

# Preliminary Study of $D^0 \rightarrow K\pi\pi^0$ Decays with Dalitz Plots

Moira Gresham

ERULF

Reed College

Stanford Linear Accelerator Center

Menlo Park, CA

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### **Abstract**

Preliminary Study of  $D^0 \rightarrow K\pi\pi^0$  Decays with Dalitz Plots. MOIRA GRESHAM (Reed College, Portland, OR 97202) RAY COWAN (MIT Laboratory for Nuclear Science, Cambridge, MA 02139)

Particle physicists study the smallest particles and most basic rules of their interactions in humankind's current scope. The Charm Analysis Working Group (CWG) of the BaBar Collaboration studies decays involving the charm quark. They currently study *mixing* in  $D$  decays, an interesting and poorly understood phenomenon in current physics models. We, as part of the CWG, investigated the plausibility of using Dalitz plots and the BaBar analysis framework to study mixing in Wrong Sign (WS)  $D^0 \rightarrow K\pi\pi^0$  decays. Others in the CWG have studied mixing in the 2-body decay,  $D^0 \rightarrow K\pi$ . The 3-body decay analyzed with the RooFitDalitz analysis package and Dalitz plots provides more information and another way of separating Doubly Cabibbo Suppressed Decays (DCSD) from mixing -- which share the same end products. Through doing many simulations, we have demonstrated the usefulness of this approach. We selected  $D^0 \rightarrow K\pi\pi^0$  events from Simulation Production run #4 (SP4) and BaBar's run 1 and run 2. We made Dalitz plots with this data. Now that we better understand Dalitz plots and software, we plan to select WS  $D^0 \rightarrow K\pi\pi^0$  events and perform rate fits as discussed in BaBar Analysis Document (BAD) #443, as well as fits for several different decay times and resonances, in order to further distinguish DCSD from mixing.

## Introduction

Fruitful scientific research happens on frontiers. Elementary particle physics sits on the primordial frontier that separates us from nature's deepest secrets. The smallest, most fundamental particles found by humans are quarks (named up, down, charm, strange, top, and bottom – u, d, c, s, t and b), leptons (electron, muon, tau, and their associated neutrinos), and force-mediating particles (photon, gluon, Z, and W). These particles and the accepted rules for their interactions form the *Standard Model* of particle physics. Humans instinctively make models to explain nature. To better understand nature, we must constantly push and test our models. The Standard Model of particle physics is no exception! It is *not* in stasis, and *not* entirely convincing or well understood. Interesting physics lurks.

We find quarks bound together as *mesons* (in groups of two) or *baryons* (in groups of three). Mesons and baryons are classified as hadrons, particles that interact via the strong force. Quarks carry properties such as spin (1/2), charge ( $\pm 1/3$  or  $2/3$ ), and mass. Their properties propagate; we observe their combined spin, charge, and mass in mesons and baryons -- just as we observe the combined properties of baryons and leptons (for example the proton, neutron, and electron in atoms) in larger bound groups. Two quarks with equal and opposite charge compose a neutral meson. Neutral mesons have revealed a fundamental asymmetry in our universe -- Charge-Parity (CP) violation. CP violation may account for the abundance of matter (as opposed to anti-matter) in our universe; it is one phenomenon that 'allows' us to exist.  $K^0/\bar{K}^0$ ,  $D^0/\bar{D}^0$ , and  $B^0/\bar{B}^0$  (the "0" implies neutral; the bar implies antimatter) are the interesting neutral mesons that exhibit

mixing. The mixing phenomenon interests us on its own; in addition, through mixing we observe CP violation and other interesting physics (Griffiths 1987, Perkins 2000).

Particles decay. Following conservation laws, they spontaneously ‘break apart’ and recombine into other particles. Richard Feynman thought of a revealing way to describe these processes using diagrams. Any given particle can decay in many different ways, or channels; different Feynman diagrams describe the probability of each decay channel. We should interpret the decay of any given particle as a combination of all possible decay paths. Interestingly, there is a decay channel open to neutral mesons in which, for example,  $K^0$  changes to  $\bar{K}^0$  and vice versa. Consequently, we should view neutral mesons as a linear combination -- a *mix* -- of  $M^0$  and  $\bar{M}^0$ . (I use “ $M$ ” for Meson, in general.) We call this phenomenon *mixing* (See Fig. 1).

We may peek at interesting elementary mechanisms through the study of mixing.  $D^0$  meson mixing is difficult to observe (compared to that of  $K$  and  $B$  mesons) because it is predicted to be very small ( $R_{\text{mixing}} \sim 0-10^{-6}$  in the Standard Model) and because  $D$  particles have a short lifetime compared with their mixing period. Physicists do not understand  $D^0$  mixing well; it "carries a large potential for discovery of new physics" (Liu, 1995 p.2). Poorly determined parameters exist in the equations describing  $D^0$  mixing. Also, other kinds of  $D^0$  decays produce the same end products and further inhibit our ability to measure  $D^0$  mixing.

$D^0$  decays can be categorized as right sign (RS) or wrong sign (WS). Right sign decays occur most often. For example, the end products of a RS decay for the  $D^0$  – the Cabibbo Favored (CF) decay -- are  $K^-\pi^+$ . (Written  $D^0 \rightarrow K^-\pi^+$ , where  $+$ ,  $-$ ,  $0$  are their respective charges in units of one electron charge.) Likewise, the end products of a RS

$\bar{D}^0$  decay are  $K^+\pi^-$  (see Fig. 2). The end products of a  $D^0$  WS, Doubly Cabibbo Suppressed Decay (DCSD) are  $K^+\pi^-$  (see Fig. 3). This kind of WS decay is approximately 400 times less likely. Inconveniently, DCSD decays produce the same products as mixing. Because they have the same decay end-products, distinguishing DCSD events from mixing events is challenging. When the goal is to study mixing, DCSD events are considered primarily as "annoying ... background" (Liu, 1995, p.3).

Particle physicists have speculated about  $D$  mixing since the discovery of the charm quark (Gaillard, 1975). In the first few years of  $D^0/\bar{D}^0$  mixing studies, people searched for  $D^0 \rightarrow K^+\pi^-$  assuming mixing was the only contributor; this led to false confidence in a mixing rate ( $R_{\text{mixing}}$ ) due to the contribution of that "annoying" DCSD background. Later, researchers moved to studying semileptonic (for example,  $D^0 \rightarrow K^- e^+ \nu_e$ ) decays, which are not subject to the DCSD background (Liu, 1995, p.3). However, interest in  $D^0$  hadronic decays (i.e.  $D^0 \rightarrow \text{hadrons}$ ) has recently reemerged (Liu, 1995). Since BaBar (the big particle detector at SLAC) has recorded many  $D^0$  decays in the last several years, we have the capability to measure  $D$  mixing through hadronic decays with enhanced sensitivity. With such a large data set, we can expect the mixing parameters to show up in the interference term of the equation describing the  $D^0$  hadronic decay rate (Liu, 1995). See the appendix for rate equations.

We must know how a decay starts (i.e. was the original particle  $D^0$  or a  $\bar{D}^0$ ?) in order to determine whether DCSD or mixing occurred. Therefore, we 'tag' our events using the decays  $D^{*+} \rightarrow D^0 \pi^+$  and  $D^{*-} \rightarrow \bar{D}^0 \pi^-$ . The 'extra' charged pion tells us whether a particle started as  $D^0$  or  $\bar{D}^0$ .

To extend the current BaBar mixing analysis using two-body,  $D^0(\bar{D}^0) \rightarrow K^\mp \pi^\pm$  decays, we study mixing by analyzing 3-body hadronic decays ( $D^0(\bar{D}^0) \rightarrow K^\mp \pi^\pm \pi^0$ ) (see Fig. 4). The third ‘body’ of the  $D^0$  decay allows for Dalitz analysis. Dalitz analysis involves plotting the squared invariant mass (inv. mass<sup>2</sup> = Energy<sup>2</sup> - momentum<sup>2</sup>) of the three particles, in combinations of two, in a two-dimensional scatter plot (see Figs 5A & B). For example, in the decay,  $D \rightarrow ABC$ , we might plot  $\text{Mass}_{AB}^2$  versus  $\text{Mass}_{BC}^2$ . A Dalitz plot reveals the substructure of a decay. In other words, if the density at a certain mass (say M, along the AB axis) is high, we say that there was a resonance at M and the decay probably happened as  $D \rightarrow MC$ ,  $M \rightarrow AB$  (which ‘sums’ to  $D \rightarrow ABC$ ). We can make such a plot of an event for several different times and see how the resonances change with time. We have enough knowledge of possible resonances for mixing and DCSD to distinguish between the two types. Remember that a particle decays through every possible mechanism, and the relative densities for each resonance give the relative probabilities (Amplitude<sup>2</sup>) for different decay mechanisms. Thus, we can measure amounts of mixing and DCSD with time through *time dependent* Dalitz analysis. We venture out onto the primordial frontier of elementary particles to test the Standard Model by studying poorly understood mixing quantities.

## **Materials and Methods**

### **Dataset**

Since elementary particles are very small -- and most ‘live’ for a very short period of time (small fractions of a second) -- we have to use sophisticated, sensitive instruments to produce and detect them. BaBar, which stands for  $B/B\bar{b}$ , is one such instrument

(BaBar, B.Aubert *et al.*, 2002). ( $B$ 's are mesons with  $b$  quarks, and are produced when  $e^+e^- \rightarrow b\bar{b}$ .) It is the detector "attached" to the PEP-II rings at the end of the Stanford Linac (linear accelerator). BaBar was designed, primarily, to detect  $B$  meson decays. The energy at which positrons and electrons (coming from the PEP-II rings) collide is optimal for the production of  $B$  mesons. BaBar and the PEP-II project was designed to catalog and analyze many, many  $B$  decays in order to study CP (Charge-Parity) violation. In addition, since the  $c\bar{c}$  (the state that precedes  $D$  meson events) production cross-section is just as large as that of  $b\bar{b}$ , BaBar also has a large sample of  $D$  meson events. (Cross-section relates to the probability with which an event occurs. Larger cross-section implies higher probability.) The Charm Analysis Working Group (CWG) at BaBar analyzes these  $D$  events to search for new and interesting physics.

High Energy physicists often use computer simulations -- also known as Monte Carlo -- to better understand all of the complicated effects associated with extracting unique physics numbers from complex particle detectors. A Monte Carlo simulation 'knows', in good detail, the geometry and other properties of the BaBar detector. In Monte Carlo simulations, using random number generators and known physics models, the computer generates events and passes them through the detector simulation. This allows one to understand, with some confidence, how the detector and analysis programs perform.

We use both simulated Monte Carlo data and 'real' BaBar/PEP-II data. Our  $40 \text{ fb}^{-1}$  of simulated data comes from BaBar's Simulation Production run #4 (SP4). SP4, created in 2001, is the latest simulation available. We use Monte Carlo data from generic  $c\bar{c}$

modes. Our  $57.1 \text{ fb}^{-1}$  of ‘real’ BaBar/PEP-II data comes from BaBar’s run 1 (2000) and part of run 2 (before December, 2001).

## Analysis Method

### **Dalitz Analysis Technique**

We use a relatively new method for charm mixing analysis; we examine the phases and amplitudes of resonances for our decay as a function of time in Dalitz plots. Others in the CWG have studied hadronic  $D$  decays with Dalitz plots in search of mixing -- but not  $D^0 \rightarrow K\pi\pi^0$  decays. In general, a three-body decay can be characterized by two variables (Perkins, 1987, p.130; Podolsky D. 1998). Given a decay,  $P \rightarrow ABC$ , we typically choose any two of the variables:  $m_{AB}^2$ ,  $m_{BC}^2$ , and  $m_{CA}^2$  (where  $m_{ij}^2 = (\mathbf{p}_i + \mathbf{p}_j)^2$ , the squared invariant mass of particles  $i$  and  $j$ ) as the axes for our Dalitz plot. As discussed below, this allows one to read the invariant mass of an intermediate resonance directly from the plot. If only kinematics (that is, conservation of momentum and energy) determines the decay of a parent particle,  $P$ , then a Dalitz plot, by construction, should show a uniform distribution over the *kinematically* allowed region (Perkins, 132). In the relativistic limit, this region resembles a triangle with rounded edges (see Fig. 6). One can easily determine the upper, lower, right, and left asymptotes for such a plot using conservation of energy. In the rest frame of the parent particle, the upper/right and lower/left asymptotes ( $m_{ij\max}$  and  $m_{ij\min}$ , respectively) are:  $m_{ij\max} = m_P - m_k$  and  $m_{ij\min} = m_i + m_j$ . (For an exact specification of kinematic boundaries in Dalitz plots, see Jackson, 2000.) We are most interested in the *dynamics* of decays. If there exists substructure in a decay (in other words, if a particle decays through intermediate channels) we detect it in a Dalitz plot, since, in this case, the probability of finding an

event with a given mass distribution is *not* uniform across phase space. The substructure of a decay is determined by intermediate resonances. Particle physicists care about the relative probabilities of resonances – given by the Amplitude<sup>2</sup> of the resonance.

Typically, Breit-Wigner distributions are used to describe resonances (Jackson, 1964). If a resonance exists, it will show up on a Dalitz plot as a band of higher density centered on<sup>1</sup> the mass of the intermediate particle. The relative densities of bands correspond to relative Amplitude<sup>2</sup> (see Fig. 7). If intermediate particles have intrinsic spin, additional angular momentum constraints exist for the corresponding resonance. This translates to ‘lobes’ in the resonance bands of Dalitz plots (see Fig. 8). For example, if a spin zero parent particle decays through a spin-1 (say,  $AB$ ) resonance to spin zero daughters, it is very *unlikely* to find particle  $C$  moving perpendicular to  $A$  and  $B$  but very likely to find it moving parallel or anti-parallel. This corresponds to a ‘hole’ near the middle of the resonance band and a reinforcement at the ends; the density along the band varies like  $\cos^2\theta$  where  $\theta$  is the angle between particle  $C$  and particle  $A$  (or  $B$ ) in the parent’s center-of-mass (CMS) frame. Since Breit-Wigners -- which describe particle resonances -- contain complex amplitudes and phases, different resonances can interfere and show up as depleted or enhanced areas on the Dalitz plot. The Dalitz plot beautifully demonstrates that, to fully describe these directly *unobservable* decay resonances, we must use complex wave functions rather than probabilities (see Fig. 8). Importantly, looking at Dalitz plots of a certain decay at several different decay times should show changes in resonant contributions with time.

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<sup>1</sup> Actually, to be exact, the band is centered slightly below the mass. See the Jackson reference.

Specifically, we wish to examine the wrong sign (WS) 3-body decays,  $D^0 \rightarrow K^+\pi^-\pi^0$  and  $\bar{D}^0 \rightarrow K^-\pi^+\pi^0$  (the charge conjugate). In these WS decays, DCSD and mixing occur at different rates. Thus a change, with time, in  $D^0 \rightarrow K^+\pi^-\pi^0$  (or  $\bar{D}^0 \rightarrow K^-\pi^+\pi^0$ ) substructure, as well as interference between mixing and DCSD, can allow us to identify and study  $D$  mixing (Liu, T. 1995). However, before we study the WS decays exclusively, we plan to include the much more abundant  $D^0 \rightarrow K^-\pi^+\pi^0$  and  $\bar{D}^0 \rightarrow K^+\pi^-\pi^0$  RS decays in our dataset and fits in order to check that our analysis technique works for  $D^0/\bar{D}^0$  3-body decays in general.

#### **Generate/Fit method – tools:**

We plan to fit for the relative amplitudes and phases of resonances in the decay mode of interest. To check that our fitting method is accurate, we will eventually compare fits with real data to the fits with simulated data. We will use BaBar analysis tools to accomplish this. RooFitDalitz is the new tool of interest.

RooFitDalitz (RFD) is a package for doing Dalitz plot fits. It is based on the RooFit toolkit, and also relies on EvtGen - a BaBar event generator package (Dvoretiskii, A. 2002). The RooFit packages provide a toolkit for modeling the expected distribution of events in a physics analysis. Models can be used to perform likelihood fits, produce plots, and generate "toy Monte Carlo" samples for various studies. The RooFit tools are integrated with the object-oriented and interactive ROOT graphical environment (Kirkby, 2001). EvtGen (Ryd et al, 2001) is a package for simulating physics processes of most known particle decays. The output of EvtGen is a set of 4-vectors and vertices for the decay products (Ryd, 2001).

In this preliminary study, we made several cuts on the data to find a clean sample of  $D^0 \rightarrow K\pi\pi^0$  events. See table 1 for a description.

## **Process**

Because we had not used Dalitz plots and the RooFitDalitz package for studying  $D^0 \rightarrow K\pi\pi^0$  decays before, it was necessary to educate ourselves about the package. We needed to determine the plausibility of an analysis with RooFitDalitz for our decay, as well as educate ourselves about Dalitz plots. After I had familiarized myself with basic BaBar computer skills and programs, we experimented with event selection by using and editing the example event selection macro, NTrkExample. We observed how various cuts reduce the bad combinatorics, and improve reconstruction and particle identification quality. Then we learned about RooFitDalitz by examining and tweaking the example RooFitDalitz macros and other RooFitDalitz code created by Alexei Dvoretzskii. We added and subtracted intermediate resonances, changed amplitudes, changed phases, and changed decays. We made many plots. After feeling confident that we knew how RooFitDalitz worked, we made cuts on the aforementioned data and generated Dalitz plots from this data. We have not performed fits yet, due to lack of time.

## **Results**

Figures 6 through 10 contain some Dalitz plots that we generated, as exercises, with RooFitDalitz. Plot 6, generated from macro 2, demonstrates phase space. Plots 7 A,B, and C demonstrate resonances of spin 0, 1, and 2 particles. Figure 8, generated with macro 2, demonstrates resonance interference. Figures 9 and 10 show plots generated from an adapted version of macro 2. They show several individual resonances (9) and

plots with increasing numbers of resonances (10). See the figure captions for more details.

Figure 5B shows a Dalitz plot for  $D^0 \rightarrow K\pi\pi^0$ , selected from reconstructed SP4 Monte Carlo simulation data. The cuts applied to the simulated events are detailed in table 1.

### **Discussion and Conclusions**

This was a preliminary study to find the plausibility of using BaBar analysis tools and Dalitz plots to measure mixing in  $D^0 \rightarrow K\pi\pi^0$ . Much of the substance of this project did not materialize into formal results. We did not find new physics, but, through the new technical knowledge of the experimenters, increased the *potential* for finding new physics. Event selection took longer than expected. Thus, we made Dalitz plots of our ‘clean’  $D^0 \rightarrow K\pi\pi^0$  reconstructed events from SP4, but did not have time to fit any data yet. We determined that RooFitDalitz works as we would hope for our decay. The plots look reasonable. We now understand RooFitDalitz, the example macros, and how to write our own macros. This kind of analysis carries potential, indeed. An eventual examination of mixing in our decay mode will require selecting wrong sign events, plotting on a Dalitz plot, and fitting with RooFitDalitz. Among other factors, we must account for efficiency in later fits. A time dependent fit will give more information about  $D^0$  decays than has been used for previous  $D^0 \rightarrow K\pi\pi^0$  mixing analyses (Liu, 1995).

The most substantial part of my project involved familiarizing myself with BaBar software and analysis tools – as well as learning about particle physics in general and, in specific: mixing, charm physics, CP violation, and Dalitz plots. The collective BaBar analysis framework/software is a very large, powerful, and complicated beast (in my opinion, at least). At SLAC, I learned about very cool physics!

### **Acknowledgements**

I thank the Department of Energy and National Science Foundation for organizing and financing the ERULF program.

I also thank my mentor, Ray Cowan. I learned an enormous amount from him. He spent much time working with and teaching me -- always with patience and kindness. He made my ERULF experience wonderful.

Thanks very much to Helen Quinn for organizing the ERULF program at SLAC.

Thanks also, Sekazi Mtingwa, for directing this program.

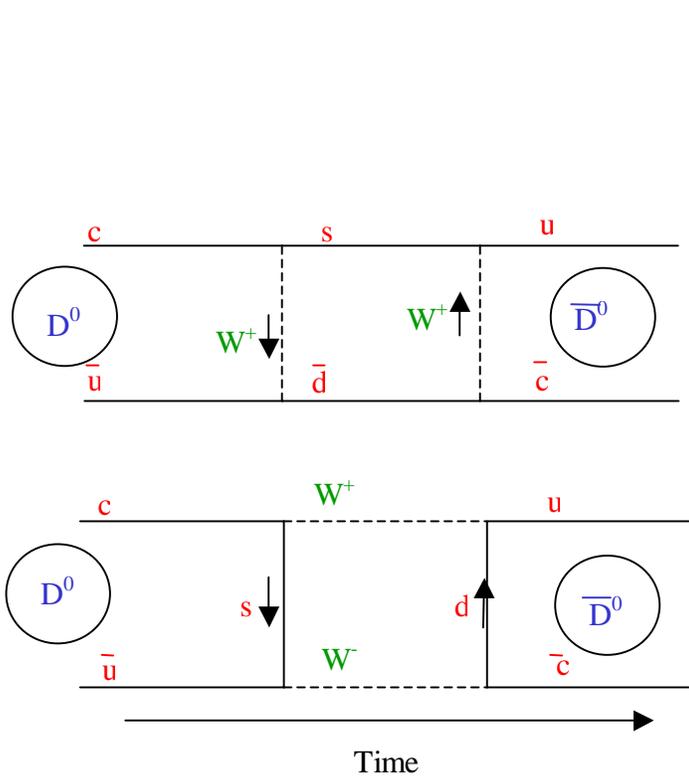
I thank Pat Burchat for kindly sharing her knowledge of Dalitz plots.

I extend thanks to Michael Spitznagel and Ben Brau of the MIT LQS group for their help and kindness.

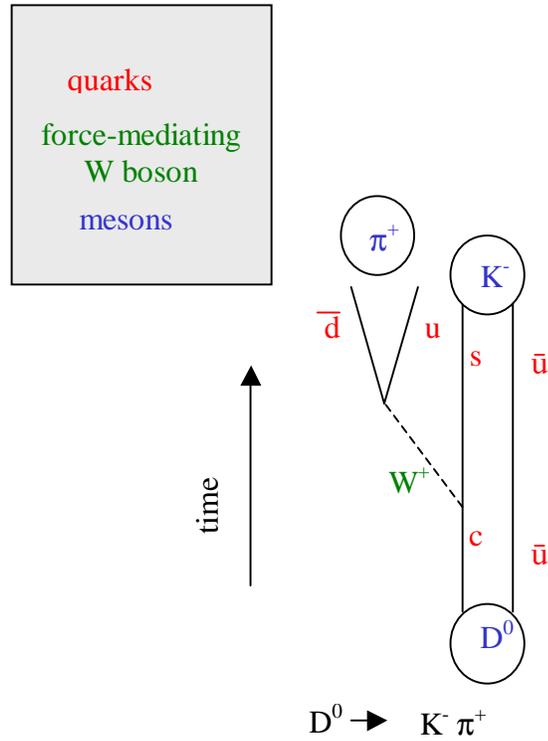
## Tables and Figures

Cut Name	Cut Specifics/ Purpose
“D <sup>0</sup> ” mass	Rejects D <sup>*0</sup> particles that were misidentified as D <sup>0</sup> s. Require the D <sup>0</sup> mass to be $1.864 \pm 0.03$ GeV.
Very_tight particle ID requirement on K <sup>±</sup> and π <sup>±</sup>	Rejects π's misidentified as K's. Rejects e's and μ's misidentified as π's.
Momentum On all particles	Rejects fake tracks (i.e. those made up of unrelated points). Require all particles to have momentum of at least 100 MeV in the laboratory frame.
K/D <sup>0</sup> angle	Rejects π's (from the uds continuum) misidentified as K's. Require cosine of the angle between K and D <sup>0</sup> flight Directions in CMS to be less than 0.75.
D <sup>0</sup> reconstruction Probability	Rejects wrong combinations of daughter particles. Require probability that the D0 was properly reconstructed from the daughter tracks to be greater than 0.05.
π <sup>0</sup> reconstruction Probability	Same idea as D <sup>0</sup> reconstruction Probability. Require Prob(proper π <sup>0</sup> reconstruction) > 0.05.

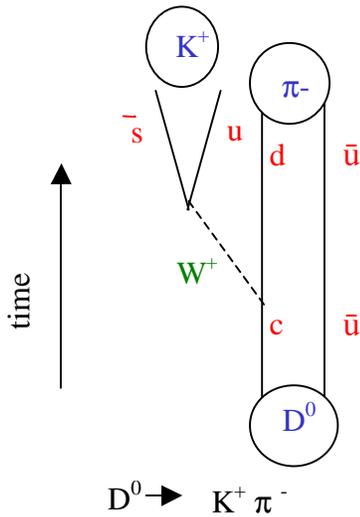
**Table 1:** Summary of the major data cuts done on the SP4 and BaBar datasets that we used.



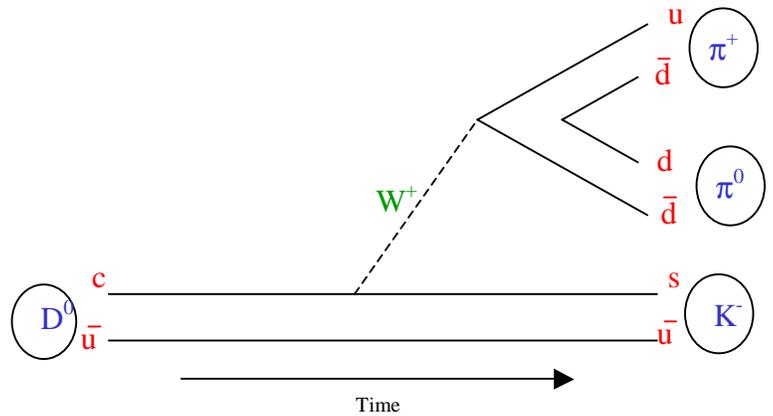
**Figure 1:** Two Feynman diagrams contributing to  $D^0$  mixing.  $\bar{D}^0$  can change to  $D^0$  through the same diagram, but with all charge conjugates.



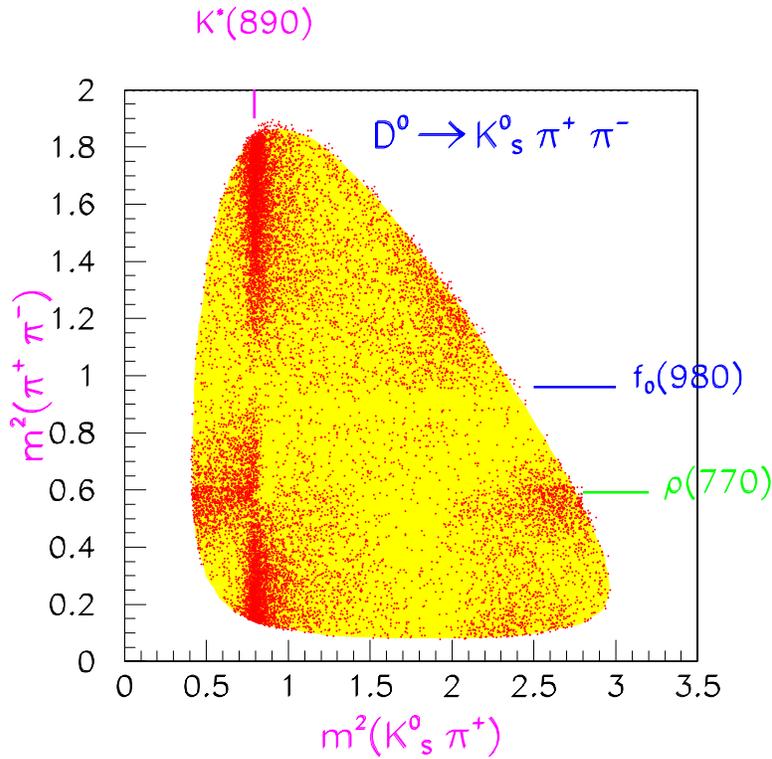
**Figure 2:** Feynman diagram showing the 2-body Doubly Cabibbo Suppressed decay for  $D^0$ . This is a Right Sign decay.



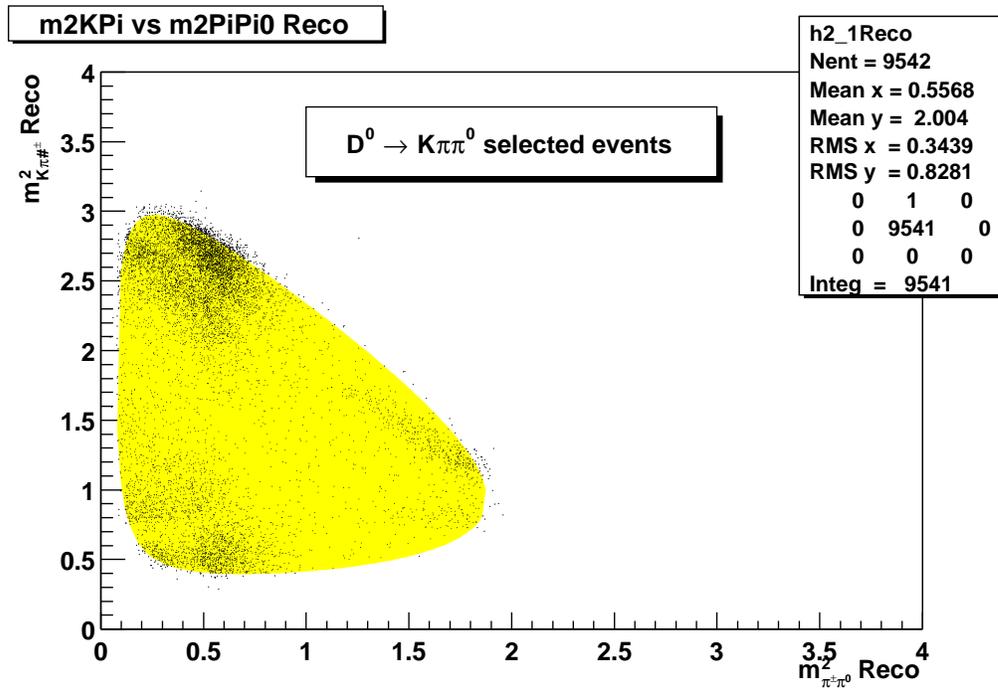
**Figure 3:** Feynman diagram showing the 2-body Cabibbo Favored decay for  $D^0$ . This is a Wrong Sign decay.



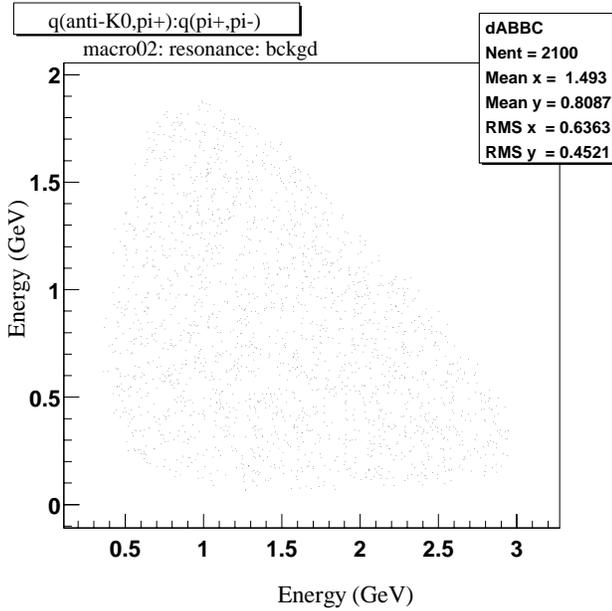
**Figure 4:** Feynman diagram contributing to the RS 3-body decay,  $D^0 \rightarrow K^- \pi^+ \pi^0$ .



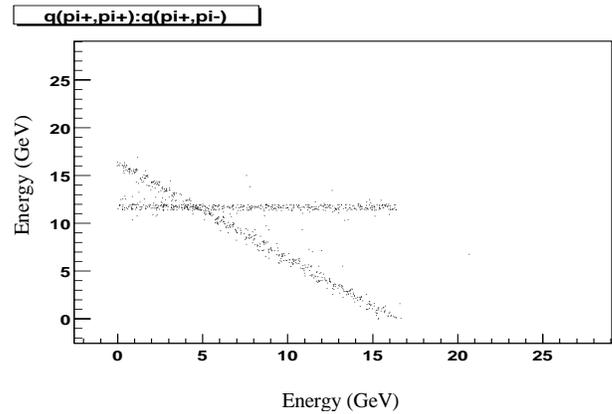
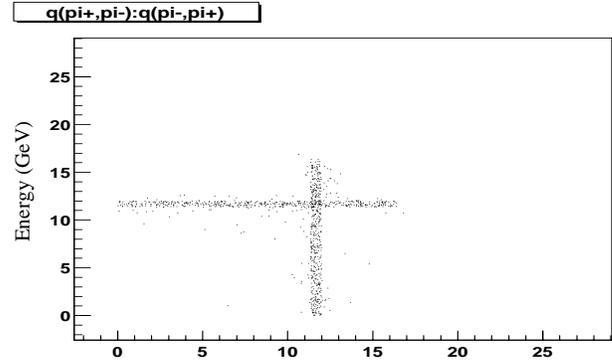
**Figure 5A:** A BaBar Dalitz plot with ‘real’ data. Note the 3 resonance bands and interference. This plot was taken from Palano, 2001.



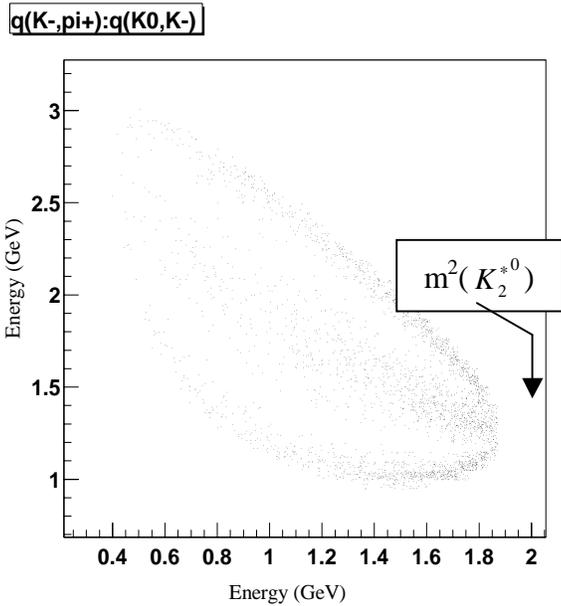
**Figure 5B:** Dalitz plot with ‘clean’  $D^0$  events from Simulation Production Run #4. The cuts used to obtain our clean data are detailed in table 1. The yellow region is the calculated kinematically allowed region. There are three resonant bands:  $\rho^{+/-}$  (vertical),  $K^{*0}$  (horizontal), and  $K^{*+/-}$  (diagonal).



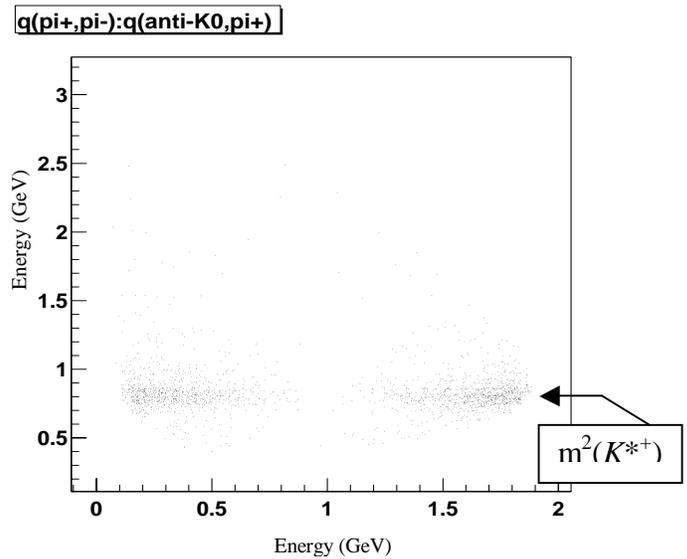
**Figure 6:**  $m^2(\pi^+\pi^-)$  vs  $m^2(\bar{K}^0\pi^+)$  with non-resonant background, only. The evenly distributed points occupy the kinematically allowed region for the decay. Note the rounded triangular shape – characteristic of Dalitz plots.



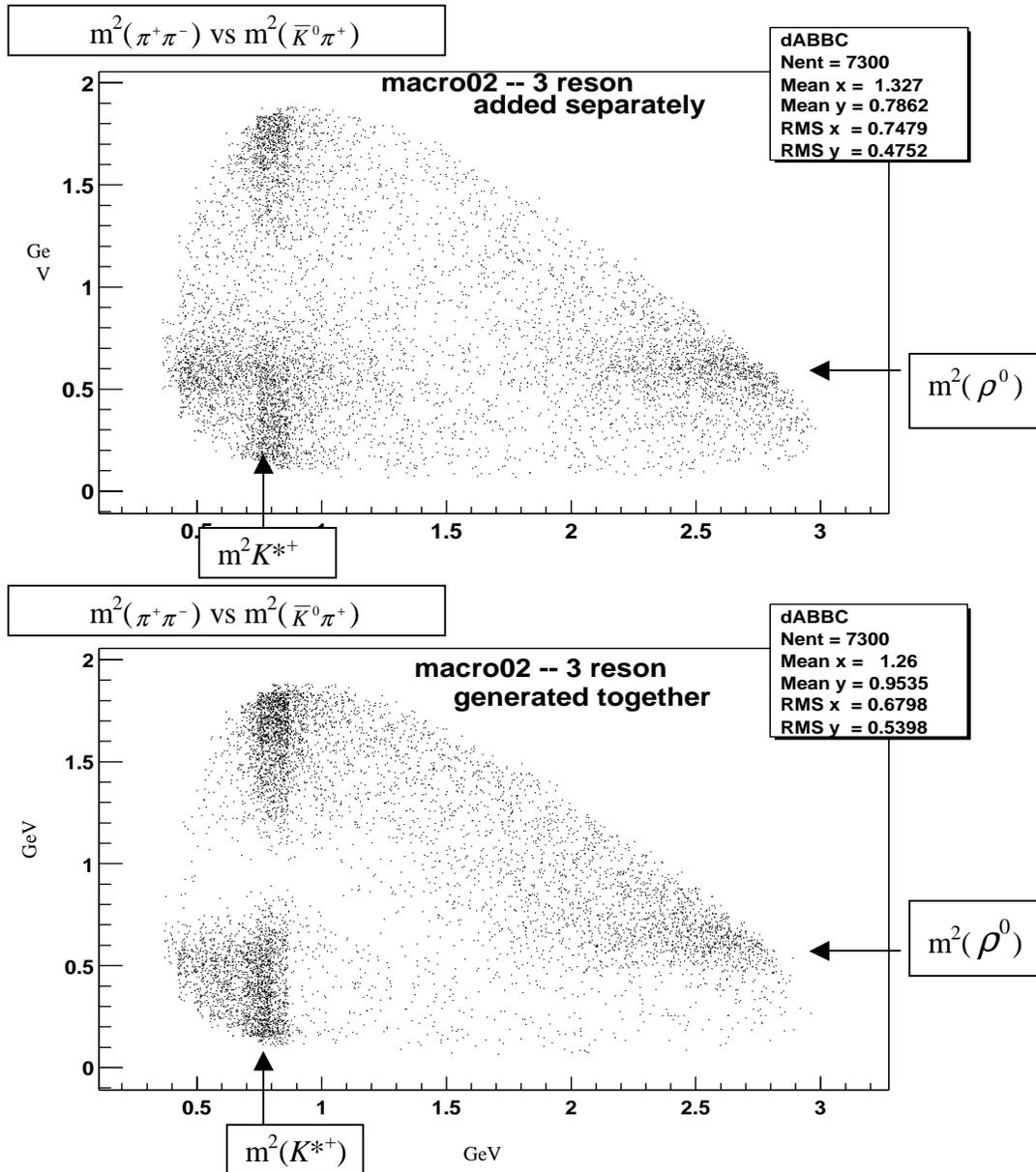
**Figure 7A:**  $m^2(\pi^+\pi^-)$  vs  $m^2(\pi^+\pi^-)$  --  $\chi_c$  resonances for the decay,  $K^+ \rightarrow \pi^+\pi^+\pi^-$ .  $\chi_c$ 's are spin-0 mesons. Note that the resonance forms a uniform band with no lobes. Also note that the top and bottom graphs represent the same decay, but with different axes.



**Figure 7C:** resonance for the decay. Note the three lobes since  $K_2^{*0}$  is a spin-2 particle. Also,  $m^2(K_2^{*0}) \approx 2$ , which is outside of the kinematic limit. However, the 'tail' of the resonance still shows up.

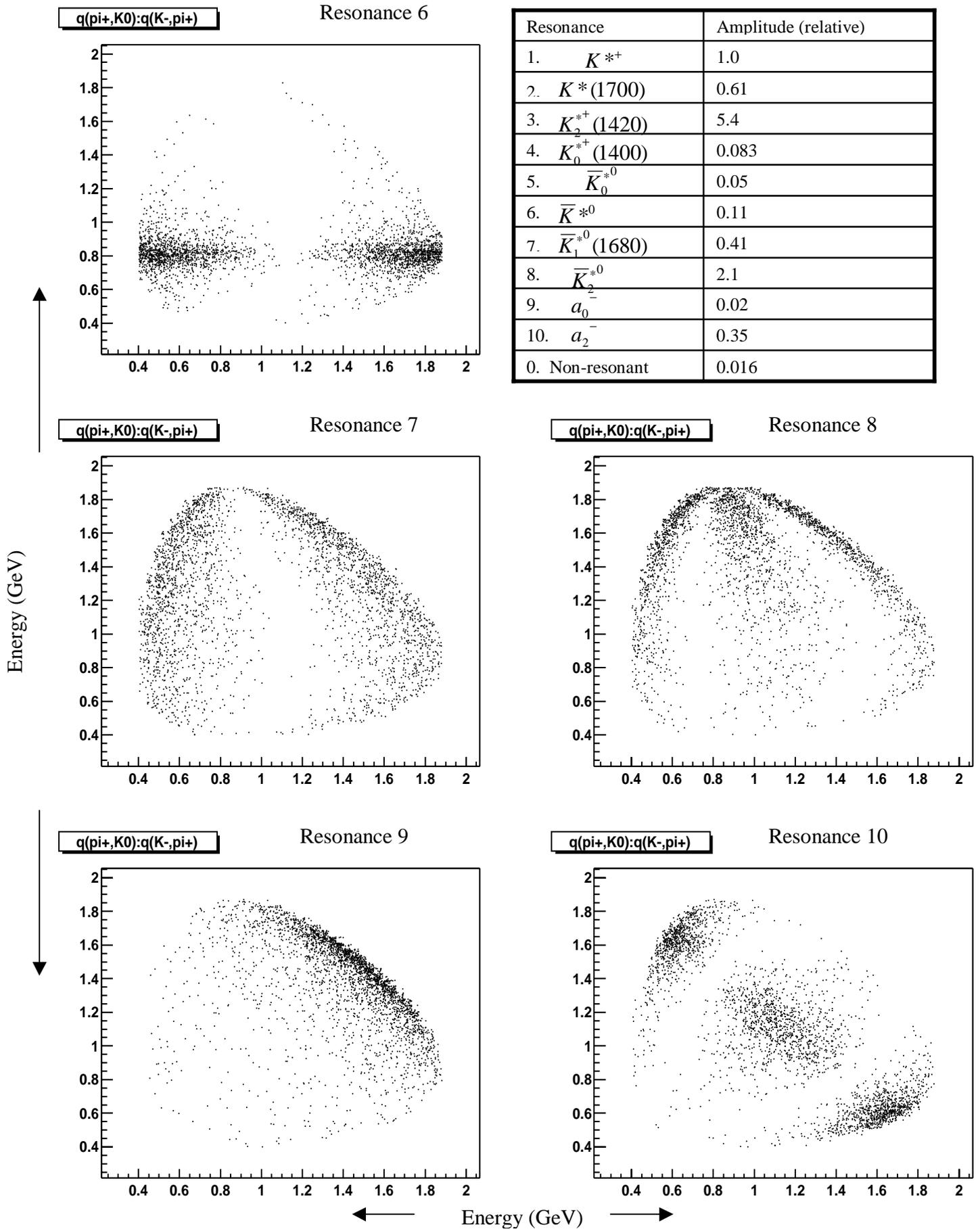


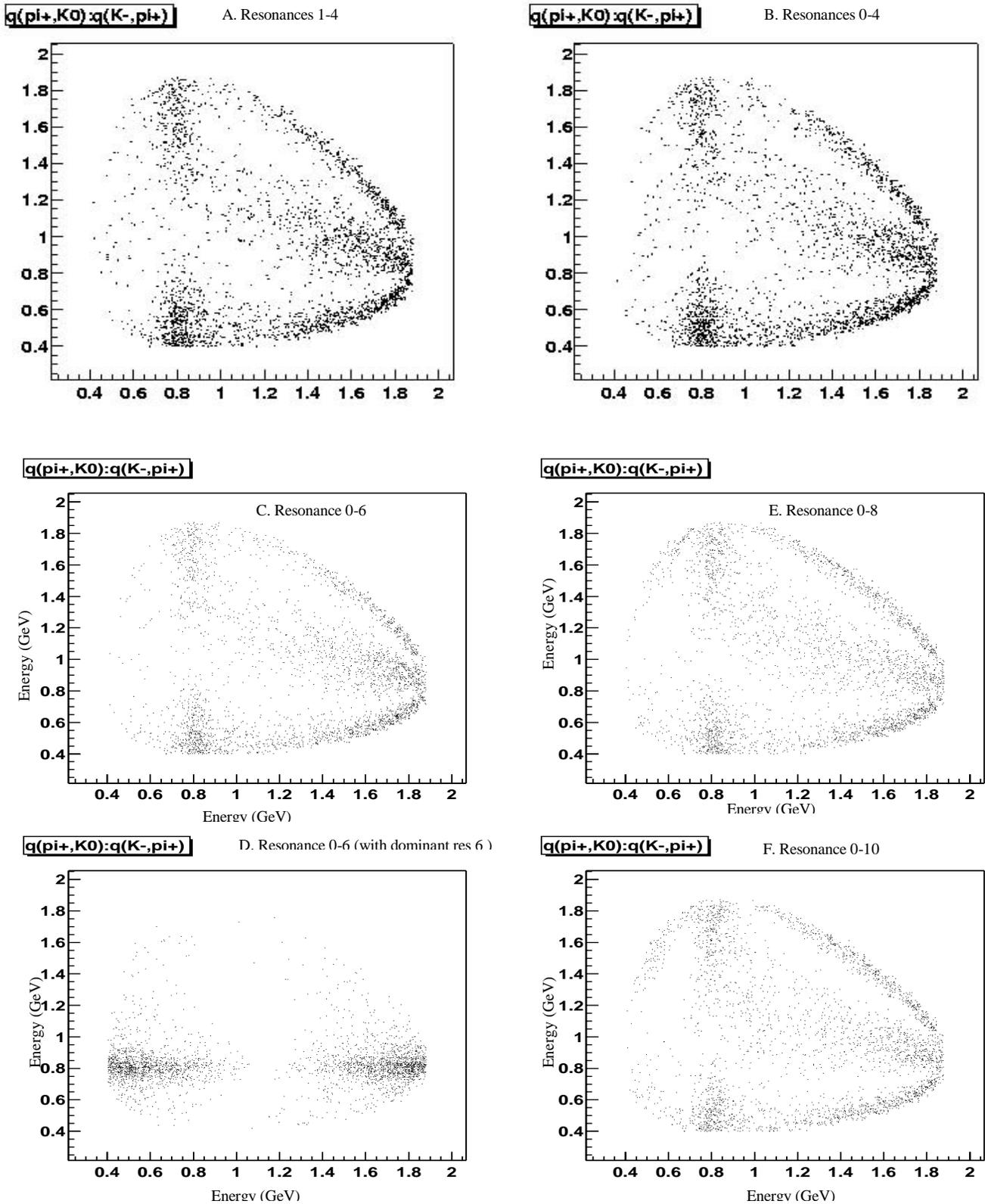
**Figure 7B:** resonance for the decay  $D^0 \rightarrow \bar{K}^0\pi^+\pi^-$ . Note the two lobes since  $K^{*+}$  is a spin-1 particle.



**Figure 8:** Dalitz plot for the decay,  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  generated from RooFitDalitz macro. Top graph: Non-resonant background, a  $K^{*+}$  resonance, and a  $\rho^0$  resonance were generated separately and then added together. The number of points plotted for each is proportional to their relative amplitudes as was specified for the bottom graph. Bottom graph: the same non-resonant background, and  $\rho^0$  and  $K^{*+}$  resonances were generated together and then plotted. Note the empty space in the bottom graph that do not appear in the upper graph. This demonstrates that resonances interfere.

**Figure 9:**  $m^2(K^- \pi^+) \text{ vs } m^2(K^0 \pi^+)$  for the decay  $D^0 \rightarrow K^0 K^- \pi^+$ . Five different contributing resonances are shown. Some combinations of these resonances are in Fig. 10.





**Figure 10:** Dalitz plots of the decay  $D^0 \rightarrow K^0 K^- \pi^+$  generated with RooFitDalitz. See table in Fig 9 for the relative amplitudes of the resonances (ordered 0-10). As more resonances are added, each is more difficult to detect, by eye, on the plot. Also note interference between the resonances. The resonances with low relative amplitude show up least on the plot. Some individual resonances are plotted in Fig. 10.

### Appendix

Mixing is characterized by the parameters,  $x \equiv \Delta M / \Gamma$  and  $y \equiv \Delta \Gamma / 2\Gamma$ , where  $\Delta M = m_1 - m_2$ ,  $\Delta \Gamma = \gamma_1 - \gamma_2$ ,  $\Gamma = (\gamma_1 + \gamma_2) / 2$ , and  $\gamma_1$  &  $\gamma_2$  are the widths of  $D_1$  &  $D_2$ .

Because there exists an unknown strong phase  $\delta_{K\pi}$  in the final-state interaction between the DCSD decay and the CF decay, particle physicists often work in terms of

$$x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi} \quad \text{and} \quad y' = -x \sin \delta_{K\pi} + y \cos \delta_{K\pi}.$$

Assuming  $x, y \ll 1$  and CP invariance, we find that the WS decay rate or  $D^0$  (or  $\bar{D}^0$  since we assume CP invariance) is given by

$$\Gamma(D^0 \rightarrow K^+ \pi^-)(t) \propto e^{-t/\tau_{D^0}} \times (R_D + \sqrt{R_D} y' t / \tau_{D^0} + \frac{x'^2 + y'^2}{4} (t / \tau_{D^0})^2)$$

where  $\tau_{D^0}$  is the  $D^0$  lifetime, and  $R_D$  is the DCSD rate (BAD#443).

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