SLAC-PUB-1943
June 1977
(T)

NONRENORMALIZABLE INTERACTIONS AND EIGENVALUE CONDITION*

Tohru Eguchi

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

It is pointed out that a certain class of nonrenormalizable theories can be made renormalizable if a theory possesses an ultraviolet stable fixed point.

As an example, four-fermion theories of Nambu-Jona-Lasinio type are considered.

(Submitted to Phys. Rev. D)

*Work supported by the Energy Research and Development Administration.

It is well known that in a class of nonrenormalizable field theories, interesting collective phenomena occur. In particular, in Nambu-Jona-Lasinio type[1] four-fermion theories, one can show the existence of collective bosonic bound states by solving Bethe-Salpeter equations within a certain approximation. Thus these models are actually interacting theories of fermions and bosons despite the fact that the original Lagrangian contains only spinor fields.

Usually these four-fermion models are, of course, regarded as nonrenormalizable. In a case of current-current type self-interaction, however, it was known for some time that one can formally develop a renormalizable perturbation series.[2,3] Here the basic idea is to expand the theory in terms of induced coupling constants between collective bosons and fermions instead of the original four-fermion coupling constant. In the model of Ref. 2 and 3, a collective bound state, a photon, appears in the vector channel. When one expands the theory in terms of photon-fermion coupling constant, one finds that the perturbation series is renormalizable and obtains the same S-matrix as in quantum electrodynamics to all orders in perturbation theory.

Recently this type of renormalizability and correspondence of four-fermion models to certain renormalizable theories has been extended to other types of four-fermion interactions.[4] The original Nambu-Jona-Lasinio model, for instance, is shown to correspond to the linear σ-model.

In these works, however, the renormalizability of perturbation series is yet formal. Although the series has a finite number of superficially divergent vertices and all the ultraviolet infinities of the theory are amalgamated into field, mass and charge renormalizations, renormalized coupling constants are not automatically guaranteed to take finite and cut-off independent values. This is because in four-fermion theories induced coupling constants become independent of the original Fermi coupling constant G after renormalization and cannot be made finite and arbitrary by making G cut-off dependent in a prescribed way. It is well known that in the lowest order, Hartree-Fock, approximation induced Yukawa coupling constant behaves as $g_R^2 \sim G/G \ln \Lambda = 1/\ln \Lambda$ (Λ is the ultraviolet cut-off). It is possible to prove that this feature persists to all orders.

In this paper we shall show that there is a Gell-Mann-Low eigenvalue condition on these radiatively created charges. If a theory possesses an ultraviolet stable fixed point, they can take well-defined and cut-off independent values. In this case Nambu-Jona-Lasinio theories become completely independent of ultraviolet cut-off and equivalent to corresponding renormalizable models. On the other hand, if a theory does not possess a fixed point, cut-off dependence persists in created charges and they vanish in the limit of infinite cut-off. In this case, four-fermion models become a free field theory of collective bosons and fermions.

Let us first consider the Nambu-Jona-Lasinio model. The Lagrangian is given by

$$\mathscr{L} = \bar{\psi} i \gamma \partial \psi - \frac{G}{2} \left\{ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right\}, \tag{1}$$

where ψ is an iso-doublet spinor field. The description of collective bound states can be facilitated if we introduce collective variables σ and π and a new Lagrangian,

$$\mathscr{L}' = \bar{\psi} i \gamma \partial \psi - g \bar{\psi} (\sigma + i \gamma_5 \vec{\tau} \vec{\pi}) \psi - \frac{\delta\mu^2}{2} (\sigma^2 + \vec{\pi}^2) \tag{2}$$

where

$$G = \frac{g^2}{\delta\mu^2} \quad .$$

g is a bare induced Yukawa coupling constant and a term proportional to $\delta\mu^2$ is interpreted as a boson self-energy counterterm. The new Lagrangian $\mathscr{L}'$ has the same dynamical content as $\mathscr{L}$ since if we perform a path integration over σ and $\vec{\pi}$ in

$$\int d\psi d\bar{\psi} d\sigma\, d\vec{\pi}\ \exp\left\{i\left(\mathscr{L}'(\psi,\bar{\psi},\sigma,\vec{\pi}) + \bar{\eta}\psi + \bar{\psi}\eta\right)\right\} , \tag{3}$$

we obtain the original generating functional,[5]

$$W[\eta,\bar{\eta}] = \int d\psi d\bar{\psi} \exp\left\{i\left(\mathscr{L}(\psi,\bar{\psi}) + \bar{\eta}\psi + \bar{\psi}\eta\right)\right\} . \tag{4}$$

A renormalized perturbation expansion for $\mathscr{L}'$ can be formally set up if we add and subtract vertices for bound states,

$$\begin{aligned}\mathscr{L}' = &\left\{\bar{\psi}_R\, i\gamma\partial\psi_R - g_R\bar{\psi}_R(\sigma_R + i\gamma_5\vec{\tau}\vec{\pi}_R)\psi_R\right\} \\ &+ \left\{(Z_2 - 1)\bar{\psi}_R\, i\gamma\partial\psi_R - g_R(Z_1 - 1)\bar{\psi}_R(\sigma_R + i\gamma_5\vec{\tau}\vec{\pi}_R)\psi_R \right. \\ &\left. - \frac{\delta\mu^2}{2} Z_3(\sigma_R^2 + \vec{\pi}_R^2)\right\} ,\end{aligned} \tag{5}$$

$$\begin{aligned} = &\left\{\bar{\psi}_R\, i\gamma\partial\psi_R - g_R\bar{\psi}_R(\sigma_R + i\gamma_5\vec{\tau}\vec{\pi}_R)\psi_R \right. \\ &+ \frac{1}{2}\left((\partial_\mu\sigma_R)^2 + (\partial_\mu\vec{\pi}_R)^2\right) - \frac{1}{2}\mu_R^2(\sigma_R^2 + \vec{\pi}_R^2) \\ &\left. - \frac{\lambda_R}{4}\left(\sigma_R^2 + \vec{\pi}_R^2\right)^2\right\} \\ &+ \left\{(Z_2 - 1)\bar{\psi}_R\, i\gamma\partial\psi_R - g_R(Z_1 - 1)\bar{\psi}_R(\sigma_R + i\gamma_5\vec{\tau}\vec{\pi}_R)\psi_R \right. \\ &- \frac{1}{2}\left((\partial_\mu\sigma_R)^2 + (\partial_\mu\vec{\pi}_R)^2\right) + \frac{1}{2}\left(\mu_R^2 - \delta\mu^2 Z_3\right)\left(\sigma_R^2 + \vec{\pi}_R^2\right) \\ &\left. + \frac{\lambda_R}{4}\left(\sigma_R^2 + \vec{\pi}_R^2\right)^2\right\}\end{aligned} \tag{6}$$

Here the notations are standard. Five terms in the first curly bracket in Eq. (6)

are regarded as parts of the renormalized Lagrangian and those in the second bracket as counterterms. g_R and λ_R are renormalized induced coupling constants. Next let us compare the above expression with that of the linear σ-model,

$$- \mathscr{L}_\sigma = \bar{\psi}\, i\gamma\, \partial\psi - g\bar{\psi}\,(\sigma + i\gamma_5 \vec{\tau}\,\vec{\pi})\psi + \frac{1}{2}\left((\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2\right)$$

$$- \frac{1}{2}\mu^2\,(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \quad , \tag{7}$$

$$= \Big\{\bar{\psi}_R\, i\gamma\, \partial\psi_R - g_R \bar{\psi}_R(\sigma_R + i\gamma_5 \vec{\tau}\,\vec{\pi}_R)\,\psi_R + \frac{1}{2}\left((\partial_\mu \sigma_R)^2 + (\partial_\mu \vec{\pi}_R)^2\right)$$

$$- \frac{1}{2}\mu_R^2\left(\sigma_R^2 + \vec{\pi}_R^2\right) - \frac{\lambda_R}{4}\left(\sigma_R^2 + \vec{\pi}_R^2\right)^2\Big\}$$

$$+ \Big\{(Z_2 - 1)\,\bar{\psi}_R\, i\gamma\, \partial\psi_R - g_R(Z_1 - 1)\,\bar{\psi}_R(\sigma_R + i\gamma_5 \vec{\tau}\,\vec{\pi}_R)\psi_R +$$

$$+ \frac{1}{2}(Z_3 - 1)\left((\partial_\mu \sigma_R)^2 + (\partial_\mu \vec{\pi}_R)^2\right)$$

$$- \frac{1}{2}\left((Z_3 - 1)\,\mu_R^2 + Z_3 \delta\mu^2\right)\left(\sigma_R^2 + \vec{\pi}_R^2\right) - \frac{\lambda_R}{4}(Z_4 - 1)\left(\sigma_R^2 + \vec{\pi}_R^2\right)^2\Big\} \quad . \tag{8}$$

We notice that equations (6) and (8) differ in the coefficient of counterterms.

The elimination of ultraviolet infinities in the linear σ-model is well understood.[6] Since divergent parts of radiative corrections have a strictly chiral symmetric structure, they can be eliminated by a common wave-function renormalization factor Z_2 for σ and π and a common mass counterterm $\delta\mu$ if we choose appropriate subtraction points. Then renormalization factors are determined as a power series in g_R, λ_R and $\ln \Lambda$,

$$Z_i - 1 + (\text{divergent part of radiative corrections})_i = 0 \quad , \tag{9}$$

$$i = 1, 2, 3, 4 \quad ,$$

$$(Z_3 - 1)\,\mu_R^2 + Z_3 \delta\mu^2 + (\text{divergent part of self energy}) = 0 \quad . \tag{10}$$

On the other hand in the Nambu-Jona-Lasinio model the elimination of infinities has an unconventional feature. It is performed by

$$Z_i - 1 + (\text{divergent part of radiative corrections})_i = 0 \quad , \qquad i = 1, 2 \quad , \tag{11}$$

$$-1 + (\text{divergent part of radiative corrections})_i = 0 \quad , \qquad i = 3, 4 \quad , \tag{12}$$

$$-\mu_R^2 + Z_3 \delta\mu^2 + (\text{divergent part of self energy}) = 0 \quad . \tag{13}$$

Here the radiative corrections in (9), (10), and (11), (12), (13) have an identical structure since they are calculated using the same renormalized Lagrangian. In Nambu-Jona-Lasinio model there are only three parameters Z_1, Z_2 and $\delta\mu$ to absorb infinities and hence the above equations (11), (12), (13) impose nontrivial restrictions on g_R and λ_R. Comparing (9) and (12) we notice that the vanishing of Z factors,

$$\lim_{\Lambda\to\infty} Z_3(g_R, \lambda_R, \Lambda) = 0 \quad , \tag{14}$$

$$\lim_{\Lambda\to\infty} Z_4(g_R, \lambda_R, \Lambda) = 0 \quad , \tag{15}$$

is needed to eliminate infinities in Nambu-Jona-Lasinio theory. When these conditions are satisfied with cut-off independent values of g_R and λ_R, the model becomes free of ultraviolet cut-off. Furthermore since the same renormalized Lagrangian is used to compute Green's functions and S-matrix in both Nambu-Jona-Lasinio and σ-model these quantities will have an identical structure when expanded in power series in g_R and λ_R. From this the equivalence of two theories follows.

The above equations (14) and (15) are nothing but familiar compositeness conditions. This is only reasonable since σ and π are fermion-antifermion composites in our theory. Although numerous literature[7] exist on the compositeness or bootstrap condition Z=0, its physical significance has been somewhat obscured due to ultraviolet divergences in the case of a relativistic field theory. Next we analyze these conditions using renormalization group equations and the idea of Gell-Mann-Low fixed point. In the following we choose subtraction points at certain nonexceptional Euclidean momenta in order to avoid possible infrared problems.

It is well known that renormalization factors obey Callan-Symanzik equations,[8]

$$\left(\Lambda \frac{\partial}{\partial\Lambda} - \beta_g \frac{\partial}{\partial g_R} - \beta_\lambda \frac{\partial}{\partial\lambda_R} + 2\gamma_M\right) Z_3(g_R, \lambda_R, \Lambda) = 0 \quad , \tag{16}$$

$$\left(\Lambda \frac{\partial}{\partial\Lambda} - \beta_g \frac{\partial}{\partial g_R} - \beta_\lambda \left(\frac{\partial}{\partial\lambda_R} + \frac{1}{\lambda_R}\right) + 4\gamma_M\right) Z_4(g_R, \lambda_R, \Lambda) = 0 \quad . \tag{17}$$

Here $\gamma_M(g_R, \lambda_R)$ is the anomalous dimension of the meson (σ and $\vec{\pi}$) field. β_g and β_λ are the β-functions related to the scale transformation of g_R and λ_R respectively.

First let us consider a possibility that β_g and β_λ have an ultraviolet stable fixed point at $(g_R^\infty, \lambda_R^\infty)$ in two-dimensional coupling constant plane,

$$\beta_g(g_R^\infty, \lambda_R^\infty) = \beta_\lambda(g_R^\infty, \lambda_R^\infty) = 0 \quad , \tag{18}$$

and renormalized coupling constants are equal to these values,

$$g_R = g_R^\infty \quad , \qquad \lambda_R = \lambda_R^\infty \quad . \tag{19}$$

Then using Callan-Symanzik equations we obtain an exponentiation of $\ln\Lambda$,

$$Z_3(g_R, \lambda_R, \Lambda) \sim \Lambda^{-2\gamma_M(g_R^\infty, \lambda_R^\infty)} \quad , \tag{20}$$

$$Z_4(g_R, \lambda_R, \Lambda) \sim \Lambda^{-4\gamma_M(g_R^\infty, \lambda_R^\infty)} \quad . \tag{21}$$

Since $\gamma_M(g_R^\infty, \lambda_R^\infty)$ is non-negative due to unitarity,[9] it follows that,[10]

$$\lim_{\Lambda\to\infty} Z_3 = 0 \quad , \tag{22}$$

$$\lim_{\Lambda\to\infty} Z_4 = 0 \quad . \tag{23}$$

Thus $g_R = g_R^\infty$, $\lambda_R = \lambda_R^\infty$ is a solution to our bootstrap condition.

Actually the above assumption (19) can be relaxed significantly. As is well known in the renormalization group analysis, as far as g_R and λ_R lie within a domain of attraction of the fixed point there are so-called running coupling constants which interpolate between (g_R, λ_R) and $(g_R^\infty, \lambda_R^\infty)$ in the coupling constant plane. When Callan-Symanzik equations are solved in terms of them, we obtain a minor correction to (20) and (21) which leaves our result (22) and (23) left unchanged. Thus the bootstrap condition can be solved for a certain range of values for g_R and λ_R if the theory possesses an ultraviolet stable fixed point.[11]

In this case we obtain an interesting phenomenon; although we start with a theory of a spinor field and dimensional constant G, we arrive at a theory of bosons and fermion with two-dimensionless numbers g_R and λ_R as well as a dimensional constant μ_R. It appears that the dimensional constant G has somehow been traded for μ_R. On the other hand parameters g_R and λ_R are independent of G and are interpreted as being radiatively created. They are determined to be a solution to bootstrap conditions, however, these equations do not completely fix them and allow them to take values within a certain range. The theory exhibits a scaling behavior at high energy with anomalous dimensions.

Such a mechanism seems to be common to most of the models for dynamical bound states. For instance in the models of Refs. 12 and 13, the analysis is based on an assumed existence of a power-behaved, nonperturbative solution to Schwinger-Dyson integral equations. The residue of a bound state becomes cut-off independent when a Bethe-Salpeter kernel possesses a certain power behavior. All of this will happen if a theory has a fixed point.

On the other hand if a fixed point is absent in the theory, cut-off dependence persists in g_R and λ_R. In this case renormalization group equation is useless and we have to directly inspect the expression of Z factors in terms of g_R, λ_R and $\ln \Lambda$. In the n-th order perturbation theory Z factors have a structure,

$$Z = 1 + \left(C_{11} f + \ldots + C_{1n} f^n\right) \ln \Lambda + \ldots + C_{n1} f^n (\ln \Lambda)^n \quad , \tag{24}$$

where f is either g_R^2 or λ_R. We notice that bootstrap conditions imply a cut-off dependence of g_R and λ_R as,

$$g_R^2 \sim \frac{1}{\ln \Lambda} \quad , \qquad \lambda_R \sim \frac{1}{\ln \Lambda} \quad , \tag{25}$$

in each order of perturbation theory. Hence both vanish as $\Lambda \to \infty$. Thus we obtain a trivial free-field result. Therefore the eigenvalue is a condition to support nontrivial values for radiatively generated charges.

Apparently the above result applies to other types of four-fermion theories as well. Unfortunately in most of nongauge Yukawa theories the origin of the coupling constant plane is ultraviolet unstable.[14] In these cases we may not be able to discuss fixed points within the realm of weak-coupling perturbation expansion. On the other hand if we let non-Abelian gauge field couple to our theories, it is possible to stabilize the origin. The analysis seems feasible in this case though it becomes complicated due to gauge-dependence of renormalization factors.

The above results become somewhat modified when one considers a four-fermi theory where a gauge field appears as collective bound states. Here the simplest example is the Abelian model of Refs. (2) and (3),

$$\mathscr{L} = \bar{\psi}\, i\gamma\,\partial\psi - m\bar{\psi}\psi - \frac{G}{2}\,(\bar{\psi}\,\gamma_\mu\psi)^2 \quad . \tag{26}$$

Using the same technique as before we can introduce a collective variable A_μ and a new Lagrangian

$$\mathscr{L}' = \bar{\psi}\, i\gamma\,\partial\psi - m\bar{\psi}\psi - e\bar{\psi}\gamma_\mu\psi A^\mu + \frac{\delta\mu^2}{2}\,A_\mu A^\mu \quad . \tag{27}$$

Here the term proportional to $\delta\mu^2$ is again interpreted as a mass counterterm. It should be adjusted to cancel the photon self-energy when we use a noncovariant momentum space cut-off. If a gauge invariant regulator or dimensional regularization is used, we should put $\delta\mu = 0$. Then the above model becomes equivalent to quantum electrodynamics if a bootstrap condition

$$\lim_{\Lambda\to\infty} Z_3(e_R, \Lambda) = 0 \quad , \tag{28}$$

is satisfied.

Now we argue that the above equation (28) may not be satisfied even by imposing an eigenvalue condition,

$$\beta(e_R^\infty) = 0 \quad . \tag{29}$$

Here the basic reason is that the anomalous dimension of a photon field vanishes at the fixed point due to gauge invariance and hence Z_3 remains nonzero in the limit $\Lambda\to\infty$.

Though this may appear obvious to those familiar with finite theory of quantum electrodynamics,[15,16] we shall give a simple argument using dimensional regularization. In 't Hooft's scheme of renormalization[17] Z_3 is expanded in a Laraunt series in n-4,

$$Z_3 = 1 + \sum_{\nu=1}^{\infty} \frac{a_\nu(e_D^2)}{(n-4)^\nu} \quad , \tag{30}$$

where a_ν has a structure,

$$a_\nu(e_D^2) = a_{\nu 1}(e_D^2)^\nu + a_{\nu 2}(e_D^2)^{\nu+1} + \ldots \quad . \tag{31}$$

a_1 is related to the β-function,

$$\frac{1}{2} e_D^3 \, a_1'(e_D^2) = \beta(e_D^2) \quad . \tag{32}$$

Then using renormalization group constraints,[17]

$$\left(-e_D^2 a'_{\nu-1} + a_{\nu-1}\right) a_1' - a_\nu' = 0 \quad , \tag{33}$$

and the fact that the zero of the β-function is an infinite-order zero,[16] one obtains,

$$a_\nu = 0 \quad , \qquad \nu = 1, 2, 3, \ldots \quad . \tag{34}$$

Hence[18]

$$Z_3(e_D^\infty) = 1 \quad .$$

This is just the other extreme of the Lehmann bound opposite to the bootstrap $Z_3 = 0$. Thus it appears unlikely that a photon (Abelian gauge field) can be interpreted as a fermion-antifermion bound state.

It is also possible to consider a non-Abelian analogue of the above example. For instance in the case of SU(2) it is given by

$$\mathscr{L} = \bar{\psi}\, i\gamma\,\partial\psi - m\bar{\psi}\psi - \frac{G}{2}\left(\bar{\psi}\,\gamma_\mu \frac{\vec{\tau}}{2}\psi\right)^2 \tag{36}$$

Introducing a field $\vec{A}_\mu$ and putting $\delta\mu = 0$ we obtain,

$$\mathscr{L}' = \bar{\psi}\, i\gamma\, \partial\psi - m\bar{\psi}\psi - g\bar{\psi}\gamma_\mu \frac{\vec{\tau}}{2}\psi\vec{A}^\mu \quad . \tag{37}$$

By adding a ghost term we write the Lagrangian as

$$\begin{aligned}
\mathscr{L}'' &= \bar{\psi}i\gamma\partial\psi - m\psi\bar{\psi} - g\bar{\psi}\gamma_\mu \frac{\vec{\tau}}{2}\psi\vec{A}^\mu - \tfrac{1}{2}\partial^\mu\vec{\phi}^*(\partial_\mu + g\vec{A}_\mu \mathrm{x})\vec{\phi} \\
&= \Big\{ \bar{\psi}_R i\gamma\partial\psi_R - m_R\bar{\psi}_R\psi_R - g_R\bar{\psi}_R\gamma_\mu \frac{\vec{\tau}}{2}\psi_R\vec{A}^\mu_R \\
&\quad - \tfrac{1}{2}\partial^\mu\vec{\phi}^*_R(\partial_\mu + g_R\vec{A}^R_\mu \mathrm{x})\vec{\phi}_R - \tfrac{1}{4}(\partial_\mu\vec{A}^R_\nu - \partial_\nu\vec{A}^R_\mu + g_R\vec{A}^R_\mu \mathrm{x}\vec{A}^R_\nu)^2 \Big\} \\
&\quad + \Big\{ (Z_F-1)\bar{\psi}_R i\gamma\partial\psi_R - m_R(Z_F-1)\bar{\psi}_R\psi_R - \delta m Z_F\bar{\psi}_R\psi_R - g_R(Z_g-1)\bar{\psi}_R\gamma_\mu \frac{\vec{\tau}}{2}\psi_R\vec{A}^\mu \\
&\quad - \tfrac{1}{2}(\tilde{Z}_3-1)\partial_\mu\vec{\phi}^*_R\partial^\mu\vec{\phi}_R - \tfrac{1}{2}g_R(\tilde{Z}_1-1)\partial_\mu\vec{\phi}^*_R\vec{A}^\mu_R \mathrm{x}\vec{\phi}_R - \tfrac{1}{4}(\partial_\mu\vec{A}^R_\nu - \partial_\nu\vec{A}^R_\mu)^2 \\
&\quad - \tfrac{1}{2}g_R(\partial_\mu\vec{A}^R_\nu - \partial_\nu\vec{A}^R_\mu)(\vec{A}^\mu_R \mathrm{x}\vec{A}^\nu_R) - \tfrac{1}{4}g^2_R(\vec{A}^R_\mu \mathrm{x}\vec{A}^R_\nu)(\vec{A}^\mu_R \mathrm{x}\vec{A}^\nu_R) \Big\}
\end{aligned} \tag{38}$$

Then, comparing Eq. (38) with the Lagrangian of an SU(2) Yang-Mills theory coupled with fermions, we notice that two theories become equivalent if a bootstrap condition $Z_1=0$, $Z_3=0$ is satisfied. Renormalization factors Z_1, Z_3 are those associated with three-vector and two-vector vertices, respectively.

Although these parameters are gauge dependent, we can solve Callan-Symanzik equations and evaluate them due to asymptotic freedom. The result depends on the choice of group and representations. It is possible to show that if quantities

$$\bar{\beta} = \frac{1}{16\pi^2}\left(\frac{11}{3}C_2 - \frac{4}{3}T_2\right) \quad , \tag{39}$$

$$\bar{\gamma} = \frac{1}{8\pi^2}\left(\frac{8}{3}T_2 - \frac{13}{3}C_2\right) \quad , \tag{40}$$

are both positive, the running coupling constant and gauge parameter go to zero, $g^\infty_R = \alpha^\infty_R = 0$, in the deep Euclidean limit.[19] Here C_2 is the value of a quadratic

Casimir operator in the adjoint representation and T_2 is that of the representation of fermions. Then by using renormalization group equations we obtain,

$$Z_1 \sim \lim_{\Lambda \to \infty} \Lambda^{-3\bar{\gamma}/4\bar{\beta}} = 0 \quad , \tag{41}$$

$$Z_3 \sim \lim_{\Lambda \to \infty} \Lambda^{-\bar{\gamma}/2\bar{\beta}} = 0 \quad . \tag{42}$$

This result is gauge independent in the sense that it holds in any gauge. Thus bootstrap conditions are satisfied. On the other hand if $\bar{\beta}>0$ but $\bar{\gamma}<0$, Z's remain nonzero in the limit of infinite cut-off. In the case of SU(2), for example, $\bar{\beta},\bar{\gamma}>0$ if there are F fermion doublets with $13/2 < F < 11$.

Our method described above has an interesting comparison with that of Wilson[20] who introduced an unconventional renormalization procedure for super-renormalizable theories in less than four dimensions. In this procedure dimensional coupling constants of a theory are let go to infinity while properly defined dimensionless couplings constants are held fixed as $\Lambda \to \infty$. Then a super-renormalizable theory is converted into a nontrivial renormalizable theory. There is an eigenvalue condition for dimensionless coupling constant and by satisfying it the theory exhibits a scaling behavior with anomalous dimensions. Here relevant eigenvalues are infrared stable fixed points.

A great virtue of this method is the guaranteed existence of a fixed point close to the origin so far as $\epsilon = 4-d$ is small. However, in the limit $\epsilon \to 0$ Wilson's prescription gives a free field theory. Therefore here the existence of an eigenvalue is achieved only by having a trivial theory at four dimensions.

Our work suggests an intimate connection between the idea of dynamical symmetry breakdown and Gell-Mann-Low fixed point. If a theory possesses an ultraviolet stable fixed point, some of its fields (presumably spin-zero mesons) may be interpreted as composites and their vertices can be eliminated from the Lagrangian without spoiling renormalizability. Then we have a smaller number of fields and vertices and a more constrained theory than the original one. Allowed phases in such a theory may well be quite restricted. Hopefully such a procedure eliminates a high degree of arbitrariness in the conventional Higgs mechanism and gives us a new approach to dynamical symmetry breakdown.

The author is indebted to G. Guralnik and N. Snyderman for discussions on the possibility of fixed points in four-fermi theories. He also thanks Y. Nambu for discussions and reading the manuscript.

After completion of this manuscript we received a preprint by C. Bender, F. Cooper, and G. Guralnik (Los Alamos Report 77-1093), where some of the materials of this paper are discussed using mean field theory.

REFERENCES

1. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
2. J. D. Bjorken, Ann. Phys. (N.Y.) 24, 174 (1963).
3. G. S. Guralnik, Phys. Rev. 136, B1414 (1964). See also I. Bialynicki-Birula, Phys. Rev. 130, 465 (1963).
4. T. Eguchi, Phys. Rev. D 14, 2955 (1976); N. Snyderman, Brown University Thesis (1976). See also, T. Eguchi in Quark Confinement and Field Theory, edited by D. R. Stump and D. H. Weingarten (John Wiley and Sons, New York, 1976); N. Snyderman and G. S. Guralnik, ibid.
5. This kind of technique has been used by various authors. S. Coleman, R. Jackiw and H. D. Politzer, Phys. Rev. D 10, 2491 (1974); D. Gross and A. Neveu, ibid., 10, 3235 (1974); K. Kikkawa, Prog. Theor. Phys. 56, 947 (1976); T. Kugo, ibid., 55, 2032 (1976); H. Kleinert, Erice Lecture Note (1976).
6. B. W. Lee, Nucl. Phys. B9, 649 (1969). J. L. Gervais and B. W. Lee, ibid., B12, 627 (1969); K. Symanzik, Commun. Math. Phys. 16, 48 (1970).
7. Earlier references are: J. C. Houard and B. Jouvet, Nuovo Cimento 18, 446 (1960); A. Salam, ibid., 25, 224 (1962); S. Weinberg, Phys. Rev. 130, 776 (1963); D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1964). See also, K. Shizuya, preprint UT-Komaba 77-3 (1977).
8. C. Callan, Phys. Rev. D 2, 1541 (1970); K. Symanzik, Commun. Math. Phys. 18, 227 (1970).
9. H. Lehmann, Nuovo Cimento 11, 342 (1954).
10. We assume $\gamma_M \neq 0$. $\gamma_M = 0$ is quite unlikely to hold since $\beta_g = \beta_\lambda = \gamma_M = 0$ is over-determinant for g_R^∞ and λ_R^∞.

11. The connection of bootstrap condition to Gell-Mann-Low fixed point is implicitly used in Migdal-Polyakov conformal bootstrap: A. A. Migdal, Phys. Lett. 37B, 386 (1971). A. M. Polyakov, Zh. ETF Pis. Red. 12, 538 (1970). (English translation: JETP Lett. 12, 381 (1970).) G. Mack and K. Symanzik, Commun. Math. Phys. 27, 247 (1972).

12. H. Pagels, Phys. Rev. D 7, 3689 (1973).

13. R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. M. Cornwall and R. E. Norton, ibid., 8, 3338 (1973).

14. A. Zee, Phys. Rev. D 7, 3630 (1973). S. Coleman and D. Gross, Phys. Rev. Lett. 31, 851 (1973).

15. K. Johnson and M. Baker, Phys. Rev. D 3, 2516 (1971).

16. S. L. Adler, Phys. Rev. D 5, 3021 (1972).

17. G. 't Hooft, Nucl. Phys. B71, 455 (1973).

18. This result may be taken with a care in view of the fact that the fixed point is an essential singularity. Dimensional regularization specifies a particular path to approach singularity.

19. A. Hosoya and A. Sato, Phys. Lett. 48B, 36 (1974). W.-C. Ng and K. Young, Phys. Lett. 51B, 291 (1974); B. W. Lee and W. I. Weisberger, Phys. Rev. D 10, 2530 (1974).

20. K. Wilson, Phys. Rev. D 7, 2911 (1973); K. Wilson and J. Kogut, Phys. Lett. 12C, 75 (1974). See also, S. S. Shei and T.-M. Yan, Phys. Rev. D 8, 2457 (1973).