generalized coordinates $q^i(t_{st})$ and the ordered set of generalized momenta $p^i(t_{st})$ for this path, called path, at that moment t_{st}. The density P shall give the probability density for the path relative to the natural (Liouville theorem) measure on phase space. Because of Liouville theorem saying that this measure is invariant under the time development, the density $P(\text{path})$ defined will for a given path be the same number independent of at which moment of time t_{st} we choose to use the phase space (canonical) density $dqdp = \prod_i (dq^i dp^i)$. So generally formulated we have almost not assumed anything but left all assumptions to be done to the selection of the functional form of $P(\text{path})$ as function of the path.

At first one would think [11] that $P(\text{path})$ should depend in a simple way only on the very first moment $t \to 0$ or $t = t_{creation}$, the creation time of the universe. However, we are with the usual law properties used as a paradigm tempted to favor a form of the probability weight factor like

$$P(\text{path}) = \exp(\int P(q(t), p(t))dt) \tag{13.1}$$

which depends in the same way on the state along the track for all times t ! But such a form immediately seems to endanger getting out a good second law, since its time translational invariance is already in danger of leading to at least some features of the path to depend is a possibly simple enough to be recognized way on even the future. Such sufficiently simple dependence on the future might be recognized as "the hand of God" or even "miraculous effects" some times. However, we believe that it is realistic with models of a reasonable nature—a reasonable nice choice of P—of this kind to in practice have so few miracles or "hand of God" effects that the model is phenomenologically viable. That was what we attempted to argue for in last article [10] and the miracles would be small under the present conditions although Higgs particles could be a special danger for them to pop up so that LHC would be a flavored target for miracles or hand of God effects. The major partly future determined effect were there suggested to be the smallness of the cosmological constant, a phenomenologically welcome "miracle".

It is the purpose of the present article to extend somewhat this cosmological constant prediction to not only having one cosmological constant or vacuum energy density being small, but to have several minima in the scalar field effective potential "landscape" being very close to zero, too.

This result of the present paper is what one of us (H. B. N.) and his collaborators have been announcing as the Multiple Point Principle[1]. Mainly it has been claimed to give phenomenologically good results and derivations have, although being similar, been in

[1] See, e.g. [2] and references therein.

principle quite different from the present one. In fact derivations have only been successful with some mild violation of the principle of locality. In that light the previous derivations cannot be extremely convincing, since after all, we otherwise do not find much evidence for violation of locality, except perhaps precisely in connection with the cosmological constant problem.

Indeed we shall in the present article argue for the multiple point principle, but only under a very important extra assumption: At least some coupling constants or mass parameters are "dynamical", or one should rather say that they are to be counted as part of the "initial conditions".

The meaning of this making the coupling constants—such as say the Higgs-quark Yukawa couplings—"dynamical" is that we consider them part of the path in the above terminology, so that $P(\text{path})$ also comes to depend on them. Thus we have to maximize the probability also allowing for the variation, and thus adjustment, of the couplings which are declared [12] "dynamical". We might either just assume then "dynamical" in this sense—really meaning counted as part of the "path"—as a brute force assumption, adding them as special generalized coordinates, or we may imagine that they in some way have come out of the ordinary dynamical variables as e.g. in baby universe theory. Really it is the way of arguing in the present article not to go into details with respect to how precisely the coupling constants became "dynamical", rather saying:

Since we seemingly had some success—solving the cosmological constant problem—in last articles by introducing the assumption that the cosmological constant was "dynamical" in this type behind second law model, it is by analogy suggested that also other couplings, quite analogous to the cosmological constant, are or "dynamical".

It is our hope that allowing several possibilities for how it came that the coupling constants became "dynamical" is the sense of depending on dynamical variables or fundamentally themselves already being "dynamical". Then the model presented has the collected probability of being true, collected from these different possible ways.

In the following section, section 2, we shall set up the formalism for the probability density, and sketch how one might ideally wish it to look very analogous with the action. In section 3 we shall make some rather general considerations about the stability and most flavored states of universe that can be relevant for surviving over exceedingly long periods of time. The main point is here to investigate, how the likelihood of a certain combination of macrostates $\langle Pe^S \rangle$ depends on the variation of the couplings, especially when a minimum in the landscape of the scalar field effective potential passes from being negative to being positive. Our point is that the minimum being close to zero is flavored. In section 4 we shortly review that the model could—as seen in last article—provide an effective Big Bang although the time before the inflation era is a crunching inflationary era with opposite second law i.e. $\dot{S} < 0$. It is thus "pre-Big Bang" one could say. In section 5 we review how this multiple point principle prediction has already been claimed to be phenomenologically a very good assumption leading to phenomenologically good predictions for relations between coupling constants in the Standard Model, especially the top quark mass is what is predicted. Also a detail difference between the present and the earlier "derivations" of the multiple point principal of degenerate vacua is put forward: In the present model many of the possible vacua are only realized over very small space time regions. Perhaps only one of the vacua are hugely realized. In the old competing derivations they all had to be realized over order of magnitude comparable space time 4-volumes. In section 6 we present the conclusion and further outlook.

13.2 Model behind second law of thermodynamics

Since the second law of thermodynamics is well-known to concern the state of the world rather than as “the other laws”, such as Hamilton equations or equivalently Newton’s second law, then a law behind this law must of course somehow assign probabilities to different states, or directly tell which one is the right one. Since the time development laws (“the other ones”) are assumed to be valid (under all circumstances) we should really think about a law behind the second law of thermodynamics as assigning probability or perhaps even validity to solutions of the equations of motion. We might to keep it very abstract think of a space of all solutions to the equations of motions. Then the law behind the second law of thermodynamics could be thought of as having the form of a probability distribution P over this space of solutions. So it (=the law behind) is required to formulate a probability measure over this space of solutions to the equations of motion. It happens that such a measure can be written down rather elegantly in as far as a solution by selection of a “standard time” t_{st} is correlated to a point in phase space namely

$$(q_1(t_{st}), q_2(t_{st}), ..., q_n(t_{st}), p_i(t_{st}), ..., p_n(t_{st})) \,. \tag{13.2}$$

Now the phase space has the “natural” measure

$$\prod_i dq_i \prod_i dp_i \tag{13.3}$$

which is the one from the Liouville theorem. It is of course suggested then to use this measure with $(q_1, ..., q_n, p_i, ..., p_n)$ taken as $(q_1(t_{st}), ..., q_n(t_{st}), p_i(t_{st}), ..., p_n(t_{st}))$ which means to use the measure

$$\prod_i dq_i(t_{st}) \prod_i dp_i(t_{st}). \tag{13.4}$$

Then one could define a density $P(\text{path})$ using (13.4) by writing the probability density for the path

$$\text{path} = (q_1, ..., q_n, p_1, ..., p_n) : \text{time axis} \to \text{“Phase Space”} \tag{13.5}$$

as

$$\text{“probability measure”} = P(\text{path}) \prod_i dq_i(t_{st}) \cdot \prod_i dp_i(t_{st}). \tag{13.6}$$

One would now fear that this probability density $P(\text{path})$ defined this way would depend on the standard moment t_{st} chosen. It is, however, trivial to see that this fear is without reason, since indeed $P(\text{path})$ will not depend on t_{st}. It is well known that the measure (13.3) or (13.4) is invariant under canonical transformations and that the time development is a canonical transformation. Thus we do not need to attach any index t_{st} to $P(\text{path})$, it is only a function of the solution “path”.

Now to really produce a guess making up a law behind the second law of thermodynamics one has to make some assumptions about the defined probability density function, P : solution space $\to R_+ V\{0\}$. Because if one do not assume anything it is a very big class of possibilities for P and there will not be much content in such a formalism. That there is not much content in just putting up such a formalism is encouraging, because it makes it (more) likely that we have not assumed anything wrong by using the formalism with such P.

In the present article it is our intention to a large extend to keep the model at this general level by making very general assumptions about P. For example we may assume that it exists for some sort of world machinery at some fundamental level, but that we do not dare to guess it—since our chance guessing it wrong by world of course be outrageously high—so that we instead should attempt to guess a statistical distribution over function of type P. Then the idea should be that we should be allowed to play with the formalism as if P were chosen as a random one from this assumed distribution of P-type functions. This way of thinking of a statistical distribution for objects—here P—that actually are thought to make a law of nature is typical for the project which one of us called "random dynamics". In this sense we can consider our last paper [10] a random dynamics derivation of the second law of thermodynamics.

Here we shall, however, not go on to put up a statistical distribution for P as a function but just keep ourselves to a rather general discussion about P. In fact we may use such argumentation as: To find a big value for $\log\langle P\rangle$ where $\langle\cdots\rangle$ denotes averaging over a region in space of solutions we have less chance to find it very big when we average over a smaller region than if we average over a bigger region. There is a bigger fluctuation for a small region and thus better chance for the outrageous average value.

¿From this kind of statistical argument we would see that it will in all likelihood help to produce a big probability if we can get arranged that the system would stands around in an appropriate (not too big) region in phase space. The smaller this region the better is the chance that we accidentally have in that region a high average P. Thus we would see that regions of phase which are metastable have high chance to have some of the highest probabilities. As the typical region of such a stable kind or rather metastable one we could think of the universe in a stable one of an only slightly excited vacuum with a limited amount of field vibration on it. It might then be metastable due to some interactions.

If we want to write down an expression for a proposal for P(path) which has symmetry and locality properties analogous with those of the time development laws, we would in a classical field theory model make a construction for log P(path) guide analogues to the action. The suggestion of such an analogy is in fact strongly suggested by for a short moment thinking about a quantized generalization of our model in a Feynman path integral formulation. It would be very strongly suggested to put the P(path) in as a factor $\sqrt{P(\text{path})}$ multiplying the path-amplitude by suggesting the replacement

$$e^{iS[\text{path}]} \overset{\text{replace}}{\longrightarrow} \sqrt{P(\text{path})}e^{iS[\text{path}]} \tag{13.7}$$

(Here, S[path] is of course the action, and not the entropy.) for the quality occurring in the Feynman path integral. To do this replacement one would of course need to have a model form for $\sqrt{P(\text{path})}$ and P(path) even for those paths which do not obey the equations of motion. In the present article it is, however, still the intention to use a purely classical description and we would not need such an extension. But only the esthetic suggestion of seeing

$$\log\sqrt{P(\text{path})} = \frac{1}{2}\log P(\text{path}) = -\text{"ImS"} \tag{13.8}$$

as really being an imaginary part of the action, so that symmetry and locality properties of log P(path) would be suggested to be taken to be just the same as for the usual—i.e. the real part of—action S(path). We would therefore, say in a general relativity setting, obtain a form

$$\log P(\text{path}) = \int d^4x\sqrt{g(x)}P(\varphi, \partial_\rho\varphi, \psi, \partial_\sigma\psi, g_{\mu\nu}, \partial_\sigma g_{\mu\nu}, \cdots). \tag{13.9}$$

Here we should of course have in mind that corresponding to a path one has a development of all the field $\varphi(x), g_{\mu\nu}(x), \psi(x), \cdots$ their derivatives $\partial_\sigma\varphi(x), \cdots$ too. Thus the expression (13.9) is a well-defined functional of the path.

We can imagine—and it would be the most esthetic an nicest—that the function P of the fields and their derivative obey all the rules required from the symmetries obeyed by the usual timedevelopment laws, the ones given by the action.

For instance since gauge transformations are supposed not to cause any physical change, we should have $\int d^4x\sqrt{g}$ be gauge invariant clearly. The form as an integral the requirement of locality and thus if we can manage to get such a form work phenomenologically we could even say that the law behind the second law of thermodynamic could obey such a locality postulate.

Such a set up with a lot of symmetry requirements might at first be somewhat difficult to check and thus remain speculations, but the real immediate worry, the reader is expected to have, is that such a form of $P(\text{path})$ will have enormous difficulty in leading to the second law. Immediately one would rather think that it would lead to mysterious regularities in what will happen both in past and future and even today in order to optimize P. If there are too many features of the actual path predicted to be destined to organize a special future or present the model may be killed immediately.

In reality we consider it a remarkable result of our previous work [10] that we argue that this type of model is not totally out, but on the contrary looks promising even without almost assuming anything about the specific form of $\log P$.

13.2.1 Example: Scalar fields, exercise

To provide us with an idea of how such a model will function let us imagine a theory with one or several scalar fields. If we add the further assumption that not only the Lagrangian density, but also the quite analogous density P has coefficients of the dimensions required by "renormalizability", then the "kinetic terms" in the density P would be quite analogous to the ones in the Lagrangian density L and no terms with higher number of derivatives would be allowed neither in L in P. Also only an up to fourth order term in the potential $V(\varphi_1, \varphi_2, \cdots)$ and the analogous "potential" term in P would be allowed.

To get an idea of what can go on we can think that if for some special value combination of the scalar fields

$$(\varphi_1, \varphi_2, \cdots) = (\varphi_1^{(0)}, \varphi_2^{(0)}, \cdots) \tag{13.10}$$

where the density P has a maximum, then a configuration with the scalar fields taking that set of values will be a priori very likely. However, we shall also have in mind that most likely the fields will not stay at just that special combination for long if it has to obey the equations of motion. Unless such a maximum in the "potential" part of P (We think of the part of P independent of the derivatives of the fields thus only depending on the values of the fields) is also an extremum for the potential part of the Lagrangian density there is no reason that standing fields should be solutions. Rather the fields will roll down say—and not even any especially slow roll a priori—.

It might actually pay better to get a high probability or likelihood if the field-combination chooses to sit at a minimum in the potential $V(\varphi_1, ...)$ from the usual Lagrangian density $L = \sum \partial_\mu \varphi_i \partial^\mu \varphi_i - V(\varphi_1, ...)$ with a relatively high but not maximal P-potential-part value. At such a place we could have the fields standing virtually externally and that would count much more than a short stay at an even higher value for the "potential" part of P.

The longer time of it staying there will give much more to the time integral form for $\log P$.

But we can investigate if it could be arranged to get the gain from the very high P near some unstable combination for a relatively short time and then at another earlier and/or later time attain for long the somewhat lower but still if well-arranged reasonably high P-value from a minimum in the potential V from L.

In the previous articles it were suggested that such a shorter time high P could well pay and be indeed the explanation that during some period in the middle of times there were an inflationlike Big Bang similar time with the scalar field at an unstable point. Our model is not really guaranteed to solve the problem of getting the roll slow enough—although we could say that meaning "it would like to if it could"— but even a shorter inflation period could at least provide a from outside (in time) seen Big Bang. Let us though stress two important deviations—none of which are so far experimentally accessible—between our simulated Big Bang and the conventional one:

1) Ours is in the "middle of times" so that there is a half time axis at the pre-Big Bang side actually with an inverted second law of thermodynamics $\dot{S} < 0$.
2) We do not have any true singularity, but rather have inflation like situation with finite energy density all through this "middle period."

13.3 The Derivation of Multiple Point Principle

13.3.1 Dynamical couplings and what to maximize

To derive the Multiple point principle it is very important that we take a series of coupling constants to be dynamical in the sense that they can be adjusted to take special values guaranteeing the many degenerate minima, which are by definition the point of the Multiple Point Principle. So we must take it that the P-probability also depends on these couplings. That means that so called different paths have as some of their degrees of freedom these couplings so that they are different for different paths.

We have already argued for that the most likely type of path i.e. development to corresponds to the scenario of an inflation era in some middle of the time axis, surrounded by asymptotic regions of an almost static big universe with thin matter and essentially zero cosmological constant operating near a minimum in the potential. Then one can get the biggest P from a long asymptotic era—which though must be at least meta stable—while still getting a high P concentrated contribution from a short "around Big Bang" era.

Now we should have in mind that the effective potential $V(\varphi_1, \varphi_2, ...)$ can and will typically have several minima. A priori, however, these minima will not be degenerate with their separate cosmological constants being zero as the Multiple Point Principle which we seek to derive.

Rather the precise height of the various minima in the effective potential will depend on the various coupling constants and mass parameters which we have just assumed that we shall —at least effectively— count as part of the "inial conditions" i.e.the solution "path". After we assumed these couplings and mass-parameters to be "dynamical" meaning here part of the path on which P depends we shall allow them to be varied too in the search for the most likely path. Now it is, however, not quite the right thing to look for just that very special path that goes with the highest P, because what we in practice are interested in is not really to know the special path but rather what class of paths not distinguishable by macroscopic observation. We rather look for describing the scenario in terms of macrostates meaning roughly that sort of states that are used in thermodynamics where one characterizes systems with huge number of degree of freedom by means of a few macro variables, energy, numbers of various types of particles and the like, entropy e.g. Even if such a macro state having a huge number of micro states collected under its heading does not contain the most likely single solution to the equations of motion if it could very well happen that the sum over all its micro states

$$P_{macro} = \sum_{path-macro} P(path) \tag{13.11}$$

could be—even much—bigger than the single $P(path_{max})$ for the uttermost scoring solution $path_{max}$. In such a case we should like in practice to consider it that the correct scenario for us as macro-beings is the one with the macro state giving the biggest sum (13.11). Rather than looking for the largest $P(path)$ we are therefore looking for the largest sum over a whole or perhaps even better a whole class of similar macro states, i.e. for the largest

$$P_{macro} = \sum_{path \in macro} P(path) = \langle P \rangle_{macro} \cdot e^S \tag{13.12}$$

where we introduced the average over the macro state notation

$$\langle P \rangle_{\text{"macro"}} = \frac{\sum_{path \in \text{"macro"}} P(path)}{\#\text{micro states in "macro"}} = \frac{\sum_{path\, \in\, \text{"macro"}} P(path)}{e^{S(\text{"macro"})}} \tag{13.13}$$

and defined the entropy of the macro state "macro" as the logarithm of the number of micro states in it

$$S(macro) \equiv \log(\#\text{micro states in "macro"}). \tag{13.14}$$

13.3.2 Central derivation of many degenerate vacua.

When one characterizes the competing classes of microstates as macrostates with some entropy S, what we really shall think of as being maximized by the model, is the quantity $\langle P \rangle e^S \mu_S^{2N}$ or we can say $\log(\langle P \rangle e^S)$. Here $\langle P \rangle$ stands for the average over the macrostate of P. This quantity $\log(\langle P \rangle L^S)$ is expected from general smoothness assumptions and assuming no fine tuning a priori to vary smoothly and with non-zero slope as a function of all the parameters, especially as a function of the various coupling constants and mass parameters. In other words these coupling constants and mass parameters should be determined together with the class of microstates to be most likely from the maximization of $\log(\langle P \rangle e^S)$ point of view.

Now, however, we have to take into account that the appearance of a minimum in the effective potential—as function of the effective (composite or fundamental) scalar fields—in addition to that minimum that leads the exceptionally high $\log(\langle P \rangle e^S)$ which gives the highly probable asymptotic behavior can cause a destabilization. In fact the appearance of a competing different minimum means when it becomes deeper than the high $\log(\langle P \rangle e^S)$ one that the latter becomes strictly speaking unstable. It can namely in principle then happen that the high $\log(\langle P \rangle e^S)$ macrostate around this latter minimum, develops into a state around the lower energy density vacuum, a state belonging to this other minimum. One should have in mind that it is the lack of energy that keeps the "asymptotic" state of the universe to remain very close to the vacuum so as to ensure the high $\log(\langle P \rangle e^S)$. If energy can be released by the scalar fields shifted to a lower/deeper minimum then this cause of stability disappears and the universe will no longer keep at the vacuum with high $\log(\langle P \rangle e^S)$ and most likely a much lower value for $\log(\langle P \rangle e^S)$ will be reached. That means that the smooth continuous variation with the coupling constants etc. as a function gets a kink, a singularity, wherever a competing minimum passes from being above the high $\log(\langle P \rangle e^S)$ one to being deeper.

There is a very high chance that the maximum achievable $\log(\langle P \rangle e^S)$ will occur just at this type of kink. All that is needed is really that as the minimum competing with the high $\log(\langle P \rangle e^S)$ as a function of some coupling, g say, is lowered—still while being above and thus no threaten to the high $\log(\langle P \rangle e^S)$ the $\log(\langle P \rangle e^S)$ is—accidentally—having appropriate sign of its rate of variation. In fact what is needed is that the $\log(\langle P \rangle e^S)$-quantity gets larger under variation of say g when the competing minimum gets lower. In such a case the largest $\log(\langle P \rangle e^S)$ will be reached by bringing the competing minimum to be as low as possible before it destabilizes the high $\log(\langle P \rangle e^S)$ vacuum and thus spoils the smooth estimation. But that means that the maximum $\log(\langle P \rangle e^S)$ meaning the most likely scenario will precisely happen when the destabilization sets in. So it is very likely that seeking—as our model does—the maximal $\log(\langle P \rangle e^S)$ scenario will lead to very likely have competing minima just with the same effective potential values as the high log $\log(\langle P \rangle e^S)$—vacuum. But this is precisely what we mean by the multiple point principle: There shall be many vacua with the same energy density or we can say same cosmological constant.

In this way out of our model we have interestingly enough derived just this principle on which one of us and his collaborators have already worked a lot, seeking to show that it has very good phenomenological fitting power.

13.4 Review of the other good features of our model

In this section we shall review and elaborate the point that our model—although it does not at first look so—indeed is to a very good approximation a law behind the second law, even with a few extra predictions.

The most surprising is that we can get the second law of thermodynamics out of an at the outset totally time reversal invariant "law behind the second law of thermodynamics" However, that can also only be done by a slight reinterpretation:

We argued that although the bulk of the—assumed infinite—time axis is taken up by eras in which roughly the maximal contribution from these bulk eras to $\log(\langle P \rangle e^S)$ is the biggest attainable for a rather limited stable region in phase space, it pays nevertheless to have a short less stable era in some smaller interval. The full development will, in this case even if not exactly, then with respect to crude features be time reversal invariant around a time-reflection point in the middle of this unstable little era. The time reversal asymmetry is now achieved by postulating that we ignore and in practical life do not take seriously one of the two half axis of the time axis. Indeed we claim that we in practice only count what happens offer the mentioned middle point of the relatively short "more unstable era". The argument was now that by finding some small subset of microstates with very high $\log P$-contribution from this "unstable" era a universe development with higher $\log(\langle P \rangle e^S)$ could likely be found with such an unstable period than as a development of the type behaving as the asymptotically stable way at all times. Typically a very small phase space volume in the central part of the "unstable era" is expected to be statistically favorable because we expected it to be easier to find an average over P to be very big if we only average over a very small region. We almost expect a state with exceptionally high P to have to be past to make the "unstable era—excursion" from the asymptotic behavior to be the very most likely. We thus see that we expect the entropy in this "unstable era" to be very low indeed. Thinking of the especially high P being achieved by going to a highest "potential" part for P and having scalar fields sliding down from there the argument for a very low entropy in the unstable era seems indeed to be justifiable in such a more concrete setting.

A priori one would now think that an analogous argumentation of most exceptionally high $\log\langle P \rangle$ occurring more likely in a small region of phase space than in a larger phase space region would also give a low entropy in the asymptotic era. Now, however, there are some phenomenological peculiarities in nature which are combined with our suggested picture of a big universe in the asymptotic era points to that for practical purposes log P gets almost constant over the relevant neighborhood or the high $\log(\langle P \rangle e^S)$ providing vacuum (minimum in the effective potential). This phenomenological peculiarity is that the universe even today already expanded so much and the parameters of the Standard Model are such that:

1) Interactions are relatively seldom—i.e. weak couplings,
2) All the particles around are in practice of the nature that they only acquire non-zero-masses by the Higgs field expectation value $\langle \varphi_{ws} \rangle \neq 0$,
3) Even this Higgs VEV is tiny from the presumed fundamental scale point of view.

As a caricature we may thus see the present era—which is already really the asymptotic era to first approximation—as an era with a big universe with a "gas" of massless weakly interacting particles only.

Further we should keep in mind that we phenomenologically have—locally at least—Lorentz invariance. This means by imagining the theory rewritten from the field theory description, used so far in this article, to a particle description that the contributions to log P should be integrals along the time tracks of the various particles with coefficients depending on which particle type provides the log P-contribution. Now, however, for massless particles the time-track is lightlike and *thus always zero*. We get therefore no such contribution from the presumably almost massless particles in the Standard Model. If this is so it means that once we have got limited the set of states at which to find the true state in the asymptotic era to those with the Lorentz invariance and masslessness properties, there is no gain for $\log\langle P \rangle$ by further diminishing the class of states included. The $\log\langle P \rangle$ would anyway remain much the same even if in the asymptotic time the photons say were removed because they due to the masslessness do not count anyway. Thus a further reduction in phase space in the future is not called for since really it is rather as earlier stressed $\log(\langle P \rangle e^S)$ which should be maximized, and we can increase this quantity by having more particles—meaning a wider range of phase space—contributing to entropy S without changing $\log\langle P \rangle$ much.

This masslessness phenomenology thus provides an argument for a much higher entropy in the future in the scenario favored by our model. Well, we should rather than future say in the numerically asymptotically big times.

In the inflation era on the other hand the typical temperatures at least after "re"heating are much higher and at least the Weinberg Salam Higgs cannot be prevented from appearing.

13.5 Multiple point principle already somewhat successful phenomenologically

Accidentally the derivation from our law behind the second law of thermodynamics of there being many minimal in the effective potential for the scalar fields—fundamental or bound state ones—having all very small cosmological constants(=potential heights) is just a hypothesis—called multiple point principle—on which one of us and his collaborators have worked a lot and claim a fair amount of phenomenological success.

In fact we started by fitting fine structure constants in model with a bit unusual gauge group by means of the phase transition couplings in lattice gauge theories. Now phase transition couplings would mean couplings for which more than one phase of the vacuum can coexist. So asking for vacua with the same cosmological constants is in fact equivalent to ask for some relevant coupling constant being at the phase transition point. So if we look the lattice gauge theory serious to really exist in nature, or just the lattice artifact monopoles which mainly determine the phase transition couplings, the above prediction of degenerate vacua would imply such phase transition coupling constant values. In the old times we had indeed a sort of historically probable success in the sense that we had the by that time unknown number of families of leptons and quarks as a fitting parameter relating the "family gauge group" gauge couplings taken to be just at the phase transition point, and we fitted it to be three. Thereby we predicted by a model that had as one of its major input assumptions the equally deep minima—although formulated rather differently—just derived. The model though is just one among many possibilities first of all characterized by having the gauge group of the Standard Model $G = SU(3) \times SU(2) \times U(1) = S(U(2) \times U(3))$ repeated at a more fundamental level near the Planck energy scale once for each family of quarks and leptons. In other words, each of the N_{gen} families of quarks and leptons supposed finally to be found had their own set of Standard Model gauge particles only acting on that "proto"family. Remarkably we predicted this number of families $N_{gen} \approx 3$ before the measurement at L.E.P. of the number of families of neutrinos.

In the pure Standard Model our requirement of same dept in this case of a second minimum in the Weinberg-Salam Higgs effective potential as the one of which we live $\langle \varphi_{ws} \rangle \approx 24\text{GeV}/\sqrt{2}$ leads to the Higgs particle to be the minimal one allowed by stability of vacuum. Without extra corrections pure renormalization group calculations lead to a prediction of the Higgs mass from this degeneracy principle to be $135\text{GeV}/c^2$. This is already good in consideration of indirect Higgs mass determinations pointing to a light Higgs mass.

In works involving one of us (H.B.N.) and C.D. Froggatt and L. Laperashvili were developed a perhaps not so trustable story of an exceptionally strongly bound highly exotic meson of 6 top quarks and 6 antitop quarks bound together by Higgs exchange just in such a way as to produce a degenerate vacuum with this type of exotic meson forming a Bose-condensate. Remarkably enough our calculations taking that sort of bound state or exotic meson serious and imposing the degeneracy of the vacua, not only leads to an only within uncertainly too high Yukawa coupling for the top quark, but also solves the problem essentially behind the hierarchy problem! Indeed the coincidence of the top-quark-Yukawa-coupling values g_t needed for

1) getting the bound state condensate just be degenerate, and for
2) getting it possible to have the second minimum in the Weinberg-Salam Higgs field effective potential degenerate with the first one;

leads to a need for the ratio of the Higgs vacuum expectation values the minima be a number given as an exponential.

That is to say that if for some reason the second minimum in the Weinberg-Salam Higgs effective potential were of the order of some grand unifying scale or the Planck scale or a fundamental scale, then the ratio of this scale to the weak scale would be explained to have to be an exponentially big ratio from the derived multiple point principle in the present article. In this sense we can claim that the multiple point principle solved the question as to why so big a scale ratio problem, a problem which is really behind the more technical hierarchy problem.

13.6 Conclusion and Outlook

We have worked further on the model that the second law of thermodynamics be caused by there existing a "fundamental" probability density functional P assigning to each possible solution "path" of the equations of motion a probability density P("path") in phase space. Without making more than even in mild form the assumption that this "fundamental" probability assignment P should obey the usual properties of laws of nature—locality (in time first of all) and translational invariance—we already got (phenomenologically) good results. In fact we roughly and practically got the second law of thermodynamics as was the initial purpose and in addition some good cosmology.

The present article obtained the further prediction of there being most likely many different states of vacuum, all having small cosmological constants. It must be admitted though that we only obtained this result with the further very important assumption that—some way or another the coupling constants and mass parameter, i.e. the coefficients in the Lagrangian density, have become or are what we call "dynamical". This meant that it somehow were themselves or depended on ordinary dynamical variables, like fields or particle positions. Now it turned out remarkably that this prediction by one of us and his collaborators had since long been argued to be a good one phenomenologically! It must be admitted though that for all its successes a bit of helping assumptions were to be used. But even with only a mild assumption that the order of magnitude of the Higgs field in the high Higgs VEV alternative vacuum we got a very good value for the top quark mass $173GeV \pm 6GeV$. Taking our previous Multiple Point fitting most seriously with three degenerate vacua in the Standard Model alone we actually could claim that Higgs-mass of $115GeV/c^2$ seemingly found at L.E.P. is quite well matching as being our prediction. It must be admitted though that especially our correction bringing the predicted down from our older prediction $135GeV/c^2$ to about the L.E.P. values is very doubtful and uncertain.

It is remarkable that we get such a funny and at least in future by Higgs mass, testable series of models higher scale of energy predictions about couplings constants as this multiple point principle out of modeling the second law of thermodynamics, an at first sight rather different branch in physics. Already this, provided it works (i.e. Higgs mass really be what the calculations will give etc.), would be a remarkable sort of unification of this second law with other physics information, seemingly at first quite unrelated! Taking into account that the major development of the universe into a low density, low temperature, large universe could—in the foregoing articles in this series—be considered the major "hand of God effects" predicted from of model we must say that it unifies quite far away features for the physical world!

As outlook we may list a few routes of making testing of our present unification:

1) In the light of the result of the present article testing of there being the many degenerate vacua in the various models beyond the Standard Model may if sufficiently successful be considered a confirmation of our "law behind the second law of thermodynamics".
2) One could seek to estimate more numerically the cosmological parameters such as what size the already argued to be "small" cosmological constant should have included here could also be if some detail concerning the inflation going on predicted could be tested by say microwave background investigations.
3) A third route of testing or checking the model would be to really find rudimentary "hand of God effects". That would of cause from the conventional theory point of view be quite shocking and thus be a strong confirmation of something in the direction of our model, if such effects were convincingly seen. It would of course be even more convincing if they were found with a predictable order of magnitude and of the right type. In previous articles we put is as an especially likely possibility that Higgs particles—special in the Standard Model by not being mass protected—were either flavored or disfavored to be produced. That is to say there would respectively happen hand of God effects seeking to enhance or to diminish the number of Higgs particles being produced.

Acknowledgments

We acknowledge the Niels Bohr Institute (Copenhagen) and Yukawa Institute for Theoretical Physics for their hospitality extended to one of them each. The work is supported by Grant-in-Aids for Scientific Research on Priority Areas, Number of Area 763 "Dynamics of Strings and Fields", from the Ministry of Education of Culture, Sports, Science and Technology, Japan.

References

1. S. Carnot, "Réflexions sur la puissance motrice du feu et sur les machines propres á dvélopper cette puissance", Bachelier, Paris 1824; see "The Second Law of Thermodynamics", translated and edited by W. F. Magie, Harper and Brothers, New York 1899, pp 3–61.
2. R. Clausius, Ann. Phys. Chem. 79 (1859) 368–397, 500–524, *ibid* 93 (1854) 418–506; *ibid* 125 (1865) 353–400; see "The Mechanical Theory of Heat", translated by W.R.Brown, Macmillan and co. London, 1897; see also W. F .Magie [1] pp 65–108.
3. W. Thomas, "Mathematical and Physical Papers", University Press Cambridge, 1882 Vol 1, (1848) pp 100-106; (1849) 133–155; (1851-1854,1878) 174–332, (with Y. P. Youle), (1852–1862) 333–455.
4. H. B. Nielsen and S. E. Rugh, "Arrows of time and Hawking's no-boundary proposal", Niels Bohr Institute Activity Report 1995.
5. H. B. Nielsen and M. Ninomiya, JHEP **03** (2006) 057–072, hep-th/0602020.
6. H. B. Nielsen and M. Ninomiya, Proceedings to the 8th Workshop 'What Comes Beyond the Standard Models', Bled, July 19. - 29., 2005, Slovenia; hep-ph/0512061, YITP-05-38, OIQP-05-06, p.88–105, hep-th/0601048.
7. H. B. Nielsen and M. Ninomiya, IJMPA Vol 21, No. 25(2006) 5151-5162 YITP-05-38, OIQP-05-08, hep-th/0601021.
8. H. B. Nielsen and M. Ninomiya, Progress of Theoretical Physics, Vol. 116, No. 5 (2006); hep-th/0509205, YITP-05-43, OIQP-05-09.
9. J. B. Hartle and S. Hawking, Phys. Rev. **D28** (1983) 2960–2975.
10. H. B. Nielsen and M. Ninomiya, Proceedings to the 9th Workshop "What Comes Beyond the Standard Models", Bled, 16 - 26 September 2006, DMFA Zaloznistvo, Ljubljana; hep-ph/0612032.
11. L. Boltzmann, Ann. Physik, **60** (1897) 392, as translated in S. G. Brush, "Kinetic Theory", (Pergamon Press, New York, 1965); J. B. Hartle, gr-qc/9712001, Talk given at Nobel Symposium on Modern Studies of Basic Quantum Concepts and Phenomena, Gimo, Sweden, 13-17 Jun 1997.
12. C. D. Frogatt, L. Lagerashville, R. Nevzorov, H. B. Nielsen and M. Sher, Phys. Rev. **D73** (2006) 095005, hep-ph/0602054.

Discussion Section

This year discussion section includes only two topics: the discussion on the dark energy and on possible interpretation of Higgs and Yukawa couplings. We hope that all the other discussions of this workshop, on these two topics as well as also on the others, will continue and will appear in the next year proceedings. The reader can see how different are the explanations what the Higgs and the Yukawa couplings are. One statement is that there is no Higgs at all, the second is trying to see how far can us bring the starting ideas of the standard model by extending the scalar with the SU(2) fermion charges, what the Higgs is, to additional heavy Higgses with family charges, breaking the family symmetry. The *spin-charge-family* theory interprets the *standard model* Higgs together with the Yukawa couplings as an effective low energy manifestation of several scalar fields with the ordinary charges and the family charges in the adjoint representations. This theory also predicts that no advise how to search for either Higgs (the scalar fields indeed) or family members can be successful if a model used does not take care of all the family members.

Let us see whose prediction is right, that is: Do we need a very new idea or can we just live with the old ones?

All discussion contributions are arranged alphabetically with respect to the authors' names.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 215)*
Bled, Slovenia, July 11-21, 2011

14 What at all is the Higgs of the Standard Model and What is the Origin of Families?

N. S. Mankoč Borštnik

Department of Physics, FMF, University of Ljubljana, Jadranska 19, 1000 Ljubljana

Abstract. The *standard model* of the elementary particles is built on several assumptions. The Higgs is assumed to be a scalar, a boson, with the charges of a fermion (in the fundamental representations of the charge groups). No explanation is offered for the existence of families of fermions – quarks and leptons– for the charges of these family members, for the appearance of the Yukawas which take care of fermion properties. The theory explaining the origin of families predicts that several scalar fields with the boson kind of the charges (in the adjoint representations of the charge groups) manifest effectively at low energies as the Higgs and the Yukawas.

14.1 Introduction

When the *standard model* of the elementary particles and fields was proposed more than 35 years ago it offered an elegant new step in understanding the origin of fermion and boson fields and the interactions among them.

It is built on several assumptions, chosen to be in agreement with the data: **i.)** There exist the massless family members - coloured quarks and colourless leptons, both ”left” and ”right handed” (handedness concerns the properties under the Lorentz transformations), the left handed members distinguishing from the right handed ones in the weak and hyper charges. **ii.)** There exist the gauge fields to the observed charges of the family members. **iii.)** There exists a boson, the Higgs, with a ”non zero vacuum expectation value”, a scalar with the charges of a fermion. Its properties are chosen to ”dress” successfully the ”right handed” family members with the weak and the appropriate ”hyper” charge so that they manifest the properties of the left handed partners. The Higgs takes care at the same time of masses of the weak gauge fields Z_m and $W^{\pm}_m$. **iv.)** There exist the families of fermions. **v.)** There exist the Yukawa couplings, distinguishing among family members (u and d quarks, e and ν leptons) to ensure right properties of families of fermions, that is of their masses and decay properties (mixing matrices).

The properties of fermions and bosons as assumed by the *standard model* are presented in tables 14.1,14.2,14.3.

While all the so far observed *fermions* are *spinors* with the *charges in the fundamental representations* of the charge groups [1] and all the so far observed *bosons* are *vectors* in the *adjoint representations with respect to the charge groups*, the *Higgs* fields are *scalars* with the

[1] The ”internal degrees” of freedom of particles and fields, that means the spin and the charges, are theoretically described by the representations of the Lie groups. The same commutation relations of the infinitesimal generators of the groups allow infinite many representations: the scalar one, the fundamental one, the adjoint one, $\cdots$.

α name	handedness $-4iS^{03}S^{12}$	weak charge τ^{13}	hyper charge Y	colour charge	elm charge Q
u^i_L	left handed (-1)	$\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$\frac{2}{3}$
d^i_L	left handed (-1)	$-\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$-\frac{1}{3}$
ν^i_L	left handed (-1)	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
e^i_L	left handed (-1)	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1
u^i_R	right handed (1)	weakless	$\frac{2}{3}$	colour triplet	$\frac{2}{3}$
d^i_R	right handed (1)	weakless	$-\frac{1}{3}$	colour triplet	$-\frac{1}{3}$
ν^i_R	right handed (1)	weakless	0	colourless	0
e^i_R	right handed (1)	weakless	-1	colourless	-1

Table 14.1. The *standard model* assumes that there are before the electroweak phase transition three ($i = 1, 2, 3$) so far observed massless families of quarks and leptons. Each family contains the left handed weak charged and the right handed weak chargeless quarks, belonging to the colour triplet $(1/2, 1/(2\sqrt{3}))$, $(-1/2, 1/(2\sqrt{3}))$, $(0, -1/(\sqrt{3}))$ and the colourless left handed weak charged and the right handed weak chargeless leptons, if in this tiny extension of the *standard model* the right handed ν is added. Originally ν^i_R were excluded since no massless ν were observed and in the *standard model* assumption all the quantum numbers of ν_R are zero. τ^{13} defines the third component of the weak charge, Y the hyper charge, $Q = Y + \tau^{13}$ is the electromagnetic charge.

name	handedness	weak charge	hyper charge	colour charge	elm charge
hyper photon	0	0	0	colourless	0
weak bosons	0	triplet	0	colourless	triplet
gluons	0	0	0	colour octet	0

Table 14.2. The *standard model* assumes that there are before the electroweak phase transition three massless vector fields, the gauge fields of the three charges - the hyper charge (Y), the weak charge ($\vec{\tau}^1$) and the colour charge ($\vec{\tau}^3$), respectively. They all are vectors in $d = (1+3)$, carrying the corresponding charges in the adjoint representations. $Q = \tau^{13} + Y$.

name	handedness	weak charge	hyper charge	colour charge	elm charge
Higgs_u	0	$\frac{1}{2}$	$\frac{1}{2}$	colourless	1
Higgs_d	0	$-\frac{1}{2}$	$\frac{1}{2}$	colourless	0

Table 14.3. The *standard model* assumes that there is before the electroweak phase transition the scalar field Higgs, a boson, which carries the hyper charge (Y) and the weak charge ($\vec{\tau}^1$) in the fundamental (spinor) representations of the charge groups. It contributes to the phase transition by gaining a non zero "vacuum expectation value" of that component which has the electromagnetic charge ($Q = \tau^{13} + Y$) equal to zero. Correspondingly it changes properties of the vacuum.The Higgs "dresses" right handed $(d^i_R\, \phi)$ and $(e^i_R\, \phi)$ with the weak and the appropriate hyper charge, the anti-Higgs "dresses" correspondingly $(u^i_R\, anti\phi)$ and $(\nu^i_R\, anti\phi)$. Higgs takes care of the masses of the superposition of the weak and hyper charge gauge bosons, leaving the electromagnetic field massless. To take care of the masses and mixing matrices of fermions in agreement with the experimental data the *standard model* postulates the existence of Yukawa couplings, which are different for different family members.

charges in the fundamental representations of the charge groups. Therefore, quite a strange object, which reminds us of a supersymmetric particle [2] (but it is not because it does not fit the so called R parity requirement for a supersymmetric particle). The labels "scalar, vector, spinor (fermion)" fields express the behaviour of a field with respect to the Lorentz transformations in the space $d = (1 + 3)$.

The *standard model* never has the ambition to explain its own assumptions, leaving the explanation of the open questions to the next step of the theory. Although the *standard model* leaves many questions unanswered, yet it is, without any doubt, a very efficient effective theory: There is so far no experiment which would help to show the next step beyond the *standard model*, no new fermions or bosons, no supersymmetric particles, even no Higgs yet.

In the literature there are several proposals trying to go beyond the standard model, most of them just extending the ideas of the *standard model*, like: i.) A tiny extension is the inclusion of the right handed neutrinos into the family (what is done in table 14.1). ii.) The $SU(3)$ group is assumed to describe – not explain – the existence of three families. iii.) Like Higgs has the charges in the fundamental representations of the groups, also Yukawas are assumed to be scalar fields, in the fundamental (fundamental for left handed family members and anti-fundamental for the right handed ones) representation of the $SU(3)$ group, belonging to different gauge groups for different family members [1–4]. vi.) Supersymmetric theories assuming the existence of partners to the existing fermions and bosons, with charges in the opposite representations, adjoint for fermions and fundamental for bosons.

The question is: What do the Higgs together with the Yukawa couplings of the *standard model* effectively represent? Is the Higgs really a scalar with the fermionic quantum numbers in the charge sector, or it is just (so far very efficient) effective representation for several scalar fields which manifest as the Higgs and the Yukawa couplings? Are extensions of the Higgs to the Yukawa scalar fields with the family charge(s) again in the fundamental representations the right way beyond the *standard model*?

To answer any question about the Higgs and the Yukawas one first needs the answer the question: Where do Yukawas originate, that is where do families originate?

Although effective interactions can have in physics many times quite unexpected shape and yet can be very useful (as it is the case, for example, with the by experiments suggested spin-spin interaction in several models in the solid state physics where the interaction of the electromagnetic origin among many electrons and nuclei involved can effectively be expressed with the spin-spin interaction) yet it is hard to accept that effective theories of the type where the $SU(3)$ groups describe the family quantum numbers, with the scalar dynamical fields which carry the family charges of the fermion kind, can make useful predictions for new experiments, where searches depend strongly on the proposed theories behind. To my understanding at this stage of physics a new more general understanding of fermion and boson fields is needed.

Any new step in theoretical explanation of the *standard model* assumptions must answer the following most urgent open questions:

- What is the origin of families? How many families there are at all?
- What is the origin of the scalar fields (the Higgs)? Where do their masses (the Higgs mass) and correspondingly the masses of the gauge fields originate? What is the origin of the fermion masses, where do Yukawa couplings originate?

[2] The supersymmetric theories assume that in the low energy regime there exist superpartners to the existing particles. The superpartners to existing bosons are fermions with the charges in the adjoint representations and the super partners to existing fermions are bosons with the charges in the fundamental representations.

- Where does the dark matter originate?

There are also several other questions which may not be so urgently answered, like: Where do dark energy originate? What is the origin of charges, and correspondingly of the gauge fields? What does cause the matter-antimatter asymmetry? And (many) others.

Let me in this discussion demonstrate that the theory unifying spin, charges and families, called the *spin-charge-family* theory [5–9], looks so far very promising in answering the above and several other open questions, and accordingly offers the next step beyond the *standard model*.

The *spin-charge-family* theory is defined in more than four dimensional space (and is accordingly of a Kaluza-Klein type) and therefore not yet acceptable for those who require for all theoretical assumptions the existing experimental confirmation. Yet this theory may teach us a lot about the open questions of the *standard model*, and correspondingly about possible origin of families of fermions and of the origin of scalar fields – the Higgs and Yukawas, if taken as it manifests at low energies.

I shall comment on the nature of the scalar dynamical fields, with the family (flavour) charges in the fundamental (fermionic) representations [1–3], extending Higgs to Yukawas from the point of view of the *spin-charge-family* theory.

14.2 Short presentation of the spin-charge-family theory

I present here the *spin-charge-family* theory from the point of view which shows up what new steps beyond the *standard model* is the theory offering and in which way. I present its starting assumptions and the effective action which the theory manifests after several breaks of the starting symmetry (that is after several phase transitions), manifesting before the electroweak break massless families of fermions and massless gauge vector bosons with the properties which the *standard model* postulates and are presented in tables 14.1, 14.2. The *spin-charge-family* theory predicts four rather than three so far observed families of quarks and leptons at the low energy regime and several scalar dynamical fields (the gauge scalar bosons) with all the charges (with the family quantum number included) in the adjoint representations. Effectively, however, these scalar fields do behave approximately as the *standard model* Higgs and Yukawas. Although the detailed calculations are not yet finished, the so far made estimations show that the *spin-charge-family* theory is in a good way to answer the urgent (presented above) and other open questions of the *standard model*.

The reader is kindly asked to look for more details in the ref. [7,10] and the references therein.

Let me start with the main assumption of the *spin-charge-family* theory.

i.) The space has more than $(1+3)$ dimensions. I made a choice of $d = (1+13)$

ii.) One of the two existing Clifford algebra objects, γ^a, is used to describe spin in $d > (1+3)$ of fermions, the other one, I called it $\tilde{\gamma}^a$, to describe families.

iii.) The simplest action for massless fermions, carrying in $d > (1+3)$ only two kinds of the spin, no charges, and for the corresponding gauge fields - the vielbeins and the two kinds of the spin connection fields - in $d > (1+3)$ is taken.

iv.) The breaks of symmetries (phase transitions) are assumed which lead in $d = (1+3)$ to the observed phenomena: To the observed massive families of quarks and leptons with the observed charges assumed by the *standard model*.

Since there exist two, only two, kinds of the Clifford algebra objects, which generate equivalent representations with respect to each other (that is independent spaces), and since there exist families of fermions, then if one of these two objects is used to describe the spin in $d > (1+3)$ (which then manifest in $d = (1+3)$ as the spin and all the charges of one family of fermions), the other has a great chance to properly describe families. Dirac used 80 years ago γ^a's to describe the spin of fermions enabling the success of quantum mechanics. Kaluza-Klein-like theories [11], assuming that space has more than (1 + 3) dimensions, suggest that the spin together with the angular momentum in higher dimensions manifests as charges in (1 + 3) [3]. As shown in the refs. [5–7] the second kind of the Clifford algebra objects has a great chance to properly describe families.

The *spin-charge-family* theory starts in $d = (1+13)$ with the simplest possible action [6,7] which takes into account both kinds of the Clifford operators, γ^a and $\tilde{\gamma}^a$ (this two kinds of generators anticommute ($\gamma^a\, \tilde{\gamma}^b + \tilde{\gamma}^b\, \gamma^a$) =0),

$$S = \int d^dx\, E\, \mathcal{L}_f + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R}). \tag{14.1}$$

The first part describes the fermion degrees of freedom and is just the action for massless fermions with the two kinds of the spin and no charges interacting correspondingly with (only) the gravitation field – the vielbeins $f^{\alpha}{}_{a}$ and the two kinds of the spin connection fields, the gauge fields of S^{ab} $(= \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a))$ and $\tilde{S}^{ab}$ $(= \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a))$, ($S^{ab}\, \tilde{S}^{cd} - \tilde{S}^{cd}\, S^{ab} = 0$),

$$\begin{aligned} \mathcal{L}_f &= \frac{1}{2}\,(E\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.}, \quad p_{0a} = f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, E f^{\alpha}{}_a\}_-, \\ p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}. \end{aligned} \tag{14.2}$$

The Lagrange density for the gauge fields is assumed to be in the starting action linear in the curvature $R = \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta})\} + \text{h.c.}$, $\tilde{R} = \frac{1}{2}\, f^{\alpha[a}f^{\beta b]}\; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^{c}{}_{b\beta}) + \text{h.c.}$. The action for fermions manifests after several breaks of symmetries (phase transitions) in $d = (1 + 3)$ before the electroweak break four families of left handed weak charged quarks and leptons and right handed weakless (without the weak charge) quarks and leptons [7], just as it is assumed by the *standard model*, except that an additional $U(1)$ charge exists and that there are also right handed neutrinos, with the nonzero value of this additional $U(1)$ charge and that there are four rather than three families. The superposition of the gauge fields $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ manifest after several phase transitions and before the electroweak break as the known gauge vector fields (if $\alpha \in (o, 1, 2, 3)$) and the scalar gauge fields (if $\alpha \in (5, 6, 7, \cdots), d$).

While $\bar{\psi}\, E\,\gamma^m p_{0m}\psi$, with $p_{0m} = p_m - g^a\,\tau^{Ai}A^{Ai}_m$, represents the usual Lagrange density for massless fermions of the *standard model*, and p_{0m} the covariant momentum with the charges A presented in table 14.1, represents $\bar{\psi}\, E\,\gamma^s p_{0s}\psi$, $s = 7, 8$; the mass term [7]: It is the operator γ^s which transforms all the charges of the right handed members into the charges of the left handed ones, doing the job for which the scalar Higgs with the fermionic charges was postulated. The scalar fields appearing in p_{0s}

$$p_{0s} = p_s - \sum_{A,i} (g^{\tilde{A}}\, \tau^{\tilde{A}i}\, \tilde{A}^{\tilde{A}i}) - (g^Q Q A^Q_s + g^{Q'} Q' A^{Q'}_s + g^{Y'} Y' A^{Y'}_s) \tag{14.3}$$

are dynamical scalar fields, which in the electroweak phase transition gain a nonzero "vacuum expectation values"[4] and determine mass matrices of fermions. The two triplet scalar fields $\tilde{A}^{\tilde{A}i}_s$ are the gauge fields of $\tilde{\tau}^{\tilde{A}}$ which determine the two $SU(2)$ charges of four

[3] In the ref. [13] possible ways of solving the Witten's [12] "no-go" theorem for the Kaluza-Klein-type of theories are presented.

[4] The "nonzero vacuum expectation values" of some fields, which break symmetries, are in the solid state physics, well known. Such an example are states of ferromagnetic or anti-ferromagnetic systems.

families, that is the family quantum numbers with the properties that each member of four families carries both quantum numbers and the massless members transform accordingly as presented in the diagram [10,7]

$$\overset{\tilde{N}^i_L}{\leftrightarrow}\begin{pmatrix} I_4 & I_3 \\ I_1 & I_2 \end{pmatrix} \updownarrow \tilde{\tau}^{1i} \quad . \tag{14.4}$$

$\tilde{N}^i_L$ are generators of one of the two $SU(2)$ groups and $\tilde{\tau}^{1i}$ the generators of another one ($\tilde{N}^i_L = \sum_{a,b=0,1,2,3} \tilde{c}^{\tilde{N}^i_L}_{ab}\, \tilde{S}^{ab}$, $\tilde{\tau}^{1i} = \sum_{a,b=5,6,7,8} \tilde{c}c^{\tilde{\tau}^{1i}}_{ab}\, \tilde{S}^{ab}$). Each family member carries in addition the spin and the charges originating in S^{ab}: the hyper ($Y = \sum_{ab} c^{Y}_{ab} S^{ab}$), weak ($\tau^{1i} = \sum_{ab} c^{1i}_{ab} S^{ab}$) and colour charges ($\tau^{3i} = \sum_{ab} c^{3i}_{ab} S^{ab}$) [5]. In the electroweak break all the scalar fields gaining "nonzero expectation values" determine masses and mixing matrices of fermions.

The calculations done so far show that the operator γ^s [6] and the scalar dynamical fields in p_{0s} (all with the charges in the adjoint representations, also with respect to the family quantum numbers) manifest [7,10] the so far observed properties of fermions and gauge fields, and explain therefore the role of the Higgs and the Yukawa couplings of the *standard model*.

The scalar dynamical fields determine also masses of massive weak boson fields, Z_m and $W^{\pm}_m$. Detailed calculations are in progress and the reader is kindly asked to see the refs. [7,10] and the references therein for more information.

14.3 Conclusions and predictions of spin-charge-family theory

The *spin-charge-family* theory predictions are so far:

- There are two groups of four families in the low energy regime. The fourth of the lowest four families waits to be measured. The fifth family of the higher group of four families is stable and it is therefore the candidate to constitute the dark matter. More about these topics can be found in the refs. [7,10,9].
- The scalar fields which are responsible for mass matrices of fermions (and correspondingly for their masses and mixing matrices) have all the charges (with the family charges included) in the adjoint representations, that is they behave like so far observed bosons with respect to the established charges and also with respect to the family charges. According to the so far made calculations [7,10] the *spin-charge-family* theory has a good chance to reproduce the experimental data. How these scalar fields can be represented effectively to manifest as the scalar Higgs field with the fermion charges and the Yukawa couplings, is explained in the ref. [7].
- It is evident from the *spin-charge-family* theory that besides the known gauge fields also the scalar fields are the interaction fields. Accordingly, since they are effectively representing the Higgs and the Yukawa couplings, also the *standard model* Yukawas are the interacting fields.

---

[5] In the *spin-charge-family* theory the family members carry one more $U(1)$ charge, $Y' = \sum_{ab} c^{Y'}_{ab} S^{ab}$ hyper charge.

[6] $\gamma^0\gamma^s$, $s = 7, 8$, can be formally represented as the operator $\sum_{i=1,2} |\psi_{Li} >< \psi_{Ri}|$, $i = 1$ stays for the u quarks and neutrinos and $i = 2$ for the d quarks and electrons, which "rotates" the right handed family members into the left handed partners.

- Searching for scalar fields will manifest that there are several dynamical scalar fields, not just one, and that each of them couples differently to different family members. More about this topic can be read in the ref. [7]. The calculations are under consideration [10].
- There are no supersymmetric partners of the so far observed fermion and boson fields, at least not in the low energy regime.
- The extensions of the *standard model* [1–3], where also Yukawas are taken as dynamical fields with the charges in the fundamental or anti-fundamental representations of several $SU(3)$ groups, distinguishing among the left handed quarks and leptons and among all the right handed family members (not quite because of the right handed neutrinos missing in almost all these kinds of models) can, according to the *spin-charge-family* theory hardly be of some help in predicting future experiments. While the *standard model* Higgs is, together with the Yukawa couplings, a simple and so far very efficient replacement for the dynamical scalar field predicted by the *spin-charge-family* theory, the extensions [1–3] start to be complicated. The experiences in the nuclear physics and solid state physics speak against such models.

There are also several other predictions, not yet enough studied to be commented here.

Let me conclude this paper with the statement: The *spin-charge-family* theory does have the answers to open questions, which are to my understanding the most urgent to be answered for any new successful step beyond the *standard model*. I doubt that trying to explain only one of the ”urgent open questions” (presented above) can bring much new insight into the assumptions of the *standard model*.

The *spin-charge-family* theory offers not only the answer to the question why we have more than one family, and how many there are, it explains also the origin of the Higgs and the Yukawa couplings, of the charges and the gauge fields.

According to these predictions there is no supersymmetric particles at the low energy regime. But, without doubts, there are several additional dynamical fields - interactions.

Work is in progress and should show (the calculations done so far are promissing) that although on the tree level the mass matrices of quarks and leptons are quite far from leading to the observed properties of quarks and leptons, loop corrections change them so that the results for the lower three families agree with the experimental data.

The question is what if the *spin-charge-family* theory is not what the nature has chosen? Even in this case the theory is teaching us how to make the models with the scalar dynamical fields which behave as bosons and which have a chance to answer all the urgent open questions.

References

1. R.S. Chivukula and H. Georgi, Phys. Lett. **B** 188 (1987) 99.
2. G. D'Ambrosio, G. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. B645 (2002) 155–187, arXiv:hep-ph/0207036.
3. R. Alonso, M.B. Gavela, L. Merlo, S. Rigolin, arXiv: 1103.2915v1[hep-ph].
4. C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B147 (1979) 277.
5. N.S. Mankoč Borštnik, Phys. Lett. **B 292** (1992) 25, J. Math. Phys. **34** (1993) 3731 Int. J. Theor. Phys. **40** 315 (2001), Modern Phys. Lett. **A 10** (1995) 587, *Proceedings of the* 13^{th} *Lomonosov conference on Elementary Physics Prticle physics in the EVE of LHC*, World Scientific, (2009) p. 371-378, hep-ph/0711.4681 p.94, arXiv:0912.4532 p.119. *Proceedings to the* 10^{th} *international workshop What Comes Beyond the Standard*

Model, 17 -27 of July, 2007, DMFA Založništvo, Ljubljana December 2007, p. 94-113, hep-ph/0711.4681, p. 94-113, http://viavca.in2p3.fr, arxiv:0812.0510[hep-ph],1-12, arXiv:0912.4532, p.119-135. *Proceedings to the* 5^{th} *International Conference on Beyond the Standard Models of Particle Physics, Cosmology and Astrophysics*, Cape Town, February 1-6, 2010, http://arxiv.org/abs/1005.2288.

6. A. Borštnik, N.S. Mankoč Borštnik, *Proceedings to the Euroconference on Symmetries Beyond the Standard Model*, Portorož, July 12-17, 2003, hep-ph/0401043, hep-ph/0401055, hep-ph/0301029, Phys. Rev. **D 74** (2006) 073013, hep-ph/0512062.
7. N.S. Mankoč Borštnik, arxiv.org/abs/1011.5765, arxiv.org/abs/1012.0224, p. 105-129.
8. G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, New J. of Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0612250, p.25-50.
9. G. Bregar, N.S. Mankoč Borštnik, Phys. Rev. **D 80** (2009) 083534.
10. A. Hernández-Galeana, N.S. Mankoč Borštnik, arXiv:1012.0224, p. 166-176. The contribution to this proceedings.
11. The authors of the works presented in *An introduction to Kaluza-Klein theories*, Ed. by H. C. Lee, World Scientific, Singapore 1983.
12. E. Witten, Nucl. Phys. **B 186** (1981) 412; Princeton Technical Rep. PRINT -83-1056, October 1983.
13. N.S. Mankoč Borštnik, H.B. Nielsen, Phys. Lett. **B 633** (2006) 771-775, hep-th/0311037, hep-th/0509101, Phys. Lett. **B 644** (2007)198-202, Phys. Lett. **B 10** (2008)1016., D. Lukman, N.S. Mankoč Borštnik, H.B. Nielsen, http://arxiv.org/abs/1001.4679v5, New J. Phys. **13** (2011) 103027.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 223)*
Bled, Slovenia, July 11-21, 2011

15 On the Problem of Quark-Lepton Families

M. Yu. Khlopov

Centre for Cosmoparticle Physics "Cosmion" and
National Research Nuclear University "Moscow Engineering Physics Institute",
115409 Moscow, Russia
APC laboratory
10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France
khlopov@apc.univ-paris7.fr

Abstract. The problem of quark-lepton families is discussed in the "bottom-up" phenomenological approach to the extensions of the Standard model. It provides the possibility of the *Horizontal unification* of the three known families on the basis of horizontal gauge flavor $SU(3)_H$ symmetry. The new generations of quarks and leptons can exist. If unstable and mixed with light fermions, they should contribute the CKM matrix. If stable and decoupled from known families, new generations can provide new candidates for dark matter.

15.1 Introduction

The origin of families and description of their properties should have fundamental theoretical basis. This description should reproduce the observed properties of the three known families and make definite predictions for the expected properties of new generations. Here we discuss some qualitative features of the approach of phenomenological bottom-up models based on the extension of the symmetry of the Standard model by the additional gauge group $SU(3)_H$ of the family symmetry. This review may be useful for the distinction of the "*spin-charge-family-theory*" [1] from other approaches to the problem of quark lepton families. On the other hand the experience of phenomenological description of flavor symmetry can be useful for further development of this theory.

The existence and observed properties of the three known quark-lepton families appeal to the broken $SU(3)_H$ family symmetry [2], which should be involved in the extension of the Standard model. It provides the possibility of the *Horizontal unification* in the "bottom-up" approach to the unified theory [3]. Even in its minimal implementation the model of *Horizontal unification* can reproduce the main necessary elements of the modern cosmology. It provides the physical mechanisms for inflation and baryosynthesis as well as it offers unified description of candidates for Cold, Warm, Hot and Unstable Dark Matter. Methods of cosmoparticle physics [4,5] have provided the complete test of this model.

The extension of the Standard model also involve new generations. Stable new generations are of special interest for cosmological consequences, since they can provide candidates for the dark matter.

Here we discuss the possibilities to link physical basis of modern cosmology to the parameters of broken family symmetry, as well as the possible physical basis and properties of fourth generation.

15.2 Horizontal unification

Model of *Horizontal unification* [3] is based on the development of gauge $SU(3)_H$ flavor model of quark-lepton families [2] and occupies a special place in the phenomenological description of known families (see [4,5] for review and references).

15.2.1 Horizontal hierarchy

The approach [2] follows the concept of local gauge symmetry $SU(3)_H$, first proposed by Chkareuli (1980). Under the action of this symmetry the left-handed quarks and leptons transform as $SU(3)_H$ triplets and the right-handed as antitriplets. Their mass term transforms as $3 \bigotimes 3 = 6 \bigotimes \bar{3}$ and, therefore, can only form as a result of horizontal symmetry breaking.

This approach can be trivially extended to the case of n generations, assuming the proper $SU(n)$ symmetry. For three generations, the choice of horizontal symmetry $SU(3)_H$ is the only possible choice because the orthogonal and vector-like gauge groups can not provide different representations for the left- and right-handed fermion states.

In the considered approach, the hypothesis that the structure of the mass matrix is determined by the structure of horizontal symmetry breaking, i.e., the structure of the vacuum expectation values of horizontal scalars carrying the $SU(3)_H$ breaking is justified.

The mass hierarchy between generations is related to the hypothesis of a hierarchy of such symmetry breaking. This hypothesis is called - the hypothesis of horizontal hierarchy (HHH) [6].

The model is based on the gauge $SU(3)_H$ flavor symmetry, which is additional to the symmetry of the Standard model. It means that there exist 8 heavy horizontal gauge bosons and there are three multiplets of heavy Higgs fields $\xi_{ij}^{(n)}$ (i,j - family indexes,$n = 1, 2, 3$) in nontrivial (sextet or triplet) representations of $SU(3)_H$. These heavy Higgs bosons are singlets relative to electroweak symmetry and don't have Yukawa couplings with ordinary light fermions. They have direct coupling to heavy fermions. The latter are singlets relative to electroweak symmetry. Ordinary Higgs ϕ of the Standard model is singlet relative to $SU(3)_H$. It couples left-handed light fermions f_L^i to their heavy right-handed partners F_R^i, which are coupled by heavy Higgses ξ_{ij} with heavy left handed states F_L^j. Heavy left-handed states F_L^j are coupled to right handed light states f_R^j by a singlet scalar Higgs field η, which is singlet both relative to $SU(3)_H$ and electroweak group of symmetry. The described succession of transitions realizes Dirac see-saw mechanism, which reproduces the mass matrix m_{ij} of ordinary light quarks and charged leptons f due to mixing with their heavy partners F. It fixes the ratio of vacuum expectation values of heavy Higgs fields, leaving their absolute value as the only main free parameter, which is determined from analysis of physical, astrophysical and cosmological consequences.

The $SU(3)_H$ flavor symmetry should be chiral to eliminate the flavor symmetric mass term. The condition of absence of anomalies implies heavy partners of light neutrinos, and the latter acquire mass by Majorana see-saw mechanism. The natural absence in the heavy Higgs potentials of triple couplings, which do not appear as radiative effects of any other (gauge or Yukawa) interaction, supports additional global U(1) symmetry, which can be associated with Peccei-Quinn symmetry and whose breaking results in the Nambu-Goldstone scalar filed, which shares the properties of axion, Majoron and singlet familon.

15.2.2 Horizontal unification

The model provides complete test (in which its simplest implementation is already ruled out) in a combination of laboratory tests and analysis of cosmological and astrophysical effects. The latter include the study of the effect of radiation of axions on the processes of stellar evolution, the study of the impact of the effects of primordial axion fields and massive unstable neutrino on the dynamics of formation of the large-scale structure of the Universe, as well as analysis of the mechanisms of inflation and baryosynthesis based on the physics of the hidden sector of the model.

The model results in physically self-consistent inflationary scenarios with dark matter in the baryon-asymmetric Universe. In these scenarios, all steps of the cosmological evolution correspond quantitatively to the parameters of particle theory. The physics of the inflaton corresponds to the Dirac see-saw mechanism of generation of the mass of the quarks and charged leptons, leptogenesis of baryon asymmetry is based on the physics of Majorana neutrino masses. The parameters of axion CDM, as well as the masses and lifetimes of neutrinos correspond to the hierarchy of breaking of the $SU(3)_H$ symmetry of families.

15.3 New generations

15.3.1 The problem of New generations

Modern precision data on the parameters of the Standard model do not exclude [7] the existence of the 4th generation of quarks and leptons. Even more new generations are possible, if their contribution to these parameters is negligible, e.g. due to decoupling of very heavy quarks and leptons of these new generations.

If the 4th generation is mixed with the three known families, quarks and leptons of this generation are unstable, as it is the case for the current implementation of the "*spin-charge-family-theory*". Then the fermions of such 4th sequential generation should contribute in the matrix of quark mixing and their effect should be observed as violation of orthogonality of the Cabibbo-Kobayashi-Maskava matrix for three known generations. Therefore such violation would favor the existence of the 4th sequential family.

The problem of 4th sequential generation is related with the 4th neutrino, which should be heavier than $m_Z/2$ (m_Z is the mass of Z boson), what follows from the Z boson width. There should be some fundamental explanation for such a great difference in mass of this neutrino and neutrinos of the three light families. In the case of stable 4th generation this fact finds natural explanation in a new conserved charge, which 4th generation possess.

15.3.2 Stable 4th generation

The hypothesis of stable 4th generation was connected in [8] with the phenomenology of superstrings. In this phenomenology the GUT symmetry has a rank higher than the rank of the symmetry of the standard model. On the other hand, the Euler characteristic of the topology of the compactified six dimensions defines in this approach the number of generations of quarks and leptons, which can be both 3 and 4. The difference in the ranks of the symmetry groups of grand unification and the standard model implies the existence of at least one new conserved charge, which may be associated with quarks and leptons of the fourth generation. This may explain the stability of the lightest quarks and leptons (massive neutrinos) of the 4th generation and provides the basis for composite dark matter model. The latter is discussed in the contribution [9,10] to these proceedings.

15.4 Conclusions

In its current implementation the qualitatively important features of "*spin-charge-family-theory*" are related with the principal existence of the 4th sequential family, which should be mixed with the 3 known families. If decoupling of the 4th family from the three known families is possible, the known families can be considered in the framework of $SU(3)_H$ flavor symmetry and the experience, gained in the development of the model of horizontal unification (MHU) will be useful. On the other hand, MHU offers guidelines, following which development of "*spin-charge-family-theory*" can give physical mechanisms of inflation, baryosynthesis and proper candidates for dark matter.

References

1. N.S. Mankoč Borštnik, *Phys. Lett.* **B 292** (1992) 25, *J. Math. Phys.* **34** (1993) 3731 Int. J. Theor. Phys. **40** 315 (2001), *Modern Phys. Lett.* **A 10** (1995) 587, *Proceedings of the* 13^{th} *Lomonosov conference on Elementary Physics Prticle physics in the EVE of LHC*, World Scientific, (2009) p. 371-378, hep-ph/0711.4681 p.94, arXiv:0912.4532 p.119. *Proceedings to the* 10^{th} *international workshop What Comes Beyond the Standard Model*, 17 -27 of July, 2007, DMFA Založništvo, Ljubljana December 2007, p. 94-113, hep-ph/0711.4681, p. 94-113, http://viavca.in2p3.fr, arxiv:0812.0510[hep-ph],1-12; arXiv:0912.4532, p.119-135; *Proceedings to the* 5^{th} *International Conference on Beyond the Standard Models of Particle Physics, Cosmology and Astrophysics*, Cape Town, February 1-6, 2010, http://arxiv.org/abs/1005.2288; Contribuion to this Volume.
2. Z.G.Berezhiani and M.Yu.Khlopov, *Sov.J.Nucl.Phys.* 51 (1990) 739; 935; *Sov.J.Nucl.Phys.* 52 (1990) 60; *Z.Phys.C- Particles and Fields* 49 (1991) 73; Z.G.Berezhiani, M.Yu.Khlopov and R.R.Khomeriki, *Sov.J.Nucl.Phys.* 52 (1990) 344.
3. A.S.Sakharov and M.Yu.Khlopov *Phys.Atom.Nucl.* 57 (1994) 651.
4. M.Yu. Khlopov: *Cosmoparticle physics*, World Scientific, New York -London-Hong Kong - Singapore, 1999.
5. M.Yu. Khlopov: *Fundamentals of Cosmoparticle physics*, CISP-Springer, Cambridge, 2011.
6. Z.G. Berezhiani, J.K. Chkareuli, JETP Lett. **35** (1982) 612; JETP Lett. **37** (1983) 338; Z.G. Berezhiani, Phys. Lett. **B 129** (1983) 99.
7. M. Maltoni *et al.*, Phys. Lett. **B 476** (2000) 107; V.A. Ilyin *et al.*, Phys. Lett. **B 503** (2001) 126; V.A. Novikov *et al.*, Phys. Lett. **B 529** (2002) 111; JETP Lett. **76** (2002) 119.
8. K.M.Belotsky, M.Yu.Khlopov and K.I.Shibaev, Gravitation and Cosmology Supplement **6** (2000) 140; K.M.Belotsky *et al.*, Gravitation and Cosmology **11** (2005) 16; K.M.Belotsky *et al.*, Phys.Atom.Nucl. **71** (2008) 147.
9. K.M.Belotsky, M.Yu.Khlopov and K.I.Shibaev, This Volume, p. 9.
10. M. Y. Khlopov, A. G. Mayorov and E. Y. Soldatov, This Volume, p. 94.

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond ... *(p. 227)*
Bled, Slovenia, July 11-21, 2011

16 Importance of Misunderstanding in Physics Illustrated by Nonexistent Higgs

R. Mirman*

14U
155 E 34 Street
New York, NY 10016

Abstract. Absurdity strengthens belief in nonsense, fantasy, so misleading physicists, officials, appropriators — wasting much time, talent (?) and money. This is illustrated by the glaring nonsense of string theory and the hugely expensive Higgs fantasy.

There has been much effort in physics studying things that do not exist, resulting in much effort, (taxpayer) money and careers wasted.A striking example is the mythical Higgs boson. What are the fantasies about it and why are they so strongly held? To study this we must consider gauge transformations. In elementary physics we learn that the position of the origin is irrelevant, has no effect. Only differences are important. Thus in dropping an object we find that the final speed is determined (for a constant field) by the distance fallen through, not by the initial and final positions. This change of origins is a gauge transformation, and the independence of the origin an example of gauge invariance. (Of course this is for 1 dimension, not the 3+1 of space, but the principle is the same). We can add an arbitrary function (subject to some conditions) to an EM field (for example) with no change in any experimental predictions.

Notice (which it seems physicists do not) that gauge transformations are possible for EM and gravity. These are both massless (have 0 rest mass, which means they can never be at rest).

Why are gauge transformations possible (for massless objects)? This has been discussed ln depth ([2]). Here we just give a brief explanation [1]). Consider an electron and a photon traveling parallel to each other (along the same line) with their spins also along this line. We now perform a transformation changing the spin direction of the electron, leaving both momentum directions unchanged.Thus for the electron, spin and momentum are no longer parallel. However for the photon, spin and momentum are required (by the mathematics, not God) to be along the same line (the EM field is transverse). Thus there are transformations that act on the electron, that do not (seem to) act on the photon. What are these?

Obviously gauge transformations.

Gauge transformations arise because there are transformations that act on massive objects that cannot act (in the same way) on massless ones. They cannot apply to massive objects because there are no extra transformations that they can be.

Yet physicists decided that gauge transformations are so wonderful that they applied them in other cases, where they cannot be applied, resulting in all objects being massless(!). Of course this is wrong, and disagrees with experiment. However physicists believe that if

* sssbbg@gmail.com

their theories disagree with nature, then nature must be wrong. (Consider all the physicists who devote their entire careers to revising nature to fit their theories). Thus they have come up with the belief that space is filled with a field producing drag giving objects mass even though they have decided objects are massless. (Is this clear? It is the logic leading to spending billions of dollars of the taxpayer 's money.)

This is the Higgs field and the particle going with it the Higgs boson, which billions of dollars are being spent looking for.

There is a simpler (and cheaper) explanation of why objects have mass: the assumption that all objects are massless is wrong. Since gauge transformations are a property of massless objects, applying it to massive ones is nonsense. Of course it turns out wrong.

There is a guide to theories: Occams razor (parsimony). Physicists are determined to flaunt their contempt for Occam producing theories luxuriant with an overgrowth of unsupported assumptions, nonsensical and wrong.

Acknowledgements

These discussions could not have existed without Norma Mankoč Borštnik.

References

1. Mirman, R. (2006), Our Almost Impossible Universe: Why the laws of Nature make the existence of humans extraordinarily unlikely (Lincoln, NE: iUniverse, Inc.)
2. Mirman, R. (1995c), Massless Representations of the Poincaré Group, electromagnetism, gravitation, quantum mechanics,geometry (Commack, NY: Nova Science Publishers, Inc.; republished by Backinprint.com. iUniverse, Inc.)

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 229)*
Bled, Slovenia, July 11-21, 2011

17 Discussion on the Cosmological Vacuum Energy

H. B. Nielsen[1⋆] and C. Balázs[2⋆⋆]

[1]Niels Bohr Institute, University of Copenhagen,
17 Blegdamsvej, DK 2100 Denmark
[2]Monash Centre for Astrophysics and
ARC Centre of Excellence for Particle Physics at the Tera-scale
School of Physics, Monash University, Victoria 3800 Australia

Abstract. The present discussion contribution is some remarks concerning and review of the proposal by one of us to explain the cosmological constant by a/the principle of entropy. Used without further comment this principle of entropy could easily lead to untrustable *nonlocalities*, but taking into account that the long range correlations are rather to be understood as due to initial condition set up the model for the cosmological constant being small by one of us becomes quite viable.

17.1 Introduction

In a recent paper one of us [1] proposed to explain the smallness of the cosmological vacuum energy based on the energy limit that general relativity imposes on any given volume. According to Einstein's theory the maximal amount of energy in a 3-dimensional volume scales with the linear size rather then the volume itself. This energy limit can be equivalently formulated as an upper limit on the entropy contained in the volume, which latter is known as the holographic entropy bound.[1] The entropy bound is well known, for example, for Schwarzschild black holes: the entropy contained within the horizon cannot exceed the quarter of the surface area (in Planck units).

From this entropy, or equivalently energy, bound it follows that there must exist an ultraviolet cut-off for fields in the region inside any volume [1]. Simply put: the entropy (energy) in the given volume cannot exceed the maximal entropy (energy) of a black hole that fills the given volume. With such a cut-off the zero point energy of any fields, that is the vacuum energy, inside any volume becomes restricted. This in turn limits the cosmological vacuum energy which otherwise would contribute to dark energy (or the cosmological constant) an enormous amount. Such a restriction is not only suitable to reduce dark energy (cosmological constant), but as explained in Ref. [1] it will ensure that the theoretical prediction of the cosmological vacuum energy will exactly match that of the experimentally measured value.

However, it is non-trivial to understand how such an energy cut-off is implemented in quantum field theory. The most naive implementation of such a cut-off would be to restrict the individual degrees of freedom within a given volume independently not to exceed their average energy. Unfortunately, this cut-off would lead to an energy limit that

⋆ e-mail: hbech@nbi.dk
⋆⋆ e-mail: csaba.balazs@monash.edu
[1] See Ref. [1] for detailed literature on the subject.

scales with the volume rather than the linear size. Moreover, in case of the cosmological vacuum energy this is experimentally excluded since locally we observe higher energy density systems in our Universe than its average dark energy density. Thus the cut-off has to vary and has to be non-trivially correlated between degrees of freedom of the Universe. This would allow for the existence of small regions with high energy densities while the rest could compensate such that the average never exceeds the ratio of the maximal allowed energy and the volume.

But this sort of cut-off raises another question: How can such a correlated cut- off be consistent with the locality of quantum field theory? If no signal propagates faster than the speed of light, can potentially distant parts of the volume compensate for each other? Naturally, whether such a cut-off is consistent with locality depends on the details of the implementation of the cut-off itself. Most importantly, how the long range correlation between the allowed energy in one region depends on what goes on or what is allowed in an other region. But if the cut-off at one region depends on what goes on at an other, potentially distant, region *at the same time*, then locality can definitely get into trouble! So it can at best be a tolerable correlation of the cut-off at one locality with what went on somewhat earlier around the Universe in remote regions, otherwise causality is threatened.

To implement such a cut-off while saving locality, one may think in two different ways:

a) One could accept that locality is not necessarily a good principle and the solution necessitates "new physics". But then one is up to theories like the "complex action theory" proposed by Ninomiya and the other one of the present authors(HBN)[2].

b) An alternative solution is the use of some cosmic censorship assumption such as the non-existence of "white holes", that is time-reversed black holes. Such an assumption is needed anyway, to maintain the entropy bound [3].

It appears that in quantum field theory the entropy bound holds only if either

- the cut-off is strangely correlated between the degrees of freedom, as suggested by [1], or
- the limitation of the number of states is not just a limitation due to the cut-off of the theory but due e.g. to some special initial condition. And as an example of the latter - one of us would say more reasonable type of state limitation for the application in question - the cosmic censorship comes in.

In Section 17.2 we discuss the problems related to the argument that the cosmological vacuum energy is limited by the entropy bound. In Section 17.3 we put forward an idea of how a cut-off based on the entropy bound could be interpreted or replaced by a cosmic censorship based philosophy. This latter could, at least in a certain sense, be free of the problems with locality or causality. Finally we conclude and look out in the conclusion section 17.4.

17.2 Trouble for the Entropy Principle

In a physical system obeying the laws of thermodynamics the extensive thermodynamical variables, such as energy and entropy, typically scale with the volume containing the system. Since in quantum systems the energy, in turn, typically scales with the number of degrees of freedom, the latter is usually thought to grow with the volume. Considering a system of fields defined within a volume, without any special restrictions on the degrees of freedom, it is clear that the number of states can grow as an exponential of the volume. In field theory in any local region of space the energy density can reach that of the highest energy accelerators and beyond, and the number of degrees of freedom in a given volume is unlimited. Because of this field theory does not respect the Bekenstein-Hawking entropy bound or the Schwarzschild energy limit. As we saw before, the entropy bound as a naive, uncorrelated cut-off on any given volume is out of question in field theory. The entropy and energy bounds can only be consistent with field theory, they can only allow reaching energy densities well tested in science and in daily life, if the corresponding cut-off is highly correlated between the degrees of freedom. Thus the nature of the cut-off is such that it imposes a strong restriction on the allowed states.

About a decade ago Bousso extended the Bekenstein-Hawking entropy bound into a covariant entropy conjecture [3]. While Bousso sharpened the definition of the interior of the surface in which the entropy is limited by the enclosing area, in his derivation he also made the crucial assumption that there should be *no singularities* in the interior in question. (Cf. page 9 of [3].) Bousso also pointed out that "Because the conjecture is manifestly time reversal invariant, its origin cannot be thermodynamic, but must be statistical. It thus places a fundamental limit on the number of degrees of freedom in nature."

In case of black holes, for example, the non-singularity assumption appears to be an obvious necessity. Otherwise one can imagine white holes channeling entropy into the volume that is independent of the surface area of the black hole. This could easily violate the entropy bound.

17.2.1 Time reversal thinking on the number of states problem

It is natural to get an idea of our problem for the "entropy principle" by thinking for a moment in the time reversal way:

If we think about how one of the to a volume-behaving entropy enormously many states could have come about, we may produce the answer by thinking time-reversed: What would happen if we started with a typical state taken out of the situation with a volume proportional entropy, and reversed the Hubble expansion to be a Hubble contraction. That would mean a situation with a very high energy density over a very large extension and would of course correspond to a world that were already to be considered inside a black hole. Also it would have already so much entropy that it would be too much for a/one black hole. Rather what such a system would develop into would be many many black holes. As such a collapsing universe with a lot of energy density develops the energy density gets even bigger and after some time there will be many relatively small subregions which have both too much energy and too much entropy to avoid being black holes. So at some stage it would develop into an approximately smooth distribution of "small" black holes. This would mean a kind of piecewise collapse - even before the naively calculated total collapse, when the general size of this universe would go to zero. One could say that this in naive sense calculated collapse due to the radius going to zero never gets realized, because the piecewise collapse into separate black holes takes over effectively and forms a collapse at an earlier stage.

Now time reversing this scenario back to the real world, its means that the majority of the to the volume behaving entropy corresponding states are of such a nature, that they could only be formed from an earlier stage of the Universe containing enormously many "small" "white holes" rather than coming from a genuine Big Bang or other single or few singularity picture as usual cosmology tells.

Since the "white holes" - meaning as we just used here the time reversed black holes - are precisely the most important example of what a cosmic censorship principle should forbid, it is clear that the majority of states in the volume-based entropy scenario are *cosmic censorship forbidden* states. In this way there is at least the hope that it is the cosmic censorship that can bring the number of states down to match the Bekenstein-Hawking-area law.

17.3 Can we Rescue the Cosmological Constant Derivation?

At first there might seem to be a chance to rescue the work of one of us on deriving the cosmological constant being small derivation by just saying:

Taken as simply an ultraviolet cut off straight away it looks dangerous for locality with the correlations in the cut off needed to make the cut off match the entropy principle. However, if we now interpret the cut off as mainly due to the initial conditions occurring due to say cosmic censorship requirements it would sound much more acceptable, since it would no longer even threaten the locality in a genuine sense. You cannot really in a world, which has come from a development, in which a cosmic censorship principle were valid, send messages faster than light or the like. At least we have no experimental evidence against that we should live in a world with no white holes (unless though perhaps Big Bang itself should be considered a white hole or a cosmic censorship violating event). So a cut off considered due to a cosmic censorship would seemingly not be against what we could believe. However, then the problem would be: The main job of the cut off which we should obtain due to the entropy principle were to limit the zero-point energy of the quantized field theory of the world, say of the Standard Model. At first one would think that the cosmic censorship and other agents that could influence the initial state would not really influence the zero-point energy! One would say this because when we think of initial conditions caused by such influences as cosmic censorship or from inflations and development whatever, then one has in mind that all those high frequency modes which are at a certain time not excited by onshell particles will fluctuate nevertheless "peacefully" in their zero-point fluctuation way. We so to speak normally imagine that the zero point fluctuations for the high frequency modes just are there as in vacuum for all the frequencies higher than the ones relevant for the state being realized. In this philosophy the initial state and thus the cosmic censorship would not get true access to influence the cutting off of the high frequency modes. If we cannot get the cosmic censorship influence the higher frequencies zero mode fluctuation of course the above discussion and proposal to use cosmic censorship would not help.

17.3.1 But could initial state effects possibly influence zero-point fluctuations?

But now really the question is: Shall we take it for a good argument that zero-point oscillations of very high frequencies are organized to be present as soon as we reach temperatures where they are no longer excited? At first one would again say: yes, it is reasonable that the high frequency modes would fall to their zero point fluctuation level but no longer as the Universe expands with a very strong Hubble expansion and effectively the excitation of a mode is moved from one mode to a lower one due to this expansion. The zero point oscillation cannot be reduced by the Hubble expansion and the higher frequency modes would seemingly have to stay in their zero point fluctuation.

But are we not more and more dreaming about a fantasy world of high frequencies which never according to the entropy principle should even have a chance to be realized? If one turned the philosophy a bit around one would say: We have this fantastic dream of there existing a number of possible states of the Universe system which is the number of states corresponding to an entropy going with the *volume* of the Universe, but on the other we know from entropy principle or essentially equivalently from the cosmic censorship that it is only a very tiny minority of these states that have a true chance to be realized. In a way it would be most sensible if in an ontological way only the states that have at least the chance corresponding to the entropy principle would exist in a sense of being present in the most fundamental theory. But if that were so then all these fantasy states making up the to volume proportional entropy ought not to be there. That would of course make it more strange to worry about the zero point energy involved in the many high energy modes which can essentially ever be excited, or at least almost never. It should namely be had in mind that at colliders like LHC we do excite very high modes which are normally - i.e. in the almost empty universe - never excited.

Shall we really imagine that in the fundamental theory at the ontological level which we shall may be once find the degrees of freedom relevant for the LHC are in some strange way being built up from some degrees of freedom that at first looked like being made for a bit microwaves in a low frequency passing far behind the Moon? Shall we really imagine that ontologically at the end the degrees of freedom are being shuffled around so that when the LHC needs some more degrees of freedom it collects them up from perhaps big distances away ? Although it sounds a great challenge to construct just a model showing that such an idea is possible in a local way, it may not be totally excluded since either a clever way may be found or nature might at the root of it not respect our usually expected principles of locality.

17.4 Conclusion

We have discussed some problems with the model of one of us solving the cosmological constant problem - of the surprisingly small size of the cosmological constant found experimentally - by using the entropy principle (of the entropy only going as the surrounding area). The major problem is really a problem with the entropy principle rather than only with the proposed solution to the cosmological constant. You namely cannot interpret the entropy principle at all as a restriction given on the number of states as due to some conventional cut off. So either you must say that the entropy principle has nothing to do with the number of states allowed by an ultraviolet cut off - but is say a question of the initial state only (perhaps via cosmic censorship)- or we must be satisfied by an ultraviolet cut off that at least at first looks rather complicated with one would say mysterious correlations. It may be that these “mysterious correlations” could sound sensible from a speculative fundamental physics point of view.

17.4.1 Outlook and hope

It looks that our discussion is driving us in the direction of asking how much reality there is at the fundamental level in the zero point fluctuations of the various fields. For instance in last years discussions there were a contribution by one of us(H.B.N.) Moultaka and Nagao and Norma Mankoč Borštnik[4] related to the quantum mechanics philosophy going back to De Broglie. The crux of the matter is that the quantum system *has* a position even when it is not in a position eigenstate! Translated into field theory we might take this to maean that the fields *have* values even when they are not in an eigenstate field values. This is of course crazy and in disagreement with Heisenberg uncertainty principle, but for Bohm and De Broglie the philosophy is different. If we bought the theory of Bohm and De Broglie for fields we would not have to believe that there were truly(ontologically) zero point fluctuations, but could leave it as a more difficult physical question to be answered by a deeper understanding of the more fundamental theory we are looking for. But to by such a means get the contribution to the cosmological constant from the high frequency modes be negligible as is hoped to get rid of the cosmological constant problem it would e needed that this hoped for theory behind (or at the end at the most fundamental level) would put the energy of the high frequency modes to be lower than what is possible in quantum mechanics with its minimum at the zero point energy for a harmonic oscillator. We would have to put the harmonic oscillator corresponding to the high frequency modes of the fields - for which we want to get rid of the contribution to the cosmological constant - to have *both* zero momentum and zero position rather exactly! For Heisenberg impossible, but for Bohm and De Broglie the question is more to be studied with more details added.

17.4.2 The more Private hope using the Complex Action Model

Since as the discussion above has shown the proposed model for solving the cosmological constant problem has been threatened - although not definitively killed on that ground - by locality principle. If it should at the end turn out to be indeed needed to give up such principles and for instance go to a model like the "complex action model" by one of us (H.B.N.) and Ninomiya - originally based on ideas developed by H.B.N. and Don Bennett - which were the model used in the above mentioned discussion contribution from last year by Moultaka et. al. we might use such a model to suggest what should be the classical values of the high frequency fields. In other words we might now ask in the complex action model for how the in this model essentially classically standing fields behave (it means they do not respect Heisenberg in the way we ask for their fundamental values). We can almost immediately guess the answer: In the complex action model the guiding principle is that the initial conditions get set so as to minimize the imaginary part of the action. Now this must looking at world as it is mean that to have the vacuum we live in is extremely favourable to lower this imaginary part of the action. Then presumably if the "God"(having such a minimization principle arranging the quantity the imaginary part of the action S_I to be minimal is almost like having a "God" in quotation marks governing the world to make "His" deficit S_I as small as possible, preferably negative) behind the governing of the Universe were so keen to make so much vacuum, "He" should be even more keen to push the vacuum the last little bit by putting the fields that in our usual vacuum picture are in their zero point fluctuation states the last bit so as to have both momentum and position go to the bottom. If the imaginary part of the action S_I is just a reasonably smooth function(al) of the field configurations and their conjugate momenta and it seems that the most beloved state (by "God", meaning giving the most favoured meaning low S_I) is the vacuum then if it were possible almost certainly the classical replacement for the vacuum having the fields exactly zero would have an even lower S_I and thus be even more beloved! So our complex action model would indeed predict that the values of the fields - only being allowed by De Broglie and Bohm - would be so that the zero point energy would be killed at the fundamental level! That would as the reader can immediately understand be wonderful for the cosmological constant model we have discussed: we suggested in the last years discussion that the complex action model could function approximately as a model behind the Bohm-De Broglie picture, and now that the prediction from complex action would then be that the vacuum fields would be - at least when not too much disturbed - be put to zero exactly (contrary to what Heisenberg uncertainty would allow, but that is o.k. in Bohm De Broglie and in complex action interpreted the right way as being "by hindsight", i.e. including knowledge collected by a measurement) as well as the conjugate momentum to the field modes in question.

Now using the complex action might however be an almost too high price in the sense that Ninomiya and one of us (H.B.N.) already have an article suggesting that this complex action model is good for helping with the cosmological constant problem [5]. In the kind of thinking in the articles seeking to solve cosmological constant problem in the complex action model or related models previously the philosophy were however quite a bit different in as far as in that sort of works it would rather be assumed that there is presumably very big *bare cosmological constant*, which simply gets adjusted by essentially the already mentioned "God" in quotation marks so as to minimize the imaginary part of the action. If "He" for some reason should want a Universe avoiding collapse but not expanding faster than necessary "He" could easily arrive to vote for a small cosmological constant. But if "He" has power to adjust the *bare* cosmological constant, "He" hardly need to for that reason go into adjusting zero-point fluctuating modes, but "He" according to the above does it anyway.

Acknowledgement

One of us (H.B.N.) wants to thank Keiichi Nagao of Ibaraki University for discussions about the problems of number of states relatively time ago at a conference in Holbaek.

References

1. C. Balazs, I. Szapudi, AIP Conf. Proc. **903** (2007) 560-563. [hep-th/0605190].
2. H. B. Nielsen and M. Ninomiya, Proceedings to the 9th Workshop "What Comes Beyond the Standard Models", Bled, 16 - 26 September 2006, DMFA Zaloznistvo, Ljubljana; hep-ph/0612032.
3. R. Bousso, JHEP **9907** (1999) 004 [arXiv:hep-th/9905177].
4. H. B. Nielsen , N. S. Mankoč Borštnik, K. Nagao and G. Moultaka, Bled Proceeding on "What comes beyond the Standard Model " 2010 (these proceedings last year) , p. 211.
5. H. B. Nielsen and M. Ninomiya, Progress of Theoretical Physics, Vol. 116, No. 5 (2006); hep-th/0509205, YITP-05-43, OIQP-05-09.

Virtual Institute of Astroparticle Physics Presentation

BLED WORKSHOPS
IN PHYSICS
VOL. 12, NO. 2

Proceedings to the 14th Workshop
What Comes Beyond . . . *(p. 239)*
Bled, Slovenia, July 11-21, 2011

18 VIA Discussions at XIV Bled Workshop

M. Yu. Khlopov[1,2,3]

[1]Moscow Engineering Physics Institute (National Nuclear Research University), 115409 Moscow, Russia
[2] Centre for Cosmoparticle Physics "Cosmion" 125047 Moscow, Russia
[3] APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. Virtual Instutute of Astroparticle Physics (VIA), integrated in the structure of Laboratory of AstroParticle physics and Cosmology (APC) is evolved in a unique multifunctional complex of $e-science$ and $e-learning$, supporting various forms of collaborative scientific work as well as programs of education at distance. The activity of VIA takes place on its website and includes regular videoconferences with systematic basic courses and lectures on various issues of astroparticle physics, regular online transmission of APC Colloquiums, participation at distance in various scientific meetings and conferences, library of their records and presentations, a multilingual Forum. VIA virtual rooms are open for meetings of scientific groups and for individual work of supervisors with their students. The format of a VIA videoconferences was effectively used in the program of XIV Bled Workshop to provide a world-wide participation at distance in discussion of the open questions of physics beyond the standard model. The VIA system has demonstrated its high quality and stability even for minimal equipment (laptop with microphone and webcam and WiFi Internet connection).

18.1 Introduction

Studies in astroparticle physics link astrophysics, cosmology, particle and nuclear physics and involve hundreds of scientific groups linked by regional networks (like ASPERA/ApPEC [1]) and national centers. The exciting progress in these studies will have impact on the fundamental knowledge on the structure of microworld and Universe and on the basic, still unknown, physical laws of Nature (see e.g. [2,3] for review).

In the proposal [4] it was suggested to organize a Virtual Institute of Astroparticle Physics (VIA), which can play the role of an unifying and coordinating structure for astroparticle physics. Starting from the January of 2008 the activity of the Institute takes place on its website [5] in a form of regular weekly videoconferences with VIA lectures, covering all the theoretical and experimental activities in astroparticle physics and related topics. The library of records of these lectures, talks and their presentations is now accomplished by multi-lingual Forum. In 2008 VIA complex was effectively used for the first time for participation at distance in XI Bled Workshop [6]. Since then VIA videoconferences became a natural part of Bled Workshops' programs, opening the virtual room of discussions to the world-wide audience. Its progress was presented in [8,9]. Here the current state-of-art of VIA complex, integrated since the end of 2009 in the structure of APC Laboratory, is presented in order to clarify the way in which VIA discussion of open questions beyond the standard model took place in the framework of XIV Bled Workshop.

18.2 The current structure of VIA complex

18.2.1 The forms of VIA activity

The structure of VIA complex is illustrated on Fig. 18.1. The home page, presented on

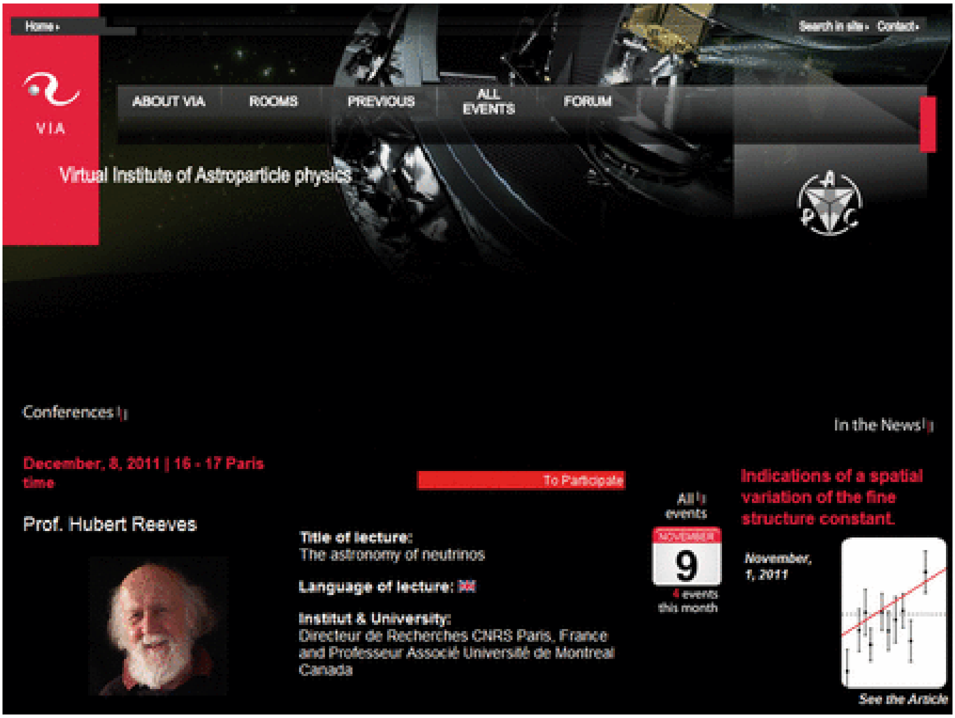

Fig. 18.1. The home page of VIA site

this figure, contains the information on VIA activity and menu, linking to directories (along the upper line from left to right): with general information on VIA (About VIA), entrance to VIA virtual lecture hall and meeting rooms (Rooms), the library of records and presentations (Previous) of VIA Lectures (Previous → Lectures), records of online transmissions of Conferences(Previous → Conferences), APC Seminars (Previous → APC Seminars) and APC Colloquiums (Previous → APC Colloquiums) and courses, Calender of the past and future VIA events (All events) and VIA Forum (Forum). In the upper right angle there are links to Google search engine (Search in site) and to contact information (Contacts). The announcement of the next VIA lecture and VIA online transmission of APC Colloquium occupy the main part of the homepage with the record of the most recent VIA events below. In the announced time of the event (VIA lecture or transmitted APC Colloquium) it is sufficient to click on "to participate" on the announcement and to Enter as Guest in the corresponding Virtual room. The Calender links to the program of future VIA lectures and events. The right column on the VIA homepage lists the announcements of the regularly up-dated hot news of Astroparticle physics.

In 2010 special COSMOVIA tours were undertaken in Switzerland (Geneve), Belgium (Brussels, Liege) and Italy (Turin, Pisa, Bari, Lecce) in order to test stability of VIA online transmissions from different parts of Europe. Positive results of these tests have proved the stability of VIA system and stimulated this practice at XIII Bled Workshop. These tours assumed special equipment, including, in particular, the use of the sensitive audio system KONFTEL 300W [10]. The records of the videoconferences at the previous XIII Bled Workshop are available on VIA site [11].

In 2011 VIA facility was effectively used for the tasks of the Paris Center of Cosmological Physics (chaired by G. Smoot), for the public programme "The two infinities" (conveyed by J.L.Robert) for Post-graduate programme assumed by the agreement between the University Paris Diderot and the University of Geneva. It has effectively supported participation at distance at meetings of the Double Chooz collaboration: the experimentalists, being at shift, took part in the collaboration meeting in such a virtual way.

It is assumed that the VIA Forum can continue and extend the discussion of questions that were put in the interactive VIA events. The Forum is intended to cover the topics: beyond the standard model, astroparticle physics, cosmology, gravitational wave experiments, astrophysics, neutrinos. Presently activated in English, French and Russian with trivial extension to other languages, the Forum represents a first step on the way to multi-lingual character of VIA complex and its activity.

One of the interesting forms of Forum activity is the educational work. For the last four years M.Khlopov's course "Introduction to cosmoparticle physics" is given in the form of VIA videoconferences and the records of these lectures and their ppt presentations are put in the corresponding directory of the Forum [12]. Having attended the VIA course of lectures in order to be admitted to exam students should put on Forum a post with their small thesis. Professor's comments and proposed corrections are put in a Post reply so that students should continuously present on Forum improved versions of work until it is accepted as satisfactory. Then they are admitted to pass their exam. The record of videoconference with their oral exam is also put in the corresponding directory of Forum. Such procedure provides completely transparent way of estimation of students' knowledge.

18.2.2 VIA lectures, online transmissions and virtual meetings

First tests of VIA system, described in [4,6,8,9], involved various systems of videoconferencing. They included skype, VRVS, EVO, WEBEX, marratech and adobe Connect. In the result of these tests the adobe Connect system was chosen and properly acquired. Its advantages are: relatively easy use for participants, a possibility to make presentation in a video contact between presenter and audience, a possibility to make high quality records and edit them, removing from records occasional and rather rare disturbances of sound or connection, to use a whiteboard facility for discussions, the option to open desktop and to work online with texts in any format. The regular form of VIA meetings assumes that their time and Virtual room are announced in advance. Since the access to the Virtual room is strictly controlled by administration, the invited participants should enter the Room as Guests, typing their names, and their entrance and successive ability to use video and audio system is authorized by the Host of the meeting. The format of VIA lectures and discussions is shown on Fig. 18.2, illustrating the talk "New physics and its experimental probes" given by John Ellis from CERN in the framework of XIV Workshop. The complete record of this talk and other VIA discussions are available on VIA website [13].

The ppt or pdf file of presentation is uploaded in the system in advance and then demonstrated in the central window. Video images of presenter and participants appear in the right window, while in the upper left window the list of all the attendees is given. To protect the quality of sound and record, the participants are required to switch out their microphones during presentation and to use lower left Chat window for immediate

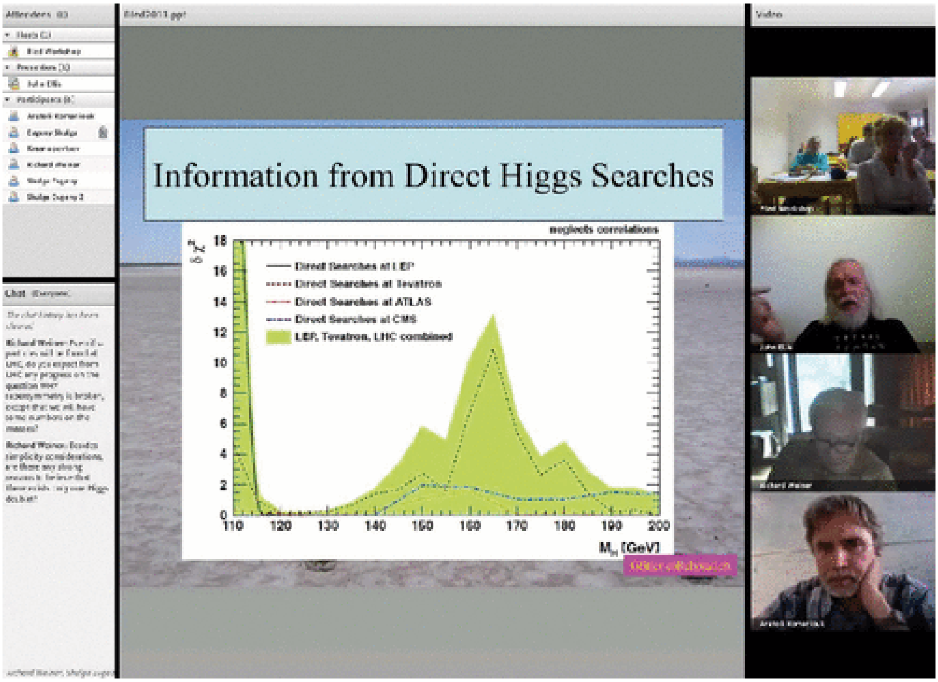

Fig. 18.2. Videoconference Bled-Marburg-Liege-Geneve-Moscow-Paris with lecture by John Ellis, which he gave from his office in CERN, Switzerland, became a part of the program of XIV Bled Workshop.

comments and urgent questions. The Chat window can be also used by participants, having no microphone, for questions and comments during Discussion. The interactive form of VIA lectures provides oral discussion, comments and questions during the lecture. Participant should use in this case a ”raise hand” option, so that presenter gets signal to switch our his microphone and let the participant to speak. In the end of presentation the central window can be used for a whiteboard utility as well as the whole structure of windows can be changed, e.g. by making full screen the window with the images of participants of discussion.

Regular activity of VIA as a part of APC includes online transmissions of all the APC Colloquiums and of some topical APC Seminars, which may be of interest for a wide audience. Online transmissions are arranged in the manner, most convenient for presenters, prepared to give their talk in the conference room in a normal way, projecting slides from their laptop on the screen. Having uploaded in advance these slides in the VIA system, VIA operator, sitting in the conference room, changes them following presenter, directing simultaneously webcam on the presenter and the audience.

18.3 VIA Sessions at Bled Workshop

18.3.1 The program of discussions

In the course of XIV Bled Workshop meeting the list of open questions was stipulated, which was proposed for wide discussion with the use of VIA facility.

The list of these questions was put on VIA Forum (see [14]) and all the participants of VIA sessions were invited to address them during VIA discussions. Some of them were covered in the VIA lecture "New physics and its experimental probes" given by John Ellis (see the records in [13]). During the XIV Bled Workshop the test of minimal necessary equipment was undertaken. VIA Sessions were supported by personal laptop with WiFi Internet connection only. It proved the possibility to provide effective interactive online VIA videoconferences even in the absence of any special equipment. Only laptop with microphone and webcam together with WiFi Internet connection was shown to be sufficient not only for attendance, but also for VIA presentations and discussions.

Another application at Bled Workshop was related with VIA records of closed meetings. The presentation was given in the regime of VIA online transmission and recorded, but the admission to the virtual room was restricted by a very short list of distant participants and the link to the record was available to a restricted list of users. Such use of VIA facility may be of interest for closed collaboration meetings.

18.3.2 VIA discussions

VIA sessions of XIV Bled Workshop have developed from the first experience at XI Bled Workshop [7] and their more regular practice at XII and XIII Bled Workshops [8,9]. They became a regular part of the Bled Workshop's programme.

In the framework of the program of XIV Bled Workshop, John Ellis, staying in his office in CERN, gave his talk "New physics and its experimental probes" and took part in the discussion, which provided a brilliant demonstration of the interactivity of VIA in the way most natural for the non-formal atmosphere of Bled Workshops. The advantage of the VIA facility has provided distant participants to share this atmosphere and contribute the discussion. VIA sessions were finished by the discussion of puzzles of dark matter searches (see [13]). N.S. Mankoč Borštnik and G. Bregar presented possible dark matter candidates that follow from the spin-charge-family theory approach, unifying spins and charges, and Maxim Khlopov presented composite dark matter scenario, mentioning that it can offer the solution for the puzzles of direct dark matter searches as well as that it can find physical basis in the above approach. H.B.Nielsen informed about his macroscopic candidate for dark matter. The comments by Rafael Lang from his office in USA were very important for clarifying the current status of experimental constraints on the possible properties of dark matter candidates (Fig. 18.3).

VIA sessions provided participation at distance in Bled discussions for John Ellis and A.Romaniouk (CERN, Switezerland), K.Belotsky, N.Chasnikov, A.Mayorov and E. Soldatov (MEPhI, Moscow), J.-R. Cudell (Liege, Belgium), R.Weiner (Marburg, Germany) H.Ziaeepour (UK), R.Lang (USA) and many others.

18.4 Conclusions

Current VIA activity is integrated in the structure of APC laboratory and includes regular weekly videoconferences with VIA lectures, online transmissions of APC Colloquiums and Seminars, a solid library of their records and presentations, together with the work of multi-lingual VIA Internet forum.

The Scientific-Educational complex of Virtual Institute of Astroparticle physics can provide regular communications between different groups and scientists, working in different scientific fields and parts of the world, get the first-hand information on the newest scientific results, as well as to support various educational programs at distance. This

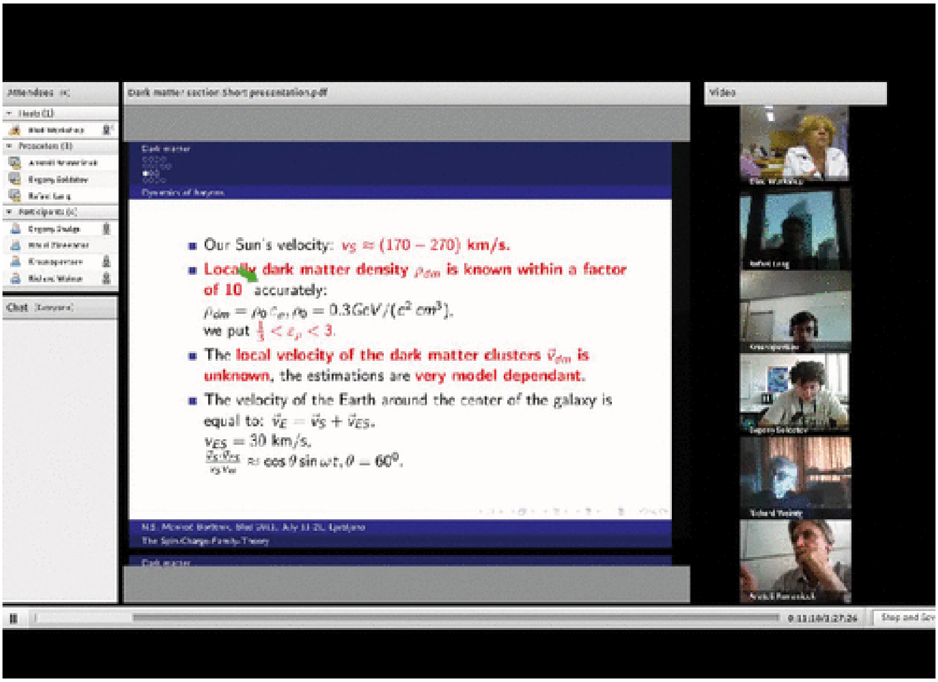

Fig. 18.3. Bled Conference Discussion Bled-Moscow-CERN-UK-Marburg-Liege-USA

activity would easily allow finding mutual interest and organizing task forces for different scientific topics of astroparticle physics and related topics. It can help in the elaboration of strategy of experimental particle, nuclear, astrophysical and cosmological studies as well as in proper analysis of experimental data. It can provide young talented people from all over the world to get the highest level education, come in direct interactive contact with the world known scientists and to find their place in the fundamental research. VIA applications can go far beyond the particular tasks of astroparticle physics and give rise to an interactive system of mass media communications.

VIA sessions became a natural part of a program of Bled Workshops, opening the room of discussions of physics beyond the Standard Model for distant participants from all the world. The experience of VIA applications at Bled Workshops plays important role in the development of VIA facility as an effective tool of $e - science$ and $e - learning$.

Acknowledgements

The initial step of creation of VIA was supported by ASPERA. I am grateful to P.Binetruy, J.Ellis and S.Katsanevas for permanent stimulating support, to J.C. Hamilton for support in VIA integration in the structure of APC laboratory, to K.Belotsky, A.Kirillov and K.Shibaev for assistance in educational VIA program, to A.Mayorov, A.Romaniouk and E.Soldatov for fruitful collaboration, to M.Pohl, C. Kouvaris, J.-R.Cudell, C. Giunti, G. Cella, G. Fogli and F. DePaolis for cooperation in the tests of VIA online transmissions in Switzerland, Belgium and Italy and to D.Rouable for help in technical realization and support of VIA complex. I express my gratitude to N.S. Mankoč Borštnik, G.Bregar, D. Lukman and all Organizers of Bled Workshop for cooperation in the organization of VIA Sessions at XIV Bled Workshop.

References

1. http://www.aspera-eu.org/
2. M.Yu. Khlopov: *Cosmoparticle physics*, World Scientific, New York -London-Hong Kong - Singapore, 1999.
3. M.Yu. Khlopov: *Fundamentals of Cosmoparticle physics*, CISP-Springer, Cambridge, 2011.
4. M. Y. Khlopov, arXiv:0801.0376 [astro-ph].
5. http://www.cosmovia.org/
6. M. Y. Khlopov, Bled Workshops in Physics **9** (2008) 81.
7. G. Bregar, J. Filippini, M. Khlopov, N. S. Mankoc-Borstnik, A. Mayorov and E. Soldatov, *In *Bled Workshops in Physics* **9** (2008) 67.
8. M. Y. Khlopov, Bled Workshops in Physics **10** (2009) 177.
9. M. Y. Khlopov, Bled Workshops in Physics **11** (2010)227.
10. http://www.konftel.com/default.asp?id=8581
11. In http://www.cosmovia.org/ Previous - Conferences - XIII Bled Workshop
12. In http://www.cosmovia.org/ Forum - Discussion in Russian - Courses on Cosmoparticle physics
13. In http://www.cosmovia.org/ Previous - Conferences - XIV Bled Workshop
14. In http://www.cosmovia.org/ Forum - CONFERENCES BEYOND THE STANDARD MODEL - XIV Bled Workshop ”What Comes Beyond the Standard Model?”

BLEJSKE DELAVNICE IZ FIZIKE, LETNIK 12, ŠT. 2, ISSN 1580-4992

BLED WORKSHOPS IN PHYSICS, VOL. 12, NO. 2

Zbornik 14. delavnice 'What Comes Beyond the Standard Models', Bled, 11. – 21. julij 2011

Proceedings to the 14th workshop 'What Comes Beyond the Standard Models', Bled, July 11.–21., 2011

Uredili Norma Susana Mankoč Borštnik, Holger Bech Nielsen in Dragan Lukman

Publikacijo sofinancira Javna agencija za knjigo Republike Slovenije

Brezplačni izvod za udeležence

Tehnična urednica Tadeja Šekoranja

Založilo: DMFA – založništvo, Jadranska 19, 1000 Ljubljana, Slovenija

Natisnila tiskarna Birografika Bori v nakladi 160 izvodov

Publikacija DMFA številka 1855