

Thick de Sitter brane solutions in higher dimensions

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Abstract

We present thick de Sitter brane solutions which are supported by two interacting *phantom* scalar fields in five-, six- and seven- dimensional spacetime. It is shown that for all cases regular solutions with anti-de Sitter asymptotic (5D problem) and a flat asymptotic far from the brane (6D and 7D cases) exist. We also discuss the stability of our solutions.

1 Introduction

In recent years, there is growing interest in higher-dimensional cosmological models inspired by the recent progresses in string theory. In these models, branes which are submanifolds embedded into the higher-dimensional spacetime play an important (even essential) role, for instance, in a description of the confinement of the nongravitational interactions and/or stabilization of the extra dimensions. Much effort to reveal cosmology on the brane has been given in the context of five-dimensional spacetime (see the review [1]). But of course there is no particular reason to restrict the number of dimensions to be five. In recent years, the focus has been turned to higher-dimensional braneworld models.

It is still unclear how the gravity and cosmology on the brane embedded in the higher-dimensional spacetime behave. In higher dimensions, there would be many kinds of classes of possible braneworld models. Here we focus on the generic feature of higher-dimensional braneworld models and from this we deduce the subject to be revealed in this paper.

It is commonly assumed that a brane is an infinitely thin object. In five dimensions, this is a good approximation as long as one is interested in the scales larger than the brane thickness. Then, the cosmology on the brane in five dimensions, i.e., dynamics of the brane in one bulk spacetime, is uniquely determined by the Israel junction conditions once the matter on the brane is specified. But, in higher dimensions the situation is quite different. It happens because self-gravity of an infinitesimally thin three-brane in a higher-dimensional spacetime develops a severe singularity and one cannot put any kind of matter on the brane if the number of codimensions is larger than 2. Also in the case of a codimension-two brane, one cannot put any kind of matter on the brane other than the pure tension. The cosmology on such a brane must be investigated under some prescription of the singular structure of the brane. The well-motivated prescription is to regularize the brane by taking its microscopic structure into consideration. However, in the case of six-dimensional models, the cosmology on the brane strongly depends on the way of regularization. This implies that there is no unique brane cosmology in the case of higher codimensions.

Thus, in this paper we take a different point of view. We shall take the brane thickness into account *from the beginning* and look for the regular braneworld solutions with finite thickness, called *thick brane solutions*, in a field theory model. The thickness of the brane may be very close to the length scale of the quantum gravitational theory, e.g., the string length scale. As we mentioned, the brane thickness must be more essential in higher-dimensional spacetimes, than in the five-dimensional one. Thus, we look for the thick brane solutions in higher dimensions as well as in five dimensions. Bearing the cosmological

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applications, we focus on the thick brane solutions which have de Sitter geometry in the ordinary four-dimensions.

The thick brane solutions and their properties in five dimensions have been explored in the literatures (see Ref. 3 and 4 in [2]). In this paper, we consider thick brane solutions supported by two interacting scalar fields ϕ and χ , whose potential is given by

$$V(\phi, \chi) = \frac{\lambda_1}{4}(\phi^2 - m_1^2)^2 + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2 + \lambda_3\phi^2\chi^2 + \Lambda. \quad (1)$$

A stronger interaction between two scalar fields develops the local metastable vacua, which may be able to support our brane universe, other than the global one. Several works on thick brane solutions in such a setup have already been performed. Minkowski brane solutions were derived in [3]. Bearing cosmological applications in mind, our goal in this work is to find thick de Sitter brane solutions.

In Ref. [4] there are some arguments in favor of the fact that the scalar fields, used in [3], are a quantum nonperturbative condensate of a $SU(3)$ gauge field. Briefly these arguments consist of the following: components of the $SU(3)$ gauge field can be divided in some natural way into two parts. The first group contains those components which belong to a subgroup $SU(2) \in SU(3)$. The remaining components belong to a factor space $SU(3)/SU(2)$. Following Heisenberg's idea [5] about the nonperturbative quantization of a nonlinear spinor field the nonperturbative quantization for the $SU(3)$ gauge field is being carried out as follows. It is supposed that two-point Green functions can be expressed via the scalar fields. The first field ϕ describes two-point Green functions for $SU(2)$ components of a gauge potential, and the second scalar field χ describes two-point Green functions for $SU(3)/SU(2)$ components of the gauge potential. It is supposed further that four-point Green functions can be obtained as some bilinear combination of the two-point Green functions. Consequently, the Lagrangian of the $SU(3)$ gauge field takes the form (2). It allows us to regard such sort of thick brane solutions as some defect in a spacetime filled by a condensate of the gauge field living in the bulk.

2 Basic theory

Our interest is in the $D = (4 + n)$ -dimensional Einstein-scalar theory whose action is given by

$$S = \int d^D x \sqrt{-g} \left\{ -\frac{M_{n+4}^2}{2} R + \epsilon \left[\frac{1}{2} \partial_A \phi \partial^A \phi + \frac{1}{2} \partial_A \chi \partial^A \chi - V(\phi, \chi) \right] \right\}, \quad (2)$$

where M_{n+4} is the gravitational energy scale in the spacetime with n extra dimensions. The potential energy V is defined by Eq. (1). $\epsilon = +1(-1)$ corresponds to the case of normal (phantom) scalar fields and Λ is an arbitrary constant. As we mentioned in the introduction, in Ref. [4], it was argued that such a theory composed of two interacting scalar fields coupled to the remaining $U(1)$ degrees of freedom appears as a result of the gauge condensation in the original $SU(3)$ gauge theory. In this case, the scalar fields ϕ and χ correspond to the vacuum expectation values of the $SU(2)$ and $SU(3)/(SU(2) \times U(1))$ gauge sectors, respectively.

We assume that the generalized D -dimensional metric has the static form

$$ds^2 = a^2(r) \gamma_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta - \lambda(r) (dr^2 + r^2 d\Omega_{n-1}^2), \quad (3)$$

where $d\Omega_{n-1}^2$ is the solid angle for the $(n-1)$ sphere. $\gamma_{\alpha\beta}$ is the metric of the four-dimensional de Sitter space whose scalar curvature is given by $R_{\mu\nu}[\gamma] = 3H^2 \gamma_{\mu\nu}$. The detailed forms of Einstein and scalar field equations for our ansatz can be found in Ref. [2].

The extremum of the bulk potential are located at the following four points

$$\begin{aligned} & \left(\phi = \pm\phi_0 := \pm\sqrt{\frac{2\lambda_3\lambda_2 m_2^2 - \lambda_1\lambda_2 m_1^2}{4\lambda_3^2 - \lambda_1\lambda_2}}, \chi = \pm\chi_0 := \pm\sqrt{\frac{2\lambda_3\lambda_1 m_1^2 - \lambda_1\lambda_2 m_2^2}{4\lambda_3^2 - \lambda_1\lambda_2}} \right) \\ & \quad (\phi := 0, \chi := 0) \\ & \quad (\phi := \pm m_1, \chi := 0) \\ & \quad (\phi := 0, \chi := \pm m_2) \end{aligned} \quad (4)$$

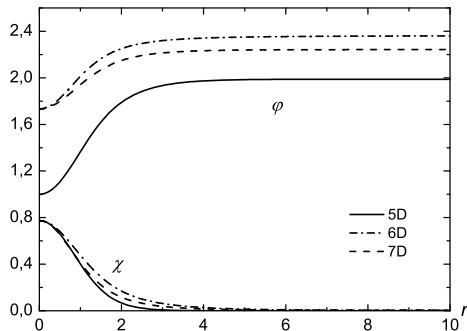


Figure 1: The scalar field configurations ϕ and χ are shown as functions of the dimensionless r .

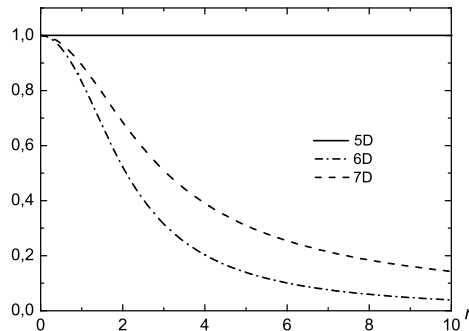


Figure 2: The metric function $\lambda(r)$ is shown as functions of dimensionless r .

We call these points 0, 1, 2 and 3, respectively, with corresponding potentials V_i ($i = 0, 1, 2, 3$). Clearly, $\max(V_2, V_3) \leq V_1$. The potential (1) has two global minimums 3 and two local minima 2 at values of the parameters λ_1, λ_2 used in this paper. The conditions for existence of local minima are $\lambda_1 > 0, m_1^2 > \lambda_2 m_2^2 / (2\lambda_3)$, and for global minima $\lambda_2 > 0, m_2^2 > \lambda_1 m_1^2 / (2\lambda_3)$. From comparison of the values of the potential in global and local minima we have (taking into account that $V_{loc} = 0$ at $V_0 = \lambda_2 m_2^4 / 4$, see below for the six- and seven-dimensional cases) $V_{gl} < V_{loc} \implies V_{gl} < 0$, i.e. $\lambda_2 m_2^4 > \lambda_1 m_1^4$. Besides two local and two global minima, four unstable saddle points (0) and the local maximum (1) exist. In the next sections, we look for thick brane solutions starting and finishing in one of local minima 2 of the potential (1).

For our purpose, in the following sections the bulk coordinate and scalar field variables are rescaled to made them dimensionless as $r \rightarrow H^{-1}r$, $\phi \rightarrow M_{n+4}^{-(n+2)/2}\phi$ and $\chi \rightarrow M_{n+4}^{-(n+2)/2}\chi$. Correspondingly, the potential and its parameters are also rescaled as $V \rightarrow H^2 M_{n+4}^{n+2}V$, $\lambda_i \rightarrow \lambda_i H^2 / M_{n+4}^{n+2}$ and $m_j^2 \rightarrow M_{n+4}^{n+2}m_j^2$, where $i = 1, 2, 3$ and $j = 1, 2$.

Then, the boundary conditions at $r \rightarrow 0$ are given by $a'(0) = 0$, $\phi' = 0$ and $\chi' = 0$. And from the constraint relation of Einstein equations, we find the boundary value of scale factor $a(0) = \sqrt{6\epsilon/V(0)}$. At the asymptotic infinity, the scalar fields approach the local minimum where $\phi'(\infty) = \chi'(\infty) = 0$. Also at the asymptotic infinity, the bulk geometry is assumed to be anti-de Sitter (AdS) for the five-dimensional model and Minkowski for the six- and seven-dimensional models.

3 Solutions

We present the numerical examples of thick de Sitter brane solutions for the case of phantom scalar field $\epsilon = -1$. We solved the coupled Einstein-scalar system by the iteration method. The detailed descriptions of the method can be found in Ref. [2, 3].

We set the parameters as: $\lambda_1 = 0.1$, $\lambda_2 = 1.0$, $\lambda_3 = 1.0$. For the five-dimensional case we have chosen $\phi(0) = 1.0$, $\chi(0) = \sqrt{0.6}$ and $\Lambda = -3.6$. For these parameters we solved the nonlinear eigenvalue problem for m_1 and m_2 . After 11 iterations the following eigenvalues $m_1 \approx 1.989512$ and $m_2 \approx 1.964764$ were found. In Figs. 1-4 we showed our numerical solutions. The spacetime is an asymptotically anti-de Sitter one: the asymptotic value of the potential (1) $\epsilon V_\infty = \epsilon(\lambda_2 m_2^4 / 4 + \Lambda) < 0$ plays the role of a negative cosmological constant.

For the six-dimensional case we take the boundary condition $\phi(0) = \sqrt{3} \approx 1.73205$ and $\chi(0) = \sqrt{0.6} \approx 0.774597$. We also choose the bulk cosmological constant as $\Lambda = -\lambda_2 m_2^4 / 4$, in order for our solutions to be asymptotically flat in the bulk. After 10 iterations, we find the eigenvalues $m_1 \approx 2.3590$ and $m_2 \approx 3.0599$. In Figs. 1-4 we showed our numerical solution for a given set of parameters.

For the case of the seven-dimensional case, we also choose the bulk cosmological constant as $\Lambda =$

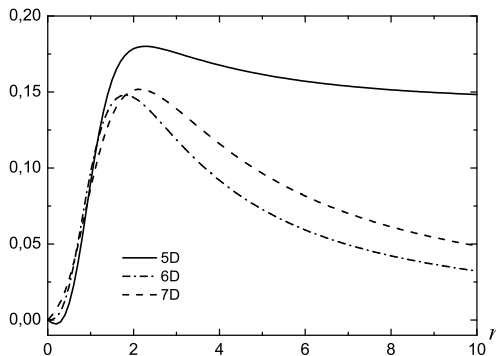


Figure 3: The function a'/a is shown as functions of dimensionless r .

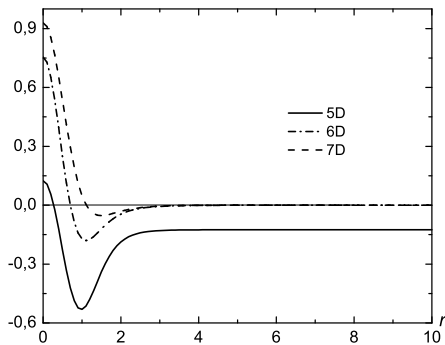


Figure 4: The energy density $T_0^0 = \epsilon \left[\frac{1}{2\lambda} (\phi'^2 + \chi'^2) + V \right]$ is shown as functions of dimensionless r .

$-\lambda_2 m_2^4/4$. After 10 iterations, we find the eigenvalues $m_1 \approx 2.24633$ and $m_2 \approx 3.11911$. The obtained solutions for this case are presented in Fig. 1-4.

4 Summary and Discussions

In this presentation, we have presented five-, six- and seven-dimensional thick de Sitter brane solutions supported by two interacting (*phantom*) scalar fields. The special form of the potential (1) allows us to find regular solutions with finite energy density. It was shown that asymptotically there exist anti-de Sitter spacetime for the five-dimensional case and flat spacetime for the six- and seven-dimensional cases.

Also we have confirmed that our solutions in five-dimensional spacetime are stable. It is also quite reasonable to expect that our thick brane solutions in higher dimensions are also stable since the spacetime and vacuum structures are essentially the same as in the case of five spacetime dimensions. For more details, see [2].

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