

PAPER • OPEN ACCESS

## Neutrino spin-flavor oscillations derived from the mass basis

To cite this article: R Fabbriatore *et al* 2016 *J. Phys.: Conf. Ser.* **718** 062058

View the [article online](#) for updates and enhancements.

### Related content

- [Random magnetic fields inducing solar neutrino spin-flavor precession in a three generation context](#)  
M M Guzzo, P C de Holanda and O L G Peres
- [Sterile Neutrinos](#)  
Antonio Palazzo
- [Low-energy neutrinos](#)  
Livia Ludhova

### Recent citations

- [Neutrino spin and spin-flavor oscillations in transversal matter currents with standard and nonstandard interactions](#)  
Pavel Pustoshny and Alexander Studenikin
- [Neutrino spin and spin-flavour oscillations in transversally moving or polarized matter](#)  
Alexander Studenikin
- [A QFT-induced phase in neutrino flavour oscillations](#)  
Dharam Vir Ahluwalia and Cheng-Yang Lee



**IOP | ebooks™**

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

# Neutrino spin-flavor oscillations derived from the mass basis

R Fabbriatore<sup>1</sup>, A Grigoriev<sup>2</sup>, A Studenikin<sup>1,3</sup>

<sup>1</sup> Department of Theoretical Physics, Faculty of Physics, Lomonosov Moscow State University, Moscow, 119991, Russia

<sup>2</sup> Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, 119991, Russia

<sup>3</sup> Joint Institute for Nuclear Research, Dubna, Moscow Region, 141980, Russia

E-mail: studenik@srd.sinp.msu.ru

**Abstract.** We reconsider the standard scheme for description of neutrino spin-flavor oscillations aiming at a rigorous derivation of evolution equation for the mixed flavor neutrino states in magnetic field. For this purpose we obtain the evolution equation in the physical basis of massive neutrinos and then trace its transformation into the flavor basis. The effective Hamiltonian of the resulting equation relevant to interaction with the magnetic field differs from the standard one by several entries. The approach leads to interesting relations of the neutrino magnetic moments defined in the two neutrino bases and to some additional subtle properties of the formalism.

## 1. Introduction

Significant advances in neutrino physics, performed during the last decades, do not leave any doubt that neutrino is subjected to oscillations and has nonzero mass [1]. The latter fact leads to the well-known possibility for neutrino to have non-trivial electromagnetic properties [2], which brought forth the research area that was investigated in details by numerous authors (see recent review [3] and references therein). In the course of these studies many phenomena that may appear in electromagnetic fields have been recognized and described thoroughly. Among them the neutrino spin-flavor oscillations is the one, featuring both the above mentioned basic neutrino aspects – nonzero mass and electromagnetic properties from the one side and mixing from another [4]. Owing to this, in spite of being a longstanding problem, the spin-flavor oscillations can reveal some new aspects of existence of neutrino mass and electromagnetic properties. These concern the problems of neutrino parameters relation in neutrino physical (mass) and flavor bases and, generally speaking, of accurate establishment of the formulas used to describe oscillations.

In this way, in this short note we would like to give an advanced view on the standard scheme of neutrino spin-flavor oscillations description aiming at solid determination of parameters involved in the formalism and its rigorous derivation. Starting from the mass basis we obtain neutrino evolution equation with effective Hamiltonian written through fundamental electromagnetic parameters, and then perform transformation to flavor basis. A direct comparison of the result with the standard picture known for the flavor basis gives relation of the neutrino electromagnetic parameters among the bases. The resulting spin-flavor oscillation



pattern must contain nothing but fundamental parameters, defined in the mass basis. In conclusion to our note, features of the spin-flavor oscillations arising due to the transformations are highlighted.

## 2. Neutrino spin oscillations in mass basis

As usual, for the sake of brevity, we will consider only two neutrino physical states,  $\nu_1$  and  $\nu_2$ , having masses  $m_1$  and  $m_2$ . We also restrict ourself by the Dirac neutrino nature and assume fundamental interactions introduced into the Standard Model such way that neutrino acquire electromagnetic interaction via magnetic moment matrix  $\mu_{\alpha\beta}$ ,  $\alpha, \beta = 1, 2$ :

$$H_{EM} = \frac{1}{2} \mu_{\alpha\beta} \bar{\nu}_\alpha \sigma_{\mu\nu} \nu_\beta F^{\mu\nu} + h.c. , \quad (1)$$

where  $F^{\mu\nu}$  is the electromagnetic field tensor,  $\sigma_{\mu\nu} = i/2(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$  and  $\gamma_\mu$  being the Dirac matrices. In a uniform magnetic field the Hamiltonian (1) becomes

$$H_{EM} = -\mu_{\alpha\alpha'} \bar{\nu}_\alpha \Sigma \mathbf{B} \nu_{\alpha'} + h.c., \quad (2)$$

where

$$\Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \quad (3)$$

and  $\sigma_i$  are the Pauli matrices.

In the neutrino oscillation framework, one is interested in evolution of chiral neutrino components within the common neutrino beam state. Since in the ultrarelativistic limit the latter are approximated by free neutrino states with definite helicity  $s = \pm 1$  the 4-component basis  $(\nu_{1,s=1}, \nu_{1,s=-1}, \nu_{2,s=1}, \nu_{2,s=-1})$  is adopted to describe neutrino beam. With the standard column vector notation,  $\nu_m \equiv (\nu_{1,s=1}, \nu_{1,s=-1}, \nu_{2,s=1}, \nu_{2,s=-1})^T$  the neutrino evolution equation relevant to electromagnetic interaction has the Schrödinger-like form:

$$i \frac{d}{dt} \nu_m(t) = H_{eff} \nu_m(t). \quad (4)$$

The effective Hamiltonian consists of the vacuum and interaction parts:

$$H_{eff} = H_{vac} + H_B. \quad (5)$$

where the interaction part is composed of matrix elements of the field interaction Hamiltonian taken over the helicity neutrino states:  $H_B = \langle \nu_{\alpha,s} | H_{EM} | \nu_{\alpha',s'} \rangle$ .

Let us calculate the effective interaction Hamiltonian under assumption that neutrino moves along the  $z$ -axis. From the magnetic field interaction Hamiltonian (2) we have:

$$H_{\alpha,s;\alpha',s'}^B = \langle \nu_{\alpha,s} | H_{EM} | \nu_{\alpha',s'} \rangle = -\frac{\mu_{\alpha,\alpha'}}{2} \int d^3x \nu_\alpha^\dagger \gamma_0 \begin{pmatrix} \Sigma \mathbf{B} & 0 \\ 0 & \Sigma \mathbf{B} \end{pmatrix} \nu_{\alpha'}. \quad (6)$$

For the spinors representing the free neutrino states we take:

$$\nu_{\alpha,s} = C_\alpha \sqrt{\frac{E_\alpha + m_\alpha}{2E_\alpha}} \begin{pmatrix} u_s \\ \frac{\Sigma \mathbf{p}_\alpha}{E_\alpha + m_\alpha} u_s \end{pmatrix} e^{i\mathbf{p}_\alpha \mathbf{x}}, \quad (7)$$

where  $\mathbf{p}_\alpha$  is the neutrino  $\nu_\alpha$  momentum. The two-component spinors  $u_s$  define neutrino helicity states, and are given by

$$u_{s=1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_{s=-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (8)$$

Recall that in the ultrarelativistic limit these are correspondent to the right-handed  $\nu_R$  and left-handed  $\nu_L$  chiral neutrinos, respectively.

Substituting (7) into the effective Hamiltonian formula (6) we get

$$H_{\alpha,s;\alpha',s'}^B = -\frac{1}{2}\mu_{\alpha\alpha'}C_\alpha C_{\alpha'} \int d^3x \mathbf{B} \begin{pmatrix} u_s^\dagger & \frac{\Sigma \mathbf{p}_\alpha}{E_\alpha + m_\alpha} u_s^\dagger \\ 0 & -\Sigma \end{pmatrix} \begin{pmatrix} u_{s'} \\ \frac{\Sigma \mathbf{p}_{\alpha'}}{E_{\alpha'} + m_{\alpha'}} u_{s'} \end{pmatrix} \\ \times \frac{\sqrt{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}}{2\sqrt{E_\alpha E_{\alpha'}}} \exp(i\Delta p x). \quad (9)$$

Decomposing the magnetic field vector into longitudinal and transversal with respect to neutrino motion components  $\mathbf{B} = \mathbf{B}_\parallel + \mathbf{B}_\perp$  it is possible to show that

$$\mathbf{B} \begin{pmatrix} u_s^\dagger & \frac{\Sigma \mathbf{p}_\alpha}{E_\alpha + m_\alpha} u_s^\dagger \\ 0 & -\Sigma \end{pmatrix} \begin{pmatrix} u_{s'} \\ \frac{\Sigma \mathbf{p}_{\alpha'}}{E_{\alpha'} + m_{\alpha'}} u_{s'} \end{pmatrix} = \\ u_s^\dagger \left( \Sigma \mathbf{B}_\parallel \left( 1 - \frac{\mathbf{p}_\alpha \mathbf{p}_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})} \right) + \Sigma \mathbf{B}_\perp \left( 1 + \frac{\mathbf{p}_\alpha \mathbf{p}_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})} \right) \right) u_{s'}. \quad (10)$$

Let us apply the ultrarelativistic condition  $\frac{m_\alpha}{E_\alpha} \ll 1$  to the part of the integrand of Eq. (9):

$$\left( 1 - \frac{\mathbf{p}_\alpha \mathbf{p}_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})} \right) \frac{\sqrt{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}}{2\sqrt{E_\alpha E_{\alpha'}}} \approx \frac{1}{2} \left( \frac{m_\alpha}{E_\alpha} + \frac{m_{\alpha'}}{E_{\alpha'}} \right) \equiv \gamma_{\alpha\alpha'}^{-1}, \quad (11)$$

where the quantity  $\gamma_{\alpha\alpha'}$  we will call the transition gamma-factor. Similarly, it is also possible to show that

$$\left( 1 + \frac{\mathbf{p}_\alpha \mathbf{p}_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})} \right) \frac{\sqrt{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}}{2\sqrt{E_\alpha E_{\alpha'}}} \approx 1. \quad (12)$$

Introducing an angle  $\beta$  between the  $\mathbf{B}$  and  $\mathbf{p}_\alpha$  vectors and assuming that  $\mathbf{B}_\perp$  is aligned along the  $x$ -axis we further obtain:

$$u_{s=1}^\dagger \Sigma \mathbf{B} u_{s=1} = (1 \ 0) \Sigma \begin{pmatrix} 1 \\ 0 \end{pmatrix} (\mathbf{B}_\parallel + \mathbf{B}_\perp) = \\ (1 \ 0) \sigma_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} B \cos \beta + (1 \ 0) \sigma_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} B \sin \beta = B \cos \beta, \quad (13)$$

and similarly

$$u_{s=1}^\dagger \Sigma \mathbf{B} u_{s=-1} = B \sin \beta, \quad (14)$$

$$u_{s=-1}^\dagger \Sigma \mathbf{B} u_{s=1} = B \sin \beta, \quad (15)$$

$$u_{s=-1}^\dagger \Sigma \mathbf{B} u_{s=-1} = -B \cos \beta. \quad (16)$$

As it was expected, in neutrino transitions without change of helicity only the  $B_\parallel = B \cos \beta$  component of the magnetic field contribute to the effective potential, whereas in transitions with change of the neutrino helicity the transversal component  $B_\perp = B \sin \beta$  matters.

Performing the remaining simple algebra one can readily write out the  $H_B$  matrix. However we wish to write at once the evolution equation (4) in which the whole effective Hamiltonian is obtained by adding the diagonal vacuum part  $H_{vac}$ :

$$i \frac{d}{dt} \begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} = \begin{pmatrix} E_1 + \mu_{11} \frac{B_{\parallel}}{\gamma_{11}} & \mu_{11} B_{\perp} & \mu_{12} \frac{B_{\parallel}}{\gamma_{12}} & \mu_{12} B_{\perp} \\ \mu_{11} B_{\perp} & E_1 - \mu_{11} \frac{B_{\parallel}}{\gamma_{11}} & \mu_{12} B_{\perp} & -\mu_{12} \frac{B_{\parallel}}{\gamma_{12}} \\ \mu_{12} \frac{B_{\parallel}}{\gamma_{12}} & \mu_{12} B_{\perp} & E_2 + \mu_{22} \frac{B_{\parallel}}{\gamma_{22}} & \mu_{22} B_{\perp} \\ \mu_{12} B_{\perp} & -\mu_{12} \frac{B_{\parallel}}{\gamma_{12}} & \mu_{22} B_{\perp} & E_2 - \mu_{22} \frac{B_{\parallel}}{\gamma_{22}} \end{pmatrix} \begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix}. \quad (17)$$

This equation governs all possible oscillations of the four neutrino mass states determined by the masses  $m_1$  and  $m_2$  and helicities  $s = 1$  and  $s = -1$  in the presence of a magnetic field. The form of the equation enables us to make some general suggestions on influence of an arbitrary magnetic field on neutrino evolution:

1) the longitudinal magnetic field component  $B_{\parallel}$  coupled to the corresponding magnetic moment shifts the neutrino energy,

2) in case of nonzero transition magnetic moment  $\mu_{12}$ , the mixing between neutrino states with different masses is induced by  $B_{\parallel}$  also.

At the same time, we note that general previously known features of the spin oscillations are clearly emerge here: first, the change of helicity upon mixing is due to the corresponding magnetic moment (or transition magnetic moment) interaction with the transversal magnetic field  $B_{\perp}$ , and, second, the suppression of terms with longitudinal field component in the ultrarelativistic limit.

### 3. Transition to flavor basis

Once having physics in the mass basis in hands, our next step is to bring it to observational terms. This means that we must elaborate a generalization of the mixing matrix for transitions between neutrino vector written in two four-component bases  $\nu_m$  and  $\nu_f \equiv (\nu_e^R, \nu_e^L, \nu_{\mu}^R, \nu_{\mu}^L)^T$  so that

$$\nu_f = U \nu_m. \quad (18)$$

This procedure appears to be not quite direct since we should hold the condition that polarization of the fields must preserve under transformation of the bases elements. That is why we put (still keeping in mind that chiral components are almost helicity ones):

$$\begin{cases} \nu_e^{R,L} = \nu_{1,s=\pm 1} \cos \theta + \nu_{2,s=\pm 1} \sin \theta, \\ \nu_{\mu}^{R,L} = -\nu_{1,s=\pm 1} \sin \theta + \nu_{2,s=\pm 1} \cos \theta. \end{cases} \quad (19)$$

Then, using Eqs. (18) and (19), it is easy to obtain that

$$U = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & -\sin \theta & 0 & \cos \theta \end{pmatrix}. \quad (20)$$

Given the transition matrix (20), derivation of the evolution equation in the flavor basis is straightforward:

$$i \frac{d}{dt} \nu_f = U H U^{\dagger} \nu_f, \quad (21)$$

so that the effective interaction Hamiltonian  $H_B^f \equiv UH_B U^\dagger$  has the structure:

$$H_B^f = \begin{pmatrix} \tilde{\mu}_{ee} \frac{B_{\parallel}}{\gamma_{ee}} & \mu'_{ee} B_{\perp} & \tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} & \mu'_{e\mu} B_{\perp} \\ \mu'_{ee} B_{\perp} & -\tilde{\mu}_{ee} \frac{B_{\parallel}}{\gamma_{ee}} & \mu'_{e\mu} B_{\perp} & -\tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} \\ \tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} & \mu'_{e\mu} B_{\perp} & \tilde{\mu}_{\mu\mu} \frac{B_{\parallel}}{\gamma_{\mu\mu}} & \mu'_{\mu\mu} B_{\perp} \\ \mu'_{e\mu} B_{\perp} & -\tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} & \mu'_{\mu\mu} B_{\perp} & -\tilde{\mu}_{\mu\mu} \frac{B_{\parallel}}{\gamma_{\mu\mu}} \end{pmatrix}. \quad (22)$$

Here we have introduced the following formal notations intended to manifest an analogy with the standard spin-flavor oscillation formalism (see below):

$$\begin{aligned} \mu'_{ee} &= (\mu_{11} \cos^2 \theta + \mu_{22} \sin^2 \theta + \mu_{12} \sin 2\theta) \\ \mu'_{e\mu} &= (\mu_{12} \cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11}) \sin 2\theta) \\ \mu'_{\mu\mu} &= (\mu_{11} \cos^2 \theta + \mu_{22} \sin^2 \theta - \mu_{12} \sin 2\theta) \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\tilde{\mu}_{ee}}{\gamma_{ee}} &= \left( \frac{\mu_{11}}{\gamma_{11}} \cos^2 \theta + \frac{\mu_{22}}{\gamma_{22}} \sin^2 \theta + \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta \right) \\ \frac{\tilde{\mu}_{e\mu}}{\gamma_{e\mu}} &= \left( \frac{\mu_{12}}{\gamma_{12}} \cos 2\theta + \frac{1}{2} \left( \frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}} \right) \sin 2\theta \right) \\ \frac{\tilde{\mu}_{\mu\mu}}{\gamma_{\mu\mu}} &= \left( \frac{\mu_{11}}{\gamma_{11}} \cos^2 \theta + \frac{\mu_{22}}{\gamma_{22}} \sin^2 \theta - \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta \right) \end{aligned} \quad (24)$$

It should be noted that the Eqs. (23) have been in principle defined in [3] but in the general form through transition relation among the neutrino bases (the final form like that of Eqs. (23) was not established).

#### 4. Discussion

According to the aims of the paper let us now confront the obtained effective Hamiltonian of interaction with the magnetic field with the one typically written straight in the flavor basis:

$$H_B^f = \begin{pmatrix} \mu_{ee} \frac{B_{\parallel}}{\gamma} & \mu_{ee} B_{\perp} & 0 & \mu_{e\mu} B_{\perp} \\ \mu_{ee} B_{\perp} & -\mu_{ee} \frac{B_{\parallel}}{\gamma} & \mu_{e\mu} B_{\perp} & 0 \\ 0 & \mu_{e\mu} B_{\perp} & \mu_{\mu\mu} \frac{B_{\parallel}}{\gamma} & \mu_{\mu\mu} B_{\perp} \\ \mu_{e\mu} B_{\perp} & 0 & \mu_{\mu\mu} B_{\perp} & -\mu_{\mu\mu} \frac{B_{\parallel}}{\gamma} \end{pmatrix}, \quad (25)$$

where  $\gamma$  is the common neutrino gamma-factor.

First of all, we can ascertain that the structure of the obtained expression (22) is consistent with the “standard” Hamiltonian (25). At that, the magnitudes (23) well correspond to neutrino magnetic moments in flavor basis. The magnitudes (24), however, do not coincide with the due structures in (25), but resemble them. In the limit of ultrarelativistic neutrinos all the values for  $\gamma_{11}$ ,  $\gamma_{22}$  and  $\gamma_{12}$  in (24) obviously become equal leading to complete correspondence. The presence of these magnitudes in the Hamiltonian means that the spin-flavor oscillations depend on neutrino masses – an interesting feature, though useless due to the gamma-factor suppression.

Also, in general, all the comments made at the end of Section 2 remain true given that the second item from there is to be read with using of  $\mu'_{e\mu}$  magnitude and is concerned the mixing between flavor neutrino states. To make the meaning of this item more clear it is interesting

to observe that purely hypothetically, for the very specific set of parameters which we will not discuss here, the Hamiltonian structure can be

$$\begin{pmatrix} a & 0 & b & 0 \\ 0 & -a & 0 & -b \\ b & 0 & c & 0 \\ 0 & -b & 0 & -c \end{pmatrix}, \quad (26)$$

so that neutrino states with different flavor and same chirality decouple and form subsystems independently mixed by the field. For example, one would have two states  $(\nu_e^L, \nu_\mu^L)$  mixed in accordance with the equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e^L \\ \nu_\mu^L \end{pmatrix} = - \begin{pmatrix} \tilde{\mu}_{ee} \frac{B_{\parallel}}{\gamma_{ee}} & \tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} \\ \tilde{\mu}_{e\mu} \frac{B_{\parallel}}{\gamma_{e\mu}} & \tilde{\mu}_{\mu\mu} \frac{B_{\parallel}}{\gamma_{\mu\mu}} \end{pmatrix} \begin{pmatrix} \nu_e^L \\ \nu_\mu^L \end{pmatrix}. \quad (27)$$

In this way, the system would change flavor and preserve chirality i.e. would oscillate in flavor in the usual sense by means of magnetic field.

In conclusion we would like to summarize that going into the physical neutrino basis gives some new possibilities for the neutrino spin-flavor oscillation phenomenon to reveal its features. On this basis a new (secondary) oscillation channels can be established and a connection with the fundamental electromagnetic neutrino parameters can be achieved.

### Acknowledgments

One of the authors (A.S.) is thankful to Nicolao Fornengo and Carlo Giunti for the kind invitation to participate in the 14th International Conference on Topics in Astroparticle and Underground Physics. The work on this paper was partially supported by the Russian Basic Research Foundation grants no. 14-22-03043-off-m, 15-52-53112-gfen and 16-02-01023-a.

### References

- [1] Fukuda Y *et al* (Super-Kamiokande Collaboration) 1998 *Phys. Rev. Lett.* **81** 1562; Ahmad Q R *et al* (SNO collaboration) 2001 *Phys. Rev. Lett.* **87** 071301; Ahmad Q R *et al* (SNO collaboration) 2002 *Phys. Rev. Lett.* **89** 011301
- [2] Fujikawa K and Shrock R 1980 *Phys. Rev. Lett.* **45** 963
- [3] Giunti C and Studenikin A 2015 *Rev. Mod. Phys.* **87** 531 (*Preprint* arXiv:1403.6344 [hep-ph])
- [4] Cisneros A 1971 *Astrophys. Space Sci.* **10** 87; Schechter J and Valle J 1981 *Phys. Rev.* **D 24** 1883; Voloshin M B, Vysotsky M I and Okun L B 1986 *Sov. Phys. JETP* **64** 446; Akhmedov E K 1988 *Sov. J. Nucl. Phys.* **48** 382; Lim C-S and Marciano W J 1988 *Phys. Rev. D* **37** 1368