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Medium-induced radiation beyond the Poisson approximation

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Abstract. We present a novel technique for the calculation of the probability for emission of gluons, radiated from a high- p_T quark, through a medium (QGP). Our work is an extension of the maximal helicity violating method to compute the non-Abelian correction for 2 gluon emissions.

1. Introduction

The experiments conducted at RHIC (relativistic heavy ion collision) and LHC (large hadron collider) on nucleus-nucleus collisions, also known as heavy ion collisions, provide a new arena to understand the properties of QCD (quantum chromodynamics). One of the greatest challenges in QCD is the confinement phenomenon, which explains why quarks and gluons cannot be isolated. However, data taken in heavy ion collision experiments provides strong evidence that deconfined QCD matter, also known as QGP (quark gluon plasma), has been produced [1]. Note that in this QGP state, quarks and gluons are able to move freely for a very short time.

In nucleus-nucleus collisions the three main experimental observations that confirm the observation of QGP are: 1) *elliptic flow* which is the anisotropic collective flow of a fireball matter (QGP) measured in non-central heavy ion collisions[2]; 2) *collective flow* in which heavy quarks, charm and bottom, are observed to be dragged along by the collective expansion that leads to a large elliptic flow [3]; 3) *jet quenching* where high energetic partons passing through the medium lose large amounts of energy [4].

In this work we are going to focus on the theoretical understanding of the third observation, jet energy loss. Jets are known as the manifestation of high- p_T partons produced in nucleus-nucleus collisions. In the presence of QGP medium these high- p_T partons experience multiple interactions and lose energy through scattering, and mainly through gluon radiation prior to fragmentation and hadronisation[5, 6, 7]:

$$\text{Jet}(E) \longrightarrow \text{Jet}(E - \Delta E) + \text{Radiation}(\Delta E). \quad (1)$$

In practice we cannot detect these radiations, since they are either emitted with a very low energy compared to the resolution of the detectors (*soft*), or are emitted at a very small angle and cannot be resolved from the parent jet (*collinear*). But on the theoretical side the soft-collinear behavior



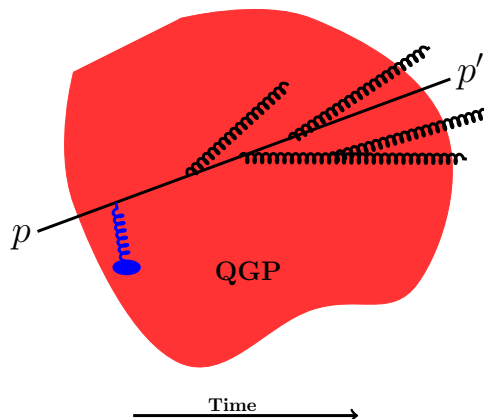


Figure 1: A parton radiating through a medium.

of these radiations provides all the necessary tools to compute the probability distribution of gluon emission. However, the non-Abelian nature of QCD complicates the computation, also the distribution is often approximated as a Poisson distribution, even if such approximation is equivalent to neglecting the gluon interferences. Therefore the main goal of this work is to present a new technique to compute the non-Abelian correction (gluon interferences) to the Poissonian approximation,

$$dP_{\text{improved}} = dP_{\text{poisson}} + dP_{\text{correction}}. \quad (2)$$

As such we want to know if the non-Abelian effect is negligible or not? For that we consider a light quark, which interacts softly with a medium by a single gluon exchange, and study the correlation between emitted gluons. In the next section we give a short introduction to the maximal helicity violating (MHV) technique that we are going to use, then we review the single emission using the new set of variables in MHV, and finally we compute the non-Abelian correction for the two gluon distribution in order to construct the correlation function.

2. Maximal Helicity Violating

This section is a short introduction for the MHV techniques that will be used through out this work in order to compute scattering amplitudes [10, 11, 12]. As physicists, it is very natural to use Feynman diagram techniques to compute amplitudes, but we know that the computation becomes complicated as the number of particles increases. In the table below we can see how the number of diagrams increases with the number of gluons emitted, for quark anti-quark production:

# Gluon	1	2	3	...	n
# Diagram	2	8	48	...	$2^n n!$

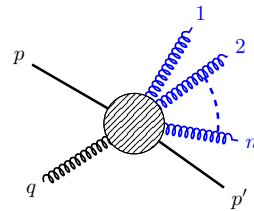
However it turns out the analytical expression of the amplitudes are extremely simple using a different set of variables, the spinor variables. It is even simpler when the conservation of helicity is maximally violated, i.e. $\Delta h = |h_i - h_f|$ is maximal. The spinor helicity formalism is one of the main ingredients of the MHV techniques, and in this formalism the momenta will be represented with a two by two matrix given by $P^{a\dot{a}} = (\sigma_\mu)^{a\dot{a}} p^\mu$. The invariant mass of a given particle will be the determinant of the 2×2 matrix that represents its momentum. For massless particles, since the invariant mass is zero, the momenta can be parametrized as follows

$$P^{a\dot{a}} = (\sigma_\mu)^{a\dot{a}} p^\mu = \lambda^a \tilde{\lambda}^{\dot{a}}, \quad (3)$$

where λ and $\tilde{\lambda}$ are called the spinor helicity variables and transform, respectively, as left and right handed spinors. Using these new variables we can define two different Lorentz invariant products that are both antisymmetric:

$$\langle 12 \rangle = \epsilon_{ab} \lambda_1^a \lambda_2^b \quad \text{and} \quad [12] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_1^{\dot{a}} \tilde{\lambda}_2^{\dot{b}}. \quad (4)$$

These products, angle and square brackets, can be related to the four vector scalar product as $2p_1 \cdot p_2 = \langle 12 \rangle [12]$. In terms of these new variables the MHV amplitude corresponding to an excited quark that absorbs a gluon from a medium and radiates n gluons is given by the partial amplitude,



$$= g^{n+1} \frac{\langle pq \rangle^3 \langle qp' \rangle}{\langle pp' \rangle \langle p'q \rangle \langle q1 \rangle \langle 12 \rangle \dots \langle np \rangle}. \quad (5)$$

The full amplitude is obtained by summing all the permutations of the gluon indices over the color string basis $(T_{a_1} T_{a_2} \dots T_{a_n})$, which is known as color kinematic decomposition of QCD.

3. Review of a single gluon emission

Generally speaking the momentum distribution of multiple radiations is obtained in two steps. First, we factorize the soft-collinear behavior from the Born amplitude using the following property of scattering amplitudes

$$\mathcal{M}_{n\text{-gluon}} = \mathcal{M}_{\text{born}} \times J_n(\{k_i\}), \quad (6)$$

where $\mathcal{M}_{\text{born}}$ is the Born amplitude and J_n is called the eikonal factor. The distribution will be given by the absolute square of J_n . The case of single gluon emission is a well known result, so in this section we are not going to rederive that computation, instead we are going to use the known result and translate it in the language of spinor helicity so the reader will be familiarized with the spinor variables. Recall that the eikonal factor for the single emission is given by $J_1(k_1) \sim j_\mu(k_1) \varepsilon^\mu(k_1)$ [13, 14], where $j^\mu(k_1)$ is the emission current and $\varepsilon^\mu(k_1)$ the associated gluon polarization vector. The probability distribution is defined as

$$\frac{dP^{(1)}}{d\Phi_1} = \frac{C_A}{2} |j_\mu(k_1) \varepsilon^\mu(k_1)|^2 \quad (7)$$

where $d\Phi_1$ is the differential phase space of a single emission of momentum k_1 , and C_A is the Casimir operator in the adjoint representation of $SU(3)$. Let us now use the spinor helicity formalism, where any four vector V^μ will be represented by a two by two matrix $V^{a\dot{a}}$ that carries a left and right handed spinor indices. In the same way the polarization vector $\varepsilon^\mu(k_1)$ are represented as

$$\varepsilon_-^{a\dot{a}}(k_1, r) := \frac{\lambda_1^a \tilde{\lambda}_r^{\dot{a}}}{[1r]} \quad \text{and} \quad \varepsilon_+^{a\dot{a}}(k_1, r) := \frac{\lambda_r^a \tilde{\lambda}_1^{\dot{a}}}{\langle 1r \rangle}, \quad (8)$$

where $\varepsilon_\pm^{a\dot{a}}(k_1, r)$ is the gluon polarization associated to a gluon with momentum k_1 and helicity $h = \pm 1$. The reference momentum $r^{a\dot{a}} = \lambda_r^a \tilde{\lambda}_r^{\dot{a}}$ is arbitrary, where the two different choices of reference are related by a gauge transformation. For the case of the emission current we have

$$j^\mu(k_1) = \frac{p^\mu}{2p \cdot k_1} - \frac{q^\mu}{2q \cdot k_1} \longrightarrow j^{a\dot{a}}(k_1) = \frac{\lambda_p^a \tilde{\lambda}_p^{\dot{a}}}{\langle p1 \rangle [p1]} - \frac{\lambda_q^a \tilde{\lambda}_q^{\dot{a}}}{\langle q1 \rangle [q1]}, \quad (9)$$

where $p^\mu \rightarrow \lambda_p^a \tilde{\lambda}_p^{\dot{a}}$ is the momentum of the highly energetic quark, and $q^\mu \rightarrow \lambda_q^a \tilde{\lambda}_q^{\dot{a}}$ is the momentum of the gluon from the medium. For the emission of positive helicity gluons the eikonal factor is equal to

$$j_{a\dot{a}}(k_1) \varepsilon_+^{a\dot{a}}(k_1, r) = \frac{\langle pq \rangle}{\langle p1 \rangle \langle 1q \rangle}, \quad (10)$$

and as we can see the left hand side of the equation depends on the reference momentum r , however the right hand side doesn't, that is, the eikonal factor is a gauge invariant quantity. In the next section we will use the MHV formula from the previous section to fully compute the two gluon emission distribution, and show that the Poisson approximation is only valid in some region of the phase space.

4. Non-Abelian correction for the two gluon emissions

To compute the full amplitude in the MHV, we need to perform a sum over the permutation of gluon indices weighted by the color string bases in which we apply the factorization. After computation the eikonal factor, J_2 , can be written in two terms, which are respectively symmetric and antisymmetric under the permutation of 1 and 2,

$$J(1, 2) = C_{12}^s J^s(1, 2) + C_{12}^a J^a(1, 2). \quad (11)$$

The symmetric part contains the independent emission, as in the Poisson approximation, and the antisymmetric contains information on the gluon interference that breaks the Poisson approximation,

$$|J_2(1, 2)|_{\text{new}}^2 = \frac{1}{4} [3 + F(1, 2)] |J_2(1, 2)|_{\text{poisson}}^2. \quad (12)$$

The function $F(1, 2)$ is what we call the non-Abelian correction, or gluon interference. The fact that this correction is scale invariant, i.e. conformal, allows us to express F only in terms of angles

$$F(\theta_1, \theta_2, \phi) = \frac{1 - \cos \theta_1 \cos \theta_2}{1 - \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \Delta \phi}, \quad (13)$$

θ_1 and θ_2 are the emission angles of the two gluons, and $\Delta \phi$ is the separation angle between the two emission planes, as shown in Figure 2.

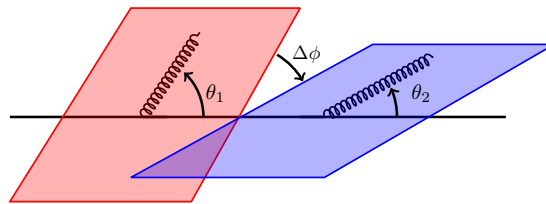


Figure 2: Two gluons emitted in two different planes.

We can also compute the two particle correlation function $C(1, 2)$ between the two emitted gluons as in [15]. And in order to simplify the expression let us introduce the follow change of variables: $\eta_i = -\ln [\tan(\theta_i/2)]$, here η_i is the analogue to the pseudorapidity associated to the i -th gluon in which the quark line direction is taken as the beam direction to define η . The correlation is given by

$$C(1, 2) = \frac{|J_2|_{\text{new}}^2 - |J_2|_{\text{poisson}}^2}{|J_2|_{\text{poisson}}^2} = \frac{1}{4} \frac{\cos \Delta \phi}{\cosh \Delta \eta - \cos \Delta \phi}, \quad (14)$$

where $\Delta\eta = \eta_1 - \eta_2$ the relative difference between the pseudorapidity of the two gluons. In Figure 3 we can see that the two gluons are strongly correlated around $(\Delta\eta = 0, \Delta\phi = 0)$, where the correlation tends to zero as the separation angle $\Delta\eta$ increases. However, the correlation also decreases as $\Delta\phi$ increases up to $\pi/2$, then it grows negatively up to π , then increases again.

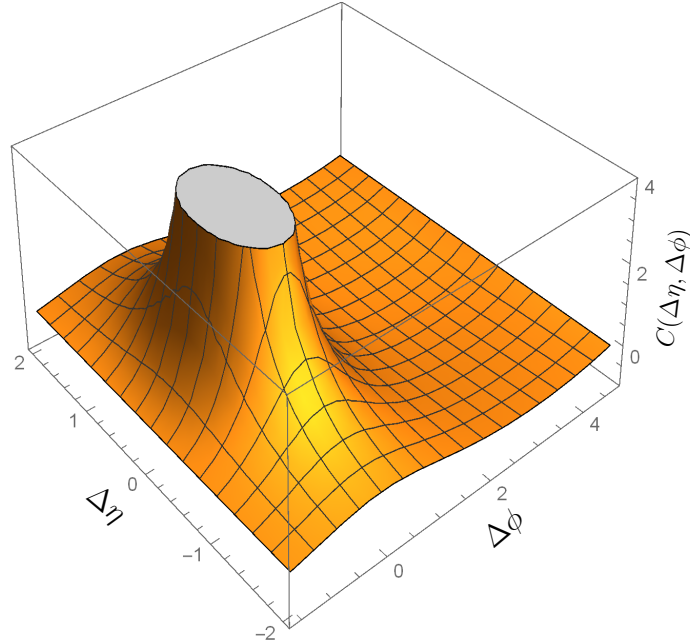


Figure 3: Two particle correlation function.

5. Conclusion

To conclude let us remind ourselves that we want to improve the momentum distribution for emitting soft-collinear gluons for a quark passing through a QGP. For that we performed a full QCD calculation using the MHV techniques in order to avoid diagrammatic complications. And for the two gluon emission case we computed the non-Abelian piece $F(1, 2)$, which is a correction to the Poissonian distribution and depends only on three variables: the two emission angles and the angle between the emission planes.

We also showed that the non-Abelian correction can be translated into two particle correlations which depend only on the separation angle $\Delta\phi$, and the pseudorapidity separation $\Delta\eta$ of the two gluons. We saw that the correlation decays as the two gluons are separated in η and ϕ , and the distribution tends to be Poisson. However the two particles are strongly correlated in η , and since we are in the situation where $\theta_1 \sim \theta_2$ (collinear behavior), then we can say that the non-Abelian effect cannot be suppressed in the computation of the momentum distribution of the soft-collinear gluon radiation. This work can be extended in many different ways, one of them is to study the non-flow v_2 generated by the correlation, and see how much of the non-flow contributes in the measured v_2 in pp -collisions.

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