Crystalline Undulator: Current Status and Perspectives

A. Kostyuk, A. Korol, A. Solov'yov and W. Greiner

Abstract Recent advances in the theory of novel sources of hard electromagnetic radiation,—a crystalline undulator (CU) and a Crystalline Undulator based Laser (CUL), are reviewed. The operating principle of CU is based on the channeling phenomenon. Channeling takes place if a particle enters a crystal at small angle to major crystallographic planes (or axes). The particle becomes confined by the planar or axial potential and move preferably along the plane or axis following its shape. If the planes or axes are periodically bent, the particles move along nearly sinusoidal trajectories. Similarly to what happens in an ordinary undulator, relativistic charged particles radiate electromagnetic waves in the forward direction. The advantage of CU is that due to extremely strong electrostatic fields inside the crystal the particles are steered much more effectively than by the field of the most advanced superconductive magnets. This allows one to make the period of CU two or even three orders of magnitude smaller then that of the conventional undulator. As a result the frequency of the radiation can reach the hard X-ray and gamma ray range. Similarly as it takes place in an ordinary free electron laser (FEL), the radiation becomes more powerful and coherent if the density of the particle beam is modulated along the beam direction with the period equal to the wavelength of the produced radiation.

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1 Introduction

The aim of this chapter is to review recent results in the theory of the crystalline undulator—a novel source of high energy photons. The *feasibility* of this idea was explicitly demonstrated in [1, 2]. The history of the idea as well as newer results was reviewed in [3].

The term 'crystalline undulator' stands for a system which consists of two essential parts: (a) a crystal whose crystallographic planes are bent periodically, and (b) a bunch of ultra-relativistic positively charged particles undergoing planar channeling in the crystal (see Fig. 1). In such a system there appears, in addition to a well-known channeling radiation, the radiation of an undulator type which is due to the periodic motion of channeling particles which follow the bending of the crystallographic planes.

The (yz)-plane in Fig. 1 is a cross section of a periodically bent crystal (PBCr), and the z-axis represents the beam direction. Two sets of filled circles denote the nuclei which belong to the periodically bent neighbouring planes spaced by the interplanar distance d. The planes form a periodically bent channel whose shape is defined by $y_B(z) = a\cos(2\pi z/\lambda_u)$. The amplitude of the bending, a, is defined as a maximum displacement of the deformed midplane (the thin dashed line) from its position in a straight crystal. The quantity λ_u stands for a spatial period of the bending. The quantities d, a and λ_u satisfy strong double inequality: $d \ll a \ll \lambda_u$. Typically $d \sim 10^{-8}$ cm, $a \sim 10 \dots 10^2\, d$, and $a \sim 10^{-5} \dots 10^{-4}\, \lambda_u$.

The operational principle of a CU does not depend on the type of a projectile. Provided certain conditions are met (see Sect. 2), the particles, injected into the crystal, will undergo channeling in PBCr. Thus, the trajectory of a particle contains two elements, which are illustrated by Fig. 1 where the solid wavy line represents

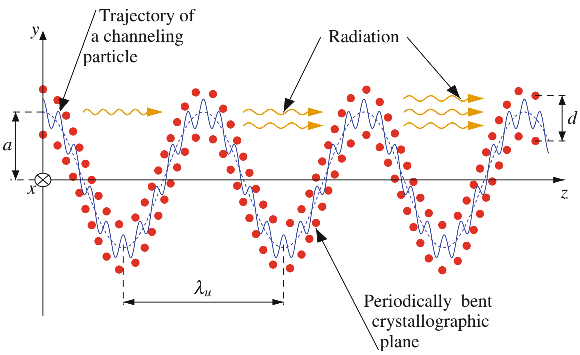

Fig. 1 Schematic representation of the crystalline undulator. *Closed circles* mark the atoms belonging to two neighbouring crystallographic planes (separated by the interplanar distance d) which are periodically bent with period λ_u and amplitude a (the y- and z-scales are incompatible!). *Thin solid line* illustrates the trajectory of the particle, which propagates along the centerline (the undulator motion) and, simultaneously, undergoes channeling oscillations

the trajectory of the particle. Firstly, there are oscillations inside the channel due to the action of the interplanar potential,—the channeling oscillations. This mode is characterized by a frequency $\Omega_{\rm ch}$ dependent on the projectile type, energy, and the parameters of the interplanar potential. Secondly, there are oscillations caused by the periodicity of the bent channel,—the undulator oscillations, whose frequency is $\omega_0 \approx 2\pi c/\lambda_{\rm u}$ (c is the velocity of light which approximately is the velocity of an ultra-relativistic particle).

Spontaneous emission of photons which appears in this system is associated with both of these oscillations. Typical frequency of the emission due to the channeling oscillations is $\omega_{\rm ch} \approx 2\gamma^2\Omega_{\rm ch}$ where γ is the relativistic Lorenz factor $\gamma = \varepsilon/mc^2$. The undulator oscillations give rise to the photons with frequency $\omega \approx 4\gamma^2\omega_0/(2+K^2)$ where the quantity K, a so-called undulator parameter, is related to the amplitude and the period of bending, $K = 2\pi\gamma(a/\lambda_{\rm u})$.

If strong inequality $\omega_0 \ll \Omega_{\rm ch}$ is met than the frequencies of the channeling radiation (ChR) and the undulator radiation (UR) are also well separated, $\omega \ll \omega_{\rm ch}$.

The scheme presented in Fig. 1 leads also to the possibility of generating a stimulated emission of the FEL type. Thus, it is meaningful to discuss a novel source of electromagnetic radiation in hard X and gamma range,—a *Crystalline Undulator Laser* (CUL) [1–9]. Specific features of CUL as well as quantitative estimates of the parameters of stimulated emission are presented below in Sect. 5.

There is essential feature which distinguish a seemingly simple scheme presented in Fig. 1 from a conventional undulator based on the action of periodic magnetic field on the projectile. In the latter the beam of particles and the photon flux move in vacuum whereas in the proposed scheme they propagate through a crystalline media. The interaction of both beams with the crystal constituents makes the problem much more complicated from theoretical, experimental and technological viewpoints.

2 Feasibility of CU

The conditions, which must be met to treat a *CU as a feasible scheme* for devising a new source of electromagnetic radiation, are as follows:

$$\begin{cases} C = 4\pi^2\varepsilon a/U'_{\rm max}\lambda_{\rm u}^2 < 1 & -\text{stable channeling,} \\ d < a \ll \lambda_{\rm u} & -\text{large-amplitude regime,} \\ N = L/\lambda_{\rm u} \gg 1 & -\text{large number of periods,} \\ L \sim \min\Big[L_{\rm d}(C), L_{\rm a}(\omega)\Big] & -\text{account for dechanneling and photon attenuation,} \\ \Delta\varepsilon/\varepsilon \ll 1 & -\text{low energy losses.} \end{cases} \tag{1}$$

The formulated conditions are of a general nature since they are applicable to any type of a projectile undergoing channeling in periodically bent channel (PBCh). Their application to the case of a specific projectile and/or a crystal channel allows one to analyze the feasibility of the CU by establishing the ranges of ε, a, $\lambda_{\rm u}$, L, N and $\hbar\omega$ which can be achieved.

- A *stable channeling* of a projectile in a PBCh occurs if the maximum centrifugal force F_{cf} is less than the maximal interplanar force U'_{max}, i.e. $C = F_{cf}/U'_{max} < 1$. Expressing F_{cf} through the energy ε of the projectile, the period and amplitude of the bending one formulates this condition as it is written in (1).
- The operation of a CU should be considered in the *large-amplitude regime*. The limit $a/d > 1$ accompanied by the condition $C \ll 1$ is mostly advantageous, since in this case the characteristic frequencies of UR and ChR are well separated: $\omega_u^2/\omega_{ch}^2 \sim Cd/a \ll 1$. As a result, the channeling motion does not affect the parameters the UR, the intensity of which can be comparable or higher than that of ChR. A strong inequality $a \ll \lambda_u$ ensured elastic deformation of the crystal.
- The term "undulator" implies that the *number of periods, N, is large*. Only then the emitted radiation bears the features of an UR (narrow, well-separated peaks in spectral-angular distribution). This is stressed by the third condition.
- A CU essentially differs from a conventional undulator, in which the beams of particles and photons move in vacuum, In CU the both beams propagate in crystalline medium and, thus, are affected by *the dechanneling and the photon attenuation*. The dechanneling effect stands for a gradual increase in the transverse energy of a channeled particle due to inelastic collisions with the crystal constituents [10]. At some point the particle gains a transverse energy higher than the planar potential barrier and leaves the channel. The average interval for a particle to penetrate into a crystal until it dechannels is called the dechanneling length, L_d. In a straight channel this quantity depends on the crystal, on the energy and the type of a projectile. In a PBCh bent channel there appears an additional dependence on the parameter C. The intensity of the photon flux, which propagates through a crystal, decreases due to the processes of absorption and scattering. The interval within which the intensity decreases by a factor of e is called the attenuation length, $L_a(\omega)$. This quantity is tabulated for a number of elements and for a wide range of photon frequencies (see, e.g., Ref. [11]).
  The fourth condition in (1) takes into account severe limitation of the allowed values of the length L of a CU due to the dechanneling and the attenuation.
- Finally, let us comment on the last condition, which is of most importance for light projectiles, positrons and electrons. For sufficiently large photon energies ($\hbar\omega \gtrsim 10^1 \dots 10^2$ keV depending on the type of the crystal atom) the restriction due to the attenuation becomes less severe than due to the dechanneling effect. Then, the value of $L_d(C)$ effectively introduces an upper limit on the length of a CU. Since for an ultra-relativistic particle $L_d \propto \varepsilon$ (see, e.g., [12]), it seems natural that to increase the effective length one can consider higher energies. However, at this point another limitation manifests itself [13]. The coherence of UR is only possible when the energy loss $\Delta\varepsilon$ of the particle during its passage through the undulator is small, $\Delta\varepsilon \ll \varepsilon$. This statement, together with the fact, that for ultra-relativistic electrons and positrons $\Delta\varepsilon$ is mainly due to the photon emission, leads to the conclusion that L must be much smaller than the radiation length L_r, the distance over which a particle converts its energy into radiation.

For a positron-based CU a thorough analysis of the system (1) was carried out for the first time in Refs. [1–3, 13–15]. Later on, the feasibility of the CU utilizing the planar channeling of electrons was demonstrated [16]. Recently, similar analysis was carried out for heavy ultra-relativistic projectiles (muon, proton, ion) [17].

3 Positron-Based Crystalline Undulator

To illustrate the crystalline undulator radiation phenomenon, let us consider the spectra of spontaneous radiation emitted during the passage of positrons through PBCr.

The calculated spectra of the radiation emitted in the forward direction (with respect to the z-axis, see Fig. 1) in the case of $\varepsilon = 0.5$ GeV planar channeling in Si along (110) crystallographic planes and for the photon energies from 45 keV to 1.5 MeV are presented in Fig. 2 [15]. The ratio a/d was varied within the interval

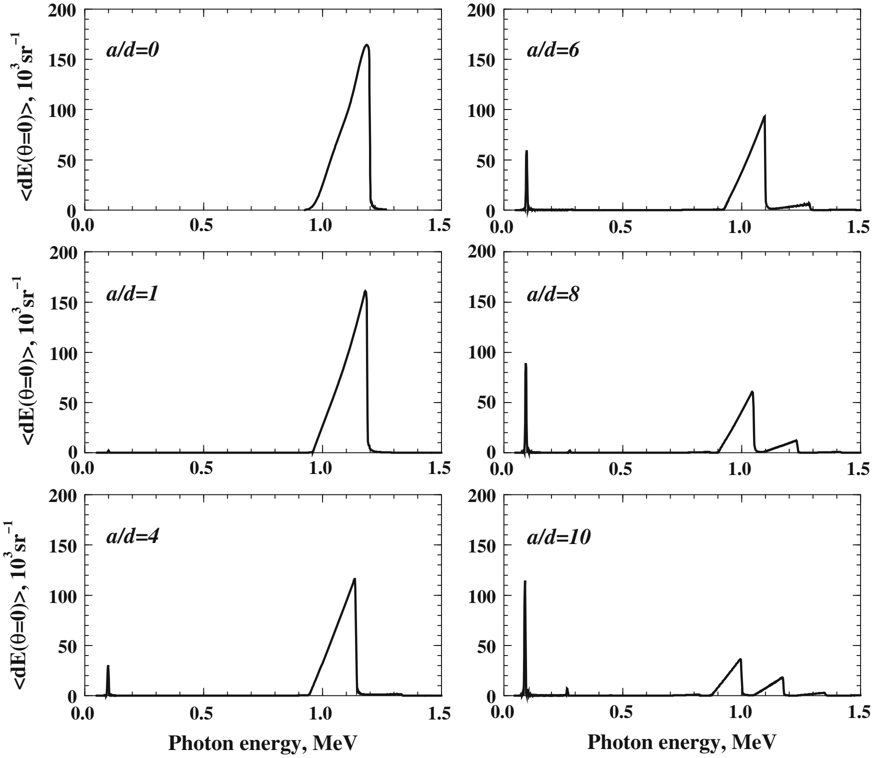

Fig. 2 Spectral distribution of the total radiation emitted in the forward direction ($\vartheta = 0°$) for $\varepsilon = 0.5$ GeV positron channeling in Si (110) crystallographic plane calculated at different a/d ratios. Other parameters are given in the text

$a/d = 0 \dots 10$ (the interplanar spacing is 1.92 Å). The case $a/d = 0$ corresponds to the straight channel. The period λ_u used for these calculations equals to 2.33×10^{-3} cm. The number of undulator periods and crystal length were fixed at $N = 15$ and $L = N\lambda_u = 3.5 \times 10^{-2}$ cm.

The spectra correspond to the total radiation, which accounts for the two mechanisms, the undulator and the channeling. They were calculated using the quasi-classical method [18, 19]. Briefly, to evaluate the spectral distribution the following procedure was adopted (for more details see [15, 20, 21]). First, for each a/d value the spectrum was calculated for individual trajectories of the particles. These were obtained by solving the relativistic equations of motion with both the interplanar and the centrifugal potentials taken into account. We considered two frequently used [22] analytic forms for the continuum interplanar potential, the harmonic and the Molière potentials calculated at the temperature $T = 150$ K to account for the thermal vibrations of the lattice atoms. The resulting radiation spectra were obtained by averaging over all trajectories. Figure. 2 correspond to the spectra obtained by using the Molière approximation for interplanar potential.

The first graph in Fig. 2 corresponds to the case of zero amplitude of the bending (the ratio $a/d = 0$) and, hence, presents the spectral dependence of the ordinary channeling radiation only. The asymmetric shape of the calculated channeling radiation peak, which is due to the strong anharmonic character of the Molière potential, bears close resemblance with the experimentally measured spectra [23]. The spectrum starts at $\hbar\omega \approx 960$ keV, reaches its maximum value at 1190 keV, and steeply cuts off at 1200 keV. This peak corresponds to the radiation into the first harmonic of the ordinary channeling radiation (see e.g. [24]), and there is almost no radiation into higher harmonics.

Increasing the a/d ratio leads to the modifications in the radiation spectrum. The changes which occur are: (i) the lowering of the channeling radiation peak, (ii) the gradual increase of the intensity of UR due to the crystal bending.

The decrease in the intensity of the channeling radiation is related to the fact that the increase of the amplitude a of the bending leads to lowering of the allowed maximum value of the channeling oscillations amplitude $a_{\rm ch}$ (this is measured with respect to the centerline of the bent channel) [13, 25]. Hence, the more the channel is bent, the lower the allowed values of $a_{\rm ch}$ are, and, consequently, the less intensive is the channeling radiation, which is proportional to $a_{\rm ch}^2$ [18].

The UR related to the motion of the particle along the centerline of the PBCh bent channel is absent in the case of the straight channel (the graph $a/d = 0$), and is almost invisible for comparatively small amplitudes (see the graph for $a/d = 1$). Its intensity, which is proportional to $(a/d)^2$, gradually increases with the amplitude a. For large a values ($a/d \sim 10$) the intensity of the first harmonic of the UR becomes larger than that of the ChR. The undulator peak is located at much lower energies, $\hbar\omega \approx 90$ keV, and has the width $\hbar\Delta\omega \approx 6$ keV which is almost 40 times less than the width of the peak of ChR.

4 Electron-Based Crystalline Undulator

Initially, it was proposed to use positron beams in the crystalline undulator. Such undulator has been considered in the previous section. Positively charged particles are repelled by the crystal nuclei and, therefore, they move between the crystal planes, where there are no atomic nuclei and the electron density is less than average. This reduces the probability of random collisions with the crystal constituents. Hence, the transverse momentum of the particle increases slowly and the particle travels a longer distance in the channelling regime and performs more undulator oscillations.

In contrast, negatively charged particles are attracted by the crystal nuclei and therefore they have to cross the crystallographic plane in the cause of the channeling oscillation. Therefore, the probability of random collisions of the projectile with the crystal constituents is strongly enhanced. The negative particles dechannel very quickly. Typically the dechanneling length for electrons is two orders of magnitude shorter then for positrons at the same conditions.

On the other hand, the electron beams are easier available and are usually of higher intensity and quality. Therefore, from the practical point of view, electron based crystalline undulator has its own advantages and deserves a thorough investigation.

It has been found [16] that an electron based crystalline undulator is also feasible. However, it requires the electron beam energy of a few tens of GeV.

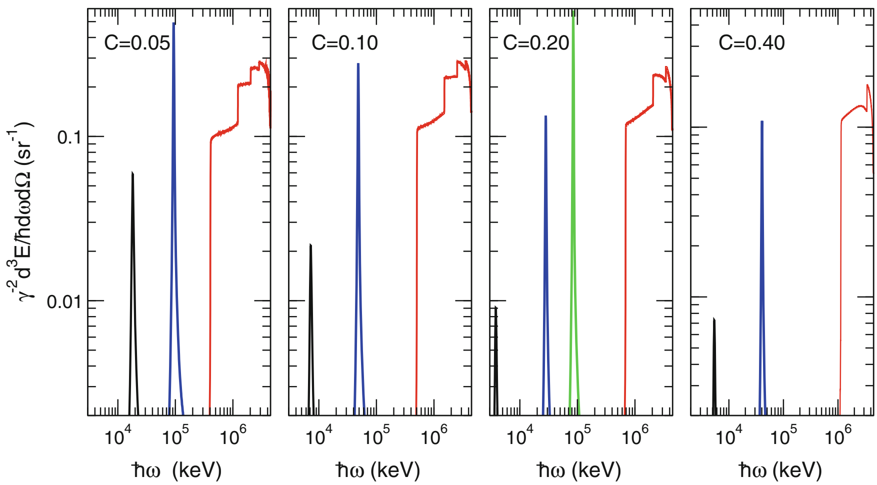

Fig. 3 Spectral distributions (scaled by γ^2) of UR and ChR (wide peaks) emitted in the forward direction by a 50 GeV electron in Si (111). Each graph corresponds to the indicated value of parameter C. Narrow peaks stand for the spectral distribution of UR in the vicinity of the fundamental harmonics for nine different undulators. In each graph the leftest narrow peak corresponds to $N_d L_d / \lambda_u = 5$, the second peak—to $N_d = 10$, and the third peak (only for C = 0.20)—to $N_d = 15$

The graphs in Fig. 3 illustrate that by changing parameters of the undulator one can vary the frequency and the peak intensity of the UR over wide ranges. However, it is important to compare these quantities with the characteristics of the ChR.

The wide peak in each graph stands for the spectral distribution of the ChR in the forward direction. Figure 3 clearly demonstrates that by tuning the parameters of bending it is possible to separate the frequencies of the UR from those of the ChR, and to make the intensity of the former comparable or higher than of the latter.

The present technologies enable one to construct the periodically bent crystalline structures with the required parameters. These include making regularly spaced grooves on the crystal surface either by a diamond blade [26, 27] or by means of laser-ablation [28], deposition of periodic Si_3N_4 layers onto the surface of a Si crystal [27], growing of $Si_{1-x}Ge_x$ crystals[29] with a periodically varying Ge content x [21, 30].

Similar to the case of a positron-based undulator [14], the parameters of high-energy electrons beams available at present [11] are sufficient to achieve the necessary conditions to construct the undulator and to create, on its basis, powerful radiation sources in the γ-region of the spectrum.

5 Crystalline Undulator Based Gamma Laser

The proposed gamma laser combines a crystalline undulator with a free electron laser (FEL) [8], see Fig. 4.

The first essential element of the apparatus is a CU in which the radiation with the wavelength λ is formed. The CU has to be manufactured in such a way that charged particles, when enter the crystal from the appropriate direction, move inside the crystal along the periodically bent planes or axes in channeling regime as it described in the previous sections. The parameters of the crystal bending, the period and the amplitude, have to be chosen to satisfy the resonance condition for the desirable wavelength λ of the produced radiation at given average particle energy ε of the beam in the FEL undulator. The crystal length in the beam direction has to be comparable to or smaller than the attenuation length of the radiation with the wavelength λ in the crystal material. Choosing the type of crystal and the plane or axis with largest *demodulation length* (see [4, 5] for the definition) is preferable: larger

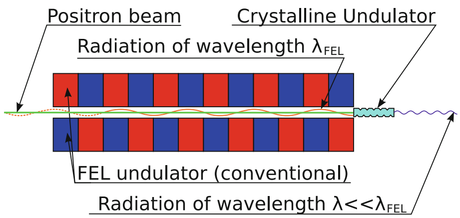

Fig. 4 A scheme of the crystalline undulator-based gamma laser

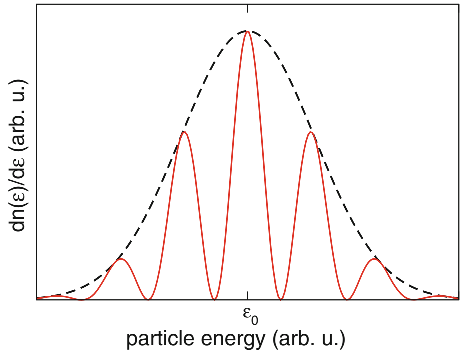

Fig. 5 The Gaussian energy distribution (*dashed curve*) and the layered energy distribution (*solid curve*) of the beam particles

number of undulator periods per demodulation length improves the performance of the device. The use of a positron beam is preferable over an electron beam.

The second key element is conventional FEL which must be tuned to the wavelength $\lambda_{\rm FEL}$ larger than the wavelength of the produced radiation: $\lambda_{\rm FEL} > \lambda$. The ratio $\lambda_{\rm FEL}/\lambda$ has to be an integer.

The PBCr is placed at the exit from the FEL and exposed to the particle beam that has traveled some distance inside the FEL undulator.

The beam source of FEL has to be modified in such a way that the distribution of the particles with respect to their energy becomes *layered*, i.e. the distribution has a number of maxima separated by minima (an example is shown in Fig. 5). The distances between the maxima has to be optimized to enhance the radiation of CUL at the wavelength λ.

It is known from the theory of the conventional undulator that its radiation becomes much more intense and less divergent if the particle density in the beam is modulated with the period approximately equal to the wavelength of the produced radiation [31]. This phenomenon is known as coherent emission and is used in free electron lasers [32–35]. It has been demonstrated recently [4, 5] that the initially modulated beam can preserve its modulation at sufficient depth while channeling in the crystal. Therefore, coherent emission takes place also in the crystalline undulator, provided that it is fed by a modulated particle beam whose modulation period is close to the wavelength of the produced radiation. This effect is utilized in the proposed apparatus.

In the proposed device, the free electron laser is used as a source of the modulated beam. It is tuned to the wavelength $\lambda_{\rm FEL}$ which is larger than the wavelength of the produced radiation λ: $\lambda_{\rm FEL} > \lambda$. According to the theory of FEL (see, e.g., [34, 35]), the particle beam while traveling through the FEL undulator becomes modulated (micro-bunched) with the period $\lambda_{\rm FEL}$.

If the energy distribution of the particles in the beam that enters the undulator of FEL is *layered*, the shape of micro-bunches will have maxima and minima whose position depends on the position of the maxima and minima in the layered particle distribution with respect to the energy (see Ref. [36] for the details). In this case, the

Fourier expansion of the dependence of particle density in the beam on the spatial coordinate along the beam direction contains also a higher harmonic with the period approximately equal to the distances between maxima (or minima) of the bunch shape. If the crystalline undulator is tuned to this harmonic, an enhanced coherent emission will be observed.

The proposed gamma laser can be used in scientific laboratories, in particular for nuclear physics and plasma physics laboratories. It may be used in medicine, e.g. as a diagnostic tool or for cancer therapy. It can be used for nondestructive analysis of isotope composition of various objects. Other applications can be found in future.

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