Axion BEC Dark Matter

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Cold dark matter axions thermalize through gravitational self-interactions and form a Bose-Einstein condensate when the photon temperature reaches approximately 500 eV. Axion Bose-Einstein condensation provides an opportunity to distinguish axions from the other dark matter candidates on the basis of observation. The rethermalization of axions that are about to fall in a galactic potential well causes them to acquire net overall rotation, whereas ordinary cold dark matter falls in with an irrotational velocity field. The inner caustics of galactic halos are different in the two cases.

1 Introduction

Both axions and weakly interacting particles (WIMPs) are considered forms of cold dark matter (CDM). Furthermore, until recently, axions and WIMPs were thought to be indistinguishable on observational grounds, i.e. indistinguishable on the basis of purely astronomical data. The discovery [1] that dark matter axions form a Bose-Einstein condensate (BEC) has changed this view since axion BEC is claimed to have observable consequences [1, 2, 3]. This raises the question: When do axions behave as ordinary CDM and when do they not?

To start off it is worth emphasizing that, at the fundamental level, axions and WIMPs are very different. The surprise is really that they have similar properties as far as large scale structure is concerned. Both axions and WIMPs are described by quantum fields. Furthermore, both are excellently described by classical limits of quantum fields. But the classical limits are different in the two cases: WIMPs are in the classical particle limit whereas (decoupled) axions are in the classical field limit. In the classical particle limit one takes $\hbar \to 0$ while keeping $E = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$ fixed. Since $\omega, \vec{k} \to \infty$, the wave nature of the quanta disappears. WIMPs are to excellent approximation classical point particles. In the classical field limit, on the other hand, one takes $\hbar \to 0$ for constant ω and $\vec{k}$. $E = \mathcal{N}\hbar\omega$ and $\vec{p} = \mathcal{N}\hbar\vec{k}$ are held fixed by letting the quantum state occupation number $\mathcal{N} \to \infty$. This is the limit in which quantum electrodynamics becomes classical electrodynamics. It is the appropriate limit for (decoupled) cold dark matter axions because they are a highly degenerate Bose gas. The axion states that are occupied have huge occupation numbers, $\mathcal{N} \sim 10^{61}$ [1]. The need to restrict to *decoupled* axions will be explained shortly.

So axions and WIMPs are fundamentally different even if both can legitimately be called CDM. The distinction is not just academic, and is certainly important if axions thermalize, i.e. if axions find a state of larger entropy through self-interactions. Recall that, whereas statistical mechanics makes sense of the behaviour of large aggregates of classical particles (it was invented by Boltzmann to derive the properties of atoms in the gaseous state) it fails to make sense of classical fields. In thermal equilibrium every mode of a classical field would have average energy $k_B T$. As Rayleigh pointed out, the energy density is infinite then at finite temperature due to the contributions from short wavelength modes. Thus the application of statistical mechanics to classical field theory (classical electrodynamics in particular) is in direct disagreement with observation. As is well-known, the disagreement is removed because of, and only because of, quantum mechanics.

In summary, if the axions are decoupled (i.e. do not interact and hence do not thermalize), they behave to excellent approximation like a classical field. However, if the axions thermalize, they are not described by a classical field. Instead they form a Bose-Einstein condensate, an essentially quantum-mechanical phenomenon.

2 Axion Bose-Einstein Condensation

Axions were originally introduced [4] as a solution to the strong CP problem. It was later found that they are a cold dark matter candidate [5]. Cold axions are produced when the axion mass turns on during the QCD phase transition. The critical time, defined by $m(t_1)t_1 = 1$, is $t_1 \simeq 2 \cdot 10^{-7}$ sec $(f/10^{12}\ \text{GeV})^{\frac{1}{3}}$, where f is the axion decay constant. The zero temperature axion mass is given in terms of f by

$$m \simeq 6 \cdot 10^{-6}\ \text{eV}\ \frac{10^{12}\ \text{GeV}}{f} \quad . \tag{1}$$

At temperatures well above 1 GeV, the axion mass is practically zero. It increases from zero to m during the QCD phase transition. The cold axions are the quanta of oscillation of the axion field that result from the turn on of the axion mass. They have number density [5]

$$n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left(\frac{f}{10^{12}\ \text{GeV}}\right)^{\frac{5}{3}} \left(\frac{a(t_1)}{a(t)}\right)^3 \tag{2}$$

where $a(t)$ is the cosmological scale factor. Because the axion momenta are of order $\frac{1}{t_1}$ at time t_1 and vary with time as $a(t)^{-1}$, the velocity dispersion of cold axions is

$$\delta v(t) \sim \frac{1}{m t_1} \frac{a(t_1)}{a(t)} \tag{3}$$

if each axion remains in whatever state it is in, i.e. if axion interactions are negligible. We refer to this case as the limit of decoupled cold axions. If decoupled, the average state occupation number of cold axions is

$$\mathcal{N} \sim\ n\ \frac{(2\pi)^3}{\frac{4\pi}{3}(m\delta v)^3} \sim 10^{61} \left(\frac{f}{10^{12}\ \text{GeV}}\right)^{\frac{8}{3}} \quad . \tag{4}$$

Clearly, the effective temperature of cold axions is much smaller than the critical temperature

$$T_c = \left(\frac{\pi^2 n}{\zeta(3)}\right)^{\frac{1}{3}} \simeq 300\ \text{GeV} \left(\frac{f}{10^{12}\ \text{GeV}}\right)^{\frac{5}{9}} \frac{a(t_1)}{a(t)} \tag{5}$$

for BEC. Axion number violating processes, such as their decay to two photons, occur only on time scales vastly longer than the age of the universe. The only condition for axion BEC that is not manifestly satisfied is thermal equilibrium.

Axions are in thermal equilibrium if their relaxation rate Γ is large compared to the Hubble expansion rate $H(t) = \frac{1}{2t}$. However, the usual techniques of non-equilibrium statistical mechanics are not applicable to dark matter axions. On the one hand, cold axions are highly condensed in phase space ($\mathcal{N} >> 1$), which greatly exaggerates the quantum effect of Bose-enhancement in their scattering processes. On the other, because their energy dispersion is very small, they are outside the 'particle kinetic regime'. The picture of instantaneous collisions breaks down, and the usual Boltzmann equation no longer applies. The particle kinetic regime is defined by $\delta\omega >> \Gamma$ where $\delta\omega$ is the energy dispersion of the particles. Axions are in the opposite regime, $\delta\omega << \Gamma$, which we call the 'condensed regime'. Thermalization in the condensed regime is discussed in detail in ref. [6], which gives an estimate for the relaxation rate of cold dark matter axions due to their gravitational self-interactions:

$$\Gamma \sim 4\pi G n m^2 \ell^2 \tag{6}$$

where $\ell \sim (m\delta v)^{-1}$ is the axion correlation length. $\Gamma(t)/H(t)$ is of order $5 \cdot 10^{-7}(f/10^{12}\ \text{GeV})^{\frac{2}{3}}$ at time t_1 but grows as $ta^{-1}(t) \propto a(t)$. Thus gravitational interactions cause the axions to thermalize and form a BEC when the photon temperature is of order 500 eV $(f/10^{12}\ \text{GeV})^{\frac{1}{2}}$.

The question is then whether axion BEC has observable consequences. It is shown in refs. [1, 6] that cold dark matter axions behave as ordinary cold dark matter on all scales of observational interest when they are non-interacting. Observable differences between cold axions and ordinary CDM occur only when the axions self-interact or interact with other species. Before Bose-Einstein condensation, cold axions are described by a free classical field and are indistinguishable from ordinary cold dark matter on all scales of observational interest. After Bose-Einstein condensation, almost all axions are in the same state. In the linear regime of evolution of density perturbations and within the horizon, the lowest energy state is time independent and no rethermalization is necessary for the axions to remain in the lowest energy state. In that case, axion BEC and ordinary CDM are again indistinguishable on all scales of observational interest [1]. However, beyond first order perturbation theory and/or upon entering the horizon, the axions rethermalize to try and remain in the lowest energy available state. Axion BEC behaves differently from CDM then and the resulting differences are observable.

An example of an observable distinction between axion BEC and ordinary CDM is given in the next section. Another possible observable distinction is the cooling of cosmic photons by thermal contact with the axion BEC. If this happens, the baryon-to-photon ratio at nucleosynthesis and the effective number of neutrinos (a measure of the radiation density at recombination) are modified compared to their values in the standard cosmological model [3].

3 Tidal torquing with axion BEC

Let us consider axion BEC dark matter as it is about to fall into the gravitational potential well of a galaxy. The gravitational field of neighbouring galaxies applies a tidal torque [7] to the axion BEC. Under what conditions is thermalization by gravitational self-interactions sufficiently fast that the condensed axions remain in the lowest energy available state as the space-time background evolves? Following the arguments or ref. [6], we expect that the axion BEC rethermalizes provided the gravitational forces produced by the BEC are larger than the typical rate $\dot{p}$ of change of axion momenta required for the axions to remain in the lowest energy state. The gravitational forces are of order $4\pi G n m^2 \ell$. In this case, the correlation length ℓ must be taken to be of order the size L of the region of interest since the gravitational fields due to axion BEC outside the region do not help the thermalization of the axions within the region. Hence the condition is

$$4\pi G n m^2 L \gtrsim \dot{p} \quad . \tag{7}$$

The self-similar infall model [8] was used [6] to estimate L and $\dot{p}$ as functions of time. Furthermore, assuming that most of the dark matter is axions, the Friedmann equation implies

$$4\pi G n m \simeq \frac{3}{2} H(t)^2 \simeq \frac{2}{3t^2} \tag{8}$$

after equality between matter and radiation. It is found [6] that Eq. (7) is satisfied at all times from equality till today by a margin of order 30.

We conclude that the axion BEC does rethermalize before falling into the gravitational potential well of a galaxy. Most axions go to the lowest energy state consistent with the total angular momentum acquired from neighboring inhomogeneities through tidal torquing [7]. That state is a state of rigid rotation on the turnaround sphere, implying $\vec{\nabla} \times \vec{v} \neq 0$ where $\vec{v}$ is the velocity field of the infalling axions. In contrast, the velocity field of WIMP dark matter is irrotational. The inner caustics of galactic halos are different in the two cases. Axions produce caustic rings [9, 10] whereas WIMPs produce the 'tent-like' caustics described in ref. [11]. There is evidence for the existence of caustic rings in various galaxies at the radii predicted by the self-similar infall model. For a review of this evidence see ref. [12]. It is shown in ref. [2] that the phase space structure of galactic halos implied by the evidence for caustic rings is precisely and in all respects that predicted by the assumption that the dark matter is a rethermalizing BEC.

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