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Elastic *pp* Scattering in AdS/CFT

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Abstract. This work is geared towards studying *pp* collisions using the AdS/CFT correspondence. It is inspired by the work of Richard C. Brower, Joseph Polchinski, Matthew J. Strassler and Chung I Tan (arXiv:1204.0472v1) on particle interactions. A simple extension is provided by calculating the scattering amplitudes of *pp* collisions using the BPST Pomeron and comparing it to the well known empirical result of elastic *pp* scattering.

1 Introduction

Our understanding of Quantum Chromodynamics in terms of perturbation theory is limited due to the running nature of the coupling constant. Gauge/gravity dualities such as AdS/CFT correspondence offer ways to perform calculations on strongly interacting systems. The leading order exchange (Pomeron) for cross-sections in this framework was identified by Brower, Polchinski, Strassler and Tan [5]. The total cross-section is given using the optical theorem and the eikonal approximation as

$$\sigma_{tot} = \frac{1}{s}\mathrm{Im}\,A(s,t=0) = \int \mathrm{d}^2b \int \mathrm{d}z\mathrm{d}z' P_{13}(z)P_{24}(z')\,\mathrm{Re}(1-\mathrm{e}^{\mathrm{i}\chi(s,b,z,z')}) \tag{1}$$

where

P_{ij} = product of wavefunctions for the external states i and j,

$\mathrm{Im}\chi(s,b,z,z') = \frac{g_0^2}{16\pi}\sqrt{\frac{\rho}{\pi}}\mathrm{e}^{(1-\rho)\tau}\frac{\zeta}{\sinh\zeta}\frac{\exp(-\frac{\zeta^2}{\rho\tau})}{\tau^{3/2}}$,

$\rho = \frac{1}{\sqrt{\lambda}}$,

λ = 't Hooft coupling,

$\tau = \log(\frac{\rho}{2}zz's)$,

$\zeta = \log(1+v+\sqrt{v(2+v)}$,

v = chordal distance in AdS_3.

However, there is a need to accommodate for certain properties of QCD such as confinement, which is done by modifying the geometry of the AdS space. This is done by introducing a cut-off $z_0 \sim \frac{1}{\Lambda_{QCD}}$ on the AdS space, where Λ_{QCD} is the confinement scale.

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2 The Cross-section

2.1 Conformal AdS Model

The b space integral for single Pomeron exchange can be calculated as in [2].

$$\int \mathrm{d}^2 b \, \mathrm{Im}\chi(s,b,z,z') = \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}(zz')\mathrm{e}^{(1-\rho)\tau}\frac{\exp(\frac{-(\log z - \log z')^2}{\rho\tau})}{\tau^{1/2}} \tag{2}$$

$$\Rightarrow \sigma_{tot} \approx \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}\int \mathrm{d}z\mathrm{d}z' P_{13}(z)P_{24}(z')(zz')\mathrm{e}^{(1-\rho)\tau}\frac{\exp(\frac{-(\log z - \log z')^2}{\rho\tau})}{\tau^{1/2}} \tag{3}$$

For $pp \to pp$ collisions, the wavefunctions P_{13} and P_{24} both correspond to protons and can be approximated by delta functions. Therefore,

$$P_{13}(z) \approx \delta(z - 1/Q), \tag{4}$$

$$P_{24}(z') \approx \delta(z' - 1/Q'). \tag{5}$$

Here, Q and Q' characterise the respective "sizes" of the two protons. Note that the effects of confinement should be significant only for $z_0 \geq \frac{1}{Q}, \frac{1}{Q'}$. The total cross-section in the conformal model is therefore given by

$$\sigma_{tot} \approx \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}\left(\frac{1}{QQ'}\right)\mathrm{e}^{(1-\rho)\tau}\frac{\exp(\frac{-(\log Q' - \log Q)^2}{\rho\tau})}{\tau^{1/2}}. \tag{6}$$

2.2 Hardwall Model

For the hardwall model, we introduce a cut-off on the AdS space z_0 (corresponding to the confinement scale Λ_{QCD}) by means of a second term resulting in the total cross-section of the form

$$\sigma_{tot} \approx \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}\left(\frac{1}{QQ'}\right)\mathrm{e}^{(1-\rho)\tau}\left(\frac{\exp(\frac{-(\log(Q'/Q))^2}{\rho\tau})}{\tau^{1/2}} + \mathcal{F}(Q,Q',\tau)\frac{\exp(\frac{-(\log(1/QQ'z_0^2))^2}{\rho\tau})}{\tau^{1/2}}\right) \tag{7}$$

where $\mathcal{F}(Q,Q',\tau)$ is some smooth function such that $\mathcal{F} \to 1$ as $\tau \to 0$ and $\mathcal{F} \to -1$ as $\tau \to \infty$. Therefore, at very large values of s, where the effects of confinement become sizable, there is a partial cancellation of the cross-section growth rate.

3 Results and Discussion

The following figure plots the empirical fit [3, 4] for the scattering cross-section of $pp \to pp$ scattering along with the above found conformal result.
As can be seen, at high energies the growth of the cross-section according to our model is more rapid than what is dictated by experiments. This is a due to the fact that our model violates the Froissart bound for large s owing to the single Pomeron calculation. Further, disagreement at low energies can be ascribed to electromagnetic interaction between the protons which has not been taken into account. The parameter values are well within our expectation as $Q, Q' \sim O(1)$ GeV, i.e., we expect Q, Q' to be of the order of the proton mass.
Another important note is that the process considered here is simplified as we approximate the wavefunctions with Dirac delta peaks. The results are therefore promising as the fit is good regardless of the simplification made. A detailed analysis with data has been left out at this point and is in progress.

Table 1. Model parameter values

ρ	g_0^2	Q	Q'
0.88	1252.087	0.872	0.863

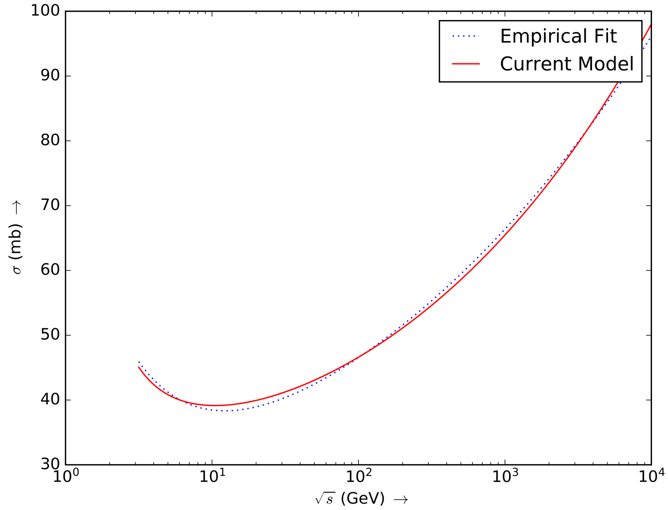

Figure 1. The blue line corresponds to the true fit while the red line corresponds to our current model. The values found for the model parameters are tabulated above.

4 Conclusion and Future Work

We calculated the total cross-section for $pp \to pp$ process using the BPST Pomeron in AdS. We then fit our result to the well known empirical curve for the total cross-section given by $\sigma_{tot} = 21.70s^{0.0808} + 56.08s^{-0.4525}$ [3, 4]. The model parameters are found to be reasonable within our assumptions.

As discussed, our current model is a rough sketch of the full picture. Following is the future scope of this research.

1. The hardwall model has not been explored here. Interesting insight could be derived by looking at how the values of the parameters change.

2. To avoid violating the Froissart bound, the full eikonal approximation should be taken into account instead of a single Pomeron exchange.

3. The proton wavefunctions should be explicitly calculated for higher energies to accommodate for internal structure. In the process, coupling with QED and the photon should be taken into account.

4. Elastic pp scattering is a first step towards studying production processes at current LHC energies. The phenomenology of charged-particle multiplicity using AdS/CFT can shine some light on non-perturbative QCD processes.

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