

Sum rules for tetraquark decay in broken $SU(3)$ symmetry

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Abstract.

We give sum rules for tetraquark decay coupling constants, taking into account the $SU(3)$ symmetry breaking interactions to first order.

1. Introduction

Recently [1], the Belle Collaboration has found the decay of heavy mesons of masses 10.61 GeV and 10.65 GeV into $\Upsilon + \pi^\pm$, referred to as $Z_b(10610)$ and $Z_b(10650)$, respectively. In this note we suggest that these new states are tetraquarks made of $(b\bar{b}q\bar{q})$ with $q = u, d, s$. We work out the consequences of broken $SU(3)$ symmetry to derive sum rules for tetraquark decay coupling constants.

2. Notation

Let us consider a general tetraquark $T(Q\bar{Q}M(q\bar{q}))$ with Q ($= b$ or c) quark a $SU(3)$ flavour singlet and $M(q\bar{q})$ made of the light quarks q ($= u, d$, and s) a $SU(3)$ flavour octet. $M(q\bar{q})$ could be the 0^- pseudoscalar octet (π^\pm, K^\pm , etc.) or the 1^- vector octet ($\rho^\pm, K^{*\pm}$, etc.).

The natural decay mode of the tetraquark $T(Q\bar{Q}M(q\bar{q}))$ would be

$$T(Q\bar{Q}M(q\bar{q})) \rightarrow H(Q\bar{Q}) + M(q\bar{q}), \quad (1)$$

where $H(Q\bar{Q})$ is a heavy meson since $Q = b$ or c . Furthermore, one expects that $H(Q\bar{Q})$ and $M(q\bar{q})$ in the final state have the same J^P they have in the tetraquark. This is the simplest possibility.

3. Sum rules

If the decay interaction is flavour $SU(3)$ singlet, then all the eight decays will have the same coupling constant G_0 . However if one considers a first order breaking interaction transforming like λ_8 (or T_3^3) which conserves isospin invariance then this will give rise to two $SU(3)$ breaking coupling constants, G_F and G_D , which arise from the antisymmetric and symmetric octets



resulting from the breaking interaction and the $M(q\bar{q})$ octet in the final state. Thus, the eight decays will be given in terms of three coupling constants. However, since isospin invariance is present, there will be four independent coupling constants namely, when the $M(q\bar{q})$ meson is (K^+, K^0) , (\bar{K}^0, K^-) , (π^+, π^0, π^-) , and $s\bar{s}$. These four coupling constants will be linear combinations of G_0 , G_F , and G_D , and hence, we expect a sum rule among the four coupling constants of the observable decays [2, 3, 4].

a) Since the $SU(3)$ calculation is analogous to the calculation of the meson masses with λ_8 breaking, one obtains a coupling constants sum rule analogous to the Gell-Mann-Okubo mass formula for mesons [5, 6]. Thus, if our interpretation of the observed particles as tetraquarks is right we expect,

$$2G [T(Q\bar{Q}K) \rightarrow H(Q\bar{Q}) + K] + 2G [T(Q\bar{Q}K) \rightarrow H(Q\bar{Q}) + \bar{K}] = \\ G [T(Q\bar{Q}\pi) \rightarrow H(Q\bar{Q}) + \pi] + 3G [T(Q\bar{Q}s\bar{s}) \rightarrow H(Q\bar{Q}) + s\bar{s}]. \quad (2)$$

Here $K = K^+$ or K^0 , $\bar{K} = \bar{K}^0$ or K^- , $\pi = \pi^+$, π^- , or π^0 . The heavy quark Q is a $SU(3)$ singlet and could be either b or c quark.

b) The above formula for decay coupling constants is also valid for tetraquarks $T(Q\bar{Q}'M(q\bar{q}))$ where $Q = b$ (or c) with $Q' = c$ (or b). As noted earlier, the light quark octet $M(q\bar{q})$ could be the vector octet (ρ , K^* , etc.). The sum rule would be valid with the obvious change $\pi \rightarrow \rho$, $K \rightarrow K^*$, etc.

The sum rules contain $\eta = s\bar{s}$. However, as is well known, both for pseudoscalar and vector mesons, $s\bar{s}$ is a combination of the $SU(3)$ singlet η_1 and the eighth component η_8 . This mixing must be taken into account in testing our coupling constants sum rule. Specifically, in terms of the $SU(3)$ flavor states η_1 and η_8 ,

$$s\bar{s} = \frac{1}{\sqrt{3}}\eta_1 - \frac{2}{\sqrt{6}}\eta_8, \quad (3)$$

where

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \quad \text{and} \quad \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}). \quad (4)$$

For $J^P = 0^-$, $\eta = \eta_8 \cos \theta - \eta_1 \sin \theta$ and $\eta'(958) = \eta_8 \sin \theta + \eta_1 \cos \theta$, with $\theta = -11.5^\circ$. For $J^P = 1^-$, $\eta \rightarrow \phi(1020)$ and $\eta'(958) \rightarrow \omega(782)$, with $\theta = 38.7^\circ$ [7].

c) In case there are baryonic pentaquarks of the form $P(Q\bar{Q}B)$ where B is usual baryon octet then our sum rule will be valid for the decay coupling constants of

$$P(Q\bar{Q}B) \rightarrow H(Q\bar{Q}) + B, \quad (5)$$

with appropriate changes, namely,

$$2G [P(Q\bar{Q}N) \rightarrow H(Q\bar{Q}) + N] + 2G [P(Q\bar{Q}\Xi) \rightarrow H(Q\bar{Q}) + \Xi] = \\ G [P(Q\bar{Q}\Sigma) \rightarrow H(Q\bar{Q}) + \Sigma] + 3G [P(Q\bar{Q}\Lambda) \rightarrow H(Q\bar{Q}) + \Lambda]. \quad (6)$$

The above sum rule with appropriate changes would be valid for pentaquarks $P(Q\bar{Q}'B)$, where $Q = b$ (or c) with $Q' = c$ (or b).

4. Conclusions

The above sum rules were obtained by taking into account the $SU(3)$ symmetry breaking only up to the first order. At present we do not have data to extract the coupling constants involved in the sum rules. Hopefully, we will have sufficient data in the near future to enable us to check the sum rules given here.

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