DYNAMICAL SU(3) MODEL FOR STRONG INTERACTIONS AND ψ PARTICLES

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Abstract :

We propose as a mechanism for the internal excitation of the hadrons a simple model based on the group embedding SU(3) $\rightarrow$ SU(3.1). Ex cited states of the quark (antiquark) belong to the representation (ℓ0) ((0ℓ)) of the maximal compact subgroup SU(3). The scheme leads to a definite predictions for the vector meson spectrum produced in the e^+e^- - annihilation. The SU(3.1) interpretation of ψ particles is briefly discussed.

The discovery of massive particles[1] $\psi(3105)$, $\psi'(3695)$ and a new member called ?(4.1)[2] has invited us to further theoretical speculations. Extremely narrow widths of ψ and ψ' strongly indicate the existence of a new degree of freedom which may or may not be a charm quantum number. The purpose of this report is to draw physicists' attention to the dynamical SU(3)[3] scheme and its predictions for the e^+e^- annihilation. For the sake of clarity, we focus our attention onto the internal symmetry. Spins and the spatial part of the entire wave function may be furnished later. The model is then based on the group embedding SU(3) $\to$ SU(3.1) which is one of the two dynamical groups for the 3-dimensional harmonic oscillator[4]. The role of the dynamical group for the hydrogen atoms is well known[5]; the energy levels of the hydrogen atoms can be changed by the generalized shifting operators of the dynamical group O(4.2). The fact that SU(3) is the invariance group for the 3-dimensional harmonic oscillator is also well known, but perhaps it should be emphasised here again. The highly degenerate spectrum of the 3-dimensional harmonic oscillator with the degeneracy $\frac{1}{2}(n+1)(n+2)$ can be described completely by the totally symmetric (antisymmetric) representations (n0) ((0n)) of SU(3)[6]. Increasing (decreasing) the total number of quanta by one, we obtain the next energy level of the harmonic oscillator $\hbar\omega(n \pm 1 + \frac{3}{2})$. This, on the other hand, corresponds to changing the total number of quarks or antiquarks by one. Since the SU(3) representation $(\ell 0)$ $((0\ell))$ can be realized by the quark (antiquark) basis of polynomials of degree ℓ, the next energy levels of the oscillator correspond to the representation $(\ell \pm 1,0)$ $((o,\ell \pm 1))$, i.e. higher SU(3) representations $(\ell 0)$ and (0ℓ) are obtained in such a way that the higher energy levels of the 3-dimensional harmonic oscillator are reached by the dynamical group. Besides exceptional groups[7], SU(3) can be embedded in only two rank 3 groups, SU(3.1) and SP(3.R).[8] Let us take the non-hermitian Cartan-Weyl basis, $E_{\alpha\beta}$ that satisfy the commutation relation and the hermiticity conditions,

$$[E_{\alpha\beta}, E_{\gamma\delta}] = \delta_{\alpha\delta} E_{\gamma\beta} - \delta_{\beta\gamma} E_{\alpha\delta}\ , \qquad \alpha\ \beta\ \gamma\ \delta = 1\ 2\ 3\ 4 \tag{1a}$$

$$(E_{ij})^\dagger = E_{ji}\ ,\ (E_{i4})^\dagger = -E_{4i}\ ,\ (E_{\alpha\alpha})^\dagger = E_{\alpha\alpha} \tag{1b}$$

Among 15 generators of SU(3.1)[9] six shifting operators and two diagonal generators span the maximum compact subgroup SU(3),

$$\sqrt{2}\, F_i^{+} = E_{jk}\ ,\ \sqrt{2}\, F_i^{-} = E_{kj} \qquad ;\ i\ j\ k = 1.2.3.$$

$$H_1 = \tfrac{1}{2}(E_{11} - E_{22})\ ,\ H_2 = \tfrac{1}{3}(E_{11} + E_{22} - 2E_{33}) \tag{2}$$

There exist six generalized shifting operators and one diagonal generator,

$$\sqrt{2}\, G_i^{+} = i\, E_{i4}\ ,\ \sqrt{2}\, G_i^{-} = i\, E_{4i}$$

$$H_3 = \tfrac{1}{4}(E_{11} + E_{22} + E_{33} - 3E_{44}) \tag{3}$$

Introducing $F_i^3 = \frac{1}{2}(E_{jj} - E_{kk})$, the commutation relations are

$$[F_i^{+}, F_i^{-}] = F_i^{3}\ ,\ [F_i^{3}, F_i^{\pm}] = \pm F_i^{\pm} \tag{4a}$$

from which we see that $\{F^+, F^-, F^3\}_i$ span I,U,V spin of SU(3). Similarily with $G_i^3 = \frac{1}{2}(E_{ii} - E_{44})$, we have

$$[G_i^{+}, G_i^{-}] = G_i^{3}\ ,\ [G_i^{3}, G_i^{\pm}] = \pm G_i^{\pm} \tag{4b}$$

$\{G^+, G^-, G^3\}_i$ span three non-independent, non-compact subgroups which are locally isomorphic to SU(1.1). Note that $(F_i^{+})^{\dagger} = + F_i^{-}$ and $(G_i^{+})^{\dagger} = -G_i^{-}$. The linear sum of the eigenvalues of three diagonal generators leads to the generalized Nishijima-Gell-Mann relation,

$$\widetilde{Q} = I_3 + \tfrac{1}{2}Y + Z(\ell)\ , \tag{5}$$

where Z is the eigenvalue of H_3, which depends on the SU(3) symmetry label ℓ.[10] Denoting the general unitary representation of SU(3.1) by D(IYZ)[11], we take the lowest representation $D^{\pm}(00Z^{\pm})$ $(Z^+ \geqslant +\frac{3}{4},\ Z^- \leqslant -\frac{3}{4})$ which correspond to the quark-antiquark tower, respectively (Fig. 1). The SU(3.1) IR then decomposes into an infinite direct sum of totally symmetric or anti-symmetric representation of SU(3) without multiplicity. Each SU(3) representation is equally separated along the Z axis and related to each other by the matrix elements of the generalized shifting operators, $G_0^{\pm} = \sum_i G_i^{\pm}$,

$$G_0^{\pm}|I_3YZ\rangle = \sum_{I_3',Y'} \langle I_3'Y'Z\pm1|\, G_0^{\pm}\, |I_3YZ\rangle\, |I_3'Y'Z\pm1\rangle. \tag{6}$$

The normalized quark, antiquark basis are[12]

$$\left|I_3 Y Z\right\rangle_Q = \left[(I+I_3)!(I-I_3)!(I-\tfrac{3}{2}Y)!\right]^{-\frac{1}{2}}\left[(-\theta-1)!\right]^{\frac{1}{2}} \cdot p^{I+I_3}\,n^{I-I_3}\,\lambda^{I-\frac{3}{2}Y}\,\gamma^{\theta}, \tag{7a}$$

$$\left|I_3 Y Z\right\rangle_{\bar{Q}} = \left[(I+I_3)!(I-I_3)!(I+\tfrac{3}{2}Y)!\right]^{-\frac{1}{2}}\left[(-\tau-1)!\right]^{\frac{1}{2}} \cdot \bar{p}^{I+I_3}\,\bar{n}^{I-I_3}\,\bar{\lambda}^{I+\frac{3}{2}Y}\,\bar{\gamma}^{\tau}, \tag{7b}$$

where $\theta = I - \frac{1}{2}Y - \frac{4}{3}Z$ and $\tau = I + \frac{1}{2}Y + \frac{4}{3}Z$. We associate with $(\ell 0)$ $((0\ell))$ the the ℓ-th excited states of quarks (antiquarks) and as a meson structure the bound state of them, i.e. mesons $\sim \bar{Q}^{\ell} Q^{\ell'} \delta_{\ell\ell'}$. Then they belong to the product representation $(\ell 0)$ x (0ℓ) . For $\ell=1$ we obtain everything we know from the eight-fold way, i.e. the familiar meson nonet. For $\ell=2$ we have

$$(22) + (11)^* + (00)^* . \tag{8}$$

Once expressed in terms of the quark-antiquark basis (7), the starred representations in (8) carry an extra power of the general SU(3) scalars, $S \sim \frac{1}{\sqrt{3}}(p\bar{p} + n\bar{n} + \lambda\bar{\lambda})$. Note that SU(3) representations multiplied by powers of S are usually ignored in order to make the representation space orthogonal. If we assign[13] $\rho'(1600)$ and $\omega(1675)$ to the $(11)^*$ in (8), we obtain for the SU(3) scalar mentioned above, $M_S \sim 0.8$ GeV. At this stage it is tempting to speculate a chain structure for lower resonances (Fig.2). That is, if states for the $(22)^*$ representation from $\ell=3$ chain exist, the mass differences from ψ's would likely be $O(M_S)$, and possibly there are broad resonances like ρ'. In fact, $\frac{1}{2}(M_\psi + M_{\psi'}) + M_S = 4.2$ GeV. This might suggest that the observed broad resonance at $M \sim 4.1$ GeV with the width, $200 \sim 250$ MeV belong to the degenerate state of $(22)^*$ from $\ell=3$. However, it should be remembered that the the present scheme tells us nothing about radial or orbital excitation except equal mass separations for lower resonances at the exact symmetry limit. In order to include these effects, an appropriate spatial group must be introduced. Right now the experimental evidences are not clear for ρ'', ω'', ϕ'' like resonances which may correspond, respectively, to the $(11)^{**}$ and $(00)^{**}$ representation of $\ell=3$ chain; they could be broad in width and lower in intensity. Assuming the one photon exchange dominance in the present e^+e^- -annihilation experimant, we should expect 1^- vector mesons with $I_3 = 0$, $I = 0$, 1 only.

Using the notation $\left|\begin{smallmatrix} P & Q \\ Y\ I & I_3 \end{smallmatrix}\right\rangle$, candidates for ψ's are therefore[14] ,

$$B_2 \sim \left|\begin{smallmatrix} 2 & 2 \\ 0\ 1 & 0 \end{smallmatrix}\right\rangle = \frac{1}{\sqrt{10}}\left\{-pp\bar{p}\bar{p} + nn\bar{n}\bar{n} + 2p\lambda\bar{p}\bar{\lambda} + 2n\lambda\bar{n}\bar{\lambda}\right\}, \tag{9a}$$

$$D \sim \left| \begin{matrix} 22 \\ 000 \end{matrix} \right\rangle = \frac{1}{\sqrt{30}} \left\{ pp\bar{p}\bar{p} - pn\bar{p}\bar{n} + nn\bar{n}\bar{n} - 3p\lambda\bar{p}\bar{\lambda} + 3n\lambda\bar{n}\bar{\lambda} + 3\lambda\lambda\bar{\lambda}\bar{\lambda} \right\}, \qquad (9b)$$

$$g_3 \sim \left| \begin{matrix} 22 \\ 020 \end{matrix} \right\rangle = \frac{1}{\sqrt{6}} \left\{ pp\bar{p}\bar{p} + 2pn\bar{p}\bar{n} + nn\bar{n}\bar{n} \right\}. \qquad (9c)$$

Among several possibilities the following assignment is perhaps the most attractive one; ψ (3105) and ψ'(3695) belong, respectively, to the $I = 1$ and $I = 0$ members of the 27-dimensional SU(3) representation (22). In each excited level of SU(3), assuming the symmetry breaking of the order of the Gell-Mann-Okubo type, we obtain $M(g_3) \sim 1.9$ GeV from (9). Observed narrow widths of the ψ particles may be attributed to an approximate SU(3.1) invariance of the strong interaction, i.e. $|\Delta\ell| \neq 0$ selection rule (this is suggested by (5)). Assuming $\pi\pi$ channel dominance, the order of magnitude estimation for $|\Delta\ell| \neq 0$ suppression gives:

$$\frac{G^2(\ell=2)}{G^2(\ell=1)} = \frac{\Gamma(\psi\to\pi\pi)}{\Gamma(\rho\to\pi\pi)} \left(\frac{M_\psi}{M_\rho}\right)^2 \frac{(\text{phase space } \rho)}{(\text{phase space } \psi)} \simeq 1.3 \times 10^{-4} \qquad (10)$$

where $G(\ell)$ is the coupling strength defined by the effective hamiltonian $\mathcal{H} = G(\ell)\, \vec{\psi}_\lambda (\vec{\varphi} \times \partial_\lambda \vec{\varphi})$. Note that in our assignment the observed suppression of of the order $\sim 10^{-2}$ for the process $\psi' \to \psi\pi\pi$, compared with the typical strong decay $\rho' \to \rho\pi\pi$, is due to the G-parity-violating decay, and it should be compared more appropriately with the process $\omega \to \rho\pi\pi$, which is, however, energetically forbidden. For leptonic decays the third diagonal generator does not correspond to an independent current like the charm current in the SU(4)scheme[15], since Z in (5) depends on the SU(3) symmetry label. Here we simply assume that the electromagnetic current, $J_\mu \sim F^3_\mu - \frac{1}{\sqrt{3}} F^8_\mu$ decouples from the possible ℓ-conserving current[16]. Then we should expect the ratio for the coupling strength of ψ's into lepton pairs, $f^{-2}_{\psi'} : f^{-2}_{\psi} \simeq f^{-2}_{\omega} : f^{-2}_{\rho}$.
Ignoring a possible mixing of the states, we obtain

$$\frac{\Gamma(\psi' \to \bar{\ell}\ell)}{\Gamma(\psi \to \bar{\ell}\ell)} = \left(\frac{f_{\psi'}}{f_\psi}\right)^2 \frac{M_{\psi'}}{M_\psi} \sim 0.13 \; , \qquad (11)$$

which should be compared with the experimental value ~ 0.4. These results encourage us to make further predictions for higher excited states. We already know that the simple non-relativistic quark model gives a surprisingly good estimate for the low-lying hadron spectra and for some interactions among them. Furthermore, we expect that the relativistic effect, if any[17], will be minimum for the e^+e^--annihilation. Therefore, we justify ourselves in making the 0-th order estimate by taking only the bare quark masses into account, and here we do not assume any particular form for the interaction between two quarks. Then the bare mass of the ℓ-th excited state quark and the threshold energy of the ℓ-th level mesons[18] are approximately:

$$M_\ell = \ell \mu_0 \qquad (12a)$$

$$E_\ell = 2D(\ell 0)M_\ell = \ell(\ell + 1)(\ell + 2)\mu_0 \qquad (12b)$$

Taking $\mu_0 \sim 150$ MeV, we obtain $E_1 = 0.9$, $E_2 = 3.6$, $E_3 = 9$ GeV, for whic h $R(q^2 \to \infty) = \sum_i Q_i^2$ is $\frac{2}{3}$, 4, 14, respectively. In conclusion, we should expect at least two more ψ-like narrow resonances (even narrower) at $M \sim 9$ GeV. They could be associated with broad resonances from the $\ell = 4$ chain, and between E_2 and E_3 no more sharp resonances will be seen, but some relatively broad resonances may be observed corresponding to the higher harmonics states. As a final remark we want to point out that although the e^+e^- annihilation experiments are informative, we should be aware of their limitations; for example, g_3 which is the $I = 2$ member of the (22) representation will never be observed unless it can be produced by the 2-photon exchange processes.

REFERENCES AND FOOTNOTES

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11. We are interested, of course, only in the discrete unitary representation of SU(3.1) here.
12. States similar to expression (7) are also considered by Bacry and Chang in connection with the relativistic 3-dimensional harmonic oscillator. They correspond to the coherent states in the cms. See H.Bacry, the 3rd International Colloquium on Group Theoretical Methods in Physics, Marseille , 1974. This point will be discussed in detail in a forthcoming paper, M.Hongoh, in preparation.
13. In Ref.3 these heavier mesons belonging to the starred representations are called the higher harmonics states; they have the same SU(3) quantum number as the mesons of the original multiplet.
14. g_3 is the $I_3 = 0$, $I = 2$ member of the (22) representation, therefore, it cannot be a candidate for ψ's, nevertheless listed here for completeness.
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16. From this point of view, we do not consider that the evidence for $\Delta S = -\Delta Q$ currents recently reported by E.G.Cazzoli et al. immediately concludes the existence of charmed baryons.
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17. In diffractive excitations and also in deep inelastic scatterings, we should expect the Lorentz contraction effect on the extended hadronic matters.
18. For example, ψ's are $\ell = 2$ level mesons in the present scheme.

FIGURE AND TABLE CAPTIONS

Fig. 1: The lowest states of $D^+(00Z^+)$ and $D^-(00Z^-)$.
Fig. 2: Equal mass-separation for lower resonances.
Table 1: Non-vanishing matrix elements for the generalized shifting operator

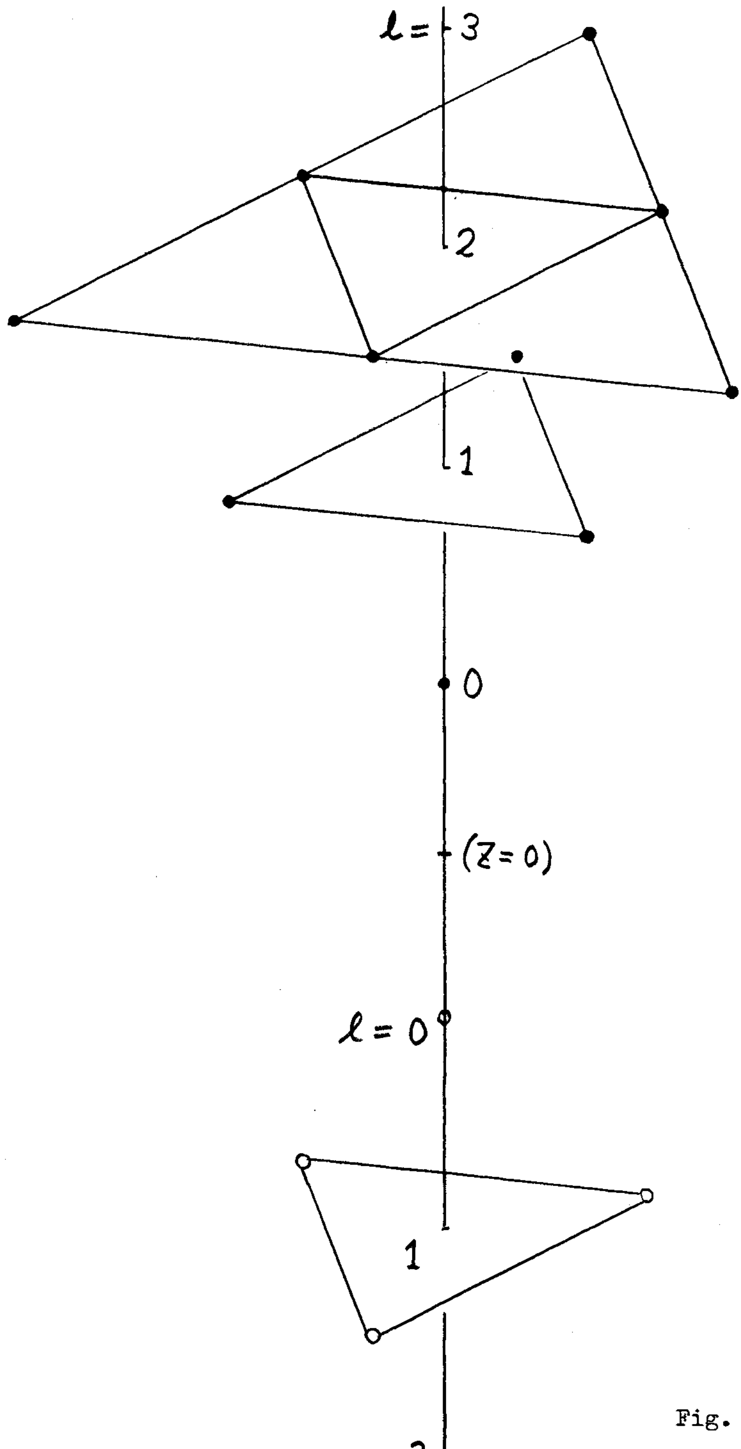

Fig. 1

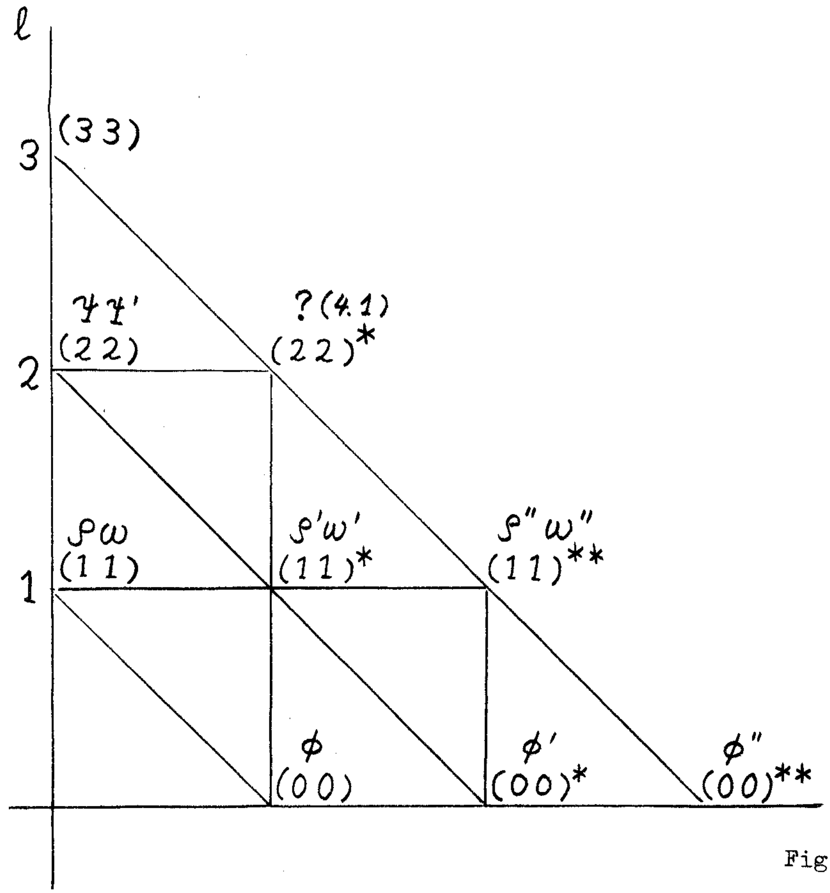

Fig. 2

TABLE 1

Non-vanishing matrix elements	Quark State	Anti-Quark State
$\left\langle I_3 + \frac{1}{2}, Y + \frac{1}{3}, Z+1 \middle| G_o^+ \middle| I_3, Y, Z \right\rangle$	$-((I+I_3+1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$	$((I+I_3)(-I + \frac{1}{2}Y + \frac{4}{3}Z+1)/2)^{\frac{1}{2}}$
$\left\langle I_3 - \frac{1}{2}, Y - \frac{1}{3}, Z-1 \middle| G_o^- \middle| I_3, Y, Z \right\rangle$	$((I+I_3)(-I + \frac{1}{2}Y + \frac{4}{3}Z-1)/2)^{\frac{1}{2}}$	$((I+I_3-1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$
$\left\langle I_3 - \frac{1}{2}, Y + \frac{1}{3}, Z+1 \middle| G_o^+ \middle| I_3, Y, Z \right\rangle$	$-((I-I_3+1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$	$((I-I_3)(-I + \frac{1}{2}Y + \frac{4}{3}Z+1)/2)^{\frac{1}{2}}$
$\left\langle I_3 + \frac{1}{2}, Y - \frac{1}{3}, Z-1 \middle| G_o^- \middle| I_3, Y, Z \right\rangle$	$((I-I_3)(-I + \frac{1}{2}Y + \frac{4}{3}Z-1)/2)^{\frac{1}{2}}$	$((I-I_3-1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$
$\left\langle I_3, Y - \frac{2}{3}, Z+1 \middle| G_o^+ \middle| I_3, Y, Z \right\rangle$	$-((I-\frac{3}{2}Y+1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$	$((I-\frac{3}{2}Y)(-I + \frac{1}{2}Y + \frac{4}{3}Z+ 1)/2)^{\frac{1}{2}}$
$\left\langle I_3, Y + \frac{2}{3}, Z-1 \middle| G_o^- \middle| I_3, Y, Z \right\rangle$	$((I-\frac{3}{2}Y)(-I + \frac{1}{2}Y + \frac{4}{3}Z-1)/2)^{\frac{1}{2}}$	$((I-I_3-1)(-I + \frac{1}{2}Y + \frac{4}{3}Z)/2)^{\frac{1}{2}}$