

Laboratory tests of gravity

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New Journal of Physics **7** (2005) 205

Received 01 April 2005

Published 29 September 2005

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/7/1/205

Abstract. Experimental tests of gravity are probes for new physics. Theoretical speculations ranging from string theory scenarios to explanations of cosmological phenomena have led to many new experiments. Recent laboratory searches for equivalence principle violations and tests of Newton's $1/r^2$ -law at short distances are reviewed.

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1. Introduction, motivation

In this review, occurring 100 years after the onset of Einstein's discoveries, we will summarize some of the recent developments in laboratory tests of gravity in the search for hints of a grand unification of all forces, Einstein's unfulfilled dream. Newton's revelations about gravity were motivated by the availability of quantitative observational data of the solar system. His theory seemed to be a complete and well-proven concept and it held for centuries. It was not until the last century that Einstein derived gravitation from the underlying spacetime concept. Einstein's work was not provoked by observational facts, but was motivated on a purely theoretical basis, which makes his formalism and prediction particularly elegant. General Relativity presented a big change in concept beyond Newton and it predicted specific phenomena. These effects have been verified experimentally and to this date are being tested to higher and higher accuracy. The classical tests were: light bending, time dilation, gravitational redshift and observation of gravitational waves. Einstein's General Relativity corrections are now being used on an every-day basis. A typical example is the global positioning system (GPS), where General Relativity effects yield a combined effect of $38 \mu\text{s}$ per day. The accuracy to which General Relativity can be confirmed is entirely limited by experimental uncertainties and none of the uncertainty originates from the theoretical description, since General Relativity has no adjustable parameters. It seems that to date, gravity is exactly described by General Relativity.

One may wonder why an additional pursuit proving (or disproving) the correctness of General Relativity is warranted or desired. Why not simply apply Occam's razor to this stage

of theory and declare General Relativity to be exact and final? History has proven that, while applying Occam's razor to mathematics often leads to a practical and correct convergence, this is not so for nature. In the past, physicists had to discover that no matter how perfect and beautiful a theory has seemed initially, it usually reflected the limiting case of an underlying, even more complete and general explanation that was discovered subsequently. It is thought that General Relativity is no exception to this rule.

It is certain that at least one more level of unification must exist, i.e. we firmly believe that a fusion of gravity and the rest of physics, unified in the Standard Model, exists and can be found. We have had these two finely distilled models, General Relativity and the Standard Model, for several decades now and each of them has passed its respective experimental tests with flying colours, but they do remain separate at a conceptual level.

The urge for a unification of all the forces in some form of quantum gravity is strong. Perhaps the first theoretical hint of this grand unification is already emerging in the form of string theory, which includes the gauge interactions and gravity. Or perhaps some of the stunning cosmological observations which are telling us that the dynamics of the universe is dominated by stuff that we cannot explain (dark matter and dark energy) can be explained by modifying General Relativity and particle physics.

Within the past decade, experimental testing of gravity has emerged as one of the most rapidly growing subfields of modern physics. In this paper, we will describe several contemporary laboratory (and some space-based experiments) designed to test the foundations of General Relativity and to probe theories that predict deviations from General Relativity.

Precision laboratory tests of gravity are extremely challenging because of the intrinsic weakness of gravity. While many of these experiments are conceptually simple, each of them must look for very subtle effects.

With the lack of one guiding theory, the field of searching for new physics or deviations from General Relativity is quite broad. Some of the most actively pursued tests that can be carried out in the laboratory are: tests of the equivalence principle and tests of the $1/r^2$ -law at short ranges. The most recent versions of these tests will be described in this paper.

Gravitational waves are another area of intense experimental gravity research. These waves have not been detected directly but it is expected that this will happen in the next few decades. Gravitational waves will be used as a medium to observe some of the most extreme and bizarre events in the universe such as black holes colliding and coalescing with other black holes. The possibility of observing stochastic gravitational wave remnants of the big bang era are perhaps even more exciting. Such gravitational radiation may contain direct signatures of the universe's inflationary period or of the electroweak phase transition or ultimately may present direct traces of quantum gravity. In this paper we will not review these fascinating experiments for direct detection of gravitational waves since they are described by others [1] in this Focus Issue.

2. Tests of the equivalence principle

Tests of the equivalence principle anecdotally date back to at least the time of Galileo. Albert Einstein used the hypothesis of the complete equivalence of gravity to an accelerated reference frame, [2] called Einstein's equivalence principle (EEP). Realizing this fundamental symmetry of the equivalence principle was Einstein's 'happiest thought' [3] (1907), which he brilliantly expanded into the theory of General Relativity. Since the equivalence principle is of empirical

nature and since it forms the fundamental basis on which the exact mathematical body of General Relativity is based, it is imperative to test it experimentally to the highest possible precision. In the past, the apparent high accuracy of the equivalence principle was often taken for granted and its exactness was implied. With the advent of grand unification scenarios, however, it has become increasingly evident that the equivalence principle must be violated at some level. As a matter of fact, practically all attempts to extend the present framework of physics require violations in the form of new scalar or vector fields and violations of the equivalence principle are considered to be by far the most sensitive low-energy probes for new physics [2].

A direct consequence of the EEP is the universality of free fall (UFF), which states that any point particle experiences exactly the same acceleration at the same location in a gravitational field. The emphasis is on any meaning that the acceleration is not a function of the composition and specifically that no new (unknown) scalar- or vector-charge coupled interactions exist.

In the famous UFF experiments by Braginsky [5] and Dicke and co-workers [4] in the 1960s and 1970s it was assumed that the length scale of the equivalence principle violation is equal to the scale of Newtonian gravity, namely that it also has infinite range. This assumption is also referred to as the Newtonian equivalence principle stating that gravitational mass is exactly equal (or identical) to inertial mass. Therefore, Newtonian equivalence principle tests were carried out using accelerations to objects that are far away. Braginsky as well as Dicke chose to compare the gravitational acceleration of two closely spaced test masses in the field of the sun. The choice of the sun as the source of the gravitational field was obvious, as the Newtonian gravity field is very uniform over the separation of the test masses and even more important because the earth's rotation modulates the direction of an equivalence principle violation signal. This signal modulation allowed extraction of a small signal from much larger force backgrounds and noise.

For a more general test of the equivalence principle, it must be assumed that the violation may not have a infinite range. A finite range is natural for violations of the equivalence principle in terms of new scalar or vector fields that are mediated by massive particles. In the quantum world, a nonzero rest mass requires the new scalar (spin = 0) or vector (spin = 1) particles that mediate the interaction to have a finite range as expressed in the new interaction's Yukawa exchange potential:

$$V_{1,2} = \frac{g^2}{4\pi} (q_1)(q_2) \frac{e^{-r/\lambda}}{r}.$$

The range is inversely proportional to the mass of the new particle mediating the interaction: $\lambda = \hbar/m_b c$. Precision tests of the equivalence principle can probe ranges from $\lambda = \text{mm}$ to $\lambda = \infty$, thus they are searches for massive exchange particles below $\approx 0.1 \text{ meV}$. q_1 and q_2 are the new types of charges that the equivalence principle violating interaction must couple to.

2.1. University of Washington equivalence principle tests

At the University of Washington, we are stepping in the footsteps of Baron von Eötvös, who around the time of Einstein's happiest thought, tested the equivalence principle experimentally. Similarly to Eötvös, we have developed highly sensitive torsion balance instruments for these tests. Our torsion balances have become exquisitely sensitive detectors for differential accelerations that act sideways to the torsion fibre axis. Our instruments have been refined such that they can now resolve horizontal force differences that are 10^{16} times smaller than the gravitational force (weight) in the vertical direction. To test the equivalence principle, two

masses, called ‘test masses’, made of different materials are placed on opposite sides of the ‘pan’ of the torsion balance. A small differential attraction of the test masses to a ‘source mass’ located sideways to the pendulum causes the pendulum to twist. The challenge in these measurements is to resolve tiny pendulum rotations and to suppress or account for all other small differential forces that may also cause a pendulum rotation.

2.1.1. Rotating torsion balance experiment. One of our most significant improvements to torsion balance equivalence principle tests was to move this signal from dc or near dc to a higher frequency. This was accomplished by placing the entire torsion balance on a continuously rotating turntable. We have developed sophisticated turntables that rotate smoothly enough so that the instrument operates within a few per cent of its theoretical thermal noise limit. Since the balance is operated in a vacuum, the noise budget is mostly due to the thermal noise caused by friction within the thin torsion fibre; the rotation of the instrument practically adds no extra noise (figure 1). Interestingly, the leading systematic uncertainty in many of our measurements is due to Newtonian gravitational coupling of the pendulum to gravitational gradients from the nearby environment. To understand and analyse this unwanted coupling, we developed a rigorous multipole formalism similar to that used in electromagnetism. The gravitational coupling was minimized by constructing a highly symmetric pendulum and by reducing the ambient gravitational gradient with massive compensator masses. The gradients are measured by replacing the equivalence principle pendulum with special pendulums that have deliberate large gravitational moments. For our latest experiment, 780 kg of lead were shaped to eliminate the lowest order of the ambient gravitational gradient ($l = 2$, $m = 1$ in the multipole formalism). Residual gradients were periodically measured to track changes in the gravitational gradient. We found that the biggest changes were due to changes in the water content of the soil surrounding our building. We also measure residual gravitational moments of the equivalence principle pendulum that are due to small machining imperfections. These moments are measured by placing the gravity gradient compensator masses on the opposite side of the torsion pendulum, so that they double the local gravity gradient rather than cancel it. Small trim screws on the pendulum body are used to reduce the pendulum’s susceptibility to gravitational gradients.

We also had to combat several other unwanted couplings due, e.g., to magnetic, thermal and tilt issues. One systematic coupling arose because the turntable rotation axis was not perfectly aligned with local vertical axis. We developed a special levelling system that uses feedback to a tilt sensor on the rotating platform to control the temperature of the feet on which the turntable sits. The thermal expansion of the feet continuously adjusts the rotational axis to within a few nanoradians of the true vertical axis. The torsion angle of the pendulum is measured optically with an autocollimator (figure 2).

A wide variety of test mass materials have been used. The motivation for choosing the test mass is as follows: the known interactions (gravity, electromagnetism, strong interaction, etc) are associated with conserved quantities as their charges (mass–energy, charge, colour, etc). It is therefore suggestive that an equivalence principle violation or new interaction is also associated with a conserved charge. Baryon number (B) is a conserved quantum number. In 1986, a reanalysis of Eötvös data by Fischbach *et al* [6] found a striking correlation of an apparent equivalence principle violation to baryon number. While the specific strength of the interaction predicted from the reanalysis was soon ruled out, the focus on baryon number as the charge remained. Other likely charges include lepton number (L) and the combination of $B-L$, the latter remains conserved in GUT. Most materials that can be used to make test masses have

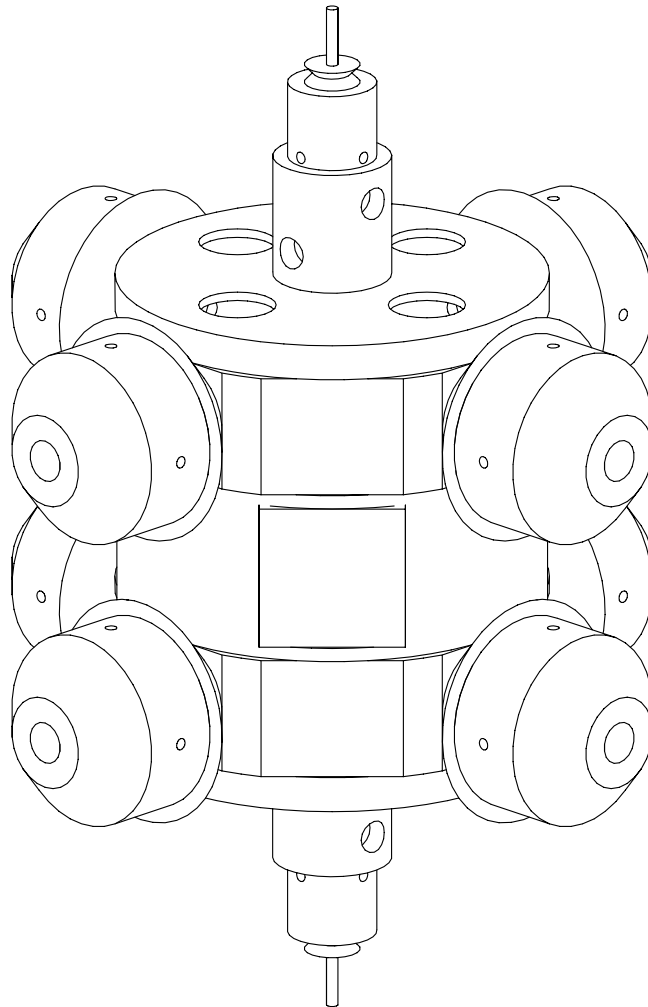


Figure 1. The pendulum body used in the Eöt-Wash equivalence principle test. Four testbodies of one material are on one side and four of the other material on the other side of the pendulum body. Currently, we are using a Ti-Be dipole. The pendulum weighs 70 g and is suspended by a $20\ \mu\text{m}$ W-wire. The pendulum is designed so that the first non-vanishing gravitational moment that occurs at the signal frequency is at $l = 7$. Small trim screws at the top and bottom are used to eliminate small residual gravitational moments. The pendulum is operated inside a continuously rotating vacuum chamber. The statistical noise performance is given by the thermal kT noise of the torsion fibre.

a very similar number of baryons per unit mass. However due to differences in nuclear binding energy, material pairs that differ by about 0.1% in their baryon number content can be found. For our most recent experiments, we chose to compare the accelerations of beryllium and titanium. (We shied away from iron because of magnetism.) In the past, we have also conducted several other experiments in which we used Al, Si, Cu and Pb as test bodies.

Our experiment can probe the equivalence principle for a great variety of Yukawa strengths, ranges and new charges. The local topology, the entire earth itself, the sun, our galaxy or the Great

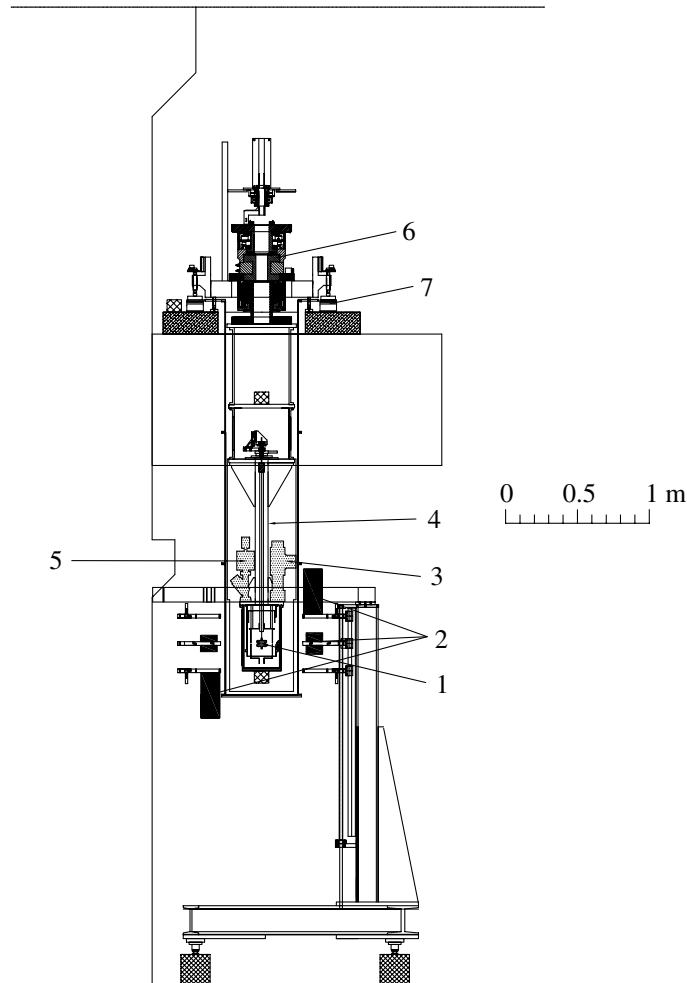


Figure 2. Cross-sectional view of the equivalence principle apparatus of the Eöt-Wash group. The torsion balance vacuum chamber (4) is suspended from an airbearing turntable. The turntable (6) rotates constantly at a frequency of about 1 mHz. Uniform rotation is achieved by using feedback to a high-quality angle encoder. 900 kg of gradient compensators (2) are used to null gravitational gradients. Multiple layers of thermal and magnetic shielding are being used. Other items are (1) pendulum, (3) autocollimator, (5) ion pump and (7) thermal expansion levelling system.

Attractor all serve as source masses. Earth fixed signals are modulated at the rotation frequency of our instrument, signals towards the Sun carry an additional daily and seasonal variation, and signals towards the Milky Way or space-fixed directions are modulated at the sidereal frequency combined with the turntable frequency. We also use the phase of a signal to associate it with a certain source in a certain direction. Differential accelerations towards the centre of our galaxy are particularly interesting as they allow us to probe the equivalence principle between ordinary and galactic dark matter [7]. With our latest measurements, we can conclude that the acceleration difference of Be-Ti towards the centre of our galaxy is less than $2.5 \times 10^{-15} \text{ ms}^{-2}$, which results in $\Delta a_{\text{Be-Ti}}/a_{\text{darkmatter}} < 5 \times 10^{-5}$.

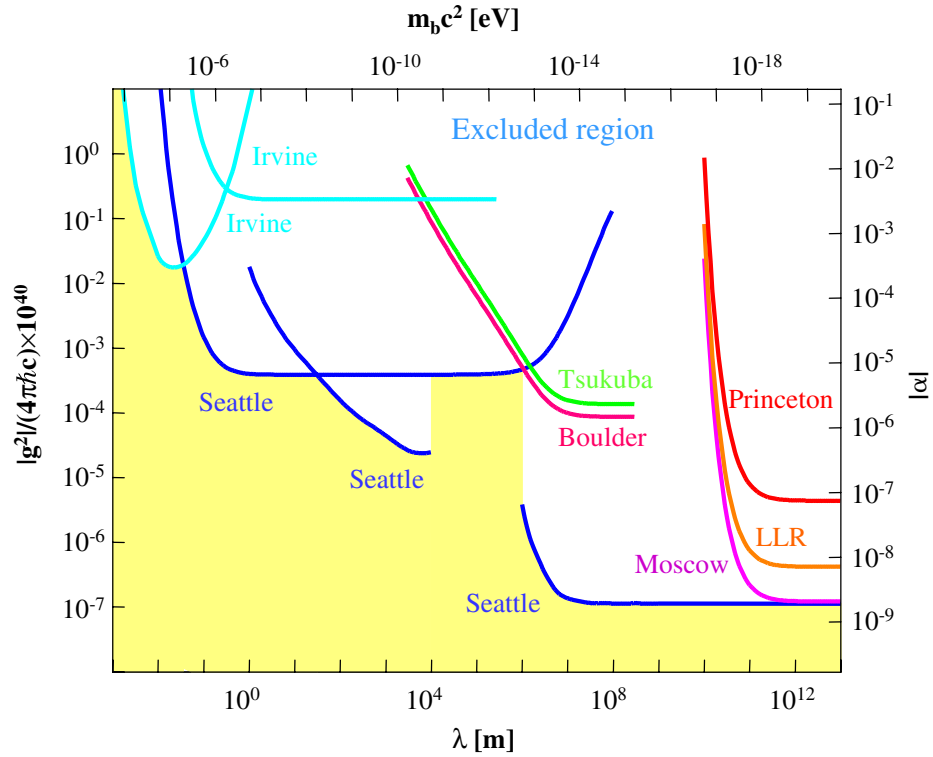


Figure 3. Published 2- σ limits on the strength of a Yukawa type equivalence principle violation coupled to baryon number, B . Interactions with strengths below the lines are still allowed. Data are from Irvine [39], Seattle [7], Boulder [10], Tsukuba [11], Princeton [3], Moscow [4] and LLR (lunar laser-ranging) [18].

2.1.2. Rotating source experiment. In another experiment [8], we rotated a 3 ton ^{238}U attractor around a stationary torsion balance carrying a Cu-Pb composition dipole. This instrument was designed to test the UFF at short ranges, and to close a ‘gap’ at ranges between 10 and 1000 km where Eötvös-type experiments using the earth as the attractor have poor sensitivity. This insensitivity is due to uncertainties in our model of the mass distribution of the earth. The ^{238}U attractor also allowed us to test for interactions coupled to isospin charge (N/Z ratio $\propto (B - 2L)$). We could not effectively use the earth for this test since the earth’s mantle is approximately isospin neutral. Because of uranium’s high density, this allowed us to place 3 ton of mass within a radius of 50 cm of the torsion balance. This instrument provided useful constraints for ranges down to 1 cm, i.e. exchange-boson masses up to 2×10^{-5} eV.

Figure 3 summarizes the limits of all recently published results on equivalence principle violations. For figure 3 we used the Yukawa parametrization with the charge being baryon number, B .

2.2. Free-fall experiments

In these Galilean tests, two test masses of different compositions are dropped simultaneously and their differential acceleration in the field of the entire earth is recorded. Free-fall experiments have much higher intrinsic sensitivity compared to torsion balance experiments: torsion balances

are only sensitive to sideways forces. For interaction ranges from $\lambda \approx 1$ to 10 000 km, for which the entire earth is used, only 0.17% of the earth's mass lies sideways to a torsion balance. This 0.17% arises due to the earth's rotation. In addition, the sideways mass distribution for ranges of 10 km contains large uncertainties due to geological models. In contrast, free-fall experiments can utilize the entire mass of the earth as a source and the vertical mass distribution of the earth is relatively well known. The disadvantage of laboratory free-fall experiments is that the typical free-fall time for each drop in a laboratory cannot be much larger than 1 s. Elaborate schemes are used to repeat the experiments, so that the total accumulated free-fall time becomes considerable. In all free-fall experiments, a co-falling (servoed) chamber shields the test masses from effects such as residual gas. The acceleration difference is measured using interferometric techniques. A common challenge to most free-fall experiments is the release of the test masses which may impart unwanted motion to them. Two free-fall experiments [10, 11] are currently setting their best limits on equivalence principle violations for ranges between 10 and 1000 km (figure 3).

Several new free-fall measurements are in preparation. In one experiment [12], a test mass assembly is launched upwards resulting in double the free-fall time. The group is attempting to discern acceleration differences as small as $\Delta g/g = 5 \times 10^{-14}$.

At the Centre of Applied Space Technology and Microgravity (ZARM) [13], a drop tower is used to lengthen the free-fall time to 4.5 s. The differential acceleration of two test masses is being measured using SQUID-sensor.

Another group [14] will drop a superconducting differential accelerometer from a stratospheric balloon ($h = 40$ km). The accelerometer is released inside a drop capsule which shields it from air drag. A usable free-fall time of 25 s can be attained. During the drop the accelerometer rotates at a velocity of 1 Hz to modulate an EP-violating signal. A differential acceleration sensitivity of $\Delta g/g = \text{several} \times 10^{-15}$ is this group's goal.

2.3. Space experiments to test the equivalence principle

Equivalence principle experiments in space are essentially free-fall experiments with very long free-fall times. To date no space equivalence principle test has been carried out, but several have been proposed and one is in preparation. Another advantage of experiments in zero-g over torsion balances is that noise associated with the test mass suspension is (nearly) absent. Space experiments of the equivalence principle have so far not been carried out because of their large cost and performance risk, but a French (CNES, ONERA) space mission called MICROSCOPE is scheduled for launch in 2007/2008.

Common to all space tests are two test bodies, made from different materials which are placed in orbit around the earth. The satellite body surrounds the test mass pairs and shields them from external influences such as solar wind. The satellite activates small thrusters to exactly follow the test masses, leaving the test masses freely floating and 'drag-free'. The displacement of the test masses relative to each other would constitute an equivalence principle violation. In the MICROSCOPE experiment, the displacement between Ti and Pt is measured capacitively. The satellite rotates so that an equivalence principle violating signal is modulated at the rotational frequency of the space craft. The displacement readout is capacitive. The expected statistical noise floor of the instrument is dominated by thermal noise generated by a thin Au wire that is necessary to keep the test masses electrically grounded. A differential acceleration sensitivity of 10^{-15} g is expected.

A similar experiment proposed by Stanford University, called STEP consists of four (for redundancy) accelerometers (test mass pairs) which are housed in one quartz block and operated at cryogenic temperatures. An exquisitely sensitive SQUID readout system is planned to detect relative motions as small as 10^{-15} m. The proposed acceleration difference that STEP would be able to achieve is 10^{-18} g [15].

An Italian group is proposing a somewhat different experiment called GG (Galileo Galilei) [16]. Two concentric cylinders rotate together about their symmetry axes. The rotational frequency is fast, at a few Hz. The signal is a small periodic displacement of the cylinders sensed in the rotating frame using capacitance measurements. The fast rotation rate reduces many problems that other equivalence principle measurements operated near dc encounter. Ground testing of this novel accelerometer appears promising. For a space flight their proposed sensitivity is 10^{-17} g.

2.4. Lunar laser-ranging to test the equivalence principle

Laser-ranging to the retro reflectors installed by the Apollo 11, 14 and 15 astronauts and by the Soviet Luna 17 and 21 missions on the lunar surface allows for a broad number of gravitational tests, including long-range measurements of the equivalence principle. An equivalence principle violation that would act differently on the moon than on the earth would result in a polarization of the moon's orbit around the earth in the direction of the sun. The astounding accumulated precision with which this polarization is resolvable is ± 4 mm [17]. The differential acceleration of the earth and the moon in the field is smaller than $\Delta a_{\text{earth-moon}}/a_{\text{sun}} = 1.4 \times 10^{-13}$ [20].

Lunar laser-ranging allows for a particularly interesting variant of an equivalence principle test answering this question: does gravitational energy itself obey the equivalence principle? [21]. This is the essence of the strong equivalence principle (SEP), which is an even more general statement than the EEP. A test of the SEP is possible since the fraction of gravitational self-binding energy of the earth is 4.6×10^{-10} while the moon's contribution is only 0.2×10^{-10} . A complete violation of the SEP would result in a polarization of the earth-moon orbit towards the sun of 13.1 m. The lunar laser-ranging 4 mm precision sets a limit on violations of the SEP of $\approx 3 \times 10^{-4}$.

For a strict interpretation of the lunar laser-ranging data, as SEP limits the possibility of a compensating composition-dependent violation of the equivalence principle, must be eliminated (also pointed out by Nordtvedt). We have carried out a torsion balance test of the WEP using test masses that mimicked the composition of the earth and that of the moon. Our test has sufficient sensitivity to eliminate this possibility and combining with the lunar laser-ranging results establishes an unambiguous test for gravitational self-energy at the $\approx 4.4 \times 10^{-4}$ level [17, 22].

The precision of lunar laser-ranging has been steadily improving. In the near future, a highly improved ranging station using the 3.5 m Apache Point telescope in Arizona will come on-line (APOLLO) [18]. This new station will receive several photons per laser pulse emitted compared to a small fraction of a photon on average in previous measurements. This allows observations at full moon where the equivalence principle signal is maximal [19]. APOLLO is expected to reach mm-precision which will increase the demand on modelling [17].

2.5. $\vec{\sigma} \cdot \vec{r}$ -tests

It has been suggested that spin, $\vec{\sigma}$, as a charge could violate the equivalence principle or that spin may play a role in gravity. Of particular interest are interactions as discussed by Moody and

Wilczek [25] that couple proportional to $\vec{\sigma} \cdot \vec{r}$. Here $\vec{r}$ represents the vector from a spin particle to a mass point. The pseudo-scalar $\vec{\sigma} \cdot \vec{r}$ -coupling is CP-violating. This interaction may in principle be long-range and could therefore manifest a macroscopic CP-violation. The axion is a candidate particle that would carry such an interaction.

In order to detect a force in a macroscopic sample, the material must have a net spin polarization. The challenge for all tests using macroscopic amounts of aligned spins is to suppress the much stronger electromagnetic coupling of spin.

Ni *et al* at the Tsing Hua University in Taiwan have carried out a number of tests involving spin. In one experiment Ni [26] rotated a copper mass about a sample of the paramagnetic salt terbium fluoride (TbF_3). A $\vec{\sigma} \cdot \vec{r}$ -interaction coupling to the copper mass would induce a slight change in magnetization of the TbF_3 sample. This change is picked up by a coil wound around the sample and amplified using a SQUID. Several layers of μ -metal and superconducting shielding were used. Ni *et al*'s measurement sets stringent limits to $\vec{\sigma} \cdot \vec{r}$ -interactions with ranges from a few mm to about 1 m.

In another experiment [27], the same group suspended a $\text{Dy}_6\text{Fe}_{23}$ sample inside a rotating torsion balance, primarily to test local Lorentz invariance, which is a consequence of the Einstein's equivalence principle. In the $\text{Dy}_6\text{Fe}_{23}$ sample, the spin magnetic moment is intrinsically cancelled by orbital magnetic moments. Hou *et al* [27] analysed their data with respect to fixed directions in space and found that the energy splitting of a single spin was less than a few times 10^{-19} eV.

In Seattle we also used a rotating torsion balance instrument [7] to carry out a test using a spin polarized torsion pendulum [28]. Our pendulum is constructed of rings of which one half is made from AlNiCo with its magnetization mostly due to electron spin and the other half of the ring is made from SmCo magnets of which $\approx 60\%$ of the magnetization is due to orbital angular momentum. The magnetic field is mostly contained in the ring, but the excess of the spins of the AlNiCo over the SmCo spins results in a large fraction of aligned spins. The pendulum is made of four such rings that are stacked together with the AlNiCo and SmCo magnets on alternating sides. While the rings contain a field of about 1 T, the field 1 cm away from the ring is about 5×10^{-9} T. A thin μ -metal shield on the pendulum body attenuates the field. Three layers of μ -metal magnetic shielding corotate with the pendulum on the rotating platform. Stationary Helmholtz coils are used to reduce the bulk of the earth field. We have successfully reduced magnetic coupling so that the coupling due to gravitational moments is a bigger concern. The experiment sets the most competitive limits on $\vec{\sigma} \cdot \vec{r}$ -interactions for ranges larger than 1 m and it allows us to search for preferred reference frames (i.e. Lorentz invariance violations) with a sensitivity $\approx 10^{-19}$ eV for a single electron spin.

3. Tests of Newton's $1/r^2$ -law

Newton's universal law of gravity is a direct consequence of Gauss's law in three dimensions. For weak field gravity, as most of the gravity around us, it appears to be very adequate to use Newton's law, while for very strong gravitational fields General Relativity must be used. Quantitative tests of Newton's law in the laboratory began with the experiments by Cavendish [24] at the end of the 1700s. Today's measurements are substantially better but progress over the two centuries has been surprisingly slow (as compared to the precision that has been achieved with electromagnetic measurements). The slow rate of progress in gravitational measurements at the laboratory scale

is mostly due to intrinsic weakness of gravity. The gravitational force between an electron and a proton separated by 1 Å is equal to the electrostatic force of a proton and an electron at more than ten times the distance between the earth and the moon!

In this paper, we will concentrate on possible $1/r^2$ -violations for distances of about 1 mm and shorter. Tests for this length-scale have become particularly interesting as several tantalizing speculations have been made.

In 1998 several theorists [29] argued that the extra dimensions needed in string theory do not have to be compactified or curled up at Planck-scale radii. In string theory gravitons are closed loop strings. These loops are free to move in all of string theory's 10 dimensions. This dilutes the strength of gravity compared to the strength of the Standard Model interactions whose particles consist of open loop strings that must be anchored in our common 3 + 1-dimensional subspace (brane). It is therefore expected that additional gravitational strength would be observed for distances shorter than the size of the compactified extra dimensions. With the unification scale at the Planck scale, and hence the compactification radius at the Planck length, gravity's full strength could not be observed for distances larger than 10^{-33} cm. If, however, the unification scale is as close as the weak energy scale ($\approx$ TeV), this would allow for macroscopic compactification radii for some of the extra dimensions. For example, for two such 'large' extra dimensions, the $1/r^2$ -law would turn into a $1/r^4$ -law for distances closer than 1 mm. This scenario is testable with table-top experiments and we will review these tests below. For more than two dimensions being 'macroscopic', a $1/r^2$ -violation would not be detectable directly, but it would lead to an apparent violation of momentum conservation that may be observable in high-energy particle collisions, as the graviton carries momentum away into the other dimensions. Such searches are currently being carried out [30]. The 'large' extra dimension scenario is attractive since it naturally eliminates the apparent lack of new physics over many orders of magnitude in energy scale between the weak energy scale and the Planck scale (or why gravity appears to be so weak). This is often referred to as the hierarchy problem.

String theory also predicts scalar particles, called dilaton and moduli, that couple to matter. With supersymmetry breaking at low energies, these particles would have Compton wavelengths in the mm-ranges [31]–[33] and would also violate the $1/r^2$ -law (and the equivalence principle).

Another very intriguing suggestion for short-range $1/r^2$ -violations is associated with the dark energy density or the cosmological constant, $\Lambda \approx 3 \text{ keV cm}^{-3}$. Combining Λ with $\hbar c$, one can construct a length scale: $(\hbar c/\Lambda)^{1/4} \approx 0.1 \text{ mm}$ [34, 35]. If the cosmological constant is indeed fundamental, one may expect new gravitational phenomena. Should the graviton be unable to mediate gravity for distances less than 0.1 mm ('fat graviton') [36], this could explain why the observed vacuum energy density is as small as the measured value of Λ and not hundred orders of magnitude larger as a theoretically constructed vacuum energy density would be (cosmological constant problem) [37].

From an experimentalist's point of view, the fact that gravity is tested at 10^{-11} -precision [38] for solar system length scales, but had not even been shown to exist for distances less than 2 mm is reason enough to undertake precision measurements of gravity at sub-millimetre separations and in particular to test the $1/r^2$ -behaviour.

A finite range equivalence principle violation would also violate the $1/r^2$ -law. Equivalence principle tests have been carried out for ranges as short as 5 mm, and no violation was found [8]. On the other hand, most $1/r^2$ -tests can be interpreted as equivalence principle tests, especially if several materials are used. Should a $1/r^2$ -violation be observed, one would have to repeat the test with different materials or conduct an equivalence principle test so that it can be decided if

the violation is due to new geometry or if it is due to the existence of a new particle-mediated force.

Similar to the equivalence principle tests, departures from $1/r^2$ -gravity are also best parametrized using a Yukawa description with a range parameter λ and a strength α : $V(r) = V_N(r)(1 + \alpha e^{-r/\lambda})$ and V_N being the Newtonian potential.

Because of their unsurpassed sensitivity to small forces on small (10 g) objects, the short range region near 1 mm is best tested with specific torsion balances. Currently, torsion balance experiments can actually probe for coupling strength equal to the strength of gravity itself down to ranges of about 80 μm . For ranges below that, micro machined oscillators and cantilevers are the instruments of choice. Common to all instruments is that extremely small forces between two very closely spaced mass distributions must be measured. For small ranges, the amount of active mass, and therefore the size of any possible signal, diminishes exponentially. It becomes essential to space the sensor and attractor masses within a distance of the order of a few λ . Because of this close spacing, a host of intrinsically much stronger non-gravitational interactions arise. Most of the measurements are not limited by their force sensitivity but rather by the ability to convincingly suppress or understand such competing forces. Therefore, the experimental limits on the coupling constant α weaken quickly as the range becomes much smaller than 1 mm.

Substantial experimental progress has been made recently in testing gravity at short ranges. Below we will discuss a variety of modern experiments that set the most current limits around 1 mm and below, with the results summarized in figure 9.

3.1. Irvine $1/r^2$ -tests

The group at UC-Irvine led by Newman established the best limits on $1/r^2$ -violations in the range from 3 mm to 3 cm. Even though the experiment was carried out more than 20 years ago, the limits it set have not been surpassed. In this experiment [39], a small cylinder made from very pure copper was suspended from one side of a torsion balance inside a long tube made from stainless steel. If the tube had infinite length, Gauss's law in three dimensions dictates that there cannot be any field inside the cylinder. Since the inner diameter of the tube was much bigger than the copper cylinder, the tube could be moved back and forth by 3.4 cm. Apart from small edge effects, the inside of a long cylinder is force free if $1/r^2$ holds. This made the experiment a very sensitive (and beautiful) null experiment. The tube was moved back and forth periodically and indeed no force (other than predicted residual gravitational coupling) on the torsion balance was registered. The torsion balance was encased in a vacuum chamber maintained at 3×10^{-7} Torr. This vacuum chamber also served as an electrostatic and thermal shield between the copper cylinder mass and the stainless steel tube, which was located outside the vacuum chamber. Ultimately, the experimental sensitivity was limited by systematic effects such as small tilt coupling due to the motion of the tube. Other uncertainties included residual gas effects, magnetic effects etc. This classic experiment was published [39] together with another $1/r^2$ -measurement which had optimal sensitivity for distances of about 20 cm. In quite a different, very sophisticated cryogenic torsion balance experiment, this group, together with the Boynton's group, is now working on a new test targeted for the 10 cm range region.

3.2. Seattle $1/r^2$ -tests

We, the Eöt-Wash group at the University of Washington, are also using the torsion balance approach but with a very different geometry. In our first iterations of experiments, we used a

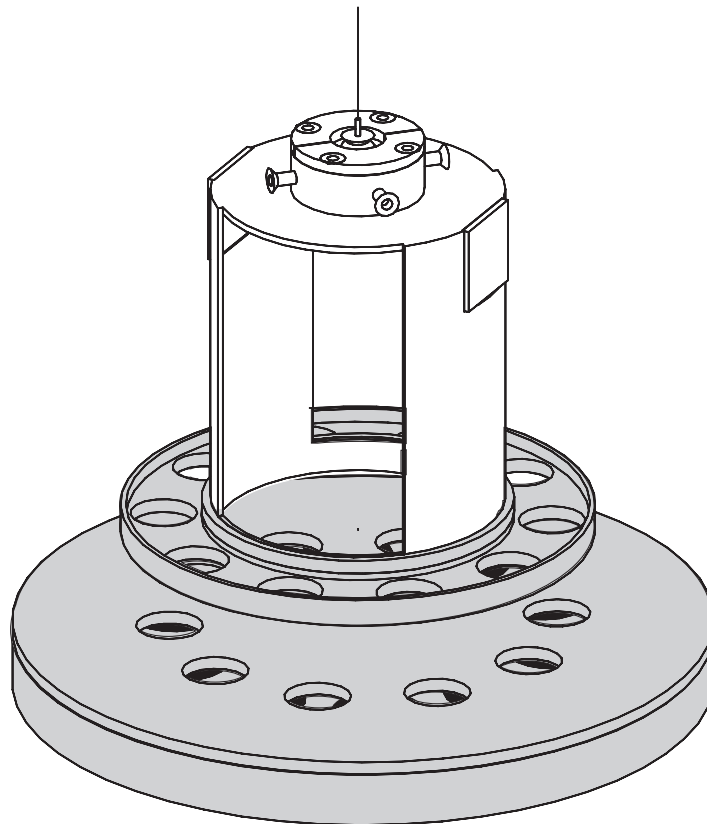


Figure 4. Schematic view of the $1/r^2$ -test pendulum of the Eöt-Wash group. The pendulum has a ring with 10 holes. Right below the pendulum is a plate that slowly rotates and also has 10 holes in it. The ‘missing mass’ of the holes in the pendulum and the attractor plates interact gravitationally. A second rotating plate is under the first plate. It also has 10 holes in it, but these holes are placed out of phase so that they cancel the gravitational effect of the upper plate. Therefore, no torque at 10 times per revolution should be observed by the pendulum, as long as gravity is a $1/r^2$ -force. A thin metal foil (not shown) is stretched between the pendulum and the rotating plates to avoid electrostatic coupling. The ring of the pendulum is about 6.5 cm in diameter.

pendulum which consisted of a horizontally suspended ring that had 10 holes drilled through it (figure 4). The pendulum was held by a $20\ \mu\text{m}$ thick, 80 cm long tungsten torsion fibre. Coaxially below the pendulum disk was a disk with the same hole pattern in it. This disk below the pendulum was rotated slowly so that the gravitational field of the ‘missing mass’ of the holes caused a small varying torque on the pendulum. For one complete revolution of the plate, the pendulum would be deflected 10 times. We then turned the experiment into an approximate null experiment by placing a second disk, also with 10 holes underneath the first ‘attractor’ plate, also on the turntable. This disk’s holes were placed exactly out of phase with the upper attractor plate (i.e. displaced by 18° with respect to the upper plate’s holes). The size of the holes was designed so that their Newtonian gravitational interaction with the pendulum would largely cancel the Newtonian interaction of the upper plate with the pendulum. For $1/r^2$ -violations, with the force

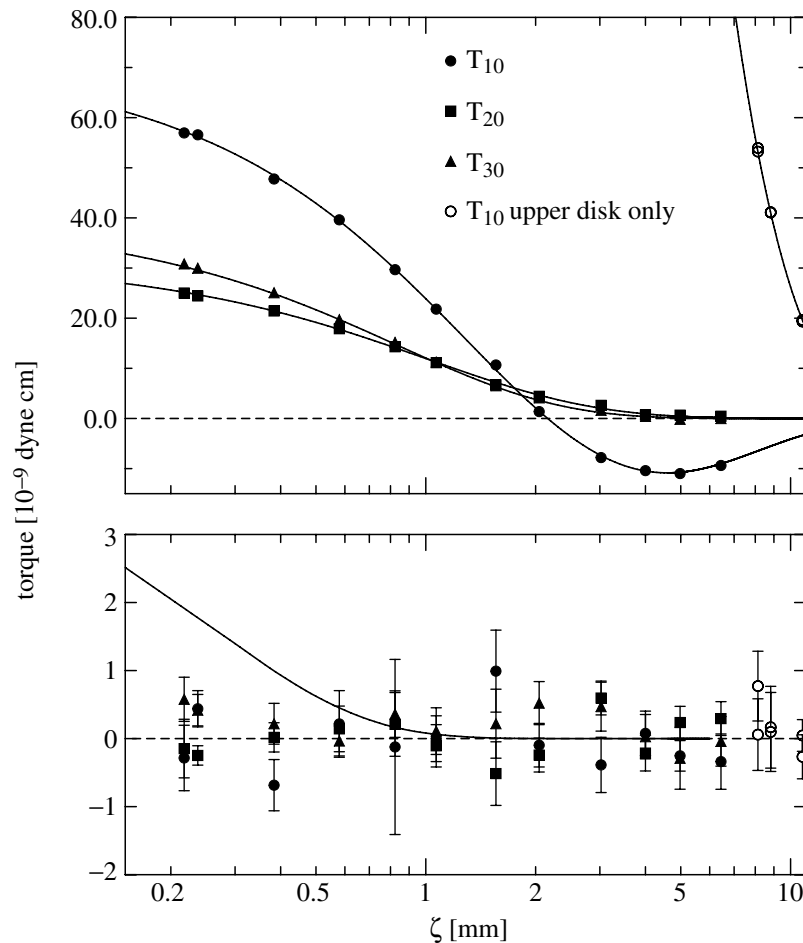


Figure 5. Eöt-Wash $1/r^2$ -test data. Presented are the measured and predicted amplitudes of the 10ω , 20ω and 30ω pendulum response as a function of the gap between the pendulum plate and the upper attractor plate. At a certain distance (≈ 2 mm), the experiment becomes a null experiment and the 10ω signal vanishes. The residuals between data and calculation are displayed in the bottom panel. The data demonstrate that Newton's $1/r^2$ -law holds for distances well below 1 mm. Also shown is the prediction for a Yukawa signal with $\lambda = 250 \mu\text{m}$ and $\alpha = 1$, which is clearly inconsistent with the data.

being stronger at short ranges, the upper plate would exert more torque than the lower plate. We varied the distance at which the pendulum was suspended over the attractor mass arrangement and compared the resulting Newtonian torque to detailed calculations. Any departure from $1/r^2$ -gravity would show up in the difference between the calculated torque at 10 times per revolution of the attractor (10ω) and the measured torques. We also calculated and measured torques that occur at harmonics of the 10ω signal (figure 5). The separation of the rotational frequency and the signal frequency at 10 times per revolution of the attractor plates strongly suppressed parasitic couplings such as vibrational, magnetic or thermal effects associated with the rotation of the attractor plates. Crucial to the success of our measurement was a thin metallic shield between the attractor discs and the pendulum. Such a shield is necessary to avoid electrostatic

interactions and to minimize effects due to residual gas. We stretched a 20 μm thick gold-coated BeCu foil in the small gap between the pendulum and the attractor discs. The entire pendulum was gold-coated and enclosed in a gold-coated housing to minimize other electrostatic torques. The twist of the torsion angle of the pendulum was registered with an autocollimator. We operated the instrument at the thermal limit given by a small energy dissipation in the torsion fibre. Within a day of operation, twist angles of about 1 nrad could be resolved. Figure 5 shows the amplitude of the measured torques at 10ω and its first two harmonics together with the predicted torques using the $1/r^2$ -gravity. Our data were quite consistent with $1/r^2$. Also shown in figure 5 is the torque prediction using a Yukawa interaction with $\alpha = 1$ and $\lambda = 250\text{ }\mu\text{m}$, which is clearly ruled out by those data. We used holes for the pendulum and attractor mass distribution because the metrology on the masses was easy. Nevertheless, metrology issues posed one of the leading systematic uncertainties in our error budget.

Since our first measurements [40, 41] described above, we have built several newer and improved versions using the same technique. In order to maximize the signal, we have operated pendulums with many more holes and much higher density materials. Our sensitivity was increased so that we are now able to detect forces with the strength of gravity for ranges of only 80 μm .

For ranges much shorter than the hole diameter, our active mass consists mostly of the radially running edges of the holes. To maximize the sensitivity to shorter ranges, we are currently developing a new pendulum and attractor mass that has many radially running ridges.

3.3. Colorado $1/r^2$ -tests

Long *et al* [42] at the University of Colorado developed a high-frequency torsional oscillator instrument targeted at the region below 100 μm . This region is of particular interest because of predictions for the moduli [32] and the dilaton [31]. At the heart of their test are two very closely spaced parallel plates, one of them being the attractor mass and the other being the sensing mass (figure 6). If the separation between the plates is much smaller than the size of the plates, Gauss's law in three dimensions predicts a force between the plates that is independent of the separation of the plates. Neglecting edge effects, no field change would be observed if the separation between the attractor plate and the sensor plate is changed. For a $1/r^2$ -violation, however, a force would vary and, in particular, for a Yukawa interaction the force would be:

$$F_y = 2\pi\alpha G\rho_a\rho_s\lambda^2 A_p(1 - e^{-t_a/\lambda})(1 - e^{-t_s/\lambda})e^{-d/\lambda}.$$

Here ρ_a , ρ_s , t_a and t_s are the densities and thicknesses of the attractor and sensor plates and d is the separation between the plates. This parallel plate geometry offers the highest ratio of active to passive mass, slowest falloff at short ranges and it is in principle a null $1/r^2$ -test for gravity (if the experiment had enough sensitivity to resolve gravitational forces). The Colorado group constructed their sensor mass as part of a high-Q-torsional oscillator machined out of tungsten and annealed at 1300 $^\circ\text{C}$. The attractor mass consisted of a tungsten cantilever (springboard) that was driven to oscillate at about 1 kHz. The 1 kHz frequency coincided with the fifth resonance frequency of the sensing torsional oscillator, which would resonate if a $1/r^2$ -violation was present. The active plate area, A_p , was about 58 mm^2 and the separation was as short as 108 μm .

Two experimental details were of extreme importance: (1) vibration isolation of the attractor and the detector to avoid mechanical coupling and (2) placing a stiff shield between the attractor

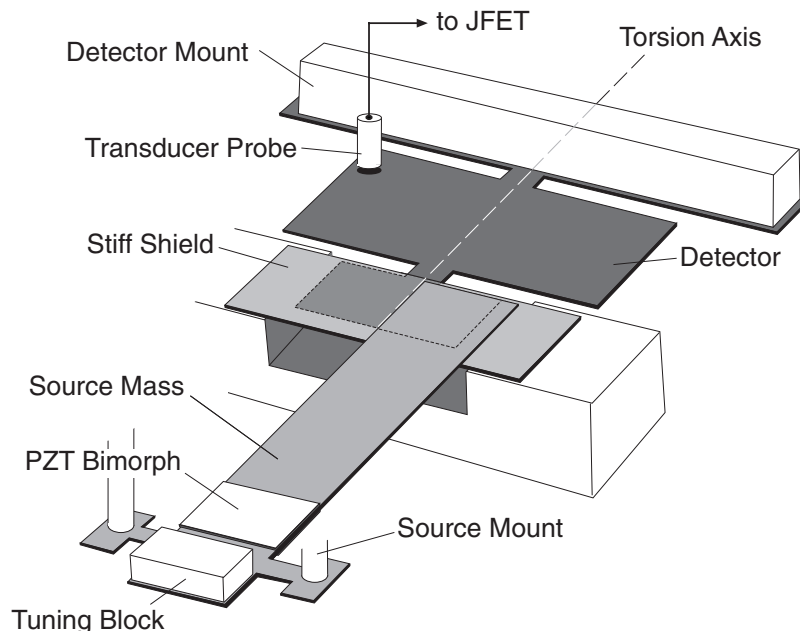


Figure 6. Schematic view of the apparatus of the Colorado group. The mass at the end of the spring board moves up and down over on half of a torsional oscillator. If the $1/r^2$ -law is violated, the oscillator will feel a force and will get driven in a resonance (1 kHz). Because of the high frequencies there is very little background noise but parasitic coupling between the attractor and the sensor requires multiple layers of vibrational isolation. A metalized sapphire shield is used to avoid electrostatic coupling.

and the detector to shield against electrostatic effects and effects due to residual gas. These two measures were also technically challenging given the high frequency and the short separations. The attractor and the detector were installed on separate stacks of vibrational isolation. A shield made from a gold-coated $60\ \mu\text{m}$ thick sapphire plate was placed into the gap between the plates.

No coupling that was consistent with a $1/r^2$ -violation was observed. The group's limits are included in the α - λ -plot (figure 9).

3.4. Stanford $1/r^2$ -test

A group at Stanford led by Kapitlnik has built a cryogenic microscopic high-frequency experiment [43], designed to test the $\lambda < 10\ \mu\text{m}$ region. Their experiment consists of a silicon nitride microcantilever with a $50\ \mu\text{m}$ gold square and $30\ \mu\text{m}$ thick ($1.4\ \mu\text{g}$), attached on its free end (figure 7). This 'spring board' had a high Q value and was used as the detector. The attractor mass arrangement consisted of five gold strips interlaced with five silicon strips. It was actuated in oscillatory motion transverse to the direction of the strips using a bimorph. The source mass moved past the cantilever at a distance as close as $25\ \mu\text{m}$. The alternating strip design permitted the source to be driven at one-third of the cantilever resonance frequency so that vibrational or other spurious coupling of the attractor mass motion to the pendulum was strongly reduced, while a $1/r^2$ -violation signal would have been on resonance. The 'spring board' motion of the sensor was read via an optical fibre interferometer. Operating at 9–11 K, the system obtained a

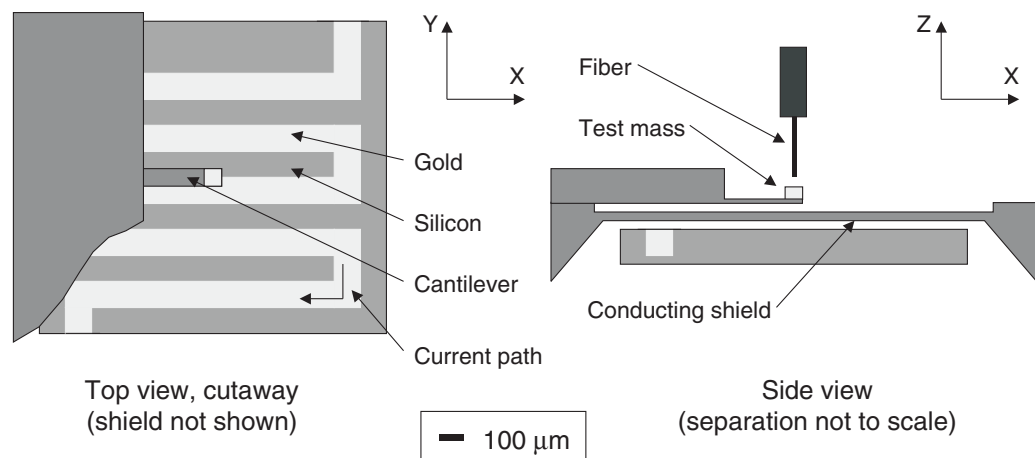


Figure 7. Schematic view of the apparatus of the Stanford group. A small test mass is attached to thin cantilever. Perpendicular to the sensitive axis of the cantilever and only $25\ \mu\text{m}$ away from the cantilever moves an attractor mass that has alternating densities of the Si and Au. The frequency at which the attractor moves back and forth is only one-third the frequency at which the cantilever resonates, but the frequency at which the density profile moves is at the resonance. A metalized SiN shield between the detector and the moving mass is necessary.

sensitivity of around $\approx 10^{-16}\ \text{N}$ at the closest distance. A $3\ \mu\text{m}$ thick gold-coated silicon nitride membrane was placed between the attractor and the sensor ($15\ \mu\text{m}$ from the sensor).

During operation a spurious signal was observed. While this signal was inconsistent with a $1/r^2$ -violation signal, the authors were conservative and set their upper limits on α - λ using the magnitude of the spurious signal [43]. It is suspected that the spurious signal may be due to electrostatic forces between the attractor mass and the shield thus deflecting the shield, which in turn exerts an electrostatic force on the sensor mass. Other known interactions such as gravity itself, Casimir forces, vibrational coupling and magnetism could be ruled out. The measurement rules out a significant phase space for moduli.

3.5. Casimir force range tests $1/r^2$ -tests

Below the range of a few micrometers, the Casimir force is the overwhelming force. Since the size scale of micrometers is too small to fabricate a shield, $1/r^2$ tests have to be performed in the presence of the Casimir force. In the last 10 years, the Casimir force has been studied intensively by Lamoreaux [44], Mostepanenko and Novello [45], Bordag *et al* [46] and Decca *et al* [47]. To interpret these measurements as $1/r^2$ -tests of gravity, one has to rely on detailed models of the Casimir force. A collaboration involving Indiana University, Purdue University, Lucent Technologies and Wabash College [48] has built microelectromechanical-based system to set constraints on $1/r^2$ -violations of gravity below $\lambda = 1\ \mu\text{m}$, which elegantly avoids uncertainties due to Casimir force models.

In their latest and most sensitive iteration of their instrument (figure 8), the group compares forces acting between a small gold-coated sphere ($d = 600\ \mu\text{m}$) and either a small plate of gold-coated Ge or a small plate of gold-coated Au (figure 8). The gold-coating on the samples

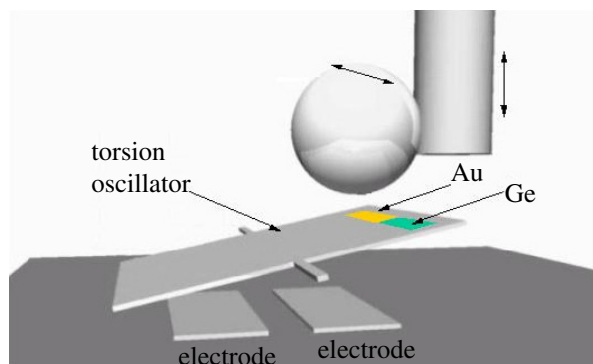


Figure 8. Schematic view of the apparatus of the Purdue group [48]. A gold-coated sphere is brought over a small torsional oscillator which serves as the force sensor. Attached to the torsional oscillator are plates of different density, one plate made from Ge and the other plate made from Au. Both plates are covered with Au so that the Casimir force of the sphere interacting with each of these plates is the same. The sphere oscillates up and down over the plates and at the same time it oscillates laterally from being over one plate to being over the other. The instrument shown is about 1 mm in size.

was thick enough to make the Casimir force equal for both surfaces, but it was thin enough to sense $1/r^2$ -violating effects from the underlying Ge or Au substrates. These two plates are attached at some moment arm on a small torsional oscillator, which serves as a force sensor. The deflection of the torsional oscillator is measured capacitively. The gold sphere is brought to be as close as 150 nm to one of the plates and oscillates up and down over the plate. The sphere is then moved laterally over to the other plate. The frequency of vertical oscillation and lateral motion is chosen so that a difference in force due to a $1/r^2$ -violation would drive the high- Q micromechanical torsion oscillator on resonance. In the end the group finds some differential force but its characteristics are not consistent with a Yukawa force, but may be due to a residual Casimir force effect. It was shown that new hypothetical Yukawa forces (figure 9) have to be smaller than $\alpha \approx 10^{12}$ at $\lambda \approx 200$ nm [48].

3.6. $1/r^2$ -tests in preparation

Several other new experiments are in preparation. At the University of Düsseldorf, Schiller, Haiberg and colleagues have built a high purity 5 kHz silicon oscillator. They are using the parallel plate geometry with the attractor mass being a variable density rotor, ‘toothed wheel’. A thin conducting plate separates the detector and the rotor. Ultimately, the system will be operated at cryogenic temperatures to take advantage of highly increased Q values.

A collaboration based at the University of Padova has built a Casimir force experiment involving parallel conducting plates. Most previous Casimir force experiments (including the one described above) have used crossed cylinders [45] or a plate and a sphere [44, 47]. The parallel plates geometry increases the signal at the expense of trickier alignment. The group has made an AFM part of their setup, which will improve the alignment and find (and remove) irregularities and dirt particles [49].

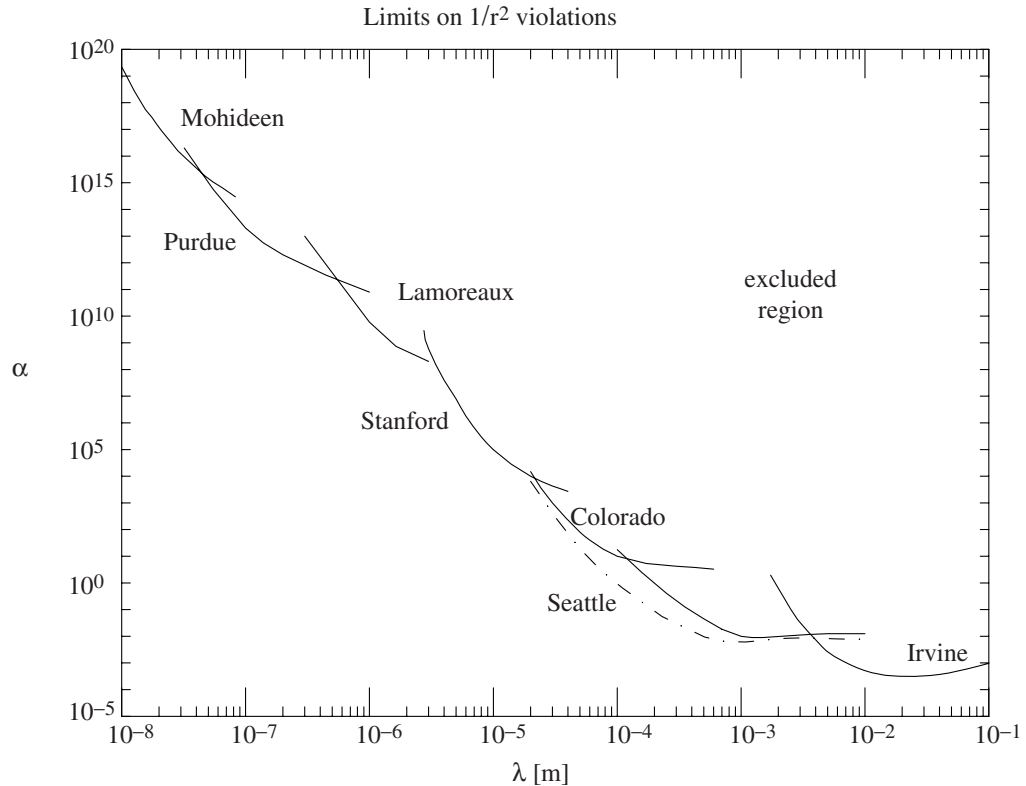


Figure 9. 2σ limits on $1/r^2$ -violations. The violation is described as a new interaction that has the form of a Yukawa interaction. The strength parameter α compares the magnitude of a violation to that of regular gravity. The parameter λ is the length scale above which the interaction has $1/r^2$ -character. At short ranges, the experimental sensitivity falls off roughly as λ^4 . The region below 1 mm has been the subject of intense research because of its large potential for observing new physics. The constraints are from: Mostepanenko [45], Purdue [48], Lamoreaux [44], Stanford [43], Colorado [42], Seattle [40] and Irvine [39]. The tightest limit at $100\ \mu\text{m}$ ($\cdot - \cdot - \cdot$) is preliminary data of the Eöt-Wash group, Seattle.

3.7. Ultracold neutron $1/r^2$ -tests

Ultracold neutrons present a very interesting system in which quantum states have macroscopic dimensions. A group from Heidelberg has used ultracold neutrons at a research reactor in Grenoble. They realized that ultracold neutrons that have barely enough kinetic energy to reach a table, a horizontal neutron mirror, must exhibit bound quantum mechanical states in the potential given by the impenetrable table and the earth's gravitational potential. The first quantum level ($n = 1$, 1.44 peV) occurs at a height of $13.7\ \mu\text{m}$, the second ($n = 2$, 2.53 peV) at $24.1\ \mu\text{m}$ etc above the table. Their data agree well with the prediction of quantum mechanics using only the earth's potential. The data can therefore be used to exclude short-range attractions between the table and the neutrons, that would have to be added to the earth's potential. The bound that can be set is currently not competitive compared to other $1/r^2$ -tests, but is nevertheless interesting because it is obtained with a quantum mechanical system [50].

4. Conclusions

Practically a century after Einstein's 'happiest thought', the search for evidence of unification of all forces remains at the heart of fundamental physics. Numerous theoretical speculations predicting violations of the equivalence principle or violations of the $1/r^2$ -law have motivated a variety of table-top precision tests of gravity. In a hopeful manner, we consider these laboratory gravitational physics experiments the low-energy frontier of particle physics. For the first time, string theorists and experimentalists are attending the same conferences. At the moment, no deviations from ordinary gravity have been found, but discoveries could be right around the corner.

Acknowledgments

I thank all the members of the Eöt-Wash group at the University of Washington. In particular I am indebted to C D Hoyle and D Kapner for helping with the preparation of this paper. The Eöt-Wash group's work is funded by grants from the NSF and NASA.

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